Parton Branching results discussion in GEN-22-001

Hannes Jung (Emeritus, II Institut f. Theoretische Physik, DESY, Hamburg)

- based on:
  - Bubanja, I. et al, arXiv: 2312.08655
  - M. Mendizabal arXiv: 2309.11802
  - Bubanja, I. et al, arXiv: 2404.04088
- together with co-authors:
  - I. Bubanja, A. Bermudez Martinez, L. Favart, F. Guzman, F. Hautmann, A. Lelek, M. Mendizabal, K. Moral Figueroa, L. Moureaux, N. Raicevic, M. Seidel, S. Taheri Monfared

The role of soft gluons

Hannes Jung (Emeritus, II Institut f. Theoretische Physik, DESY, Hamburg)

- Recap of PB method
- issues of consitency
- Drell-Yan production
- Importance of soft gluons

Parton Branching approach - recap

DGLAP evolution – solution with parton branching method

$$f(x,\mu^2) = f(x,\mu_0^2)\Delta_s(\mu^2) + \int^{z_M} \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z}, \mu'^2\right)$$

- solve integral equation via iteration:

$$f_0(x,\mu^2) \quad = \quad f(x,\mu_0^2)\Delta(\mu^2)$$

- with Sudakov form factor

$$\Delta_s(\mu^2) \quad = \quad \exp\left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mathbf{q}'^2}{\mathbf{q}'^2} \int_0^{z_{\mathrm{M}}} dz \; z \; P_{ba}^{(R)}(\alpha_s, z) \right)$$

DGLAP evolution – solution with parton branching method

$$f(x,\mu^2) = f(x,\mu_0^2)\Delta_s(\mu^2) + \int^{z_M} \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z}, \mu'^2\right)$$

- solve integral equation via iteration:

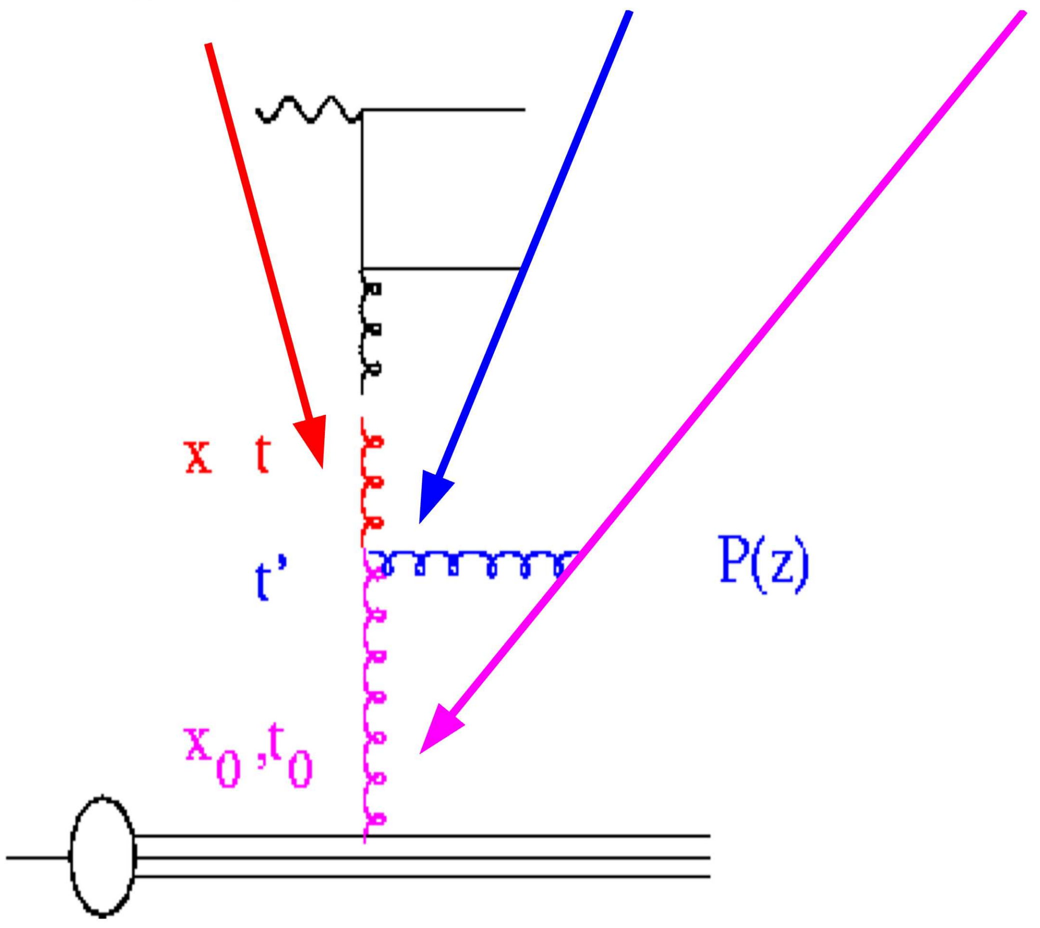
$$f_0(x,\mu^2) = f(x,\mu_0^2)\Delta(\mu^2)$$

$$f_1(x,\mu^2) = f(x,\mu_0^2)\Delta(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta(\mu^2)}{\Delta(\mu'^2)} \int^{z_M} \frac{dz}{z} P^{(R)}(z) f(x/z,\mu_0^2)\Delta(\mu'^2)$$

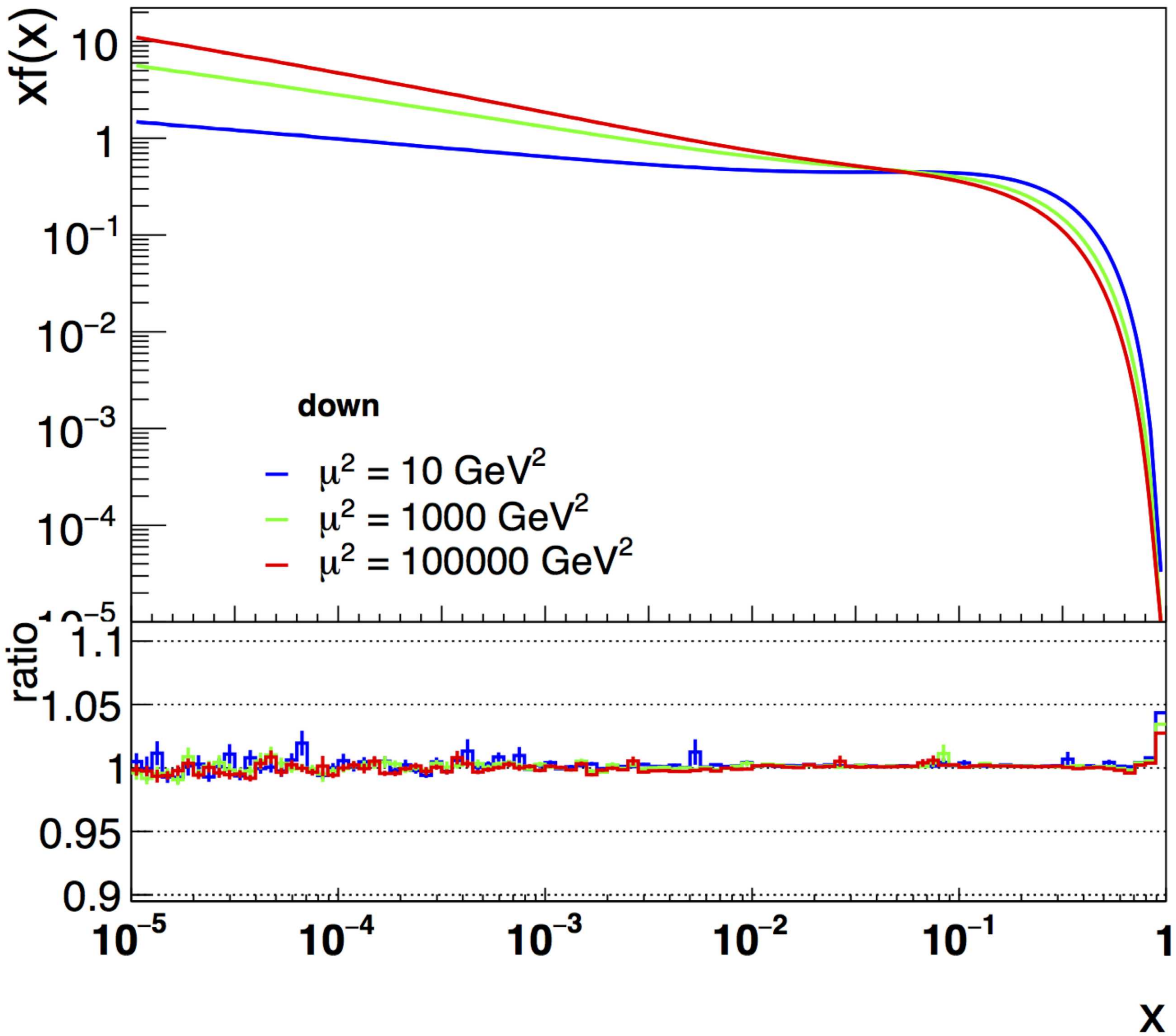
from μ' to μ w/o branching

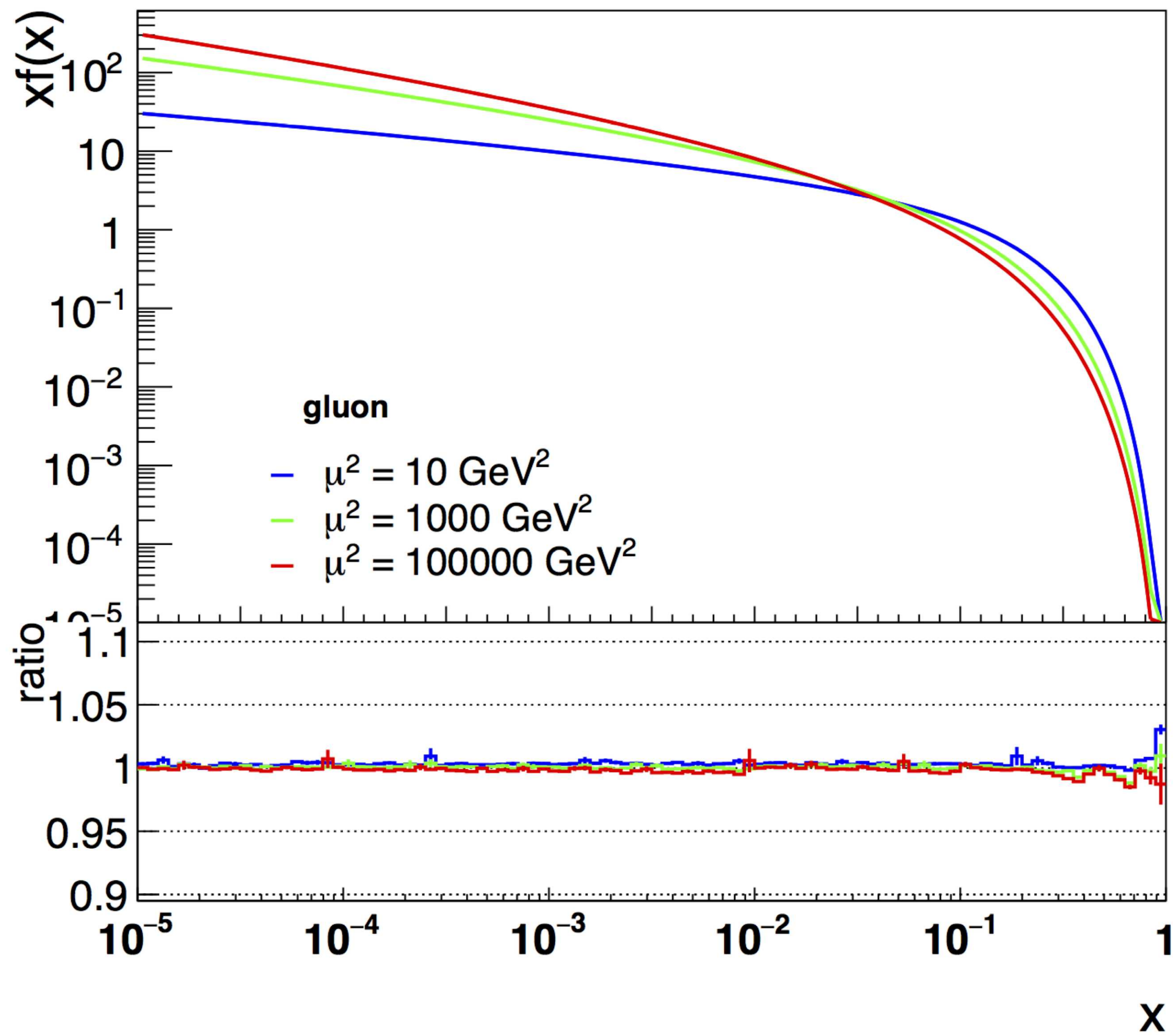
branching at μ'

from μ to μ' w/o branching



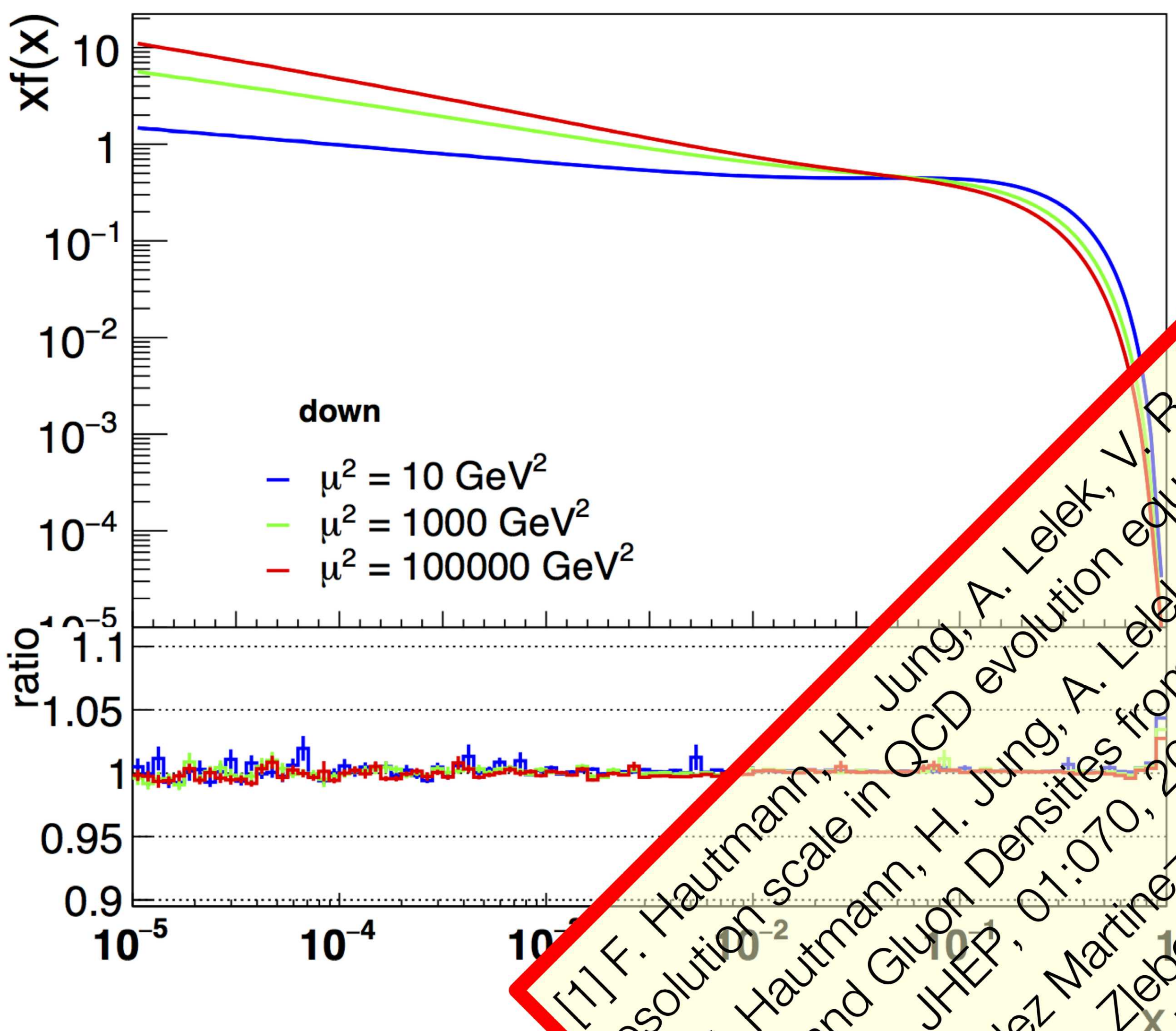
Validation of method with QCDnum at NLO

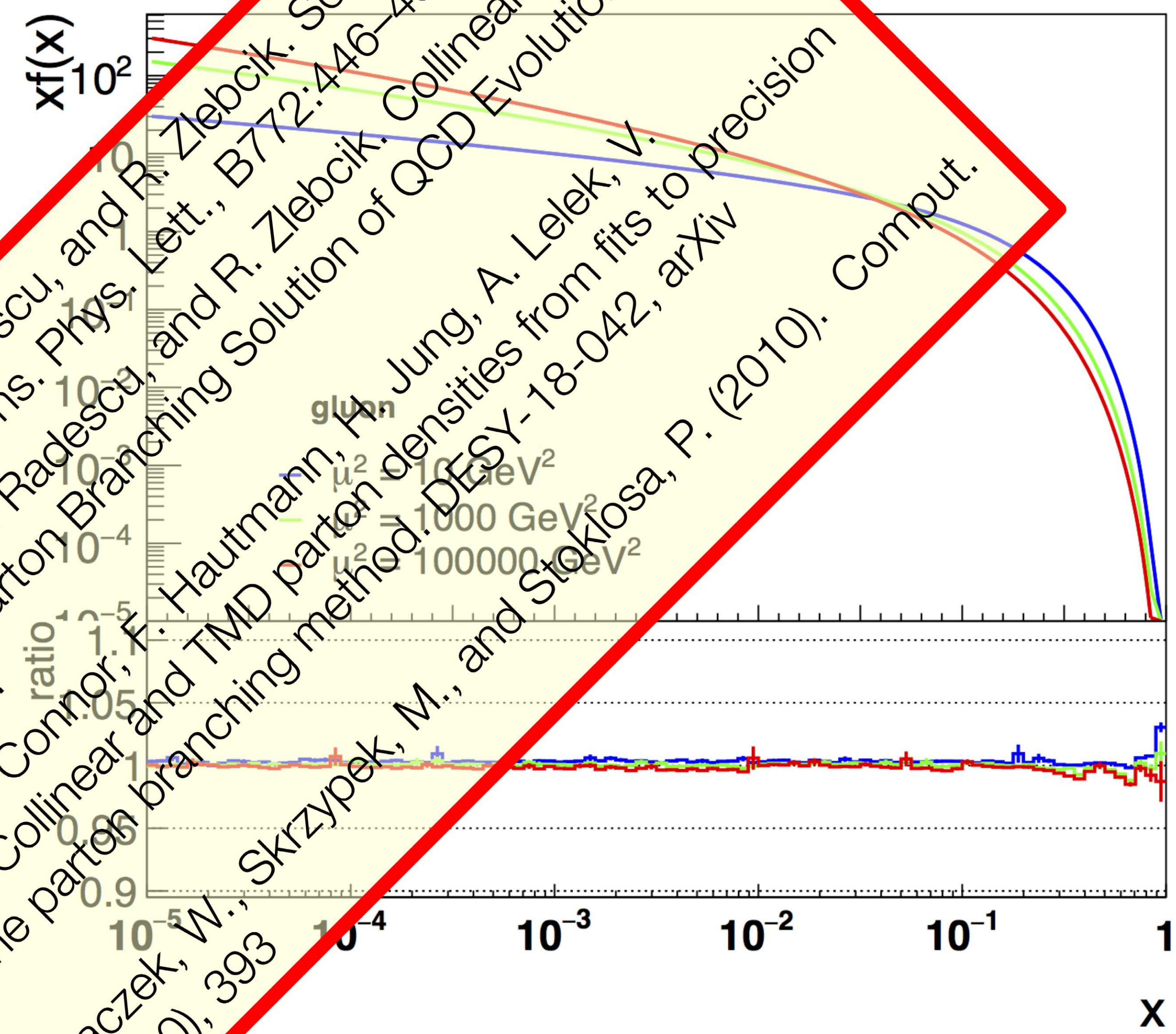




- Very good agreement with NLO - QCDnum over all x and μ^2
  - the same approach works also at NNLO !

Validation of method with QCDnum at NLO

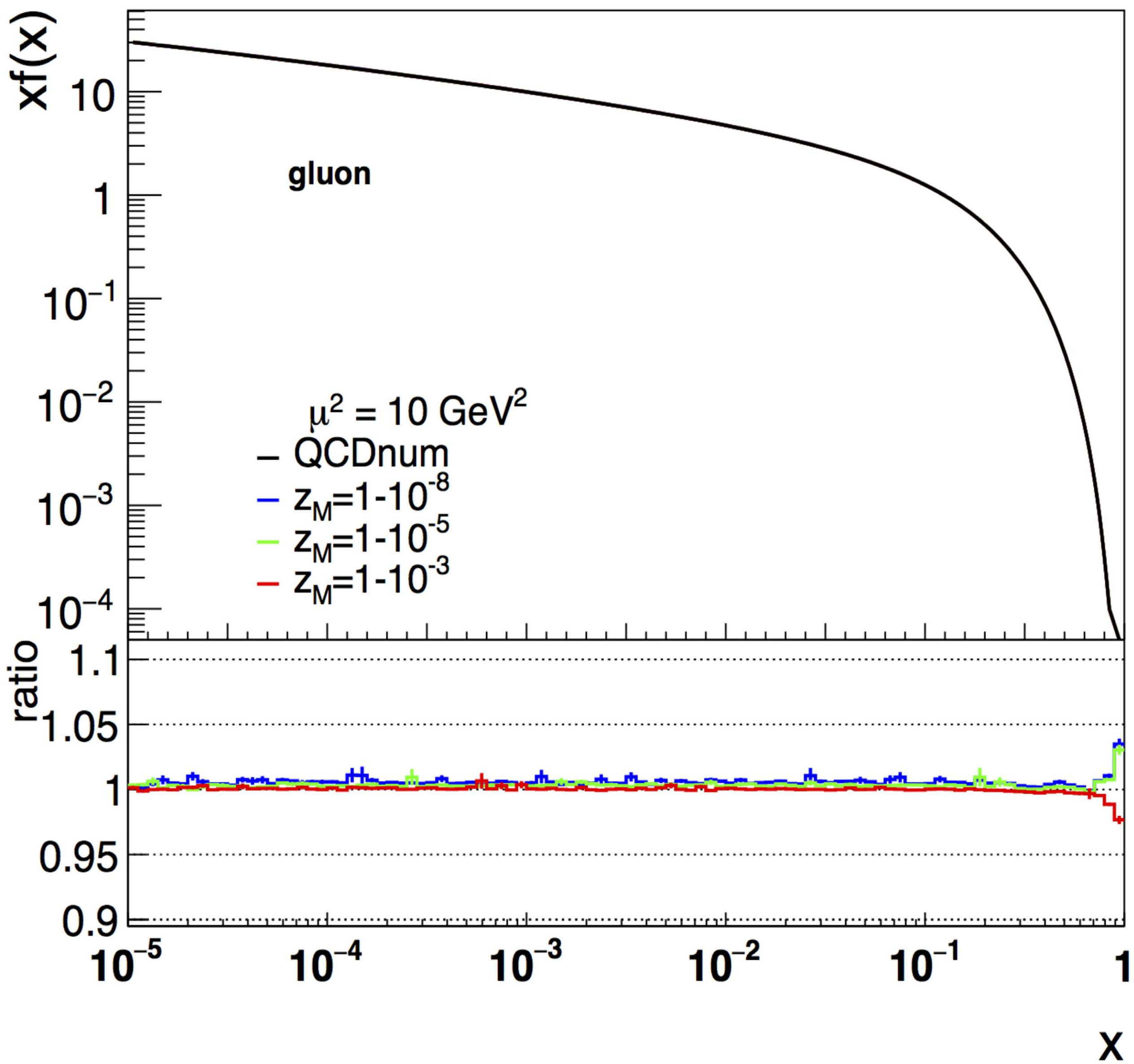


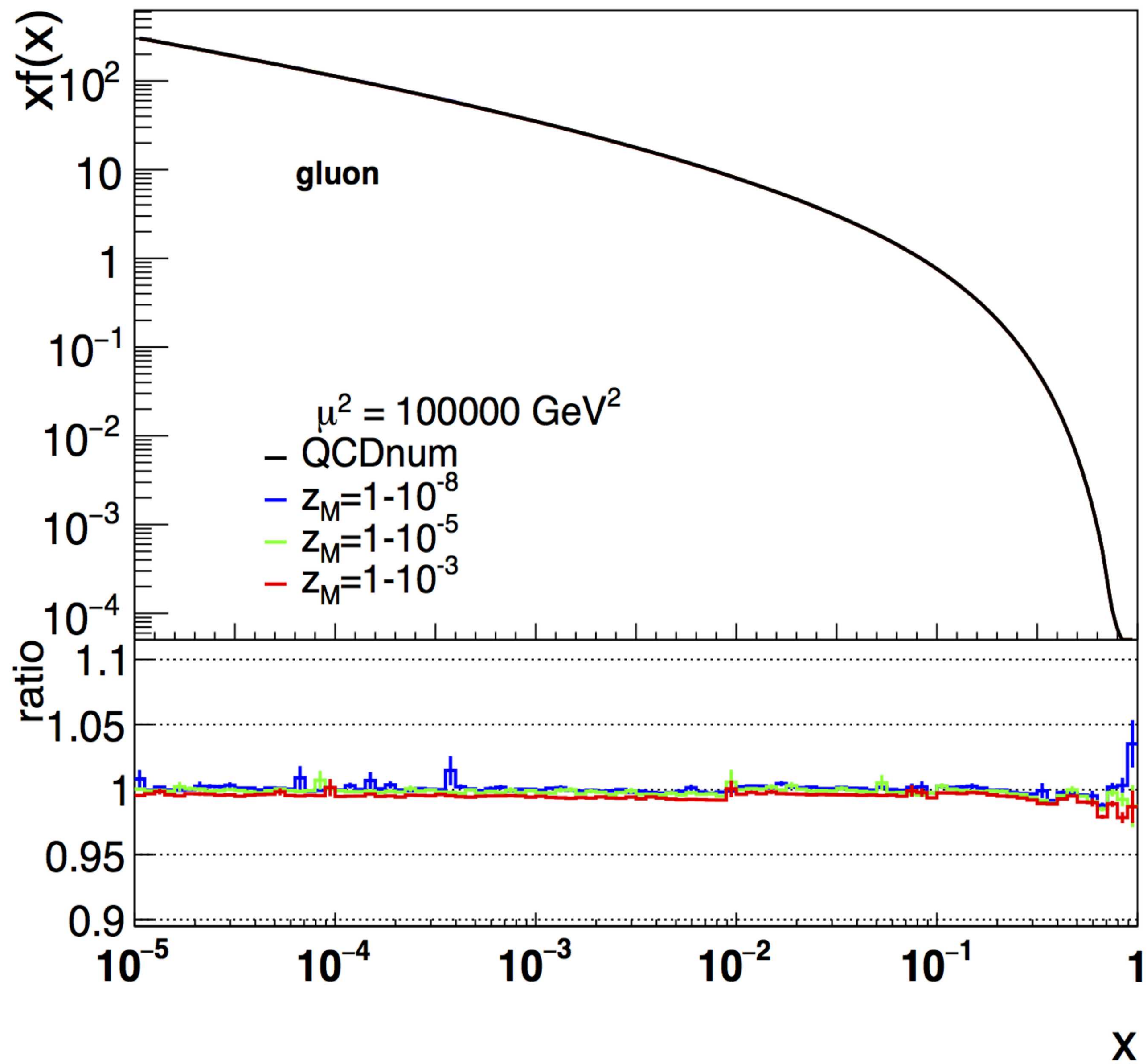


[1] F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Soft-gluon resolution scale in QCD evolution equations. Phys. Lett., B772:446–451, 2017.
[2] F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Collinear and TMD Quark and Gluon Densities from Parton Branching Solution of QCD Evolution Equations. JHEP, 01:070, 2018.
[3] A. Bermudez Martinez, P. Connor, F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Collinear and TMD parton densities from fits to precision DIS measurements in the parton branching method. DESY-18-042, arXiv 1804.11152
see also Jadach, S., Placzek, W., Skrzypek, M., and Stoklosa, P. (2010). Comput. Phys. Commun., 181(2010), 393

- Very good agreement with NLO - QCDnum over all x and μ^2
  - the same approach works also at NNLO !

Validation of method at NLO: z_M - dependence





- No dependence on z_M if z_M is large enough
- Very good agreement with NLO - QCDnum

PDFs from Parton Branching method: fit to HERA data

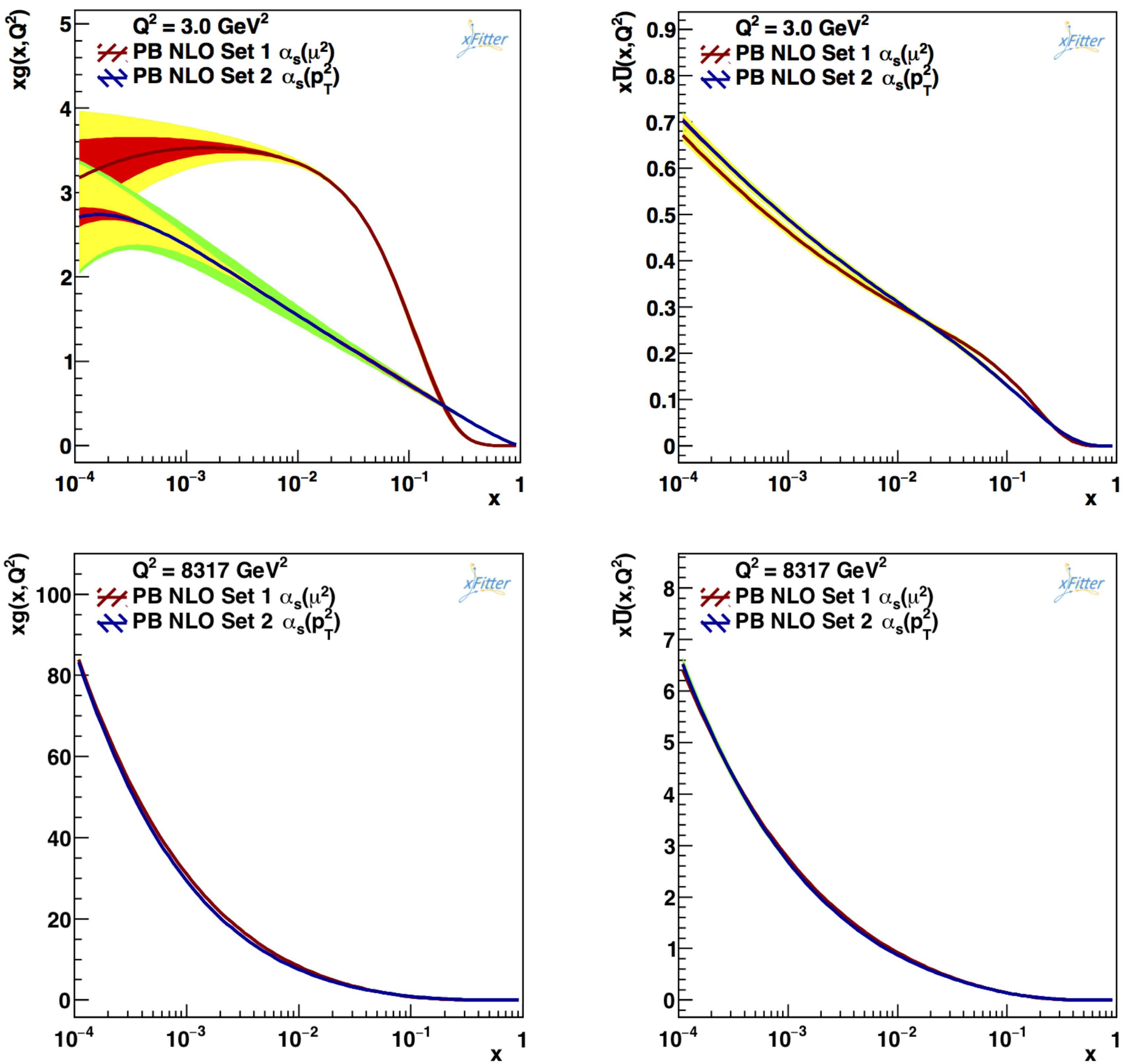
- Convolution of kernel with starting distribution

$$
\begin{aligned}
xf_a(x,\mu^2) &= x\int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b\left(x'',\mu^2\right)\delta(x'x''-x) \\
&= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'}\, \tilde{\mathcal{A}}_a^b\left(\frac{x}{x'},\mu^2\right)
\end{aligned}
$$

- Fit performed using xFitter frame (with collinear Coefficient functions at NLO)
  - using full HERA I+II inclusive DIS (neutral current, charged current) data
    - in total 1145 data points
      - $3.5 \leq Q^2 \leq 50000\ \mathrm{GeV}^2$
      - $4 \cdot 10^{-5} < x < 0.65$
      - using starting distribution as in HERAPDF2.0
      - $\chi^2/ndf = 1.2$

➔ Can be easily extended to include any other measurement for fit !

Collinear parton distributions after fit

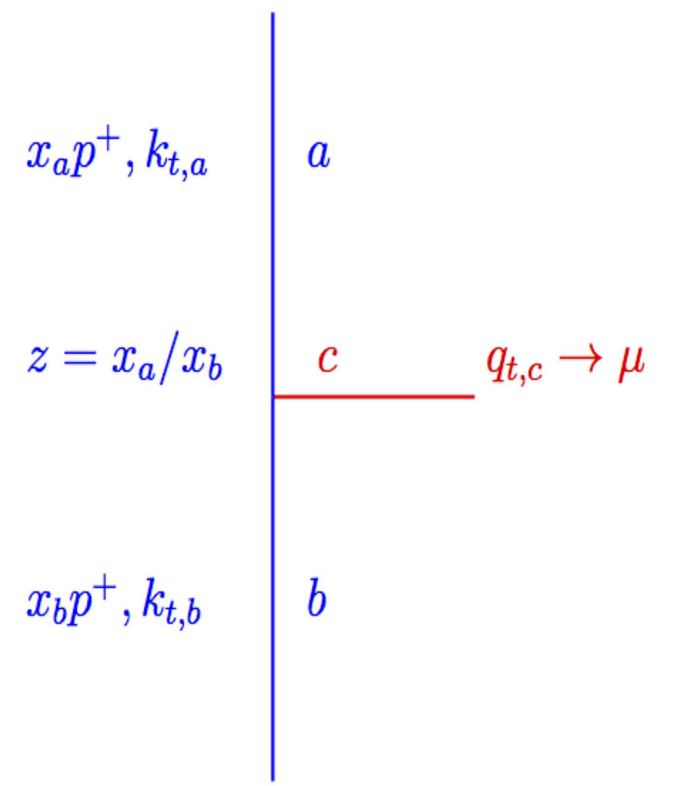


- fit 1 with $\alpha_s(q)$
  - as good as HERAPDF2.0 $\chi^2/ndf = 1.2$
- fit 2 with $\alpha_s(q(1-z))$
  - $\chi^2/ndf = 1.21$
- very different gluon distribution obtained at small Q^2

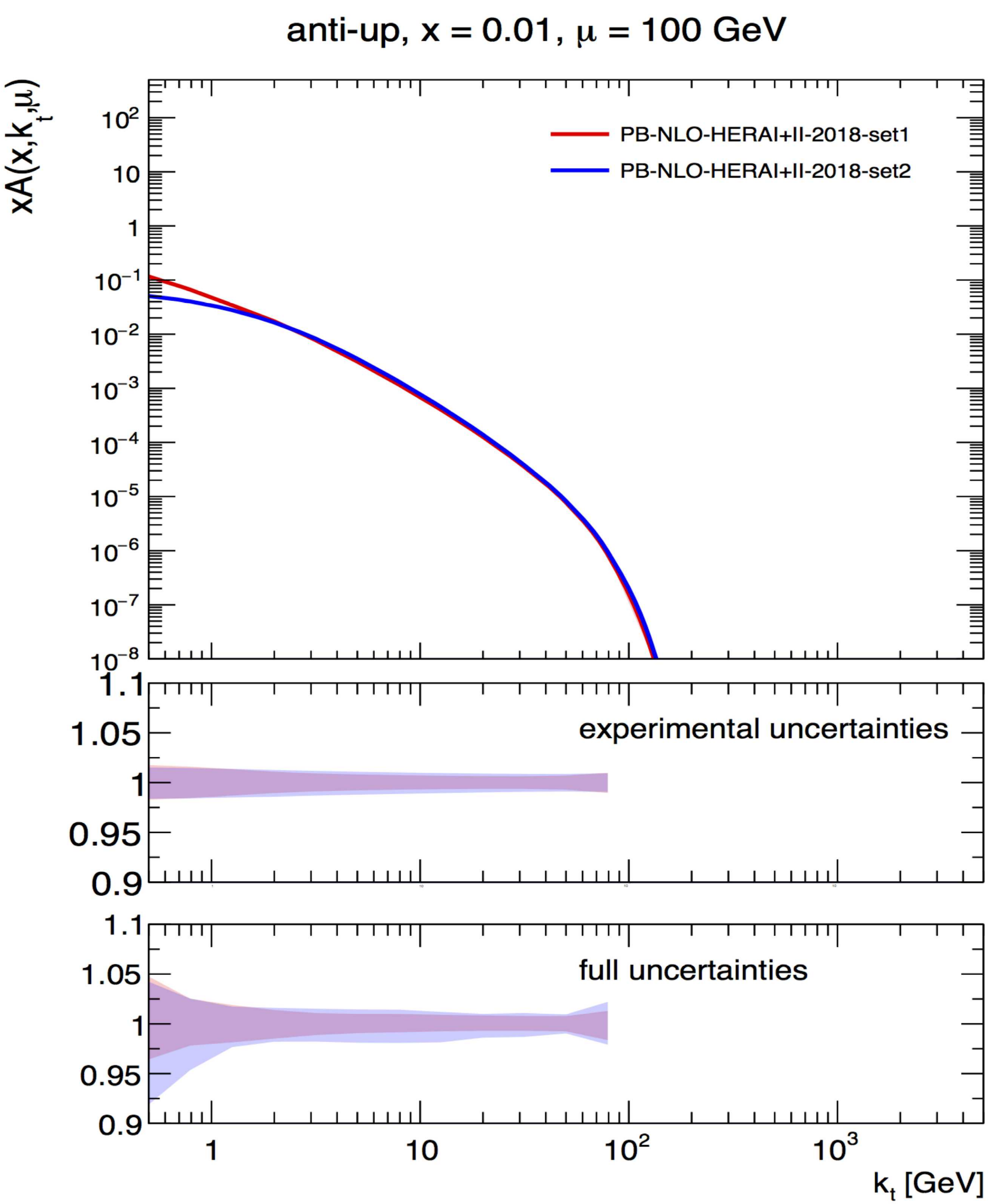
Transverse Momentum Dependence

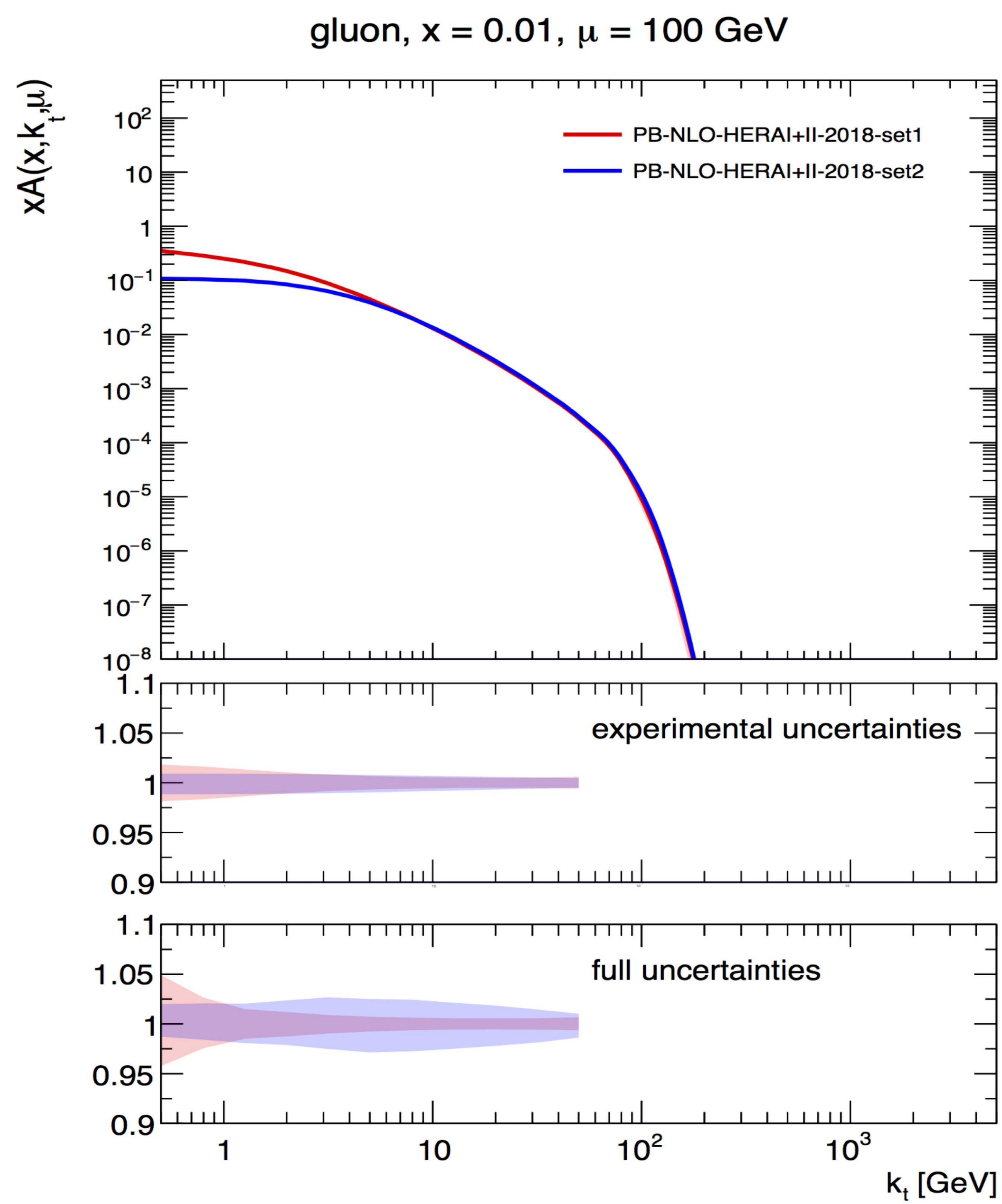
- Parton Branching evolution generates every single branching:
  - kinematics can be calculated at every step
- Give physics interpretation of evolution scale:
  - angular ordering:

    $\mu = q_T/(1-z)$



TMD distributions from fit to HERA data



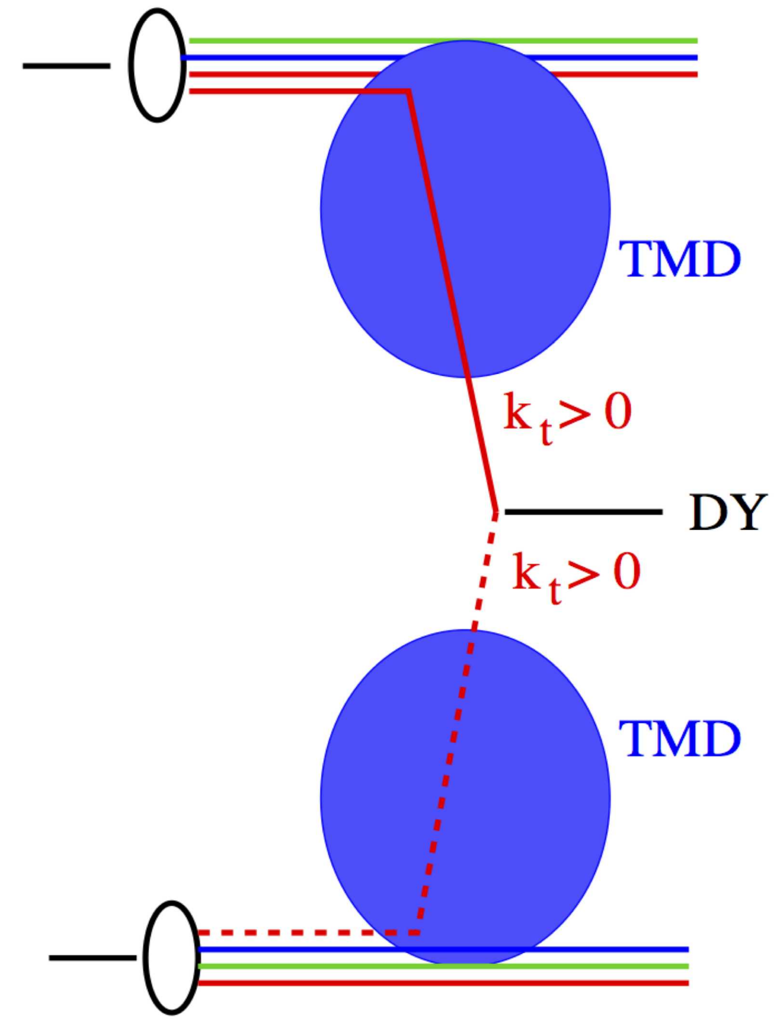


- model dependence larger than experimental uncertainties

Application of PB TMDs

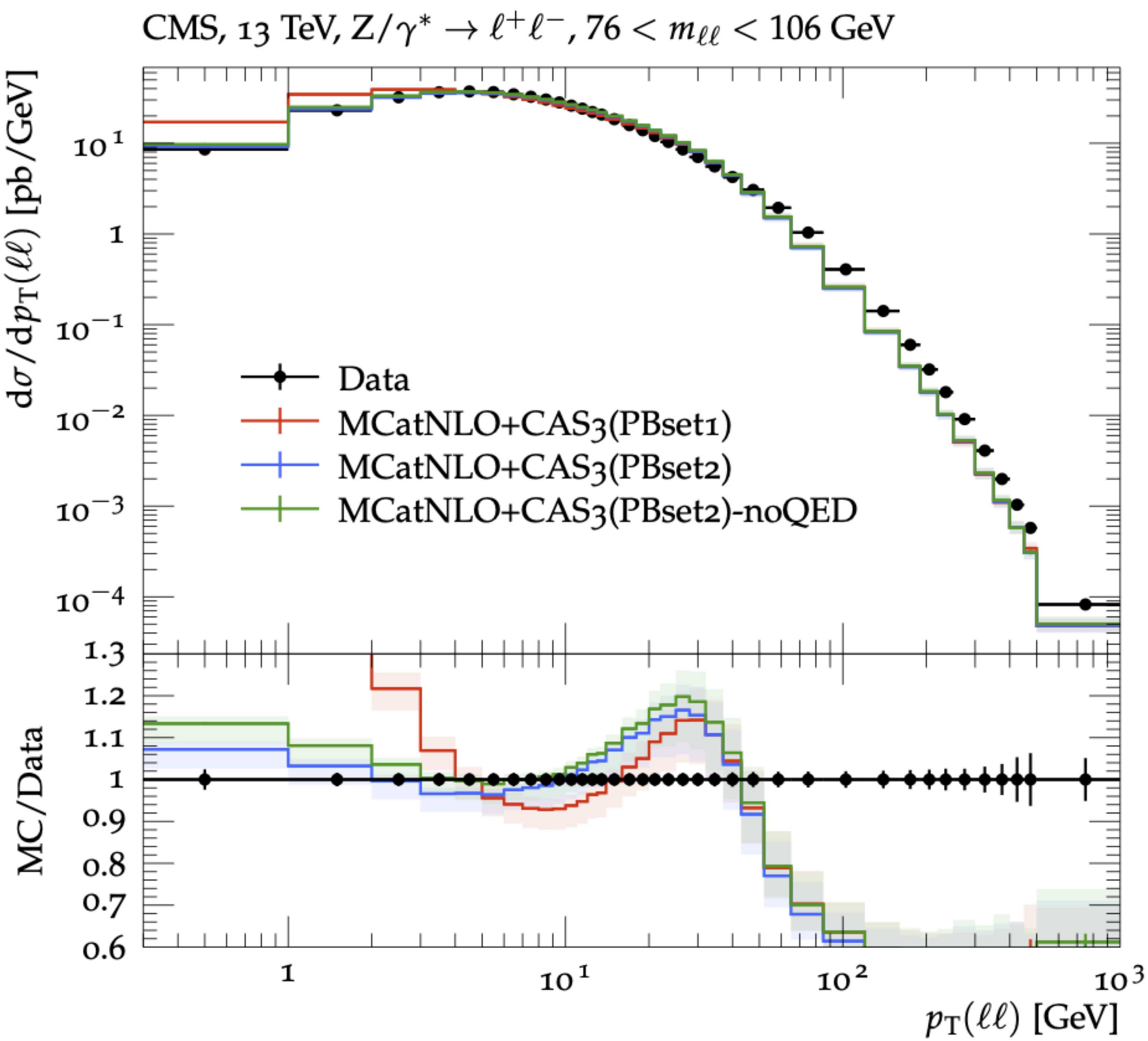
Drell-Yan production: q_T - spectrum

- DY production
  - $q\bar{q} \to Z_0$
  - use NLO calculations: MC@NLO
  - add k_t for each parton as function of x and μ according to TMD
    - keep final state mass fixed
    - preserve rapidity
      - but x_1 and x_2 (light-cone fraction) are different after adding k_t



Z - production at 13 TeV (CMS)

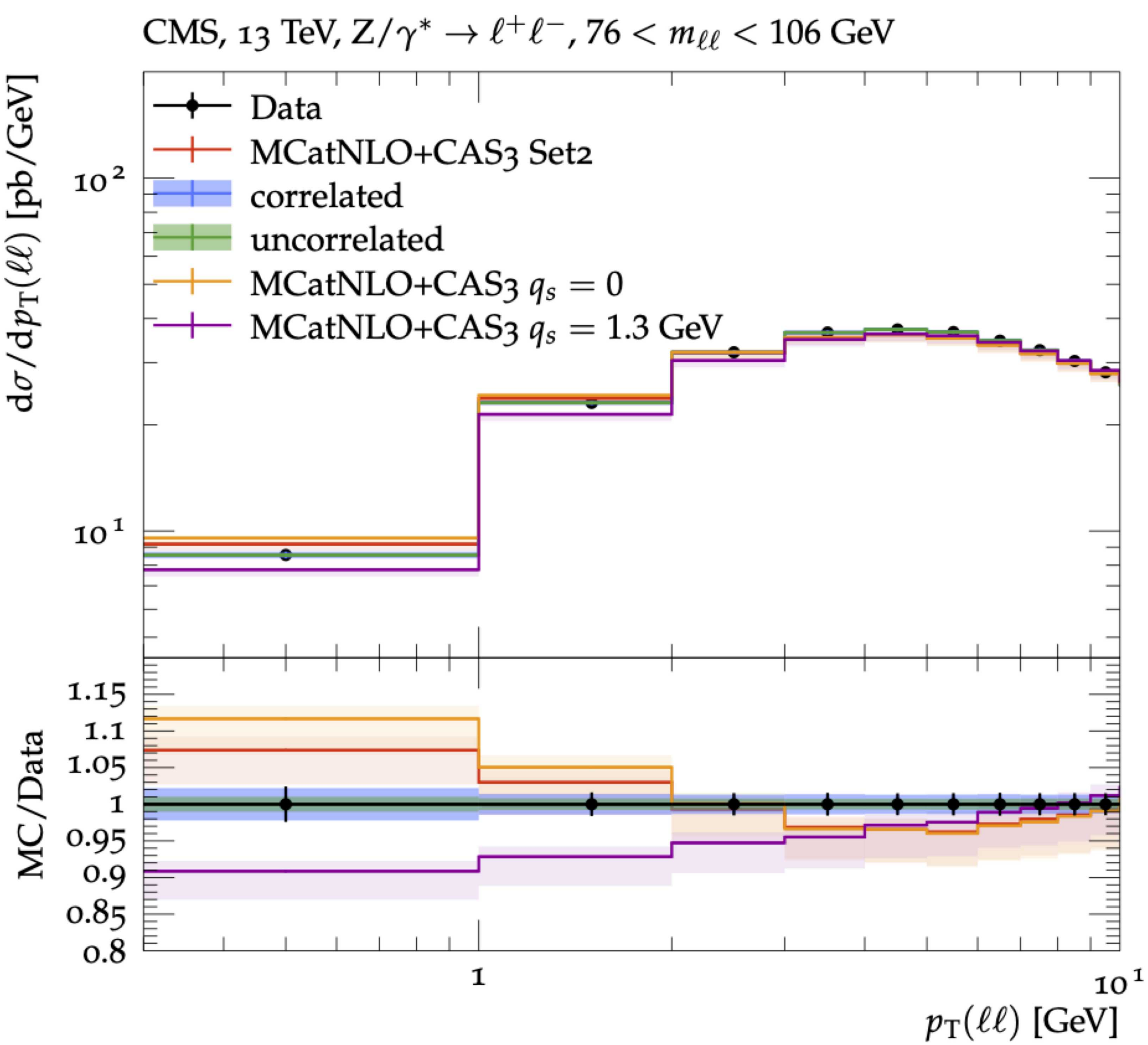
Bubanja, I. et al, arXiv: 2312.08655



- very good description of low p_T region with PB-set 2 (with $\alpha_s(q(1-z))$)
  - at larger p_T contribution from higher order matrix elements important
- Uncertainties in PB method mainly from scale of MC@NLO matrix element

Intrinsic k_T in DY - production at 13 TeV (CMS)

Bubanja, I. et al, arXiv: 2312.08655

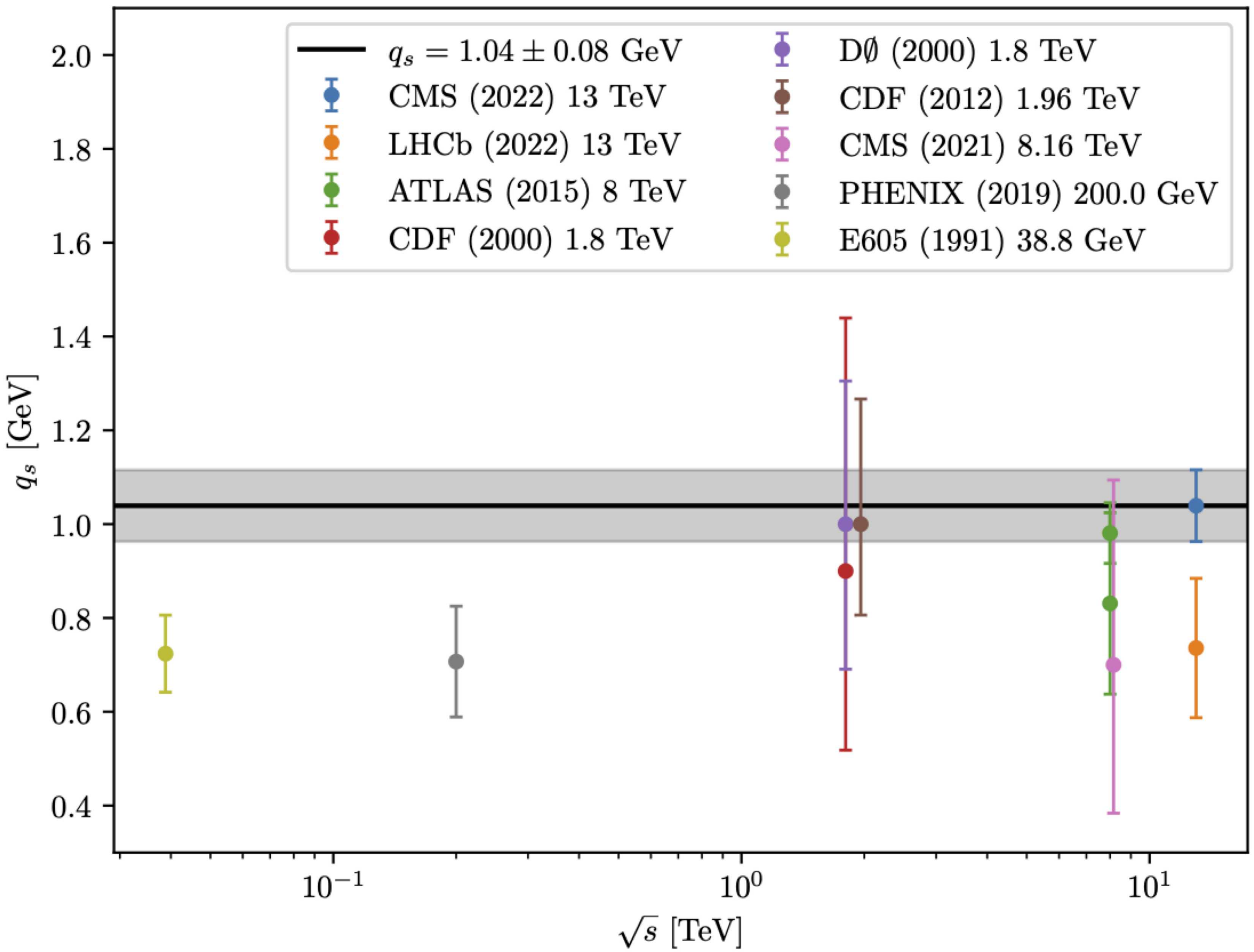


- in TMD, intrinsic k_T distribution:
  - Gauss with zero mean, width q_s

$$\sim \exp\left(-|k_T^2|/q_s^2\right)$$

- Focus on small k_T region:
  - **in lowest** p_T bin, sensitivity to intrinsic k_T
- Use DY production at different m_{DY} and $\sqrt{s}$ to determine q_s
- Is intrinsic k_T dependent on m_{DY} and $\sqrt{s}$?

Fit of Intrinsic k_T in DY – production vers $\sqrt{s}$



Bubanja, I. et al, arXiv: 2312.08655

- Gauss with zero mean, width q_s

$\sim \exp\left(-|k_T^2|/q_s^2\right)$

Fit to determine q_s of intrinsic k_T distribution from DY production as a function of $\sqrt{s}$

- obtain q_s rather independent on $\sqrt{s}$

The role of soft gluons

Reminder: DIS Jet Cross Sections in O(α_s)

- From differential x-section

$$\frac{d\sigma(\gamma g \to q\bar{q})}{dx\,dz\,d\hat{t}} = K \sum_{\text{quarks } a} e_a^2 \frac{\alpha_s(Q^2)}{4\pi^2} \frac{2z}{Q^2} z f_g(\frac{x}{z}, Q^2) \times$$
$$\times \left\{ \frac{1}{4} \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - 2\frac{Q^2}{\hat{u}\hat{t}} + 4\frac{\hat{s}Q^2}{(\hat{s}+Q^2)^2} \right) \right\}$$

➔ obtain:

J. Collins JHEP 0005:004,2000

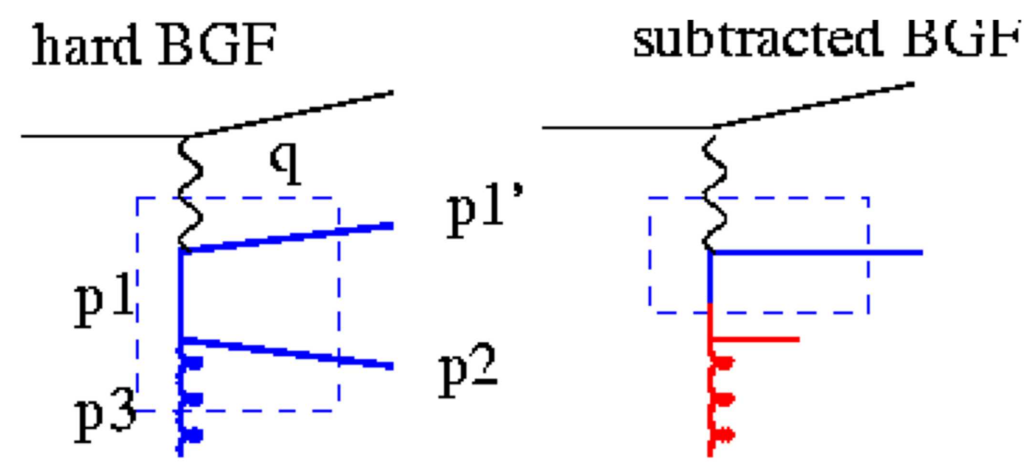
$$\frac{d\sigma(\gamma g \to q\bar{q})}{dx\,dz\,d\cos\theta} = K \sum_{\text{quarks } a} e_a^2 \frac{\alpha_s(Q^2)}{4\pi^2} z f_g(\frac{x}{z}, Q^2) \times$$
$$\times \left\{ P(z) \left[\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} \right] - \frac{1}{2} + 3z(1-z) \right\},$$

Reminder: NLO and subtraction

- and obtain singular piece separately:

$$u = \frac{-Q^2(1+cos\theta)z}{2x}$$

$$z = \frac{p_3 q}{pq}$$



➔ problem: kinematics ... avoid to subtract too much

$$\frac{d\sigma^{\rm subtract}(\gamma g\to q\bar{q})}{dx\,dy\,dz\,d\cos\theta} = K\sum_{\text{quarks } a} e_a^2\frac{\alpha_s(Q^2)}{4\pi^2} z f_g(\frac{x}{z},Q^2)P(z)\left[\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}\right]$$

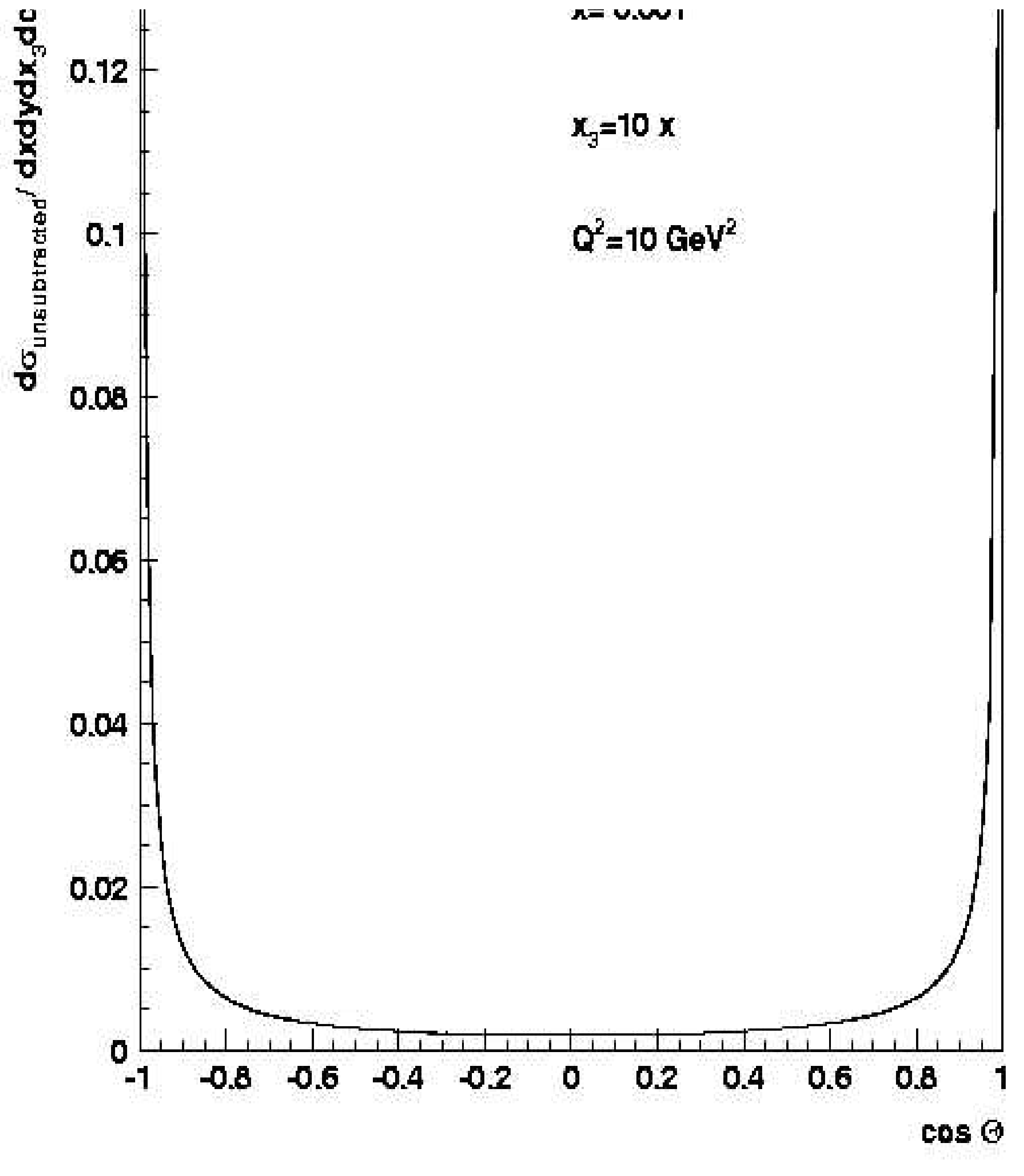
with
$$\frac{d\sigma_{\rm hard}}{dx\,dy\,dz\,d\cos\theta} = K\sum_{\text{quarks } a} e_a^2\frac{\alpha_s(Q^2)}{4\pi^2} z f_g(\frac{x}{z},Q^2)\times$$

$$\times\left\{\frac{P(z)(1-C(-t))}{1-\cos\theta}+\frac{P(z)(1-C(-u))}{1+\cos\theta}-\frac{1}{2}+3z(1-z)\right\}$$

$$C(a) = \Theta(Q^2-a)$$

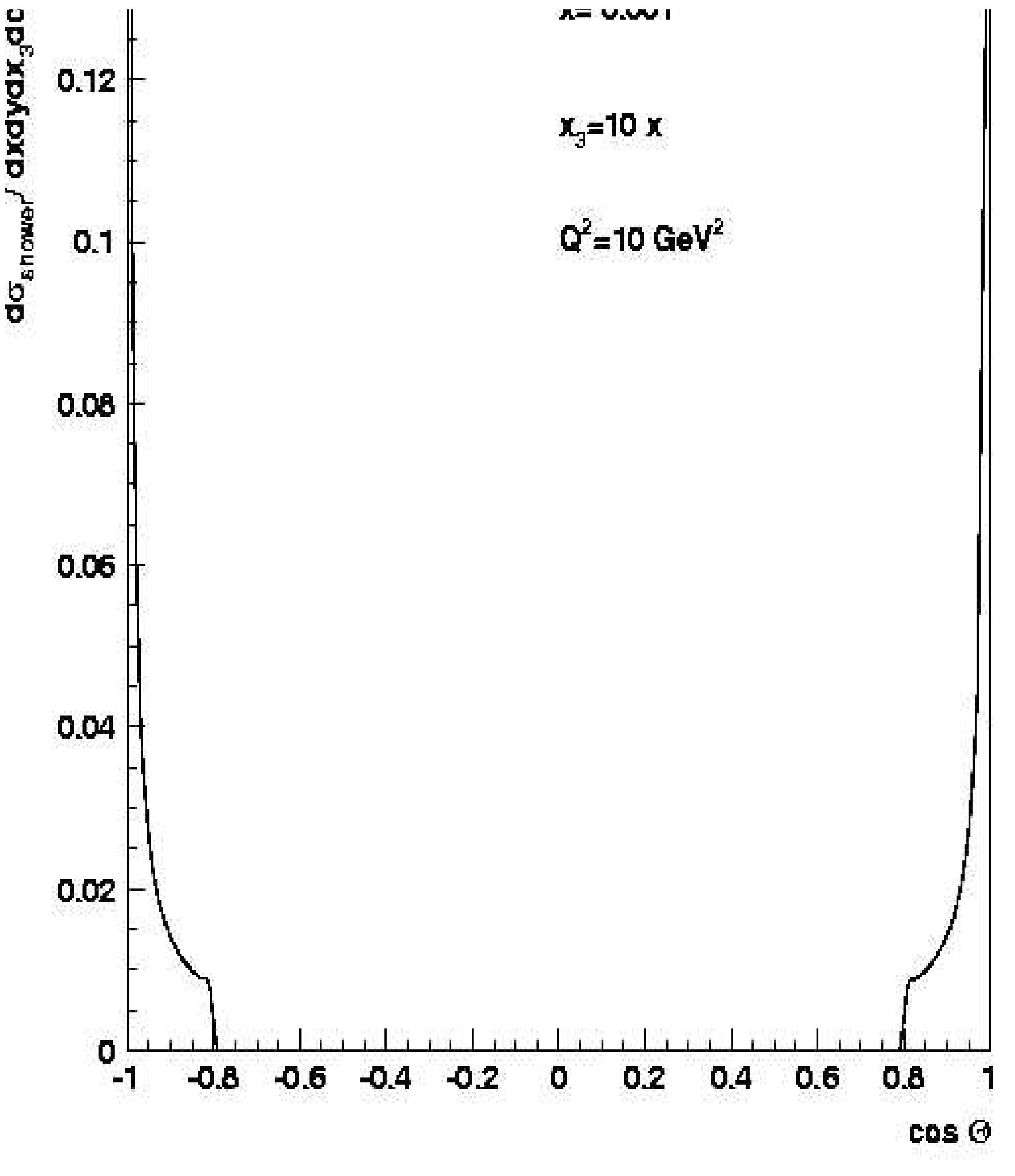
Reminder: NLO and subtraction

full BGF contribution



collinear contribution

$$C(a) = \Theta(Q^2 - a)$$



Reminder: NLO x-section

M. Mendizabal arXiv: 2309.11802

- DIS x-section at NLO:

$$F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} q(\xi,Q^2)\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} C_{\overline{\mathrm{MS}}}\left(\frac{x}{\xi}\right) + \cdots\right]$$

- with

$$C_q^{\overline{\mathrm{MS}}}(z) \quad = \quad C_F\left[2\left(\frac{\log(1-z)}{1-z}\right)_+ - \frac{3}{2}\left(\frac{1}{1-z}\right)_+ - (1+z)\log(1-z) \right.$$
$$\left. -\frac{1+z^2}{1-z}\log z + 3 + 2z - \left(\frac{\pi^2}{3}+\frac{9}{2}\right)\delta(1-z)\right]$$

- integral $d\ \xi$ has to extend up to 1 for consistency !

Reminder of NLO calculations

- Calculations at NLO (and higher order) require (integrated) parton densities with $z_M \to 1$ (if used in MSbar factorization scheme)
  - if $z_M \ll 1$, inconsistencies appear.

- Issue of $z_M \to 1$ was already discussed in Z. Nagy, D.Soper Phys. Rev. D102 (2020) 1

- Issue discussed in detail in M. Mendizabal, F. Guzman, H. Jung, S. Taheri Monfared arXiv 2309.11802

- S. Frixione, B Webber (arXiv 2309.15587) propose calculating NLO ME's with $z_M \ll 1$, huge enterprise, as all ME's need recalculation

Reminder of NLO calculations

Integration $z_M \to 1$ is essential for consistency

- Split Sudakov form factor into perturbative and non-perturbative part:

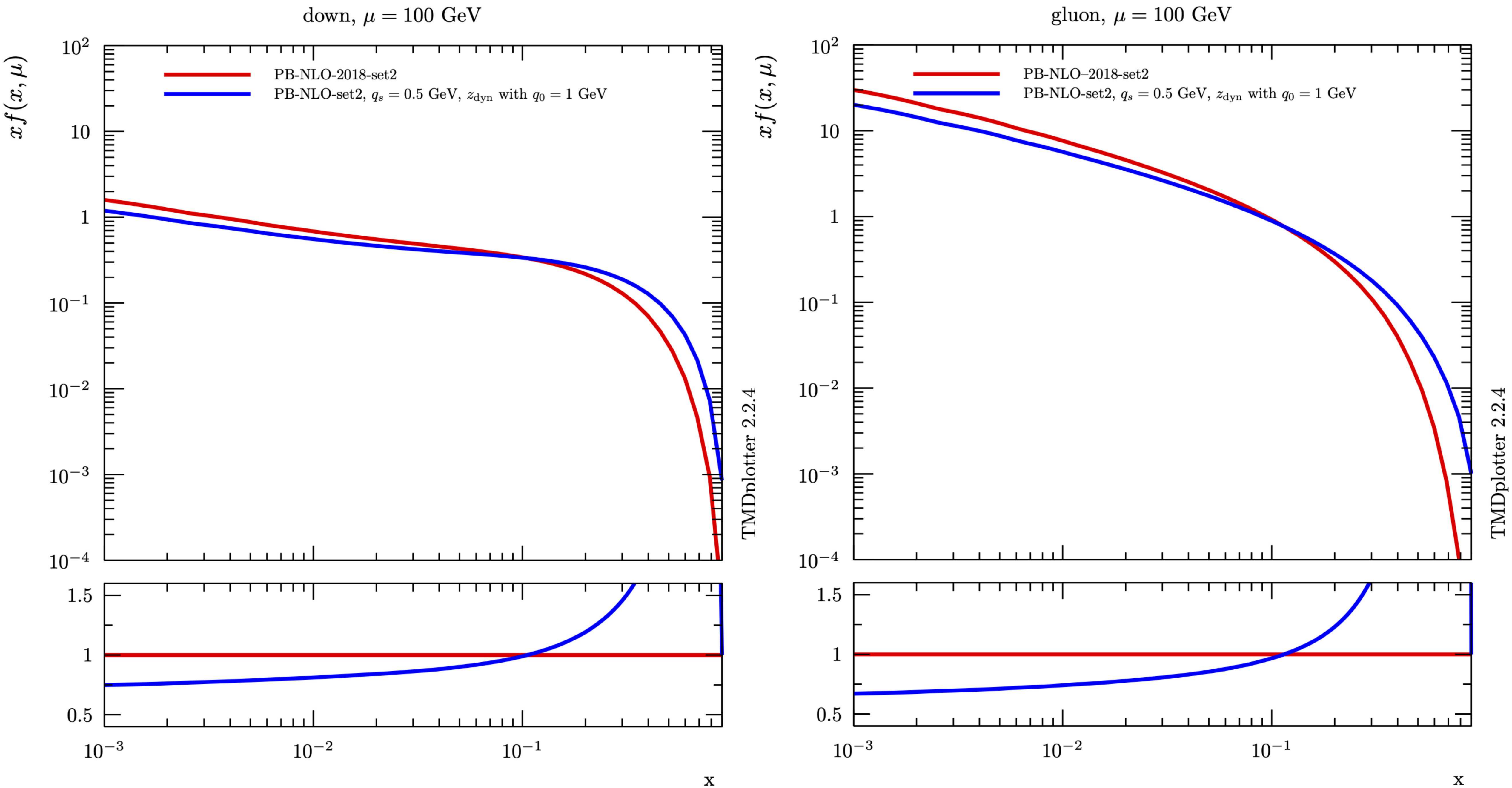
$$
\begin{aligned}
\Delta_s(\mu^2) &= \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mathbf{q}'^2}{\mathbf{q}'^2} \int_0^{z_\mathrm{M}} dz\ z\ P_{ba}^{(R)}(\alpha_s, z)\right) \\
&= \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mathbf{q}'^2}{\mathbf{q}'^2} \int_0^{z_\mathrm{dyn}} dz\ z\ P_{ba}^{(R)}(\alpha_s, z)\right) \\
&\quad \times \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mathbf{q}'^2}{\mathbf{q}'^2} \int_{z_\mathrm{dyn}}^{z_\mathrm{M}} dz\ z\ P_{ba}^{(R)}(\alpha_s, z)\right) \\
&= \Delta_s^{(\mathrm{P})}\left(\mu^2, \mu_0^2, q_0^2\right) \cdot \Delta_s^{(\mathrm{NP})}\left(\mu^2, \mu_0^2, q_0^2\right)
\end{aligned}
$$

- with $z_\mathrm{dyn} = 1 - q_0/q'$

Importance of non-perturbative gluons

Bubanja, I. et al, arXiv: 2312.08655

- Apply $z_{\rm dyn} = 1 - q_0/q'$ with $q_0 = 1$ GeV on inclusive distributions



Parton Shower MC event generators

- Parton shower follows backward evolution:

$$\Pi = \exp\left[- \int_{\mu_l^2}^{\mu_h^2} \frac{d\mu'^2}{\mu'^2} \int^{z_{\rm dyn}} \frac{dz}{z} \hat{P}(z) \frac{f(x/z,\mu^2)}{f(x,\mu^2)} \right]$$

- Emitted partons should have *resolvable* energy (or p_T) with: $p_T > q_0 \sim 1 \mathrm{G}eV$

$$z_{\rm dyn} = 1 - \frac{q_0}{\mu'}$$

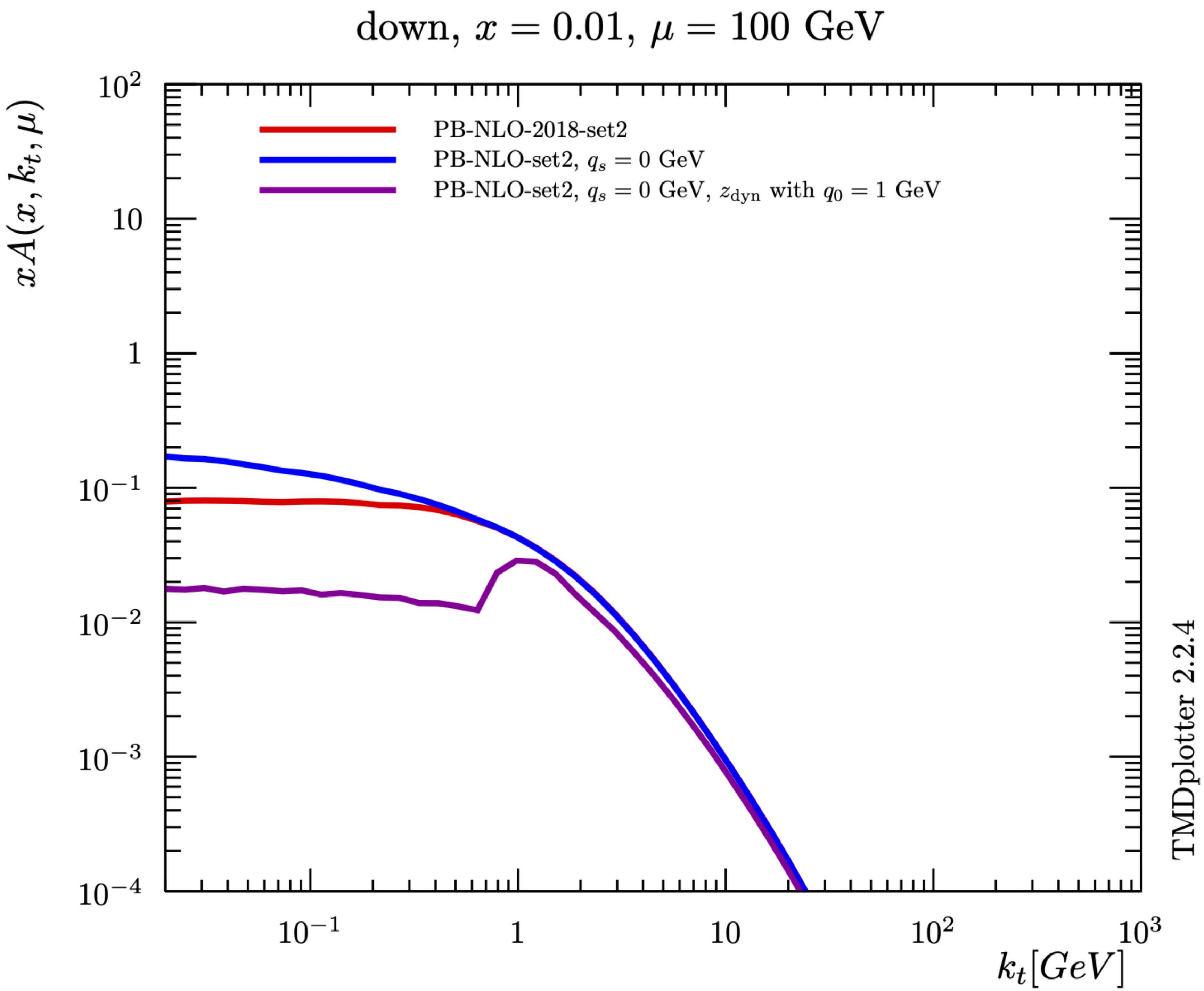
- With $z_{\rm dyn} \ll 1$ soft gluons with $p_T < 1 \mathrm{G}eV$ are neglected.

- What is the role of these soft gluons ?

Role of soft gluons in TMD distributions

Bubanja, I. et al, arXiv: 2312.08655

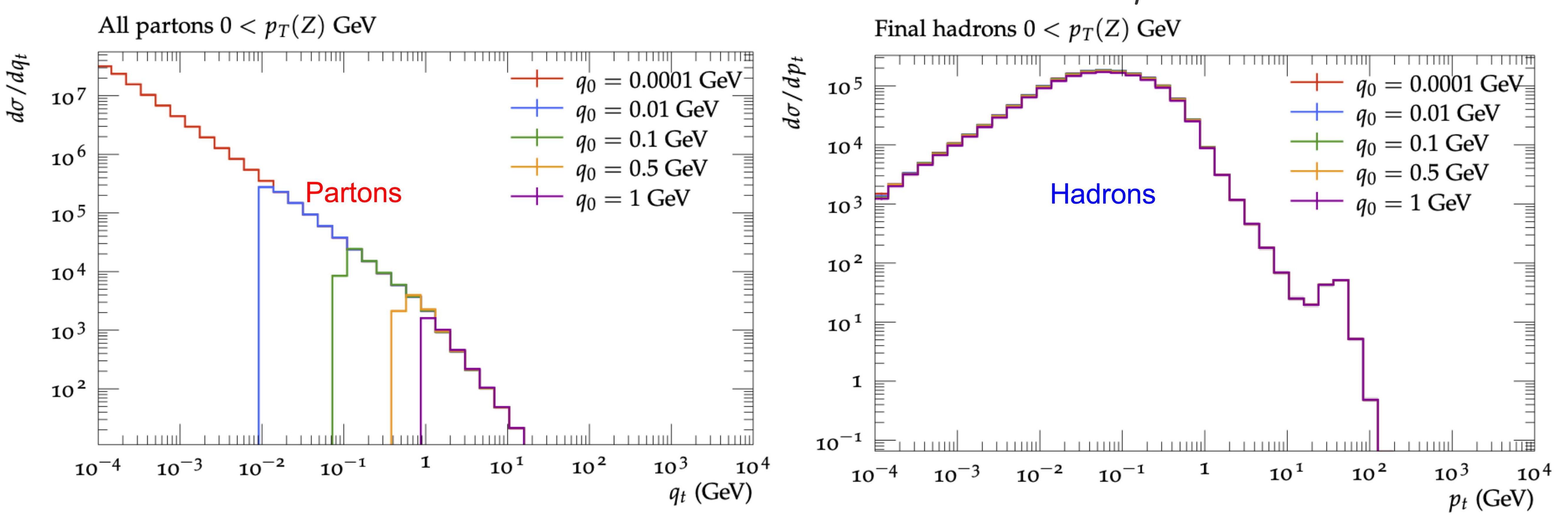
- Perform evolution with PB method with and without cut $z_{\rm dyn} = 1 - \frac{q_0}{\mu'}$



Soft gluons in Parton Shower

M. Mendizabal arXiv: 2309.11802

- With PB-TMD parton shower study effect of $z_{\rm dyn} = 1 - \frac{q_0}{\mu'}$



- in Lund string fragmentation these soft partons do not change hadron spectra
  - **no issue for hadrons or jets from parton shower !**

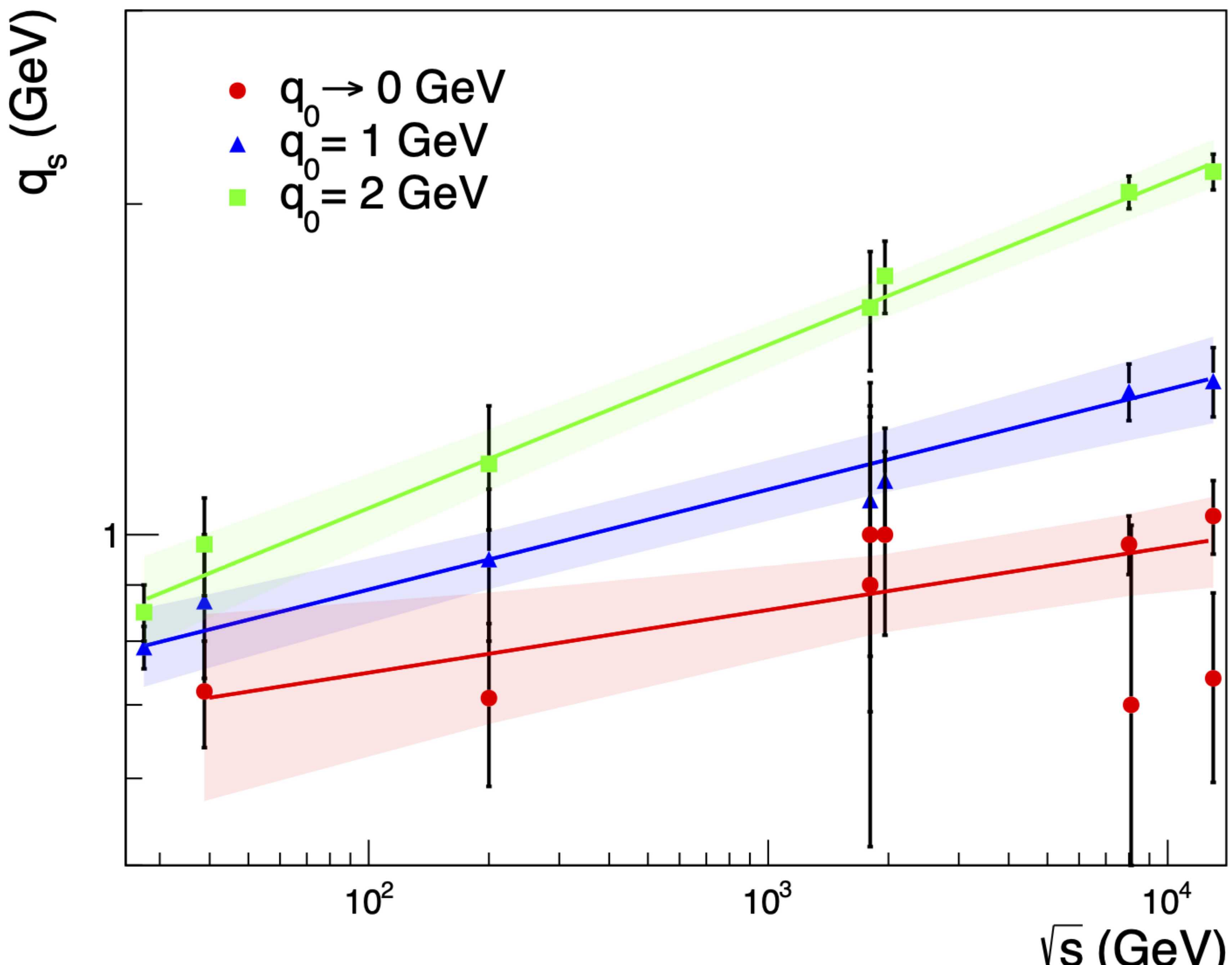
- no effect on hadron spectra !

Effect of soft gluons in inclusive DY pt spectra

Effect of removing soft gluons from TMD

- Perform evolution with PB method with cut $z_{\rm dyn} = 1 - \frac{q_0}{\mu'}$
  - to mimic parton shower approach in MC event generators
- Determine width of intrinsic kt distribution

Bubanja, I. et al, arXiv: 2404.04088



CSS formalism

Collins, Rogers arXiv 1705.07167

- Collins Soper Sterman (CSS) formalism for pt spectrum of DY production

$$
\begin{aligned}
\frac{d\sigma}{dQ^2\,dy\,dq_T^2} = {} & \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} e_j^2 \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T} \\
& \times \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\text{CSS1, DY}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
& \times \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\text{CSS1, DY}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
& \times \exp\left\{ -\int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left[A_{\text{CSS1}}(a_s(\mu'); C_1) \ln\left(\frac{\mu_Q^2}{\mu'^2}\right) + B_{\text{CSS1, DY}}(a_s(\mu'); C_1, C_2) \right] \right\} \\
& \times \exp\left[-g^{\text{CSS1}}_{j/A}(x_A, b_T; b_{\max}) - g^{\text{CSS1}}_{\bar{j}/B}(x_B, b_T; b_{\max}) - g^{\text{CSS1}}_K(b_T; b_{\max}) \ln(Q^2/Q_0^2) \right] \\
& + \text{suppressed corrections.}
\end{aligned}
$$

intrinsic k_T distribution

non-perturbative Sudakov form factor

intrinsic k_T and non-pert Sudakov must be determined by measurements

Correspondence of PB – TMDs with CSS

- Check correspondence of PB Sudakov form factor with CSS
- use only $P_{qq}(z)$ in large z limit: $P_{qq}(z) \sim \frac{1+z^2}{1-z} + \frac{3}{2}\delta(1-z) \to \frac{2}{1-z} + \frac{3}{2}\delta(1-z)$
- apply angular ordering constraint for z_M $\quad z_{\rm dyn} = 1 - q_0/q$

$$\begin{aligned}\Delta_s^{(\rm P)}(\mu) &= \exp\left(-\frac{\alpha_s}{2\pi}\left[\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\int_0^{z_{\rm dyn}} dz P(z) + \frac{3}{2}\right]\right) \\ &= \exp\left(-\frac{\alpha_s}{2\pi}\left[\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2} 2\log\frac{\mu^2}{\mu'^2} + \frac{3}{2}\right]\right)\end{aligned}$$

- perturbativeSudakov from PB is Sudakov from CSS
- PB give also prediction on non-pert Sudakov:

$$\Delta_s^{(\rm NP)}(\mu^2) = \times \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2}\frac{d\mathbf{q}'^2}{\mathbf{q}'^2}\int_{z_{\rm dyn}}^{z_{\rm M}} dz\ z\ P_{ba}^{(R)}(\alpha_s, z)\right)$$

Conclusion

- Parton Branching method to solve DGLAP equation at LO, NLO and NNLO
  - method directly applicable to determine k_T distribution

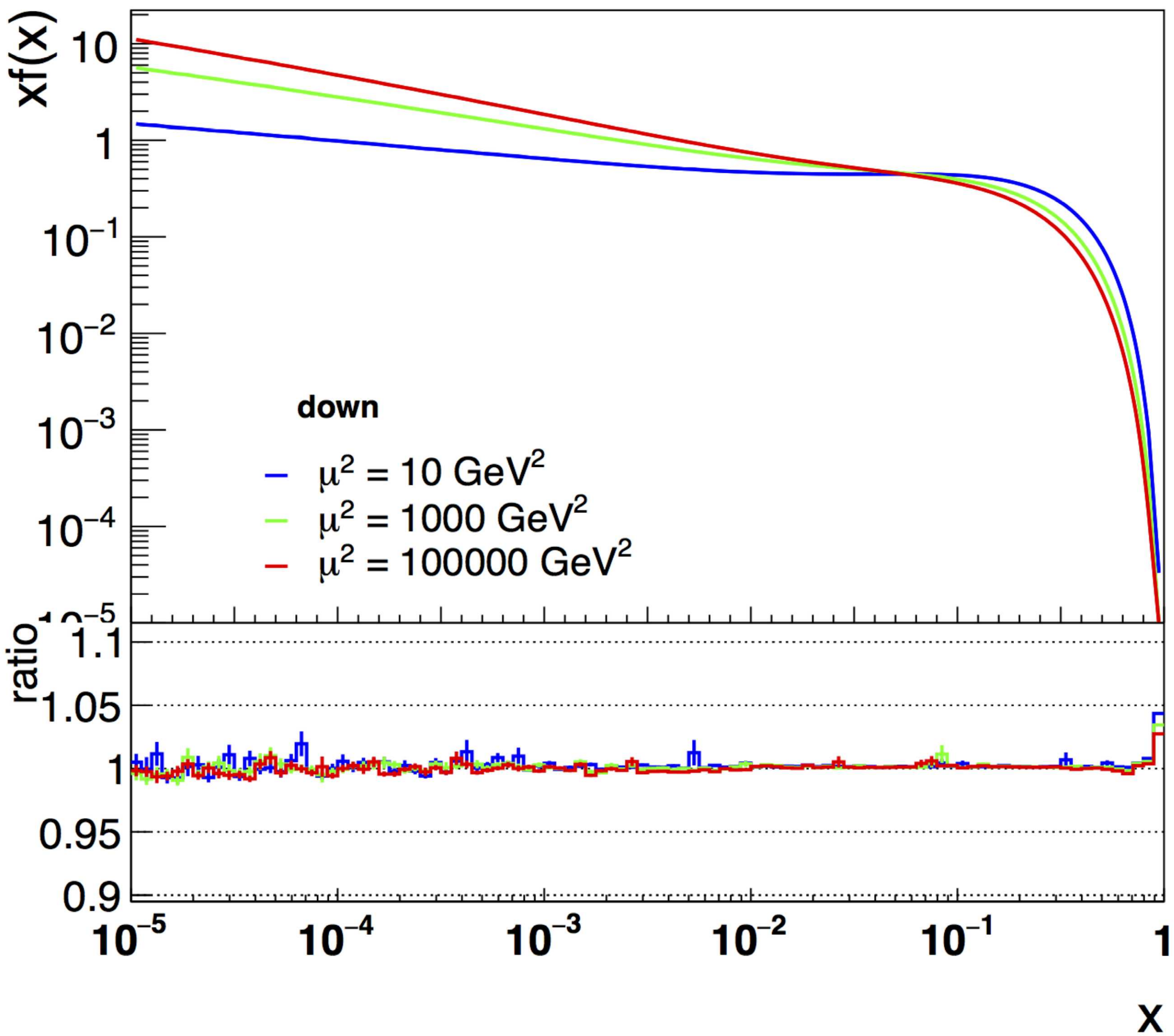
- Application to inclusive DY processes in pp at different energies and masses:
  - intrinsic k_T distribution determined over large range of m_{DY} and $\sqrt{s}$
    - no or very mild $\sqrt{s}$ dependence observed, in contrast to MCEG

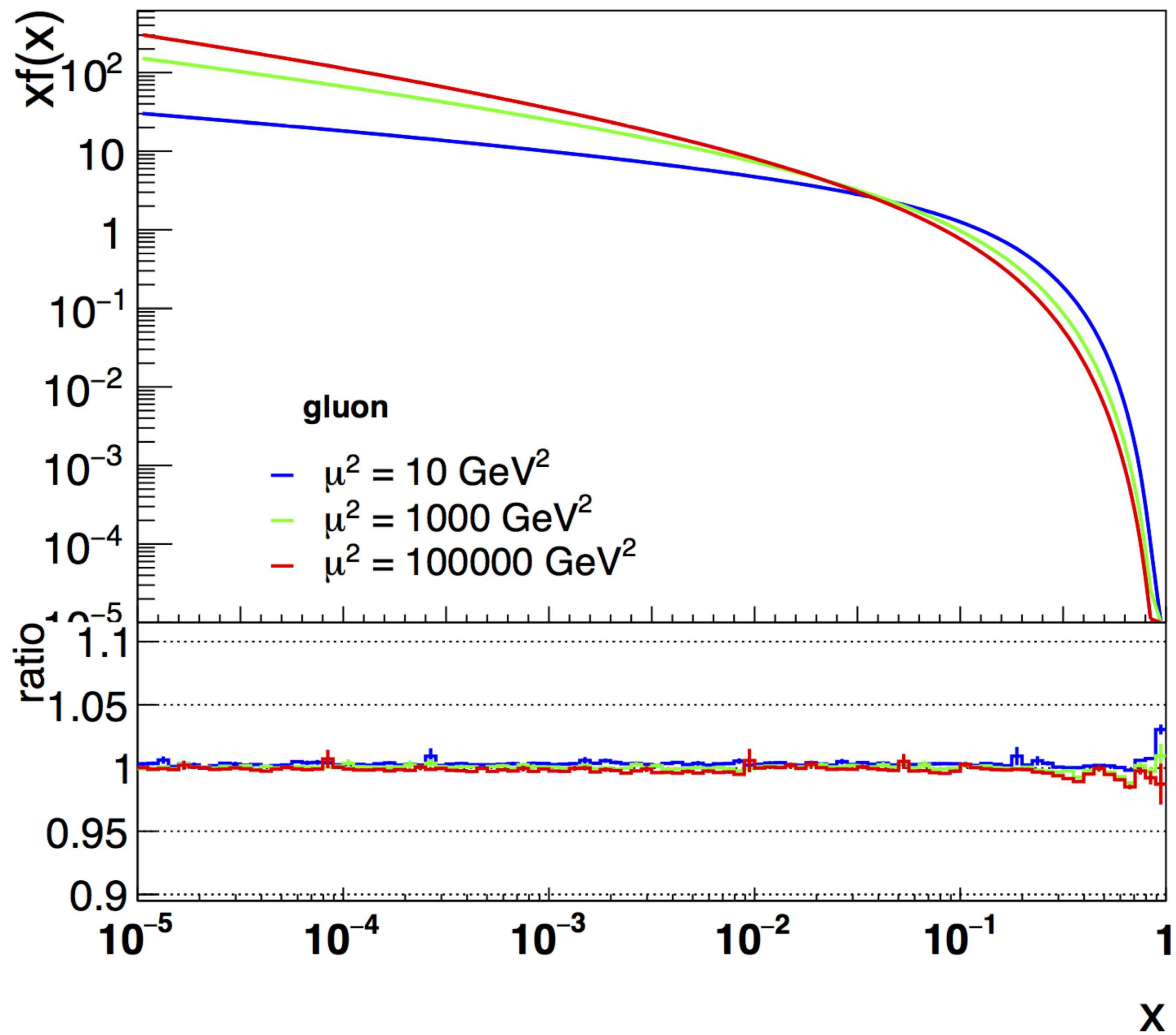
- Importance of soft gluons established:
  - essential for consistency of NLO matrix elements and pdfs !
    - otherwise new factorization scheme to be defined and all NLO ME's must be recalculated.
  - essential for inclusive parton densities (DGLAP requires $z_M \to 1$)
  - essential for inclusive TMD distributions, e.g. DY q_T spectra

- Soft gluons are not important for final state jets or hadrons from parton shower

Appendix

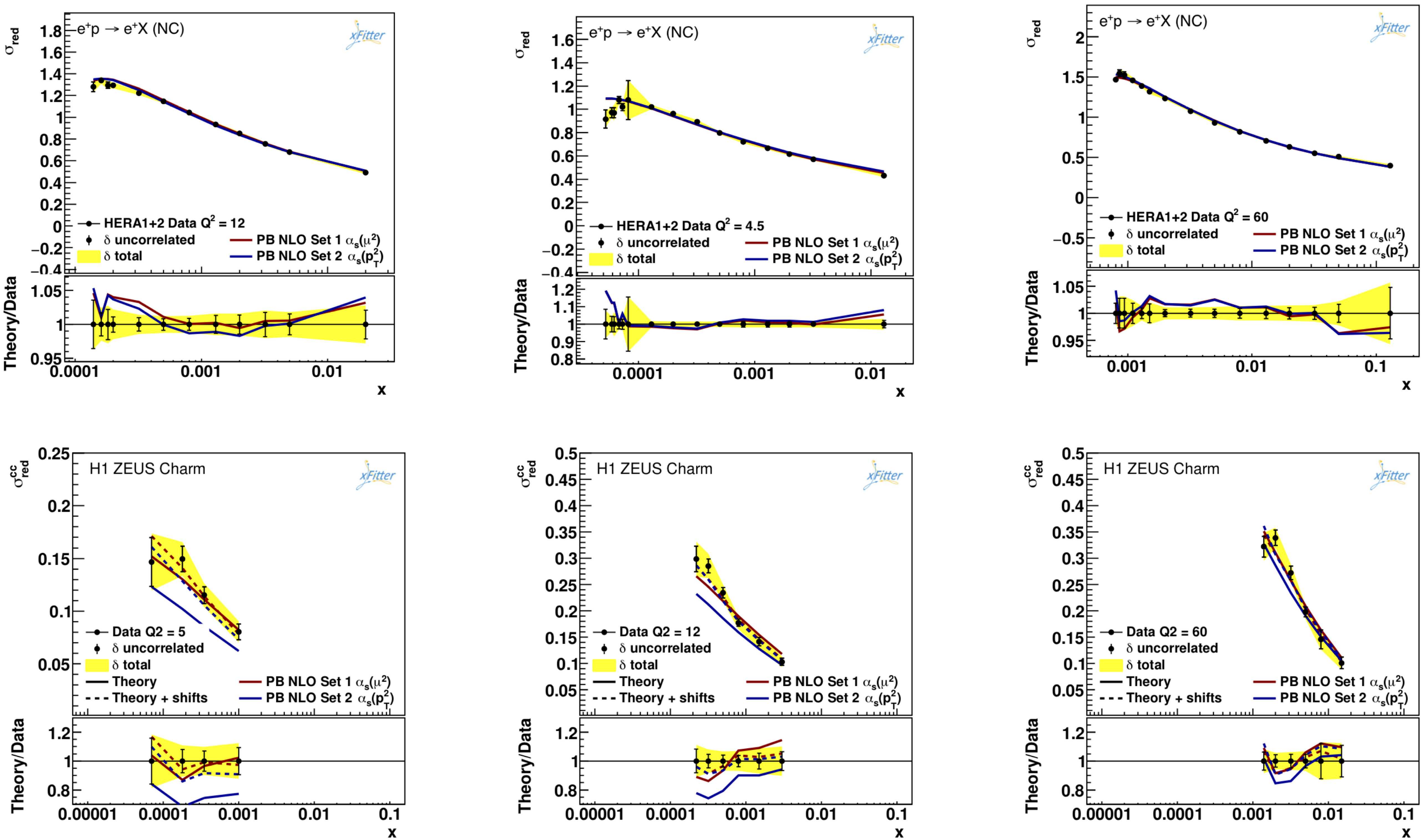
Validation of method with QCDnum at NLO





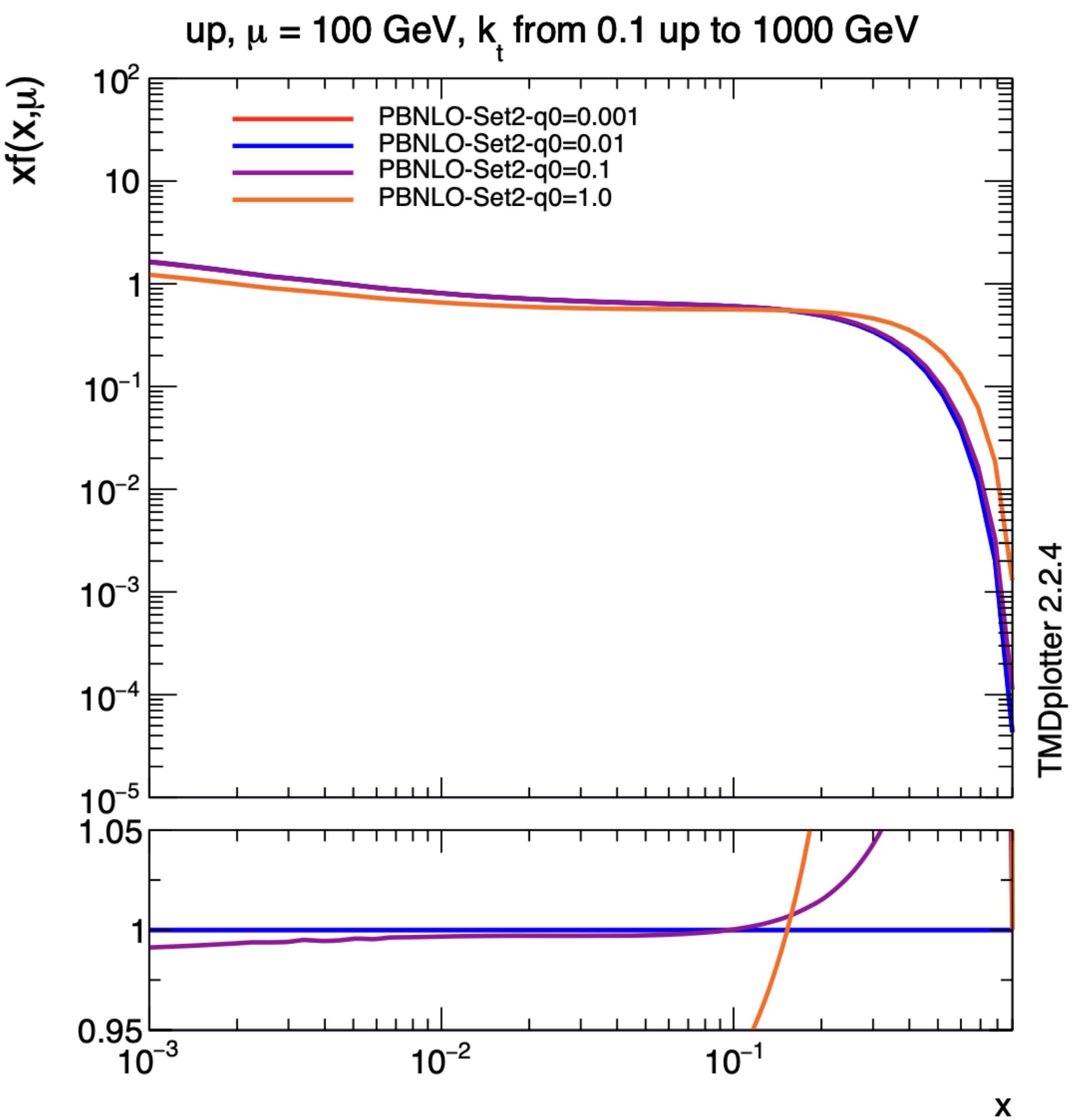
- Very good agreement with NLO - QCDnum over all x and μ^2
  - the same approach works also at NNLO !

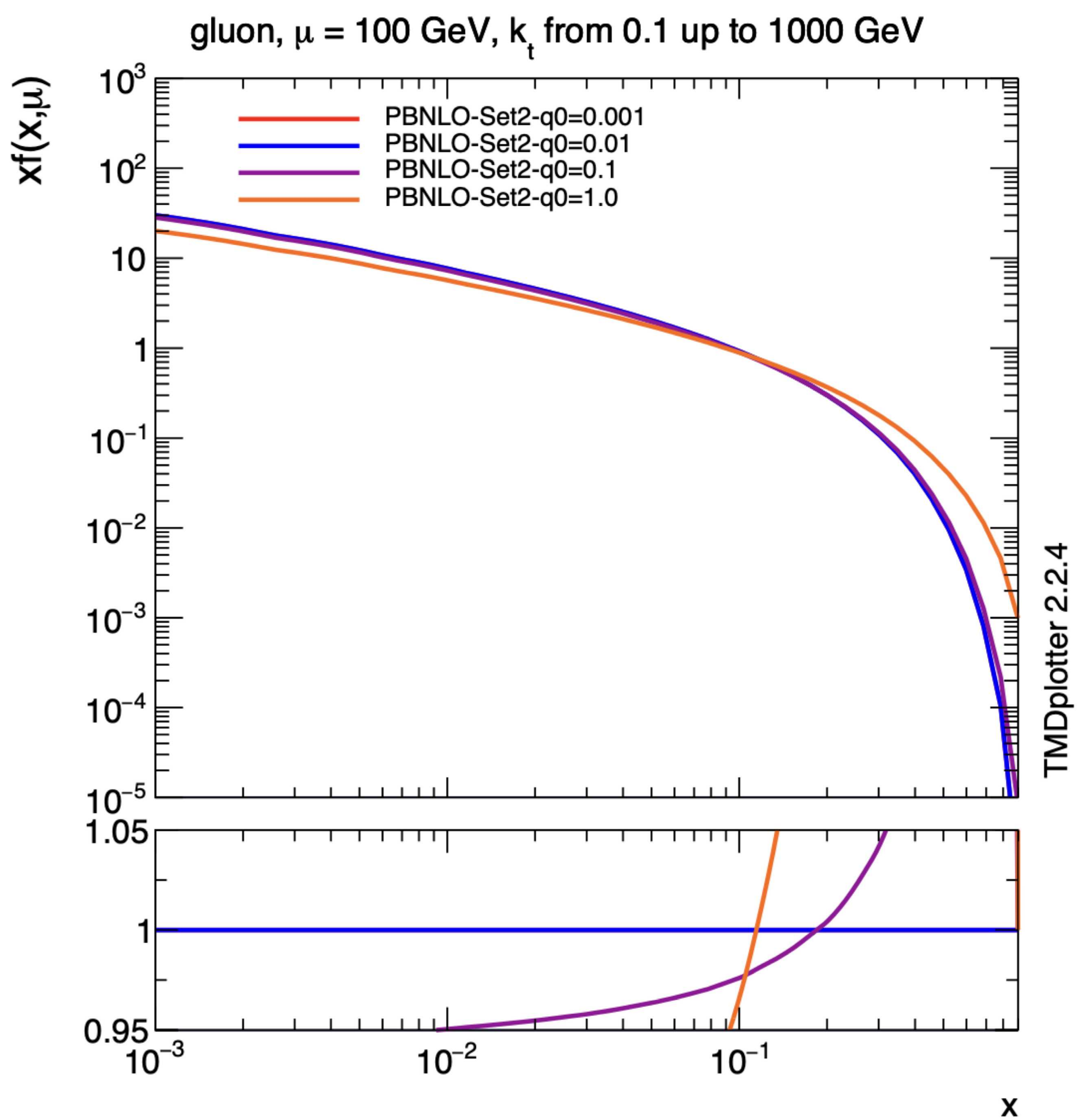
Fits to DIS x-section at NLO: F_2 and F_2^c



Role of soft gluons in inclusive distributions

- Perform evolution with PB method with and without cut $z_{\rm dyn} = 1 - \frac{q_{t\ {\rm cut}}}{\mu'}$

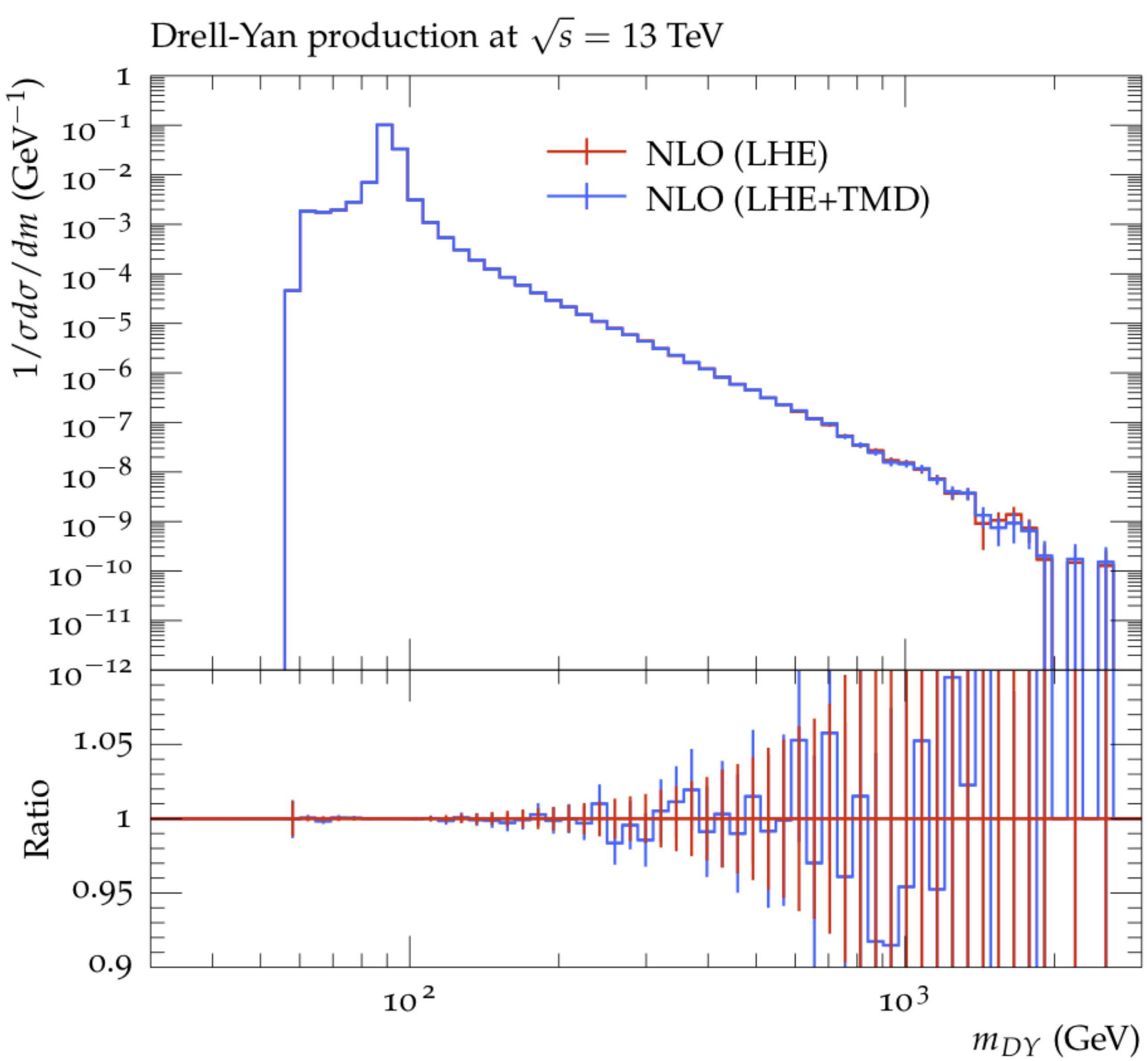


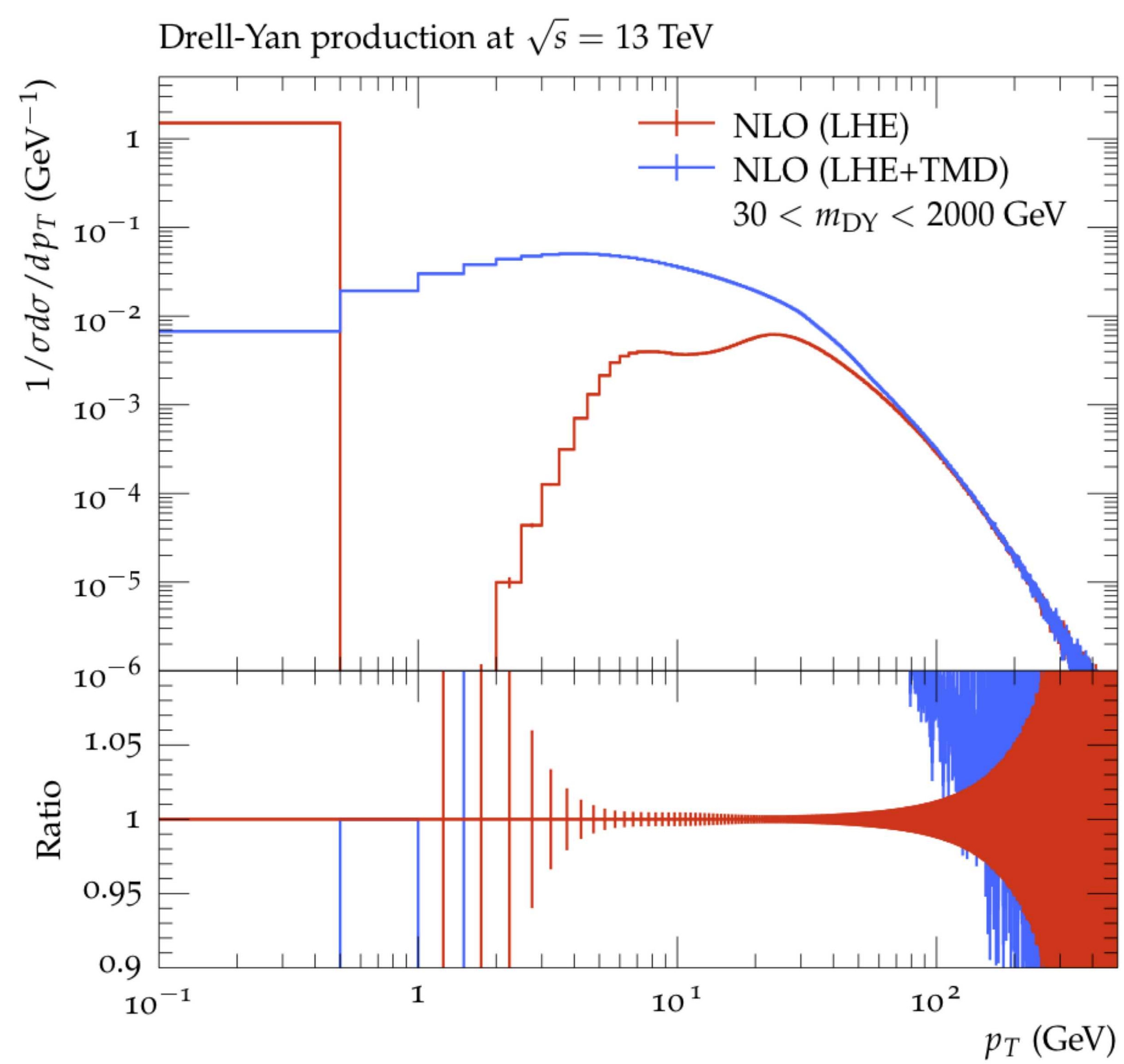


Including TMDs for DY production with MC@NLO

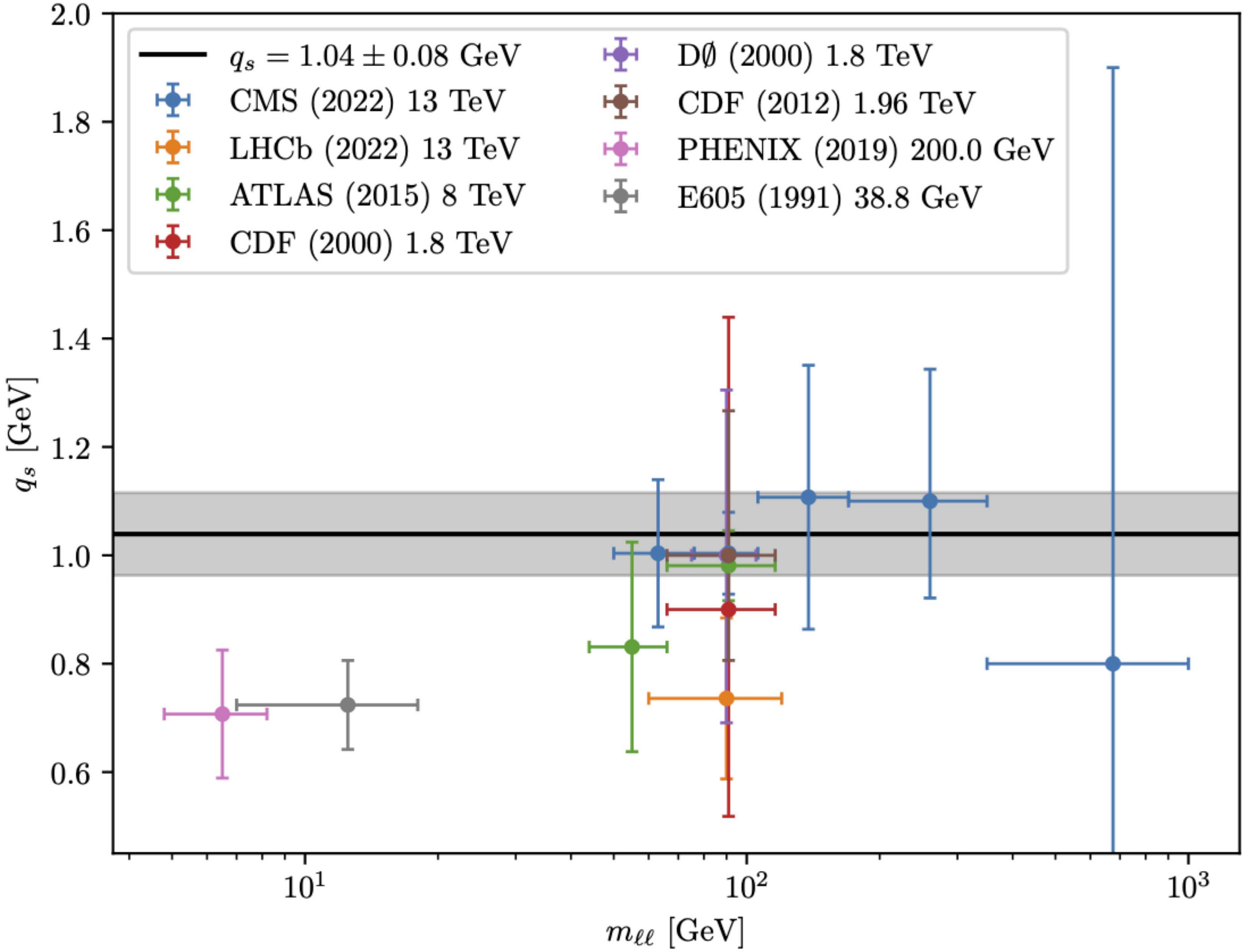
DY production: Bermudez Martinez, A. et al, arXiv 1906.00919, 2001.06488
CASCADE3 S. Baranov et al, arXiv 2101.10221

- MC@NLO subtracts soft & collinear parts from NLO (added back by TMD and/or parton shower)
  - MC@NLO without shower and/or TMD unphysical (here herwig6 subtraction)





Fit of Intrinsic k_T in DY – production vers m_{DY}



Bubanja, I. et al, arXiv: 2312.08655

- Gauss with zero mean, width q_s

$\sim \exp\left(-|k_T^2|/q_s^2\right)$

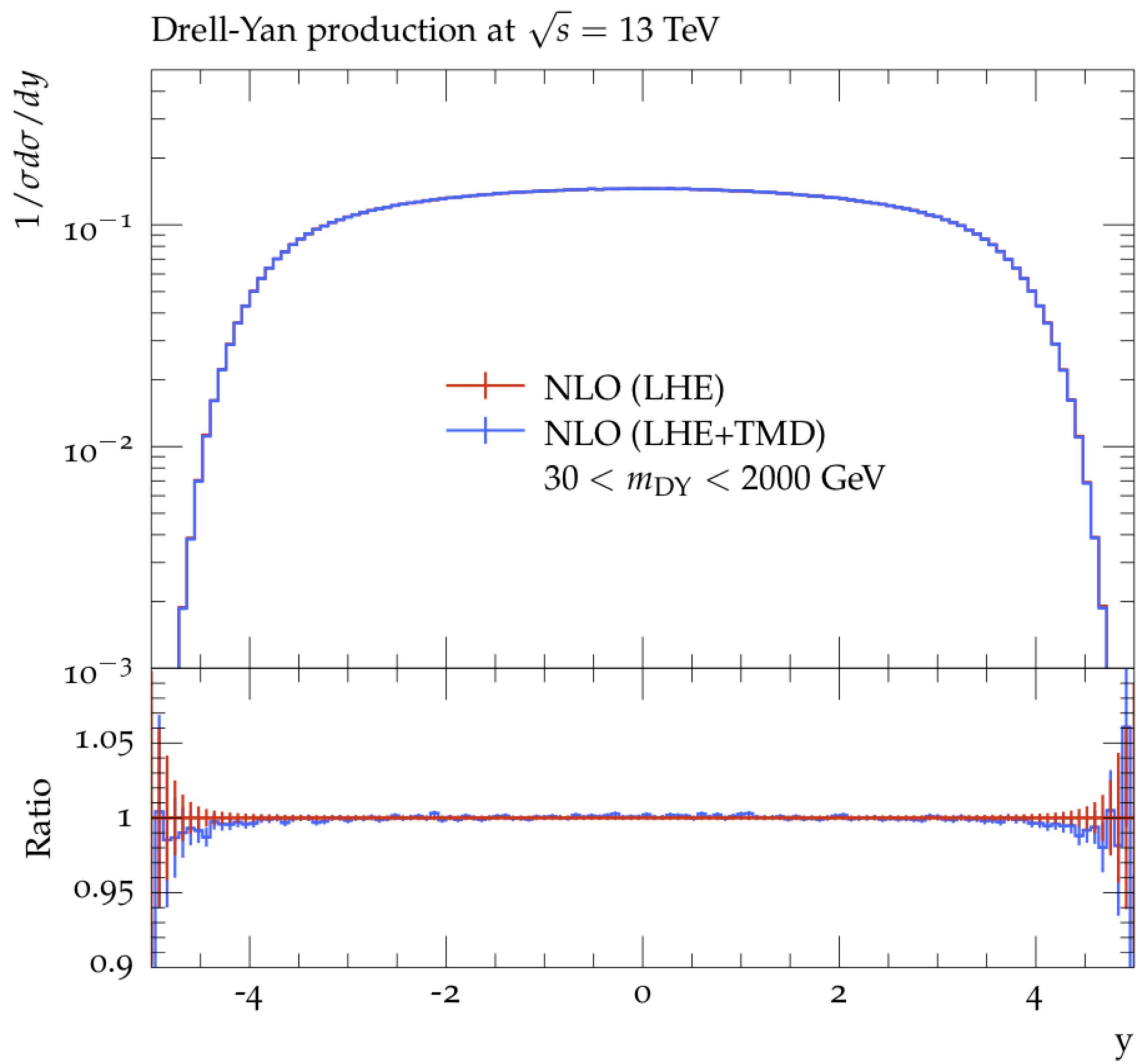
Fit to determine q_s of intrinsic k_T distribution from DY production as a function of m_{DY}

- obtain q_s rather independent on m_{DY}

Including TMDs for Z production with MC@NLO

- Are other features of DY production preserved ?

DY production: Bermudez Martinez, A. et al, arXiv 1906.00919, 2001.06488
CASCADE3 S. Baranov et al, arXiv 2101.10221



- Rapidity of DY pair not changed … (but x_1 and x_2)