Notes on the Island proposal

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ABSTRACT: In these notes we explore the basics idea of the Island proposal, its relation with AdS/CFT and information paradox. Notes prepared for the Desy theory Workshop (7.05.2024) and Desy theory workshop (14.05.2024).

Contents

1 Introduction and Page curve

In these notes, we will explore the recent *Island* proposal to compute the Entropy of a black hole system. This consist in a formula for the (generalized) von-Neumann entropy [1]

$$S(\rho_{\mathcal{I}}) = \min_{\mathcal{I}} \text{ext}_{\mathcal{I}} \left(\frac{\text{Area}(\partial \mathcal{I})}{4G_N} + S(\tilde{\rho}_{\mathcal{I}\cup R}) \right) \ . \tag{1.1}$$

The details of this formula will be given later. The above proposal was first motivated by AdS/CFT [1] and then argued from saddles of the gravitational path integral by using the replica trick [2, 3]. The goal of these notes is to give an introduction to the proposal and its use for the computation of the entropy of a black hole system.

Before going on it is worth to remember what the information paradox is and how it is related to the entropy. We remember that the black hole is radiating as first observed by Hawking. This radiation can be understood as a production of particle/anti-particle state near the horizon [4]. We can imagine that the particle escapes from the black hole, while the anti-particle, due to tunneling, is eaten by the black hole (for a more careful analysis based on this idea see [5]). As a conclusion an observer at infinity will effetely experience a thermal state due to the radiation of the black hole. The expected number of radiating

[1] We explicitly write here G_N in the formula: in the following we will use $G_N = 1$ where not necessary for the understanding.

3 Towards a proof for the Islands formula

The aim of this section is to give a conceptual explanation of where the island formula comes from. Concretely, we would like to motivate the following

Claim. The deviation between the results for BH/Hawking radiation entropies of the semi-classical analysis of Hawking on the one hand and the geometric prediction from the island formula on the other can be understood as a consequence of contributions to Renyi entropies that are produced by certain saddle points of the gravitational path integral, known as replica wormholes.

Said differently, the island formula gives a better approximation to the entropy of black holes than Hawking's computation, because it accounts for contributions to the gravitational path integral that were previously missing. To understand what this claim means, we approach it through the following four questions

- **What does the gravitational path integral compute?** To begin with, we need to think about the question what the geometric gravitational path integral computes in the first place. Answering this question is the goal of Section 3.1. The key take home message here will be that one should think about the gravitational path integral as a coarse grained description of gravity computing averages over ensembles of transition amplitudes as opposed to overlaps between individual microstates.
- **How should we determine entanglement entropy?** After having reflected upon what the main tool at our disposal, i.e. the gravitational path integral, can do for us in general, we are ready to think about how it can be applied to solve the particular problem at hand, namely to compute the entropy of Hawking radiation. In search for an answer to this question, Section 3.2 will cover how replica geometries naturally arise in path integral computations of entanglement entropies independently of whether the system under consideration contains dynamical gravity or not. We will then see that if we turn on dynamical gravity in a certain region of space time, new configurations will emerge that can be interpreted as wormholes connecting the gravitational regions of different replicas. Including these "replica wormholes" into the computation of the entropy will allow us to recover the Page curve and resolve the tension between unitary time evolution and thermal evaporation of black holes.
- **When do we recover the geometrical island prescription?** The prescription of averaging over different geometries in the path integral computation of the entropy at first glance seems to be incompatible with the island formula. How can an extremisation procedure in a fixed geometry give the same result as an average over all possible geometries? In Section 3.3, we will discuss how the island formula arises from the path integral as semi-classical limit of the entropy in a regime where maximally connected replica geometries are dominant.

- **Is the information paradox solved now?** Finally, we will take a step back and think about where we have arrived at. We try to assess, at least to the extend permitted by the length of these notes and the knowledge of their authors, what has been achieved and what question remain unanswered.

3.1 What does the gravitational Path Integral compute?

Let us consider a simple toy model for the evaporation of a black hole (the setup is motivated by [16], Section 2): Suppose we have a black hole B that we describe with states from a Hilbert space $\mathcal{H}_B$. We would like to couple the black hole to some quantum mechanical system R with Hilbert space $\mathcal{H}_R$, which we think of as describing radiation far away form the black hole.

Lets say we start off in a state

$$|\Psi\rangle_1 = |\psi_1\rangle \otimes |1\rangle \in \mathcal{H}_B \otimes \mathcal{H}_R, \tag{3.1}$$

in which there has not jet been any radiation emitted. As time goes on, more and more radiation will be emitted and our system will evolve into an increasingly complicated state with more and more entanglement between the black hole and the radiation. So after a while, $|\Psi\rangle_1$ should evolve into a state of the form

$$|\Psi\rangle_k = \frac{1}{\sqrt{k}} \sum_{i=1}^{k} |\psi_i\rangle \otimes |i\rangle \tag{3.2}$$

with large k. As an observer outside the black hole, we only have access to the radiation, so our measurements can only inform us about data encoded in the density matrix

$$\rho_k = \mathrm{Tr}_B\, |\Psi\rangle_k \langle\Psi|_k = \frac{1}{k} \sum_{i,j=1}^{k} |j\rangle \langle i| \langle\psi_i|\psi_j\rangle\,. \tag{3.3}$$

The transition amplitudes $\langle\psi_i|\psi_j\rangle$ are to be computed via a putative gravitational path integral over all geometries relevant for the gravitational system that describes our black hole.

Ideally, we of course would like to be in the luxurious situation of having a fully microscopic description of gravity, in which case $\langle\psi_i|\psi_j\rangle$ would simply be a complex number. If, however, we work with some effective coarse grained description of gravity, then the amplitudes that we write down should rather be thought of as random variables of which we can only compute ensemble averages $\Big\langle \langle\psi_i|\psi_j\rangle \Big\rangle$.

The question of which of the two the gravitational path integral computes is of key importance if we would like to compute the von Neumann entropy of ρ_k i.e. the entropy of the Hawking radiation.

Indeed, if we work with orthonormal states $|\psi_i\rangle$ then in the case that the path integral would actually compute precise transition amplitudes between microscopic states, we would directly get

$$\rho_k = \frac{1}{k}\sum_{i=1}^{k} |i\rangle\langle i| \qquad \text{and thus} \qquad S_k = -\mathrm{Tr}(\rho_k \log \rho_k) = \log k. \tag{3.4}$$

As time goes on and k steadily increases, we would thus expect the entropy of the Hawking radiation to monotonously grow, incompatibly with the Page curve and unitary time evolution, qualitatively reproducing the problems encountered in Hawking's computation of the entropy.

In the case where the path integral only computes ensemble averages however, we can imagine that our states are only orthonormal *on average* but that higher moments of the distributions of the transition amplitudes are in fact non vanishing. Then, the correct way of computing the entropy is

$$S_k = \Big\langle -\mathrm{Tr}(\rho_k \log \rho_k) \Big\rangle \neq -\mathrm{Tr}(\langle \rho_k \rangle \log \langle \rho_k \rangle). \tag{3.5}$$

Qualitatively, we can easily convince ourselves that this has a chance of explaining the Page transition and reproducing the results of the island formula: Imagine the norms of the black hole states are all identically distributed s.t. $\langle ||\psi_i||^n \rangle \sim 1$. Furthermore, suppose the off-diagonal transition amplitudes are suppressed by some small parameter ϵ such that $\langle ||\psi_i||^n \langle \psi_j|\psi_l\rangle\rangle^m \sim \epsilon^m$. Then, to linear order in ϵ, the nth Renyi entropy of ρ_k is given by

$$\Big\langle \mathrm{Tr}[\rho_k^n] \Big\rangle \sim \Big\langle \sum_{i=1}^{k} \frac{||\psi_i||^{2n}}{k^n} + \sum_{i\neq j} |\langle \psi_i|\psi_j\rangle|^2 \frac{1}{k^n} + \dots \Big\rangle \sim \frac{1+\epsilon^2 k}{k^{n-1}} \tag{3.6}$$

For $1 \ll k \ll \frac{1}{\epsilon^2}$, the Renyi entropy and thus also the von Neumann entropy should match with the expectation from eq. (3.4). However, as k grows and eventually becomes comparable to $\frac{1}{\epsilon^2}$, a phase transition occurs and the contributions of the off-diagonal transition amplitudes become dominant. A careful analysis shows, that this transition can be identified as the Page transition.

3.2 How should we determine entanglement entropy?

We now explore the consequences of the ensemble average interpretation of the gravitational path integral given in Section 3.1 for the computation of the entropy of Hawking radiation. For this, we first review the computation of von Neumann entropy from Renyi entropies in QFT using replicas and twist fields. Then, we turn on dynamical gravity and discuss how the computation qualitatively changes.

3.2.1 Density matrices, replica geometries & symmetric product orbifolds

Recall that we can think about states in QFT as prepared by path integrals with an open cut. Two prominent examples for this that appeared several times throughout the workshop seminar are the Minkowski vacuum $|0\rangle$ and the thermofield double state $|\beta\rangle_{TFD}$ of the eternal Schwarzschild black hole, see Figure 8.

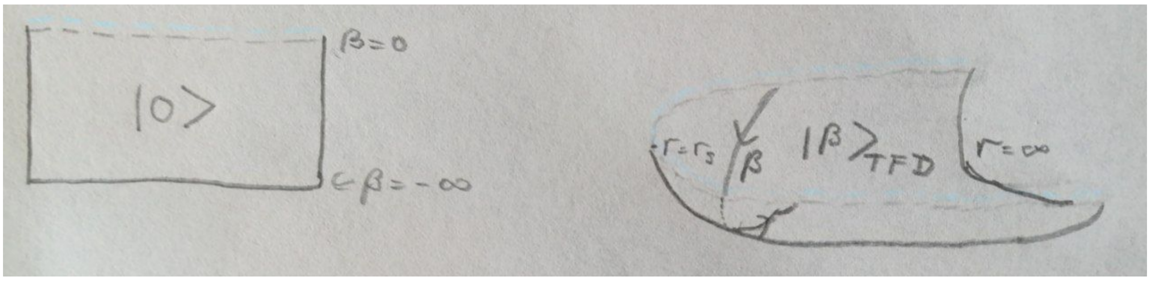


Figure 8: Pictorial representation of the Minkowski vacuum and Schwarzschild thermo field double state.

In this framework, operators are represented by path integrals over regions with two cuts. Especially, one can obtain the density matrix of a pure state by taking the disjoint union of two copies of the path integral that prepares the state. Tracing over the Hilbert space associated to the complement of a region R, generates the density matrix ρ_R relevant for the computation of the entanglement entropy of that region. Graphically, performing the trace corresponds to gluing, as illustrated for the Minkowski vacuum in Figure 9.

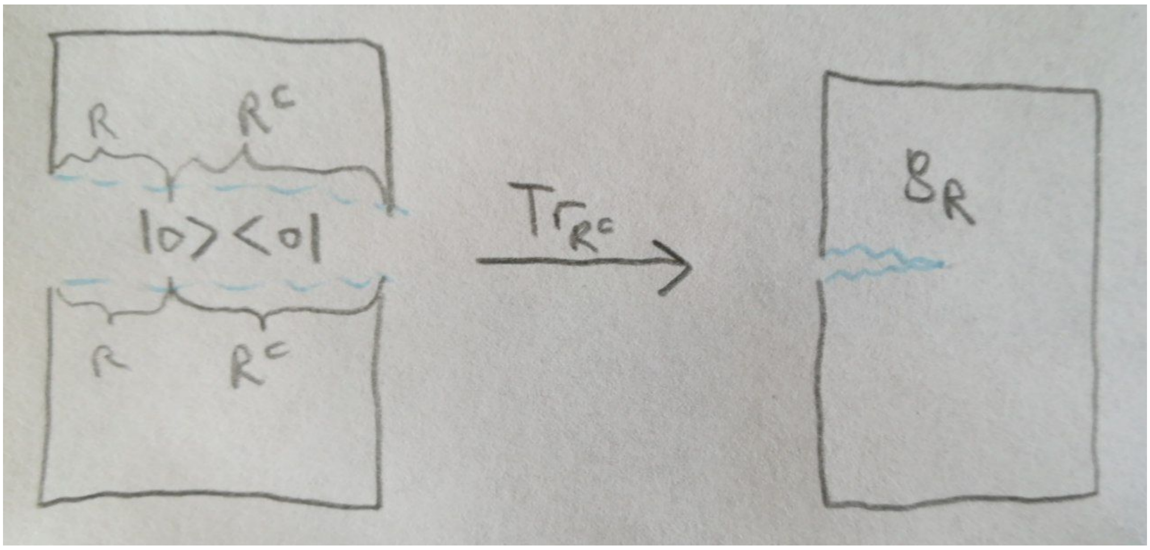


Figure 9: Density matrix associated to tracing over the complement of a region R in the Minkowski vacuum.

As already reviewed last week, the von Neumann entropy associated to a density matrix ρ_R is defined as

$$S_R = \mathrm{Tr}(\rho_R \log \rho_R) \tag{3.7}$$

and can conveniently be computed by analytic continuation in n of the nth Renyi entropy (i.e. $\mathrm{Tr}(\rho_R^n)$) as

$$S_R = \lim_{n\to 1} \frac{\mathrm{Tr}(\rho_R^n)/(\mathrm{Tr}(\rho_R))^n - 1}{n-1}. \tag{3.8}$$

With canonical normalisations, $\mathrm{Tr}(\rho_R)$ should just be 1. However, below it will be convenient to allow for unnormalised density matrices and devide out the normalisation in the computation of the entropy explicitly. In the path integral picture, we can compute Renyi entropies by taking n copies of the path integral that defines our density operator and gluing together consecutive cuts as shown in Figure 10. Note in particular that the emergent geometry $\widetilde{\mathcal{M}}_n$ has $\mathbb{Z}_n$ symmetry.

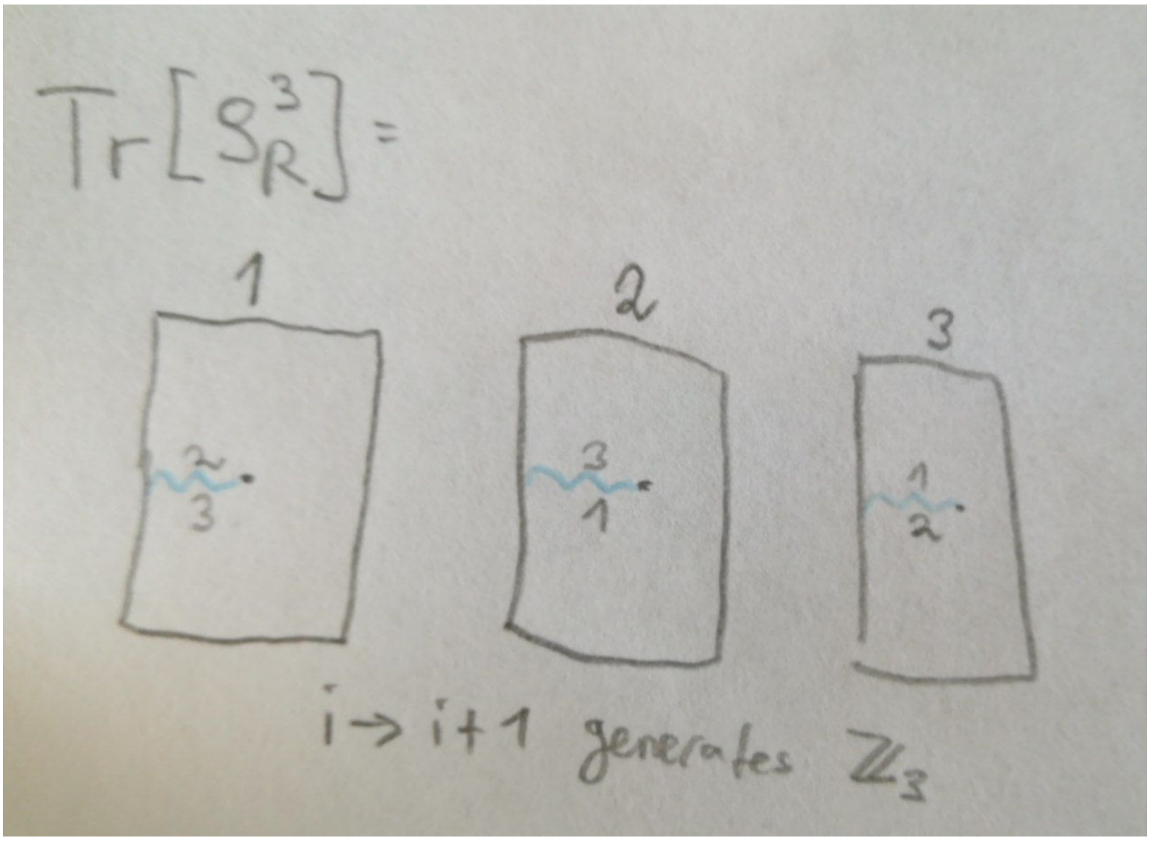


Figure 10: Computing $\mathrm{Tr}[\rho_R^3]$ for the density matrix associated to tracing over the complement of a region R in the Minkowski vacuum.

We can use a trick to evaluate the Renyi entropy through the path integral of n copies of our original QFT on the quotient $\mathcal{M}_n = \widetilde{\mathcal{M}}_n/\mathbb{Z}_n$ as opposed to a single copy of the QFT on the replicated geometry. We simply need to cut the replica geometry and impose boundary conditions on the cuts that relate the different copies to each other as shown in Figure 11. The cuts end on branch points that should be thought of as created through the insertion of local operators, namely the twist fields associated to the S_n symmetry that permutes the different copies of the QFT.

3.2.2 Turning on gravity... in some places

The approach to states, operators and entropy taken in the previous subsection appears to be incompatible with dynamical gravity. It relies a lot on the perspective that geometry is something fixed and non dynamical. However, we can give up the rigidity of the geometry

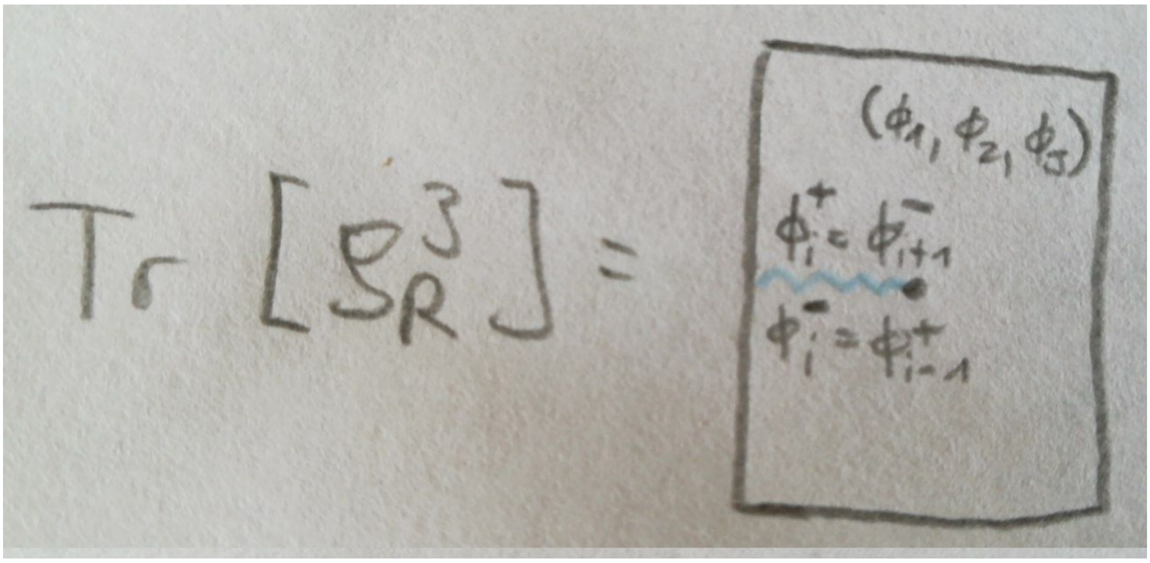


Figure 11: Computing $\mathrm{Tr}[\rho_R^3]$ for the density matrix associated to tracing over the complement of a region R in the Minkowski vacuum by using three copies of our original theory together with a branchcut that relates them to each other.

in regions far away from the cuts that define states and operators. Said the other way round, instead of providing all of the geometry connecting the cuts, we can choose to specify the geometry only in some region $\widetilde{\mathcal{M}}_n^f$ around them, in which we consider gravity to be non dynamical, and then leave boundary conditions at the boundary $\partial\widetilde{\mathcal{M}}_n^f$ of the fixed geometry region, which we consider to be dynamically completed by gravity with some $\widetilde{\mathcal{M}}_n^g$ to the full replica geometry $\widetilde{\mathcal{M}}$ (or more precisely, a gravitational path integral over different $\widetilde{\mathcal{M}}_n^g$). This procedure is illustrated in Figure 12.

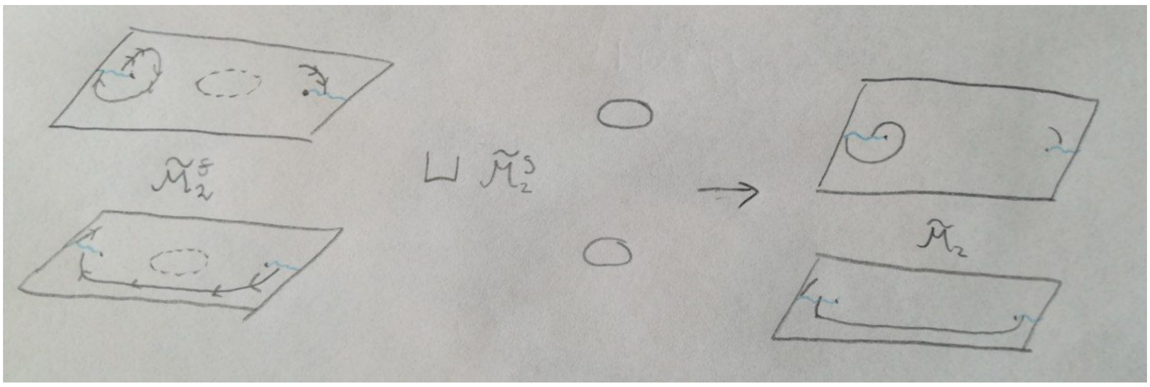
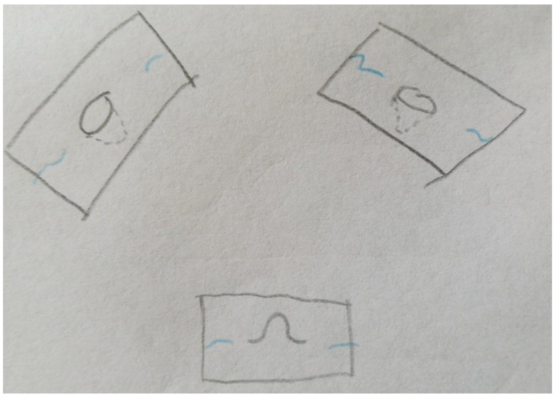
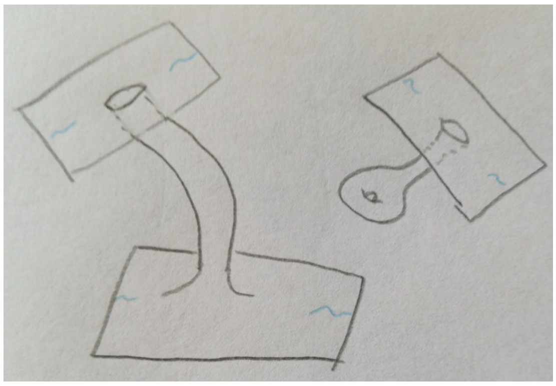


Figure 12: $\widetilde{\mathcal{M}}_2^f$ consists of two copies of a region far away from the black hole, where gravity is negligible. Each copy has a hole that should be filled out by gravity. A simple way to fill the holes is by gluing in disks. For better visualisation of the additional gluing that happens at the branchcuts, we drew some continuous curve in the replica geometry.

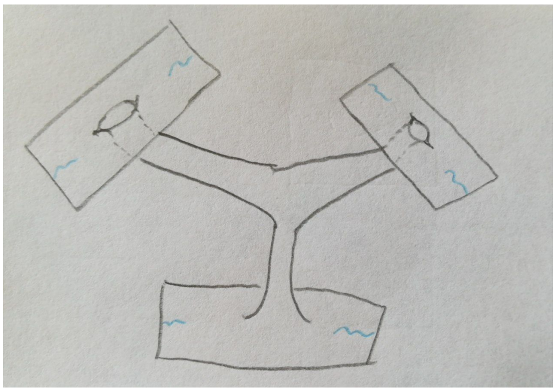
At this stage, our discussion of what the gravitational path integral computes (Section



(a) Maximally disconnected.



(b) Two connected components.



(c) Maximally connected.

Figure 13: Different contributions to $\langle \mathrm{Tr}[\rho_R^3] \rangle$.

3.1) becomes important: If we (wrongfully) identify the Renyi entropies with $\mathrm{Tr}(\langle \rho_R \rangle^n)$, the dynamical gravity regions of the different replicas cannot connect to each other. Evaluating the path integral through saddle point approximation, we will ultimately recover Hawkings result for the entropy and run into the quantum information paradox.

If, however, we compute the Renyi entropies as

$$\langle \mathrm{Tr}(\rho_R^n) \rangle = \int_{\partial \widetilde{\mathcal{M}}_n^g \simeq \partial \widetilde{\mathcal{M}}_n^f} D\widetilde{\mathcal{M}}_n^g \int_{\Phi \text{ on } \widetilde{\mathcal{M}}_n} D\Phi e^{-S[\widetilde{\mathcal{M}}_n^g, \Phi]}, \tag{3.9}$$

then the gravitational regions of different replicas are able to connect to each other, leading to qualitatively completely different new contributions to the entropy. Some examples are visualised in Figure 13.

In simple toy models, where the gravitational path integral can explicitly be evaluated, one can verify exactly that the extra contributions from geometries that connect different replicas lead to an overall entropy that recovers the Page curve and is in agreement with unitary time evolution [16]. However, here we shall take a different approach to convincing ourselves that the paradox is indeed resolved, namely by showing emergence of the island formula.

3.3 When do we recover the geometrical island prescription?

We will now motivate the island formula from the qualitative description of von Neumann entropy computations given in the previous subsection. The glaring first question that needs to be resolved for this is: *How can the island formula description of computing the entropy from a single fixed extremal geometry be compatible with the statement that the entropy is rather to be computed as a weighted sum over all geometries?*

The answer to this question is saddle point approximation. In fact, applying saddle point approximation to the path integral over geometries in eq. 3.9 directly gives us an extremisation prescription

$$\langle Tr(\rho_R^n)\rangle \approx \min\left(\underset{\partial\widetilde{\mathcal{M}}_n^g \simeq \partial\widetilde{\mathcal{M}}_n^f}{\text{ext}}\left(\int\limits_{\Phi \text{ on } \widetilde{\mathcal{M}}_n} D\Phi e^{-S[\widetilde{\mathcal{M}}_n^g,\Phi]}\right)\right), \tag{3.10}$$

analogous to that of the island formula. Here, the extremisation is over the effective action

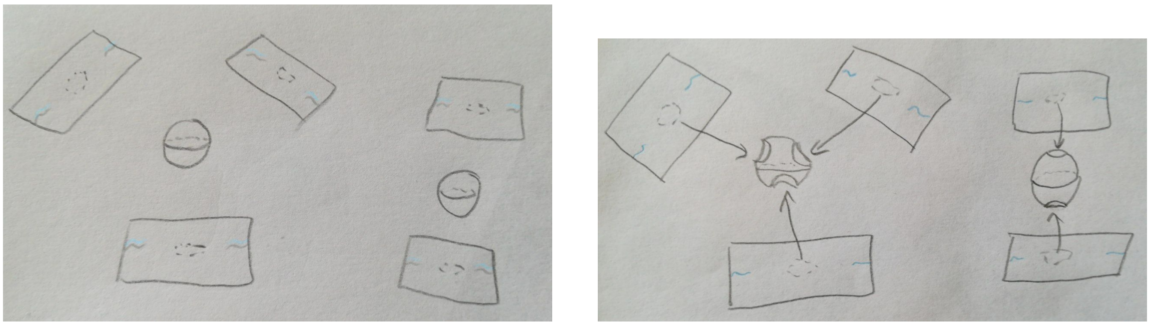
$$S_{eff}[\widetilde{\mathcal{M}}_n^g] = -\log\left(\int\limits_{\Phi \text{ on } \widetilde{\mathcal{M}}_n} D\Phi e^{-S[\widetilde{\mathcal{M}}_n^g,\Phi]}\right) \tag{3.11}$$

for the geometry that can be obtained by integrating out the matter propagating on it. This already looks more like the island formula, but there are still two issues. First, we are extremising over some replica geometries and not over islands in the original geometry. Second, we are extremising the effective action for the geometry and not the generalised entropy.

To make some progress towards a solution of the two issues, we should think about what saddles can really appear in (3.10). The qualitative picture, that we would like to paint here is that there are two effects competing against each other to determine which geometry is favoured. These are

- **Effect 1: Gravity likes it simple.** In quantum gravity, geometries with nontrivial homotopy groups should be suppressed by the loop counting parameter $1/G_N$. Thus, the gravitational contribution to the action tries to make $\widetilde{\mathcal{M}}_n^g$ as close to being contractible as possible.

- **Effect 2: Matter likes it pure.** Forget about gravity for a moment and go back to the computation of the entanglement entropy of some region R without it. The purer the state that we are in, the larger $\text{Tr}[\rho_R^n]$ should be. For sure, increasing the size of the region R makes the state under consideration more pure. But in the replica geometry picture, increasing the size of R is doing nothing else but increasing the region in which the replicas are glued together. Said differently, it makes the different replicas more connected to each other. Hence, if we turn on gravity, we should expect that the matter path integral contribution to the effective action of $\widetilde{\mathcal{M}}_n^g$ should favour geometries where the replicas are highly connected.



(a) Taking two spheres. (b) Adding punctures and gluing.

Figure 14: Construction of topologically simple replica geometries from spheres.

If we take these two effects as our guidelines, the rules of the game for determining the relevant contributions to the gravitational path integral are the following: For a given $\widetilde{\mathcal{M}}_n^f$, we should focus on those geometries $\widetilde{\mathcal{M}}_n^g$ that for a fixed number of connected components $1 \leq k \leq n$ have the simplest topology. These can be constructed by taking k spheres, punching holes into those spheres for each replica that they connect and then gluing the replicas to the holes as illustrated in Figure 14.

We already now that if we only take the maximally disconnected contribution into account, i.e. if we are in a regime where effect 1 dominates and effect 2 is negligible, we will reproduce Hawkings answer for the entropy. But as time goes by, the entropy of the radiation grows (in fact, this is what led to the information paradox in the first place) or said differently, the density matrix of the radiation will become less and less pure. But this means that effect two at the same time becomes more and more relevant until at some point the entropical gain from strengthening the connection between different replicas becomes larger than the topological cost of adding punctures to a sphere! Thus, our simple minded qualitative discussion in fact predicts a transition where Hawkings answer for the radiation gradually should be replaced by contributions to the Renyi entropy coming from more and more connected geometries. This looks promising. But, *is this transition really the Page transition?*

Answering this question in general by studying the whole transition process in detail is extremely challenging. However, we can do something simpler, which should already give us a good idea of whether we are on the right track: We know that the transition that we would like to understand starts from Hawkings answer for the entropy. But where does it take us from there? Since at late times, we end up in a regime dominated by a single saddle – namely the fully connected one – answering this question should not be harder than Hawking's original computation.

A sequence of partition functions on different geometries seems to be hard to analytically continue. Partition functions coming from a sequence of actions on a single fixed geometry however are easy to deal with: We just need to analytically continue the actions. Luckily, the fully connected geometry $\widetilde{\mathcal{M}}_n$ is $\mathbb{Z}_n$ symmetric. Thus, we can work with the orbifold $\mathcal{M} = \widetilde{\mathcal{M}}_n/\mathbb{Z}_n$, which is an n-indepent geometry with a boundary I that itself has a boundary ∂I, as illustrated in Figure 15. The boundary of the boundary is a singularity of the orbifold geometry that arises from fixed points of the $\mathbb{Z}_n$ action.

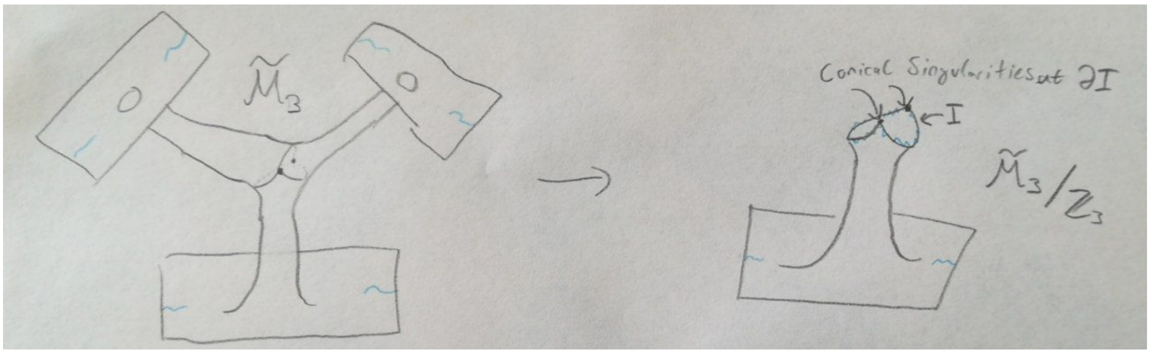


Figure 15: This figure illustrates the replica symmetry quotienting for $\widetilde{\mathcal{M}}_3$.

Note that the shape of the region I is not fixed by the arguments we gave. We should determine it by extremising $\mathrm{Tr}(\rho_R^n)$. To be more precise, we are interested in the linear term in the expansion

$$\mathrm{Tr}(\rho_R^n)/(\mathrm{Tr}(\rho_R))^n = 1 + (n-1)S_R + \ldots, \tag{3.12}$$

so what we should really extremise is $S_R[I]$. Now for a fixed I there are two contributions to $S_R[I]$:

First of all, there is a conical singularity on ∂I: Going in a circle around ∂I, we get rotated only by an angle of $2\pi/n$ instead of 2π. But this means that the scalar curvature must have a singularity

$$R = R_{smooth} + \delta(\partial I)(1 - \frac{1}{n}) \tag{3.13}$$

on ∂I (since the angular defect of a sphere is given by the integral of the scalar curvature in its interior). Since we integrate over n-copies of the fields of our theory, including the metric, this leads to an overall contribution of

$$n \cdot (1 - \frac{1}{n})\frac{\mathrm{Area}[\partial I]}{4G_N} = (n-1)\frac{\mathrm{Area}[\partial I]}{4G_N} \tag{3.14}$$

from the Einstein-Hilbert term in the action.

Secondly, we need to insert twist fields that relate different copies of our theory not only on the boundary of R but also on ∂I. But once we have done this, we can forget about where the twist fields originally came from and then realise that the path integral we are doing is, up to the term from the curvature singularity mentioned above, the same one that we would have done if we wanted to compute $\mathrm{Tr}[\rho_{R\cup I}]$ on a fixed geometry. But we know what the order $(1-n)$ contribution would be in that case: The fixed geometry entropy $S_f[R \cup I]$.

Combining the two contributions, we can therefore conclude

$$S_R[I] = \frac{\mathrm{Area}[\partial I]}{4G_N} + S_f[R \cup I]. \tag{3.15}$$

In other words, the entropy that we compute using the maximally connected replica worm hole is the entropy that the island formula predicts.

3.4 Is the information paradox solved now?

With the island formula and its replica wormhole derivation, physicists have made a lot of progress towards solving the quantum information paradox. The discussion of whether the paradox is truly solved though is however still not completely settled. The arguments given in favour for the replica wormhole / island solution in these notes can of course only be a small appetizer that should motivated interested readers to study the original literature. Furthermore, detailed arguments against it are beyond the reach of these notes. Hence, the concluding remarks made here can impossibly give proper account of the current status of the discussion.

Instead, let us end here by making four simple remarks that might be useful for the reader

- Some critics argue that both Hawking's computation and the island proposal are wrong and that instead the origin of the paradox is that the entanglement entropy of the Hawking radiation as considered in those approaches to the problem is an ill defined observable. [17, 18]
- So far, the extend to which the validity of the replica wormhole approach to entropy has been tested in concrete models is somewhat limited and mostly restricted to simple low dimensional theories (such as JT and other dilaton gravities).
- Even if the replica worm hole approach is correct, it does not explain what the true underlying degrees of freedom are that the gravitational path integral is coarse gaining over: Figuring out quantum gravity (obviously) remains on our collective to do list! However, its correctness does imply that a deep understanding of quantum gravity is not necessary in order to make black hole thermodynamics consistent with quantum mechanics.
- If one accepts the island proposal and its replica worm hole derivation, the paradox is indeed solved. A paradox is a situation where two statements appear to be true, yet contradictory. In the case of the quantum information paradox, these two where the statement of unitary time evolution in quantum mechanics and the statement that the entanglement entropy of Hawking radiation stays large even after the black hole has radiated away. The second statement turned out to be wrong.

References

[1] G. Penington, *Entanglement Wedge Reconstruction and the Information Paradox*, *JHEP* **09** (2020) 002 [1905.08255].

[2] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian and A. Tajdini, *Replica Wormholes and the Entropy of Hawking Radiation*, *JHEP* **05** (2020) 013 [1911.12333].

[3] G. Penington, S.H. Shenker, D. Stanford and Z. Yang, *Replica wormholes and the black hole interior*, *JHEP* **03** (2022) 205 [1911.11977].

[4] S.W. Hawking, *Particle Creation by Black Holes*, *Commun. Math. Phys.* **43** (1975) 199.

[5] M.K. Parikh and F. Wilczek, *Hawking radiation as tunneling*, *Phys. Rev. Lett.* **85** (2000) 5042 [hep-th/9907001].

[6] D.N. Page, *Information in black hole radiation*, *Phys. Rev. Lett.* **71** (1993) 3743 [hep-th/9306083].

[7] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian and A. Tajdini, *The entropy of Hawking radiation*, *Rev. Mod. Phys.* **93** (2021) 035002 [2006.06872].

[8] T. Hartman, *Black holes and quantum information*, *YouTube video lectures.* (2021) .

[9] Y. Sekino and L. Susskind, *Fast Scramblers*, *JHEP* **10** (2008) 065 [0808.2096].

[10] P. Hayden and J. Preskill, *Black holes as mirrors: Quantum information in random subsystems*, *JHEP* **09** (2007) 120 [0708.4025].

[11] T. Hartman, Y. Jiang and E. Shaghoulian, *Islands in cosmology*, *JHEP* **11** (2020) 111 [2008.01022].

[12] A. Almheiri, R. Mahajan, J. Maldacena and Y. Zhao, *The Page curve of Hawking radiation from semiclassical geometry*, *JHEP* **03** (2020) 149 [1908.10996].

[13] J.G. Russo, L. Susskind and L. Thorlacius, *The Endpoint of Hawking radiation*, *Phys. Rev. D* **46** (1992) 3444 [hep-th/9206070].

[14] P. Calabrese and J. Cardy, *Entanglement entropy and conformal field theory*, *J. Phys. A* **42** (2009) 504005 [0905.4013].

[15] T.M. Fiola, J. Preskill, A. Strominger and S.P. Trivedi, *Black hole thermodynamics and information loss in two-dimensions*, *Phys. Rev. D* **50** (1994) 3987 [hep-th/9403137].

[16] G. Penington, S.H. Shenker, D. Stanford and Z. Yang, *Replica wormholes and the black hole interior*, 2020.

[17] A. Laddha, S. Prabhu, S. Raju and P. Shrivastava, *The holographic nature of null infinity*, *SciPost Physics* **10** (2021) .

[18] H. Geng, A. Karch, C. Perez-Pardavila, S. Raju, L. Randall, M. Riojas et al., *Information transfer with a gravitating bath*, *SciPost Physics* **10** (2021) .