Small Instantons and the Post-inflationary QCD Axion in a Special Product GUT

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based on Shihwen Hor, Yuichiro Nakai, Motoo Suzuki, JX 2504.02033

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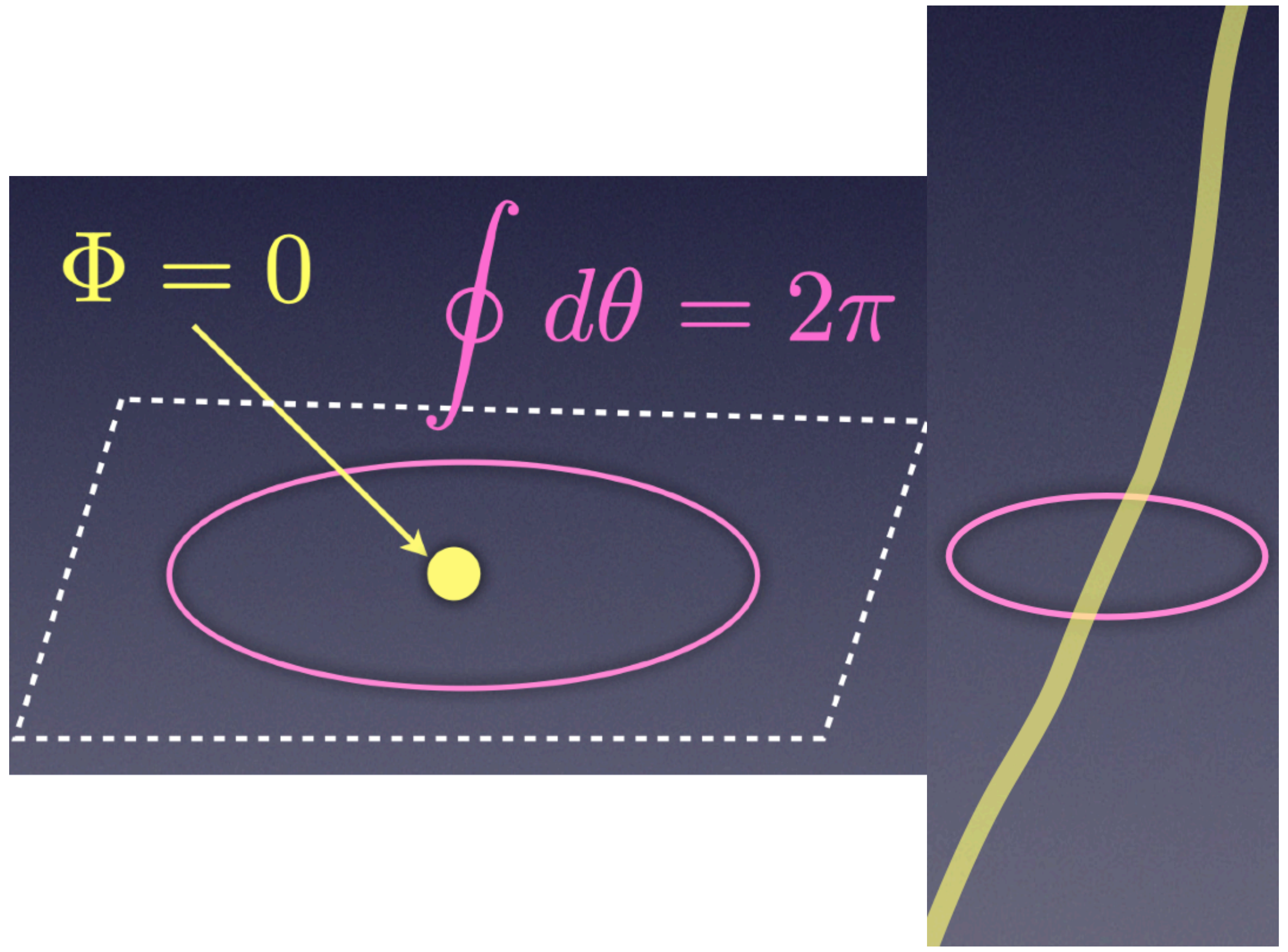
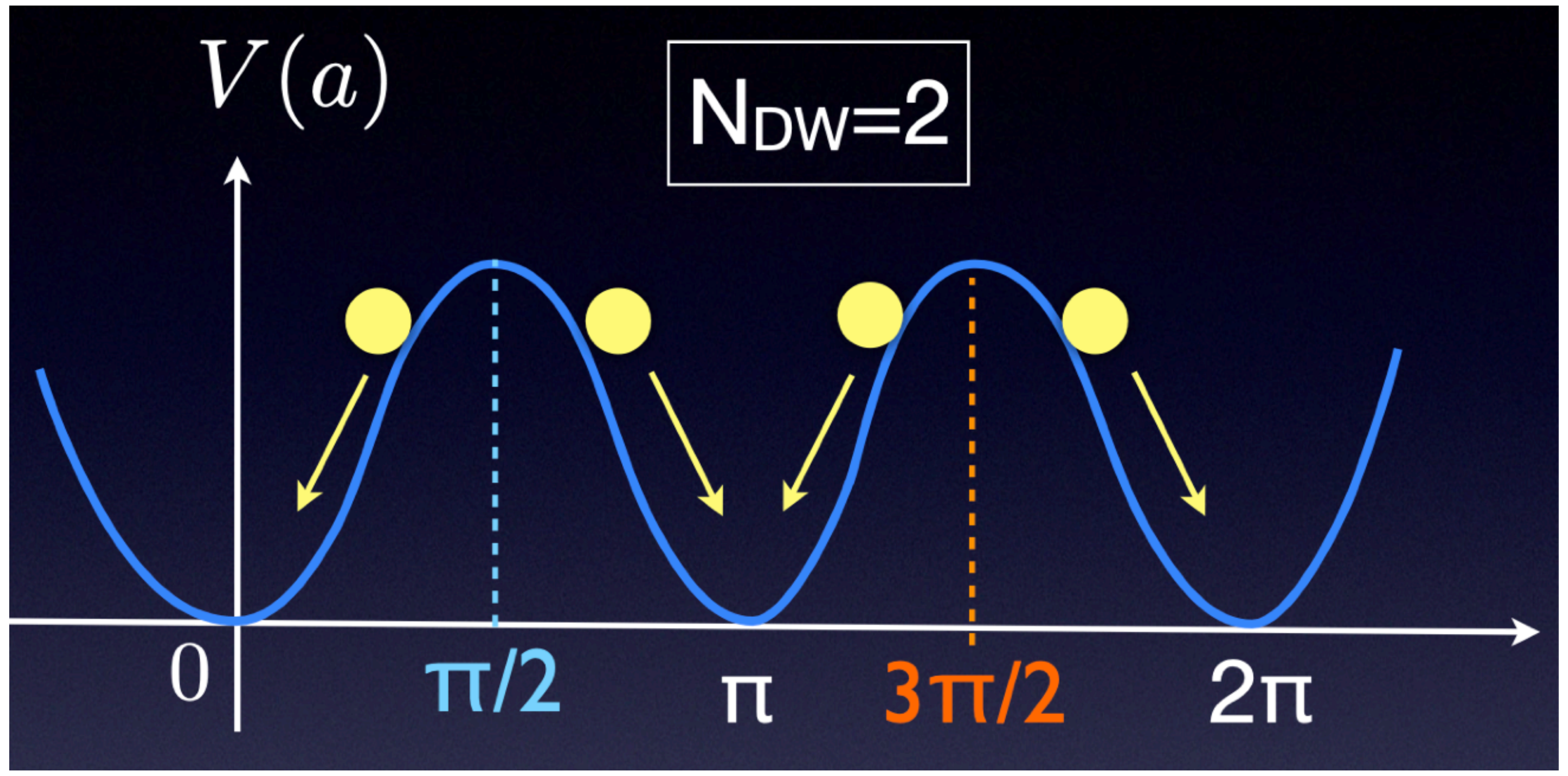
Axion Domain Wall Problem

★$U(1)_{PQ}$ breaking **after** inflation → Domain Wall Problem

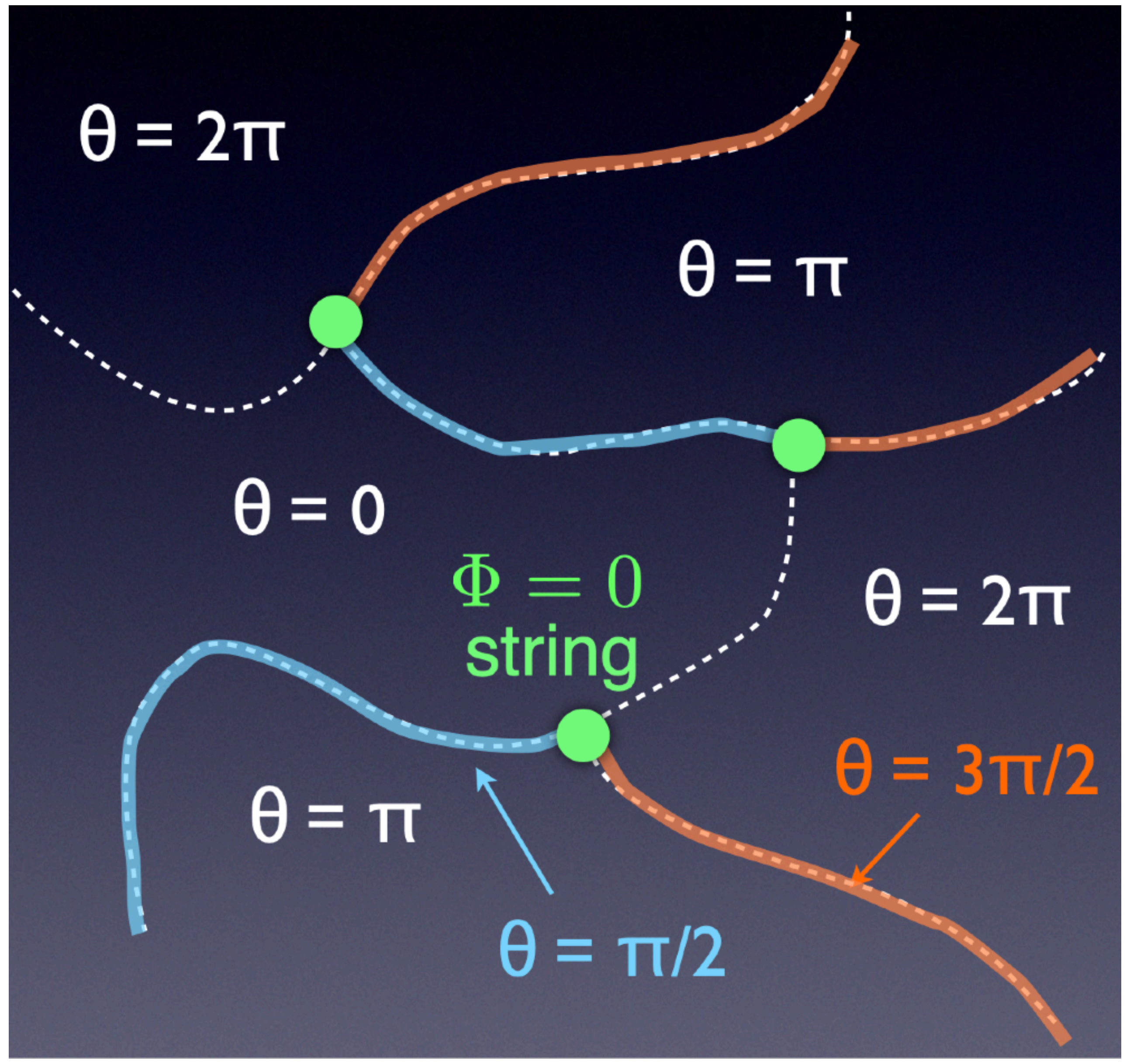
After SSB, cosmic strings are produced by Kibble mechanism.



After confinement, axion gets the potential with $N_{\rm DW}$ minima



String-domain wall system dominates the universe if $N_{\rm DW} > 1$



Masahiro Kawasaki slide

Special Product GUT

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$$SU(10) \times SU(5)_1 \to SU(5)_V \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

Axion Sector — SM

$$c \equiv \frac{T_{\rm IR}(\mathbf{r})}{1/2} = 3$$

Special Embedding

$$SU(10) \longrightarrow SU(5)_2$$

$$\mathbf{10} \longrightarrow \mathbf{10}$$

Special Product GUT

$$SU(10) \times SU(5)_1 \to SU(5)_V \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

Axion Sector ($SU(10)$) — **SM** ($SU(5)_1$)

$$c \equiv \frac{T_{\mathrm{IR}}(\mathbf{r})}{1/2} = 3$$

Special Embedding

$SU(10) \longrightarrow SU(5)_2$

KSVZ quark
(One pair)

$\mathbf{10} \longrightarrow \mathbf{10}$

$N_{\mathrm{DW}}^{(\mathrm{UV})} = 1$

UV instanton effect
$\sim \cos(\theta_a)$

$N_{\mathrm{DW}} = 3$

$V \sim \cos(3\theta_a)$

Controllable bias term

Axion Potentials

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- **QCD effects**

Axion coupling to gluons : $\int (3\theta_a + \theta_c) \frac{g^2}{8\pi^2} \mathrm{tr}(G \wedge G)$

A vector-like pair of SU(2)L doublet KSVZ (anti-)quarks and a pair of SU(2)L singlet (anti-)quarks appear after GUT breaking.

$$\Rightarrow V_{\rm QCD} \approx -\frac{m_u m_d}{(m_u + m_d)^2} m_\pi^2 f_\pi^2 \cos\left(3\theta_a + \bar{\theta}_c\right)$$

$\bar{\theta}_c \ll 10^{-2}$

due to the **SCPV**

NDW = 3

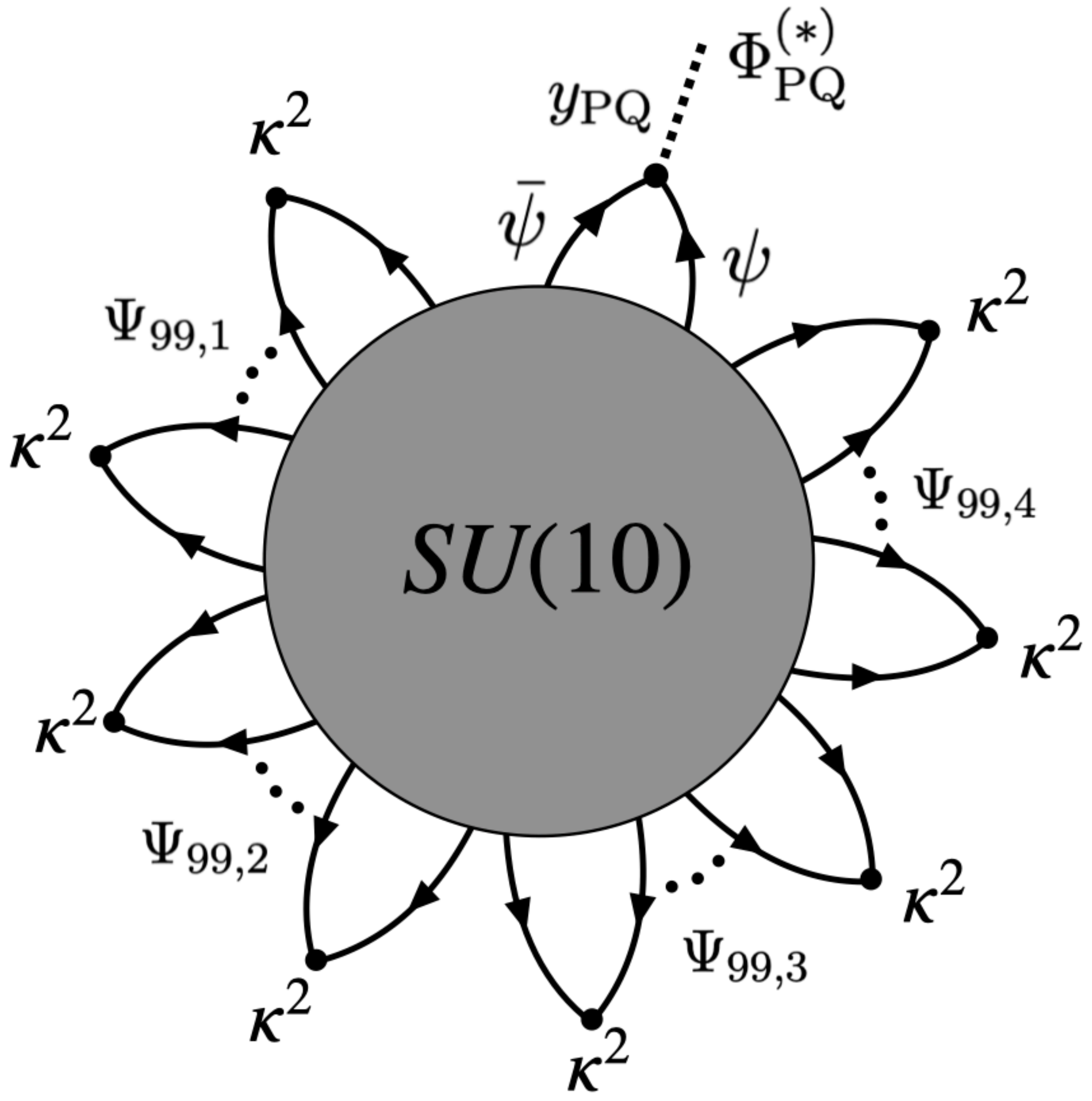
- **Small instanton effects**

Instanton NDA Csaki, D'Agnolo, Kuflik, Ruhdorfer (2024)

$$V_{\rm bias} = 3 \times 10^2 (\kappa^2)^{40} \frac{e^{-2\pi/\alpha_{\rm UV}}}{\alpha_{\rm UV}^{20}} e^{-2\pi^2} M^3 \Phi_{\rm PQ} + c.c.$$

$$= 6 \times 10^2 \epsilon \frac{e^{-2\pi/\alpha_{\rm UV}}}{\alpha_{\rm UV}^{20}} e^{-2\pi^2} M^3 v_{\rm PQ} \cos(\theta_a)$$

NDW = 1



't Hooft vertex

Axion Dark Matter

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PQ symmetry is spontaneously broken after reheating.

We focus on the scenario where <u>the axion field starts to oscillate due to the axion mass originated from</u> **<u>the bias term (before QCD phase transition)</u>** :

$$m_{\rm bias}^2 \equiv \frac{\partial^2 V_{\rm bias}/\partial\theta_a^2}{v_{\rm PQ}^2}$$

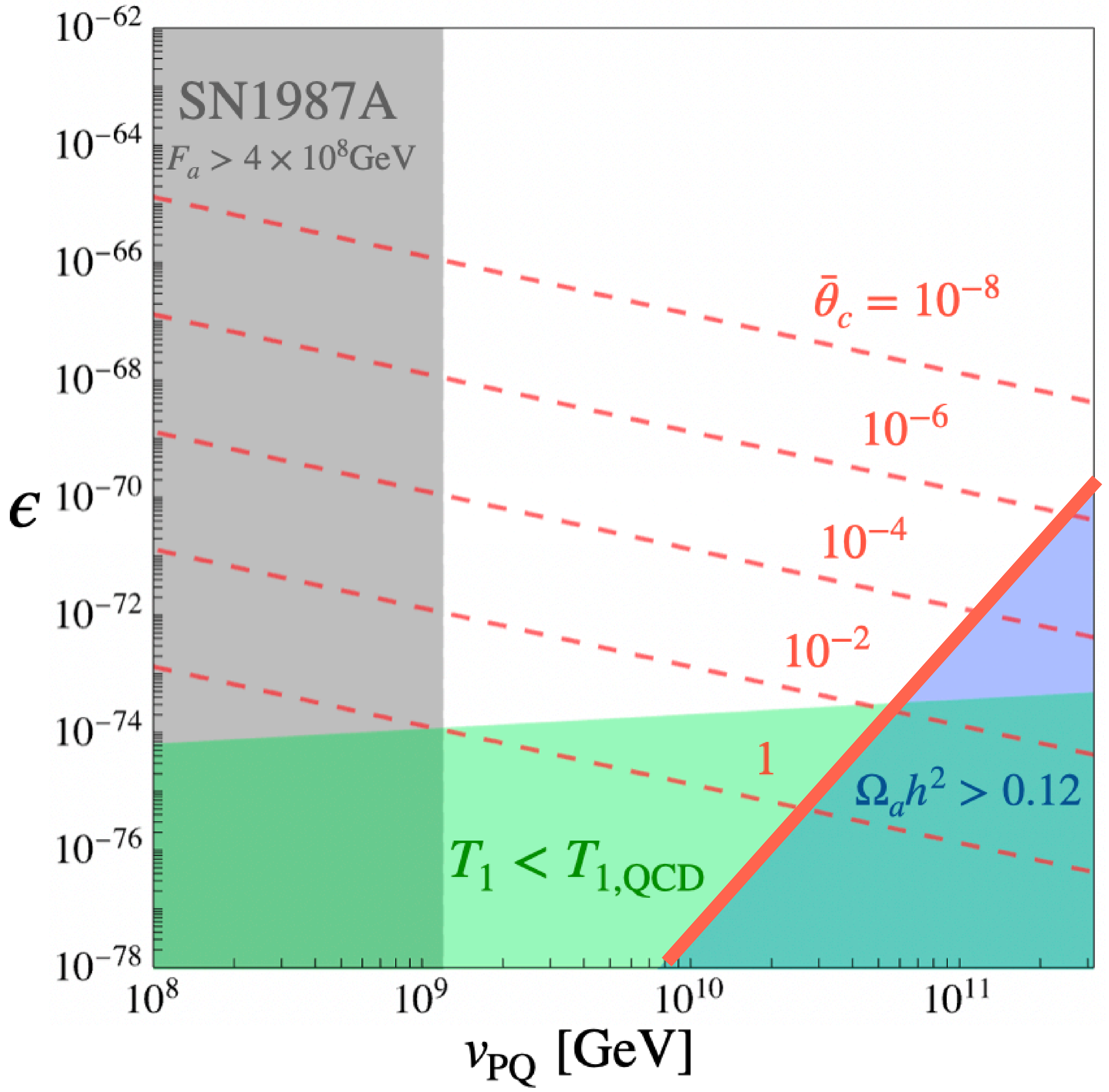
Temperature when the oscillation starts :

$$m_a(T_1) \approx m_{\rm bias} = 3H(T_1)$$

Axion abundance :

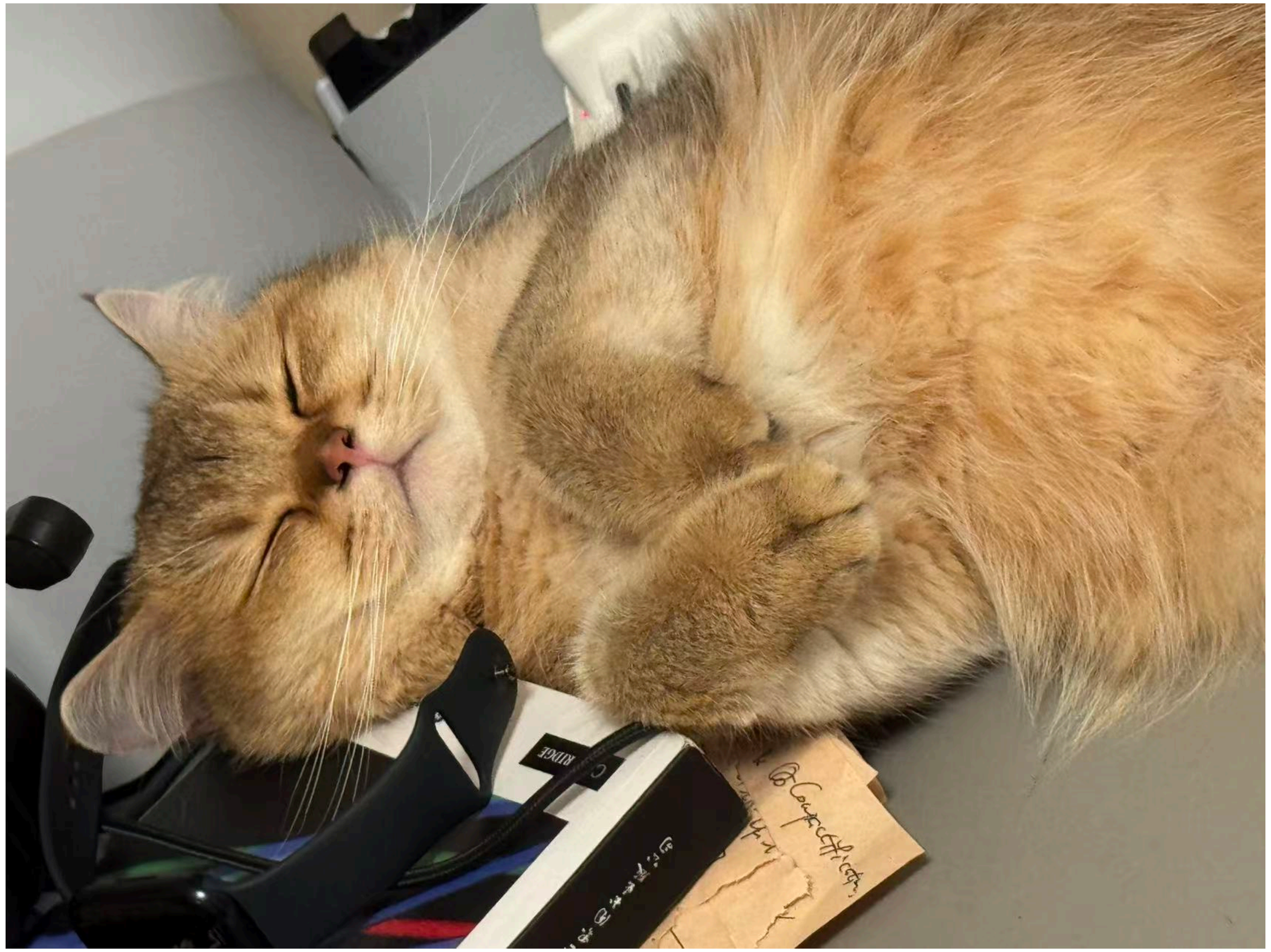
$$\Omega_a h^2 \approx 2\times 10^{-12}\,\frac{v_{\rm PQ}}{T_1}$$

$$\Delta\bar\theta \equiv \frac{m_{\rm bias}^2 v_{\rm PQ}^2}{m_a^2 F_a^2}\bar\theta_c \lesssim 10^{-10}$$



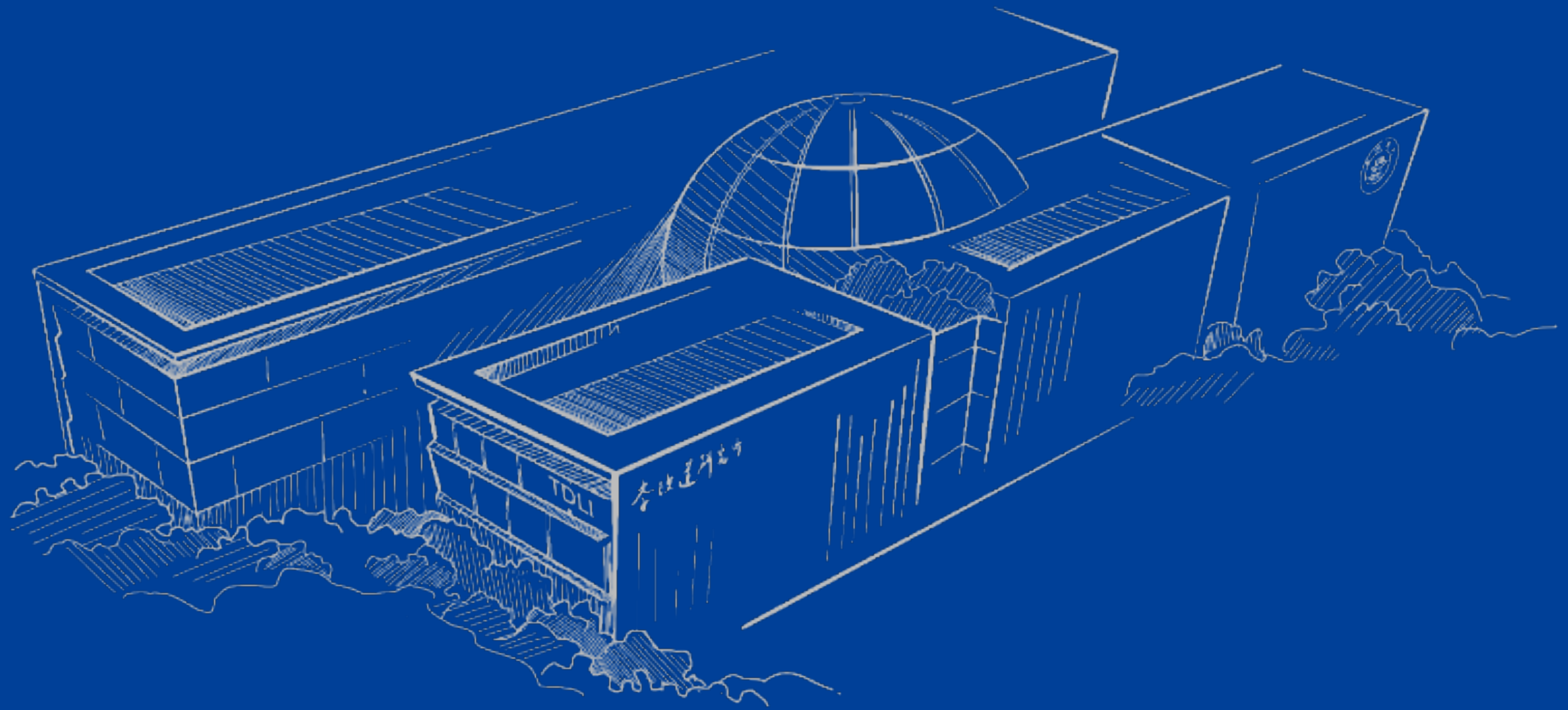
Correct axion DM abundance

$$v_{\rm PQ} \gtrsim 6\times10^{10}\ {\rm GeV}$$



Polchinski’s String Theory

Thanks!



Small Instanton

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Instanton : localized object in Euclidean spacetime, satisfying Euclidean EOM with non-trivial topology and minimizing the Euclidean action

SU(2) BPST instanton solution with $Q = 1$:

$$A^a_\mu(x)\Big|_{1-\text{inst.}} = 2\eta_{a\mu\nu}\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2}$$

Position (x_0) Instanton size (ρ)

$$\frac{g^2}{32\pi^2}\int d^4x\, F^{a\mu\nu}\widetilde{F}^a_{\mu\nu}\Big|_{\text{inst.}} = Q \quad (Q\in\mathbb{Z})$$

Instantons contribute to the axion potential

$$\propto \exp\left(-\frac{8\pi^2}{g^2(1/\rho)}\right)$$

In QCD, large-size instantons dominate due to asymptotic freedom.

A hidden gauge sector beyond QCD

GUT is a natural candidate.

↓

Small instanton effects → A possible origin of **the bias term** !

However, a naive embedding into SU(5) GUT does not work because the resulting small instanton effects **do not** lift the vacuum degeneracy.

Special Embedding

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We focus on a gauge symmetry breaking : $SU(2N) \to SU(N)$. Consider **a Weyl fermion** that transforms as the **fund** rep. of $SU(2N)$ but behaves as the **r** rep. of the $SU(N)$ subgroup :

$$\mathcal{D}_\mu \psi = \partial_\mu \psi - i g_{\mathrm{UV}} A^m_{\mathrm{UV},\mu} (T^m_{\mathrm{UV}}) \psi$$
$$\supset \partial_\mu \psi - i g_{\mathrm{IR}} A^a_{\mathrm{IR},\mu} (T^a_{\mathrm{IR}}) \psi$$

with

T^m_{UV} $(m = 1, \ldots, (2N)^2 - 1)$: $SU(2N)$ generators (**fund** rep.)

T^a_{IR} $(a = 1, \ldots, N^2 - 1)$: $SU(N)$ generators (**r** rep.)

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Special Subalgebras

r ≠ fund

$$T^a_{\mathrm{IR}} = \mathcal{O}^{am} T^m_{\mathrm{UV}}$$

Coefficients

$$\mathrm{tr}(T^m_{\mathrm{UV}} T^n_{\mathrm{UV}}) = \frac{1}{2}\delta^{mn}$$
$$\mathrm{tr}(T^a_{\mathrm{IR}} T^b_{\mathrm{IR}}) = T_{\mathrm{IR}}(\mathbf{r})\delta^{ab}$$

Embedding index

$$c \equiv \frac{T_{\mathrm{IR}}(\mathbf{r})}{1/2}$$

$$\mathcal{O}^{am} \mathcal{O}^{bn} \delta_{mn} = c\delta^{ab}$$

Special embedding corresponds to c > 1.

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Special Subalgebras

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$$\mathrm{tr}(T^m_{\rm UV} T^n_{\rm UV}) = \frac{1}{2}\delta^{mn}$$
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$$\mathcal{O}^{am}\mathcal{O}^{bn}\delta_{mn} = c\delta^{ab}$$

Embedding index

$$c \equiv \frac{T_{\rm IR}(\mathbf{r})}{1/2}$$

Special embedding corresponds to c > 1.

A part of $SU(2N)$ gauge field is expressed in terms of SU(N) gauge field :

$$A^l_{{\rm UV},\mu} = \frac{g_{\rm IR}}{g_{\rm UV}} A^a_{{\rm IR},\mu} (\mathcal{O})^{al} + \cdots$$

Canonically normalized kinetic term → $g_{\rm IR} = g_{\rm UV}/\sqrt{c}$

In our model, **r** = **10** rep. of $SU(5) \subset SU(10)$ leading to **c = 3.**

Theta term : $\int \frac{g^2_{\rm UV}}{8\pi^2} \mathrm{tr}(F_{\rm UV} \wedge F_{\rm UV}) = \int \frac{c g^2_{\rm IR}}{8\pi^2} \mathrm{tr}(F_{\rm IR} \wedge F_{\rm IR})$

Special Product GUT

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To achieve $SU(10) \times SU(5)_1 \rightarrow SU(5)_V$ we introduce a Higgs field :

$$\Phi_a^{ij} : (\mathbf{10},\ \overline{\mathbf{10}}) \qquad a\,(=1-10) \ \text{: SU(10) index} \qquad i,j\,(=1-5) \ \text{: SU(5)}_1 \text{ indices}$$

VEV of Φ is described by the embedding of the $\overline{\mathbf{10}}$ rep. of $SU(5)_1$ into the anti-fundamental rep. of SU(10) :

$$\langle \Phi \rangle = v\mathbf{1}_{10\times 10}$$

GUT Sector

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\rm PQ}$	$U(1)_\eta$
Φ	0	$\mathbf{10}$	$\overline{\mathbf{10}}$	0	0
$\mathbf{24}_H^{(1)}$	0	$\mathbf{1}$	$\mathbf{24}$	0	0

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\rm PQ}$	$U(1)_\eta$
$\mathbf{10}_f^{(1)} (f=1-3)$	$1/2$	$\mathbf{1}$	$\mathbf{10}$	0	0
$\bar{\mathbf{5}}_f^{(1)} (f=1-3)$	$1/2$	$\mathbf{1}$	$\bar{\mathbf{5}}$	0	0
$\mathbf{5}_H^{(1)}$	0	$\mathbf{1}$	$\mathbf{5}$	0	0

Axion Sector

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\rm PQ}$	$U(1)_\eta$
ψ	$1/2$	$\mathbf{10}$	$\mathbf{1}$	$+1$	-1
$\bar{\psi}$	$1/2$	$\overline{\mathbf{10}}$	$\mathbf{1}$	0	$+1$
$\Phi_{\rm PQ}$	0	$\mathbf{1}$	$\mathbf{1}$	-1	0

The PQ mechanism is implemented by a PQ breaking field and **a vector-like pair of PQ-charged fermions** (**KSVZ fermions**) that transform as (anti-)fundamental reps. under SU(10).

→ **N_{DW} = 1**

Axion Potential

- **Small instanton effects**

The instanton effects can be captured by **a local fermion operator**.

★ One flavor of KSVZ fermions in the (anti-)fundamental reps. of SU(10)

★ Four flavors of Weyl fermions in the **99** rep. of SU(10)

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\rm PQ}$	$U(1)_\eta$
$\Psi_{99,f'}$ $(f'=1-4)$	1/2	**99**	**1**	0	0
$\Psi^{(1)}_{75,f'}$ $(f'=1-4)$	1/2	**1**	**75**	0	0

We assume an approximate chiral symmetry :

$$\Psi_{99} \to \Psi_{99} e^{i\beta}, \quad \Psi^{(1)}_{75} \to \Psi^{(1)}_{75} e^{-i\beta}$$

Under $SU(5) \subset SU(10)$ $\mathbf{99} = \mathbf{75} \oplus \mathbf{24}$

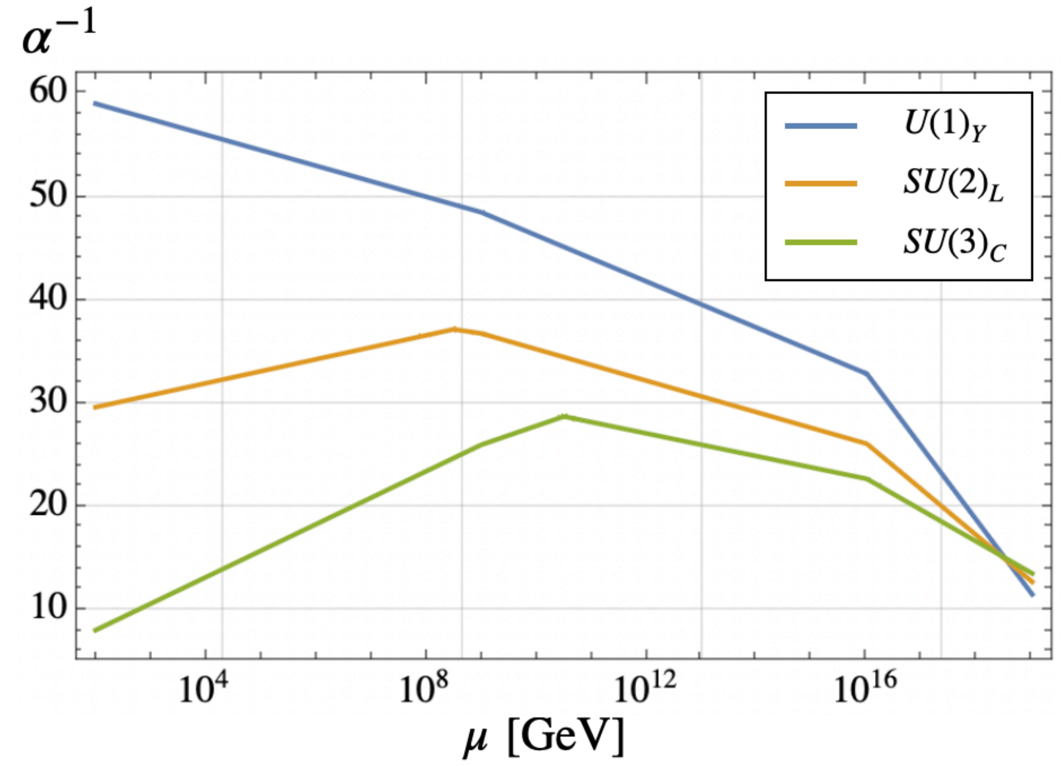
All components except the **SU(2)L triplet and SU(3)C octet** acquire masses near the Planck scale.

➡ Mass terms are suppressed by a small parameter κ ≪ 1

$$\mathcal{L} \sim \kappa^2 M (\Psi_{99})^a_b (\Psi_{99})^b_a + \kappa^{\dagger 2} M (\Psi^{(1)}_{75})^{kl}_{ij} (\Psi^{(1)}_{75})^{ij}_{kl}$$

GUT scale $M \sim M_{\rm Pl}$

➡ **Gauge coupling unification in non-SUSY model**



One-loop RGE

- Suppress the instanton effect
- **24** multiplet within **99** acquires a mass of $\mathcal{O}(\kappa^2 M)$

Axion Potential

- **Small instanton effects**

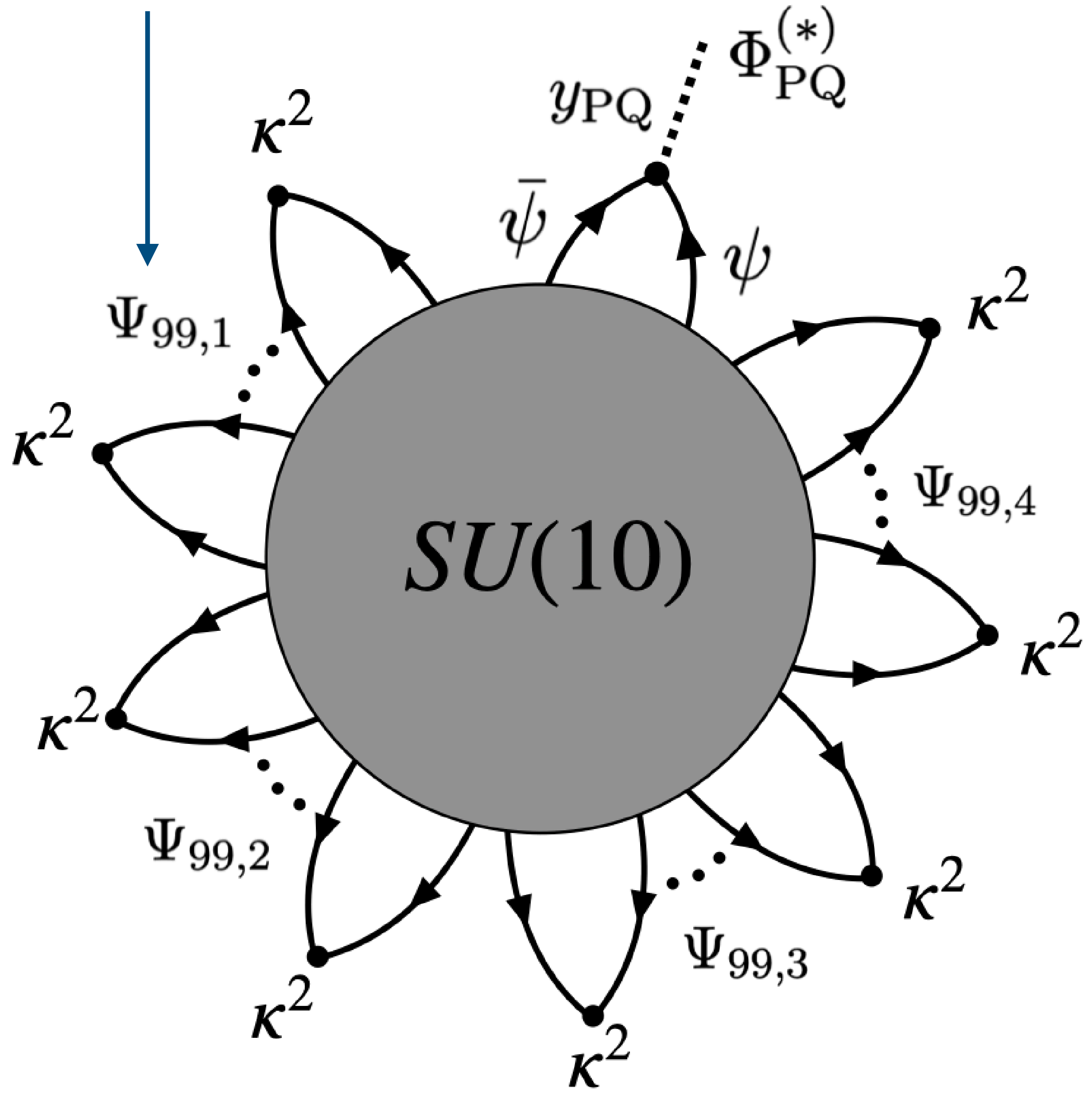
Instanton NDA

Csaki, D'Agnolo, Kuflik, Ruhdorfer (2024)

$$V_{\text{bias}} \approx C_{10} \left(\frac{2\pi}{\alpha_{\text{UV}}(M)}\right)^{2\times 10} (\Phi_{\text{PQ}} + \Phi^*_{\text{PQ}})$$
$$\times \int \frac{d\rho}{\rho^5} (\Lambda_{SU(10)}\rho)^{b_0} \underline{e^{-2\pi^2\rho^2 M^2}} y_{\text{PQ}} (\kappa^2 M\rho)^{10N_F} \rho$$
$$\approx (\kappa^2)^{10N_F} C_{10} \left(\frac{2\pi}{\alpha_{\text{UV}}(M)}\right)^{2\times 10} e^{-2\pi^2}$$
$$\times \frac{\Phi_{\text{PQ}}}{M} M^4 e^{-2\pi/\alpha_{\text{UV}}(M)} + c.c.$$

Suppression originating from SU(10) breaking

Each flavor of Ψ_{99} has 2T(Adj) = 20 legs closed by 10 mass vertices.



't Hooft vertex

$$\Lambda^{b_0}_{SU(10)} = M^{b_0} e^{-\frac{8\pi^2}{g^2_{\text{UV}}(M)}}$$

b_0 : one-loop beta function coefficient

C_{10} : SU(10) instanton density

y_{PQ}: Yukawa coupling of Φ_{PQ} and KSVZ fermions

$$V_{\text{bias}} = 3 \times 10^2 (\kappa^2)^{40} \frac{e^{-2\pi/\alpha_{\text{UV}}}}{\alpha^{20}_{\text{UV}}} e^{-2\pi^2} M^3 \Phi_{\text{PQ}} + c.c.$$
$$= 6 \times 10^2 \epsilon \frac{e^{-2\pi/\alpha_{\text{UV}}}}{\alpha^{20}_{\text{UV}}} e^{-2\pi^2} M^3 v_{\text{PQ}} \cos(\theta_a) \qquad \epsilon \equiv (\kappa^2)^{40}$$

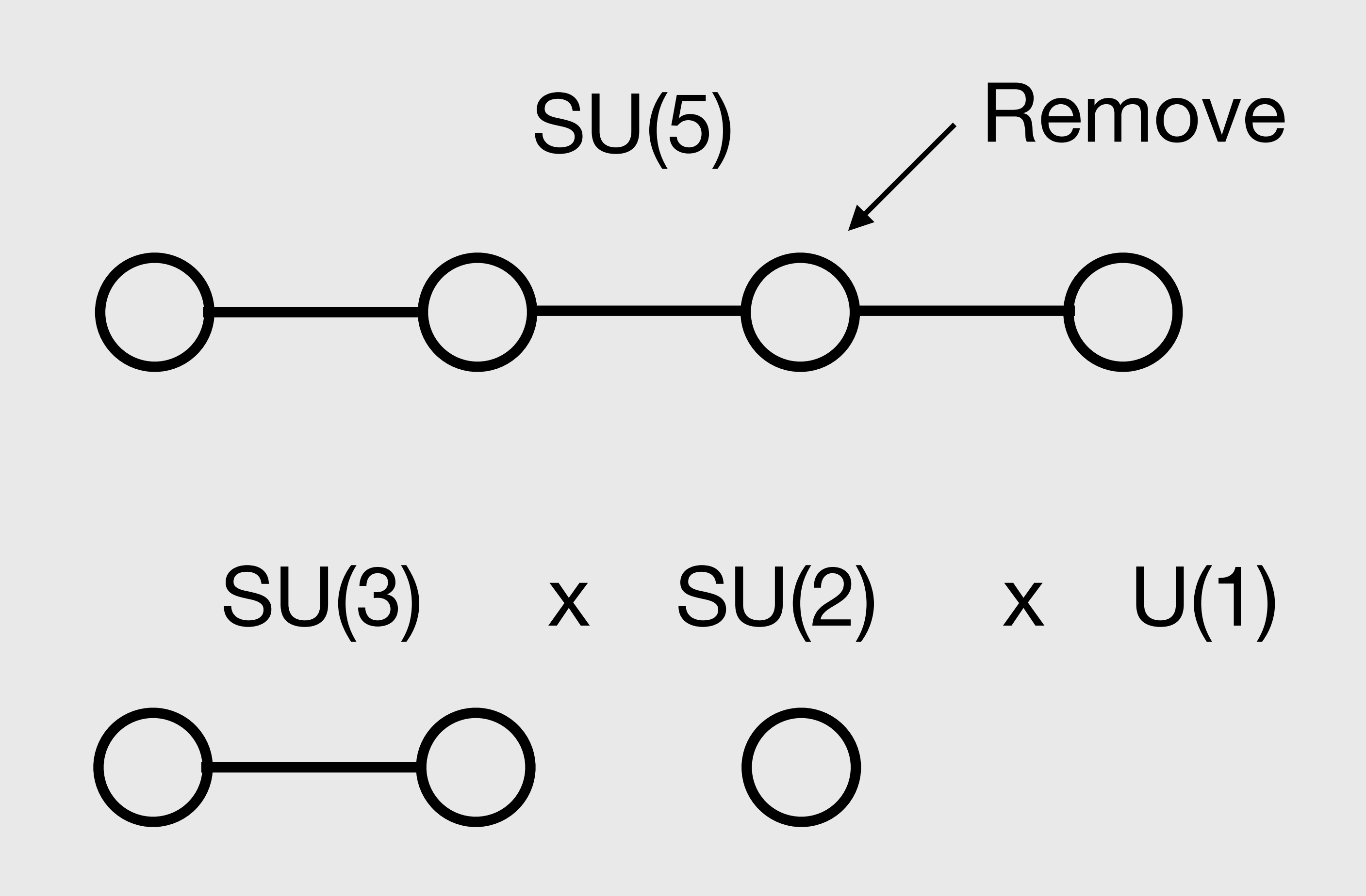
N_{DW} = 1

Axion potential from small instanton effects provides a bias term !

Special Subalgebra

Simple Lie algebras possess not only **regular** subalgebras but also **special** subalgebras.

Regular subalgebras : systematically obtained by removing nodes from Dynkin diagrams.



Special subalgebras :

do not follow this scheme !

To identify the SM gauge group as such a special subgroup of a UV gauge group is essential to obtain **small instanton effects** resolving the vacuum degeneracy of the axion potential.

Spontaneous CP Violation

The minimum of the axion potential generated by small instanton effects needs to be aligned with that of non-perturbative QCD effects.

⬆

Spontaneous CP violation

We introduce complex scalar fields with $\arg(\langle \eta_\alpha \rangle) = \mathcal{O}(1)$

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\rm PQ}$	$U(1)_\eta$
$\eta_\alpha (a = 1, 2)$	0	$\mathbf{1}$	$\mathbf{1}$	0	-1

← Forbid dangerous terms at the classical level.

To reproduce the CKM phase, η couple to the mixing term between the KSVZ fermion sector and the SM sector :

$$\mathcal{L} \sim \Phi_{\rm PQ} \psi \bar{\psi} + \sum_{f=1-3, \alpha=1,2} a^u_{\alpha f} \eta_\alpha \bar{\psi}^a (\Phi)^{ij}_a \mathbf{10}^{(1)}_{ij,f}$$

$$+ y^u_{ff'} \mathbf{10}^{(1)}_f \mathbf{10}^{(1)}_{f'} \mathbf{5}_H + y^d_{ff'} \mathbf{10}^{(1)}_f \bar{\mathbf{5}}^{(1)}_{f'} \mathbf{5}^\dagger_H$$

All coefficients are real.

Spontaneous CP Violation

The setup is similar to the **Nelson-Barr mechanism**.

Up-type quark mass matrix :

$$\mathcal{L} \sim (\underbrace{q_{uf}}_{\mathbf{10}_f^{(1)}}\ \underbrace{U}_{\bar{\psi}}\ \underbrace{Q_u}_{\psi})\mathcal{M}_u \begin{pmatrix} \bar{u}_{f'} \\ \bar{U} \\ \bar{Q}_u \end{pmatrix} \qquad \mathcal{M}_u = \begin{pmatrix} (m_u)_{ff'} & 0 & A^* \\ A^\dagger & v_{\rm PQ} & 0 \\ 0 & 0 & v_{\rm PQ} \end{pmatrix}$$

$$A^* = \sum_\alpha a^u_{\alpha f}\eta_\alpha \qquad (m_u)_{ff'} \equiv y^u_{ff'} v_{\rm SM}$$

O(1) CKM phase is properly generated when $(a^u\langle\eta\rangle)_f \gtrsim v_{\rm PQ}$

Since determinant is real, the physical θ-parameters of SU(10), SU(5)$_1$ or SU(3)$_C$ vanish at the tree-level.

Radiative corrections can still generate nonzero corrections.