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Black Holes and the Standard Model,

defining a new regime for theoretical approaches

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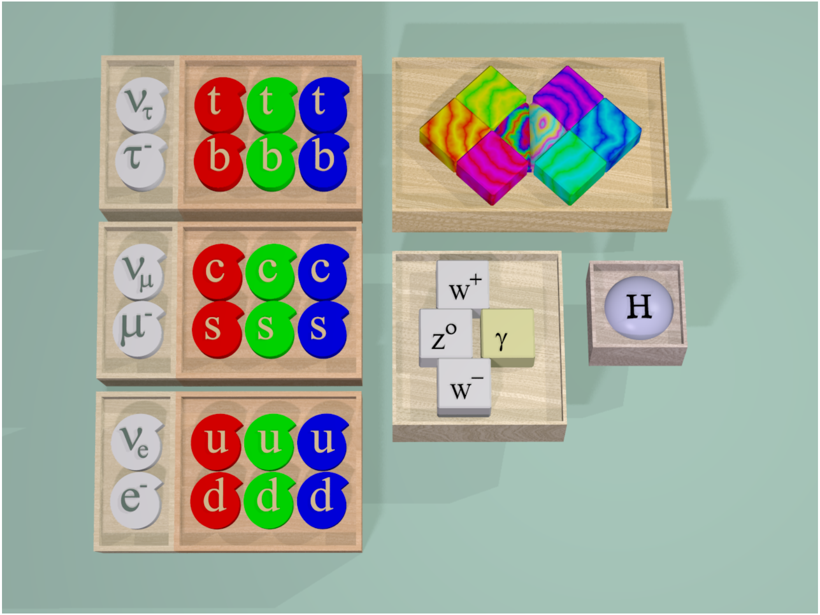
DESY Hamburg

The Standard Nodel.



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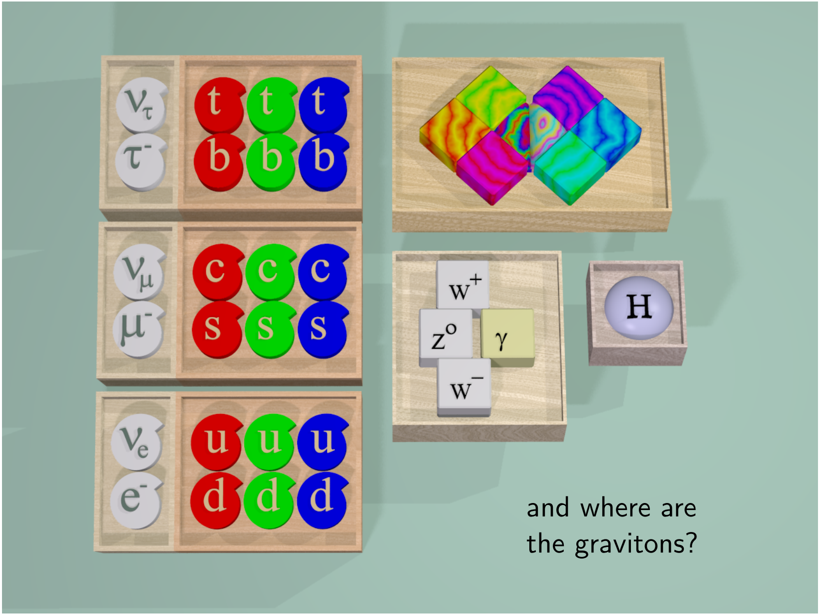

Gravity should be reconciled with particle physics.
This seems to be too difficult to accomplish, but perhaps all we need is strict logic.

Both Quantum Field Theory and gravity physics can be improved.

Gravity only depends on mass, energy and momentum, and acts only by bending space and time.

The SM has various kinds of quantum charges that carry information, while also defining mass, energy and momentum.

Even without SM, pure gravity alone generates matter. Two kinds:

- gravitons (spin 2, charge $= 0$, massless, pointlike)
- black holes (heavy, extended, massive, can have spin and charge)

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they seem to behave exotically.

Black holes are more than just odd-looking curiosities in gravity theory. They uniquely intertwine the basic principles of General Relativity with those of quantum theory.

As such, BH are more elementary than, say, the magnetic monopoles that emerge in the Standard Model.

The quantum laws inside black holes cannot be tested experimentally; it is conceivable that modifications are needed.

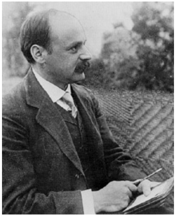
The most probable source for progress in our understanding will come from unification of the Standard Model particles, not only with gravitons but also with black holes, in one theory.

Maybe “quantum black holes” will fit in some scheme much like the periodic table of the elements – here we probably will have an infinite table, but conceptually it will be just the same idea.

Let's try.

Schwarzschild metric:

(units $c = \hbar = G_N = 1$)



$$\mathrm{d}s^2 = -\mathrm{d}t^2 (1 - \tfrac{2M}{r}) + \frac{\mathrm{d}r^2}{1 - \frac{2M}{r}} + r^2 \mathrm{d}\Omega^2$$

$$\Omega \equiv (\theta, \varphi) ,$$

$$\mathrm{d}\Omega \equiv (\mathrm{d}\theta, \sin\theta\, \mathrm{d}\varphi) .$$

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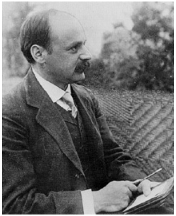

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\Omega &\equiv (\theta,\,\varphi)\ , \\
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\end{aligned}
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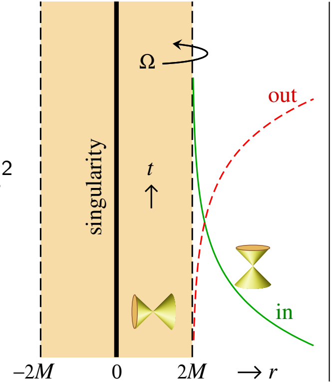

A singularity at $r = 0$, and

Schwarzschild metric:

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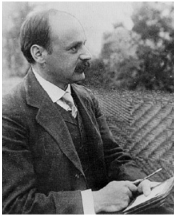

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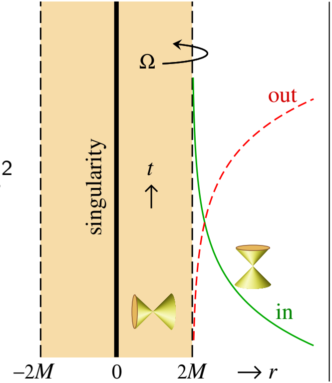

A singularity at $r = 0$, and an apparent singularity at $r \to 2M$.
To see that this is not a singularity at all, use other coordinates:

$$
\begin{aligned}
\mathrm{d}s^2 &= -\mathrm{d}t^2\,(1-\tfrac{2M}{r}) + \frac{\mathrm{d}r^2}{1-\frac{2M}{r}} + r^2\mathrm{d}\Omega^2 \\
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Kruskal-Szekeres (or 'tortoise') coordinates $x,\ y,$ replacing $(r,\ t)$

$$
\begin{aligned}
x\,y &= \left(\frac{r}{2M}-1\right)e^{r/2M} \quad ; \\
y/x &= e^{t/2M}
\end{aligned}
$$

Metric becomes:

$$\mathrm{d}s^2 \;=\; -\mathrm{d}t^2\,(1-\tfrac{2M}{r}) + \frac{\mathrm{d}r^2}{1-\frac{2M}{r}} + r^2\mathrm{d}\Omega^2$$

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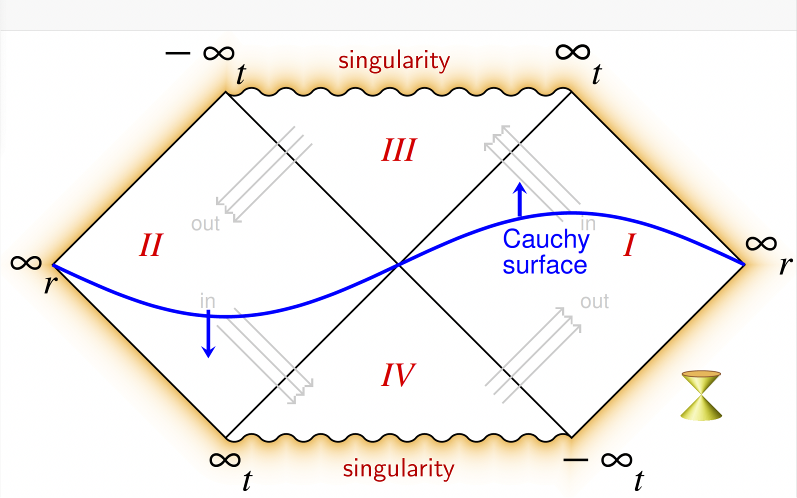
Kruskal-Szekeres (or 'tortoise') coordinates x, y, replacing $(r,\ t)$

$$x\,y \;=\; \left(\frac{r}{2M}-1\right)e^{r/2M}\quad ;$$

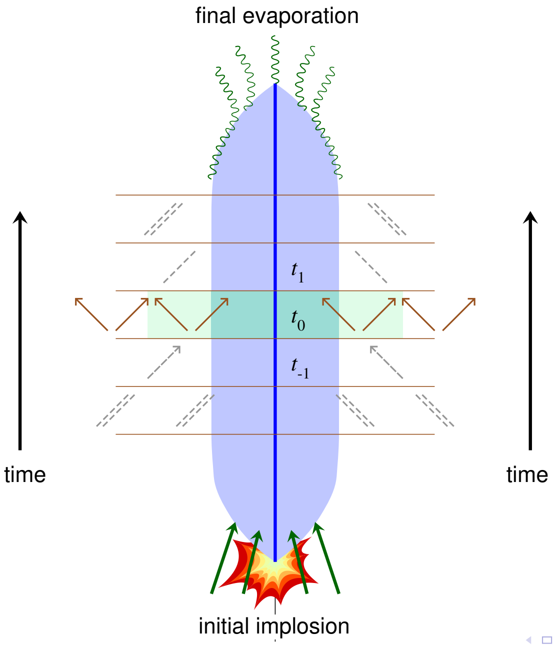
$$y/x \;=\; e^{t/2M}$$

Metric becomes:

$$\mathrm{d}s^2 \;=\; \frac{32(M)^3}{r}\,e^{-r/2M}\,\mathrm{d}x\,\mathrm{d}y + r^2\mathrm{d}\Omega^2$$



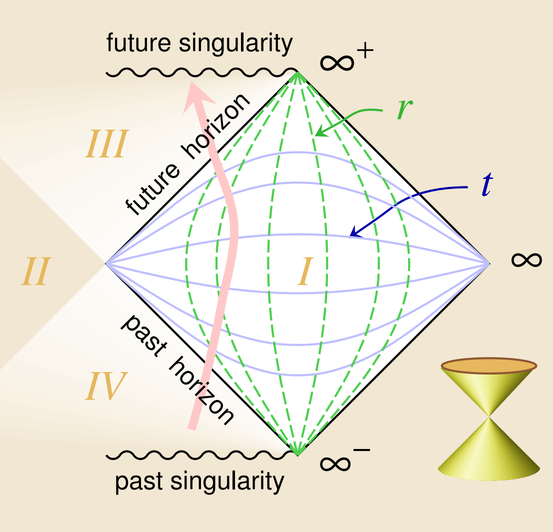
The distant observer would have to describe the "inner region", *II*, (?) as having Schwarzschild time run backwards.



We consider the Schwarzschild metric for the *stationary case* as background but this background changes slowly as time proceeds.

We treat this thing exactly as a radiating atom

The brick wall model of particles near a black hole.



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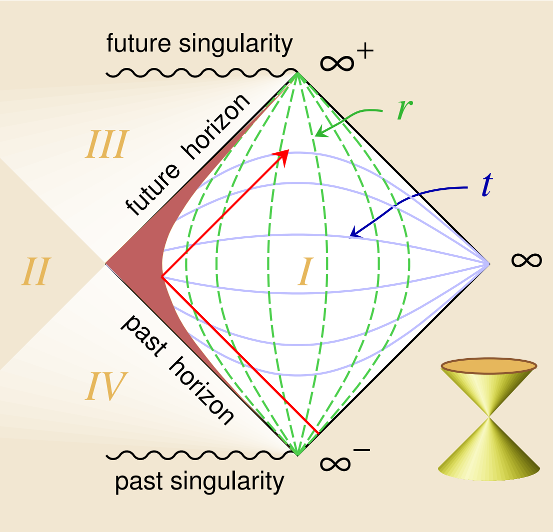

In this model, consider an ideal gas in the Schwarzschild gravitational field.

Photons escape exactly as if they were Hawking particles. Tune the position of the wall just so as to reach the right temperature.

Use this brick-wall model to compare a black hole, emitting Hawking particles, with a radio-active molecule, or a bucket of water that evaporates.

There must be an ordinary Schrödinger equation for this black hole.

The more quantum states there are, the higher the entropy, and the lower the temperature. (use $T\,\mathrm{d}S = \mathrm{d}Q$).

Hawking's derivation should be used to derive the Schrödinger equation.

Even if he didn't believe this himself!

Do we know something about quantum mechanics and gravity that we can use?

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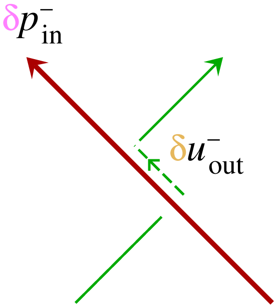
Do we know something about quantum mechanics and gravity that we can use?

> People often assume that they can use AdS/CFT, but that could be a trap!
> If we don't know the proper equations we should not assume we can apply techniques that have worked in other systems.
> This is a matter of principle that I adhere to!

Yes, there are correct equations that we can use:



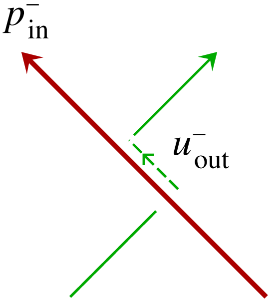
Every in-particle generates Shapiro shift

$$\delta u^{\rm out}(\Omega) \;=\; \int \mathrm{d}^2\Omega' f(\Omega,\Omega')\, p^{\rm in}(\Omega')\ ,$$

It also works the other way:

$$\delta u^{\rm in}(\Omega) \;=\; -\int \mathrm{d}^2\Omega' f(\Omega,\Omega')\, p^{\rm out}(\Omega')\ ,$$

New in-particle
generates Shapiro shift



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If we expand $u^{\mathrm{in,out}}(\Omega)$ and $p^{\mathrm{in,out}}(\Omega)$ in spherical harmonics,

$$u_{\ell,m}^{\mathrm{in,out}} \quad \text{and} \quad p_{\ell,m}^{\mathrm{in,out}} \quad \text{then } f(\Omega,\Omega') \ \rightarrow \frac{8\pi G}{\ell^2+\ell+1} \ .$$

We can write

$$u_{\ell m}^{\mathrm{out}} = \frac{8\pi G}{\ell^2+\ell+1} p_{\ell m}^{\mathrm{in}} \ .$$

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Quantum mechanics: for all particles i, j,

$$[u_i^{\pm},\ p_j^{\mp}] = i\delta_{ij} \ ,$$

we arrive at the algebra

$$[u^{+}_{\ell\, m},\ u^{-}_{\ell'\, m'}] = i\lambda_\ell\, \delta_{mm'}\, \delta_{\ell\ell'} \ ; \qquad \lambda_\ell = \frac{8\pi G}{\ell^2+\ell+1} \ .$$

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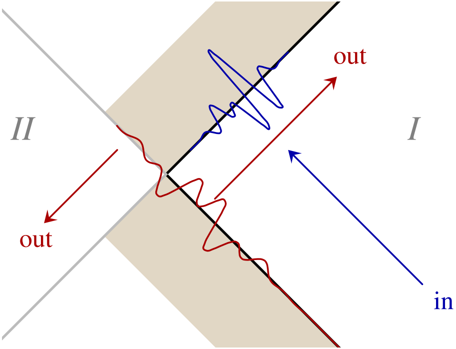
Here, the equations are just 1-dimensional (undergraduate) QM !

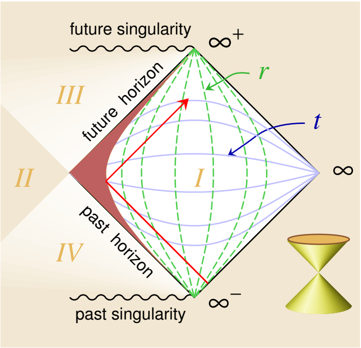
The operators $u^{\pm}_{\ell m}$ become wave functions, and u^{+} and u^{-} are each other's Fourier transform!

At every value for ℓ, m with $|m| \leq \ell$ there is a single, one dimensional quantum variable $u^{-}_{\ell m}$. The out-particle is described by the Fourier transform of the in-particle (and vice versa).

Remarkably, if we keep both the in-particles and the out-particles bounded to the inside of a box, then the Hilbert space of these states turns into a finite-dimensional vector space, just as what may happen in atoms and molecules.

The Fourier transform effectively acts exactly like our brick wall!





Unitarity can hold, but only if the Fourier transform is executed over the entire axis $-\infty < x < \infty$ onto the entire axis $-\infty < y < \infty$.

Only if we demand that region *II* describes *our* universe just as region *I* does, we can restore unitarity.

Is this a completely unitary description of a black hole?

Not quite.
Only if we identify regions I and II, we can use the fact that if a wave function is symmetric under $x \leftrightarrow -x$, its Fourier transform will be symmetric under $y \leftrightarrow -y$.

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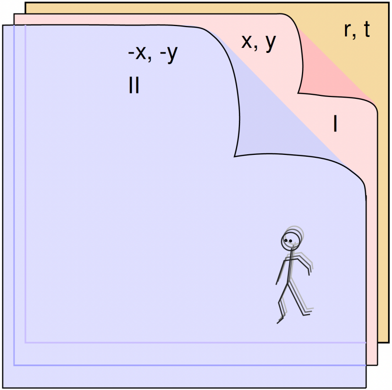
Today's favorite:

Region II is a quantum clone of region I.

The more one thinks of this, the more natural this idea looks.



In Schwarzschild coordinates, all data in region *I* are arranged exactly the same way in the coordinates $(x,\ y)$ as in $(-x,\ -y)$.

All phenomena, *including the observer*, are copied from region *I* into region *II*. But this has consequences.

Rindler space-time

And we *can* modify the notion of general coordinate invariance in states with a constant acceleration, by imposing the law that also in Rindler space, the 'back side', region *II* is to be considered as describing the quantum bra-states when the ket states are in region *I*.

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There should be a *quantum clone* of the observer in region II, runnung backwards in time! (not easy to do in experiments . . .)

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There is no place for symmetry laws such as baryon conservation.

Calculation shows that in this theory, the entropy S of a black hole is half of the value obtained originally by Hawking: The equation $T\,\mathrm{d}S = \mathrm{d}Q$ relates temperature T to the heat energy $\mathrm{d}Q$ absorbed or emitted by a black hole. $\mathrm{d}Q$ relates to total BH energy, and that cannot be changed.

Therefore, in this theory, temperature T is twice Hawking's value, so the radiation has ≈ 16 times as much energy.

When scientists do experiments in Rindler spacetime (e.g. with accelerated electrons), region II is never a clone of region I, so one finds the conventional Hawking temperature.

Remarkably, our treatment of gravity does not allow for many-particle states at a given value for the spherical wave coordinates (ℓ, m).

> *The Shapiro effect only gives us unitarity if there is exactly one particle, either going in or out, at every* (ℓ, m). *Strictly speaking, only one particle at every value of* (ℓ, m)*, during the entire lifetime of the BH.*

That sounds odd . . .

That sounds odd, but, remember that we had to divide the BH time line in short time slices; it suffices to impose the one-particle condition on every slice. $\longrightarrow$

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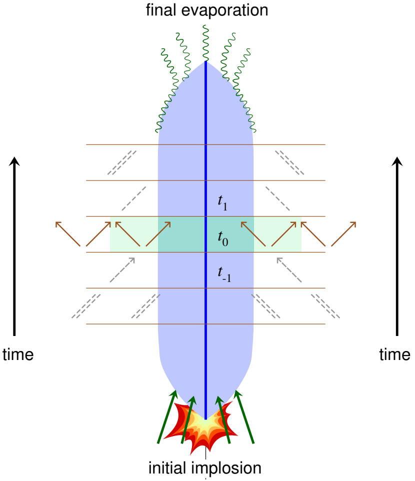
Indeed the Fourier transform is not an entirely *local* procedure here (both in space and in time), but restricting ourselves to Gaussian wave packets restores locality almost perfectly (violations only at the Planck scale).



Is this a one-particle or a two particle state?
For unitarity questions this is important.

We impose that there must be at most one particle
in every time slice

An other question:

> How to connect the one-particle states described above to the SM states living outside the black hole?

Earlier our view was that we have to look at the energy momentum operator(s) in the SM, but handling them as one-particle states seems to be more precise.

Our approach was criticised lately, but I persist that it should be followed further. For our approach no modifications are required for the equation of state, and it may well lead to a better representation of black hole quantum configurations.

Quantum mechanics: in my view, is *not* an entirely unconventional reformulation of laws of physics, but merely a superior way to handle questions of mathematics and statistics.

The evolution laws of physical phenomena are infinitely accurate and do not refer to statistics.

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Thank you

Discussion slides:

Consider quantised fields, $\Phi(\vec{x}, t)$ near the horizon. Since all points in region I are space-like separated from all points in region $II\,I$, we can derive, from Φ, creation operators $a^\dagger$ and annihilation operators a in region I that commute with those of region $II\,I$.

When we express these in terms of the creation and annihilation operators seen by a freely falling observer (inside magnifying glass), we find them to be different, in terms of a *Bogolyubov transformation* (transformation mixing a and $a^\dagger$). In short hand:

$$\begin{pmatrix} a_M(\omega) \\ a_M(-\omega) \\ a_M^\dagger(\omega) \\ a_M^\dagger(-\omega) \end{pmatrix} = C(\omega) \begin{pmatrix} 1 & 0 & 0 & -e^{-\pi\omega} \\ 0 & 1 & -e^{-\pi\omega} & 0 \\ 0 & -e^{-\pi\omega} & 1 & 0 \\ -e^{-\pi\omega} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_I(\omega) \\ a_{II}(\omega) \\ a_I^\dagger(-\omega) \\ a_{II}^\dagger(-\omega) \end{pmatrix}$$

$$C^{-1}(\omega) = \sqrt{1 - e^{-2\pi\omega}}$$

Consequently: $\quad a_M(\omega) = C(\omega)\big(a_I(\omega) - e^{-\pi\omega} a_{II}^\dagger(-\omega)\big)$, $\qquad$ etc.

See gr-qc/9607022

The vacuum state $|\Omega\rangle$ in Minkowski space obeys: $a_M(\pm\omega)|\Omega\rangle = 0$. From that, one derives that this state is a superposition of excited states in region I and II:

$$\boxed{|\Omega\rangle = C^{-1}(\omega) \sum_{n=0}^{\infty} |n\rangle_I \; |n\rangle_{II} \; e^{-\pi n\omega}}$$

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The probability for finding n particles in state $|\omega\rangle$ is:

$$C^{-2}(\omega)\, e^{-2\pi n\omega} \quad \propto \quad e^{-\beta n\omega}$$

This clearly exhibits the fact that in this state, the inverse temperature is

$$\boxed{1/kT \;= \beta = 2\pi}$$

Observe: if there are n particles in region I there are automatically also n particles in region II.

But this looks as if a better theory might apply:

The particles in region II in all respects seem to behave as the bra states $\langle n_I|$ of region I.

(running backwards in time, with negative energies)

The states $|\Omega\rangle$ neatly reflect a *density matrix* for the distant observer:

$|\Omega\rangle = C^{-1}(\omega) \sum |n\rangle \langle n| \, e^{-\pi n \omega}$. Indeed, this is a thermal density matrix.

But then, the probability of detecting state $|n\rangle$ is:

$\mathrm{Tr}\,\rho|n\rangle\langle n| \; = \; e^{-\pi n\omega} = e^{-\tilde{\beta}\, n\omega}$, so that the inverse temperature is

$$\boxed{1/k\,\tilde{T} = \tilde{\beta} = \pi}$$

This theory would give the black hole a temperature twice as high as what Hawking had derived.

Thus, once you have accepted the density matrix theory, you must conclude that the new temperature is the correct one.

One other consequence: the *entropy* of a black hole must then be half the entropy Hawking derived:

$$T\mathrm{d}S = \mathrm{d}M_{\mathrm{BH}} \quad \to \quad \mathrm{d}S = \beta \mathrm{d}M_{\mathrm{BH}}\,.$$

In statistical physics, consider the evolution in complex time:

$$t = \tau - i\beta\ , \quad U = e^{-iE\tau\, -\, \beta E} \ =\ e^{\delta S}.$$

For a particle moving near the black hole, this δS is its contribution to the entropy. The functional integral expression corresponds to the amount of space the particle has, to move around.

But if the states $|n\rangle\langle n|$ are assumed to represent the density matrix, then the states in region II are dictated by the ones in region I. The particle has only freedom to move in the domain $[0, \tilde\beta]$. This is why, in the density matrix theory, $\beta = \pi$, not 2π.

Theory 2 uses *all* information on the u^+ axis to generate the information needed on the u^- axis. This may well be a crucial advantage. But fact remains that we should be cautious with premature conclusions.

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References:

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