NNLL threshold resummation for the total top-pair production cross section

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14.10.2011

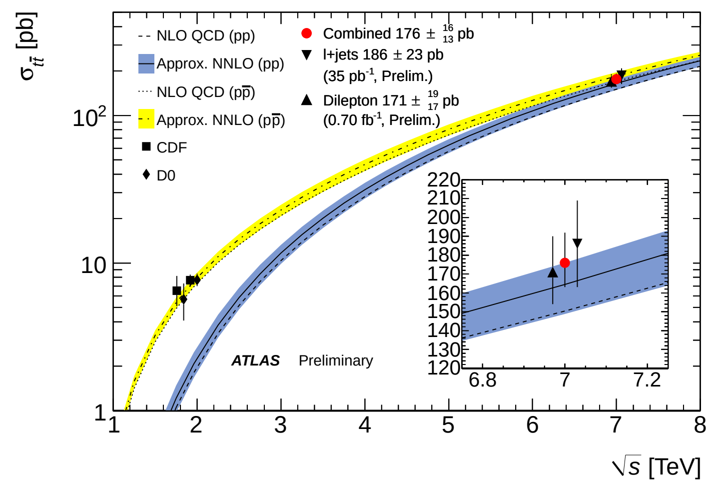
(Based on M.Beneke, P.Falgari, S. Klein, CS, arXiv:1109.1536 [hep-ph])

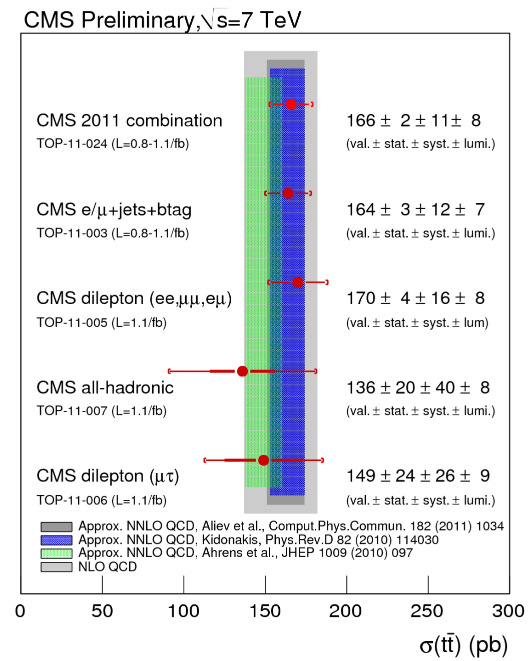
Total cross section measurements
at Tevatron and LHC

$$\Delta\sigma_{t\bar{t}}/\sigma_{tt} \sim 7\% :$$

$$\sigma_{t\bar{t}}^{\text{Tevatron}} = \begin{cases} 7.56^{+0.63}_{-0.56} & \text{(D0)} \\ 7.50^{+0.48}_{-0.48} & \text{(CDF)} \end{cases}$$

$$\sigma_{t\bar{t}}^{\text{LHC @7 TeV}} = \begin{cases} 165.8^{+13.3}_{-13.3} & \text{(CMS)} \\ 179.0^{+11.8}_{-11.8} & \text{(ATLAS)} \end{cases}$$





NLO+NLL QCD:

(Nason, Dawson Ellis 88, Bonciani et al. 98)

$$\sigma_{t\bar{t}}^{\text{Tevatron}} = 6.93^{+0.28+0.51}_{-0.50-0.45}$$

$$\sigma_{t\bar{t}}^{\text{LHC @7 TeV}} = 167.1^{+14.3+13.9}_{-15.4-13.1}$$

$\rightarrow$ Need higher-order predictions!

NNLO: in progress, several ingredients known (Czakon et al.; Bonciani et al. 08-11; Körner et al. 05-09, Anastasiou/Mert-Aybert 08;. . .)

Progress for soft gluon resummation:

massive 2-loop IR singularities (Becher/Neubert; Ferroglia et al. 09)

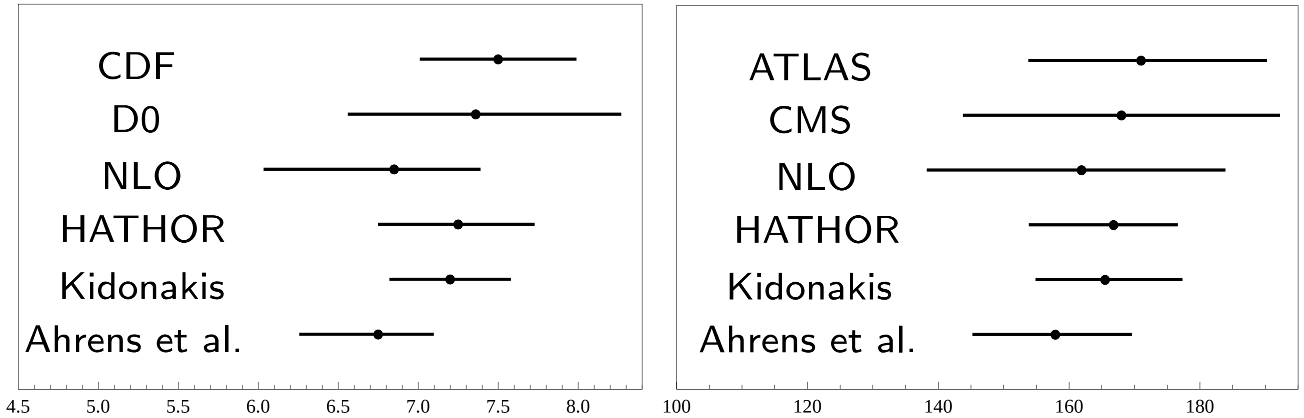
2-loop soft anomalous dimension

(Kidonakis; Mitov/Sterman/Sung; Beneke/Falgari/CS;Czakon Mitov/Sterman 09)

NNLO$_{\mathbf{app.}}$ (Moch/Uwer(Langenfeld) 08/09, Beneke et al.; Ahrens et al. 09, Kidonakis 10)

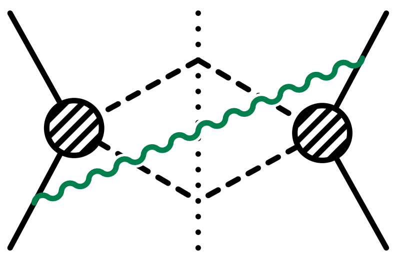
NNLL resummation (Ahrens et al. 10/11; Beneke et al. 11, Cacciari et al. 11)

Status of NNLO$_{\text{approx}}$ results (Review: Kidonakis/Pecjak 11)

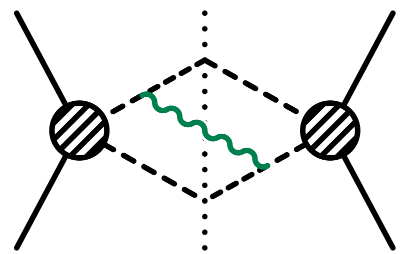


Threshold resummation

Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et al. 98, ...)

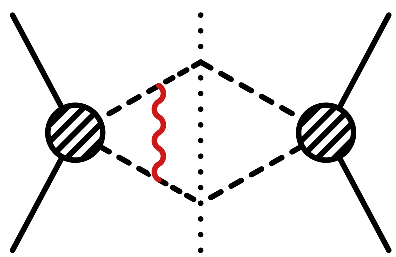


$\Rightarrow \alpha_s \; \log^2(8\beta^2)$



$\Rightarrow \alpha_s \; \log(8\beta^2)$

Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)



$\Rightarrow \alpha_s \, \frac{1}{\beta}$

Counting of threshold corrections:

$$\hat{\sigma}_{pp'} \quad \propto \quad \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \dots\} :$$

- Factorization into soft, hard and Coulomb functions

(Beneke, Falgari, CS 10)

$$\hat{\sigma}_{pp' \to HH'}|_{\hat{s} \to 4M^2} = \sum_{R,i} H_i W_i^R \otimes J^R$$

- can include Coulomb resummation to all orders

((N)LO Coulomb-Green function: Fadin/Khoze 87; Beneke/Signer/Smirnov 99,...)

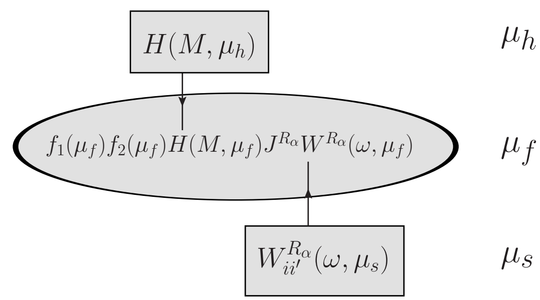
- RGEs for hard and soft function

(Becher, Neubert; Ferroglia et al.;

Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

Momentum space solution

(Becher/Neubert 06)



- $\mu_h \sim 2m_t$, Choice of μ_s:
  - RGE approach: fixed μ_s (Becher, Neubert, Xu 07)
  - Running scale $\mu_s \sim m_t \beta^2$ (Beneke, Falgari, Klein, CS 11)

  (frozen to $\mu_s \sim m_t \beta^2_{\text{cut}}$ for $\beta < \beta_{\text{cut}} = 0.35$ (Tevatron), 0.54 (LHC))

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC (7 TeV)	LHC (14 TeV)
NLO	$6.68^{+0.36+0.51}_{-0.75-0.45}$	$158.1^{+19.5+13.9}_{-21.2-13.1}$	$884^{+107+65}_{-106-58}$
NLL	$7.31^{+0.40+0.57}_{-0.54-0.54}$	$172.8^{+20.3+15.9}_{-15.5-14.6}$	$954^{+111+74}_{-76-66}$
NNLO$_{\text{app}}$	$7.06^{+0.27+0.69}_{-0.34-0.53}$	$161.1^{+12.3+15.2}_{-11.9-14.5}$	891^{+76+64}_{-69-63}
NNLL	$7.22^{+0.31+0.71}_{-0.47-0.55}$	$162.6^{+7.4+15.4}_{-7.6-14.7}$	896^{+40+65}_{-37-64}

(MSTW08PDFs, $m_t = 173.3$ GeV)

Different uncertainties for NNLL (added in quadrature above):

- Scale variation (μ_f, μ_h, μ_C):

$$\Delta_\mu \sigma_{\text{NNLL}}(\text{TeV}) = \begin{matrix}+0.21\\-0.41\end{matrix}, \quad \Delta_\mu \sigma_{\text{NNLL}}(\text{LHC7}) = \begin{matrix}+4.2\\-1.9\end{matrix}$$

- Uncertainty in resummation procedure:

  (vary β_{cut} by 20%, envelope of various approximations, ambiguity $E \leftrightarrow m_t\beta^2$)

$$\Delta_{\text{Res}} \sigma_{\text{NNLL}}(\text{TeV}) = \begin{matrix}+0.20\\-0.21\end{matrix}, \quad \Delta_{\text{Res}} \sigma_{\text{NNLL}}(\text{LHC7}) = \begin{matrix}+3.9\\-5.6\end{matrix}$$

- Estimate of missing constant at $\mathcal{O}(\alpha_s^2)$

$$\Delta_{\text{Const}} \sigma_{\text{NNLL}}(\text{TeV}) = \pm 0.10\,, \quad \Delta_{\text{Const}} \sigma_{\text{NNLL}}(\text{LHC7}) = \pm 4.7$$

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC (7 TeV)	LHC (14 TeV)
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(MSTW08PDFs, $m_t = 173.3$ GeV)

NNLL results include

- Coulomb effects beyond NNLO
- bound-state corrections below threshold

  (take $\Gamma_t \neq 0$ approximately into account)

$\Rightarrow$ small effect:

$\Delta\sigma_{\text{BS}} = 0.014$pb (Tevatron), 0.67pb (LHC7), 3.1pb (LHC14)

$\Delta\sigma_{\text{C}} = -0.052$pb (Tevatron), 0.13pb (LHC7), -0.3pb (LHC14)

Fit m_t-dependence of NNLL-cross-section:

$$\sigma^{\text{th}}_{t\bar{t}}(m_t^{\text{pole}}) = \left(\frac{172.5}{m_t^{\text{pole}}}\right)^4 \left(c_0 + c_1(m_t^{\text{pole}} - 172.5) + c_2(m_t^{\text{pole}} - 172.5)^2 + c_3(m_t^{\text{pole}} - 172.5)^3\right) \text{ pb},$$

$$c_0 = 166.5,\ c_1 = -1.15,\ c_2 = 5.1\times10^{-3},\ c_3 = 8.5\times10^{-5}$$

ATLAS-CONF-2011-121:

$$\sigma_{t\bar{t}} = 179.0 \pm 11.8\,\text{pb}$$

Dependence on m_t^{MC}:

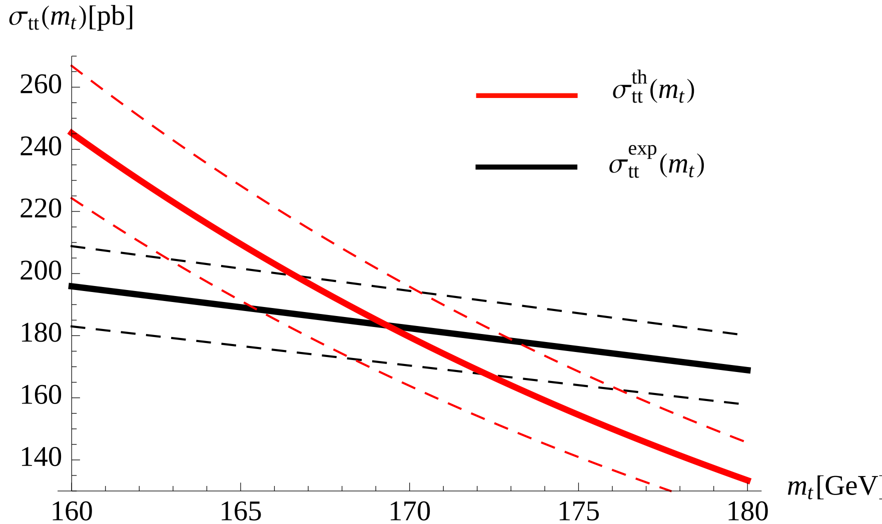
$$\sigma^{\text{exp}}_{t\bar{t}}(m_t^{\text{MC}}) = \left(411.9 - 1.35\,m_t^{\text{MC}}\right)\text{ pb}$$

maximize joined likelihood

$$f(m_t) = \int f_{\text{th}}(\sigma|m_t)\cdot f_{\text{exp}}(\sigma|m_t)d\sigma\,,$$

with normalized Gaussians $f_{\text{th/exp}}$

$$\Rightarrow m_t^{\text{pole}} = (169.8^{+5.2}_{-5.0})\text{ GeV}$$



CMS result from $\sigma_{t\bar{t}} = 169.9 \pm 18.4$ pb:

Approx. NNLO × MSTW08NNLO	m_t^{pole} / GeV	$m_t^{\overline{\text{MS}}}$ / GeV
Langenfeld et al. [7]	$170.3^{+7.3}_{-6.7}$	$163.1^{+6.8}_{-6.1}$
Kidonakis [8]	$170.0^{+7.6}_{-7.1}$	–
Ahrens et al. [9]	$167.6^{+7.6}_{-7.1}$	$159.8^{+7.3}_{-6.8}$

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC (7 TeV)	
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NNLO$_{\text{app}}$+NNLL	$7.22\,^{+0.31+0.71}_{-0.47-0.55}$	$162.6\,^{+7.4+15.4}_{-7.6-14.7}$	(Beneke, Klein, Falgari, CS 11)
NLO+NNLL$_N$	$6.72\,^{+0.24+0.16}_{-0.41-0.12}$	$158.7\,^{+12.2+4.3}_{-13.5-4.4}$	(Cacciari et al. 11)

Mellin space: $(\rho = 4m_t^2/\hat{s})$ (Sterman 87; Catani, Trentadue 89)

$$\sigma^N = \int_0^1 d\rho\rho^{N-1}\hat{\sigma}(4m_t^2/\rho)\ , \qquad \int_0^1 d\rho\rho^N\beta\log^n\beta \propto \ln^n N + \ldots$$

$\log N$-terms exponentiate: $\hat{\sigma}^N/\hat{\sigma}^{(0)N} = g^0\ \exp\left(G^{N+1}(m_t^2,\mu^2)\right)$

Inverse numerical transformation (Catani, Mangano, Nason, Trentadue 96)

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC (7 TeV)	
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Ambiguities in N-space results (Cacciari et al. 11)

- $\mathcal{O}(\alpha_s^2)$ constant in threshold expansion from Mellin transform

$$\Delta\sigma_{tt} = 0.25 \text{ pb (TeV)}, 9.2 \text{ pb (LHC 7TeV)}$$

- $1/N$ suppressed terms $\sim \left(1 - \frac{A}{A+N-1}\right)$ in NLO g_0, take $A = 0, 2$

$$\Delta\sigma_{tt} = 0.12 \text{ pb (TeV)}, 6 \text{ pb (LHC 7TeV)}$$

- Matching to NLO or NNLO$_{\text{app}}$: affects $1/N$ terms at NNLO

Comparison to other NNLO$_{\text{app}}$/NNLL predictions

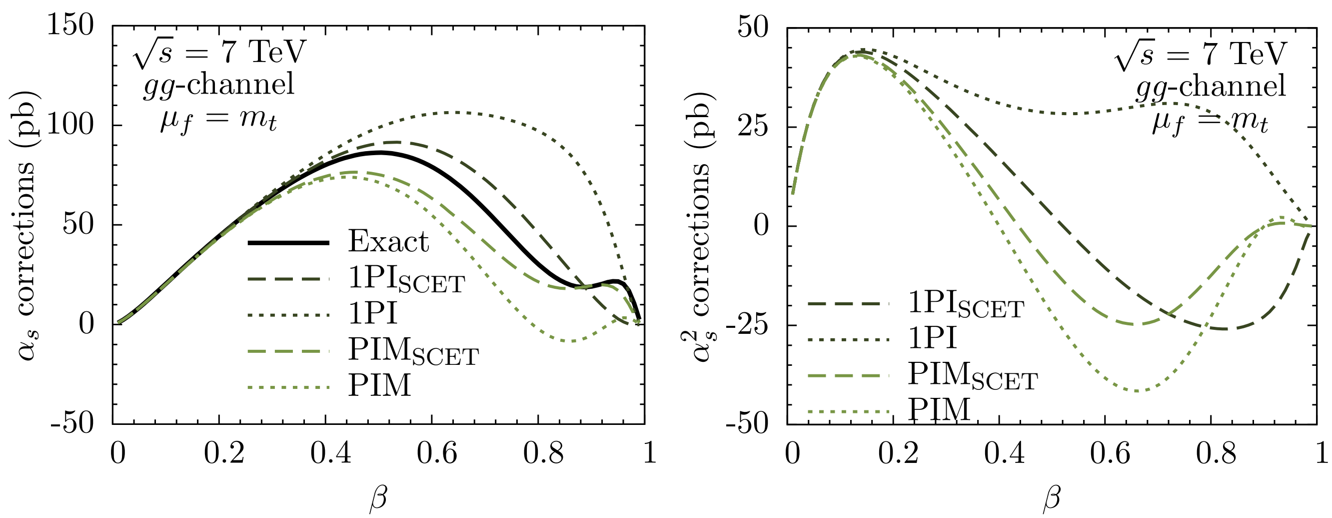
Resummation for differential cross sections

Pair invariant mass cross sections (Kidonakis,Sterman 97, Ahrens et al. 10)

$$\frac{d\hat{\sigma}(t\bar{t})}{dM_{t\bar{t}}} \Rightarrow \left[\frac{\log^n(1-z)}{1-z}\right]_+ , \; z = \frac{M_{t\bar{t}}^2}{\hat{s}} , \qquad \text{PIM}_{\text{SCET}} : \log\left(\frac{1-z}{\sqrt{z}}\right)$$

One particle inclusive cross sections: (Laenen et al. 98, Ahrens et al. 11)

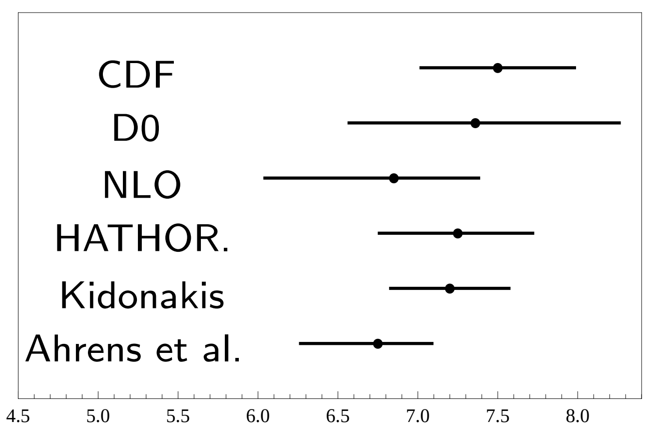
$$\frac{d\hat{\sigma}(t+X)}{ds_4} \Rightarrow \left[\frac{\log^n\left(s_4/m^2\right)}{s_4}\right]_+ ; \; s_4 = p_X^2 - m_t^2 , \qquad \text{1PI}_{\text{SCET}} : \log\left(s_4/\sqrt{m^2+s_4}\right)$$

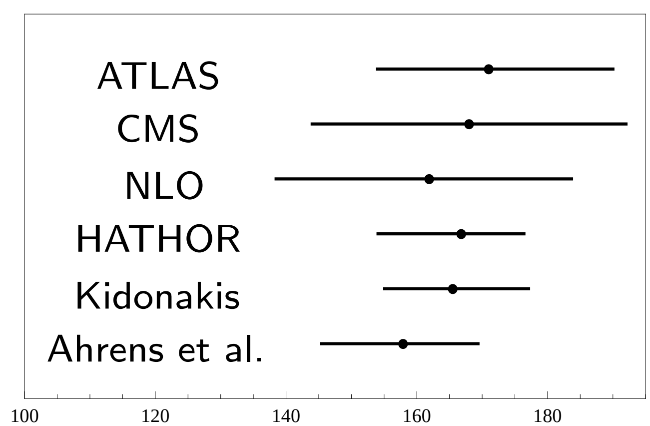


Comparison to other NNLO$_{\text{app}}$ predictions: (from Pejack/Kidonakis 11)

		Tevatron	LHC (7 TeV)
NLO		$6.74^{+0.36}_{-0.76}$	160^{+20}_{-21}
NNLO$^{\beta}_{\text{app}}$	[HATHOR]	$7.13^{+0.31}_{-0.39}$	164^{+3}_{-9}
NNLO$^{\text{1PI}}_{\text{app}}$	[Kidonakis]	$7.08^{+0.00}_{-0.24}$	163^{+7}_{-5}
NNLO$^{\text{1PI/PIM SCET}}_{\text{app}}$	[Ahrens et al.]	$6.65^{+0.08}_{-0.41}$	156^{+8}_{-9}
NNLL$^{\beta}$	[This work]	$7.29^{+0.31}_{-0.47}$	164^{+8}_{-8}

($m_t = 173$, PDF+α_s uncertainties not shown)





Summary

Threshold corrections $\sim \log^n \beta$, $\frac{1}{\beta^n}$

- Factorization of soft and Coulomb corrections
- $\log \beta$ resummation from momentum space solution to RGEs
- combined Soft and Coulomb resummation possible

NNLL resummation for $t\bar{t}$

- Estimate of residual uncertainty

  kinematic ambiguities, uncertainties in resummation procedure
- small effects beyond NNLO (2 % Tevatron, 1 % LHC)
- theory uncertainty $+4.2/-6.5\%$ Tevatron, $\pm 4.5\%$ LHC

  (PDF+α_s uncertainty larger) (Ahrens et al. 10/11)

Mass determination from $\sigma_{t\bar{t}}$:

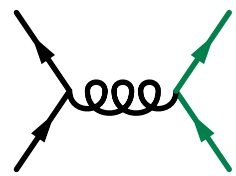
$$\Delta m_t \approx \pm 5 \text{ GeV}$$

appears viable

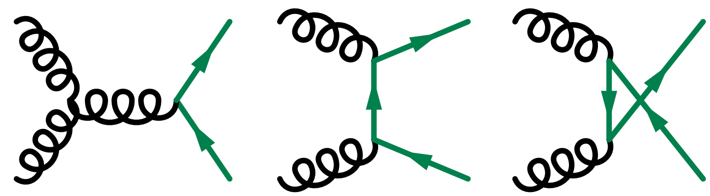
Threshold behaviour

Top-pair production: two LO subprocesses:

$$q\bar{q} \to t\bar{t}: \begin{cases} 90\% & \text{Tevatron} \\ 20-10\% & \text{LHC}7-14 \end{cases}$$



$$go \to t\bar{t}: \begin{cases} 10\% & \text{Tevatron} \\ 80-90\% & \text{LHC}7-14 \end{cases}$$



Behaviour at production threshold $\hat{s} \sim 4m_t^2$: $\left(\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}\right)$

- $q\bar{q}$ channel: colour octet, spin triplet

$$\hat{\sigma}^{(8)}_{q\bar{q}} = \frac{\pi\beta}{9m_t^2}\left[1 + \frac{\alpha_s}{4\pi}\left(\frac{-2\pi^2}{2N_c}\frac{1}{\beta} + 8C_F \log^2 8\beta^2 - (32C_F + 4N_C)\log 8\beta^2\right) + \dots\right]$$

- gg channel: colour singlet/octet, spin singlet

$$\hat{\sigma}^{(1)}_{gg} = \frac{5\pi\beta}{192m_t^2}\left[1 + \frac{\alpha_s}{4\pi}\left(2C_F\pi^2\frac{1}{\beta} + 8N_C \log^2 8\beta^2 - 32N_C \log 8\beta^2\right) + \dots\right]$$

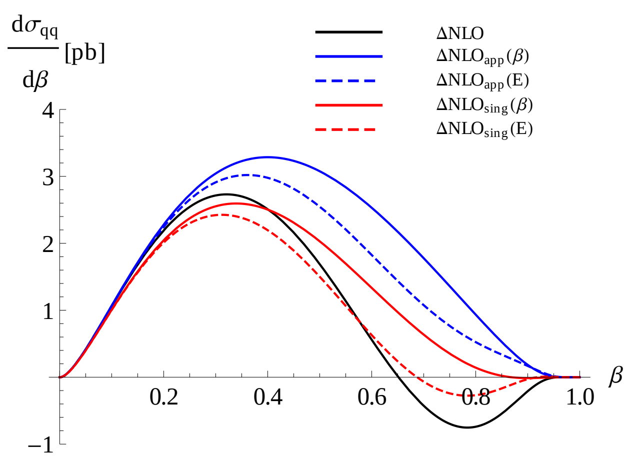
$$\hat{\sigma}^{(8)}_{gg} = \frac{\pi\beta}{96m_t^2}\left[1 + \frac{\alpha_s}{4\pi}\left(\frac{-2\pi^2}{2N_C}\frac{1}{\beta} + 8N_C \log^2 8\beta^2 - (32N_C + 4N_C)\log 8\beta^2\right) + \dots\right]$$

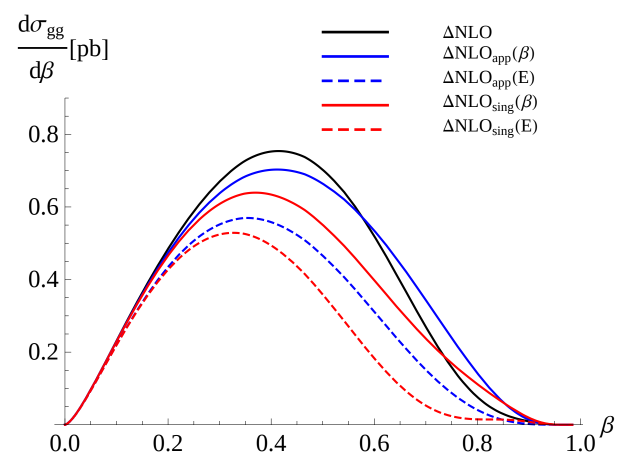
$\Rightarrow$ Universal behaviour depending on initial/final colour states

NLO$_{\text{sing}}$: only $\log\beta$, $1/\beta$-terms

NLO$_{\text{app}}$: also constant terms (hard/soft function)

Ambiguity: $E = \sqrt{\hat{s}} - 2m_t \approx m_t\beta^2$





$\frac{d\sigma}{d\beta} = \frac{8\beta m_{\tilde{q}}^2}{s(1-\beta^2)^2} L(\beta, \mu_f)\hat{\sigma}$

(Tevatron, L: parton luminosity (MSTW08))

⇒ large ambiguities, but exact result covered: ($\mu_f = \mu_r = m_t = 173.1$ GeV)

$\sigma_{\text{NLO,app}}(\text{Tev}) = 6.42 - 7.45$pb $\quad$ $\sigma_{\text{NLO}}(\text{Tev}) = 6.50$pb

$\sigma_{\text{NLO,app}}(\text{LHC7}) = 130 - 158$ pb $\quad$ $\sigma_{\text{NLO}}(\text{LHC7}) = 150$ pb

Coulomb resummation

in Coulomb Green function: (Fadin, Khoze 87; Peskin, Strassler 90)

$$\hat{\sigma}_{t\bar{t}}(\hat{s}) \sim \sum_{R=1,8} \sigma^{0,R}_{t\bar{t}}(\hat{s}) \mathrm{Im}\, G^R_{\mathrm{C}}(0,0;E+i\Gamma)$$

Singlet channel, neglecting decay width:

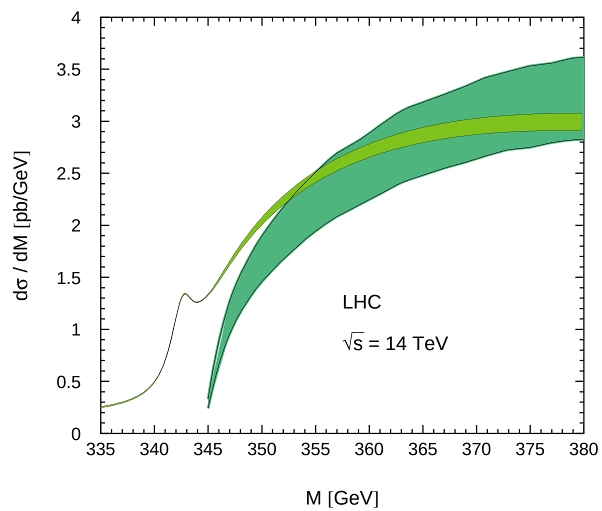
$$\mathrm{Im}\, G^1_{\mathrm{C}}(0,0;E) = \begin{cases} \frac{-m_t^2 \pi C_F \alpha_s}{4\pi} \left(e^{-\pi C_F \alpha_s \sqrt{\frac{m_t}{E}}} - 1\right)^{-1} & E > 0 \\ \sum_{n=1}^{\infty} \delta(E - E_n) R_n & E < 0 \end{cases}$$

$$E = \sqrt{\hat{s} - 2m_t} \approx m_t \beta^2.$$

Bound-state poles at

$$E_n = -\frac{\alpha_s^2 C_F^2 m_t}{4n^2}$$

smeared out by $\Gamma_t \neq 0$.



(Hagiwara et al. 08, Kiyo et al. 08)

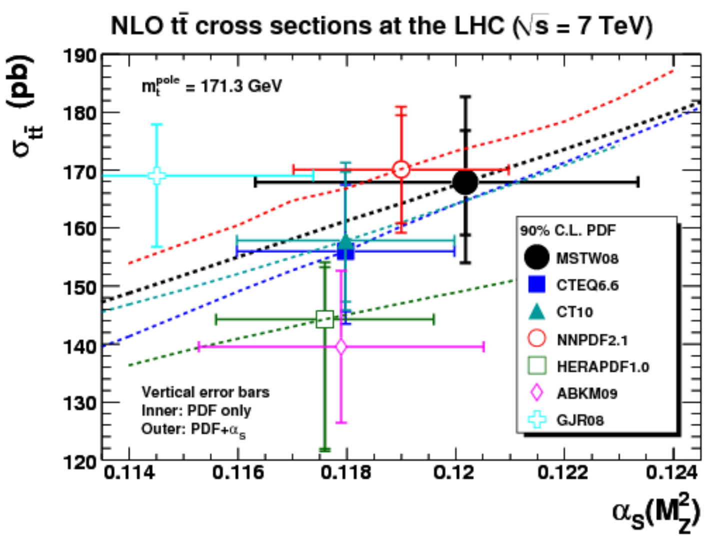
Hadron collider cross sections from QCD factorization

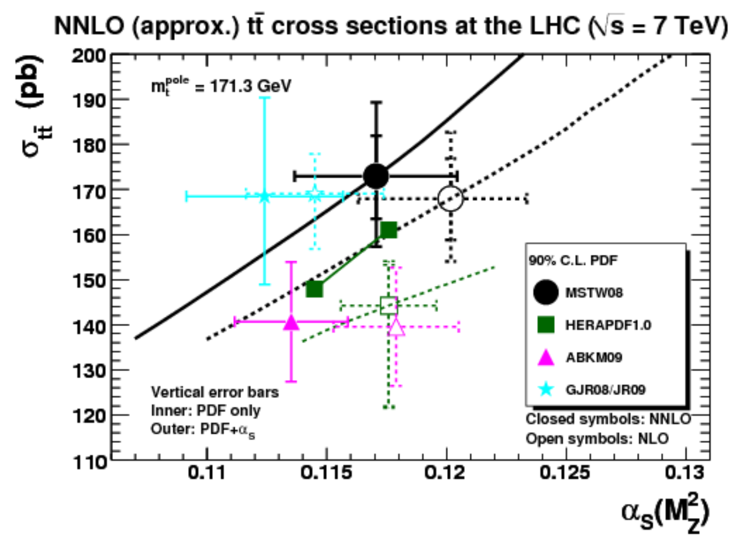
(Collins, Soper, Sterman)

$$\sigma_{NN'}(s) = \sum_{pp'} \int dx_1 dx_2 \; f_{N/p}(x_1,\mu_f) f_{N'/p'}(x_2,\mu_f)\; \hat{\sigma}_{pp'}(s x_1 x_2,\mu_f)$$

- $\hat{\sigma}_{pp'}$: partonic cross section: compute in perturbation theory
- $f_{p/N}(x)$: Parton distribution function for parton p in hadron N :

PDF uncertainties for top: (Watt, 11)



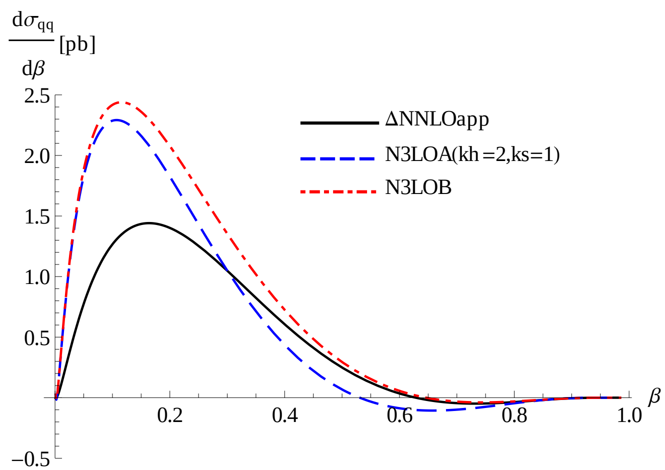


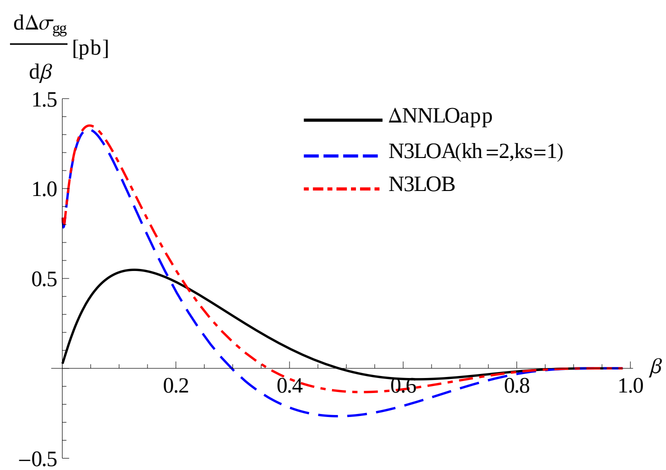
Expand NNLL to $\mathcal{O}(\alpha_s^3)$, e.g.

$$\Delta\sigma^{(3)}_{qq,\text{NNLL}} = 12945.4\log^6\beta - 37369.1\log^5\beta + 27721.4\log^4\beta + 41839.4\log^3\beta + \frac{1}{\beta}\left(-6278.5\log\beta + 3862.5\log^2\beta + 2804.7\log^3\beta - 2994.5\log^4\beta\right) + \frac{153.9\log^2\beta + 122.9\log\beta - 145}{\beta^2} + \underbrace{\left\{\log\beta^{1,2}, 1/\beta, C^{(3)}\right\}}_{\text{not known exactly}} + \text{scale dep.}$$

N^{3}LO$_A$: keep all terms, including k-dependence and constants

N^{3}LO$_B$: only keep terms known exactly





RGE approach: fixed μ_s that minimizes soft corrections to hadronic σ (Becher, Neubert, Xu 07)

Application to $t\bar{t}$:

- ambiguities in soft scale from minimising $\sigma^{\text{NLO}}_{\text{soft}}(\mu)$ (e.g. LHC7):

$$\mu_s = 58\text{GeV} \quad (\log m_t\beta^2/\mu) \Leftrightarrow \quad \mu_s = 99\text{GeV} \quad (\log E/\mu = \log(\sqrt{\hat{s}} - 2m_t)/\mu)$$

- does resum logarithms *on avarage*; appropriate for $< \beta > \approx 0.5$?

$\sigma_{t\bar{t}}$	Tevatron	LHC (7 TeV)	LHC (14 TeV)
NLO	$6.68^{+0.36}_{-0.75}$	$158.1^{+19.5}_{-21.2}$	884^{+107}_{-106}
NLL	$6.90^{+0.32}_{-0.41}$	$157.6^{+23.3}_{-20.2}$	876^{+136}_{-113}
NNLO_{app}	$7.06^{+0.25}_{-0.33}$	$161.1^{+11.4}_{-10.9}$	891^{+71}_{-63}
NNLL	$7.08^{+0.17}_{-0.28}$	$157.4^{+10.3}_{-3.6}$	868^{+69}_{-21}

(MSTW08PDFs, $m_t = 173.3$ GeV, PDF+α_s-uncertainty not included,

(N)NLL error includes μ_h/μ_f, μ_s,μ_C variation, $m_t\beta^2 \leftrightarrow \sqrt{s} - 2m_t$ difference)

Soft scale choice in momentum-space resummation

- RGE approach: fixed μ_s, vary by $0.5\ldots 2$ (Becher, Neubert, Xu 07)
- Running scale with cutoff (Beneke, Falgari, Klein, CS 11)

$$\mu_s \sim \begin{cases} m_t\beta_{\text{cut}}^2, & \text{for} \beta < \beta_{\text{cut}} \\ m_t\beta^2, & \text{for} \beta > \beta_{\text{cut}} \end{cases}$$

$\mathcal{O}(\alpha_s^2)$ **constant** in threshold expansion

$$\Delta\sigma_{tt}^{(2)}(7\text{TeV}) = \left[1.70\left(\frac{C_{qq}^{(2)}}{1000}\right) + 4.31\left(\frac{C_{gg,\mathbf{8}}^{(2)}}{1000}\right) + 1.31\left(\frac{C_{gg,\mathbf{1}}^{(2)}}{1000}\right)\right]\text{pb}$$

Estimate

$$C_2 \approx C_1^2\text{:} \qquad \Delta\sigma_{tt}^{(2)} \sim \pm 5 \text{ pb}$$

Kinematic ambiguity

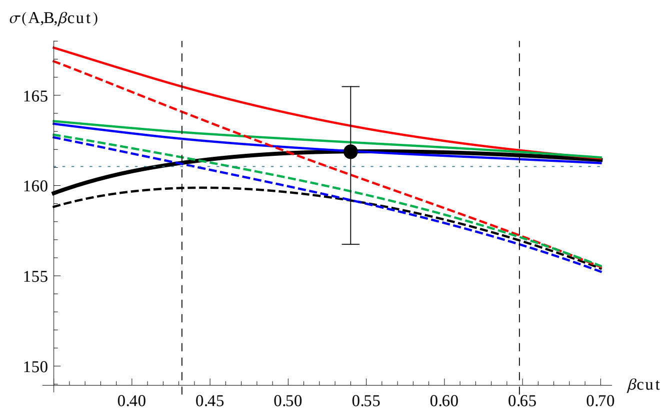
$$\log\beta \Leftrightarrow \log((\sqrt{\hat{s}} - 2m_t)/m_t) : \qquad \Delta\sigma_{tt} \sim 1 \text{ pb}$$

Running scale: Introduce β_{cut}

- allow for different implementations

  $\beta < \beta_{\textbf{cut}}$: NNLL ($\mu_s = k_s m_t \beta^2_{\text{cut}}$) with/without constant at $\mathcal{O}(\alpha_s^2)$

  $\beta > \beta_{\textbf{cut}}$: NNLL ($\mu_s = k_s m_t \beta^2$); NNLO$_{\text{approx}}$; NNNL$_3$(A/B)



- Choose β_{cut} so that not too sensitive to
  - ambiguities for $\beta \to 1$
  - breakdown of perturbation theory for $\beta \to 0$

(E.g. LHC7: $\mu_s = 2m_t\beta^2$, $\beta_{\text{cut}} = 0.54 \Rightarrow \mu_s > 100$ GeV)