A Hough Transformation for Track Finding in Time Projection Chambers (PATHFINDER)

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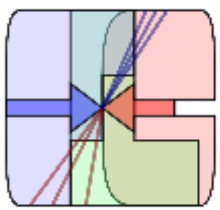


Pattern Recognition with Hough Transformation

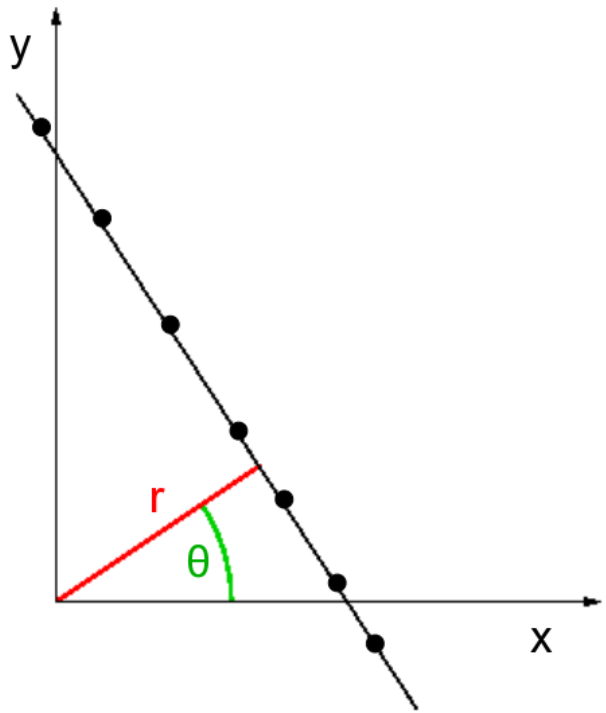
- Global method: all hits enter into the algorithm at the same time and in the same way.
- Works for any track shape which can be described by one set of parameters.
- Here: either straight line (without magnetic field) or helix (with magnetic field).
- The software package is called PATHFINDER (**PA**ckage for **T**racking by **H**ough trafo **FIND**ing including a chisquare fitt**ER**)

Hough Transformation: Straight Lines

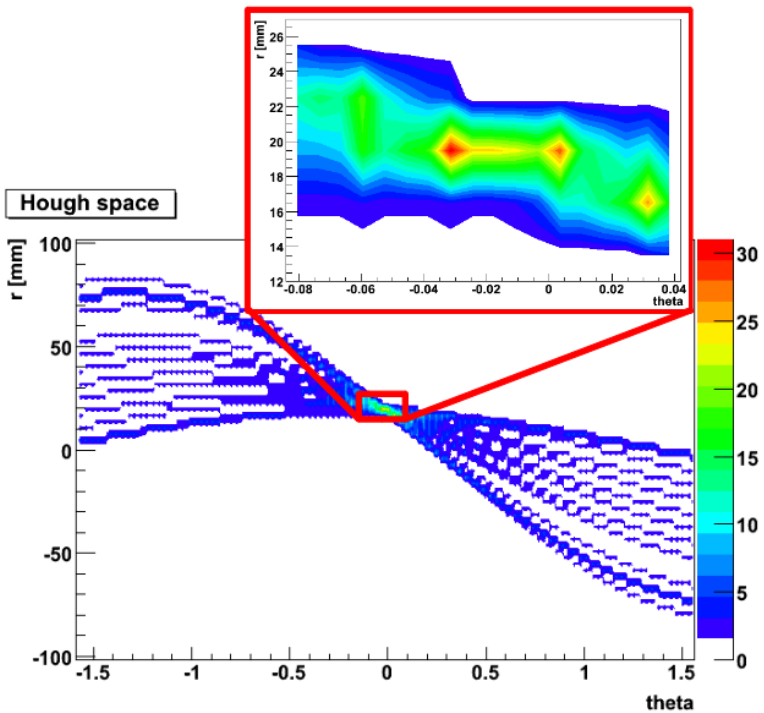
- Track Model: straight line
- Two parameters needed to describe straight line. Here: r, θ

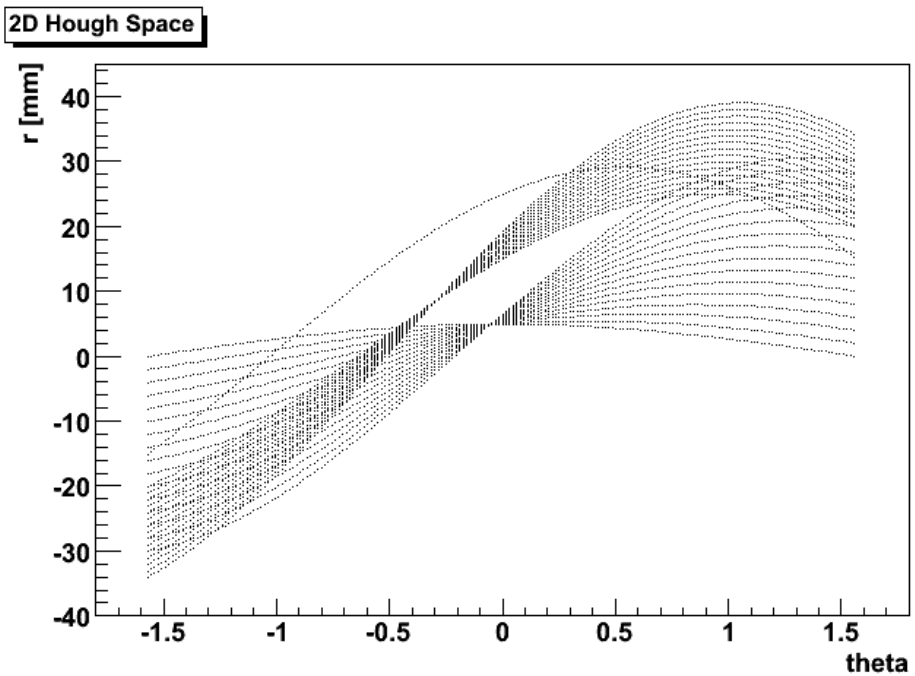
$$\begin{aligned} y(x) &= -\frac{\cos\theta}{\sin\theta} \cdot x + \frac{r}{\sin\theta} \\ \Rightarrow r(\theta) &= \cos\theta \cdot x + \sin\theta \cdot y \end{aligned}$$

- Hit positions are inserted in the function $r(\theta)$.
- If the hits are on a straight line the functions belonging to these hits intersect in one point in 2D Hough Space.
- The point of intersection delivers the parameters of the straight line.



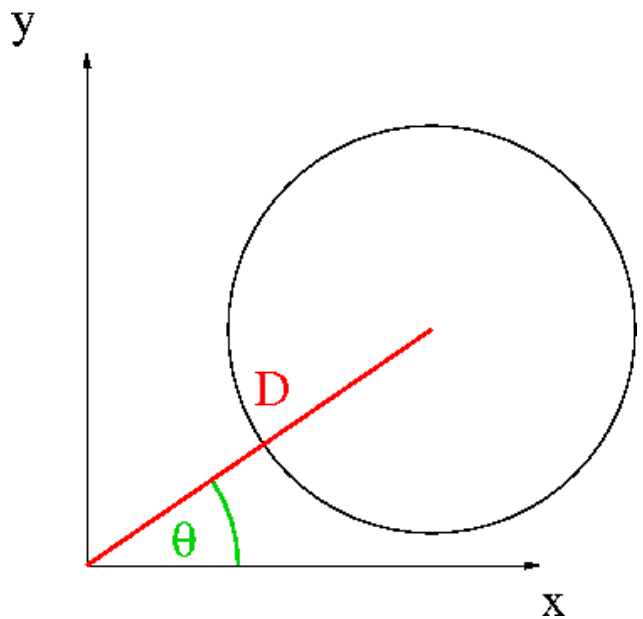
Hough Transformation: Straight Lines





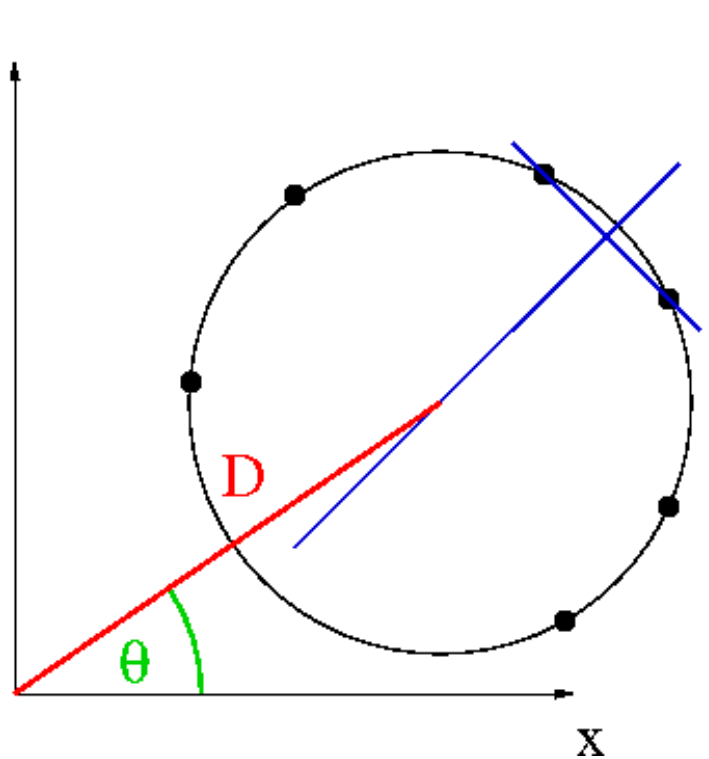
Hough Transformation: Circles

- Do the search for circles in two steps:
  - Search center of circle: 2D Hough Transformation.
  - Search radius: 1D problem.
- Parametrization of center of circle:
  - D: Distance between origin and center of circle.
  - θ: Angle between x-axis and the direction of the center of the circle.



Hough Transformation: Circles

- Two hit positions in the xy-plane: (x_1, y_1), (x_2, y_2).
- Construct a straight line between two hits and calculate the point (x_h, y_h) half way between the hits on the straight line.
- A straight line between the center of the circle and (x_h, y_h) is perpendicular to the straight line between (x_1, y_1) and (x_2, y_2).
- When the center of the circle is found, calculate the distance of each hit to the center and then find the radius of the circle.



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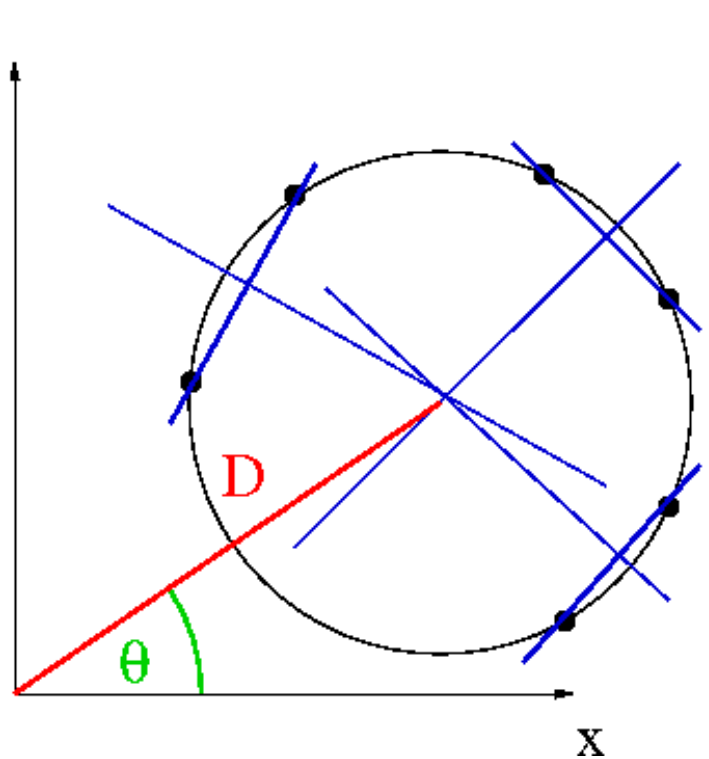

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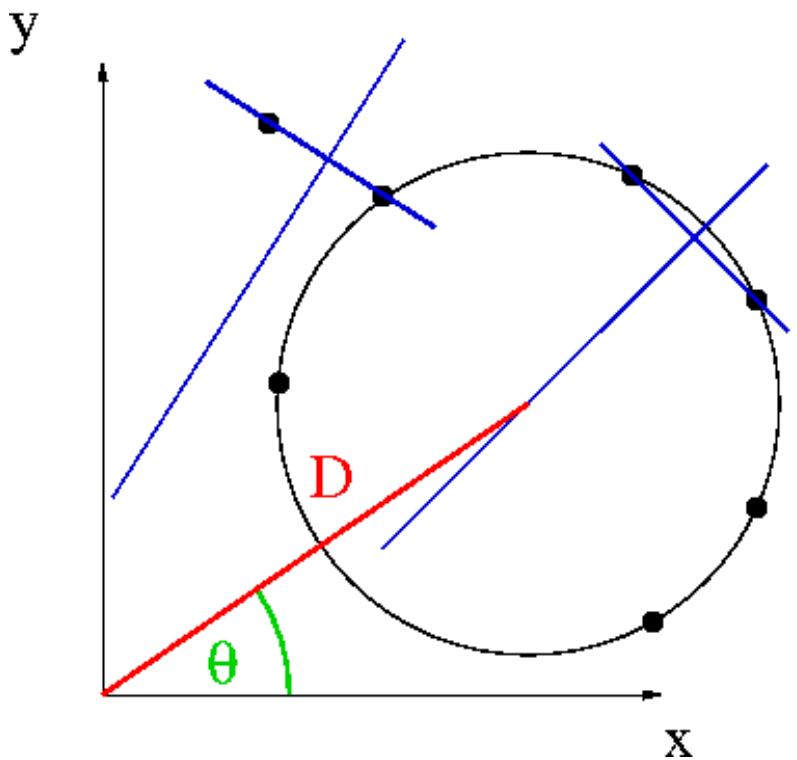
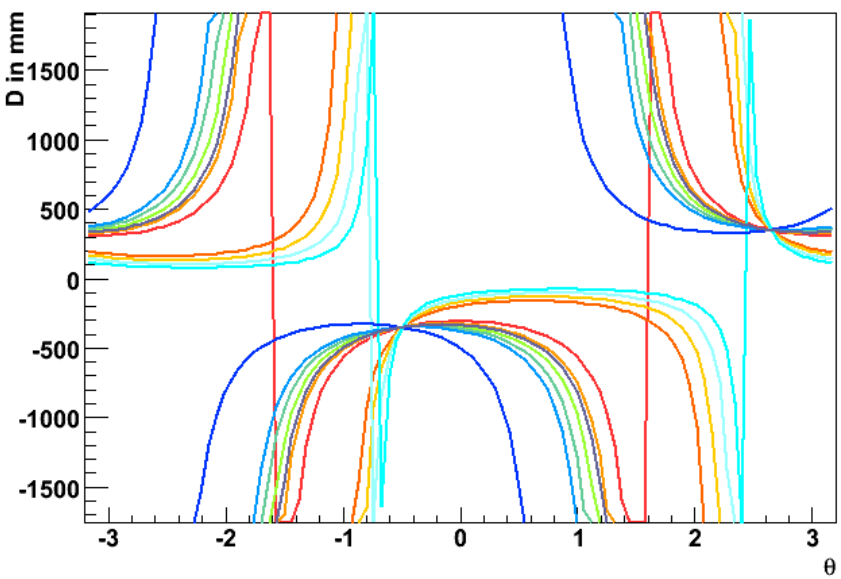
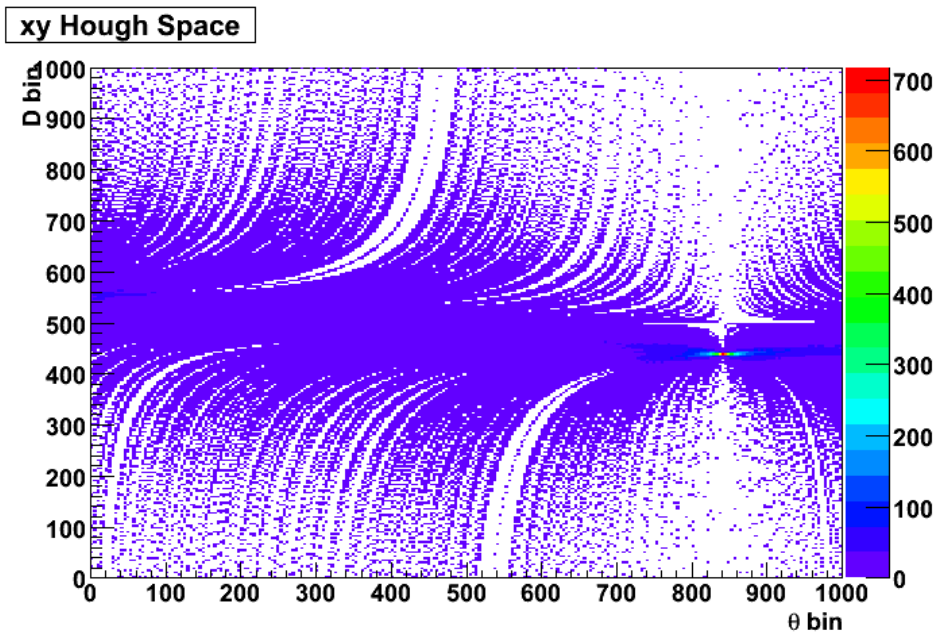


Hough Transformation: Circles

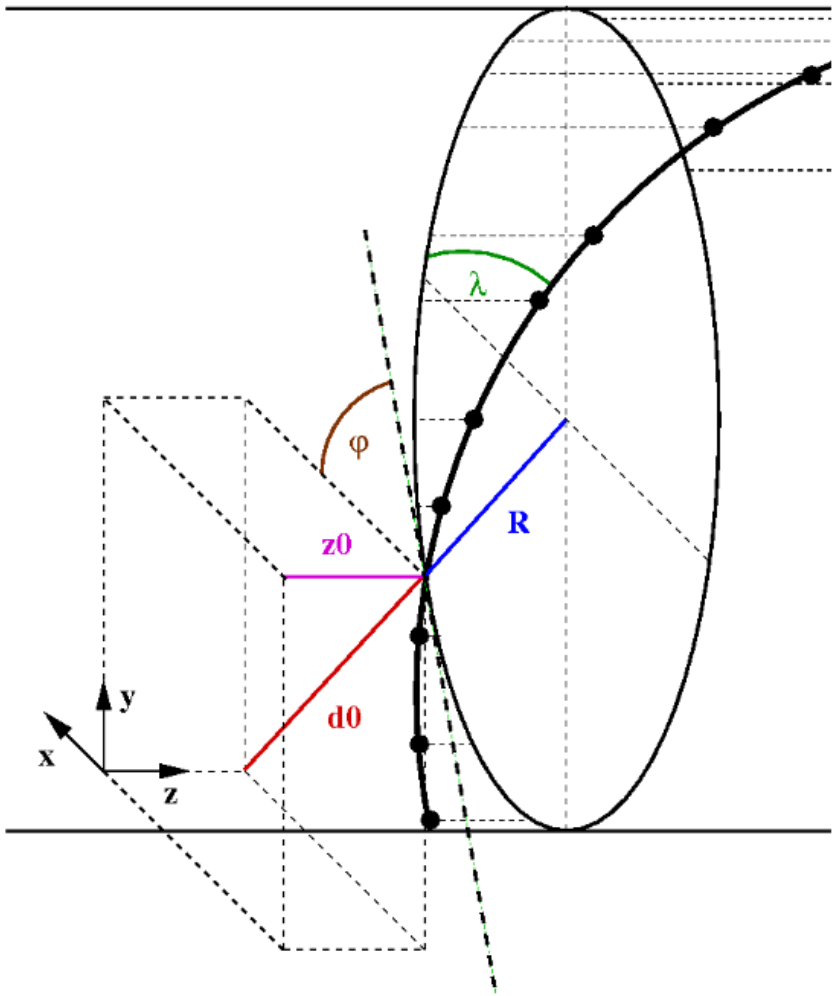
$$D(\theta) = \frac{1}{2} \cdot \frac{(y_1^2 - y_2^2) + (x_1^2 - x_2^2)}{(y_1 - y_2) \cdot \sin\theta + (x_1 - x_2) \cdot \cos\theta}$$





Hough Transformation: Helix

- Track model: helix
- Five parameters needed to describe a helix
- LCIO parameter: φ, $\Omega = 1/R$, d_0, z_0, $\tan\lambda$
- Functions would intersect in 5D Hough Space
- Search is done in two projections
  - xy projection (straight line or circle)
  - sz projection (straight line), where s is the arc length in the xy projection

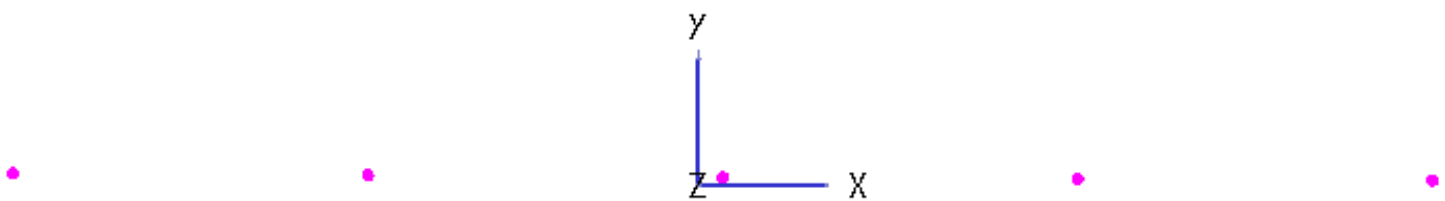


Some Words on the Simulation

- Rectangular pad plane with 100 rows parallel to the x axis, pad width in y direction 7 mm.
  - Straight Lines: 100 Hits per track.
  - Helix: depending on the radius up to 200 Hits per track.
- Parameter were chosen randomly in the following ranges:
  - $\varphi \in [-\pi; \pi]$
  - $d_0 \in [-300; 300]$
  - $R \in [-1000; 1000]$
  - $\tan\lambda \in [-2; 2]$ ($\lambda \in [-1.107; 1.107]$)
  - $z_0 \in [-300; 300]$
  - For parameter scans on parameter was always fixed, while the others were chosen randomly.
- Y positions of hits are the center of the pads.
- X and z positions of hits are calculated accordingly with the track parameters.
- No noise hits
- No smearing.
- For Helices: One loop of a helix was simulated.
- A track is defined to be found correctly if all hits were found correctly.
- track finding efficiency $= \frac{\text{number of correctly found tracks}}{\text{number of simulated tracks}}$

Parameter Scans (Straight Line)

- Tracks with $\varphi_0 = 0$ and $\varphi_0 = \pm\pi$ are parallel to the pad rows; only one hit would be reconstructed for such tracks.
- Tracks with $\lambda = \pm\frac{\pi}{2}$ are parallel to the drift direction, hits on such tracks would end up at only one pad.
- Removing the regions around $\varphi_0 = 0$ and $\varphi_0 = \pm\pi$ from the efficiency calculation leads to an efficiency of about 100 % in the remaining regions.



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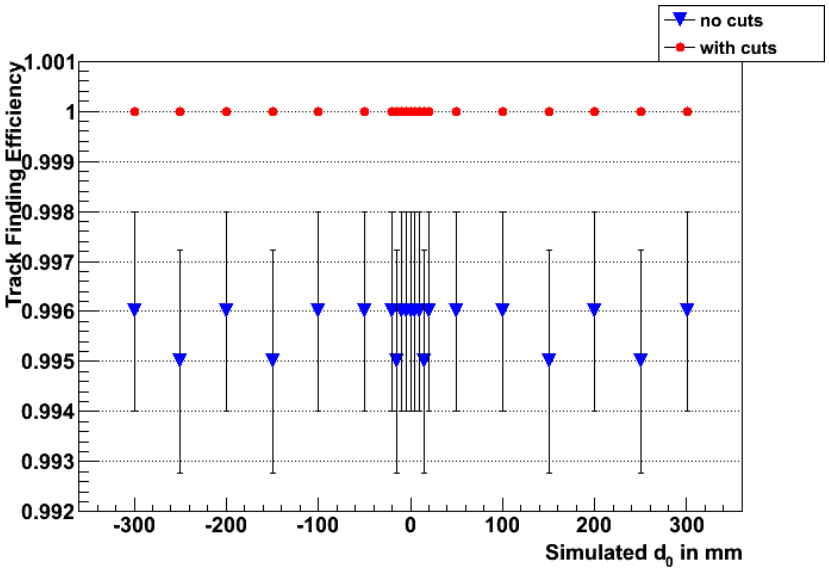
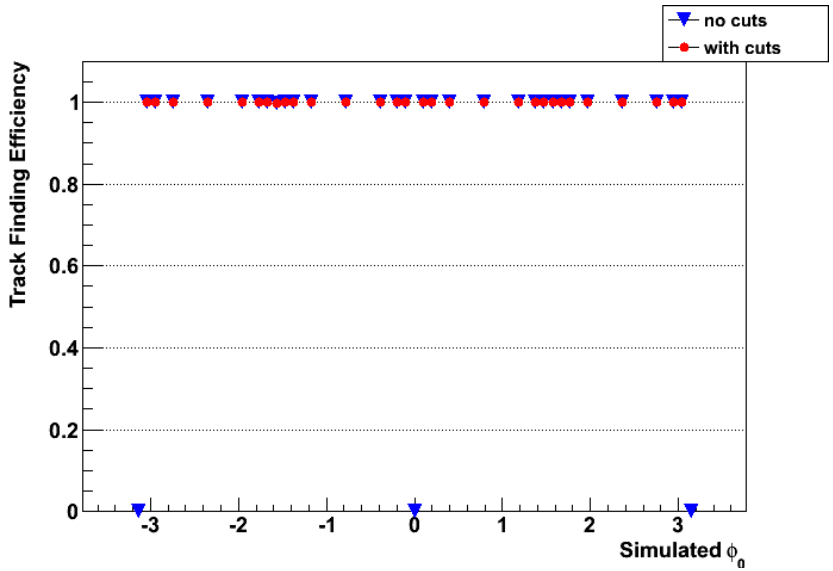
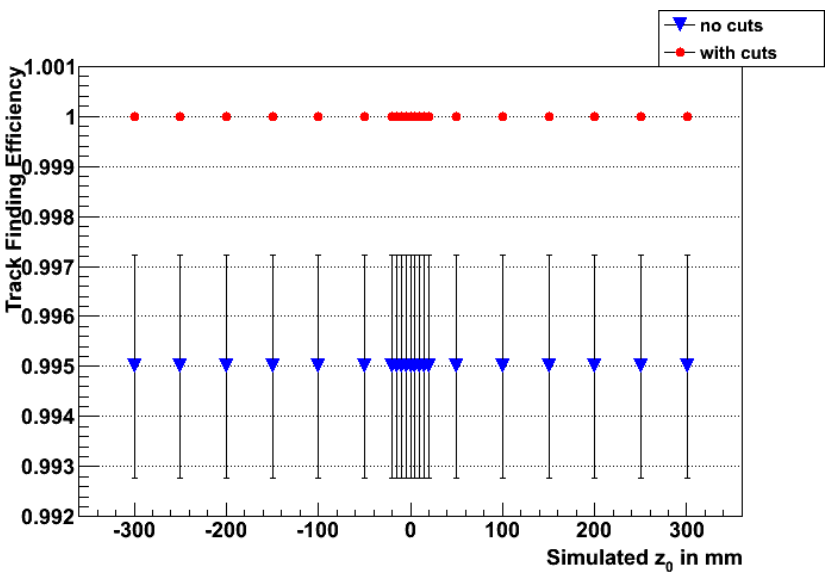
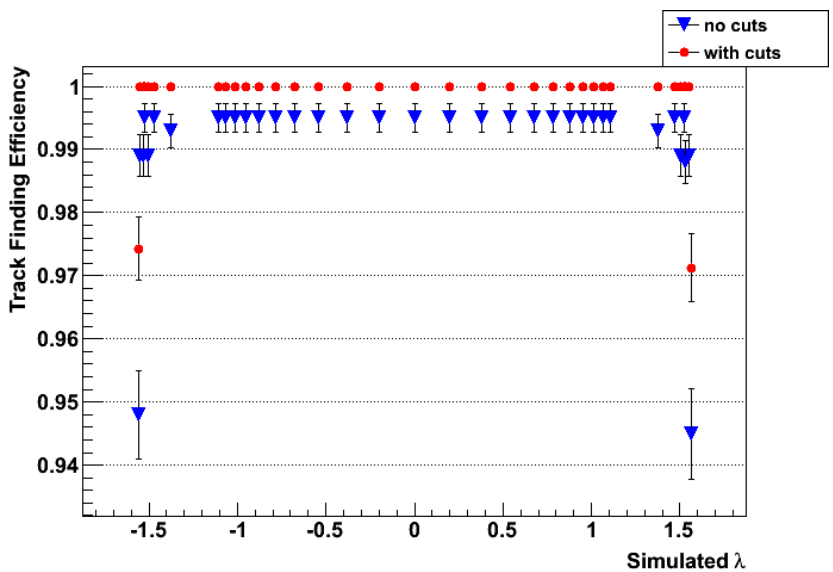


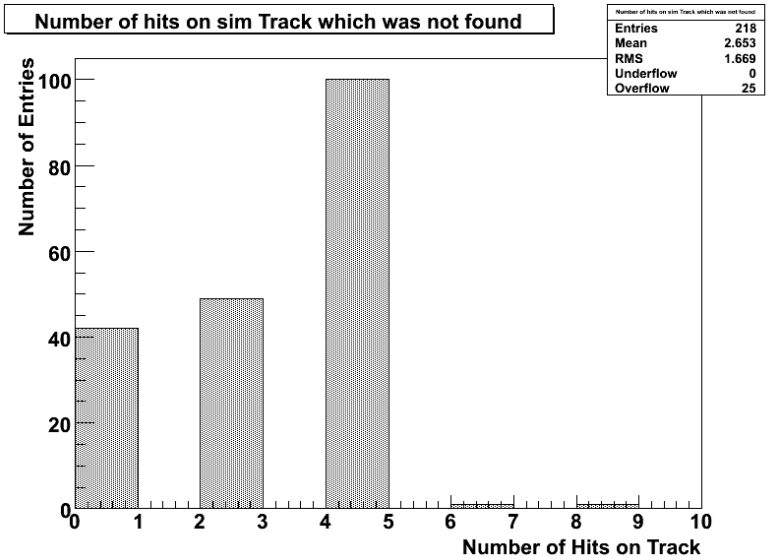


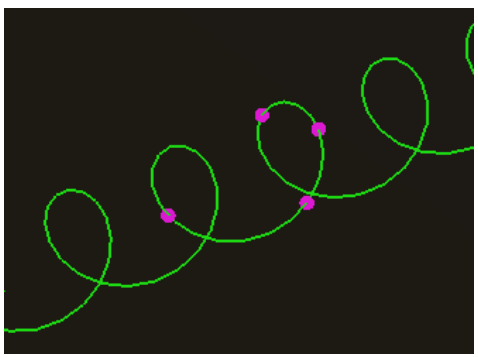


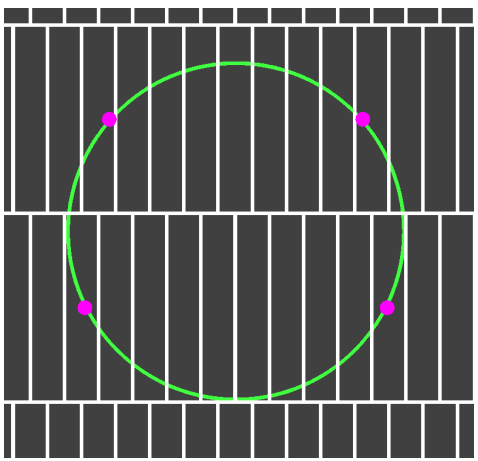


Parameter Scans (Helix)

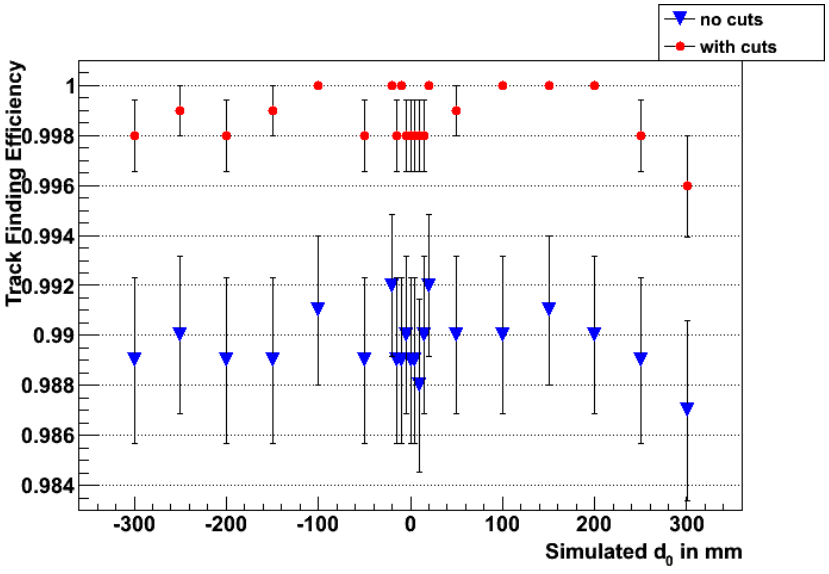
- Tracks which were not found tend to have very few hits, which corresponds to a very small radius.
- In the reconstruction it was required that a track should have at least five hits.
- Removing tracks with less than five hits (in priciple cut on R) from the efficiency calculation leads to a much higher efficiency.

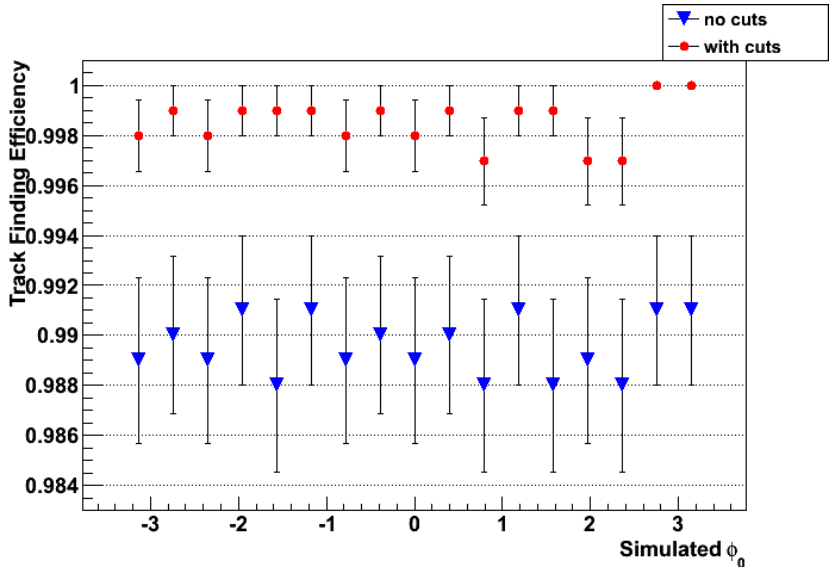


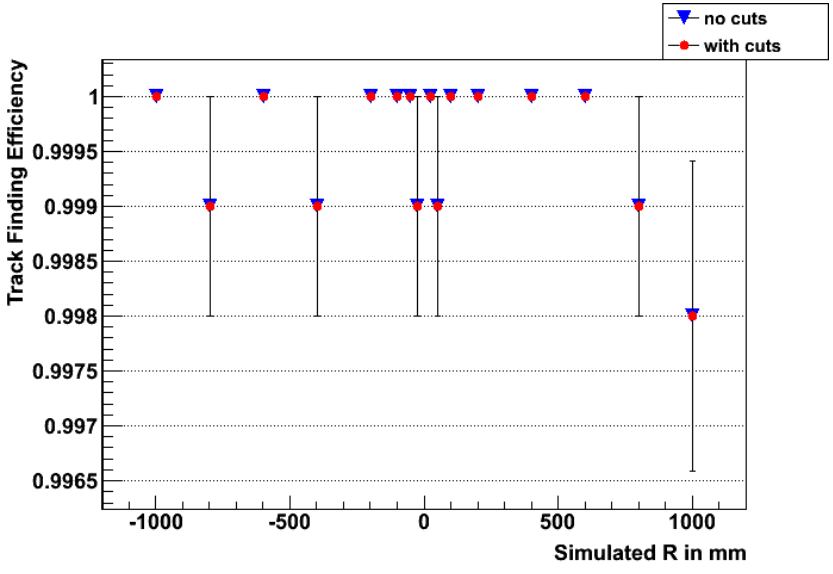




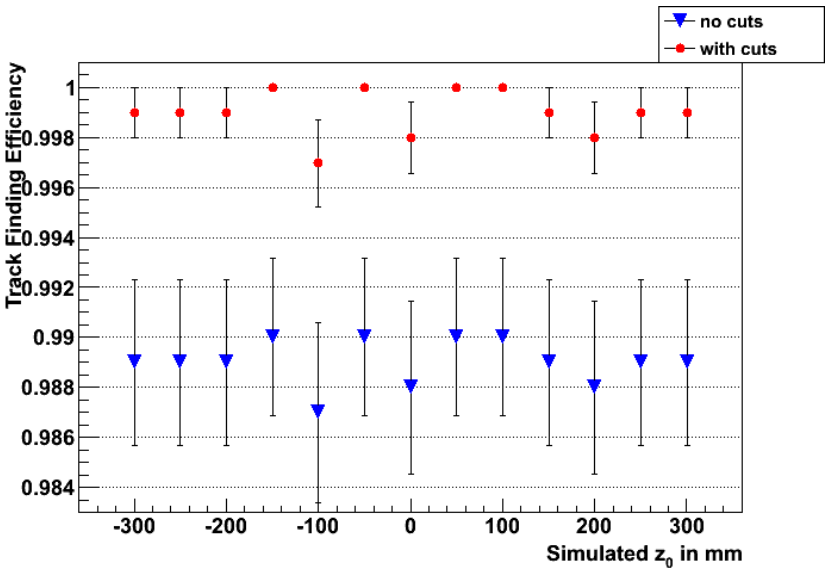
Parameter Scans (Helix, *xy* plane)

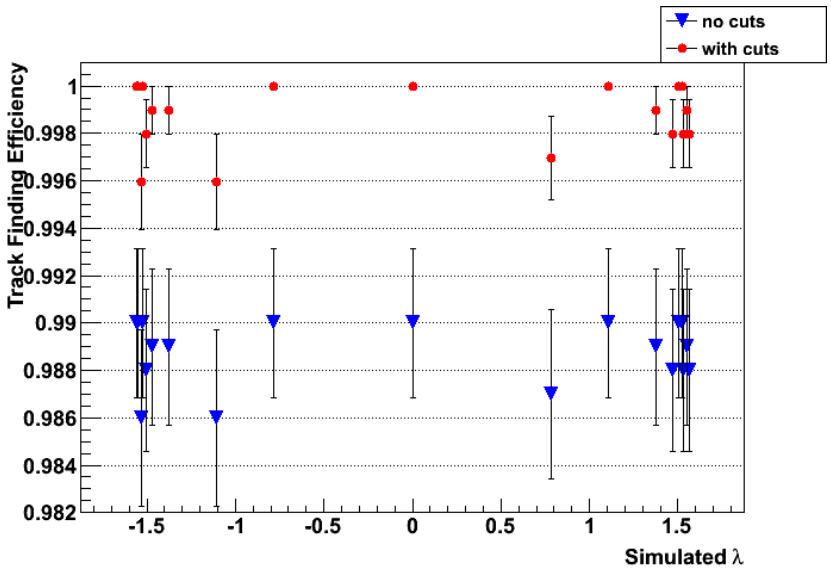






Parameter Scans (Helix, *sz* plane)





Conclusion and Outlook

Conclusion

- A Hough Transformation for track finding was implemented.
  - It works for straight lines, helix segments and curler.
  - More than one track per event can be found.
  - Noise hits are rejected.
  - Hits do not need to be exactly on the track.
- Martin Ljunggren (Lund) and Stefano Caiazza (DESY) used the Hough Transformation successfully on test beam data.

Outlook

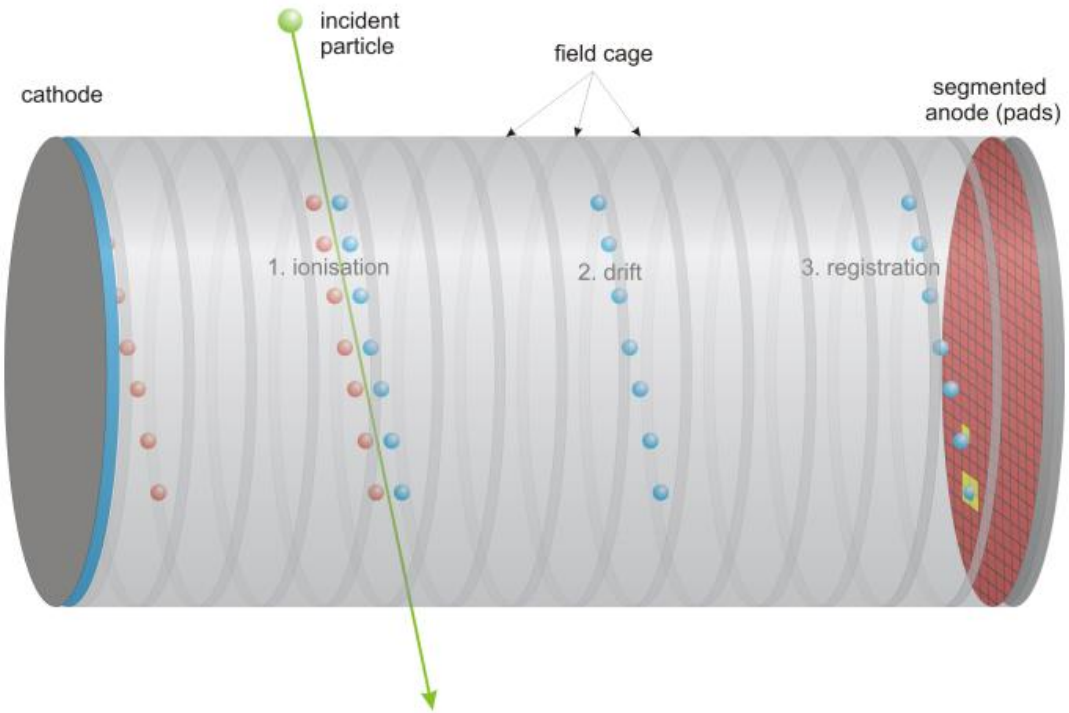
- Detailed studies on fake tracks and track separation are still to be done.
- There are some smaller issues in the code which need to be fixed.
- Making PATHFINDER publically available: January 2012.

Working Principle of Time Projection Chambers

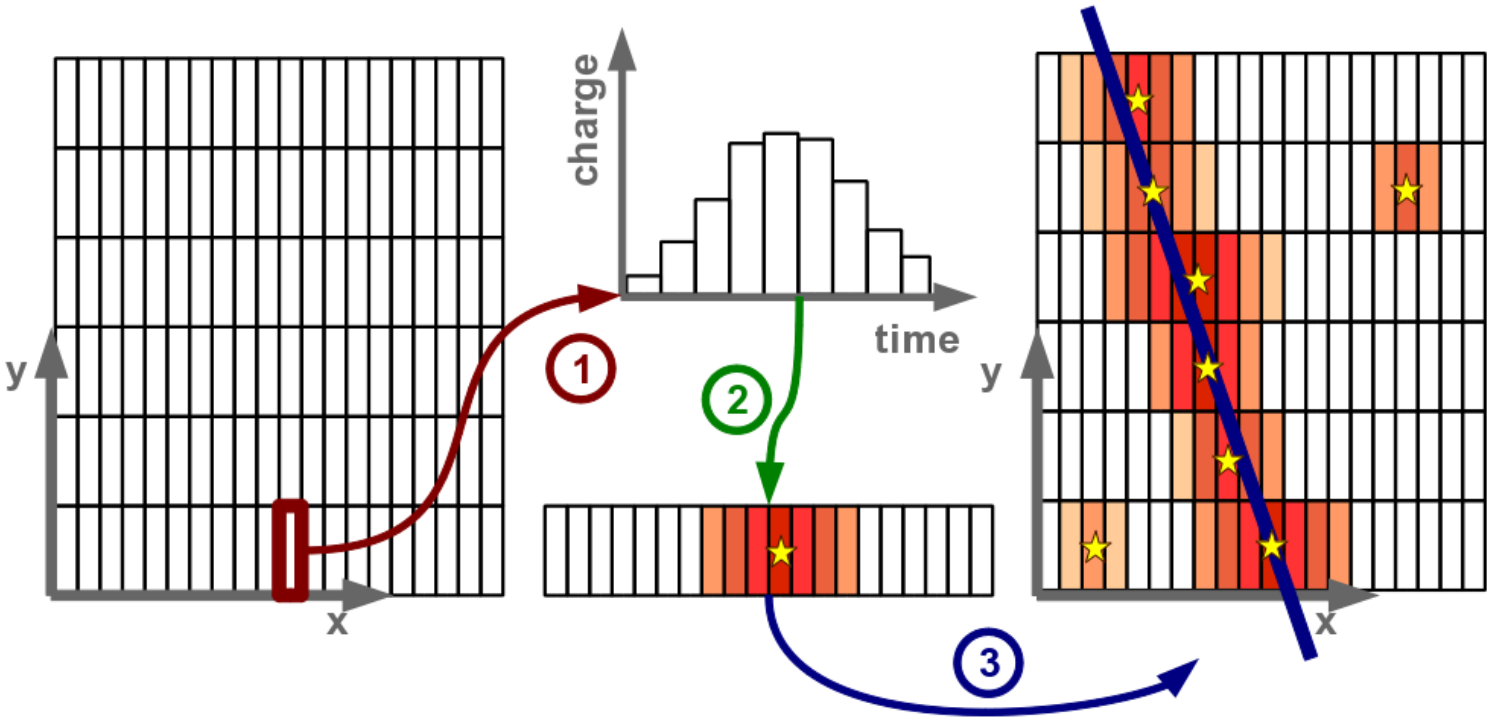
A time projection chamber (TPC) consists of a volume filled with gas with an electrical field applied to it.

Working Principle

- A charged Particle traverses the volume and ionizes the gas inside the TPC.
- Due to the electric field the electrons drift towards the anode.
- Readout of the electrons at the anode.



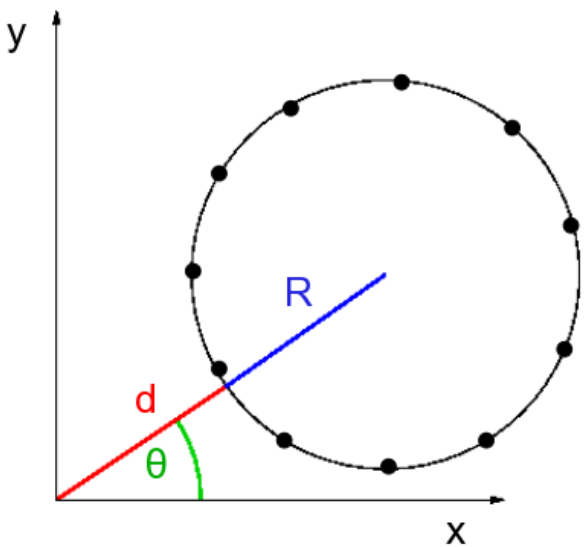
Track Reconstruction



1. For each pad a charge spectrum is recorded; search for pulses in the charge spectrum.
2. Pulses on adjacent pads in the same row are combined to hits.
3. Hits are combined to tracks; tracks are fitted.

Hough Transformation: Circles

- Track model: circle
- Three parameters needed to describe circle. Here: d, θ, R
- Same procedure as for straight lines
- Intersection of functions $R(d, \theta)$ in 3D Hough Space



$$
\begin{aligned}
R^2 &= (x - x_c)^2 + (y - y_c)^2 \\
x_c &= (R + d) \cdot \cos\theta \\
y_c &= (R + d) \cdot \sin\theta \\
\Rightarrow R(d, \theta) &= \frac{x^2 + y^2 + d^2 - 2d \cdot (x \cdot \cos\theta + y \cdot \sin\theta)}{2 \cdot (x \cdot \cos\theta + y \cdot \sin\theta - d)} \\
\Rightarrow &\quad \text{3D Hough Space}
\end{aligned}
$$

Hough Transformation: Circles

- These functions are not the best choice. They are very similar to each other in large regions. This leads to difficulties when trying to find points of intersection.