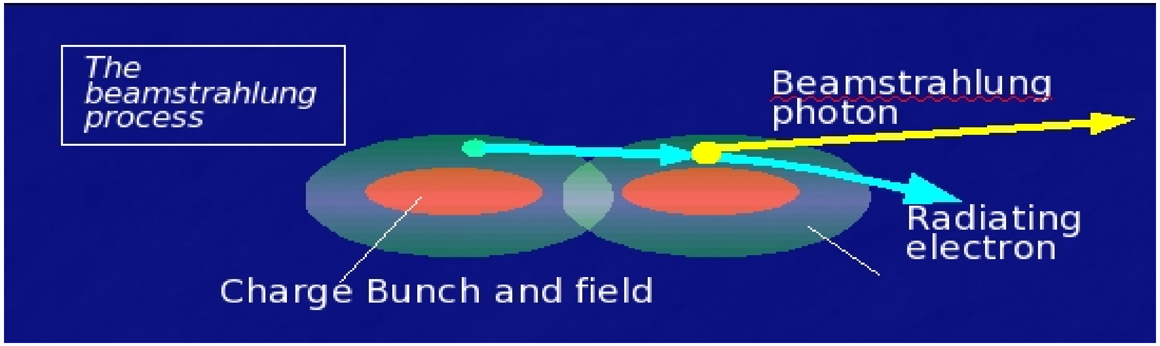
W pair production in the beam-beam field

Tony Hartin, DESY FLC, 7 December 2011

- What is a "strong field" and will there be one at ILC/CLIC?
- The "strong field" processes in the beam-beam interaction
- The "lowest order" spin tracking
- The Furry interaction picture and exact solutions
- W pair production in an external field
- -> polarization measurement uncertainty

Beam-beam processes



- Incoherent: Equivalent Photon Approximation,
  - Breit-Wheeler, Bethe-Heitler, Landau-Lifshitz, Bremstrahlung
- Coherent: Strong field (Furry picture)
  - Beamstrahlung, 1 photon pair production, AMM, higher orders
- Simulation tools: CAIN2.35 **CAIN2.35.x, GP**, GP++

Strong field parameter sets

Initial parameters

Parameter	ILC SB2011 10%BS	CLIC 04/2010
$\sqrt{s}$ (TeV)	1±1%	3±1%
N_b	$2x10^{10}$	$3.72x10^{9}$
σ_x (μm)	0.429	0.045
σ_y (μm)	0.002	0.001
σ_z (μm)	150	44
$\gamma\varepsilon_x$ (nm rad)	100	660
$\gamma\varepsilon_y$ (nm rad)	30	20
E^+ pol (%)	30±0.05	30±0.05
E^- pol (%)	80±0.2	80±0.2

simulated parameters

Parameter	CAIN	Guineapig
ILC SB2011 10%BS 1TeV		
Lumi ($x10^{34}$)	3.98	5.09
N(incoh)	389565	430361
N(coherent)	0	0
Υ(ave)	0.27	0.3
Υ(max)	0.94	0.7435
CLIC 2010 3TeV		
Lumi ($x10^{34}$)	3.6	2.4
N(incoh)	381246	196819
N(coherent)	$2.5x10^{8}$	$4x10^{8}$
Υ(ave)	3.34	
Υ(max)	10.9	11.9

IP beam-beam depolarisation

Spin-flip process

Constant crossed field – Airy functions

Spin vector

beam field tensor

$$W(\Upsilon,\boldsymbol{\xi}) = \frac{\alpha m_e^2}{\pi\epsilon}\int_0^\infty \frac{du}{(1+u)^3}\left[\frac{e}{m^3}F^{*\mu\nu}p_\mu s_\nu \frac{z\mathrm{Ai}(z)}{1+u} - \mathrm{Ai}_1(z) - \frac{2+2u+u^2}{z\,(1+u)}\mathrm{Ai}'(z)\right]$$

Spin precession

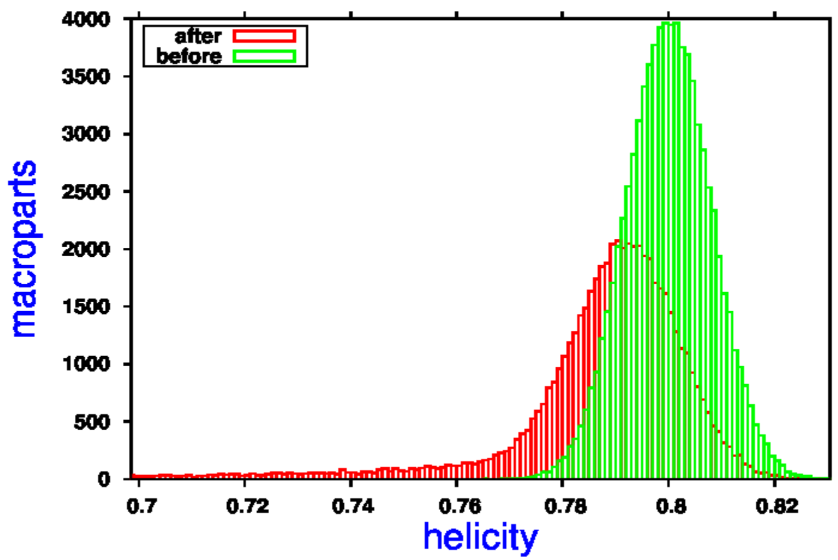
Beam field strengths

Spin vector

$$\frac{d\boldsymbol{s}}{dt} = -\left[(a+\gamma^{-1})(\boldsymbol{B}-\boldsymbol{v}\times\boldsymbol{E}) - \boldsymbol{v}\frac{a\gamma}{\gamma+1}\boldsymbol{v}\cdot\boldsymbol{B}\right]\times\boldsymbol{s}$$

Anomalous magnetic moment in external field

Sim starting helicity 0.8 vector



Parameter Set	Final ⟨helicity⟩	⟨depol⟩	Lumi weighted ⟨depol⟩
ILC SB2011	0.7818	2.27%	0.62%
CLIC 3TeV	0.69741	12.8%	3.5%

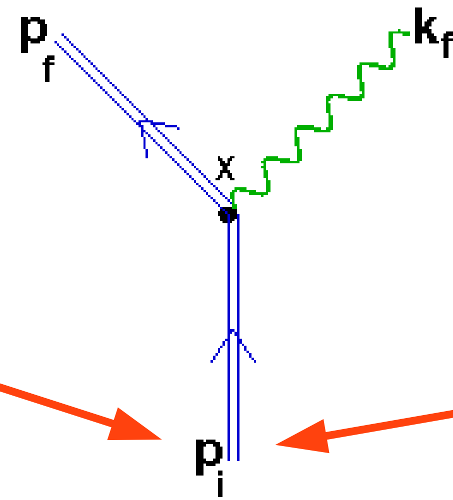
Strong field parameters Υ and χ

Vector potential of beam field

$$\Upsilon = \frac{e|a|}{m E_{cr}} (k.p_i)$$

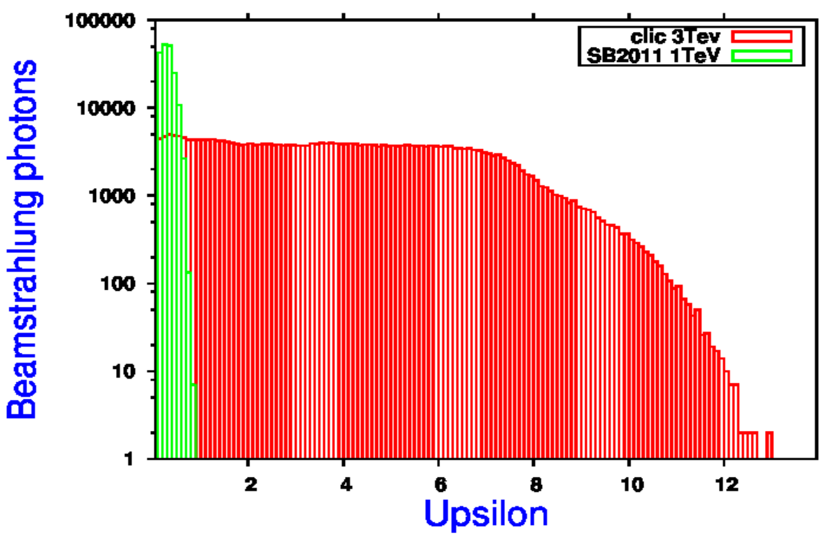
Schwinger critical field (10^{18} V/m)

momentum of beam field



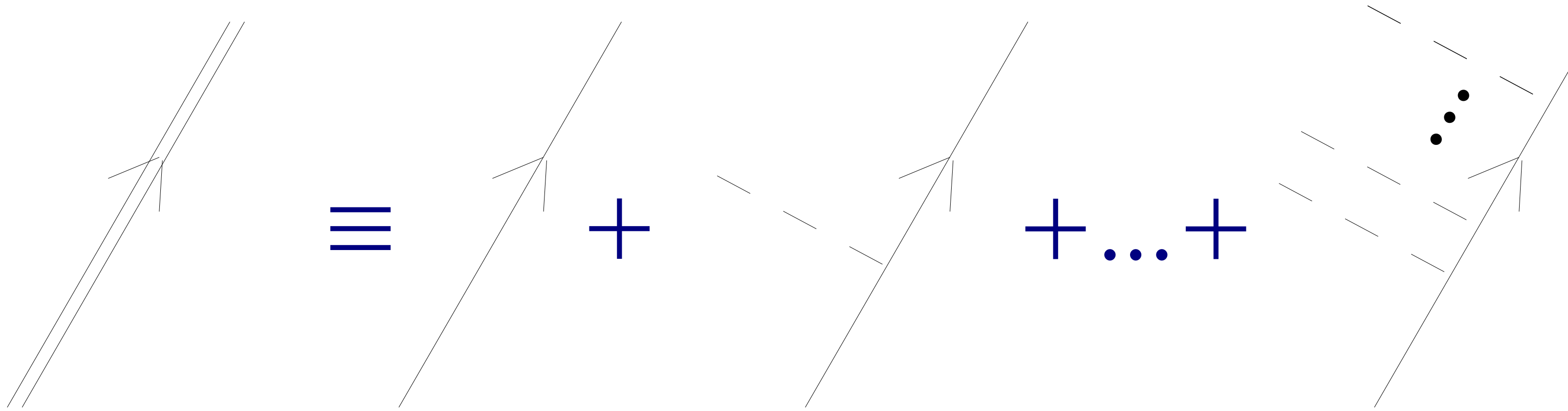
$$\chi = \frac{\omega_f}{E}$$

Ratio of emitted energy to incoming energy



- $\chi \ll 1$ **Quasi-classical method** of Baier-Katkov. Classical electron dynamics
- $\Upsilon \ll 1$ **Born approximation.** (Equivalent Photon Approx)
- $any\ \ \chi, \Upsilon$ **Furry picture.** Exact interaction with the bunch field

Furry Picture (for coherent processes)



All Feynman diagrams with double fermion lines are potential coherent processes

Exact (Volkov) solution for Dirac equation in a plane wave em field-

$$\psi^V = \left[1 + \frac{e}{2(k\cdot p)} \not{k} \not{A}^e\right] \exp\left(-i\int_0^{kx} \left[\frac{e(A^e\cdot p)}{(k\cdot p)} - \frac{e^2 A^{e2}}{2(k\cdot p)}\right] d\phi\right) \exp(-ip\cdot x)\, u_s(p)$$

Υ,χ dependent part including extra dirac matrices

An additional phase factor (Υ,χ)

The usual free fermion part

Furry picture modified vertex

Interaction Lagrangian in Furry picture constitutes a modified vertex

$$\mathcal{L}_\mathrm{I} = \bar{\psi}^V \gamma^\mu \psi^V A_\mu \equiv \bar{\psi}\, \gamma^{e\,\mu}(p,p',k\cdot x)\, \psi\, A_\mu$$

$$\psi^V \equiv E_p\, u_{p,s}\, e^{-ip\cdot x}$$

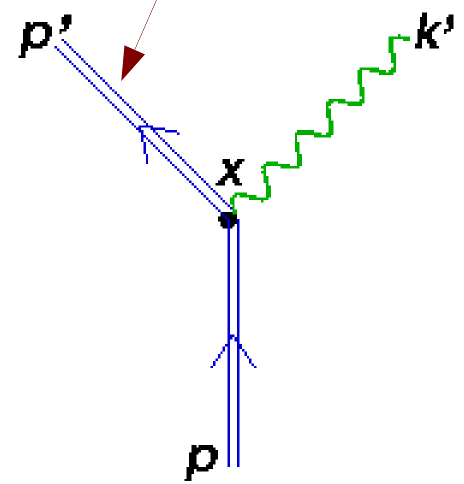
external field 4-momentum

$$\gamma^e_\mu(p,p',k\cdot x) \equiv \bar{E}_p(k\cdot x)\gamma_\mu E_{p'}(k\cdot x)$$

Transform the modified vertex to momentum space to get a contribution from external field photons

$$\gamma^e_\mu(p,p',k\cdot x) = \int_{-\infty}^{\infty} dr d\phi\, e^{-ir(k\cdot x - \phi)} \gamma^e_\mu(p,p',\phi)$$

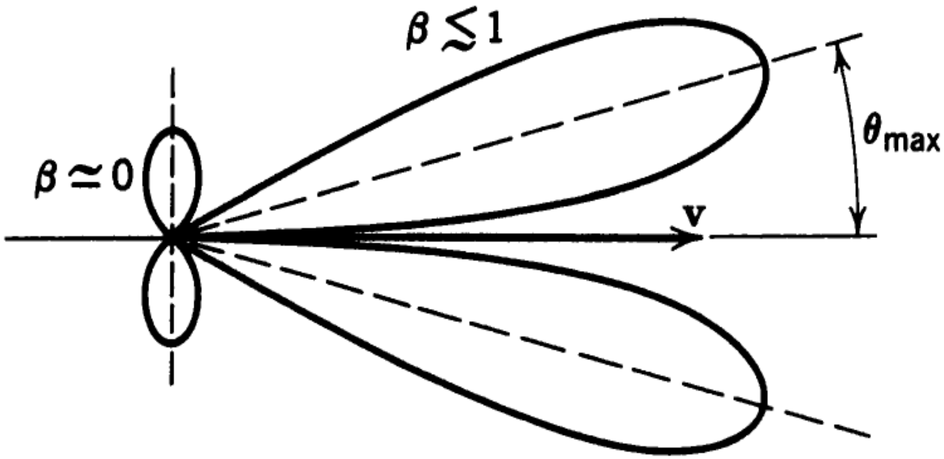
Volkov fermions represented by double lines



$$\delta(p + rk - p' - k')$$

- **New processes possible**
- **Existing processes modified**

Furry picture vertex in beam field

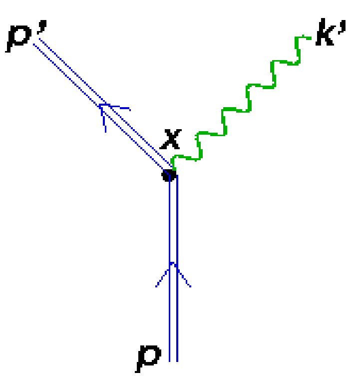


Relativistic charge emits radiation in forward direction, can be represented by constant crossed field

Temporal gauge

$$A^e_\mu = (0, \vec{a})(k \cdot x)$$

$$\gamma^e_\mu(p,p',k') = \int_{-\infty}^{\infty} dr \left[1 - i\frac{\Upsilon}{E}\hat{\not{a}}\,\hat{\not{k}}\frac{d}{dr}\right]\gamma_\mu\left[1 - i\frac{\Upsilon}{E}\hat{\not{a}}\,\hat{\not{k}}\frac{d}{dr}\right]\mathrm{Ai}(r)\,\delta\!\left(p - \frac{\Upsilon}{E}^{1/3} r k - p' - k'\right)$$



Υ ~ 1-10

E ~ 10^3-10^6

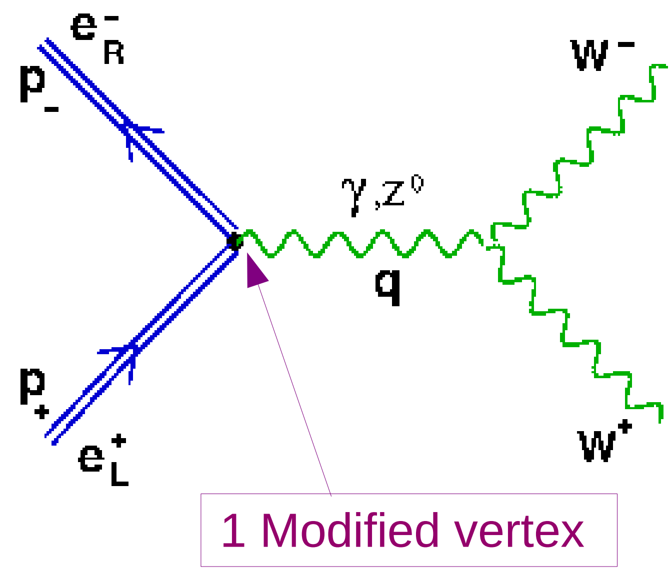
so Υ/E can be neglected

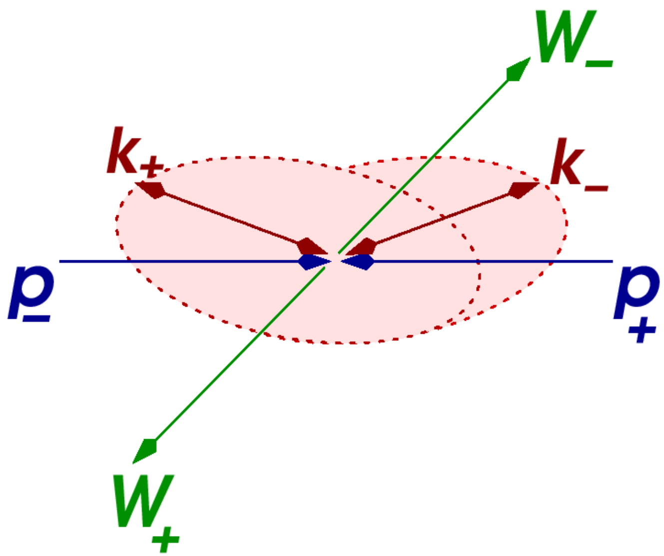
$$\gamma^e_\mu(p,p',k') = \int_{-\infty}^{\infty} dr\;\gamma_\mu\;\mathrm{Ai}(r)\;\delta\!\left(p - \frac{\Upsilon}{E}^{1/3} r k - p' - k'\right)$$

W pair production ext field – S channel

$$iM_{fi} = \frac{ie^2}{(p^- + p^+)^2}\,\bar{v}_{p^+}\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)u_{p^-} \times \text{final states}$$

$$iM^e_{fi} = \int dr\,\frac{ie^2}{(p^- + p^+ + rk)^2}$$
$$\times\,\bar{v}_{p^+}\left[1 - \frac{eA\!\!\!/ k\!\!\!/}{2(k\cdot p^+)}\right]\gamma^\mu\left[1 - \frac{eA\!\!\!/ k\!\!\!/}{2(k\cdot p^-)}\right]\left(\frac{1+\gamma^5}{2}\right)u_{p^-}$$
$$\times\,\text{extra phase} \times \text{final states}$$





- There are 2 external fields
- Assume k_+, k_-, p_+, p_- collinear – neglect crossing angle
- Exact solutions possible for the final W states as well (Obukhov et al, Rus Phys J, **30**, 398)
- There is a finite bound on the r integration – can only give up so much energy to the field

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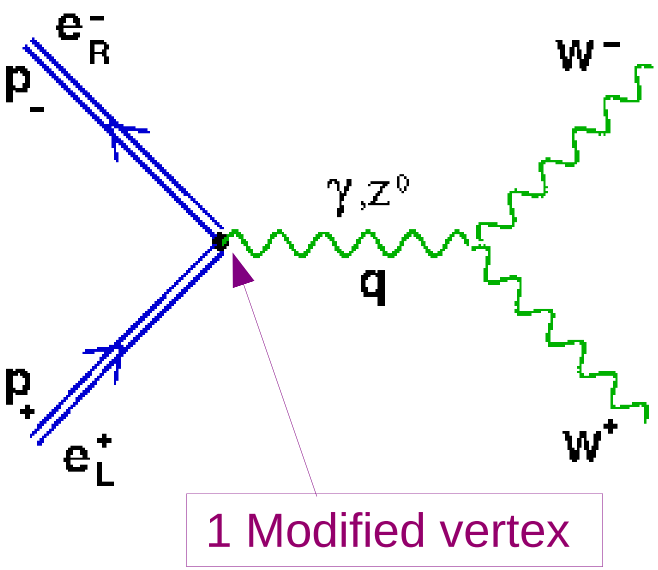
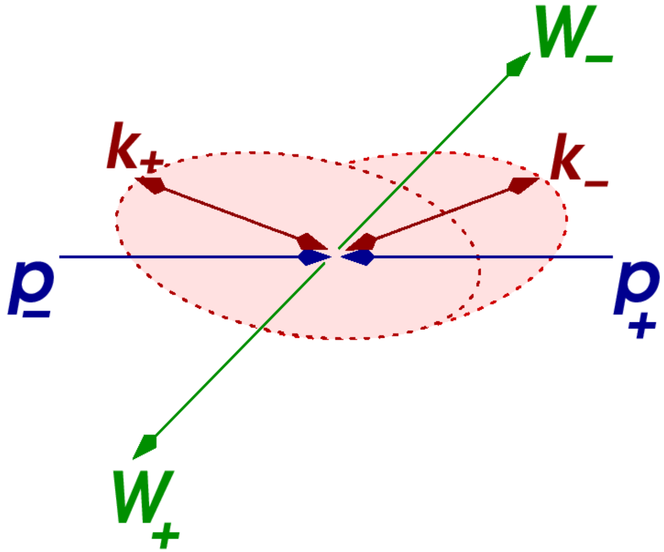
$$iM^e_{fi} = \int dr\, \frac{ie^2}{(p^- + p^+ + rk)^2}$$
$$\times\, \bar{v}_{p^+}\left[1 - \frac{e\not{A}\not{k}}{2(k\cdot p^+)}\right]\gamma^\mu\left[1 - \frac{e\not{A}\not{k}}{2(k\cdot p^-)}\right]\left(\frac{1+\gamma^5}{2}\right)u_{p^-}$$
$$\times\, \text{extra phase} \times \text{final states}$$





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W pair prod ext field – Phase Integral

Need an "auxillary" center of mass reference frame

$$\boldsymbol{p}_- + \boldsymbol{p}_+ - \frac{\Upsilon}{E}^{1/3} r(\hat{\boldsymbol{k}}_- + \hat{\boldsymbol{k}}_+) = 0$$

to integrate over r consistently we also need transformation back to "normal" CM frame

The Phase integral:

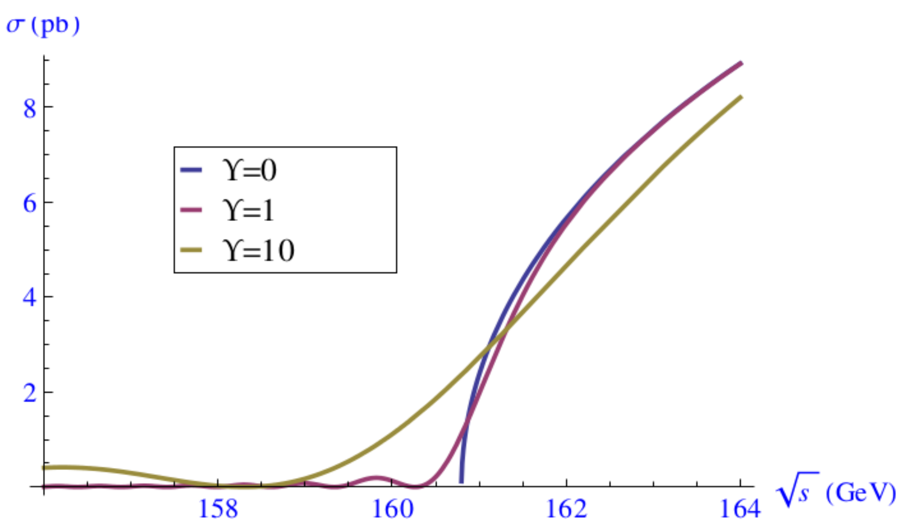
$$\int \frac{d\boldsymbol{W}_-}{2\omega^-}\frac{d\boldsymbol{W}_+}{2\omega^+}\,\delta(p_- + p_+ - \frac{\Upsilon}{E}^{1/3} r(k_- + k_+) - W^- - W^-)$$

$$= \int \frac{d\Omega}{8}\sqrt{1 - \frac{m_W^2}{(E - \frac{\Upsilon}{E}^{1/3} r)^2}}$$

$$\int \frac{d\Omega}{8}\sqrt{1 - \frac{m_W^2}{E^2}}$$

Compare with non-external field
W pair prod phase integral

Cross-section is invariant with respect to longitudinal boosts



W pair production - polarisation measurement

<u>Polarisation measurement via Blondel scheme</u>

$$| P_{e^\pm} | = \sqrt{\frac{(\sigma_{-+} + \sigma_{+-} - \sigma_{--} - \sigma_{++})(\pm\sigma_{-+} \mp \sigma_{+-} + \sigma_{--} - \sigma_{++})}{(\sigma_{-+} + \sigma_{+-} + \sigma_{--} + \sigma_{++})(\pm\sigma_{-+} \mp \sigma_{+-} - \sigma_{--} + \sigma_{++})}},$$

Beam polarization is in terms of helicity cross-sections

Require helicity amplitudes which are <u>not</u> Lorentz invariant

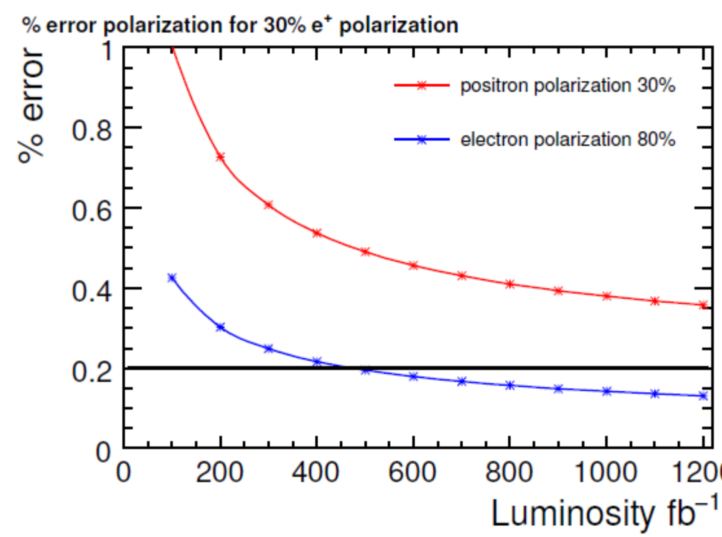
Need transformation auxillary CMA frame to "normal" CM frame

$$\text{CM frame} \quad \boldsymbol{p}_- + \boldsymbol{p}_+ = 0$$

$$\text{CMA frame} \quad \boldsymbol{p}_- + \boldsymbol{p}_+ + r\hat{\boldsymbol{k}}_- + s\hat{\boldsymbol{k}}_+ = 0$$

$$\gamma_{\text{CMA}} = 1/\sqrt{1 - \frac{(r-s)^2}{(2E+r+s)^2}}$$

Bechtle, Ehrenfeld, Marchesini LC-DET-2009



A new simulation package?

To simulate higher order processes (W pair prod)

REQUIREMENTS:

To simulate a charge bunch collision and calculate the field strength at each point of production

To have a finely scaled simulation in order to accurately model disruption, hour glass effect etc.

To perform a relatively complex cross-section calculation at each point of production

To have full spin tracking

To be flexible enough to include new higher order processes

SOLUTION:

A PIC code using the discontinuous Galerkin method for modeeling the electrodynamics – crosscheck with CAIN/GP

GPU programming using pyCUDA

T-BMT with higher order corrections to AMM, Sokolov-Ternov and higher order helicity amplitudes

Allow new processes to be loaded externally

Summary

- A strong field is indicated by $\Upsilon>1$
- Strong field leads to lumi-weighted depolarisation L_{depol}
- ILC 1 Tev has Υ->1 ,L_{depol}=0.6% . CLIC 1 Tev has Υ->10 ,L_{depol}=3.5%
- Furry picture puts ext field in Dirac Lagrangian with exact solutions
- Can obtain a modified vertex γ^e for use in Feynman diagrams
- W pair production in external field has one γ^e in s-channel and two γ^e in t-channel
- An auxillary reference frame and transformation back to CM frame is required for phase integral
- W pair production (in strong beam field) will be used to measure L_{depol} – detailed simulation required!
- A new beam-beam simulation package will be developed – cross-checks with existing packages