Remarks on the Status of Gauge Coupling Unification

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DESY Seminar
6 May 2024

Huge Success in Electroweak theory:

$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

Spontaneous Symmetry Breaking was the key

Group theory was the mathematics

Unification was the word

Keep going!

$G \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$

Lowest rank group SU(5) works, as does SO(10)

$$\bar{5} = \begin{pmatrix} \nu \\ e \\ d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

$$10 = \{ e_R, Q_L, u_R \}$$

$$1 = \{ \nu_R \}$$

$$SO(10): \; 16 = 10 + \bar{5} + 1$$

Fermions fit beautifully within complete GUT multiplets

Johannes Herms, Heidelberg Workshop, '24

(With student I obtained similar ~10^{-2} result for chiral representations.)

Result (teaser)

see you at the poster!

first naive result: $(D \leq 20, Q \leq 10)$

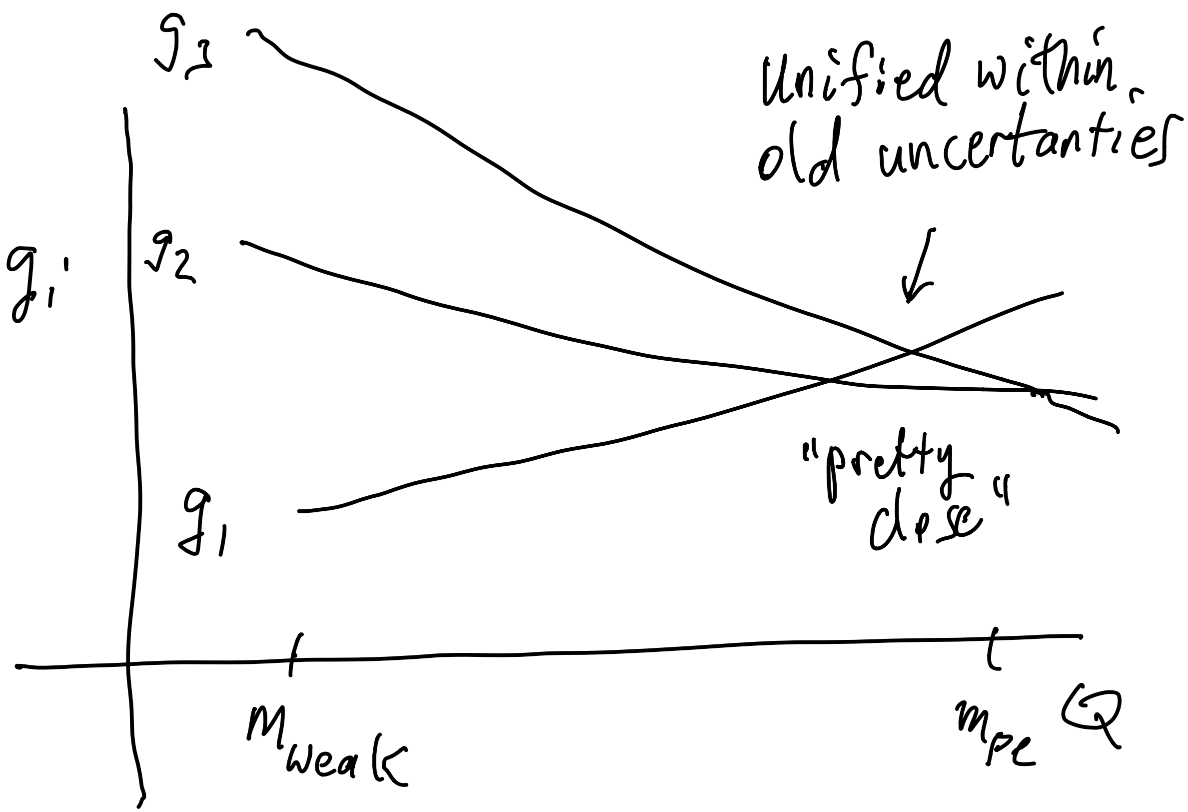
$$\frac{\#\text{neatly unifiable reps}}{\#\text{all SM-like reps}} = \frac{61}{4541567} \simeq 10^{-5}$$

most conservative result: $(D \leq 15, Q \leq 6, S \leq 1)$

$$\frac{\#\text{neatly unifiable reps}}{\#\text{all SM-like reps}} = \frac{1}{76} \simeq \mathbf{10^{-2}}$$

$\Rightarrow$ *rare at "2σ" level.*

Gauge couplings meeting (within uncertainties)



If experiment stopped in 1980, we'd all believe in GUTs.

But expt did not stop.

Desire to confirm it all by proton decay:

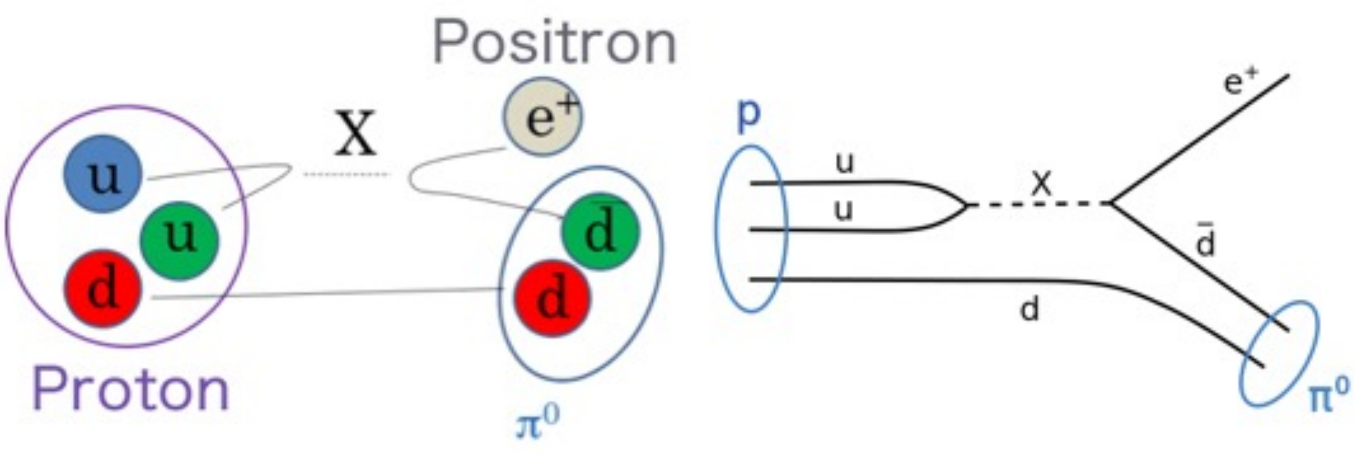


Fig.1 *Proton Decay:* $p \rightarrow e^+ + \pi^0$. **X** *is mediator;* d-*quark- Spectator.*

[Adamas university]

From Langacker Phys. Rep. 1981:

From table 4.3, I conclude (for the SU_5 model with $F = 3$ and the minimal Higgs structure)

$$\tau_p(\mathrm{yr}) = (2.4\text{–}38) \times 10^{-29} M_X^4$$
$$\tau_n/\tau_p \sim 1.1\text{–}1.5, \qquad (4.43)$$

where M_X is in GeV (τ_n/τ_p is further discussed below).

Combining this with

$$M_X = 15 \times 10^{14} \Lambda_{\overline{MS}} \times (1.5)^{\pm 1}$$
$$\Lambda_{\overline{MS}}(\mathrm{GeV}) = 0.4 \times (1.5)^{\pm 1} \qquad (4.44)$$

one has

Georgi-Glashow: $10^{30\text{-}31}$ years

$$\tau_p^{(\mathrm{yr})} = (1.2\text{–}19) \times 10^{32 \pm 0.7} \Lambda_{\overline{MS}}^4$$
$$= (3.1\text{–}49) \times 10^{30 \pm 1.4}. \qquad (4.45)$$

(4.45) can be written

$$\tau_p^{(\mathrm{yr})} = 4.8 \times 10^{32 \pm 1.3} \Lambda_{\overline{MS}}^4$$
$$= 1.2 \times 10^{(31 \pm 2)}. \qquad (4.46)$$

By 1983 the IMB (Irvine-Michigan-Brookhaven) and KamiokandeNDE (K. nucleon decay expt)
Saw nothing.

Lifetime > 10^{32} years or so.

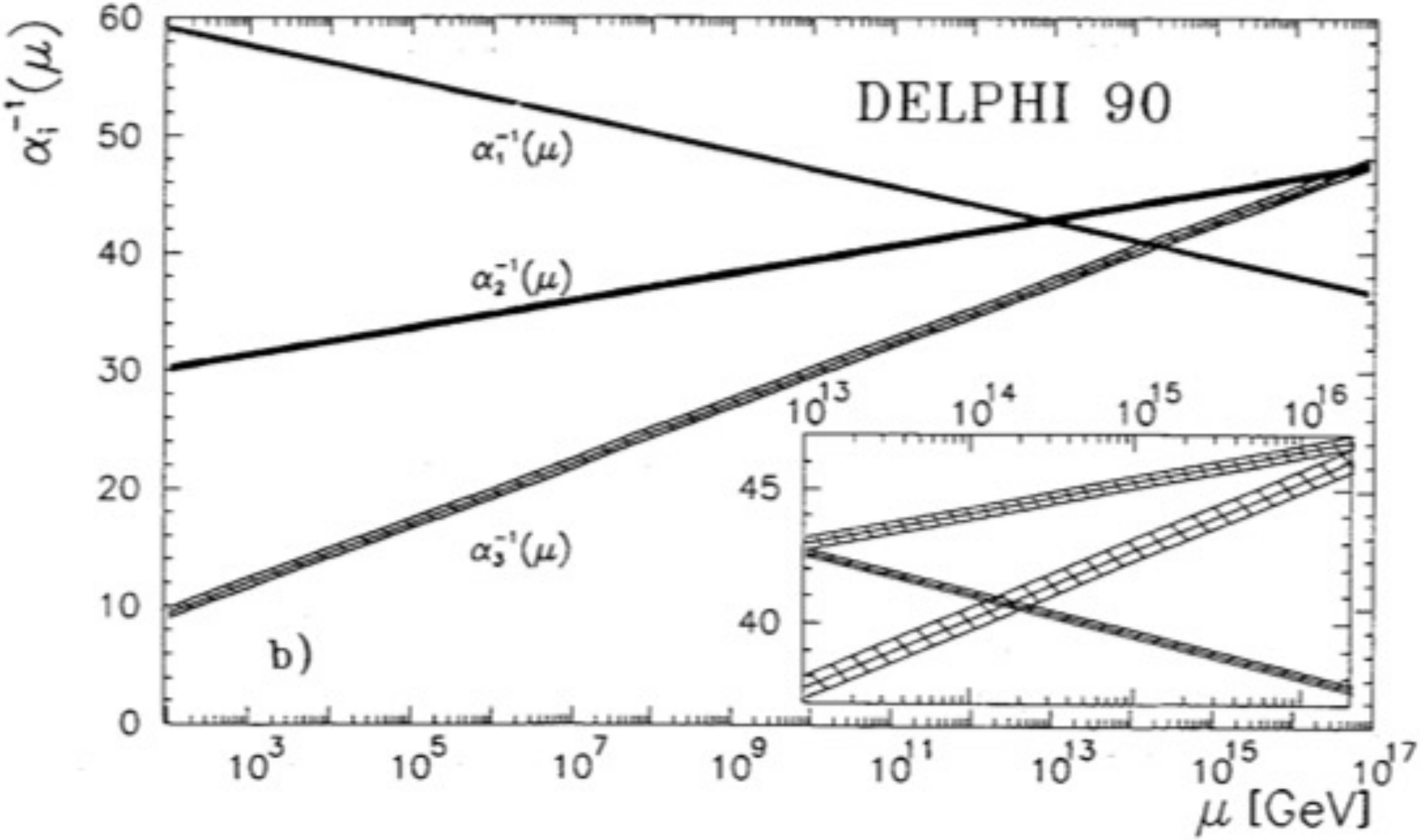
[Present limit is $\tau(p \to \pi e) > 1.6 \times 10^{34}$ years.]

Georgi 1983: “Shelly is depressed”

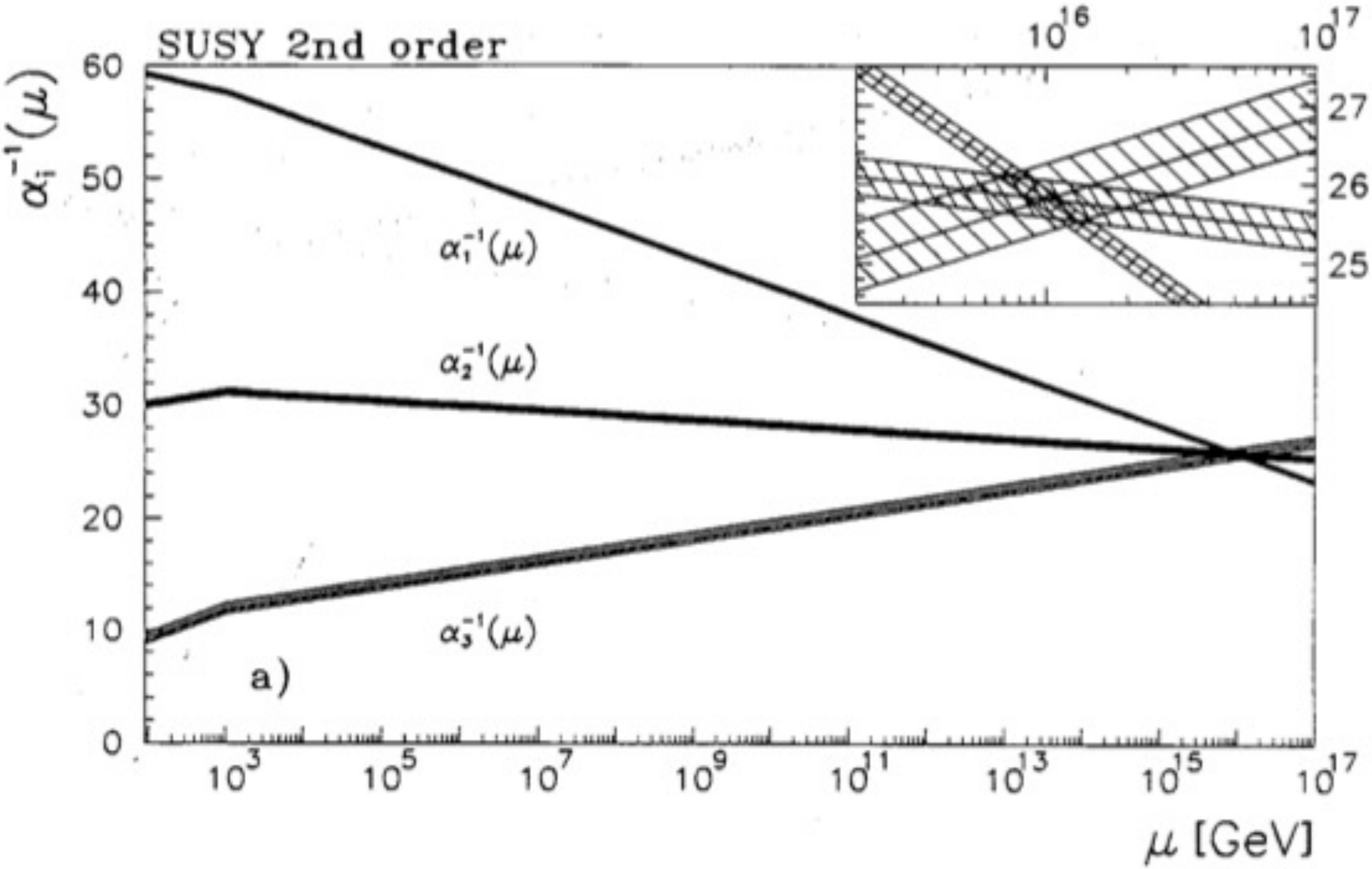
“GUT Winter” 1983-1990.

Pixels

Then LEP/SLC Z-pole precision experiments, ~1990.



Amaldi, de Boer, Fürstenau, March 1991 preprint



Amaldi, de Boer, Fürstenau, March 1991 preprint

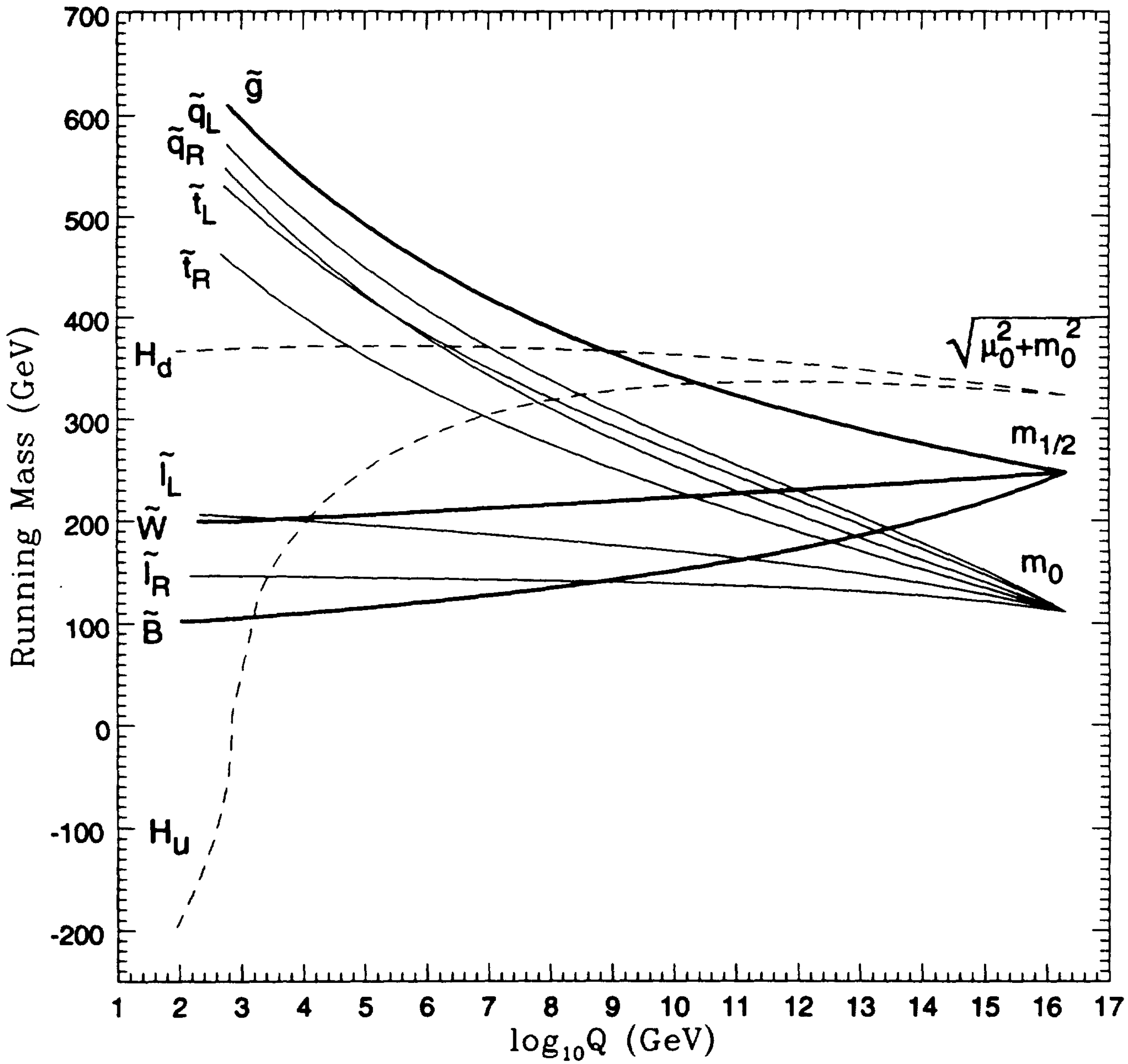
Typical viewpoint on unification, 1990-2012

theory for the strong interaction [1]. The runnings of coupling constants and mass parameters are crucial in global analysis of high precision electroweak experiments [2]. On the other hand, RGEs analysis extrapolated to extremely high energy provides a possible test for physics beyond the SM. For example, gauge couplings do not unify within the SM. This gives extra evidence against simple grand unification theories such as $SU(5)$ without supersymmetry, in addition to the non-observation of proton decay. On the other hand, gauge couplings seem to unify at a scale $\sim 2 \times 10^{16}\ GeV$ in the minimal supersymmetric standard model, which can be interpreted as an indirect evidence for supersymmetry as well as unification theories [3–5]. Comprehensive analysis can be

Luo, Xiao, "Two-loop RGEs in SM", '03

Gauge Coupling Unification / GUTs regained life!

GUT theorists headed toward SUSY adobe



Kane et al., 1994

GUT Expectations:

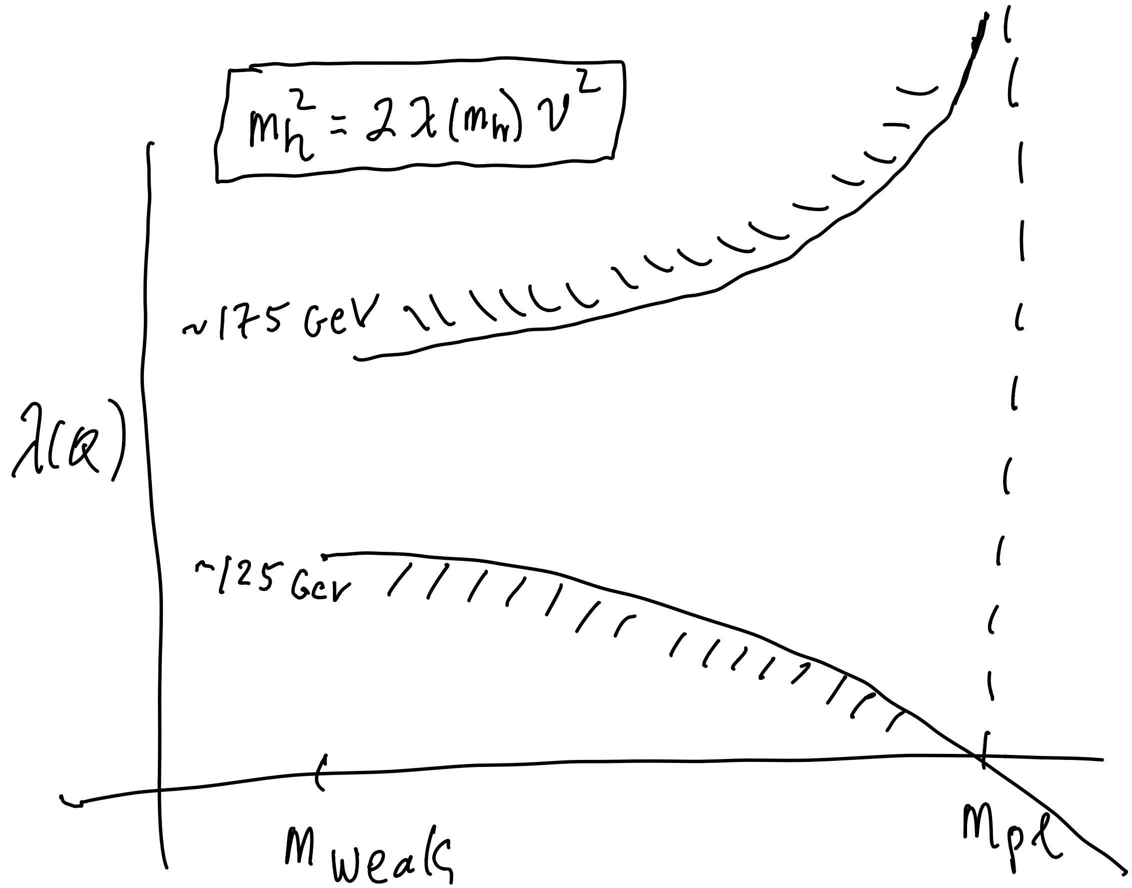
1) (*important*) Higgs boson will be found in perturbative regime (125 GeV < m_h < 175 GeV), or within SUSY expectations (100 GeV < m_h < 140 GeV)

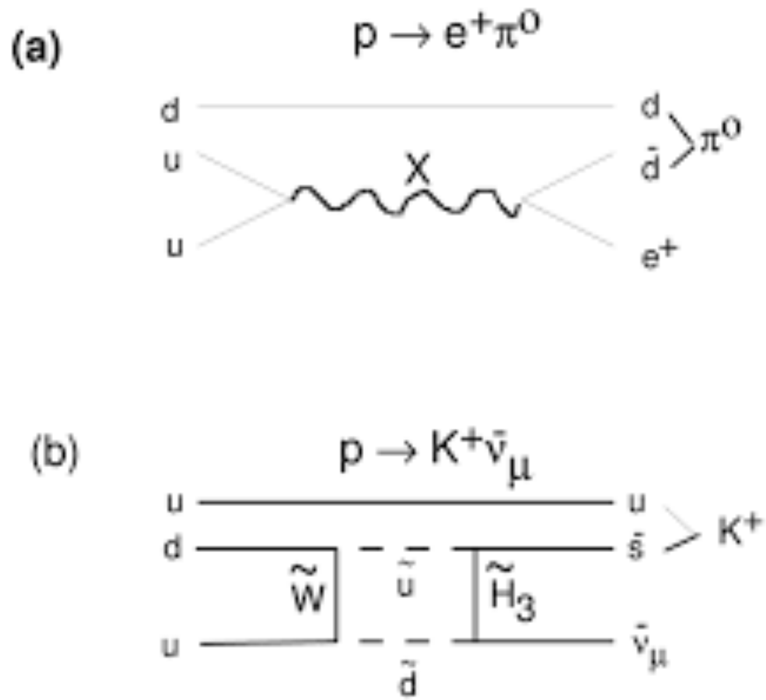
2) (*hopeful*) Proton decay may be found by future experiments

3) (*naive*) Supersymmetry will be found at LHC

$$\frac{dg_i}{d\log Q} = \beta_i(g_i, y_j, \lambda)$$

↳ β_i non-pert. if λ becomes N.P.





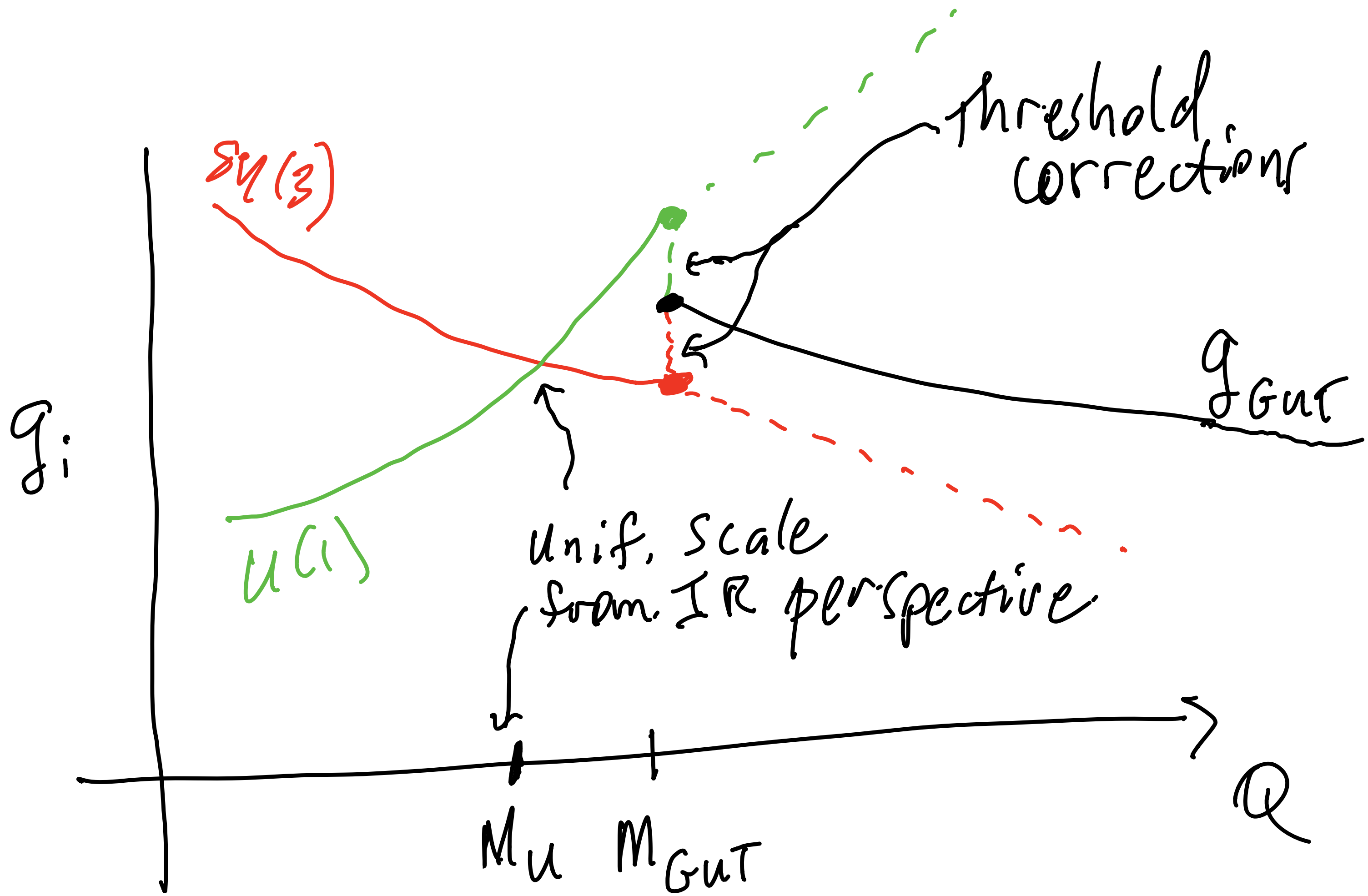
Sigel/Forbes

(a) Highly sensitive to GUT scale value.

(b) Highly sensitive to GUT scale and susy partner scale

Important question: What is the range of values of MX and Msusy and still have unification?

Proton-decay GUT-scale and IR-up G.C.U. scale not the same



Gauge couplings measured at low scale

Exact unification tests require matching at high scale and RG flow to low scale across thresholds (e.g., superpartners)

MATCHING. ($\mu_* = 10^{16}\,\mathrm{GeV}$)

$$\left(\frac{1}{g_i^2(\mu_*)}\right)_{\overline{MS}} = \left(\frac{1}{g_U^2(\mu_*)}\right)_{\overline{MS}} - \left(\frac{\lambda_i(\mu_*)}{48\pi^2}\right)_{\overline{MS}}$$

where $(\lambda_i(\mu))_{\overline{MS}} = l_i^{V_n} - 21\, l_i^{V_n} \ln \frac{M_{V_n}}{\mu} + l_i^{S_n} \ln \frac{M_{S_n}}{\mu} + 8\, l_i^{F_n} \ln \frac{M_{F_n}}{\mu}$

Relations that are independent of unified coupling:

$$\left(\frac{\Delta\lambda_{ij}(\mu_*)}{48\pi^2}\right)_{\overline{MS},\ \overline{DR}} \equiv \left(\frac{1}{g_i^2(\mu_*)} - \frac{1}{g_j^2(\mu_*)}\right)_{\overline{MS},\ \overline{DR}} = \left(\frac{\lambda_j(\mu_*) - \lambda_i(\mu_*)}{48\pi^2}\right)_{\overline{MS},\ \overline{DR}}$$

$$\left(\frac{\Delta\lambda_{ij}(\mu_*)}{48\pi^2}\right)_{\overline{MS},\ \overline{DR}} \equiv \left(\frac{1}{g_i^2(\mu_*)} - \frac{1}{g_j^2(\mu_*)}\right)_{\overline{MS},\ \overline{DR}} = \left(\frac{\lambda_j(\mu_*) - \lambda_i(\mu_*)}{48\pi^2}\right)_{\overline{MS},\ \overline{DR}}$$

The couplings $g_i(\mu_*)$ determined from flowing precision IR couplings up (including thresholds, if applicable).

What neighborhood of values of $\Delta\lambda_{ij}(\mu_*)$ do we expect?

→ Approximately Dynkin indices of GUT representations.

For minimal SU(5) models $\Delta\lambda$ ~ 10 or so

For SO(10) models $\Delta\lambda$ ~ 100 or so

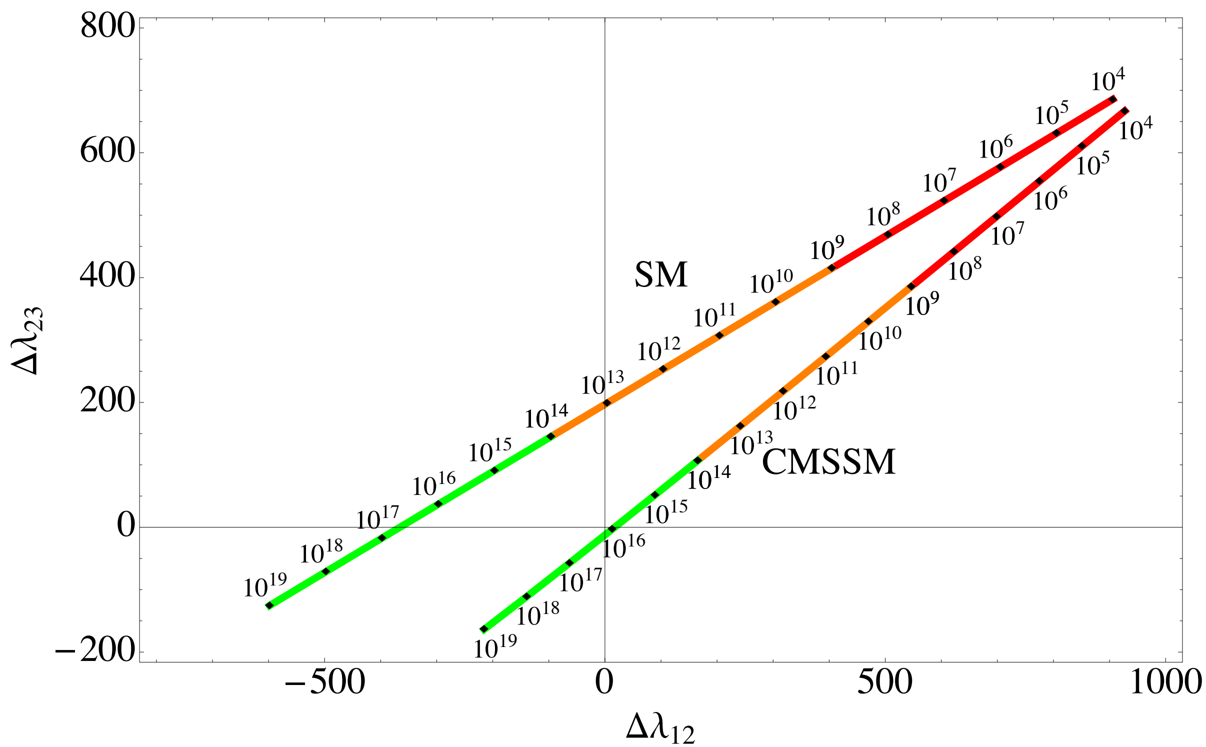


FIG. 2. This key visualization plot shows $\Delta\lambda_{23}(\mu)$ as a function of $\Delta\lambda_{12}(\mu)$ for the Standard Model and a CMSSM-like SUSY model. Labels on the line indicate the scale μ. Green regions indicate that a unification scale around those values is moderately safe from constraints. Orange indicates relatively unsafe, Red indicates very unsafe.

(Ellis, Wells, `15)

(Unexpectedly?) small thresholds needed for CMSSM

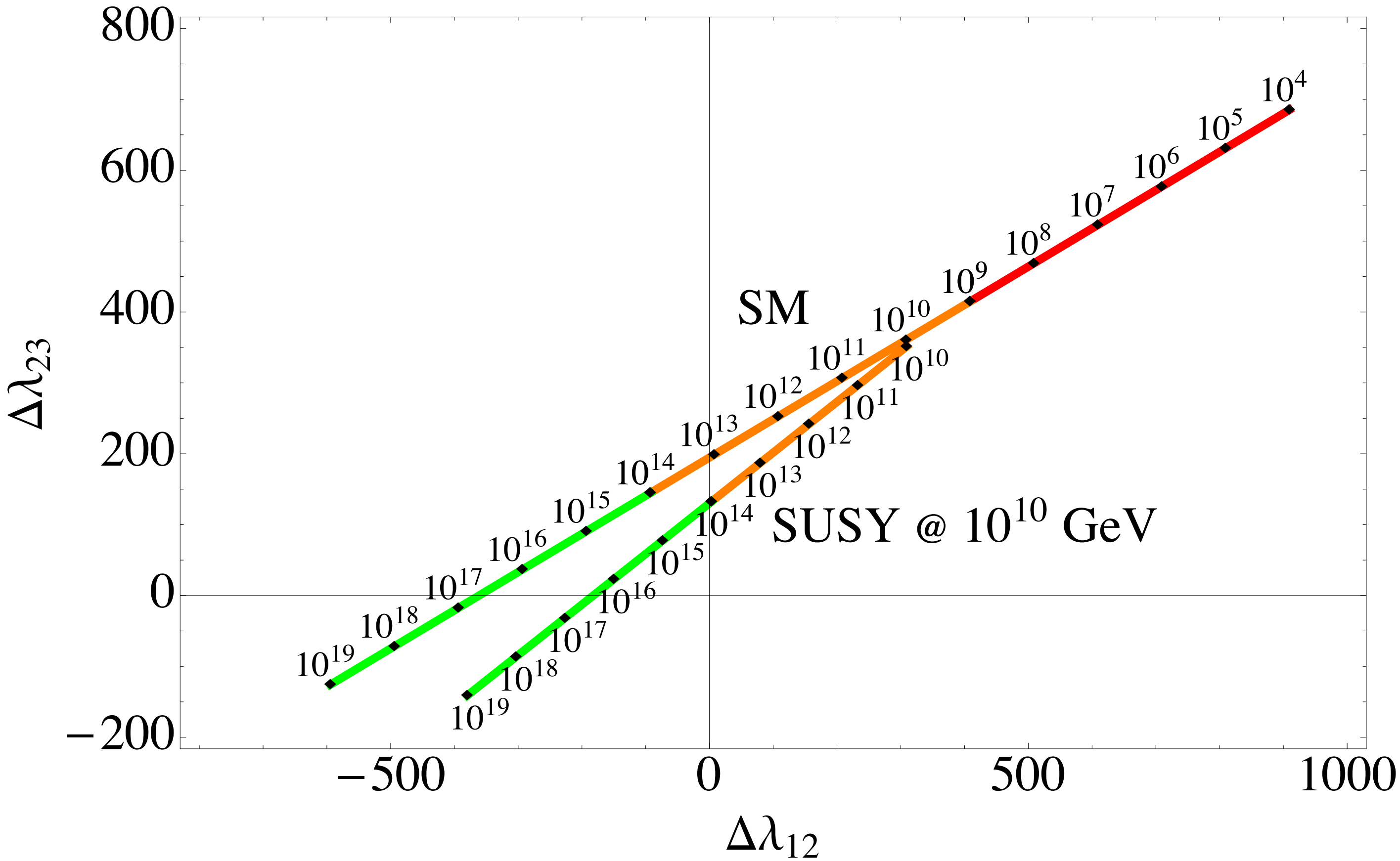


Figure 16: Plot of the threshold corrections needed for exact gauge coupling unification. The numbers along the line are the scales μ_* at which the IR couplings are evaluated for unification and at which point the needed threshold corrections are computed and then plotted in the plane. The long straight line is assuming only the SM up to the highest scale. The second line that branches downward is for the case of superpartners existing at 10^{10} GeV, which lowers the needed threshold corrections at high scales.

[Ellis, Wells, '17]

Example: Lavoura-Wolfenstein non-supersymmetric SO(10) model

Gauge Bosons			Scalars		
$SO(10)$	$SU(2)\otimes SU(3)[U(1)_Y]$	Mass	$SO(10)$	$SU(2)_L\otimes SU(2)_R\otimes SU(4)$	Mass
45	$(1,1)[0]$	M_R	**210**	$(1,1,1)$	N/A
45	$(1,1)[\sqrt{\frac{3}{5}}]$	M_R	**210**	$(2,2,6)$	Goldstone
45	$(1,1)[-\sqrt{\frac{3}{5}}]$	M_R	**210**	$(1,1,15)$	M_1
45	$(1,3)[\frac{2}{3}\sqrt{\frac{3}{5}}]$	M_R	**210**	$(2,2,10)$	M_1
45	$(1,\overline{3})[-\frac{2}{3}\sqrt{\frac{3}{5}}]$	M_R	**210**	$(2,2,\overline{10})$	M_1
45	$(2,3)[\frac{1}{6}\sqrt{\frac{3}{5}}]$	M_V	**210**	$(1,3,15)$	M_4
45	$(2,\overline{3})[-\frac{1}{6}\sqrt{\frac{3}{5}}]$	M_V	**210**	$(3,1,15)$	M_5
45	$(2,3)[-\frac{5}{6}\sqrt{\frac{3}{5}}]$	M_V	**126**	$(1,1,6)$	M_1
45	$(2,\overline{3})[\frac{5}{6}\sqrt{\frac{3}{5}}]$	M_V	**126**	$(2,2,15)$	M_1
			126	$(1,3,10)$	M_2
			126	$(3,1,\overline{10})$	M_3

TABLE I. Table showing the spectrum of superheavy particles contributing to the threshold corrections in the Lavoura-Wolfenstein SO(10) GUT, with their various masses.

the Lavoura-Wolfenstein SO(10) GUT, we obtain

$$\lambda_1^V(\mu_*) = 8 + \frac{294}{5}\log\frac{\mu_*}{M_R} + \frac{546}{5}\log\frac{\mu_*}{M_V}$$
$$\lambda_2^V(\mu_*) = 6 + 126\log\frac{\mu_*}{M_V}$$
$$\lambda_3^V(\mu_*) = 5 + 21\log\frac{\mu_*}{M_R} + +84\log\frac{\mu_*}{M_V}$$

for the contributions from vector bosons, and

$$\lambda_1^S(\mu_*) = -\frac{274}{5}\log\frac{\mu_*}{M_1} - \frac{142}{5}\log\frac{\mu_*}{M_2} - \frac{36}{5}\log\frac{\mu_*}{M_3} - \frac{114}{5}\log\frac{\mu_*}{M_4}$$
$$\lambda_2^S(\mu_*) = -50\log\frac{\mu_*}{M_1} - 40\log\frac{\mu_*}{M_3} - 30\log\frac{\mu_*}{M_5}$$
$$\lambda_3^S(\mu_*) = -62\log\frac{\mu_*}{M_1} - 17\log\frac{\mu_*}{M_2} - 18\log\frac{\mu_*}{M_3} - 12\log\frac{\mu_*}{M_4} - 12\log\frac{\mu_*}{M_5}$$

for the contributions from the scalars.

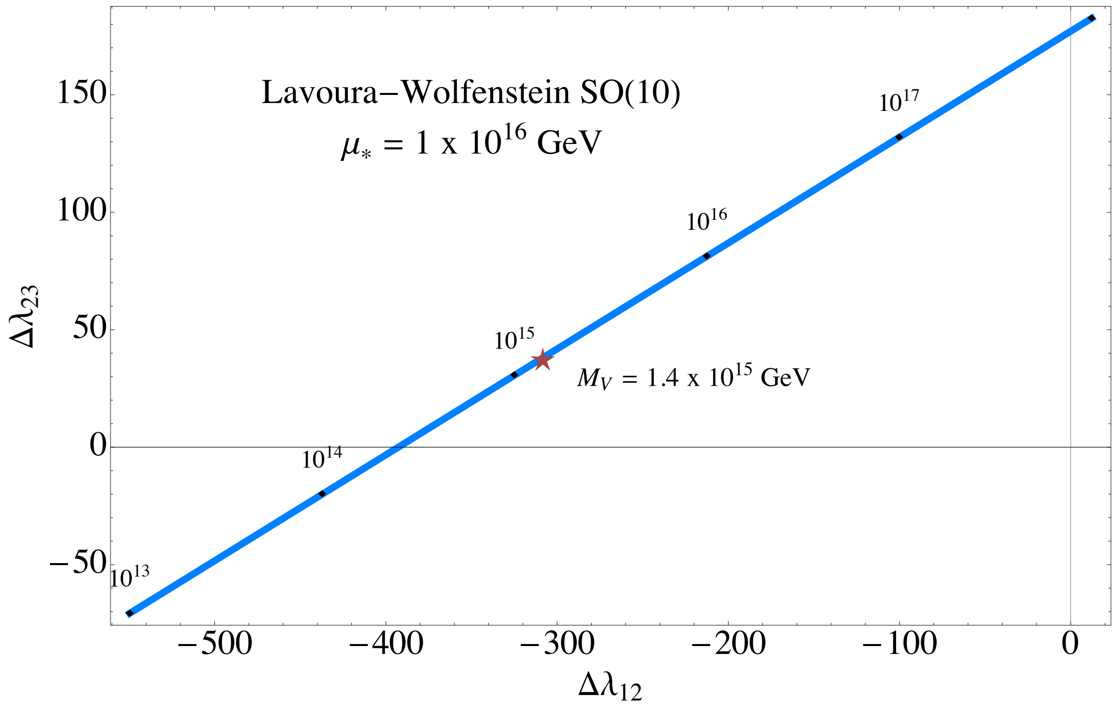
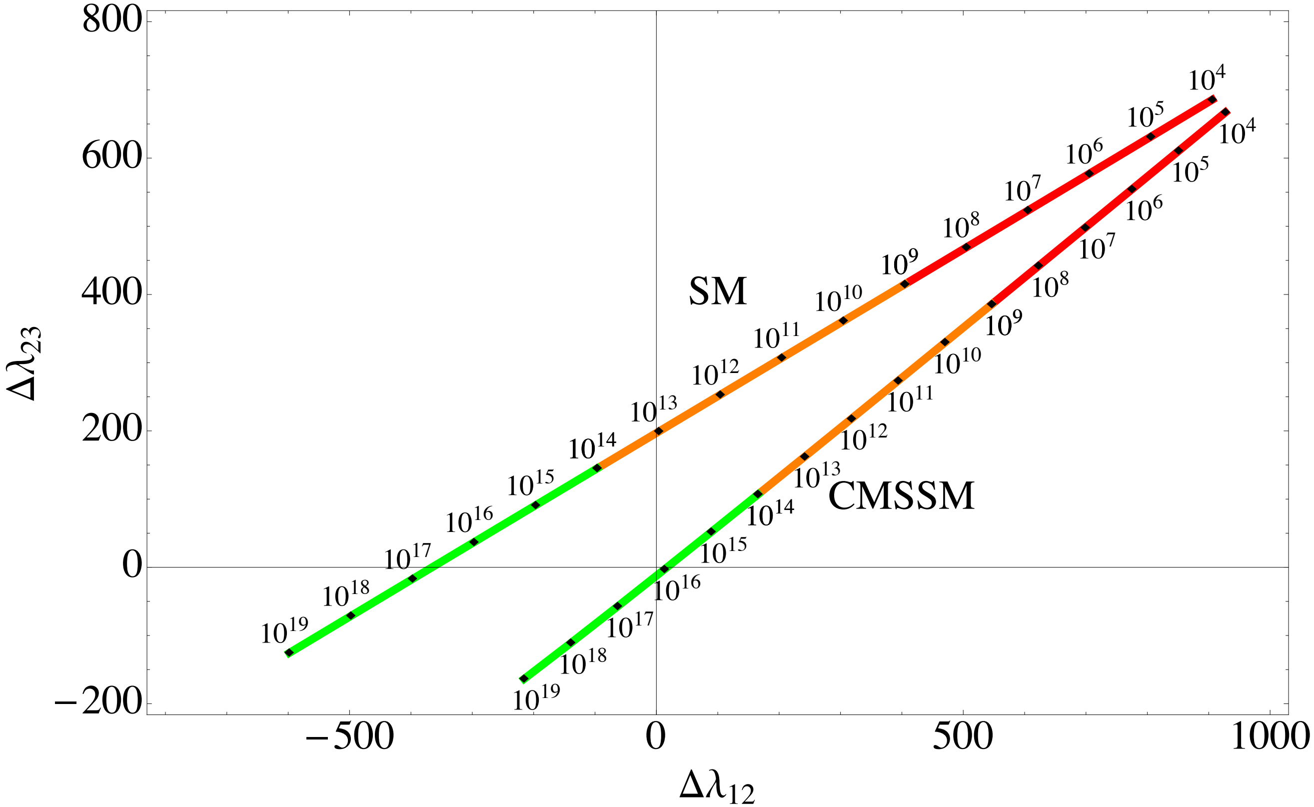


FIG. 3. Plot of $\Delta\lambda_{23}(\mu)$ as a function of $\Delta\lambda_{12}(\mu)$. Shown is the Lavoura-Wolfenstein SO(10) (blue) with $M_V/M_R = 20$, $M_V/M_1 = 3$, $M_V/M_2 = 7$, $M_V/M_3 = 8$, $M_V/M_4 = 10$ and $M_V/M_5 = 14$, with M_V varying between 10^{13} and 10^{18}. The star corresponds to the required values of $\Delta\lambda_{12}(\mu_*)$ and $\Delta\lambda_{23}(\mu_*)$ in the SM. We find that $M_V = 1.4 \times 10^{15}$ gives the desired $\Delta\lambda_{12}(\mu_*)$ and $\Delta\lambda_{23}(\mu_*)$ in the Lavoura-Wolfenstein SO(10) for the given mass ratios.

Some general points



Ellis, Wells, '15

Low-scale SUSY can tolerate surprisingly low threshold corrections at high scale

Non-susy unification possible with "big yet reasonable" threshold corrections

… And we have not even talked about gravity corrections:

We can write the gauge-kinetic function of minimal $SU(5)$ as

$$\int d^2\theta \left[\frac{S}{8M_{\rm Pl}}\mathcal{W}\mathcal{W} + \frac{y\Sigma}{M_{\rm Pl}}\mathcal{W}\mathcal{W}\right] \tag{3}$$

where $\Sigma = \mathbf{24}_H$ and $\langle S\rangle = M_{\rm Pl}/g_G^2 + \theta^2 F_S$ contains the effective singlet supersymmetry breaking. The $SU(5)$ gauge coupling is g_G and the universal contribution to the masses of all gauginos is $M_{1/2} = -g_G^2 F_S/(2M_{\rm Pl})$.

$$\langle\Sigma\rangle = v_\Sigma\,{\rm diag}\left(\frac{2}{3},\frac{2}{3},\frac{2}{3},-1,-1\right)$$

The relationships between the GUT scale gauge coupling g_G and the low-scale gauge couplings $g_i(Q)$ of the MSSM effective theory are

$$\frac{1}{g_i^2(Q)} = \frac{1}{g_G^2(Q)} + \Delta_i^G(Q) + c_i\epsilon \tag{5}$$

where $\epsilon = 8yv_\Sigma/M_{\rm Pl}$ and $c_i = \{-1/3, -1, 2/3\}$ for the gauge groups $i = \{U(1)_Y, SU(2)_L, SU(3)\}$

%-level ε is enough to “save” minimal SUSY SU(5) from too small triplet Higgs mass needed for exact unification → too fast proton decay

Tobe, JDW

Let us take exact unification – "precision unification" -- very seriously (it is telling us something?)

For some reason (agnostic) the threshold corrections at high-scale are negligible.

4D Perspective theory:

Minimal supersymmetry spontaneously broken at some scale Λ.

Big desert between here and the Planck scale.

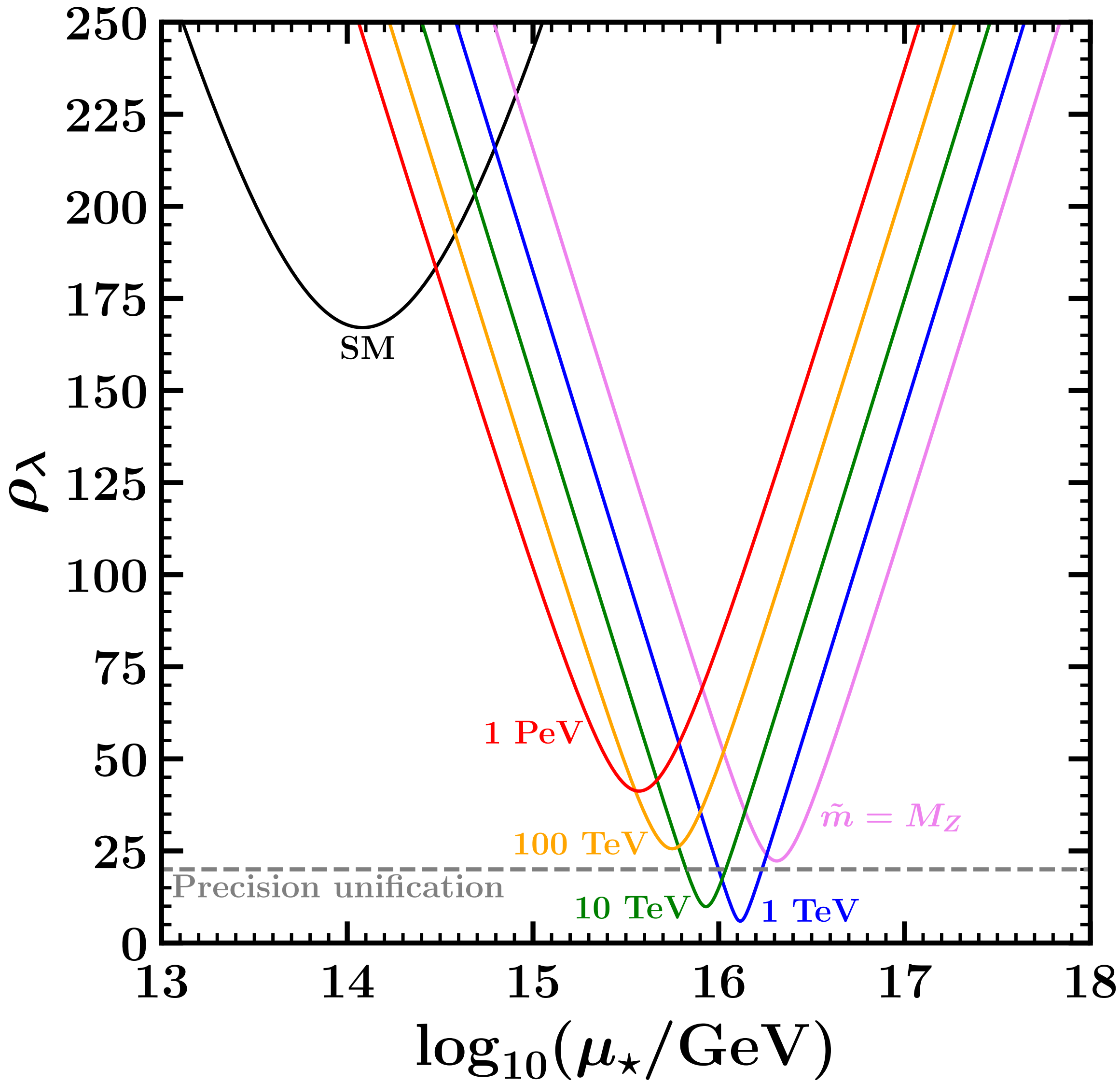
Gauge couplings unify "precisely" with negligible corrections.

First, let's define a quantity → 0 when there is precision unification:

$$\frac{\rho_\lambda}{48\pi^2} \equiv \sqrt{\sum_{i\neq j}\left(\frac{1}{g_i^2}-\frac{1}{g_j^2}\right)^2}$$

At every scale we can compute $\rho_\lambda(Q)$.

Let's do the simplest thing first with all SUSY masses equal.



The TeV scale is the scale that gives most precise unification.

Msusy=Mz → no precision unification. [Cf. worries people had of "matching alphas"]

Unification much better than SM even for PeV scale or higher.

Bhattiprolu, JDW, '24

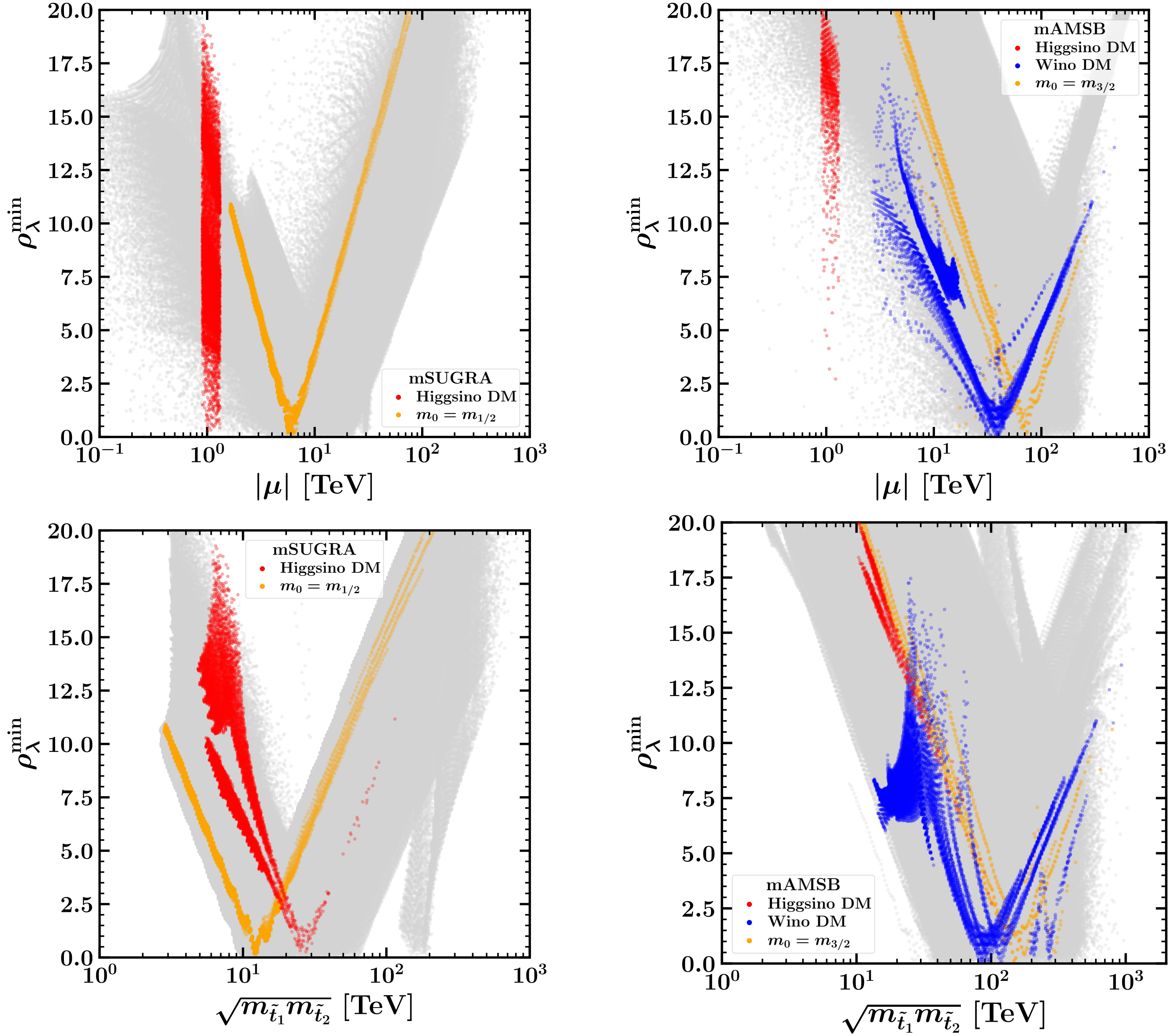


FIG. 3. $\rho_\lambda^{\min}$ plotted against the absolute value of the μ term (top panels) and the geometric mean of the top squark masses (bottom panels) in mSUGRA (left) and mAMSB (right) scenarios. The gray points correspond to all models with precise gauge coupling unification ($\rho_\lambda^{\min} < 20$) and the Higgs mass m_h within 3 GeV of 125.25 GeV. Various colored points correspond to various special cases as labeled.

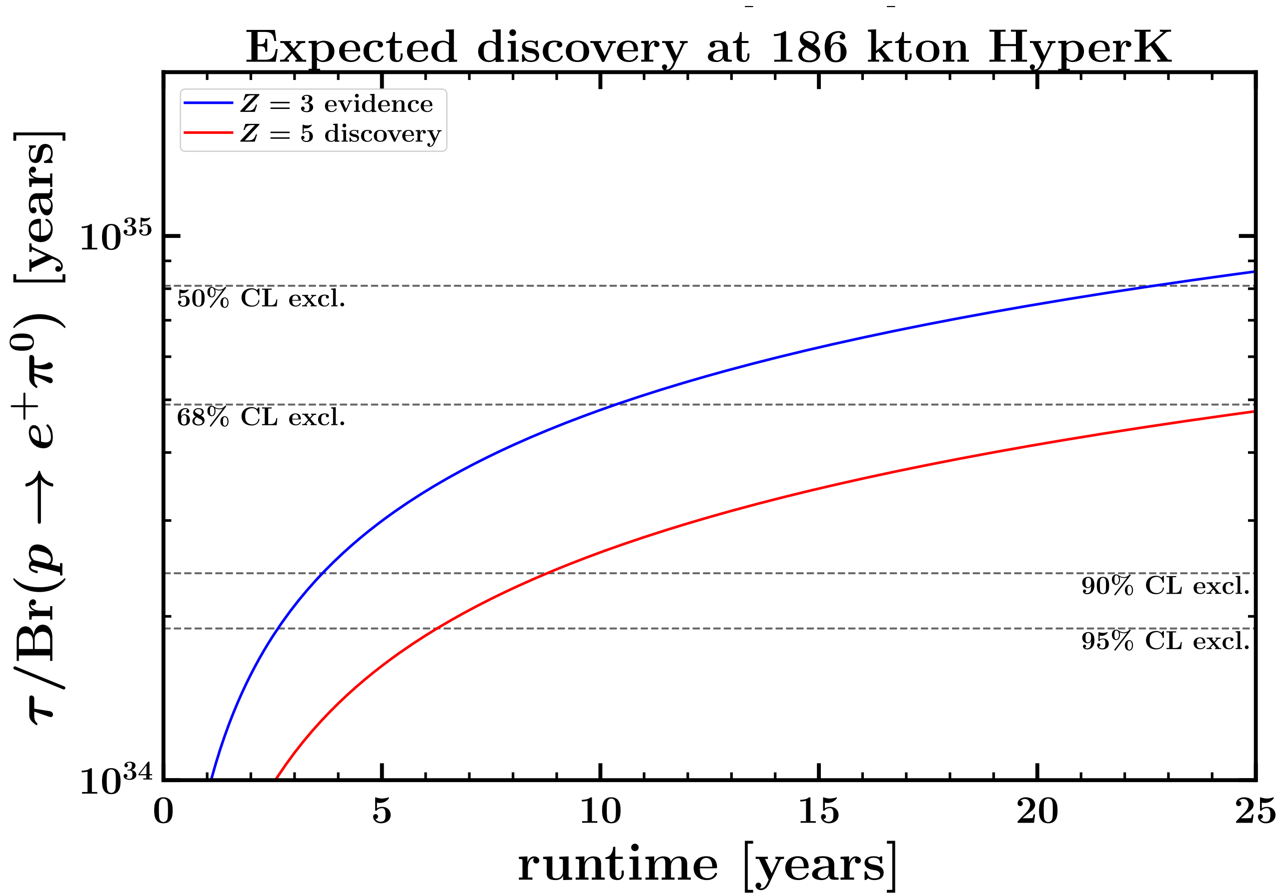


FIG. 3.8: Proton partial lifetime in the $p \to e^+\pi^0$ channel that is expected to be excluded at 90% or 95% CL [top panel; from eq. (3.10)] or discovered at $Z = 3$ or $Z = 5$ significance [bottom panel; from eq. (3.11)] at Hyper-Kamiokande with 186 kilotons of water, as a function of runtime, with the uncertainties in background and signal selection efficiency listed in Table 3.2, taken from ref. [48]. Our estimates of the current 95%, 90%, 68%, and 50% CL exclusion limit on proton partial lifetime, based on Super-Kamiokande's data from 2020 [45], are shown as horizontal dashed lines.

Bhattiprolu, Martin, JDW, '23

Conclusions

Gauge coupling unification/GUTs possible in SM-like theories

G.C.U./GUT is still rather compelling from several points of view.

Higgs mass discovery consistent with SM perturbative GUT, and SUSY GUT Higgs mass

Low-scale SUSY unifies (too?) easily

Msusy < PeV is just fine for G.C.U. and even precision G.C.U.

Proton decay is very sensitive to Msusy and MX and therefore not assured.

Only EW finetuning/hierarchy problem/naturalness concerns might keep you from believing in GUTs ...