

Form Factors and Strong Couplings of Heavy Baryons

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I. Motivation and introduction

- Weak decays of heavy-baryons are of high interest:
determination of CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$,
allow the study of spin correlations (polarization asymmetries...),
a multitude of new-physics sensitive observables ($A_{FB}...$).
- Strong coupling constants of (charmed baryon)-(charmed meson)-nucleon
are fundamental inputs in the calculations of charm production at \bar{P} ANDA.
- Available techniques to investigate heavy-to-light form factors:
a) Nonperturbative approaches: Lattice QCD, QCD sum rules,
b) Effective field theories: χ PT, HQET, SCET,
- Applications of LCSR to the meson transition in mature state:
 $\pi\gamma^* \rightarrow \gamma$, $\pi\gamma^* \rightarrow \pi$, $B \rightarrow \pi\ell\nu_\ell$, $B \rightarrow K^*\gamma$, $B \rightarrow K^{(*)}\ell^+\ell^-$,

Difficulties of heavy-baryon LCSR

- Background contribution of negative-parity baryon in the dispersion relation:

$$\langle 0 | \eta_{\Lambda_Q}^{(i)} | \Lambda_Q(P - q) \rangle = m_{\Lambda_Q} \lambda_{\Lambda_Q}^{(i)} u_{\Lambda_Q}(P - q),$$
$$\langle 0 | \eta_{\Lambda_Q}^{(i)} | \Lambda_Q^*(P - q) \rangle = m_{\Lambda_Q^*} \lambda_{\Lambda_Q^*}^{(i)} u_{\Lambda_Q^*}(P - q).$$

Fermion is not an eigenstate of parity transformation!

- Longstanding issue of interpolating current for baryons:
Ioffe current or tensor current for nucleon state (Ioffe 1981, Chung et al 1982)?
- Some attempts to avoid background pollution:
 - a) Parity projector matrix $(1 \pm \not{p})/2$ for heavy-baryon sum rule (Bagan et al 1993, ...),
 - b) Choose “old-fashioned” correlation function and construct sum rules in the complex q_0 -space in the rest frame (Jido et al 1996, ...).

Way out in the standard LCSR approach

- A natural scenario to eliminate background pollution using time-ordered correlation function in reference-frame independent way exists?
- How to construct a baryon sum rule with predictions independent on the interpolating current?
- Resolution to the two problems meanwhile:
 - a) Separating the negative-parity baryon contribution from continuum,
 - b) Constructing two independent LCSR from different kinematical structures,
 - c) Choosing a linear combination of two sum rules to remove background pollution.

II. Choices of baryonic currents

- General structure of heavy-baryon current (Shuryak, 1981):

$$\eta = \epsilon^{ijk} (q_i C \Gamma_b q'_j) \tilde{\Gamma}_b Q_k .$$

- Isospin symmetry of light diquark system:

$$(q C \Gamma_b q')_{\alpha\beta} = (-1)^{I+1} (q C \Gamma_b q')_{\beta\alpha} .$$

- Three interpolating currents of Λ_Q baryon:

$$\eta_{\Lambda_Q}^{(P)} = (u C \gamma_5 d) Q, \quad \eta_{\Lambda_Q}^{(A)} = (u C \gamma_5 \gamma_\lambda d) \gamma^\lambda Q ,$$

$$\eta_{\Lambda_Q}^{(S)} = (u C d) \gamma_5 Q \text{ (Vanishes in the heavy – quark limit!).}$$

- Two interpolating currents of Σ_Q baryon:

$$\eta_{\Sigma_Q}^{(I)} = (u C \gamma_\lambda d) \gamma^\lambda \gamma_5 Q, \quad \eta_{\Sigma_Q}^{(T)} = (u \sigma_{\mu\nu} d) \sigma^{\mu\nu} \gamma_5 Q .$$

III. LCSR of heavy-baryon form factors

- Definitions of form factors:

$$\langle \Lambda_Q(P - q) | m_Q \bar{Q} i\gamma_5 u | N(P) \rangle = (m_{\Lambda_Q} + m_N) G(q^2) \bar{u}_{\Lambda_Q}(P - q) i\gamma_5 u_N(P),$$

$$\langle \Lambda_Q(P - q) | \bar{Q} \gamma_\mu u | N(P) \rangle = \bar{u}_{\Lambda_Q}(P - q) \left\{ f_1(q^2) \gamma_\mu + i \frac{f_2(q^2)}{m_{\Lambda_Q}} \sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{m_{\Lambda_Q}} q_\mu \right\} u_N(P),$$

$$\langle \Lambda_Q(P - q) | \bar{Q} \gamma_\mu \gamma_5 u | N(P) \rangle = \bar{u}_{\Lambda_Q}(P - q) \left\{ g_1(q^2) \gamma_\mu + i \frac{g_2(q^2)}{m_{\Lambda_Q}} \sigma_{\mu\nu} q^\nu + \frac{g_3(q^2)}{m_{\Lambda_Q}} q_\mu \right\} \gamma_5 u_N(P).$$

- Introducing vacuum-to-nucleon correlation function:

$$\Pi_a(P, q) = i \int d^4 z e^{iq \cdot z} \langle 0 | T \{ \eta(0), j_a(z) \} | N(P) \rangle.$$

Weak transition current:

$$j_a = \bar{Q} \Gamma_a u, \quad \text{with } \Gamma_a = m_Q i\gamma_5, \gamma_\mu, \gamma_\mu \gamma_5,$$

- Another corrector with on-shell Λ_Q state and interpolating current for nucleon is also possible!

Hadronic dispersion relation: pseudoscalar transition

- Lorenz decomposition of correlator with EOM:

$$\Pi_5^{(i)}(P, q) = \left[\Pi_1^{(i)}((P - q)^2, q^2) + \not{q} \Pi_2^{(i)}((P - q)^2, q^2) \right] i\gamma_5 u_N(P).$$

- Hadronic dispersion relations for invariant amplitudes:

$$\begin{aligned} \Pi_1^{(i)}((P - q)^2, q^2) &= \frac{m_{\Lambda_Q}(m_{\Lambda_Q}^2 - m_N^2)\lambda_{\Lambda_Q}^{(i)}G(q^2)}{m_{\Lambda_Q}^2 - (P - q)^2} \\ &+ \frac{m_{\Lambda_Q^*}(m_{\Lambda_Q^*}^2 - m_N^2)\lambda_{\Lambda_Q^*}^{(i)}\tilde{G}(q^2)}{m_{\Lambda_Q^*}^2 - (P - q)^2} + \int_{s_0^h}^{\infty} ds \frac{\rho_1^{(i)}(s, q^2)}{s - (P - q)^2}, \\ \Pi_2^{(i)}((P - q)^2, q^2) &= -\frac{m_{\Lambda_Q}(m_{\Lambda_Q} + m_N)\lambda_{\Lambda_Q}^{(i)}G(q^2)}{m_{\Lambda_Q}^2 - (P - q)^2} \\ &+ \frac{m_{\Lambda_Q^*}(m_{\Lambda_Q^*} - m_N)\lambda_{\Lambda_Q^*}^{(i)}\tilde{G}(q^2)}{m_{\Lambda_Q^*}^2 - (P - q)^2} + \int_{s_0^h}^{\infty} ds \frac{\rho_2^{(i)}(s, q^2)}{s - (P - q)^2}. \end{aligned}$$

Contributions of higher states with the quantum numbers of $\Lambda_Q^{(*)}$ absorbed into $\rho_{1,2}^{(i)}$.

Eliminating negative-parity baryon contribution

- Each form factor enters more than one dispersion relation.
- Making a linear combination of dispersion relations:

$$\begin{aligned}
 & \frac{m_{\Lambda_Q}(m_{\Lambda_Q} + m_N)(m_{\Lambda_Q} + m_{\Lambda_Q^*})\lambda_{\Lambda_Q}^{(i)}G(q^2)}{m_{\Lambda_Q}^2 - (P - q)^2} \\
 & + \int_{s_0^h}^{\infty} ds \frac{\rho_1^{(i)}(s, q^2) - (m_{\Lambda_Q^*} + m_N)\rho_2^{(i)}(s, q^2)}{s - (P - q)^2} \\
 & = [\Pi_1^{(i)}((P - q)^2, q^2) - (m_{\Lambda_Q^*} + m_N)\Pi_2^{(i)}((P - q)^2, q^2)].
 \end{aligned}$$

containing only the hadronic matrix elements for the ground-state Λ_Q -baryon!

- Quark-hadron duality:

$$\int_{s_0^h}^{\infty} \frac{ds}{s - (P - q)^2} [\rho_1^{(i)}(s, q^2) - (m_{\Lambda_Q^*} + m_N) \rho_2^{(i)}(s, q^2)]$$

$$= \frac{1}{\pi} \int_{s_0}^{\infty} \frac{ds}{s - (P - q)^2} [\text{Im}_s \Pi_1^{(i)}(s, q^2) - (m_{\Lambda_Q^*} + m_N) \text{Im}_s \Pi_2^{(i)}(s, q^2)].$$

- Borelized sum rules for the form factor:

$$G(q^2) = \frac{e^{m_{\Lambda_Q}^2/M^2}}{m_{\Lambda_Q} (m_{\Lambda_Q} + m_N) (m_{\Lambda_Q} + m_{\Lambda_Q^*}) \lambda_{\Lambda_Q}^{(i)}} \frac{1}{\pi} \int_{m_Q^2}^{s_0} ds e^{-s/M^2}$$

$$\times [\text{Im}_s \Pi_1^{(i)}(s, q^2) - (m_{\Lambda_Q^*} + m_N) \text{Im}_s \Pi_2^{(i)}(s, q^2)].$$

- Work out the decay constants $\lambda_{\Lambda_Q}^{(i)}$ with two-point QCD sum rule following the same strategy!

Numerics

- $\Lambda_b \rightarrow p$ transition form factors:

form factors	$\eta_{\Lambda_b}^{(\mathcal{A})}$	$\eta_{\Lambda_b}^{(\mathcal{P})}$
$f_1(0)$	$0.14^{+0.03}_{-0.03}$	$0.12^{+0.03}_{-0.04}$
$f_2(0)$	$-0.054^{+0.016}_{-0.013}$	$-0.047^{+0.015}_{-0.013}$
$g_1(0)$	$0.14^{+0.03}_{-0.03}$	$0.12^{+0.03}_{-0.03}$
$g_2(0)$	$-0.028^{+0.012}_{-0.009}$	$-0.016^{+0.007}_{-0.005}$

Λ_b -baryon form factors are insensitive to the interpolating current.

- Symmetry relations in the heavy-quark limit and large-energy limit:

$$f_1(q^2) = g_1(q^2), \quad f_2(q^2) = g_2(q^2) = f_3(q^2) = g_3(q^2) = 0.$$

Only one universal form factor for $\Lambda_b \rightarrow p, \Lambda$ transitions at large recoil.

- Heavy-to-light baryonic form factors are factorizable in leading power of $1/m_b$ and in leading order of α_s from SCET analysis (arXiv:1112.0237 [hep-ph]).

IV. LCSR for the strong couplings

- Definitions of strong coupling constants:

$$\langle \Lambda_c(P - q) | D(-q) N(P) \rangle = g_{\Lambda_c ND} \bar{u}_{\Lambda_c}(P - q) i\gamma_5 u_N(P),$$

$$\langle \Lambda_c(P - q) | D^*(-q) N(P) \rangle = \bar{u}_{\Lambda_c}(P - q) \left(g_{\Lambda_c ND^*}^V \not{v} + i \frac{g_{\Lambda_c ND^*}^T}{m_{\Lambda_c} + m_N} \sigma_{\mu\nu} \epsilon^{\mu\nu} q^\nu \right) u_N(P).$$

- Heavy-mass relations:

$$g_{\Lambda_c ND} = -g_{\Lambda_c ND^*}^V, \quad g_{\Lambda_c ND^*}^T = 0,$$

$$g_{\Sigma_c ND} + 3g_{\Sigma_c ND^*}^V = \frac{3m_{\Sigma_c} + m_N - 2P \cdot v}{m_{\Sigma_c} + m_N} g_{\Sigma_c ND^*}^T.$$

- Effective Lagrangian for $\Lambda_c - N - D^{(*)}$ couplings:

$$\mathcal{L}_{\Lambda_c D^{(*)} N} = \bar{\Lambda}_c [i a_{\Lambda_c ND} \gamma_5 D + (a_{\Lambda_c ND^*}^V \gamma^\mu + \frac{a_{\Lambda_c ND^*}^T}{m_{\Lambda_c} + m_N} \sigma^{\mu\nu} \partial_\nu) D_\mu^*] N + h. c.$$

New couplings a_i are generally different from g_i !

LCSR for the strong couplings

- Strong couplings enter double dispersion relations for the same correlation function to construct the sum rule of form factors.
- Hadronic double dispersion relation:

$$\begin{aligned} \Pi_5^{(i)}(P, q) &= \frac{\lambda_{\Lambda_c}^{(i)} m_D^2 f_D m_{\Lambda_c} g_{\Lambda_c ND}}{(m_{\Lambda_c}^2 - (P - q)^2)(m_D^2 - q^2)} [(m_{\Lambda_c} - m_N) - \not{q}] i\gamma_5 u_N(P) \\ &+ \frac{\lambda_{\Lambda_c^*}^{(i)} m_D^2 f_D m_{\Lambda_c^*} g_{\Lambda_c^* ND}}{(m_{\Lambda_c^*}^2 - (P - q)^2)(m_D^2 - q^2)} [(m_{\Lambda_c^*} + m_N) + \not{q}] i\gamma_5 u_N(P) \\ &+ \dots, \end{aligned}$$

- Borelized sum rules for the strong couplings:

$$\begin{aligned} g_{\Lambda_c ND} &= \frac{e^{m_{\Lambda_c}^2/M^2} e^{m_D^2/\tilde{M}^2}}{m_{\Lambda_c} (m_{\Lambda_c} + m_{\Lambda_c^*}) m_D^2 f_D \lambda_{\Lambda_c}^{(i)}} \frac{1}{\pi^2} \int_{m_c^2}^{s_0} ds e^{-s/M^2} \\ &\times \int_{t_1(s)}^{t_2(s)} ds' e^{-s'/\tilde{M}^2} \text{Im}_s \text{Im}_{s'} [\Pi_1^{(i)}(s, s') - (m_{\Lambda_c^*} + m_N) \Pi_2^{(i)}(s, s')]. \end{aligned}$$

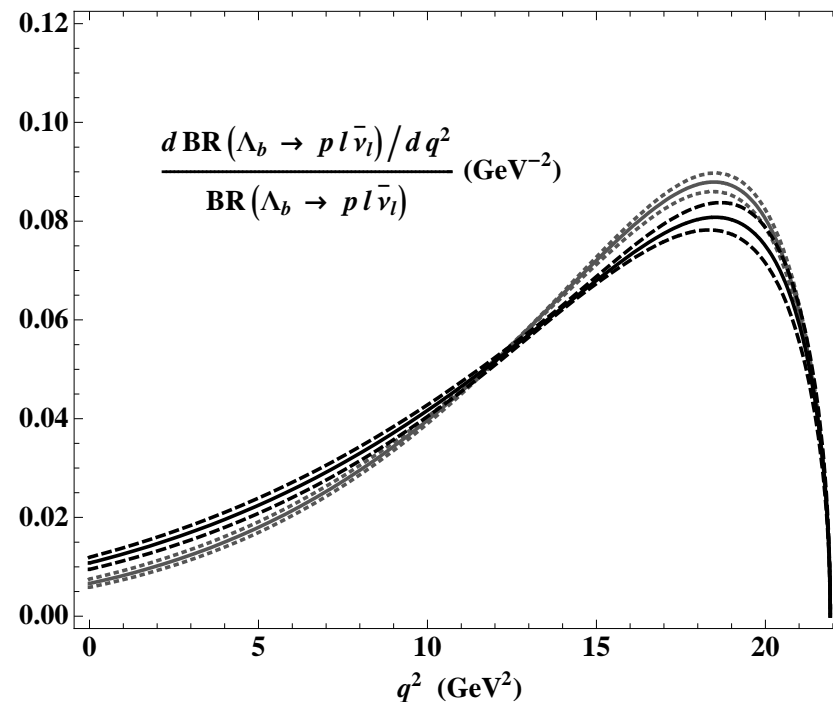
Numerics

Current Strong couplings	$\eta_{\Lambda_c}^{(A)}$ $\Lambda_c ND^*$	$\eta_{\Lambda_c}^{(P)}$ $\Lambda_c ND^*$	$\eta_{\Sigma_c}^{(T)}$ $\Sigma_c ND^*$	$\eta_{\Sigma_c}^{(T)}$ $\Sigma_c ND^*$
$g_{\Lambda_c(\Sigma_c)ND}$	$13.8^{+5.2}_{-4.1}$	$10.7^{+5.3}_{-4.3}$	$1.3^{+1.0}_{-0.9}$	$1.3^{+1.2}_{-0.8}$
$g_{\Lambda_c(\Sigma_c)ND^*}^V$	$-7.9^{+2.7}_{-3.3}$	$-5.8^{+2.1}_{-2.5}$	$1.0^{+1.3}_{-0.6}$	$0.74^{+1.08}_{-0.45}$
$g_{\Lambda_c(\Sigma_c)ND^*}^T$	$4.7^{+2.7}_{-2.0}$	$3.6^{+2.9}_{-1.8}$	$2.1^{+1.9}_{-1.0}$	$1.8^{+1.6}_{-0.8}$

- LCSR predictions of strong coupling constants are insensitive to the heavy-baryon current.
- The heavy-mass relations for the three strong couplings of Λ_c baryon are only qualitatively supported by the LCSR predictions.
- The results for $\Sigma_c ND^{(*)}$ couplings are in good agreement with the heavy mass relation.

V. Applications to exclusive Λ_b decays

- Apply the conformal mapping $q^2 \rightarrow z$ and z -series parametrization to extrapolate the form factors to the whole semileptonic $\Lambda_b \rightarrow pl\nu$ region.
- Normalized differential width of $\Lambda_b \rightarrow pl\nu_\ell$:



The enhancement in the region of large q^2 due to the growth of the form factors and the S -wave phase-space factor $\lambda^{1/2}$.

- Total branching fraction:

$$BR(\Lambda_b \rightarrow pl\nu_l) = \left\{ \begin{array}{l} (3.3_{-1.2}^{+1.5}|_{th.} \pm 0.1|_{exp.}) \\ (4.0_{-2.0}^{+2.3}|_{th.} \pm 0.1|_{exp.}) \end{array} \right\} \left(\frac{|V_{ub}|}{3.5 \cdot 10^{-3}} \right)^2 \times 10^{-4},$$

form factors from LCSR with axial-vector (pseudoscalar) Λ_b current.

About three times of $BR(B^0 \rightarrow \pi^- l^+ \nu_l) = (1.41 \pm 0.05 \pm 0.07) \times 10^{-4}$!

- Branching ratio in factorization limit:

$$BR(\Lambda_b \rightarrow p\pi) = 3.8_{-1.0}^{+1.3} (2.8_{-0.9}^{+1.1}) \times 10^{-6},$$

obtained with the axial-vector (pseudoscalar) Λ_b interpolating current.

Agree with experimental measurement (CDF, 2009):

$$BR(\Lambda_b \rightarrow p\pi) = (3.5 \pm 0.6 \pm 0.9) \times 10^{-6}!$$

Summary

- Heavy baryon form factors and strong couplings are calculated in QCD light-cone sum rule avoiding background pollution.
- Our predictions are less sensitive to the particular choice of baryon currents.
- Heavy-mass relations of form factors and strong couplings are respected by explicit LCSR calculations.
- Differential (integrated) decay width of semileptonic $\Lambda_b \rightarrow pl\nu$ predicted. Potential way to determine $|V_{ub}|$.
- Nonleptonic $\Lambda_b \rightarrow p\pi$ decay computed in the factorization limit is consistent with CDF measurement.
- More applications to rare decays $\Lambda_b \rightarrow \Lambda\gamma$ and $\Lambda_b \rightarrow \Lambda l^+ l^-$ in progress!