The Entropy of Hawking Radiation

Mostly 2006.06872 + previous workshop seminars

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1. Introduction

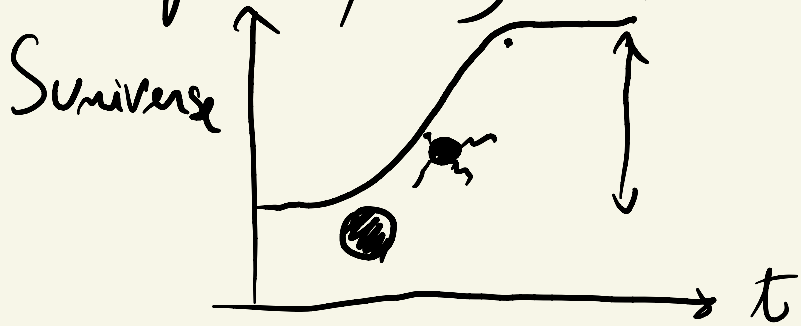
Basic question:

BHs behave as thermal objects.

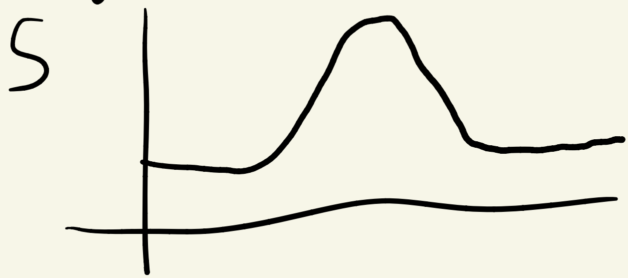
(T_{BH} (Hawking Radiation), S_{BH} (Area of Horizon))

Do they admit a statistical mechanical description? i.e.
Are they ordinary Quantum systems, at least when seen from outside?

Hawking: No, they evaporate and lose info!



Insights from many facets of String Theory (And recent work w/ gravitational path integral) Suggests information does come out:



The key step came from a formula to compute $S_{\text{von Neumann}}$ in gravitational systems.

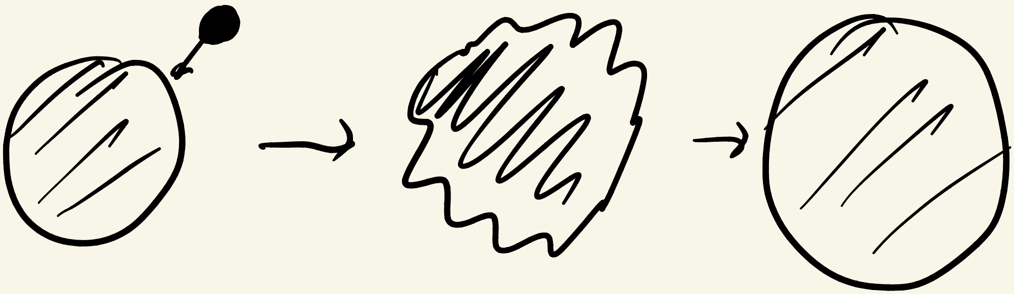
Also an area, but obtained by extremizing generalized entropy.

Ryu-Takayanagi is simplest example.

2.

2.1 BH Thermodynamics

Recall rules of BH mechanics:



$$\frac{\kappa}{8\pi G_N}\, d(\text{Area}) = dM - \Omega\, dJ$$

Postulating: $T \propto \kappa$
$S \propto A$

$$T\, dS_{BH} = dM - \Omega\, dJ,$$ suggestive of thermodynamics ($dA > 0$, classically)

Real Thermodynamics ('Hawking')

GR + QFT/Radiation $\Rightarrow$ Actual blackbody (grey body) radiation

w/ $T = \frac{\hbar \kappa}{2\pi}$

Must include full system:

$$S_{gen} = \frac{A_{hor}}{4\hbar G_N} + S_{outside/Radiation}$$

and indeed

$\Delta S_{gen} \geq 0$, like Thermodynamics

(more general, for Hawking Evap. $\Delta A < 0$)

Note That

$S \sim \frac{A}{l_p^2}$ So huge number of d.o.f.

2.2 Hawking radiation & Euclidean BHs

Recall Schwarzschild metric:

$$ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \frac{dr^2}{1-\frac{r_s}{r}} + d\Omega_2^2$$

w/ $r_s = 2G_N M$, radius of BH

Zoom in on horizon:

$$r \to r_s\left(1+\frac{\rho^2}{4r_s^2}\right), \quad t \to 2r_s\tau$$

And take $\rho \ll r_s$,

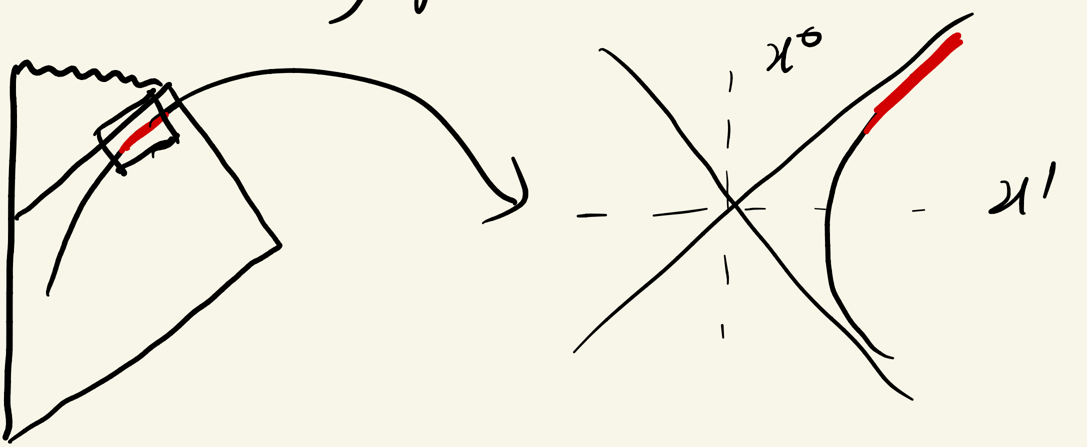
$$ds^2 \approx -\rho^2 d\tau^2 + d\rho^2$$

This is smooth flat space in Rindler coordinates;

$x^0 = \rho \sinh \tau \qquad x^1 = \rho \cosh \tau$

$ds^2 \approx -dx^{0\,2} + dx^{1\,2}$, Horizon smooth

ρ = const corresponds to floating at finite distance of Horizon. In local Rindler, uniformly accelerating observer! (Need a rocket to float at finite r away from r_s.)



Accelerated observer detects temperature!

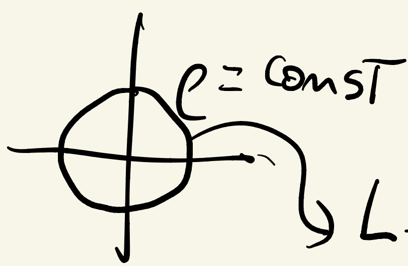
(Unruh)

<u>Trick to find</u> it: in Euclidean time, periodic propagation by L corresponds to $T = \frac{1}{L}$. (Denis's lecture)

Wick rotating: ($\tau = i\theta$, $x^0 = i x^0_E$)

$$-\rho^2 d\tau^2 + d\rho^2 \longrightarrow \rho^2 d\theta^2 + d\rho^2$$

(Flat space in polar coords)



$L = 2\pi\rho$

Hence: $$T_{accel} = \frac{1}{2\pi\rho} = \frac{a}{2\pi} = \frac{\hbar}{k_B c}\frac{a}{2\pi}$$

(Near horizon accel. observer)

Note that $\ell \to 0 \Leftrightarrow t \to \infty$.

More generally, $T_{\text{proper}}(r) \cdot \sqrt{-g_{\tau\tau}(r)} = \frac{1}{2\pi}$

So far away ($r \to \infty$): $g_{tt} = \frac{1}{4r_s^2}$ $g_{\tau\tau} = -1$

So $T(\infty) = \boxed{T_H = \frac{1}{4\pi r_s}}$.

We can also turn this around:

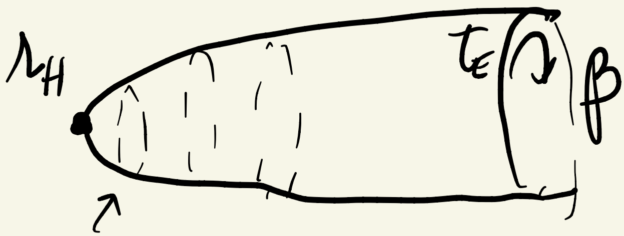
Finite Temperature $1/\beta$ amounts to $t_E \sim t_E + \beta$

hence considering the full Euclidean geometry:

$$dS_E^2 = \left(1 - \frac{r_s}{r}\right) dt_E^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + d\Omega_2^2$$

with $t_E \sim t_E + \beta$ Assymptotycally, Smoothness of the Horizon (origin in ρ, θ polar plane)

$\theta \sim \theta + 2\pi \Rightarrow T = \frac{1}{4\pi r_s}$



Cigar geometry.

$\beta_{proper} = \frac{1}{T_{proper}}$

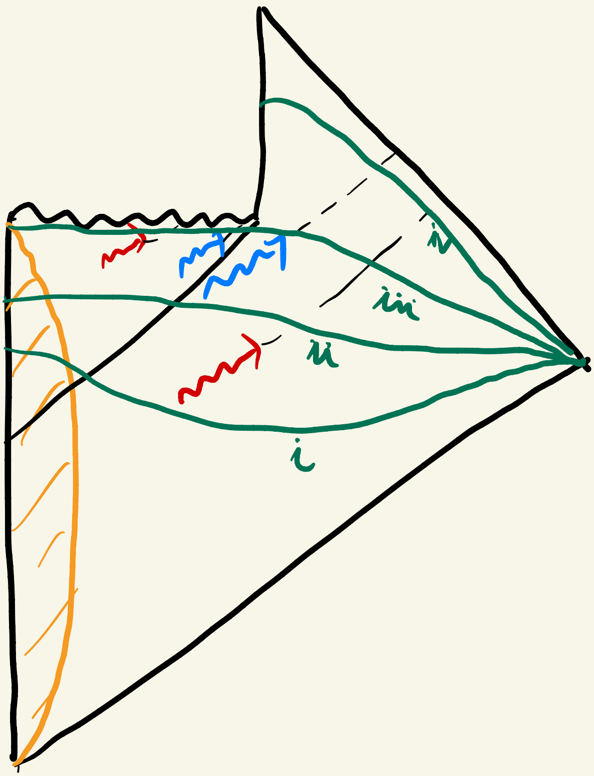
Therefore, $Z_{QFT\,in\,BH}(\beta) =$ Path integral in Cigar.

From here one derives.

$$S = (1 - \beta \partial_\beta) \log Z(\beta) = \frac{A}{4G_N} + S_{out}$$

2.3 Evaporating BHs

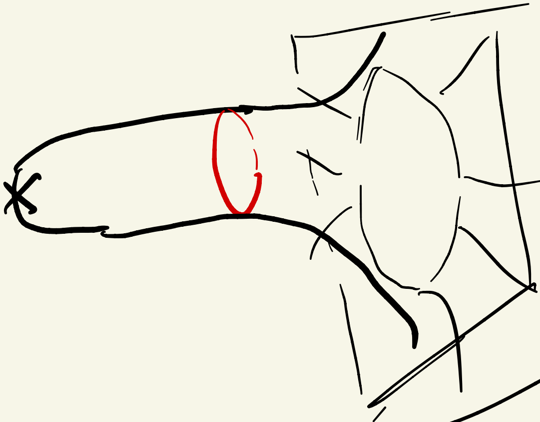
Eventually, Hawking radiation evaporates the BH away. Recall that the mechanism is "pair-creation of entangled particles". The semi-classical back-reacted space-time is:



i, ii, iii, iv

Cauchy Slices

i)



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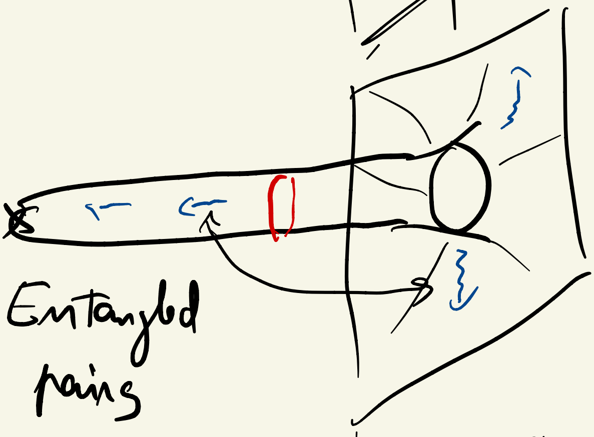

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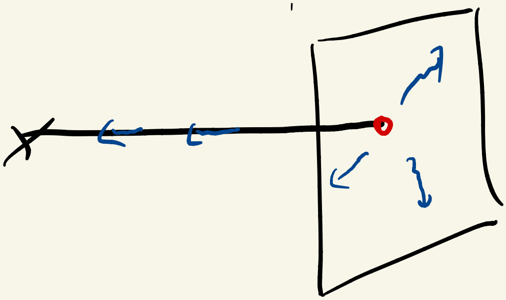

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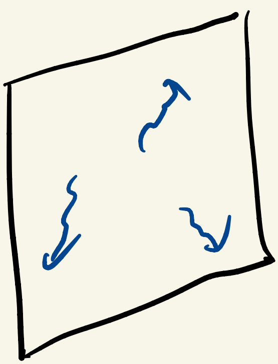


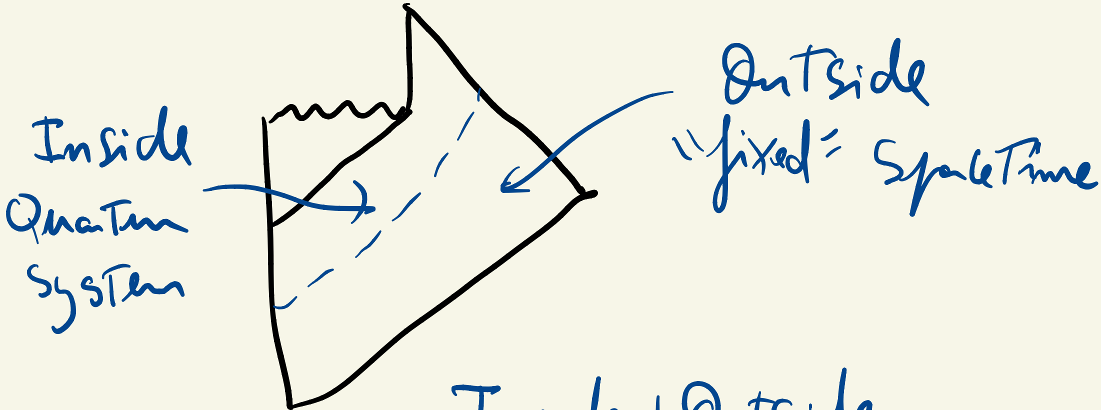
3. The BH as an ordinary Quantum System

What we have seen so far suggests BHs obey laws of Thermodynamics w/ large but finite # of d.o.fs. Central Dogma:

$$\boxed{\text{As seen from outside, a BH is just a quantum system w/ } \frac{A}{4G_N} \text{ d.o.f evolving and interacting w/ outside according to a Unitary Hamiltonian.}}$$

- No statement about interior
- $\dim \mathcal{H}_{BH} < \infty$, d.o.f not clear
- In a reflecting box (a.k.a AdS) BH evolves unitarily by itself.

In assymptotically flat space, we need to divide:

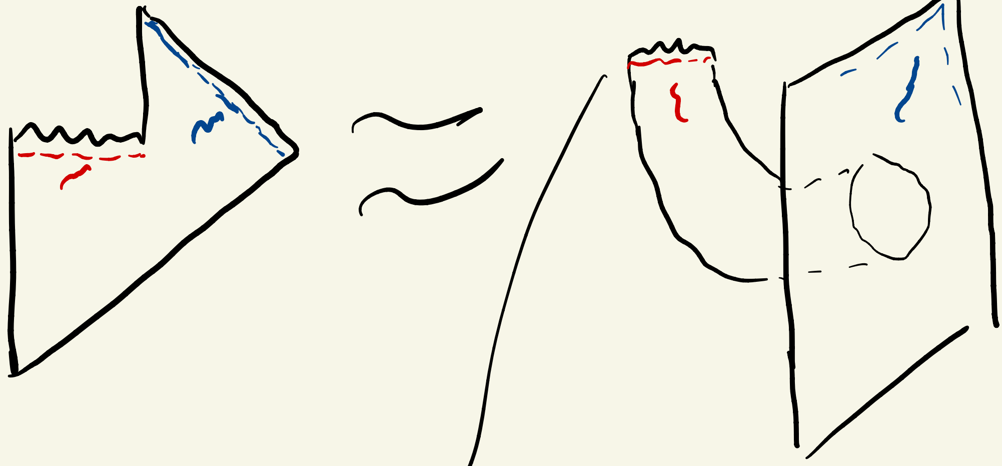


Inside + Outside evolve Unitarily.

- BH is just like a piece of rock... Outside!

- Central dogma rejected by Hawking.

Is also in Tension w/ face-value interpretation of geometry:



Like a separate future of the interior.
Why couldn't info be stuck and lost forever?

- The recent results give evidence for the central dogma, don't assume it.
  - String theory counting of d.o.f. in SUSY BHs
  - AdS/CFT provides a manifestly unitary description in the Bdry.

4. Fine-grained vs Coarse-grained Entropy

Recall:

Von Neumann a.k.a fine grained a.k.a Quantum Entropy:

$$S_{VN} = -\mathrm{Tr}(\rho \log \rho)$$

- 0 for pure state $\rho = |\psi\rangle\langle\psi|$
- Invariant under unitary time evolution
  $\rho \to U \rho U^{-1}$

Coarse-grained Entropy

Measure a few coarse-grained observables
$A_i = \{E, V, \dots\}$

take all $\tilde{\rho}$ s.t

$$\mathrm{Tr}(\tilde{\rho} A_i) = \mathrm{Tr}(\rho A_i)$$

And compute $S_{vN}(\tilde{\rho})$. Then take $\tilde{\rho}$ w/ maximal entropy. Sounds hard, but this is the definition of Thermodynamics!

We assume equal probability of all microstates compatible w/ macro-observables.

Indeed:

$$\Delta S_{coarse} \gtrsim 0 \quad \text{2nd Law}$$

- S_{gen} must be an S_{coarse}, since it increases a lot when horizon forms.

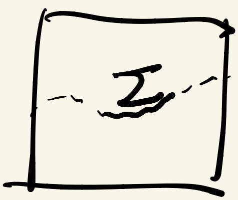
- In a two-part quantum system $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, $\rho_A = \text{tr}_B \rho$, has non-zero entropy because of entanglement w/ B, even for pure $A \cup B$.

$$S(A) = S(B) \neq 0 \; ; \; S(A \cup B) = 0$$

- $S_{vN} \leq S_{coarse}$ by definition.

Intuitively the # of total d.o.f.s gives an upper bound on the # of entangled qubits.

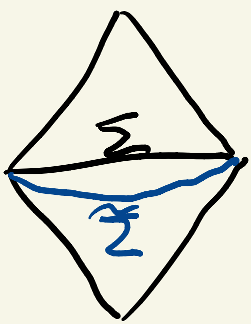
- For quantum fields, we care about entropy in spatial regions Σ.



$S_{vN}(\Sigma) = S_{vN}(\rho_\Sigma)$ (subtle)

if Σ not full Cauchy, can change w/ time.

Σ defines causal diamond:



$$S[\Sigma] = S[\tilde{\Sigma}]$$

Semiclassical Entropy in following,

We consider $S_{vN}[\Sigma]$ as fine-grained entropy of Quantum fields in fixed, "outside" curved geometry.

This is S_{matter} / $S_{outside}$ / $S_{semi\text{-}classical}$.

5. Hawking Information Paradox

This is an argument against central dogma. (otherwise no paradox, could be failure of QG)

Origin of Hawking Radiation: Entanglement

BH by collapse of pure state. why thermal radiation?

We are splitting inside $\otimes$ outside, so outside in a mixed state (Entanglement).

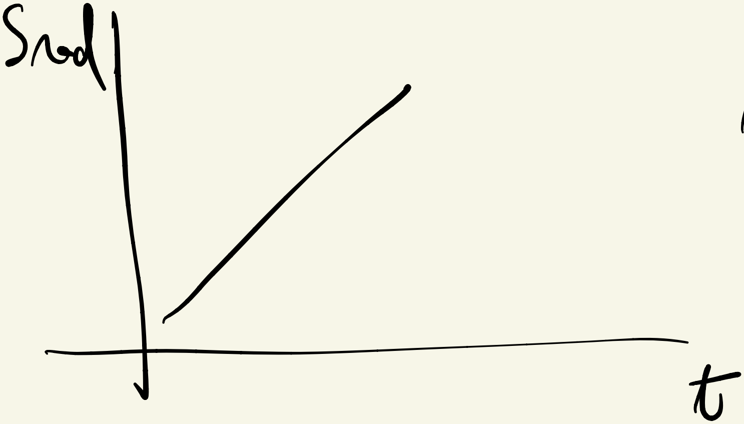
Horizon pairs, one stays, one goes:

Pure state

(ingoing Hawking Quanta) | (outgoing Hawking Quanta)

Thermal/mixed state

So far, no contradiction. In fact, pure quantum systems thermalize by emitting close to thermal rad., because of entanglement with the quantum system. So at early times



makes sense.

However, as it evaporates, BH gets smaller and smaller and we run into trouble when $S_{rad} > S_{Bek-Hawk}$

This is because there are not enough d.o.f. for the radiation to be entangled with!

if B.H d.o.f. + Rad = pure, then

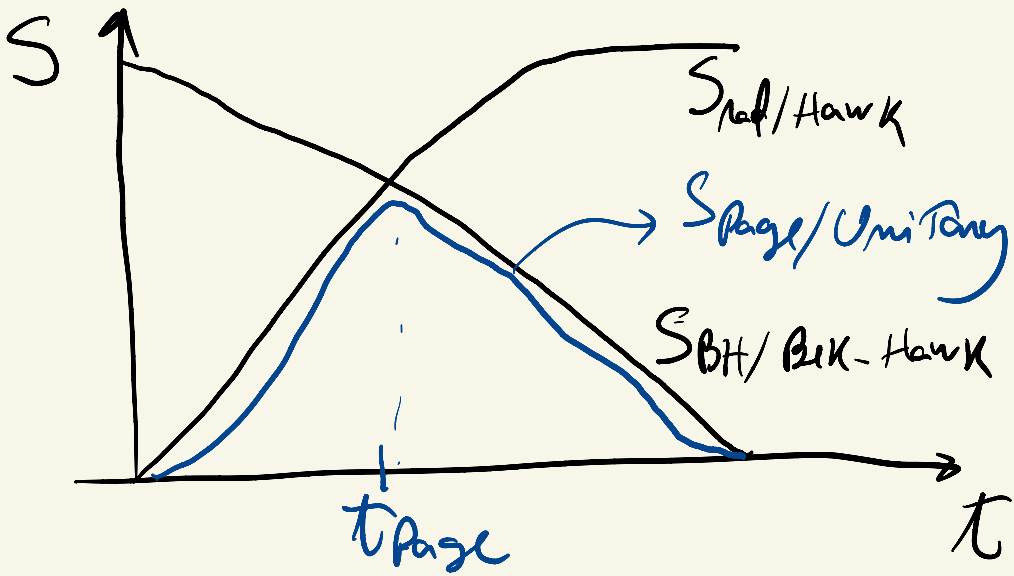
$$S_{B.H\,fine} = S_{rad}$$, but of course

$$S_{BH\,fine} \lesssim S_{BH\,coarse} = S_{Bek-Hawk}$$

If central dogma were true, at $S_{rad} = S_{Bek-Hawk}$, entropy must start decreasing!

$$S_{rad,\,fine} \lesssim S_{Bek-Hawk} \quad \text{(Page curve)}$$

t s.t. $S_{rad} = S_{Bek-Hawk}$: (Page Time)



- When BH small, Planck effects, but this is much later than T_{page}.

- Hawking's Argument robust in G_N Perturbation Theory

- We could say S_{vN} is sensitive and should not be computed semi-classically.

Bekenstein-Hawking is coarse-grained, the recent work suggests how to compute the actual fine grained entropy.
It is also a generalized entropy:

$$S_{fine} \sim \underset{\Sigma}{\mathrm{Ext}} \left[\frac{\mathrm{Area}[\Sigma]}{4G_N} + S_{out} \right]$$

New surfaces that are not Horizon!