I. Introduction (and motivation) to the topic

Lecture mainly taken from:
* 2006.02838v3, "Primordial Black Holes as Dark Matter: Recent Dev.", B. Carr
* The Astrophysical Journal, 304: 1-5, 1986 May 1: "Gravitational microlensing by the Galactic Halo", Bohdan Paczynski
* Encyclopedia of Astronomy and Astrophysics, "Microlensing"

PBHs has been a source of interest for ~50 yrs despite that we don't have evidence for them.

↳ They could be small enough for Hawking radiation to be important: e.g. PBH smaller than 10^{15} g would have evaporated by now

↳ Larger mass PBH ($>10^{15}$ g) are unaffected by Hawking radiation but they also attracted interest because of the possibility that they provide the dark matter (25% of the CRITICAL DENSITY). This because PBHs are most likely to be formed in the RADIATION DOMINATION ERA, being not subject to the BBN constraint on baryons (5% of CRITICAL DENSITY)

They could in principle explain some interesting phenomena:

* EVAPORATING PBHs → Galactic and extra-Galactic γ ray background
  → antimatter in cosmic rays
  → some short-period γ-ray bursts
  → annihilation line radiation from the Galactic centre

* Non-evaporating PBHs → lensing effects
  → heating of the stars in the Galactic disk
  → Origin of MACHOs
  → Seeds for SMBHs in Galactic nuc.
  → large scale structure formation through Poisson fluctuations
  → LIGO/Virgo GW events

usually there are other possible explanations for this features, so we can't take them as definitive evidence for PBHs.

Anyway studying these effects can be used to place constraints on the number of PBH in the halo of a certain mass M (and then constraints on cosmological model related to them)

II - Something about PBHs formation

Many different mechanisms in the Early Universe could lead to the formation of PBHs, e.g.:

* PRIMORDIAL INHOMOGINITIES
* SCALE INVARIANT FLUCTUATIONS
* COLLAPSE in a MATTER DOM. ERA
* collapse from INFLACTIONARY FLUCTUATIONS
* QUANTUM DIFFUSION
* CRITICAL COLLAPSE
* COLLAPSE at the QCD scale
* collapse of cosmic loops, through bubble collisions and DOMAIN WALLS

For them in general the cosmological energy density at early times plays a major role, giving a connection between the horizon mass and the PBH mass at formation:

$$M \sim \frac{c^3 t}{G} \sim 10^{15} \left(\frac{t}{10^{-23}\,s} \right) g$$

RAD - DOMINATED UNIVERSE

PBHs could span an enormous mass range:

* Planck time ($\sim 10^{-43}s$) $\rightarrow$ Planck mass ($10^{-5} g$)
* $t_f \sim 1s$ $\rightarrow$ $M \sim 10^5 M_\odot$ $\sim$ size of holes in the galactic nuclei

By contrast BHs formed at the present epoch from stellar collapse would not be smaller than $1 M_\odot$

<u>OBS</u>: In some models PBHs may form over an extended period, corresponding to a wide range of masses.
In our analysis of the constraints we will consider monochromatic mass functions (width $\Delta M \sim M$)

III CONSTRAINTS and CAVEATS

(FIGURE 1)

We will focus on 2 kind of constraints on PBH too large to have evaporated completely by now: EVAPORATION and MICRO (LENSING) CONSTRAINTS.

Higher mass constraints come from dynamical limits, large scale structure and accretion considerations.

NOTE that there are also 4 mass window where the PBHs could have an appreciable density in terms of dark matter (DM)

Assumptions:

* PBH cluster in galactic halo or other form of CDM
* Monochromatic mass function → mass range $\Delta M \sim M$

→ $f(M)$ can be related to $\beta(M)$ as in

CAVEATS:

→ For some constraints observation is well-understood, but there are uncertainties on the BH physics.

→ For some other the observations are not fully understood or depend on astrophysical assumptions.

→ The constraints could depend on other physical parameters not shown in the plot

* Some of the constraints could be circumvented if the PBHs have an ext. mass function.

(It's not so trivial doing it anyway)

Evaporation Constraints

PBH of initial mass M will evaporate by emission of Hawking radiation on a timescale

$$\tau \propto M^3$$

$$\rightarrow \quad \tau < t_0 \quad \text{for masses} \quad M < M_* \approx 5 \cdot 10^{14}\ g$$

We focus here on the observations from the extragalactic γ-ray background. The Galactic γ-ray background could give stronger limit but it depends sensitively on the form of the PBH mass function, so we don't discuss it here.

For PBH with a mass $M > 2M_*$, for which we can neglect the mass change, the IST. SPECTRUM (non-jet photon) is

$$\frac{d\dot{N}_\gamma^p}{dE}(M,E) \propto \frac{E^2\sigma(M,E)}{\exp(EM)-1} \propto \begin{cases} E^3M^3 & \text{for } EM<1 \\ E^2M^2\exp(-EM) & \text{for } EM>1 \end{cases}$$

$\sigma(M,E)$ absorption cross-section for photons of energy E

$$\sigma \propto \begin{cases} E^2M^4 & (EM<1) \\ M^2 & (EM>1) \end{cases}$$

This gives

$$I(E) \propto f(M) \cdot \begin{cases} E^4M^2 & \text{for } EM<1 \\ E^3M\exp(-EM) & \text{for } EM>1 \end{cases}$$

The peak goes as $E^{MAX} \propto M^{-1}$ with a value

$$I^{MAX}(M) \propto f(M)\,M^{-2}$$

Observed intensity is $I^{obs} \propto E^{-(1+\epsilon)}$ with $0.1 < \epsilon < 0.4$

$\rightarrow$ Imposing $I^{MAX} < I^{OBS}$ gives

$$f(M) < 2\cdot 10^{-8}\left(\frac{M}{M_*}\right)^{3+\epsilon}$$

The constrain is plotted for $\epsilon = 0.2$ in FIGURE 1.

NOTE that there are other constraints from evaporation coming from :

* positron data from Voyager 1 for PBH with $M < 10^{16}$ g giving $f < 0.001$

* 511 keV annihilation line radiation from the Galactic centre can constrain $10^{16} - 10^{17}$ g PBH

LENSING CONSTRAINTS

* The idea of microlensing from the Galactic Halo (PACZYNSKI, 1986)

In most cases of GR MICROLENSING of a quasar object due to a star at cosmological distance the time scale of the process is very long. If we want the time scale to be much shorter we have to look closer (stars in the halo of our Galaxy)

→ The price to pay is high: optical depth τ to gr. lensing on known stars in our galaxy is very small.

* The optical depth τ describes the probability of observing a lensing event w/ a magnification of a factor 1.34

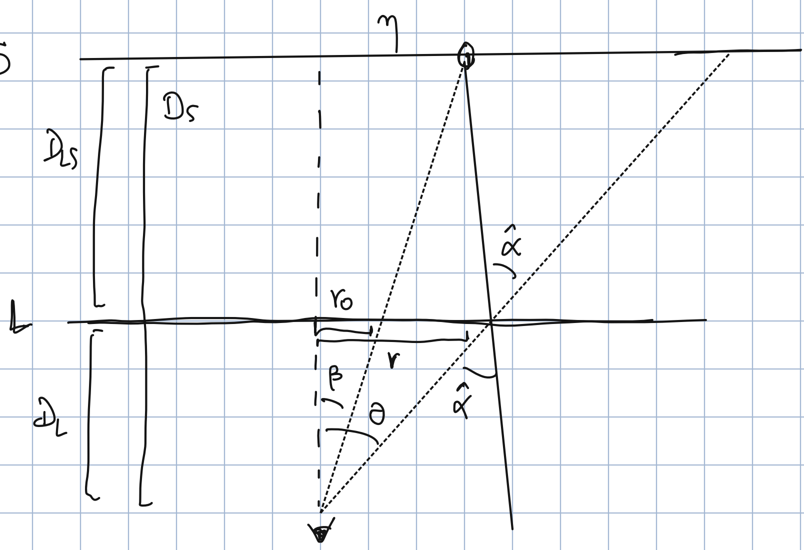
BUT most of the halo mass is believed to be in some unknown form of "dark matter".

The idea here is that if dark matter is made of massive objects, then it may give rise to gravitational lensing with a substantially higher optical depth.

* A model

It's been showed (Vietri, Ostriker 1983 and Nityananda, Ostriker 1984) that when τ is small it gives the prob. that one star strongly affects the intensity of a distant source of radiation.
Working in this regime, we consider the event due to just a single point with mass M.
We also work in the approximation of flat space and consider the source of radiation as POINTLIKE.

The lens equation can be constructed starting from a <u>geometrical consideration</u>:



DEFLECTION ANGLE

$$\underbrace{\theta D_S}_{\text{Apparent position in the source plane}} = \underbrace{\beta D_S}_{\text{REAL source position}} + \hat{\alpha} D_{LS}$$

$$\hookrightarrow \quad \boxed{\theta = \beta + \alpha}$$

<u>LENS EQUATION</u>

where

$$\alpha \equiv \hat{\alpha} \frac{D_{LS}}{D_S}$$

The "interesting" physics depends on the fact that the <u>deflection angle</u> α depends on θ

In particular, for the case a point mass lens,

$$\hat{\alpha} = \frac{4GM}{c^2 r} = \frac{4GM}{c^2 D_L \theta}$$

The LENS EQUATION becomes: $\theta = \beta + \frac{4GM}{c^2 D_L \theta} \frac{D_{LS}}{D_S}$

and projecting it in the lens plane

$$\theta^2 D_L^2 - \beta\theta D_L^2 - \frac{4GM}{c^2} \frac{D_{LS} D_L}{D_S} = 0$$

$$r^2 - r r_0 - R_0^2 = 0 \qquad (1)$$

with

$$R_0^2 \equiv \frac{4GM}{c^2} \frac{D_{LS} D_L}{D_S}$$

The angular distance associated to R_0 will be

$$\theta_E^2 \equiv \frac{4GM}{c^2} \frac{D_{LS}}{D_S D_L}$$, called EINSTEIN ANGLE

(1) has 2 solutions corresponding to the position of the two images

$$r_{1,2} = \frac{1}{2}\left[r_0 \pm \sqrt{r_0^2 + R_0^2} \right] \qquad (2)$$

* PH. MEANING of R_0: radius of the annular image forming when point source and point mass are perfectly aligned.

We can then define the amplifications of these two images as given by

$$A_{1,2} \equiv abs\left(\frac{r_{1,2}}{r_0} \frac{dr_{1,2}}{dr_0} \right) = abs\left(\frac{r_{1,2}^4}{r_{1,2}^4 - R_0^4} \right)$$

and their combined amplification as

$$A \equiv A_1 + A_2 = \frac{u^2+2}{u\sqrt{u^2+4}} \qquad \text{with} \qquad u \equiv \frac{r_0}{R_0}$$

⇝ when the source is found within a radius R_0 from the lens, then the combined amplification of the two images will be ≥ 1.34

(STRONG MICROLENSING)

When a point mass passes between the observer and the source, the apparent intensity of the source varies in proportion to A.

The effect would depend on the impact parameter of the point mass d, i.e. the minimal distance r_0 from the source (projected in the lens plane)

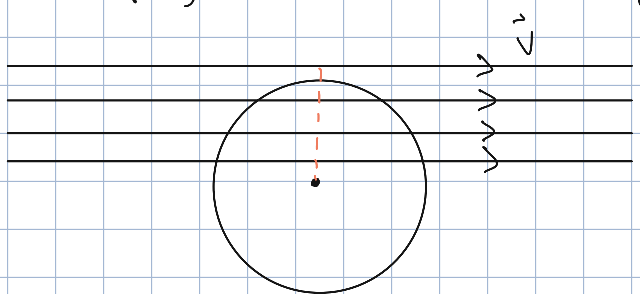


FIGURE 2.

→ The profile of the time-dependent A has two "significant" characteristic: the symmetry of the profile and the fact that it's non modifying the wavelength of the photons.

→ The intensity varies more for passages w/ a small <u>impact param</u>.

* What is the probability of this effect?

The <u>optical depth</u> is defined more generally as

$$\tau \equiv \int_0^{D_S} \frac{4\pi G D}{c^2} \rho(D_L)\, dD_L \qquad (3)$$

Ratio of surface mass density of microlensing material over the critical mass density

$$\Sigma \equiv \frac{c^2}{4\pi G D}$$

Density of microlensing material at a distance D_L from the observer

In the core of one point-like lens of mass M it reduces to

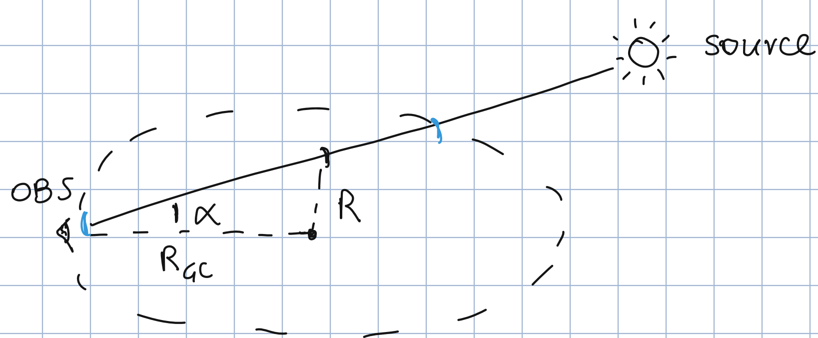
$$\frac{4\pi G D}{c^2} M = \pi R_0^2$$

Adopting an isothermal model for the mass distribution in the "dark" halo of our Galaxy, with

$$M(R) = \frac{V_{ROT}^2}{G} R \qquad \text{and} \qquad \rho(R) = \frac{1}{4\pi R^2}\frac{dM}{dR} = \frac{V_{ROT}^2}{4\pi G R^2}$$

where $V_{ROT} = 210$ km s^{-1} = constant

In general the source would be at an angular distance α from the centre of the Galaxy.



Then the density of lensing material along the line of sight will be

$$\rho(D_L) = \frac{\rho_0}{1 + d_L^2 - 2 d_L \cos\alpha}$$

$$d_L \equiv \frac{D_L}{R_{GC}} \qquad \rho_0 = \frac{V_{ROT}^2}{4\pi G R_{GC}^2}$$

ρ_0 : DM halo density close to the Sun

R_{GC} : 10 kpc

We can now estimate the optical depth for some nearby galaxies where individual stars are easily resolved and can be used as background sources, e.g. LMC, SMC, M31 and M33.

Their linear and angular distances can be taken as

$D_S \simeq 50$ kpc $\quad \alpha = 82°$ to LMC
$= 60$ kpc $\quad = 69°$ to SMC
$= 650$ kpc $\quad = 119°$ to M31
$= 730$ kpc $\quad = 127°$ to M33

Eq (3) can be rewritten as

$$\tau_0 \equiv \frac{V_{ROT}^2}{c^2} \approx 5 \cdot 10^{-7}$$

$$\tau = \frac{\tau_0}{x_s} \int_0^{x_n} \frac{(x_s - x)\, x}{1 + x^2 - 2x \cos\alpha}\, dx$$

$$x \equiv \frac{D_L}{R_{GC}} \qquad x_s \equiv \frac{D_s}{R_{GC}} \qquad x_n \equiv \frac{D_n}{R_{GC}}$$

and D_n, which represents the extent of the "dark" galactic halo is adopted as a free parameter.

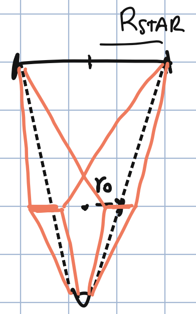
This integral can be performed analitically and studied as a function of x_n:

IN ALL CASES τ TURNS OUT TO BE $\sim 10^{-6}$.
THIS MEANS THAT THERE IS A PROBABILITY of 10^{-6} that any STAR in SMC, LMC, M31, M33 is strongly GRAVITATIONALLY MICROLENSED by a DARK OBJECT in the GALACTIC HALO.

Which mass range can we test through this effect?

All this is true provided that the lens is MASSIVE enough to amplify significantly the source, which in reality will have a small angular size.

MAXIMUM AMPLIFICATION $\rightarrow$ SOURCE, LENS and OBSERVER perfectly ALIGNED



→ CIRCULAR DISK SOURCE with RADIUS r_0 in deflector's plane

$$r_0 = R_{STAR} \frac{D_L}{D_S}$$ will give origin to

a RING LIKE image

Assuming uniform surface brightness for the star, the total intensity will be amplified by a factor

$$A = \frac{\pi r_1^2 - \pi r_2^2}{\pi r_0^2} = \sqrt{1 + 4\frac{R_0^2}{r_0^2}}$$

$r_0 < R_0$ for large max. amplification

An estimate of the minimum mass of the lensing object would be given by:

$$M_{min} \approx \frac{c^2}{4GD}\left(R_0^2\right)_{R_0 = r_0} = \frac{c^2}{4GD}\left(R_{star}\frac{D_L}{D_S}\right)^2 \approx$$

$$\approx \frac{c^2 D_L}{4G}\left(\frac{R_{star}}{D_S}\right)^2 \approx 3 \cdot 10^{-9} M_\odot \left(\frac{D_L}{10\,kpc}\right)\left(\frac{R_{star}}{R_\odot}\right)^2\left(\frac{100\,kpc}{D_S}\right)^2$$

for $D_S \gg D_L$

→ For a solar radius star we obtain

$M_{min} \approx$	$1.2 \times 10^{-8}\ M_\odot$	for LMC
	$0.8 \times 10^{-8}\ M_\odot$	for SMC
	$7 \times 10^{-11}\ M_\odot$	for M31
	$6 \times 10^{-11}\ M_\odot$	for M33

→ Low masses : possibility of discovery objects as small as asteroids

→ We can also obtain an estimate of R_0

$$R_0 \approx 1.4 \cdot 10^{14}\ \text{cm} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D_L}{10\text{kpc}}\right)^{1/2} \quad \text{for} \quad D_S \gg D_L$$

⤳ We can have lensing even for large star

and for the typical timescale of the intensity variation

$$t_0 \equiv \frac{R_0}{v} \approx 6 \cdot 10^6\ s \left(\frac{M}{M_\odot}\right)^{1/2} \approx 0.2\ yr \left(\frac{M}{M_\odot}\right)^{1/2}$$

Solar radius star in:

↳ L/SMC $t_0 \sim 10$ min if $M \sim 10^{-8} M_\odot$

M31/M33 $t_0 \sim 1$ min if $M \sim 10^{-10} M_\odot$

A solar mass "dark halo" object would microlens on a time scale of 2 months

COMMENTS: An observing program aimed at monitoring the brightness of few million stars over a period of 2 yrs could put interesting constraints on the masses of dark objects in the Galactic halo.
It could detect lensing until to sources of mass $\sim 10^2 M_\odot$
→ A single event may give only a rough estimate because to learn about the mass distribution we need to observe a large number of lensing events.

* Lensing constraints on PBHs

1) 7hrs observation of M31 with the SUBARU HYPER SUPRIME-CAM (HSC) to search for MICROLENSING of stars by PBHs lying in the halos of the Milky Way and M31 gave the bound for

$$10^{-10}\, M_\odot < M < 10^{-6}\, M_\odot$$ shown in the figure.

2) Bounds from microlensing of stars in the LMC and SMC come from:

* MACHO project detected lenses with $M \sim 0.5\, M_\odot$ but could contribute only at most 10% to the halo
* EROS project excluded $6 \cdot 10^{-8} M_\odot < M < 15\, M_\odot$ from dominating the halo
* OGLE experiment put further limits in the range $0.1\, M_\odot < M < 20\, M_\odot$

$$f(M) < \begin{cases} 1 & (6 \cdot 10^{-8} M_\odot < M < 30\, M_\odot) \\ 0.1 & (10^{-6} M_\odot < M < 1\, M_\odot) \\ 0.05 & (10^{-3} M_\odot < M < 0.4\, M_\odot) \end{cases}$$

3) femtolensing of γ-ray bursts (GRBs)

4) Lack of lensing from type Ia supernovae to constrain the PBH population

5) Millilensing of compact radio sources (weaker than the dynamical ones)

(FIGURE 1)

PBH could be a good explanation for some experimental conundra.
At the same time the same experiments tells us that PBHs cannot account for all the Dark Matter, exception made for some mass window.

FIGURE 1

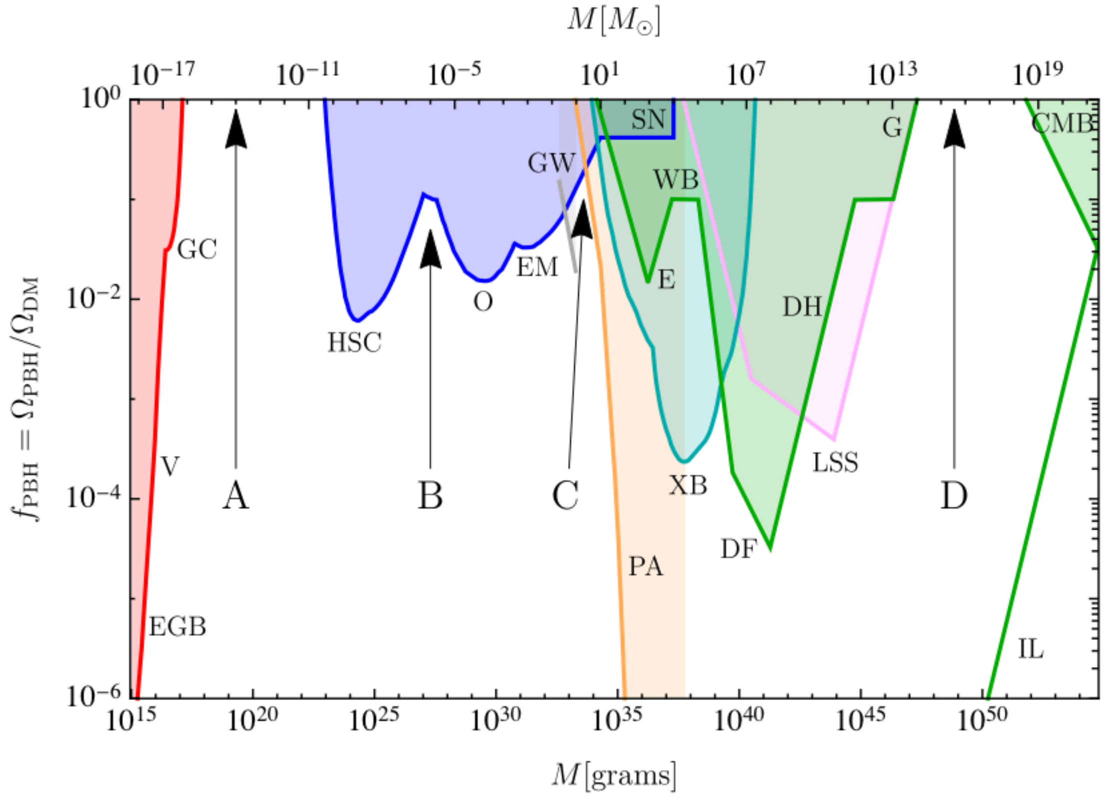


FIG. 1. Constraints on $f(M)$ for a monochromatic mass function, from evaporations (red), lensing (blue), gravitational waves (GW) (gray), dynamical effects (green), accretion (light blue), CMB distortions (orange) and large-scale structure (purple). Evaporation limits come from the extragalactic γ-ray background (EGB), the Voyager positron flux (V) and annihilation-line radiation from the Galactic centre (GC). Lensing limits come from microlensing of supernovae (SN) and of stars in M31 by Subaru (HSC), the Magellanic Clouds by EROS and MACHO (EM) and the Galactic bulge by OGLE (O). Dynamical limits come from wide binaries (WB), star clusters in Eridanus II (E), halo dynamical friction (DF), galaxy tidal distortions (G), heating of stars in the Galactic disk (DH) and the CMB dipole (CMB). Large-scale structure constraints derive from the requirement that various cosmological structures do not form earlier than observed (LSS). Accretion limits come from X-ray binaries (XB) and Planck measurements of CMB distortions (PA). The incredulity limits (IL) correspond to one PBH per relevant environment (galaxy, cluster, Universe). There are four mass windows (A, B, C, D) in which PBHs could have an appreciable density. Possible constraints in window D are discussed in Section VI but not in the past literature.

FIGURE 11

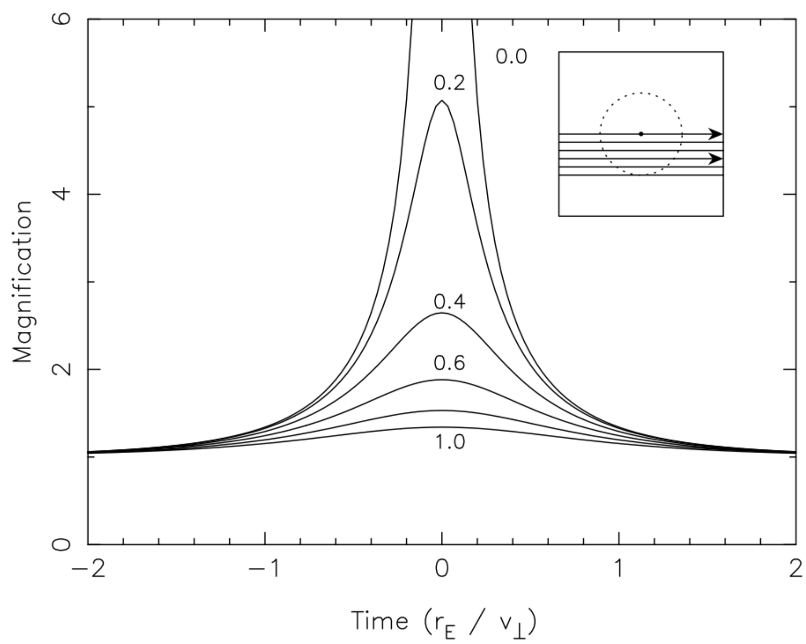


Figure 2. Microlensing event lightcurves (magnification versus time) for six values of the impact parameter $u_{\min} = 0.0, 0.2, \ldots, 1.0$ as labelled. Time is in units of the Einstein radius crossing time $r_E/v_\perp$. The inset illustrates the Einstein ring (dotted circle) and the source paths relative to the lens (dot) for the six curves.

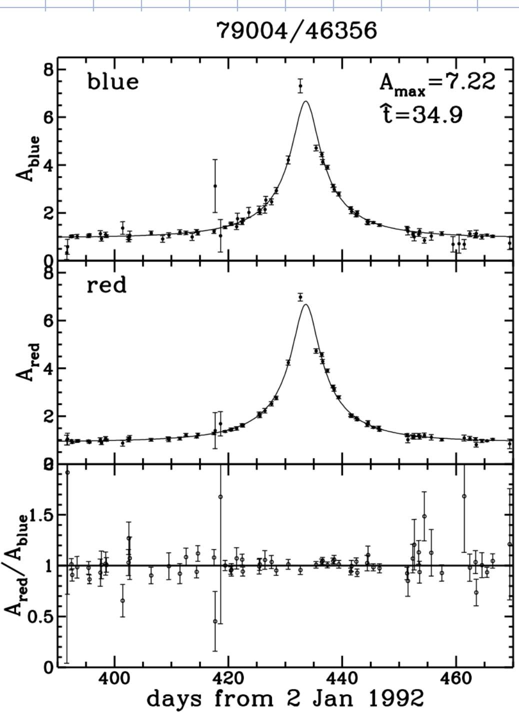


FIGURE 3

Figure 3. The first LMC microlensing candidate from the MACHO project. (Expanded view: 6 yr of constant data are outside the plot). Upper and middle panels show brightness versus time in blue and red passbands respectively, in units of the baseline value. Points with error bars are observations, the curve is the best microlensing fit. The lower panel shows the ratio of red/blue flux, illustrating the lack of color change.