Origins of Jet scaling and implications for new physics

Erik Gerwick

Georg-August-Universität Göttingen

December 7, 2011

Outline

- Multi-jet final states
- Scaling patterns
- Applications to new physics
- Conclusions and speculation

In collaboration with: Christoph Englert, Tilman Plehn, Steffen Schumann and Peter Schichtel

Based on:
EG, Plehn, Schumann; accepted to PRL hep-ph/1108.3335
Englert, EG, Plehn, Schichtel, Schumann; hep-ph/1110.1043

Also:
Englert, Plehn, Schichtel, Schumann; Phys.Rev.D83:095009,2011; hep-ph/1102.4615
Englert, Plehn, Schichtel, Schumann; hep-ph/1108.547

High multiplicity final state jets

A difficult environment for new physics

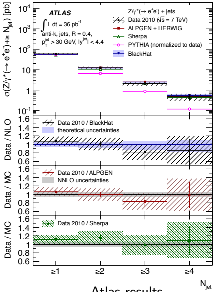
- Many jet final states crucial *e.g.* SUSY cascades, colored states, $\cdots$
- Z+jets, W+jets and pure QCD provide a large background
- Leading order calculations suffer from scale uncertainty [NLO for $n \leq 3$ in pure QCD]
- Difference between two (equally good) scale choices μ and $\bar{\mu}$

$$\sigma^{\mathrm{LO}}_{n-\mathrm{jets}}(\bar{\mu}) \; - \; \sigma^{\mathrm{LO}}_{n-\mathrm{jets}}(\mu) \quad \sim \quad \alpha_S^n \left(n \, b_0 \, \alpha_S \ln \frac{\bar{\mu}^2}{\mu^2} \right)$$

Simulations to predict multi-jet rates

- Matrix Element / Parton Shower combination robust agreement with data
- Dynamical scale choice with additional parton shower logarithms

  $\Rightarrow$ theoretical uncertainty under control
- We use Sherpa in this work [Hoeche, Krauss, Schumann, Winter]



Atlas results

Scaling: possible deeper connections between theory and experiment

We believe our simulations but how can we interpret jet rates?

- N_{jet} rates tend to follow universal scaling patterns
- Deviations from scaling patterns can help us find new physics
- Most informative presentation of N_{jet} rates are in terms of the ratios of exclusive jet multiplicities.

$$R_n \equiv \frac{\sigma_{n+1}}{\sigma_n}$$

Also ratios less sensitive to systematic uncertainties compared with rates.

Observed Scaling Patterns

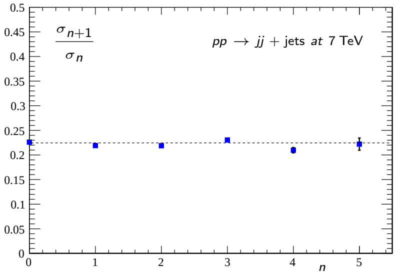
Staircase [Steve Ellis, Kleiss, Stirling (1985); Berends (1989)]

– Ratios are constant

$$\sigma_n^{\text{exclv}} \sim \; R_0^n \equiv e^{-bn}$$

$$\Rightarrow \frac{\sigma_{n+1}}{\sigma_n} = e^{-b} = R_0$$

– Observed: UA2, Tevatron, LHC



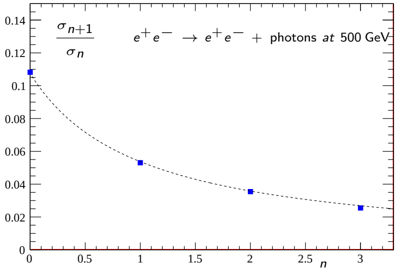
Poisson [Peskin & Schroder; Rainwater, Summers, Zeppenfeld (1997)]

– Ratios are <u>not</u> constant

$$\sigma_n^{\text{exclv}} \sim \; e^{-\bar{n}}\bar{n}^n/n!$$

$$\Rightarrow \frac{\sigma_{n+1}}{\sigma_n} = \frac{\bar{n}}{n+1}$$

– Observed: *e.g.* photons at LEP

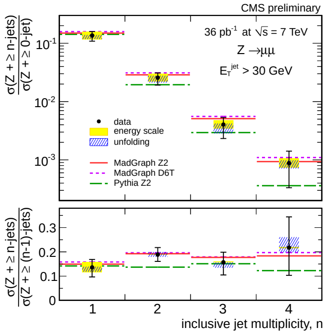


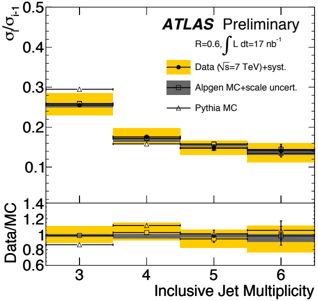
Jet scaling in the data

Theoretical and experimental basis

- Both patterns observed in QCD processes (depending on cuts)
- For staircase: $R_{excl} = R_{incl}$ [Englert, Plehn, Schichtel, Schumann]
- For Poisson:

$$R_{excl} = \frac{\bar{n}}{n+1} \quad \Longleftrightarrow \quad R_{incl} = \left(\frac{(n+1)\, e^{-\bar{n}}\, \bar{n}^{-(n+1)}}{\Gamma(n+1) - n\Gamma(n, \bar{n})} + 1 \right)^{-1}$$





QED and the emergence of Poisson scaling

Basic synopsis of Poisson radiation pattern from QED [Peskin & Schroder; Weinberg]

– Fully factorized form of the matrix element (Eikonal approximation)

$$\mathcal{M}_{n+1} = g_s T^a \epsilon^\star_\mu \bar{u}(q) \frac{q^\mu + \mathcal{O}(\not{k})}{q \cdot k + \mathcal{O}(k^2)} \mathcal{M}_n$$

– Phase space factor $1/n!$ for identical bosons in the final state

$$\Rightarrow \sigma_n \sim \frac{L^n}{n!} e^{-L} \qquad \text{with} \qquad L \sim \frac{\alpha}{\pi} \log\left(\frac{E_{hard}}{E_{soft}}\right)$$

Crucial theorem for out purposes

– Suppose two Poisson processes N_1 and N_2 with Poisson expectations $\bar{n}_1$ and $\bar{n}_2$ are independent. The counting process N defined by $N(t) = N_1(t) + N_2(t)$ is a Poisson process with rate function $\bar{n}$ given by $\bar{n} = \bar{n}_1 + \bar{n}_2$.

$\Rightarrow$ <u>All QED processes give Poisson</u> [in soft collinear limit]

Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit
2. QCD has subsequent splittings via gluon 3-point vertex
3. Different kinematics due to PDFs

Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit

   $\Rightarrow$ Corrections due to hard matrix elements

2. QCD has subsequent splittings via gluon 3-point vertex

3. Different kinematics due to PDFs

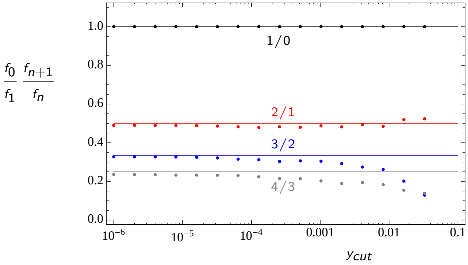
Corrections due to the hard matrix element

Require photons to be widely separated

- Scan final state with the Durham measure; *i.e.* only keep particles with

$$2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij}) > y_{cut}s.$$

- Destroys the Poisson scaling, but cannot be responsible for a staircase
- Ratios are pushed in the wrong direction (not surprising due to phase space)



Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit

   $\Rightarrow$ ~~Corrections due to hard matrix elements~~

2. QCD has subsequent splittings via gluon 3-point vertex

   $\Rightarrow$ Non-abelian contributions

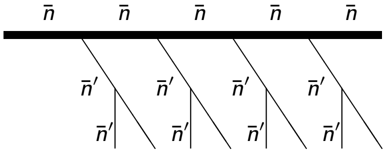
3. Different kinematics due to PDFs

Subsequent splittings and the emergence of staircase

Toy Model: An iterated Poisson process

- Extension of the (pure Poisson) exponentiation model [Rainwater, Zeppenfeld]
- Simple analogy; each emission generates a new Poisson process with separate Poisson parameter $\bar{n}'$

$$P(n,\bar{n},\bar{n}') = e^{-\bar{n}}\frac{\bar{n}}{n!} + e^{-\bar{n}-\bar{n}'}\sum_{i=1}^{n}\left(\frac{(n-1)!}{i!(n-i-1)!(n-i)!}\right)\bar{n}'^{i}\bar{n}^{n-i}$$

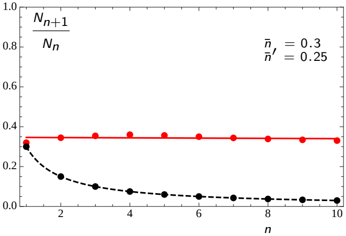


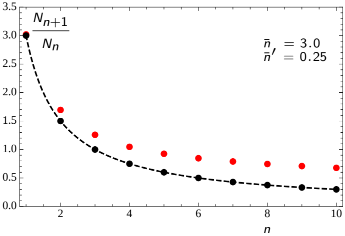
Subsequent splittings and the emergence of staircase

Toy Model: An iterated Poisson process

- Extension of the (pure Poisson) exponentiation model [Rainwater, Zeppenfeld]
- Simple analogy; each emission generates a new Poisson process with separate Poisson parameter $\bar{n}'$

$$P(n,\bar{n},\bar{n}') = e^{-\bar{n}}\frac{\bar{n}}{n!} + e^{-\bar{n}-\bar{n}'}\sum_{i=1}^{n}\left(\frac{(n-1)!}{i!(n-i-1)!(n-i)!}\right)\bar{n}'^{i}\,\bar{n}^{n-i}$$





Subsequent splittings and the emergence of staircase

More realistic model: $e^+e^- \to q\bar{q} + \text{jets}$

- Leading log and next-to-leading log jet rates available for the Durham measure (we calculate to $\mathcal{O}(\alpha^4)$) [Catani, Dokshitzer, Olsson, Turnock, Webber (1991); Webber (2010)]

$$L \equiv \log \frac{1}{y_{\text{cut}}} \qquad \text{and} \qquad a \equiv \alpha_S/\pi$$

- Purely abelian terms from qg splitting exponentiate

$$f_2^D = 1 - a\frac{C_F}{2}L^2 + a^2\frac{C_F^2}{8}L^4 - a^3\frac{C_F^3}{48}L^6 + a^4\frac{C_F^4}{384}L^8$$

$$f_3^D = a\left(\frac{C_F}{2}\right)L^2 - a^2\left(\frac{C_F^2}{4} + \frac{C_F C_A}{48}\right)L^4 + a^3\left(\frac{C_F^3}{16} + \frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{960}\right)L^6 - a^4\left(\frac{C_F^4}{96} + \frac{C_F^3 C_A}{384} + \frac{C_F^2 C_A^2}{1920} + \frac{C_F C_A^3}{21504}\right)L^8$$

$$f_4^D = a^2\left(\frac{C_F^2}{8} + \frac{C_F C_A}{48}\right)L^4 - a^3\left(\frac{C_F^3}{16} + \frac{C_F^2 C_A}{48} + \frac{7C_F C_A^2}{2880}\right)L^6 + a^4\left(\frac{C_F^4}{64} + \frac{C_F^3 C_A}{128} + \frac{C_F^2 C_A^2}{512} + \frac{C_F C_A^3}{5120}\right)L^8$$

$$f_5^D = a^3\left(\frac{C_F^3}{48} + \frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{720}\right)L^6 - a^4\left(\frac{C_F^4}{96} + \frac{C_F^3 C_A}{128} + \frac{3C_F^2 C_A^2}{1280} + \frac{41C_F C_A^3}{161280}\right)L^8$$

$$f_6^D = a^4\left(\frac{C_F^4}{384} + \frac{C_F^3 C_A}{384} + \frac{7C_F^2 C_A^2}{7680} + \frac{17C_F C_A^3}{161280}\right)L^8$$

Subsequent splittings and the emergence of staircase

More realistic model: $e^+e^- \to q\bar{q}$ + jets

- Leading log and next-to-leading log jet rates available for the Durham measure

$$L \equiv \log \frac{1}{y_{\text{cut}}} \qquad \text{and} \qquad a \equiv \alpha_S/\pi$$

- Purely Abelian terms from qg splitting exponentiate

$$f_2^D = \exp\left[-\frac{aC_F L^2}{2}\right]$$

$$f_3^D = \left(\frac{aC_F L^2}{2}\right)\exp\left[-\frac{aC_F L^2}{2}\right] - a^2\left(\frac{C_F C_A}{48}\right)L^4 + a^3\left(\frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{960}\right)L^6 - a^4\left(\frac{C_F^3 C_A}{384} + \frac{C_F^2 C_A^2}{1920} + \frac{C_F C_A^3}{21504}\right)L^8$$

$$f_4^D = \frac{1}{2!}\left(\frac{aC_F L^2}{2}\right)^2\exp\left[-\frac{aC_F L^2}{2}\right] + a^2\left(\frac{C_F C_A}{48}\right)L^4 - a^3\left(\frac{C_F^2 C_A}{48} + \frac{7C_F C_A^2}{2880}\right)L^6 + a^4\left(\frac{C_F^3 C_A}{128} + \frac{C_F^2 C_A^2}{512} + \frac{C_F C_A^3}{5120}\right)L^8$$

$$f_5^D = \frac{1}{3!}\left(\frac{aC_F L^2}{2}\right)^3\exp\left[-\frac{aC_F L^2}{2}\right] + a^3\left(\frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{720}\right)L^6 - a^4\left(\frac{C_F^3 C_A}{128} + \frac{3C_F^2 C_A^2}{1280} + \frac{41C_F C_A^3}{161280}\right)L^8$$

$$f_6^D = \frac{1}{4!}\left(\frac{aC_F L^2}{2}\right)^4\exp\left[-\frac{aC_F L^2}{2}\right] + a^4\left(\frac{C_F^3 C_A}{384} + \frac{7C_F^2 C_A^2}{7680} + \frac{17C_F C_A^3}{161280}\right)L^8$$

Subsequent splittings and the emergence of staircase

More realistic model: $e^+e^- \to q\bar{q} + \text{jets}$

- Leading log and next-to-leading log jet rates available for the Durham measure

$$L \equiv \log \frac{1}{y_{\text{cut}}} \qquad \text{and} \qquad a \equiv \alpha_S/\pi$$

- Non-abelian terms do not simply exponentiate!

$$f_2 = \exp\left[-\frac{aC_F L^2}{2}\right]$$

$$f_3 = \left(\frac{aC_F L^2}{2}\right)\exp\left[-\frac{aC_F L^2}{2}\right] - a^2\left(\frac{C_F C_A}{48}\right)L^4 + a^3\left(\frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{960}\right)L^6 - a^4\left(\frac{C_F^3 C_A}{384} + \frac{C_F^2 C_A^2}{1920} + \frac{C_F C_A^3}{21504}\right)L^8$$

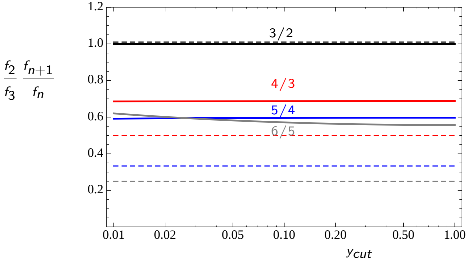
$$f_4 = \frac{1}{2!}\left(\frac{aC_F L^2}{2}\right)^2\exp\left[-\frac{aC_F L^2}{2}\right] + a^2\left(\frac{C_F C_A}{48}\right)L^4 - a^3\left(\frac{C_F^2 C_A}{48} + \frac{7C_F C_A^2}{2880}\right)L^6 + a^4\left(\frac{C_F^3 C_A}{128} + \frac{C_F^2 C_A^2}{512} + \frac{C_F C_A^3}{5120}\right)L^8$$

$$f_5 = \frac{1}{3!}\left(\frac{aC_F L^2}{2}\right)^3\exp\left[-\frac{aC_F L^2}{2}\right] + a^3\left(\frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{720}\right)L^6 - a^4\left(\frac{C_F^3 C_A}{128} + \frac{3C_F^2 C_A^2}{1280} + \frac{41C_F C_A^3}{161280}\right)L^8$$

$$f_6 = \frac{1}{4!}\left(\frac{aC_F L^2}{2}\right)^4\exp\left[-\frac{aC_F L^2}{2}\right] + a^4\left(\frac{C_F^3 C_A}{384} + \frac{7C_F^2 C_A^2}{7680} + \frac{17C_F C_A^3}{161280}\right)L^8$$

Subsequent splittings and the emergence of staircase

Numerical comparison of Non-abelian versus Abelian terms.



Rate coefficients for purely non-abelian gluon cascade [Leder]

$$f_i = \left(1, \frac{1}{2}, \frac{1}{6}, \frac{17}{360}, \frac{31}{2520}, \frac{691}{22680} \cdots \right) \quad \Rightarrow \quad R_i = (0.50, 0.33, 0.28, 0.26, 0.25 \cdots)$$

Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit; maybe this is not valid for $pp \to$ jets?

   ~~Corrections due to hard matrix elements~~

2. QCD has subsequent splittings via gluon 3-point vertex

   $\Rightarrow$ Non-abelian contributions ✓

3. Different kinematics due to PDFs

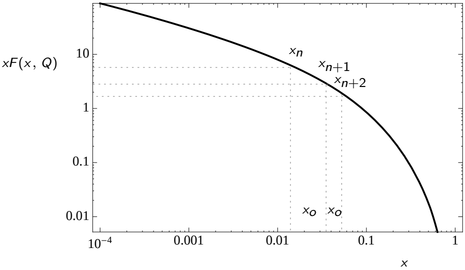
   $\Rightarrow$ Relative cost of an additional jet depends on previous jets

Effect due to PDFs

How can we estimate the effect of the PDFs on the distribution of exclusive jet multiplicities?

– Measure of the suppression on exclusive multiplicities given by the (discrete) 2nd-derivative with respect to x.

$$\text{Let} \quad B(n,Q) = \frac{|F(x_{n+1},Q_{hard})|^2}{F(x_n,Q_{hard})\,F(x_{n+2},Q_{hard})} \quad \text{with} \quad x_n = \frac{Q_{hard}}{\sqrt{s}} + n\frac{Q_{veto}}{\sqrt{s}}$$

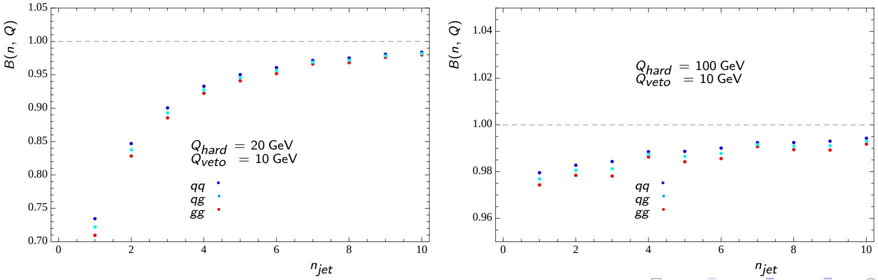


Effect due to PDFs

How can we estimate the effect of the PDFs on the distribution of exclusive jet multiplicities?

– Measure of the suppression on exclusive multiplicities given by the (discrete) 2nd-derivative with respect to x.

$$\text{Let} \quad B(n,Q) = \frac{|F(x_{n+1}, Q_{hard})|^2}{F(x_n, Q_{hard})\, F(x_{n+2}, Q_{hard})} \quad \text{with} \quad x_n = \frac{Q_{hard}}{\sqrt{s}} + n\frac{Q_{veto}}{\sqrt{s}}$$

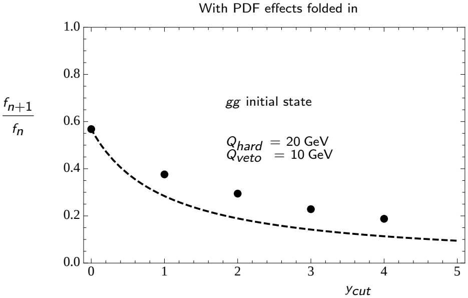


Effect due to PDFs

How can we estimate the effect of the PDFs on the distribution of exclusive jet multiplicities?

– Measure of the suppression on exclusive multiplicities given by the (discrete) 2nd-derivative with respect to x.

$$\text{Let} \quad B(n,Q) = \frac{|F(x_{n+1}, Q_{hard})|^2}{F(x_n, Q_{hard})\, F(x_{n+2}, Q_{hard})} \quad \text{with} \quad x_n = \frac{Q_{hard}}{\sqrt{s}} + n\frac{Q_{veto}}{\sqrt{s}}$$



Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit

   $\Rightarrow$ ~~Corrections due to hard matrix elements~~

2. QCD has subsequent splittings via gluon 3-point vertex

   $\Rightarrow$ Non-abelian contributions ✓

3. Different kinematics due to PDFs

   $\Rightarrow$ Relative cost of an additional jet depends on previous jets ✓

Applications

Partially explored

- Central jet vetos in Higgs searches [EG, Plehn, Schumann]
- Calibration of missing energy searches [Englert, Plehn, Schumann, Schichtel]

Speculative

- Improved data-driven extrapolation of multi-jet rates
- Reductions of experimental systematics

Central jet vetos in Higgs searches [EG, Plehn, Schumann]

Weak Boson Fusion

- Signal has high $|\eta|$ tagging jets, $Z+$jets background more likely to radiate additional jets into this region. Impose WBF cuts

  $y_1 \cdot y_2 < 0 \quad |y_1 - y_2| > 4.4 \quad m_{jj} > 600$ GeV

- Central jet veto makes this channel relevant. Veto all events with an additional jet satisfying

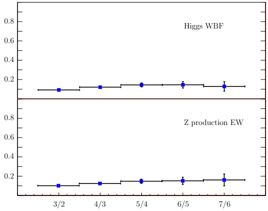
  $p_T^{\text{veto}} > 20$ GeV $\quad \min y_{1,2} < y^{\text{veto}} < \max y_{1,2}$

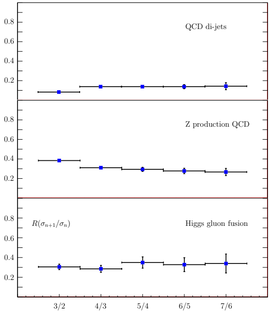
- For background, Dipole initiated shower contains a large Logarithm.

  $$\bar{n} \sim \log \frac{m_{jj}}{Q_{veto}}$$

- For signal, large log not induced.

Before WBF cuts

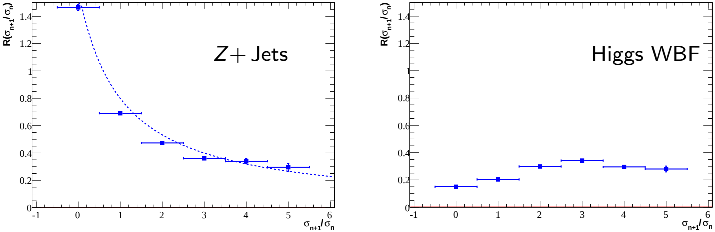




Central jet vetos in Higgs searches [EG, Plehn, Schumann]

After Weak Boson Fusion cuts

– $Z+$jets background quickly becomes Poisson, while WBF does not!



Veto Probabilities

– Can use distributions to calculate veto probability taking into account all multi-jet contributions. Poisson: $P_{veto} = e^{-\bar{n}}$.

Conclusions

- Ratio of N_{jet} distributions very interesting from a theory perspective. Would be exciting to have these in more kinematic regimes (*i.e.* Poisson-like).
- The constant ratios of exclusive jet multiplicities for a wide range of selection criteria and processes seems to be a fortuitous semi-coincidence of QCD.
- The fact that QCD cross-section do not at all follow a Poisson distribution (unless a large logarithm is present) is the result of a combination of Non-abelian splittings and PDF effects.
- We have tried to quantify these effects by considering them separately.

  1. The $e^+e^- \to$ jets allowed us to quantify the non-exponentiable terms responsible for the deviation from Poisson in the final state evolution.

  2. The PDF effects depend only on the hard-scale of the process and the veto/acceptance scale for additional jets.

- There may be other applications, we're still thinking about this!

More on Staircase

Theoretical Basis

- Black-hat + Sherpa NLO Z and W + jets (Anti-kt; $R = .4$; $E_T > 30$ GeV) [Berger et al.]
- Staircase improves for NLO versus LO
- No solid theoretical motivation

$R_n = \sigma_n/\sigma_{n-1}$	LO	NLO
R_2	.2805	.235
R_3	.2483	.223
R_4	.2394	.226

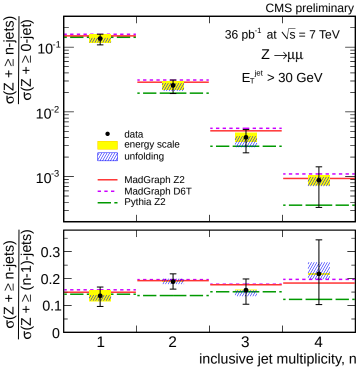
Experimental Observations

- Inclusive = exclusive ratios (for perfect staircase)

$$R_{incl} = \frac{\hat{\sigma}_{n+1}}{\hat{\sigma}_n} = \frac{\sigma_{n+1}\sum R^j_{excl}}{\sigma_n + \sigma_{n+1}\sum R^j_{excl}}$$

$$= \frac{R_{excl}\sigma_n}{(1-R_{excl})\sigma_n + R_{excl}\sigma_n}$$

$$= R_{excl}$$

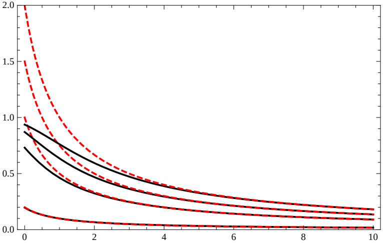


More on Poisson

Theoretical Basis (see next slide)

Experimental Observation

$$R_{excl} = \frac{\bar{n}}{n+1} \qquad R_{incl} = \left(\frac{(n+1)\, e^{-\bar{n}}\, \bar{n}^{-(n+1)}}{\Gamma(n+1) - n\Gamma(n,\bar{n})} + 1 \right)^{-1}$$



Subsequent splittings and the emergence of staircase

Toy Model: An iterated Poisson process

- Extension of the (pure Poisson) exponentiation model [Rainwater, Zeppenfeld]
- Simple analogy; each emission generates a new Poisson process with separate Poisson parameter $\bar{n}'$

$$P(n,\bar{n},\bar{n}') = e^{-\bar{n}}\frac{\bar{n}}{n!} + e^{-\bar{n}-\bar{n}'}\sum_{i=1}^{n}\left(\frac{(n-1)!}{i!(n-i-1)!(n-i)!}\right)\bar{n}'^{i}\bar{n}^{n-i}$$

$$= e^{-\bar{n}}\frac{\bar{n}}{n!} - \frac{(-1)^n e^{-\bar{n}-\bar{n}'}\bar{n}'^{(-1+n)}\left[\left(-\frac{\bar{n}}{\bar{n}'}\right)^n\bar{n}' + \bar{n}\mathtt{HG}[1-n,2,-\frac{\bar{n}}{\bar{n}'}]\right]}{n!}$$

$$\mathtt{HG}[a,b,z] \equiv \frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}dt$$