Charm-loop effects in $B \to K\ell^+\ell^-$

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Outline

- Introduction
- Separation of scales and $\mathcal{H}^{\text{eff}}_{\Delta B=1}$
- charm-loop in $B \to K\ell^+\ell^-$, $B \to K^*\ell^+\ell^-$
- factorization of matrix elements
- OPE on light cone and sum rules
- Summary

Introduction: puzzle of flavours in SM

Gauge sector in SM are aesthetically nice:
fermionic quanta and gauge principle for interactions

Masses and renormalizability require
Higgs mechanism that is “clumsy”

Fundamental Higgs allows for ad hoc Yukawa couplings

Currently under thorough study at LHC

Existence of three generations allows for explanation of
CP violation within CKM scheme

The whole scheme is rather parameterization of data

Certainly a place for BSM (NP)

Exp vs theory: quarks vs hadron flavours

LHCb experiment

“Rare” B-decays – not so rare anymore

Problem of SM precison check: QCD effects

SM in terms of quarks-qluons $\{q, g\}$ – experiment in terms of hadrons $B, K, \pi ...$

For $B \to K\ell^+\ell^-$ the underlying flavour-changing transition is $b \to s\ell^+\ell^-$

Interface: QCD in strong coupling regime – not treatable at present

Numerically: lattice

Theory tool - separation of scales

Factorization is a key word in analysis of SM
Scale separation through the OPE and notion of effective theories
EW scale of SM is $v = 250\ \mathrm{GeV}$ or in practice M_W, M_Z, m_t
For $\Delta B = 1$ processes at m_b with $\mu \sim E \sim m_b$
OPE allows for control of $\alpha_s^n ln(M_W/m_b)^k$ within pQCD ("integrating out heavy particles")

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

$C_i(\alpha_s, \mu)$ are short-distance pQCD coeffs,
$O_i(\mu)$ - composite local operators
This factorization is of pure PT origin and under control through PT series in $\alpha_s(\mu)$ for $C_i(\alpha_s(\mu))$

SM amplitudes as ME of effective operators

As for EW coupling G_F the SM amplitudes reduce to LO expression in an effective theory with Hamiltonian

$$A(B \to K^{(*)}\ell^+\ell^-) = -\langle K^{(*)}\ell^+\ell^- \mid H_{eff} \mid B\rangle + \mathcal{O}\left(\frac{m_b^2}{M_W^2, m_t^2}\right)$$

Depends on hadronic matrix elements of O_i

Dominant contributions are due to $O_{9,10}$ and $O_{7\gamma}$ with large Wilson coefficients

What are they?

Relevant operators

Tree-level like (with charm fields)

$$O_1 = (\bar{s}_L \gamma_\rho c_L)(\bar{c}_L \gamma^\rho b_L) \quad O_2 = \left(\bar{s}^j_L \gamma_\rho c^i_L\right)\left(\bar{c}^i_L \gamma^\rho b^j_L\right)$$

Through-loop generated: direct $\bar{s}b\bar{l}l$ vertices

$$O_9 = \frac{\alpha_{em}}{4\pi}(\bar{s}_L \gamma_\rho b_L)\left(\bar{l}\gamma^\rho l\right) \quad O_{10} = \frac{\alpha_{em}}{4\pi}(\bar{s}_L \gamma_\rho b_L)\left(\bar{l}\gamma^\rho \gamma_5 l\right)$$

$$O_{7\gamma} = -\frac{e}{16\pi^2}\bar{s}\sigma_{\mu\nu}(m_s L + m_b R) b F^{\mu\nu}$$

“V-A” projectors $L = (1-\gamma_5)/2$, $R = 1 - L$

Contributions of loop operators to $B \to K(K^*)\ell^+\ell^-$ are local and reduce to form factors $f^+_{BK}(q^2)$, $f^T_{BK}(q^2)$, ...

$$A(B \to K\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{\pi} V_{tb} V_{ts}^* p^\mu \times$$

$$\Big[\bar{\ell}\gamma_\mu \ell \Big(C_9 f^+_{BK}(q^2) + \frac{2(m_b + m_s)}{m_B + m_K} C_7^{eff} f^T_{BK}(q^2) \Big)$$

$$+ \bar{\ell}\gamma_\mu\gamma_5 \ell C_{10} f^+_{BK}(q^2) \Big]$$

For example, tensor $B \to K$ form factor is defined as

$$\langle K(p)|\bar{s}\sigma_{\mu\rho} q^\rho b|B(p+q)\rangle$$

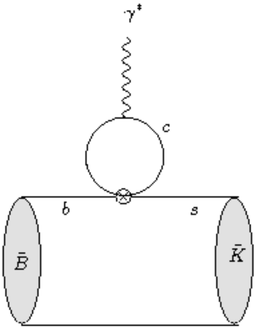
$$= \Big[q^2(2p_\mu + q_\mu) - (m_B^2 - m_K^2) q_\mu \Big] \frac{i f^T_{BK}(q^2)}{m_B + m_K} .$$

Require nonPT methods but they are local...

There appear still more complicated objects

Charm loops picture

Four-quark charm operators O_1 and O_2 lead to charm-loop interactions of the form



as top quark has but they cannot be integrated out as the scale m_c is small. The amplitude is not local

Charm-loops amplitude

Indeed, contribution to the $A(B \to K\ell^+\ell^-)$ reads

$$A^{O_{1,2}} = -(4\pi\alpha_{em}Q_c)\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{\bar{\ell}\gamma^\mu\ell}{q^2}\mathcal{H}_\mu^{(B\to K)}(p,q)$$

with $Q_c = 2/3$ and

$$\mathcal{H}_\mu^{(B\to K)}(p,q) = i\int d^4x e^{iqx} \times$$

$$\langle K(p)|T\bar{c}\gamma_\mu c(x)\Big[C_1 O_1(0) + C_2 O_2(0)\Big]|B(p+q)\rangle$$

Charm-loops amplitude essence

Key quantity

$$T_\mu = TO_1(0)J_\mu(x) = T\bar{c}\gamma_\mu c(x)O_i(0)$$

is a nonlocal amplitude that is expanded on LC

$$TO_1(0)J_\mu(x) = \bar{s}\Gamma b \otimes C(x) + \bar{s}Gb \otimes C_G(x) + ...$$

At LO in α_s and x, coeff C is given by two-point correlator

$$C \to i\int d^4x e^{iq\cdot x}\bar{c}\gamma_\mu c(x)J_\mu(0)$$

It can be computed at $q^2 \ll 4m_c^2$ that leads to fact approximation

Charm-loops amplitude at LO = factorization

At LO the amplitude

$$\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)|_{fact} = \left(\frac{C_1}{3} + C_2\right)\langle K^{(*)}(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle$$

where both O_1 and O_2 contribute and the local operator

$$\mathcal{O}_\mu(q) = (q_\mu q_\rho - q^2 g_{\mu\rho})\frac{9}{32\pi^2}\, g(m_c^2, q^2)\bar{s}_L\gamma^\rho b_L$$

reduces to the $b \to s$ current that factorizes ME to $B \to K^{(*)}$ form factors.

Factorized charm loop

The charm-loop coefficient function is given by the well-known expression:

$$g(m_c^2, q^2) = -\frac{8}{9}\ln\left(\frac{m_c}{m_b}\right) + \frac{8}{27} + \frac{4}{9}y$$

$$-\frac{4}{9}(2+y)\sqrt{y-1}\,\arctan\left(\frac{1}{\sqrt{y-1}}\right)$$

where $y = 4m_c^2/q^2 > 1$ and $\mu = m_b$
dispersion relation in the variable q^2 is valid with

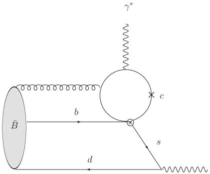
$$\frac{1}{\pi}\mathrm{Im}_s g(m_c^2, s) = \frac{4}{9}\sqrt{1-\frac{4m_c^2}{s}}(1+\frac{2m_c^2}{s})\Theta(s-4m_c^2)$$

The factorizable amplitude is often called “perturbative” or “short-distance” charm-loop effect.

NonFactorized charm loop picture

Corrections to factorization are both PT and nonPT.
Even NLO PT corrections violate factorization.
I discuss the nonPT soft gluon corrections

A.Khodjamirian,Th.Mannel,AAP,Y.-M.Wang, JHEP09(2010)089



Here $B \to K^{(*)}$ matrix element contains soft-gluon emission from the charm loop.
The c-quark loop with the emitted gluon generates the nonlocal effective operator $\widetilde{O}_\mu$

NonFactorized charm loop

One casts the soft-gluon emission part to a form

$$\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)|_{nonfact} = 2C_1\langle K^{(*)}(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle$$

where $\widetilde{\mathcal{O}}_\mu(q)$ is a convolution of the coefficient function with the nonlocal operator

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q,\omega)\bar{s}_L\gamma^\rho\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L$$

In fact this matrix element resembles a nonforward distribution with different initial and final hadrons.

Spectral density

The coeff function is given by its spectral density

$$\frac{1}{\pi}\mathrm{Im}\, I_{\mu\rho\alpha\beta}(q,\omega) = \frac{m_c^2\Theta(\tilde{q}^2 - 4m_c^2)}{4\pi^2\tilde{q}^2\sqrt{\tilde{q}^2(\tilde{q}^2 - 4m_c^2)}}$$

$$\int_0^1 du\Big\{\bar{u}\tilde{q}_\mu\tilde{q}_\alpha g_{\rho\beta} + u\tilde{q}_\rho\tilde{q}_\alpha g_{\mu\beta} - \Big[u + \frac{(\bar{u}-u)\tilde{q}^2}{4m_c^2}\Big]\tilde{q}^2 g_{\mu\alpha}g_{\rho\beta}\Big\}$$

with $\tilde{q} = q - u\omega n_-$, so that $\tilde{q}^2 \simeq q^2 - 2u\omega m_b$

Non-factorizable amplitude $\tilde{A}$

Now the amplitude reads

$$\mathcal{H}_\mu^{(B\to K)}(p,q) = \left(\frac{C_1}{3} + C_2\right)\langle K(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle$$

$$+2C_1\langle K(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle = \left[(p\cdot q)q_\mu - q^2 p_\mu\right]\mathcal{H}^{(B\to K)}(q^2)$$

and the scalar part is

$$\mathcal{H}^{(B\to K)}(q^2) = \left(\frac{C_1}{3} + C_2\right)A(q^2) + 2C_1\tilde{A}(q^2)$$

with $\tilde{A}$ being soft-gluon amplitude
$C_1 = 1.12$, $C_2 = -0.27$ that enhances $\tilde{A}$

LC sum rules

To compute $\tilde{A}$ determining the soft-gluon emission we employ LCSR with the B-meson DA's

Consider correlation function

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = i\int d^4y e^{ip\cdot y}\langle 0|T\{j^K_\nu(y)\widetilde{\mathcal{O}}_\mu(q)\}|B(p+q)\rangle\,,$$

with $j^K_\nu = \bar{d}\gamma_\nu\gamma_5 s$ and extract a residue

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = \frac{if_K p_\nu}{m_K^2 - p^2}[(p\cdot q)q_\mu - q^2 p_\mu]\tilde{A}(q^2) + cont$$

where f_K is the kaon decay constant and $cont$ accumulates higher mass states with the kaon quantum numbers, located above the threshold s_h

B-meson DA decomposition

B-meson DA is decomposed in HQET through Ψ_A, Ψ_V, X_A, Y_A as

$$\langle 0|\bar{d}_\alpha(y)\delta[\omega - \frac{(in_+\mathcal{D})}{2}]G_{\sigma\tau}(0)b_\beta(0)|\bar{B}(v)\rangle$$

$$= \frac{f_B m_B}{2}\int d\lambda e^{-i\lambda yv}\Big[(1+\not{v})\Big\{(v_\sigma\gamma_\tau - v_\tau\gamma_\sigma)\big[\Psi_A - \Psi_V\big]$$

$$-i\sigma_{\sigma\tau}\Psi_V - \frac{y_\sigma v_\tau - y_\tau v_\sigma}{v\cdot y}X_A + \frac{y_\sigma\gamma_\tau - y_\tau\gamma_\sigma}{v\cdot y}Y_A\Big\}\gamma_5\Big]_{\beta\alpha}$$

where f_B and m_B are the B-meson decay constant and mass

B-meson DA model

Model DA for B-meson

$$\Psi_A(\lambda,\omega) = \Psi_V(\lambda,\omega) = \frac{\lambda_E^2}{6\omega_0^4}\omega^2 e^{-(\lambda+\omega)/\omega_0}$$

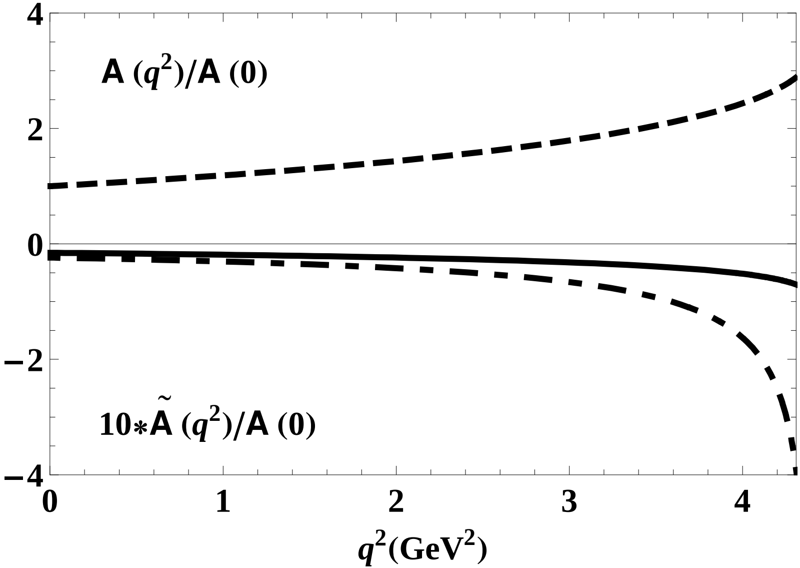
$$X_A(\lambda,\omega) = \frac{\lambda_E^2}{6\omega_0^4}\omega(2\lambda-\omega)e^{-(\lambda+\omega)/\omega_0}$$

$$Y_A(\lambda,\omega) = -\frac{\lambda_E^2}{24\omega_0^4}\omega(7\omega_0 - 13\lambda + 3\omega)e^{-(\lambda+\omega)/\omega_0}$$

Here $\omega_0 = 1/\lambda_B$ of the *B* meson two-particle DA ϕ_+^B

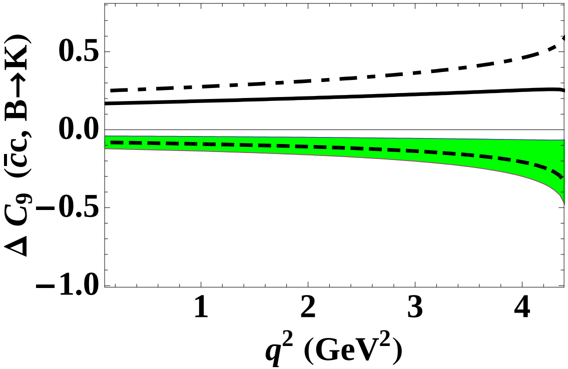
Normalization of the three-particle DA's is $\lambda_E^2 = 3/2\lambda_B^2$

Nonfactorizable $B \to K$ amplitude $\widetilde{A}(q^2)$ (solid)



and its local limit (not LC resummed) (dash-dotted), together with the factorizable amplitude $A(q^2)$ (dashed).

Results for C_9



$\Delta C_9 = (C_1 + +3C_2)g(m_c, q) +2C_1\tilde{g};$

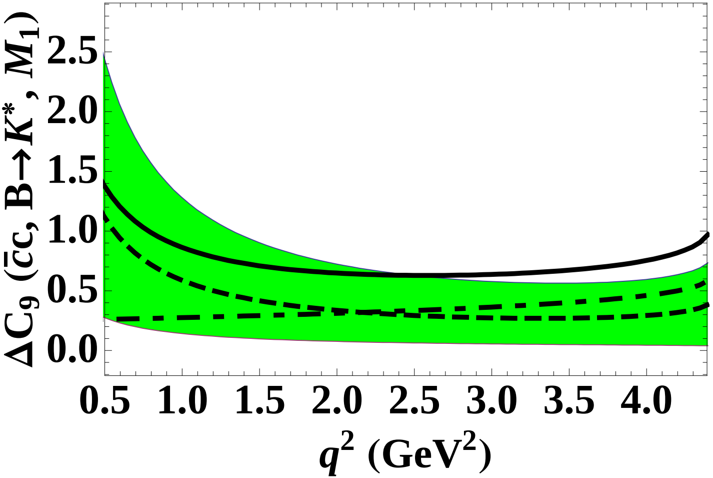
$\tilde{g} = \tilde{A}/f^+_{BK}$

$C_9 = 4.2$

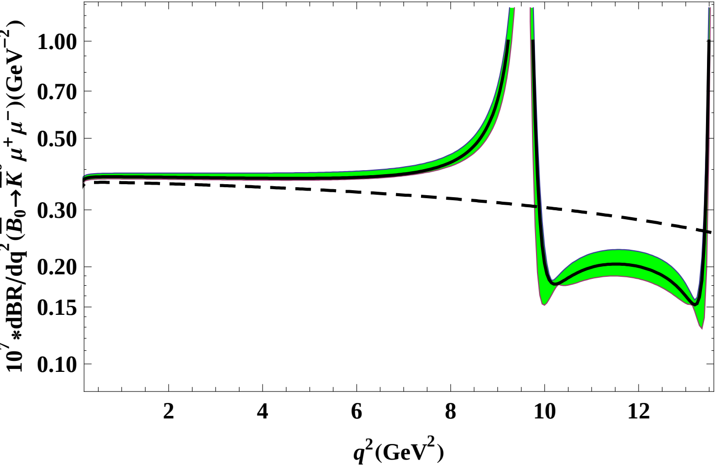
Charm-loop effect in $B \to K\ell^+\ell^-$ as a correction to the Wilson coefficient C_9 (solid),
nonfactorizable soft-gluon contribution — dashed
factorizable contribution – dash-dotted

Results for K^*

Coeff C_9 for amplitude $\mathcal{M}_1$ to the K^* decay

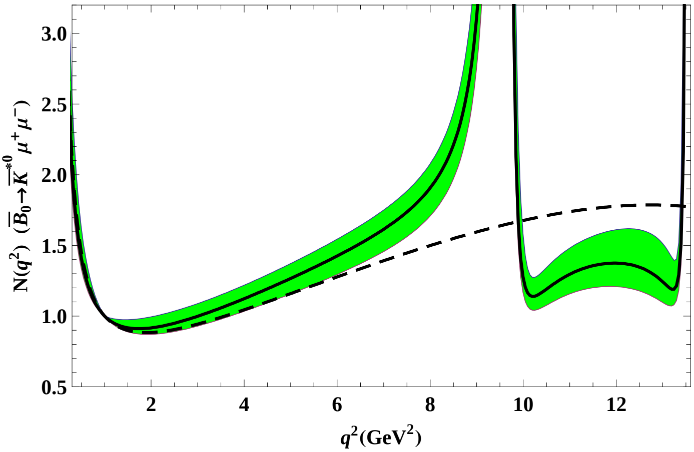


total – solid, soft-gluon – dashed
factorizable – dash-dotted

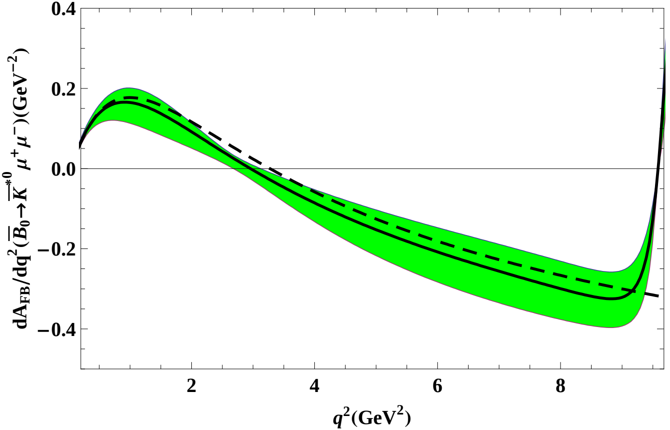


differential width of $\bar{B}_0 \to \bar{K}\mu^+\mu^-$, including the charm-loop effect (solid) and without this effect (dashed).

The differential width of $\bar{B}_0 \to \bar{K}^* \mu^+ \mu^-$ with charm-loop effect (solid) without this effect (dashed)



$q_0^2 = 2.9 \pm 0.3$ while without soft-gluon $q_0^2 = 3.2$



Forward-backward asymmetry for $\bar{B}_0 \to \bar{K}^* \mu^+ \mu^-$ decay with charm-loop effect (solid), without this effect (dashed).

Summary

For c-quark loop contribution:

- soft-gluon emission violating factorization taken into account: OPE near LC.
- LCSR with B-meson DA to calculate ME of dim 5 non-local operator. Soft-gluon contribution is enhanced by Wilson coefficient for $B \to K^* \ell^+ \ell^-$ and numerically important
- suppression of the soft-gluon contribution by $\sim 1/(4m_c^2 - q^2)$ signals that the approximation is valid at $q^2 \ll 4m_c^2$. Near $\bar{c}c$-threshold, multiple soft-gluon emission operators have to be included and one eventually looses control over OPE.