Higgs Boson Decays through the Order $O(\alpha\alpha_s)$

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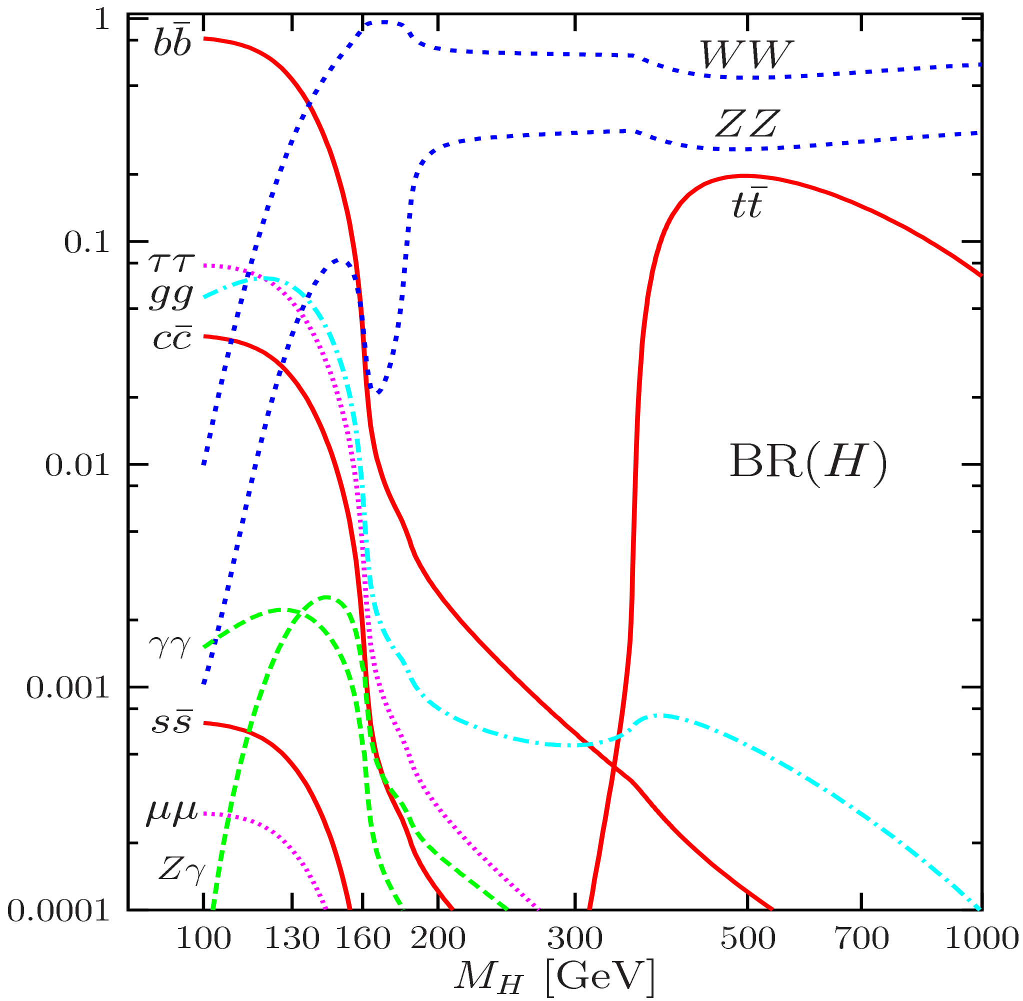
- introduction
- decays into vector bosons
- decays into fermions
- conclusions

branching ratios



decays $H \to WW$, $H \to ZZ$

- lowest order

$$\Gamma_0(H \to VV) = \delta_V \frac{G_F m_H^3}{16\sqrt{2}\pi} \sqrt{1 - \frac{4m_V^2}{m_H^2}} \left(1 - \frac{4m_V^2}{m_H^2} + \frac{12m_V^4}{m_H^4}\right), \qquad \delta_V = \begin{cases} 2 \text{ for } W \\ 1 \text{ for } Z \end{cases}$$

- EW corrections

Fleischer, Jerelehner '81, Hioki '89

Bardin et al '91, Kniehl '91

- effective theory

$\Longrightarrow$ pick up the leading corrections in the limit $m_t \to \infty$

– $O(\alpha_s G_F m_t^2)$ — Djouadi, Gambino, Kniehl, Sirlin, Spira '93,'94

– $O(\alpha_s^2 G_F m_t^2)$ — Kniehl, Steinhauser '95

– $O(G_F^2 m_t^4)$ — Djouadi, Gambino, Kniehl '98

- $O(\alpha\alpha_s)$ correction — this calculation

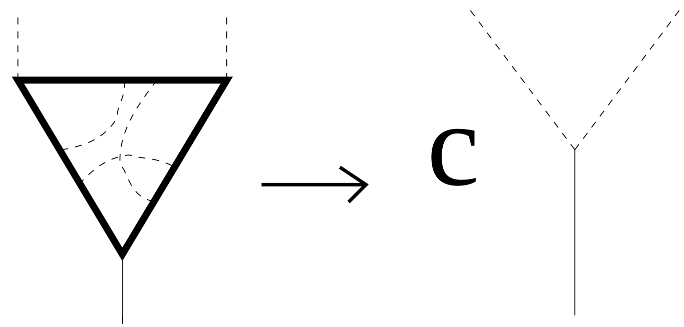
- leading terms $O(\alpha_s^j G_F m_t^2)$ can be computed whithin effective lagrangian approach In the Standard Model

$$L_{\text{Yukawa}} = -\frac{H}{v} m_t \bar{t} t$$

in the limit $m_t \to \infty$ "integrating out" t-quark $\Longrightarrow$

$$L_{\text{eff}} = -\frac{H}{v}\left(+C_g\, G_{\mu\nu}^2 + \sum_q (C_q\, m_q \bar{q} q) + C_\gamma\, F_{\mu\nu}^2 + \ldots \right)$$

with $C_j = C_j(m_t, \mu_F, \alpha_s)$ – perturbative coefficients



- subleading terms computed with large mass expansion (see later)

decays $H \to VV$. some results

$$\Gamma(H \to VV) = \Gamma^{\text{EW}}_{1\text{-loop}} \left(1 + x_t \frac{\alpha_s}{\pi} N_c C_F \Delta_{VV}\right)$$

$$x_t = G_F m_t^2 / (8\sqrt{2}\pi^2)$$

Using large mass expansion (see later):

$$\Delta_{WW} = \frac{9}{2} - \zeta_2 + \frac{m_W^2}{m_t^2}\Big[-\frac{3}{2} + \frac{m_W^2}{m_H^2}\big(-\frac{109}{1080} + \frac{\zeta_2}{2}\big)\Big] + \ldots$$

$$\Delta_{ZZ} = \frac{15}{2} - \zeta_2 + \frac{m_W^2}{m_t^2}\Big[\frac{10}{9} - \frac{31}{36\cos^2\theta_W} + \frac{16\sin^2\theta_W}{9} + \frac{m_W^2}{m_H^2}\big(\frac{37}{135} - \sin^2\theta_W + \frac{4\sin^4\theta_W}{3}\big)\Big] + \ldots$$

and similar for $H \to Z\gamma$.

decays $H \to VV$. some results

$$\Gamma(H \to VV) = \Gamma^{\mathrm{EW}}_{1\text{–loop}} \left(1 + x_t \frac{\alpha_s}{\pi} N_c C_F \Delta_{VV}\right)$$

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Using large mass expansion:

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and similar for $H \to Z\gamma$.

Analytical solutions are possible in cases of:

- gg or $\gamma\gamma$ — Spira, Harlander
- $Z\gamma$ — Spira, Zerwas

decays $H \to VV$. some results

$$\Gamma(H \to VV) = \Gamma^{\rm EW}_{1-\rm loop}\left(1 + x_t \frac{\alpha_s}{\pi} N_c C_F \Delta_{VV}\right)$$

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Using large mass expansion:

$$\Delta_{WW} = \frac{9}{2} - \zeta_2 + \frac{m_W^2}{m_t^2}\Big[-\frac{3}{2} + \frac{m_W^2}{m_H^2}\Big(-\frac{109}{1080} + \frac{\zeta_2}{2}\Big)\Big] + \ldots$$

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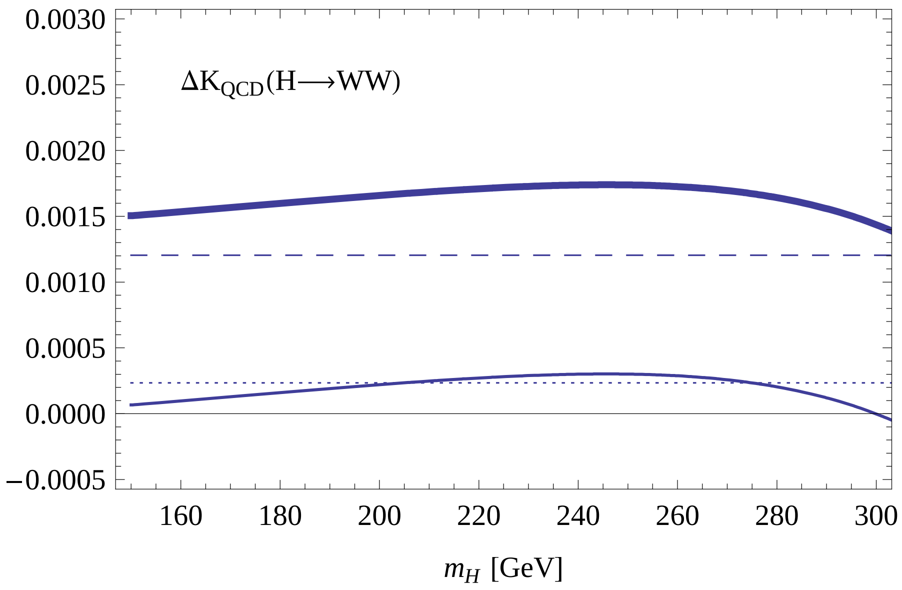
and similar for $H \to Z\gamma$.

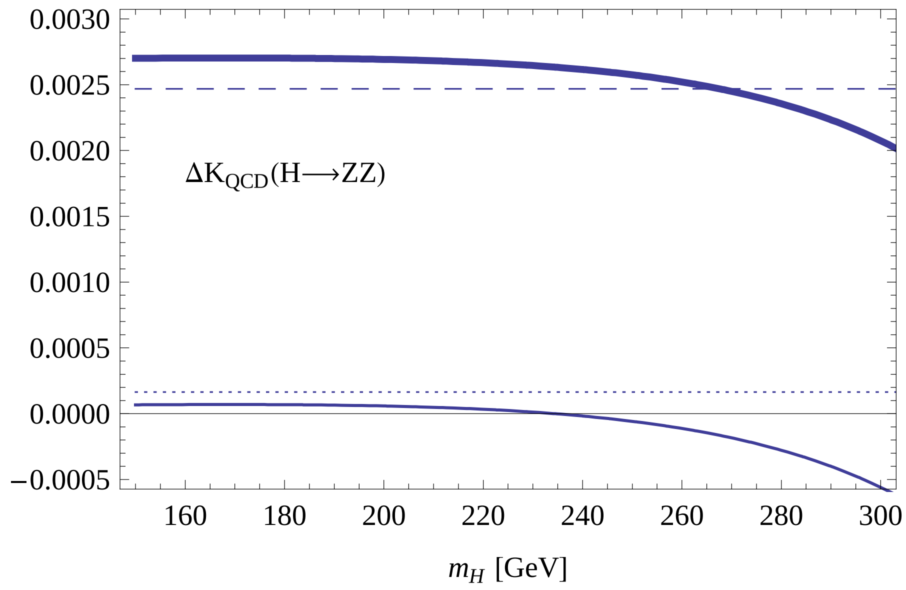
e.g. for $m_H = 200$GeV:

$$\Delta_{WW} = 2.855 + 0.629 - 0.072 - 0.030 - 0.010 + \ldots$$
$$\Delta_{ZZ} = 5.855 + 0.238 - 0.058 - 0.028 - 0.010 + \ldots$$

$(H \to VV)$ numerical evaluation

$$\Gamma = \Gamma^{EW}_{1-\text{loop}} \left(1 + \Delta K_{\text{QCD}}\right)$$





- dashed line — $\alpha_s G_F m_t^2$
- solid line — subleading $\alpha_s G_F$ correction (this calculation)
- dotted line — $\alpha_s^2 G_F m_t^2$
- bold line — full QCD correction

decay $H \to \bar{b}b$

$$\Gamma_0(H \to \bar{b}b) = \frac{3G_F m_H}{4\sqrt{2}\pi}\overline{m}_b^2(m_H)(1 + \delta_{\mathrm{EW}} + \delta_{\mathrm{QDC}} + \delta_{\mathrm{EW/QCD}} + \dots)$$

- QCD correction up to $O(\alpha_s^3)$ — Gorishny, Kataev, Larin, Surguladze '91; Larin, van Ritbergen, Vermaseren '95; Chetyrkin, Kwiatkowski '96
- EW corrections — Kniehl, Sirlin '93; Djouadi, Gambino '94
- leading heavy top terms
  $\Longrightarrow$ pick up the leading corrections in the limit $m_t \to \infty$
  - $O(\alpha_s G_F m_t^2)$ — Djouadi, Gambino, Spira '93,'94
  - $O(G_F^2 m_t^4)$ — Butenschön, Fugel, Kniehl '07
- $O(\alpha\alpha_s)$ correction — this calculation

decays $H \to \bar{b}b$

Need to evaluate:

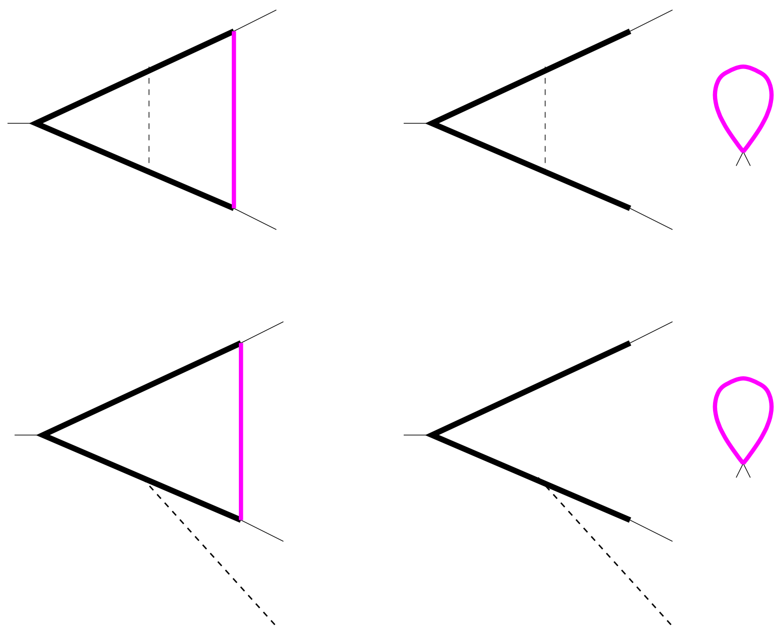
$$H \longrightarrow \bar{b}b, \quad H \longrightarrow g\bar{b}b, \quad H \longrightarrow \gamma\bar{b}b, \quad H \longrightarrow g\gamma\bar{b}b$$

diagrams of 3 types

- with γ-exchange
  $\longrightarrow$ massless diagrams
- with W-exchange (contain top-quark)
  $\longrightarrow$ large mass $(1/m_t)$ expansion
- with Z-exchange
  $\longrightarrow$ differential equations
- "universal" term (e.g. countertems, Δr, pole mass M_b)
  $\longrightarrow$ could be solved without any tricks

heavy top expansion

- consider $m_t^2 \gg m_W^2\,, m_H^2$
- in case of $H \to VV$ the actual parameter is $m_H^2/4m_t^2$
- in case of $H \to \bar{b}b$ the situation is worse
- some improvements are possible (resummations)
- examples:



differential equations

integration by parts identities (IBP):

$$\int d^d k_1 \ldots d^d k_L \ \ \frac{\partial}{\partial k_\mu} \frac{q_\mu}{D_1^{a_1} D_2^{a_2} \ldots D_r^{a_r}} = 0$$

$\Longrightarrow$ algebraic relations among integrals with different sets of $(a_1, a_2, \ldots, a_r)$ and numerators

Use IBP:

- to reduce tensor integrals to the set of *master integrals*
- to write the differential eqution

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differential equations

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$\Longrightarrow$ algebraic relations among integrals with different sets of $(a_1, a_2, \dots, a_r)$ and numerators

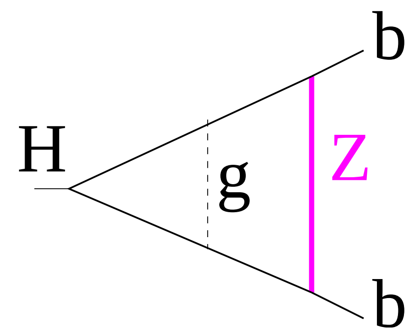
Use IBP:

- to reduce tensor integrals to the set of *master integrals*
- to write the differential eqution

$H \to \bar{b}b$

diagrams with Z-boson $\longrightarrow$ one parameter

- diff. eq. w.r.t. $z = m_H^2/m_Z^2$
- solution in terms of H-functions



relevant functions

- introduce

$$H_{a,b,\ldots,c}(z) = \int_0^z \frac{dx_1}{x_1 - a} \int_0^{x_1} \frac{dx_2}{x_2 - b} \cdots\cdots \int_0^{x_k} \frac{dx_{k-1}}{x_{k-1} - c}$$

- in case $a, b, \ldots, c = +1, -0, 1$ harmonic polylogarithms

- diagrams with HVV coupling require functions with $a, b, \ldots, c = +1, 0, -1, +e^{i\pi/3}, -e^{i\pi/3}$

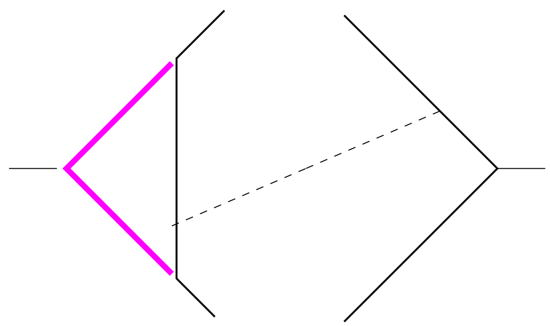
- functions up to weight $4 (= a + b + c + d)$ are required

solutions

- all 2-loop diagrams are the same as in the case of quark formfactor $Z\bar{q}q$

Kühn, Kotikov, OV '03

- some new integrals in three particle cut diagrams



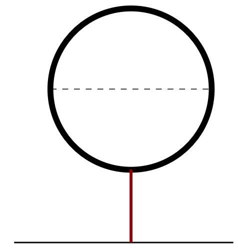
- representation of cut lines

$$\frac{1}{k^2 - m^2} \longleftrightarrow 2i\pi\delta(k^2 - m^2)$$

$\longrightarrow$ usual integration by part identities can be applied

the mass of b quark

- the are potentially large logarithms $L = \ln(m_H^2/m_b^2)$
- RG resummation with $\overline{\mathrm{MS}}$ mass $\overline{m}_b(\mu)$
- then width has tadpole contributions

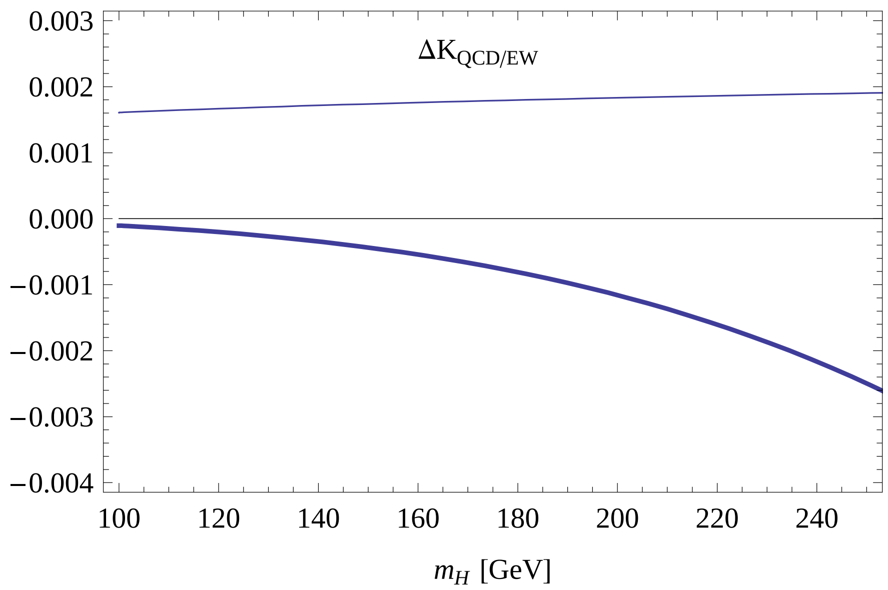


$$\sim \frac{\alpha N_c m_t^4}{\pi m_W^2 m_H^2 \sin^2\theta_W}, \qquad \sim \frac{\alpha\alpha_s N_c m_t^4}{\pi^2 m_W^2 m_H^2 \sin^2\theta_W},$$

- if tadpoles are omitted the mass *generally* is gauge dependent
- in the on-shell scheme there are no contributions from tadpoles
- but there are large logs
- mixed schemes?

$(H \to \bar{b}b)$ numerical evaluation

(preliminary, 1-loop corrections are not included)



- solid line — leading
- bold line — subleading

conclusion

- $O(\alpha\alpha_s)$ corrections are evaluated for the main Higgs boson decay modes in the Higgs boson mass range $m_H \sim 100 - 300$ GeV

- for modes $H \to VV$ the subleading $O(\alpha\alpha_s)$ terms are small but at least as large as the 3-loop leading $O(\alpha\alpha_s^2 m_t^2)$ contributions

- for $H \to \bar{b}b$ the subleading $O(\alpha\alpha_s)$ contribution is of the same order as the leading $O(\alpha\alpha_s m_t^2)$ term