Single-Top-Quark Production: Towards Increased Theoretical Precision

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in collaboration with

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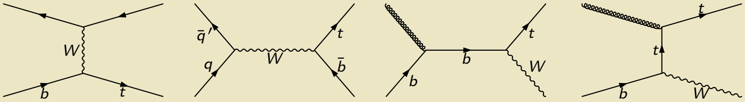
HUMBOLDT-UNIVERSITÄT ZU BERLIN

5th Annual Helmholtz Alliance Workshop
Physics at the Terascale

Single Top Production



- inspect electroweak interactions of top ($|V_{tb}|$)
  $V_{tb} \neq 1$ possible if unitarity constraints on 3×3 CKM matrix is relaxed (e.g., by fourth generation)
  [Alwall, Frederix, Gerard, Giammanco, Herquet, Kalinin, Kou, Lemaitre, Maltoni '07]
- source of polarised top quarks
- sensitive to new physics
- access to the b quark PDFs

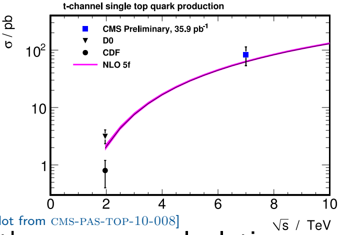
NNLO Corrections to Single Top Production

- why are NNLO corrections needed?
- how do we calculate them?
- what is the status?

Experimental and Theoretical Status

total production cross section t-channel (in pb)

experiment	measurement	theory prediction
D0	$3.14^{+0.94}_{-0.80}$	2.1 ± 0.1
CDF	$0.8^{+0.4}_{-0.4}$	
CMS	83.6 ± 29.8	64.2 ± 2.6
ATLAS	90^{+32}_{-22}	



[plot from CMS-PAS-TOP-10-008]

theory: NLO calculations and threshold corrections

[Giele, Keller, Laenen '95; Harris, Laenen, Phaf, Sullivan, Weinzierl '02; Campbell, Ellis, Tramontano '04; Campbell, Tramontano '04; Campbell, Frederix, Maltoni, Tramontano '09; Kidonakis '06 – '11]

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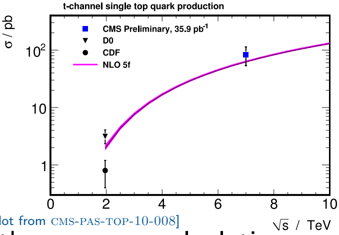

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Errors small enough?

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Error Estimates in Perturbation Theory

- observables do not depend on μ_r and μ_f, results at some fixed order do

$\Rightarrow$ scale dependence has to be canceled by higher orders

$\rightsquigarrow$ expect the higher order corrections to be comparable to the scale variation

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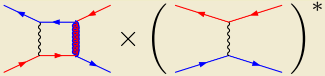
  results at some fixed order do

$\Rightarrow$ scale dependence has to be canceled by higher orders

$\rightsquigarrow$ expect the higher order corrections
to be comparable to the scale variation

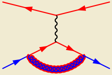
- not rigorous
- expect to fail when new features appear at the next order

no colour exchange up to NLO



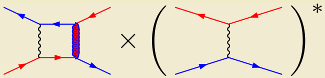
$$\propto \mathrm{tr}[T_a]\,\mathrm{tr}[T_a] = 0$$

only vertex corrections contribute



Colour Exchange Between Quark Lines

no colour exchange up to NLO



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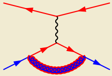
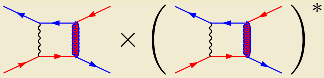
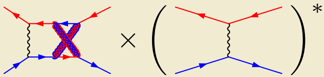


colour exchange at NNLO



$\neq 0$



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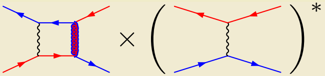

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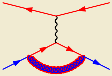

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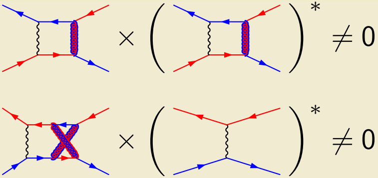


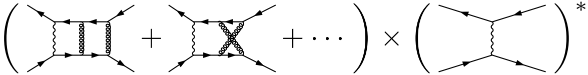
diagrams with colour exchange not covered by uncertainty estimates

- significant effect on kinematic distributions to be expected
- distributions important for detailed measurements, i.e. test $V - A$, polarisation studies, anomalous couplings, ...

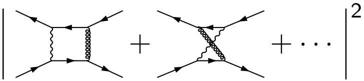
$\Rightarrow$ full NNLO calculation imperative

NNLO: Building Blocks

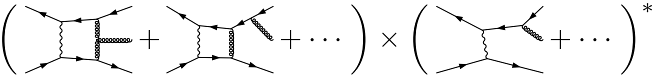
- 2-loop corrections interfered with Born

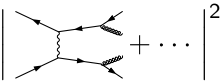


- 1-loop corrections squared



- 1-loop corrections to single top + 2 partons





Evaluation of the Virtual Corrections

1. write down all contributing diagrams

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3. reduce the number of integrals that need to be calculated
   use linear "integration-by-parts" equations between integrals
   express everything through a small set of "master integrals"

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4. evaluate the integrals

Reduction to Master Integrals

linear relations: Integration by Parts (IBP) identities

$$\int d^D k_1 \cdots d^D k_l \frac{\partial}{\partial k_i^\mu} \frac{p^\mu}{D_1^{\nu_1} \cdots D_n^{\nu_n}} = 0$$

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Laporta's algorithm: generate many equations of this type and solve the resulting overdetermined set of equations

- AIR [Anastasiou, Lazopoulos '04]
- FIRE [Smirnov '08]
- CRUSHER [Marquard, Seidel (to be published)]
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coefficients depend on kinematic invariants and D

$\Rightarrow$ difficulty rises with number of masses and legs

bottleneck

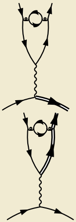
Overview: Diagrams

diagrams, grouped by colour factor

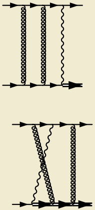
$N_c C_A C_F$

$N_c C_F^2$

$N_c T C_F$



$T C_F$



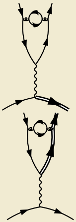
Overview: Diagrams

diagrams, grouped by colour factor

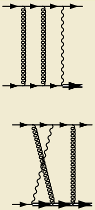
$N_c C_A C_F = 32$

$N_c C_F^2 = \frac{16}{3}$

$N_c T C_F = 2$



$T C_F = \frac{2}{3}$

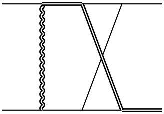


- leading colour given by vertex corrections

calculate the vertices first

Bottleneck

most complicated topology:



- two loops, seven lines (three massive)
- large number of kinematical invariants:
  - two different masses m_t, m_W
  - three independent massless external momenta $\Rightarrow s, t$
  - space-time dimension D

after rescaling: 4 invariants

Conclusions

- single top quark production:
  unique possibility to study top quark properties
- for optimal exploitation, full NNLO calculation is needed

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- for optimal exploitation, full NNLO calculation is needed
- status of calculation: leading colour in good shape
  - ✓ reduction to master integrals
  - ☐ calculation of all masters
  - ✓ performed checks with the literature
- subleading colour: work in progress