Cornering New Physics with precision calculations of Higgs-boson properties

Based mainly on

arXiv:1903.05417 (PLB), 1911.11507 (EPJC), arXiv:2202.03453 (Phys. Rev. Lett.), arXiv:2305.03015 (EPJC), arXiv:2307.14976 and ongoing works

in collaboration with Masashi Aiko, Henning Bahl, Martin Gabelmann, Shinya Kanemura, Kateryna Radchenko Serdula, Alain Verduras and Georg Weiglein

Johannes Braathen (DESY)

DESY Theory Seminar,
Hamburg, Germany | 13 May 2024

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

Outline of the talk

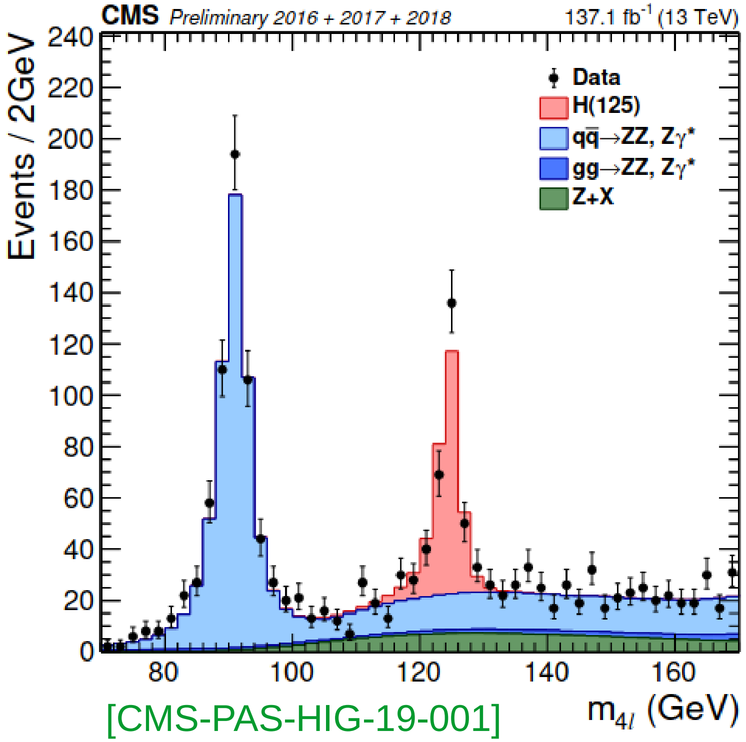
- **Introduction**
  - Why study the Higgs boson and its properties
  - Why study the trilinear Higgs coupling λ_{hhh} and how to access it experimentally
- **Part 1: Constraining New Physics with precision calculations of λ_{hhh} and $\Gamma(h \to \gamma\gamma)$**
- **Part 2: Automation & `anyH3` – a tool for calculating λ_{hhh} in arbitrary models**

Introduction

Context: a Higgs boson discovery, and where we stand now

2012: Discovery of <u>a</u> **Higgs boson** with mass **125 GeV** at the CERN LHC

- **What we know so far:**
  - Spin 0
  - Its mass **M_h = 125 GeV**, to astonishing 0.2% precision!
  - The electroweak (EW) vacuum expectation value **v = 246 GeV**
  - Not purely CP-odd



[CMS-PAS-HIG-19-001]

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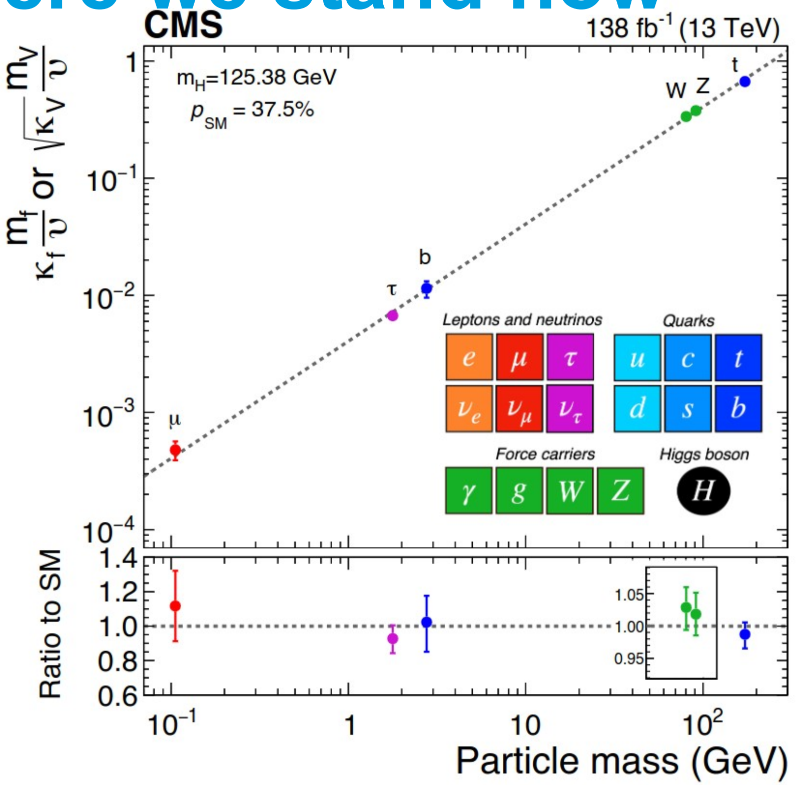

[CMS, Nature '22]

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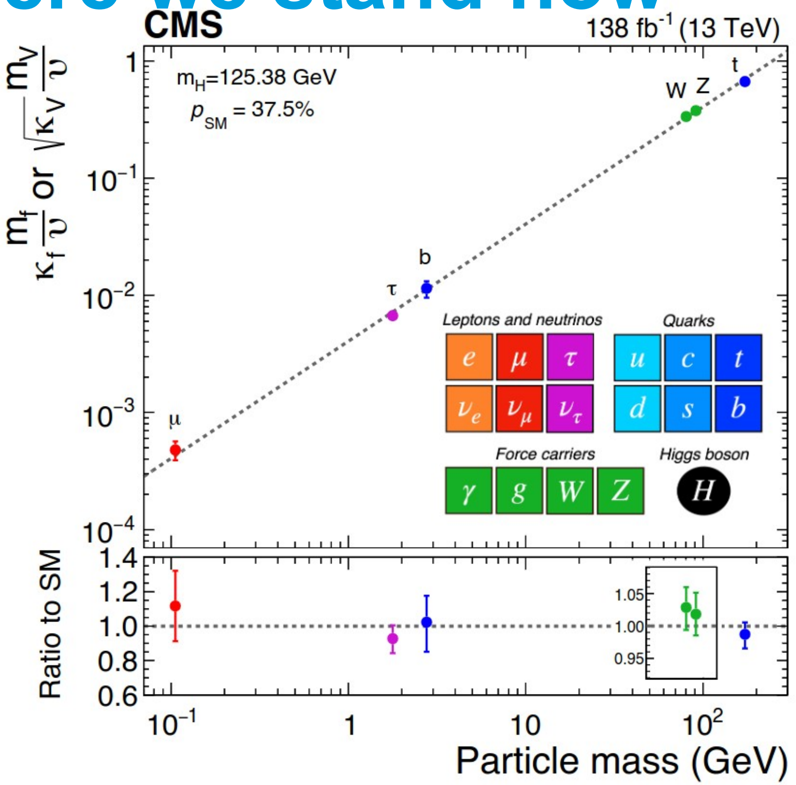

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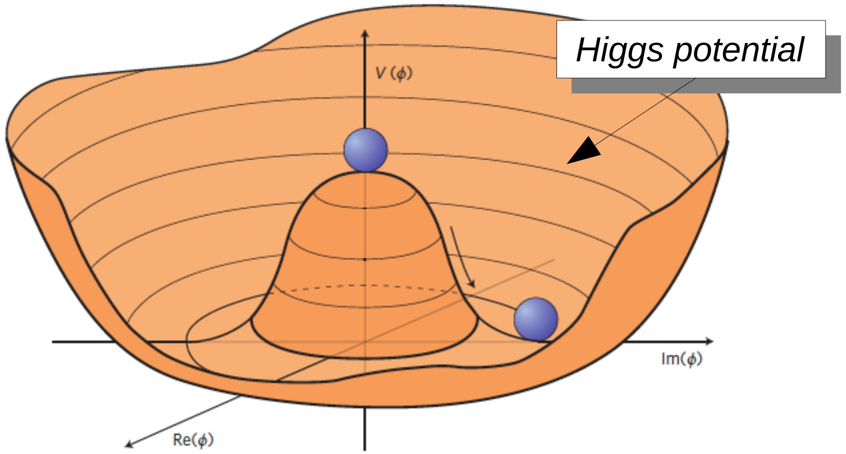

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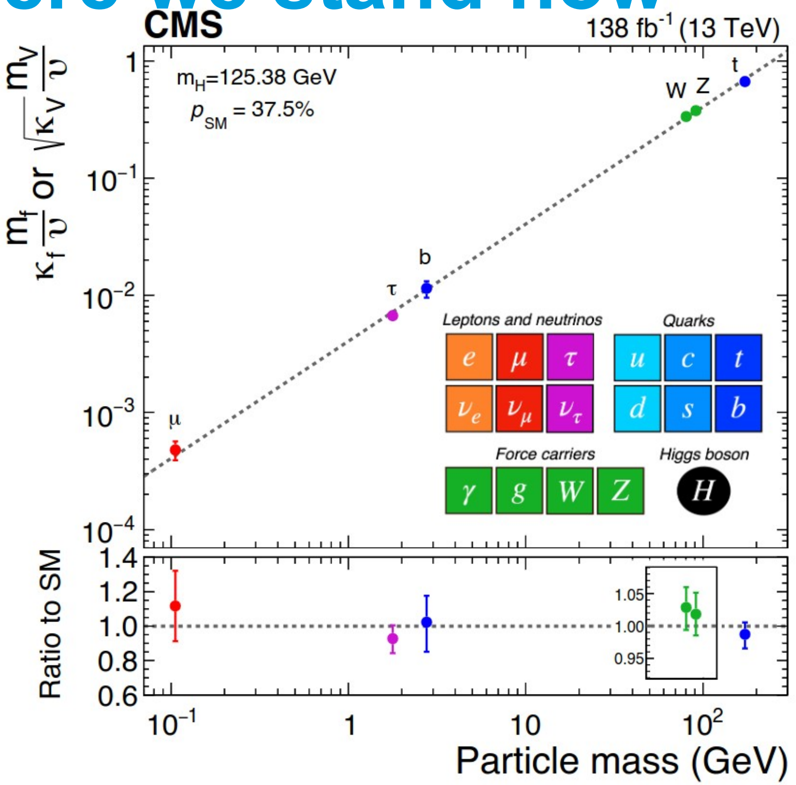
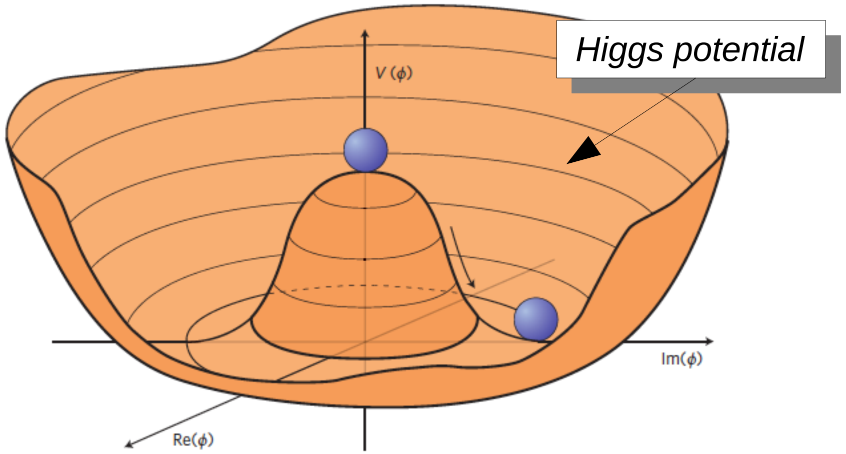


[CMS, Nature '22]

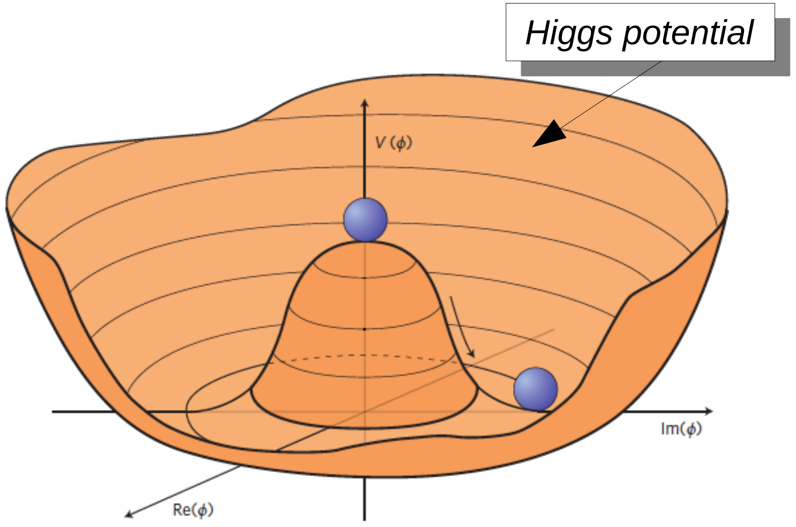
- **What we still don't know:**
  - Its coupling to 1st and 2nd gen. fermions
  - Its total width; BR(h→inv.) < ~9%
  - Its CP nature
  - Its fundamental nature? (elementary or composite)
  - The structure of the Higgs sector? (minimal or extended)
  - **The form of the Higgs potential**? *(more on this in a few slides)*



Going Beyond-the-Standard-Model

- Numerous problems **unresolved** by our current best description of High-Energy Physics (HEP), the Standard Model
  - Origin/form of Higgs potential
  - Structure of the Higgs sector
  - Hierarchy problem(s)
  - Dark Matter (DM)
  - Baryon Asymmetry of the Universe
  - Etc.

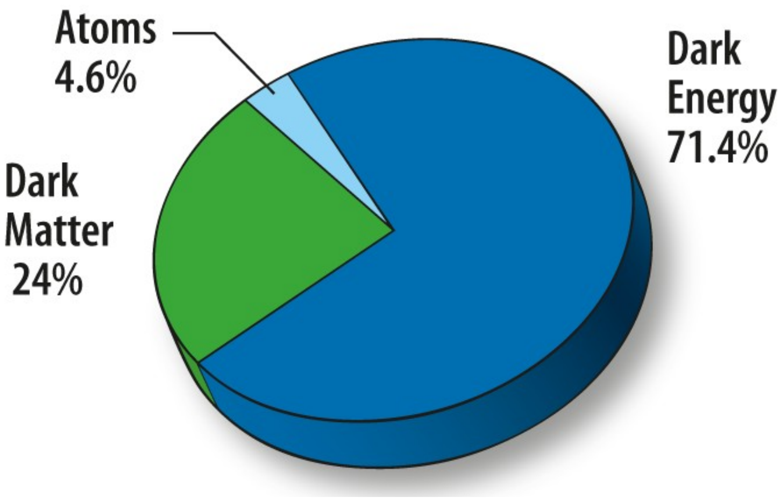
  → **Beyond-the-Standard-Model (BSM) Physics is needed!**



- *Today:*

  - probing the shape of the Higgs potential realised in Nature

  - explanation of DM with an extended Higgs sector, and how to probe such a scenario indirectly



Using the Higgs boson to search for New Physics

- Instead of direct searches (e.g. producing new states with colliders) → search for evidence of New Physics **indirectly**, via its **effects on properties of SM particles**

- Many (most) of the problems of the SM are **related to the Higgs sector**

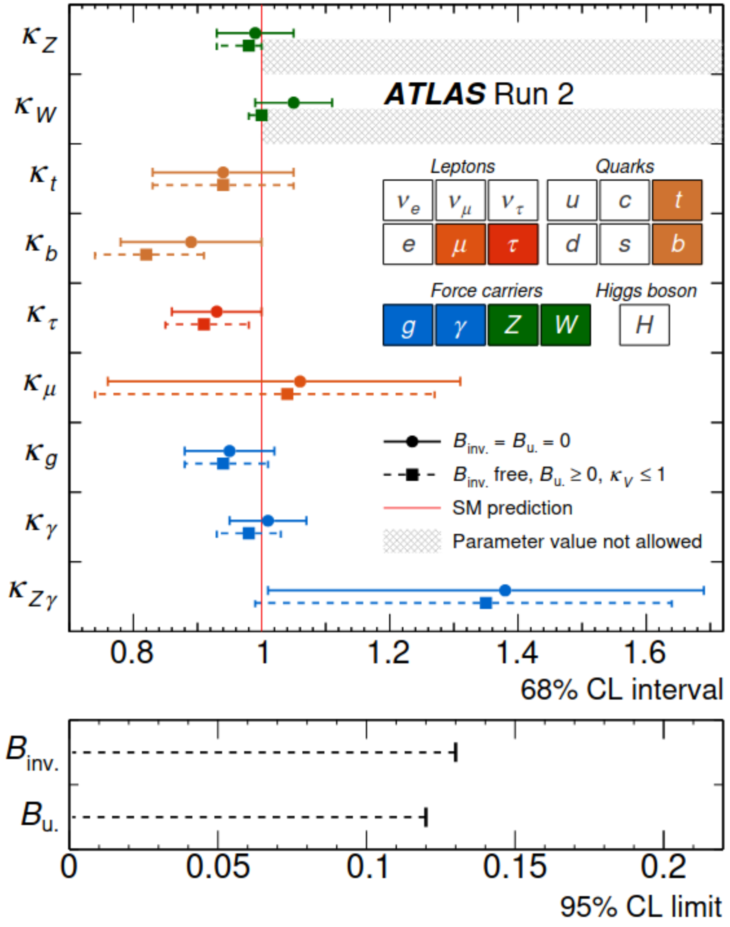
- Therefore, BSM theories often involve
  - **extended Higgs sectors**, e.g. 2nd Higgs doublet in MSSM, 2HDM, additional singlet scalars, etc. and/or
  - **states that couple to the Higgs(es)**, e.g. stops in Supersymmetry (SUSY)

- Ongoing program of **high-precision measurements of Higgs properties**, at LHC, HL-LHC, potential lepton colliders (e.g. ILC, CLIC, FCC-ee), etc.

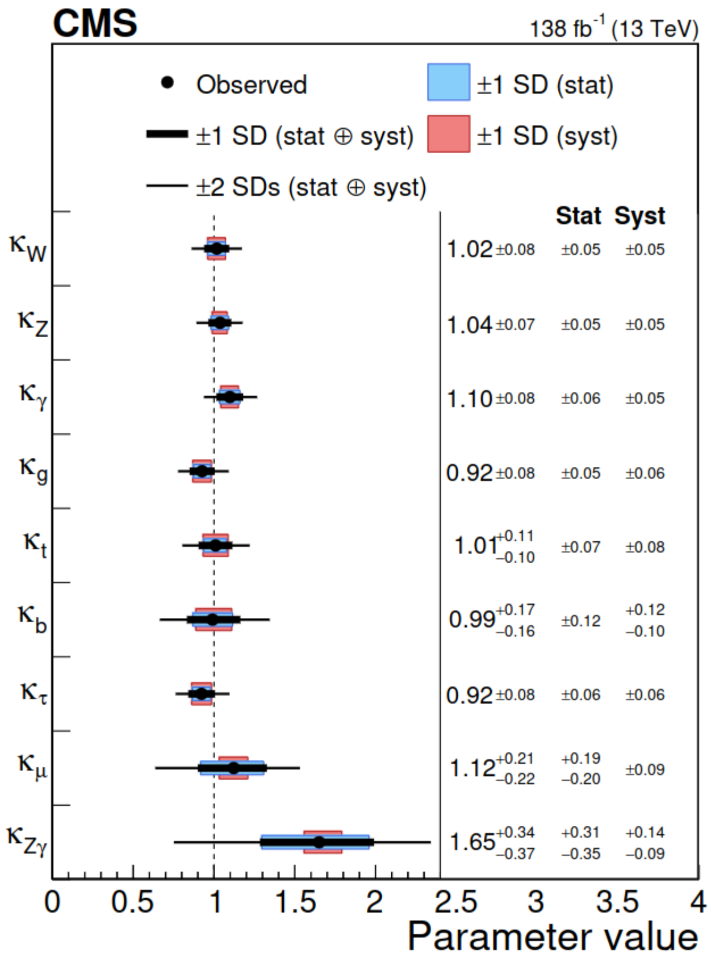
→ **Use the Higgs boson and its properties to probe signs of New Physics**

Using the Higgs boson to search for New Physics

- Determination of **Higgs couplings** currently underway, to be ***drastically improved*** in a foreseeable future



[ATLAS '22]



[CMS '22]

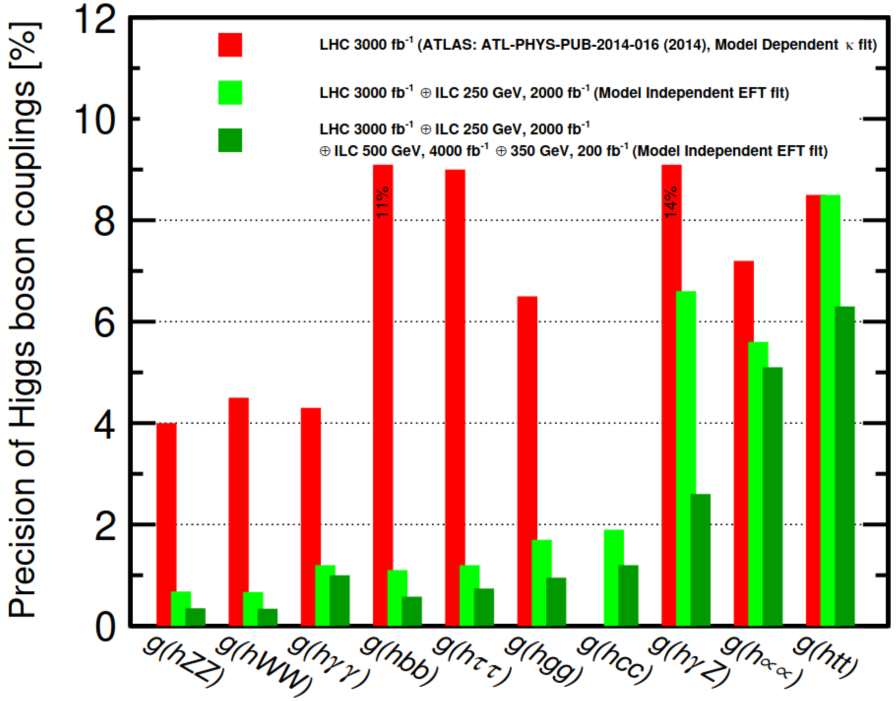


Figure from [1710.07621]
(ILC250 Physics case)

Using the Higgs boson to search for New Physics

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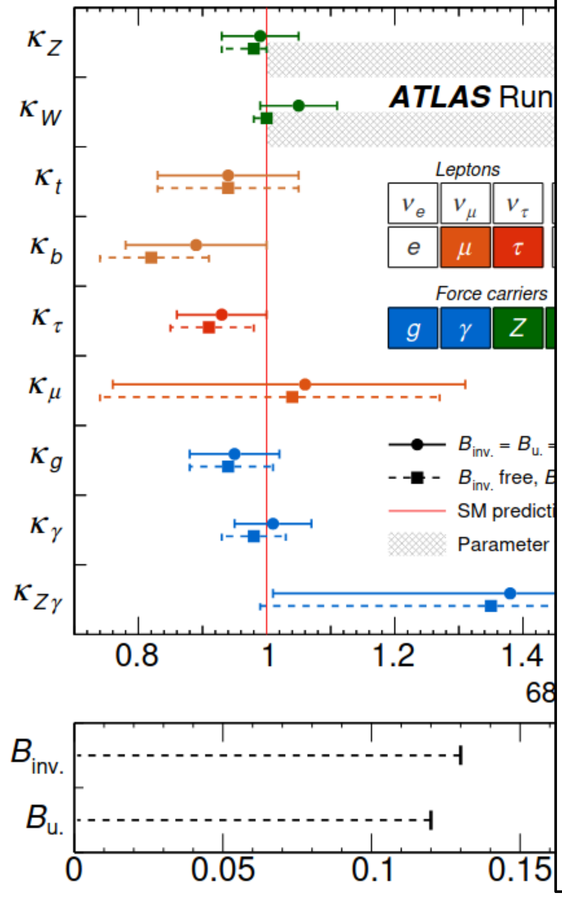


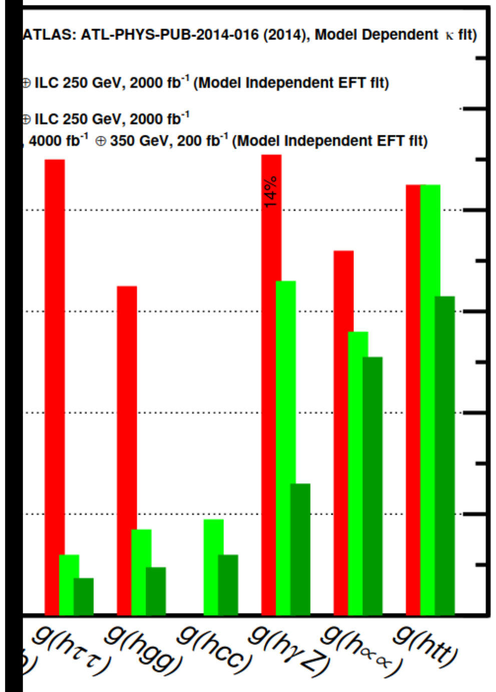
[ATLAS ‘22]

Comparing theory predictions for properties of the Higgs boson with experimental results

→ powerful tool to probe New Physics, constrain BSM parameter space and discriminate allowed/excluded scenarios

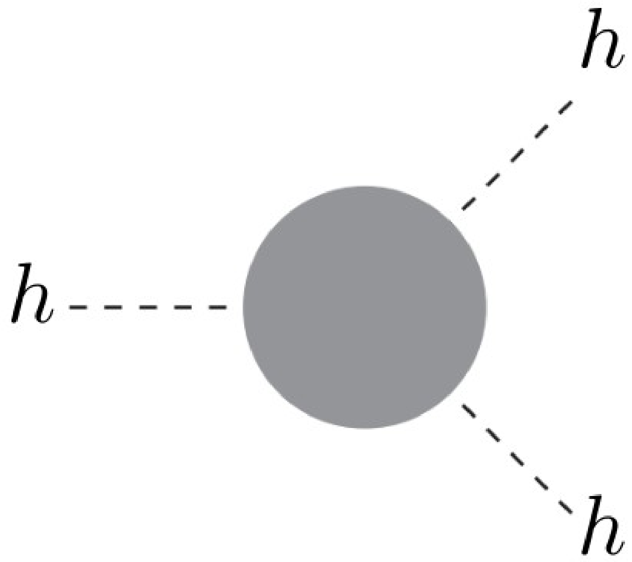
→ **today:** **trilinear Higgs coupling λ_{hhh} and $\Gamma(h\to\gamma\gamma)$**

[CMS ‘22]



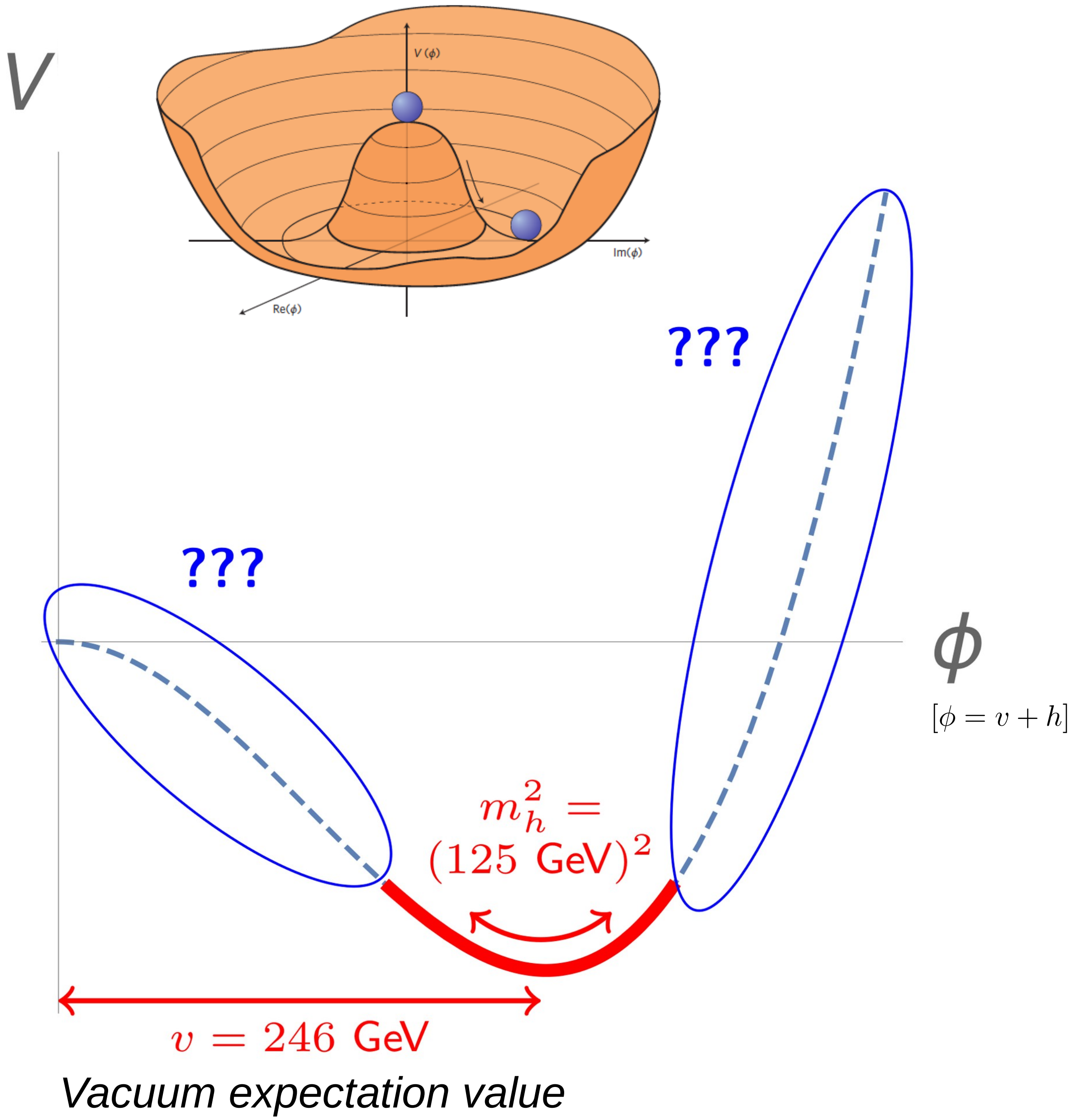
om [1710.07621]
Physics case)

Why investigate λ_{hhh}?



Form of the Higgs potential and trilinear Higgs coupling

- Brout-Englert-Higgs mechanism = **origin of electroweak symmetry breaking** ...
  … but very little known about the **Higgs potential** causing the phase transition



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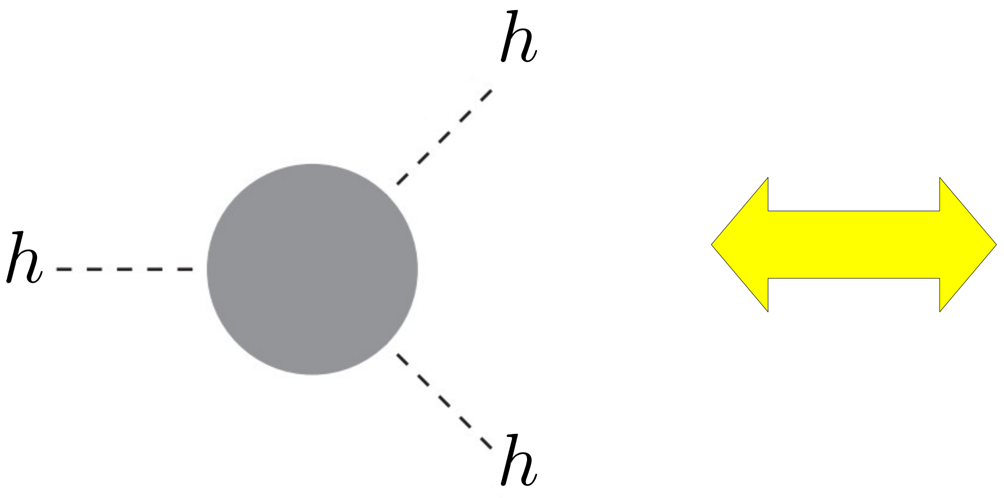
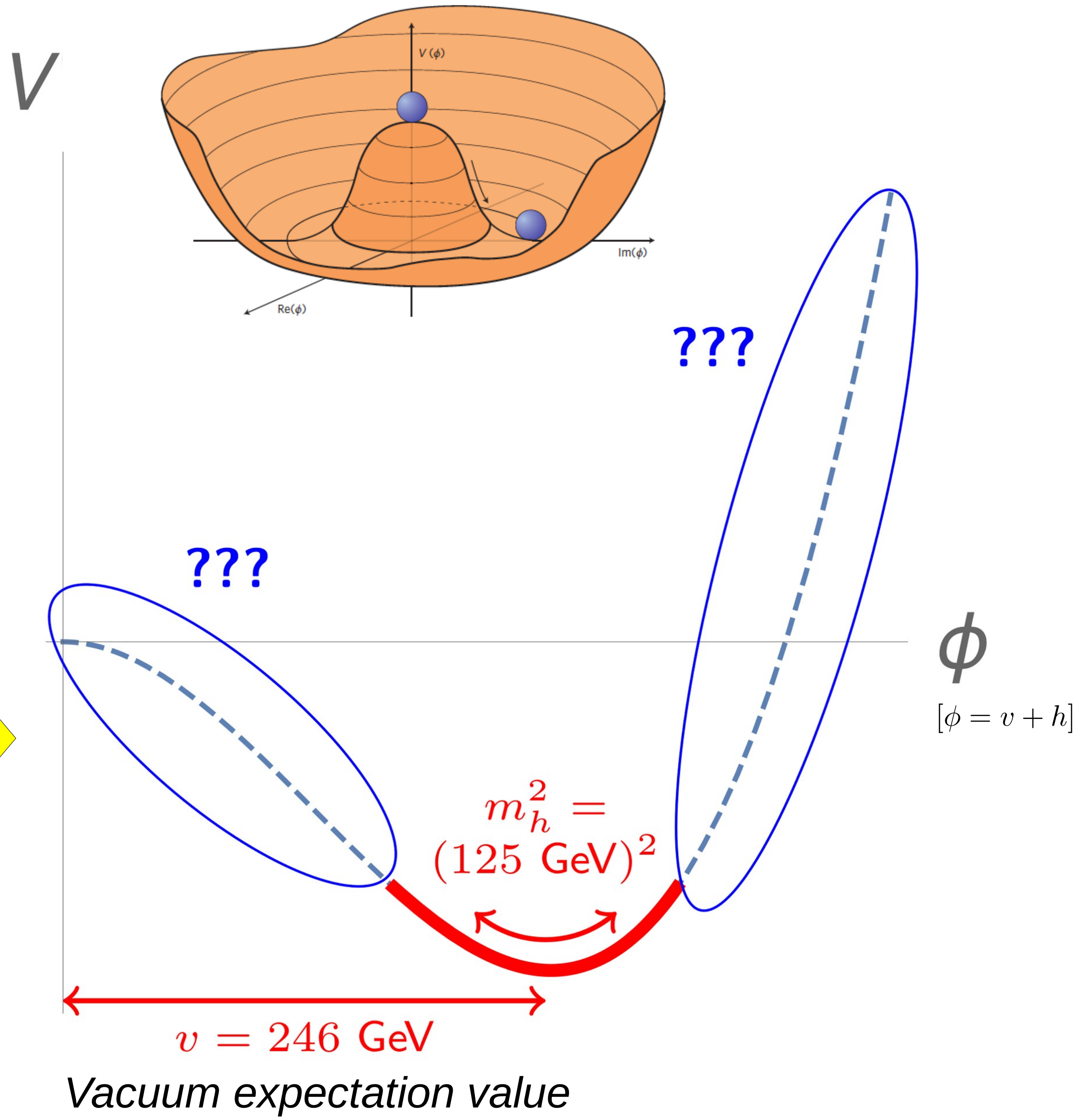
- Shape of the potential determined by **trilinear Higgs coupling λ_{hhh}**





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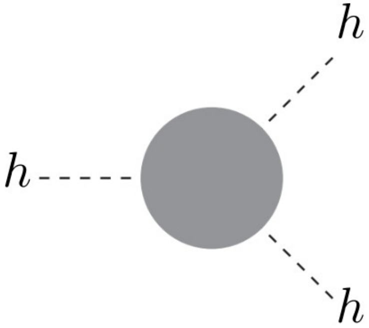
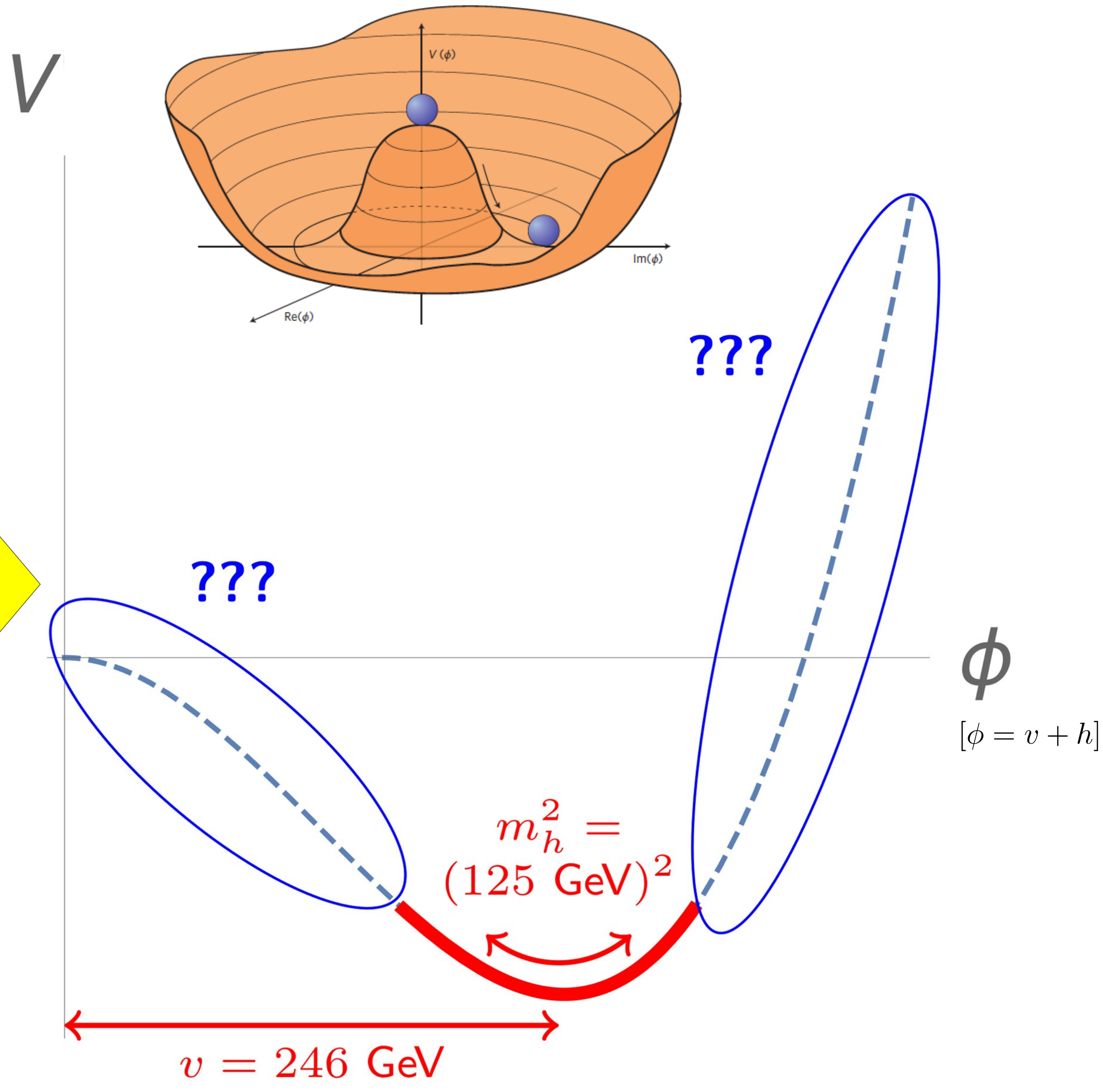


In the SM: $V^{(0)}_{\text{SM}} = \frac{1}{2}m_h^2 h^2 + \frac{1}{3!}\underbrace{\left(\frac{3m_h^2}{v}\right)}_{\equiv(\lambda^{(0)}_{hhh})^{\text{SM}}} h^3 + \frac{1}{4!}\left(\frac{3m_h^2}{v^2}\right) h^4 + \cdots$

In general:

$$V^{(0)} = \frac{1}{2}m_h^2 h^2 + \frac{1}{3!}\overbrace{\kappa_\lambda \left(\frac{3m_h^2}{v}\right)}^{\equiv\lambda_{hhh}} h^3 + \frac{1}{4!}\kappa_{\lambda_4}\left(\frac{3m_h^2}{v^2}\right) h^4 + \cdots$$

with $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda^{(0)}_{hhh})^{\text{SM}}$



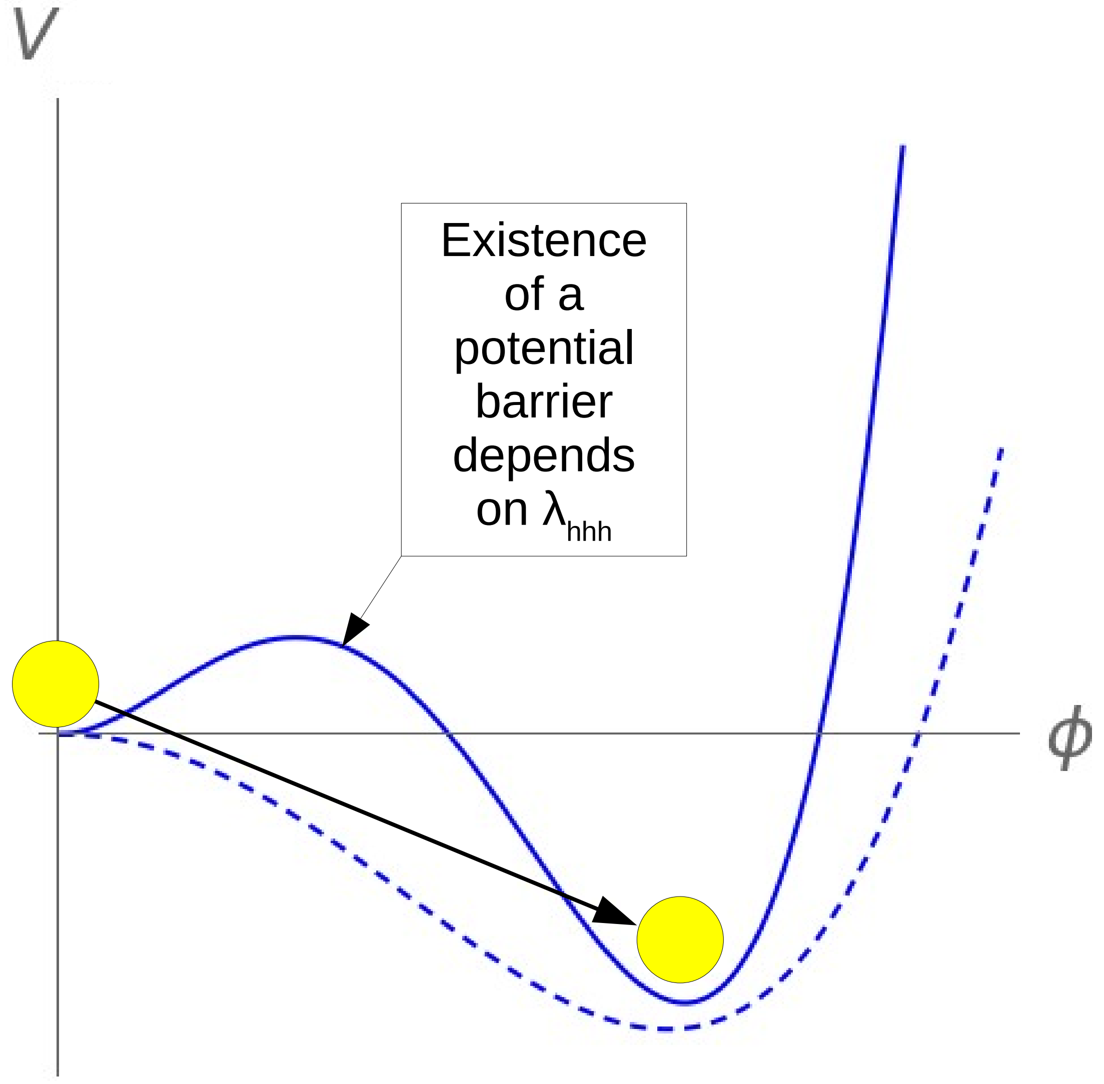
Vacuum expectation value

Form of the Higgs potential and baryon asymmetry

- Brout-Englert-Higgs mechanism = **origin of electroweak symmetry breaking** ...
  … but very little known about the **Higgs potential** causing the phase transition

- Shape of the potential determined by **trilinear Higgs coupling λ_{hhh}**

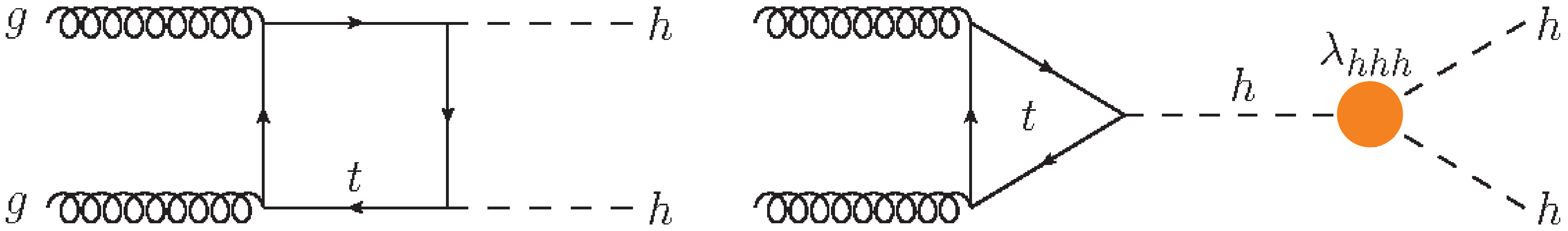
- Among **Sakharov conditions** necessary to explain **baryon asymmetry via electroweak phase transition** (EWPT):
  - ***Strong first-order EWPT***
    → barrier in Higgs potential
    → typically significant deviation in λ_{hhh} from SM



Accessing λ_{hhh} experimentally

Accessing λ_{hhh} via di-Higgs production

- **Di-Higgs production** → λ_{hhh} enters at leading order (LO) → **most direct probe of λ_{hhh}**



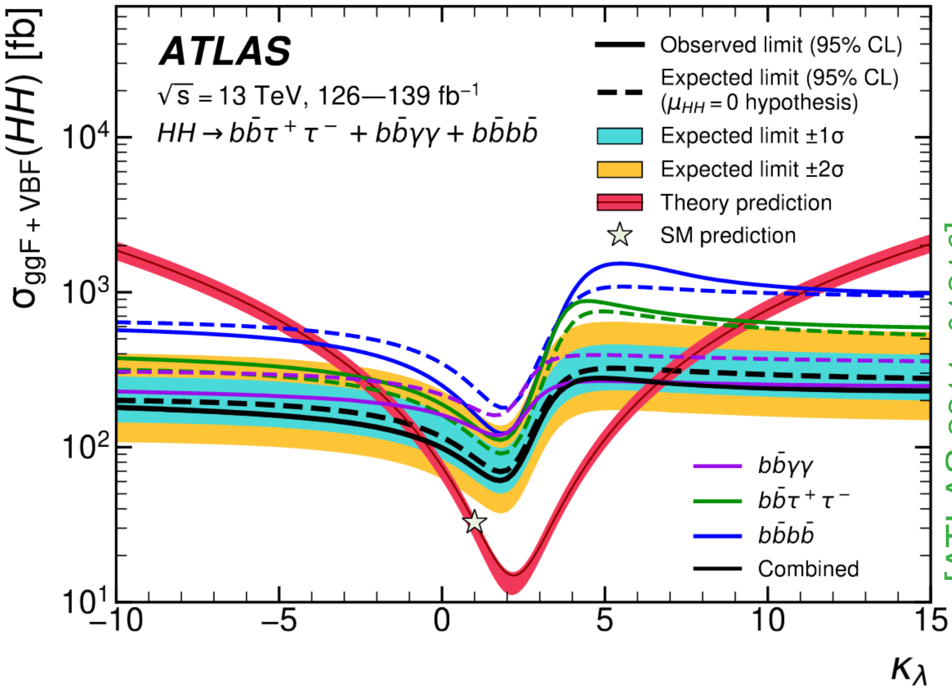
[Note: Single-Higgs production (EW precision observables) → λ_{hhh} enters at NLO (NNLO)]

- Box and triangle diagrams **interfere destructively**
  → small di-Higgs cross-section σ_{hh} in SM

  → BSM deviation in λ_{hhh} can **significantly alter di-Higgs production!**

- Upper limit on di-Higgs cross-section
  → **limits on $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$**

- κ_λ as an *effective coupling*: $\mathcal{L} \supset -\kappa_\lambda \times \frac{3m_h^2}{v^2} \cdot h^3 + \cdots$



[ATLAS 2211.01216]

Accessing λ_{hhh} via di-Higgs production

- **Di-Higgs production** → λ … enters at leading order (LO) → **most direct probe of λ**

Not…

- Bo…
  →
  →
  di-…

- Up…
  →

- κ_λ a… v^2

Recent results from ATLAS di-Higgs searches [ATLAS 2211.01216]
yield the limits:

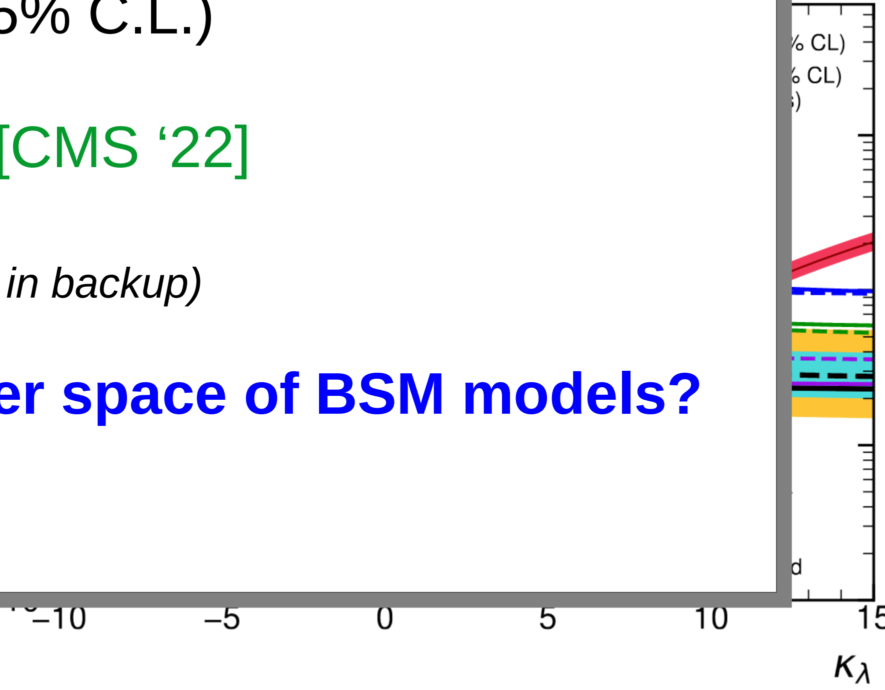
-0.4 < κ_λ < 6.3 at 95% C.L.

With κ_t floating: -1.4 < κ_λ < 6.1 (95% C.L.)

CMS: **-1.2 < κ_λ < 6.5** at 95% C.L. [CMS '22]

NB: future determination even better (details in backup)

→ **Can κ_λ now be used to constrain the parameter space of BSM models?**



[ATLAS 2211.01216]

Calculating λ_{hhh} in models with extended scalar sectors

The Two-Higgs-Doublet Model

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge ½
- CP-conserving 2HDM, with softly-broken Z_2 symmetry ($\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$) to avoid tree-level FCNCs

$$V^{(0)}_{\text{2HDM}} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2)$$
$$+ \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

$$v_1^2 + v_2^2 = v^2 = (246\ \text{GeV})^2$$

- **Mass eigenstates**:
  h, H: CP-even Higgs bosons *(h → 125-GeV SM-like state);* A: CP-odd Higgs boson;
  $H^\pm$: charged Higgs boson

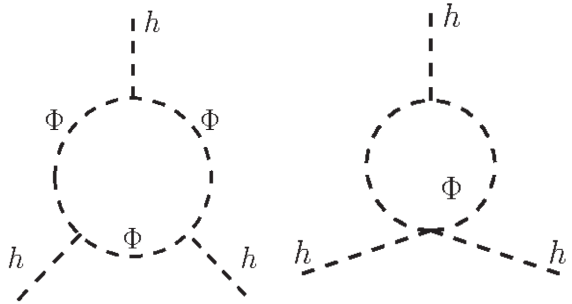
- **BSM parameters**: 3 BSM masses m_H, m_A, $m_{H^\pm}$, BSM mass scale M (defined by $M^2 \equiv 2m_3^2/s_{2\beta}$), angles α (CP-even Higgs mixing angle) and β (defined by $\tan\beta = v_2/v_1$)

- **BSM-scalar masses** take form $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2\,, \quad \Phi \in \{H, A, H^\pm\}$

- We take the **alignment limit α = β - π/2** → all Higgs couplings are SM-like at tree level
  → compatible with current experimental data

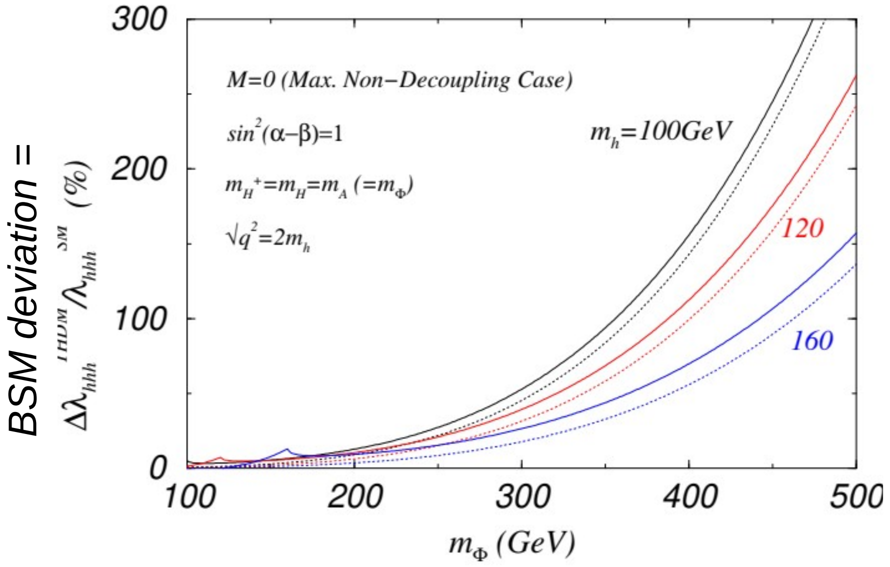
Mass splitting effects in λ_{hhh}

- First investigation of 1L BSM contributions to λ_{hhh} in 2HDM: [Kanemura, (Kiyoura), Okada, Senaha, Yuan '02, '04]



$$g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}$$

$$(\Phi \in \{H, A, H^\pm\})$$



- **Deviations of tens/hundreds of % from SM possible,** for **large $g_{h\Phi\Phi}$ or $g_{hh\Phi\Phi}$ couplings**
- **Mass splitting effects**, now found in various models (2HDM, inert doublet model, singlet extensions, etc.)

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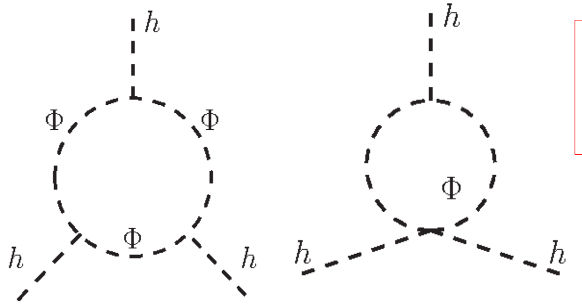

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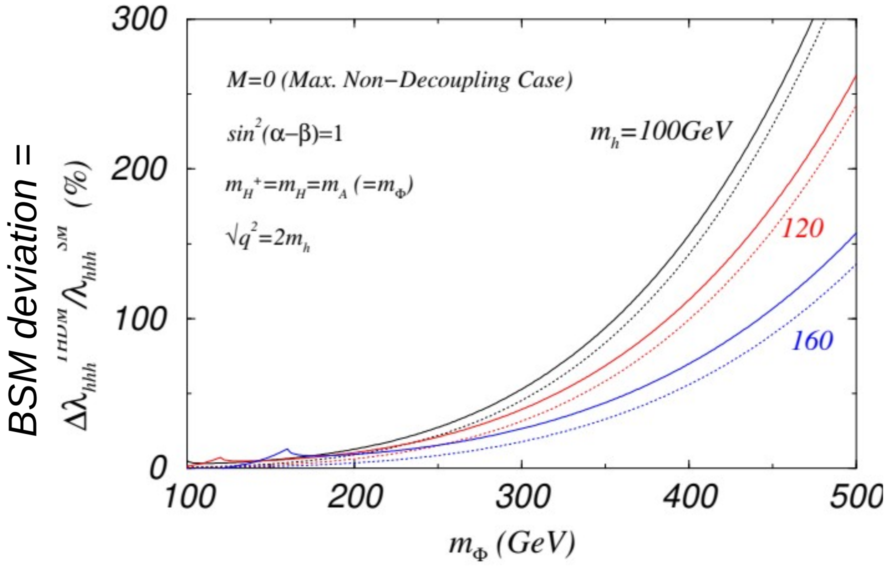
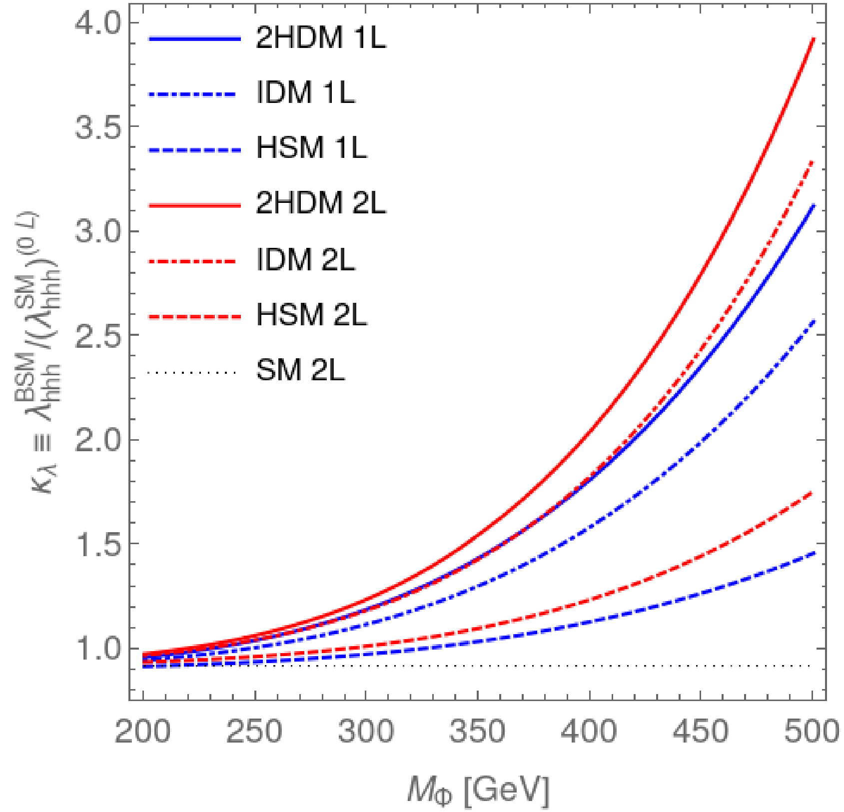
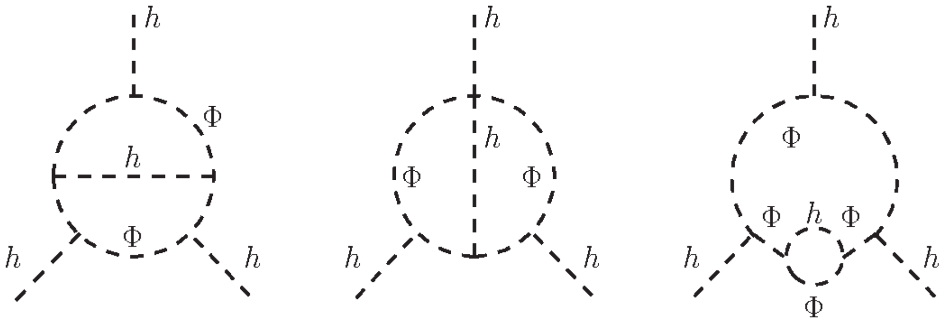


- Large effects **confirmed at 2L** in [JB, Kanemura ‘19]
→ **leading 2L corrections involving BSM scalars (H,A,H±) and top quark**, computed in effective potential approximation





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Constraining BSM models with λ_{hhh}

i. *Can we apply the limits on κ_λ, extracted from experimental searches for di-Higgs production, for BSM models?*

ii. *Can large BSM deviations occur for points still allowed in light of theoretical and experimental constraints? If so, how large can they become?*

As a concrete example, we consider an aligned 2HDM

Based on

arXiv:2202.03453 **(Phys. Rev. Lett.) in collaboration with Henning Bahl and Georg Weiglein**

Can we apply di-Higgs results for the aligned 2HDM?

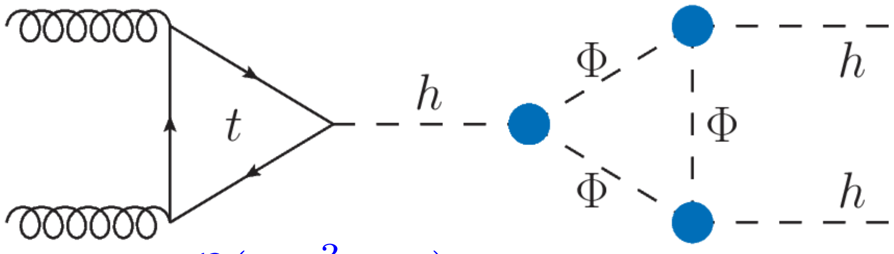
- Current strongest limit on κ_λ are from ATLAS double- (+ single-) Higgs searches

  $-0.4 < \kappa_\lambda < 6.3$ [ATLAS-CONF-2022-050]

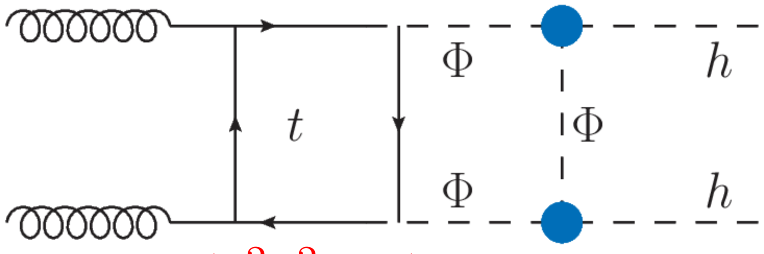
  [where $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$]

- What are the *assumptions* for the ATLAS limits?
  - All other Higgs couplings (to fermions, gauge bosons) are SM-like

    → this is **ensured by the alignment** ✓
  - The modification of λ_{hhh} is the only source of deviation of the *non-resonant Higgs-pair production cross section* from the SM



$\propto \mathcal{O}(y_t g^3_{hh\Phi\Phi})$ ***included***

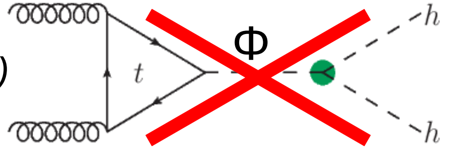


$\propto \mathcal{O}(y_t^2 g^2_{hh\Phi\Phi})$ *not included*

→ We **correctly include all leading BSM effects to di-Higgs production, in powers of $g_{hh\Phi\Phi}$, up to NNLO!** ✓

- **We can apply the ATLAS limits to our setting!**

(Note: BSM resonant Higgs-pair production cross section also suppressed at LO, thanks to alignment)



A parameter scan in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

- Our strategy:
  1. **Scan BSM parameter space**, keeping only points passing various theoretical and experimental constraints (*see below*)
  2. Identify regions with **large BSM deviations in λ_{hhh}**
  3. Devise a **benchmark scenario** allowing large deviations and investigate impact of experimental limit on λ_{hhh}
- *Here*: we consider an **aligned 2HDM of type-I**, but similar results expected for other 2HDM types, or other BSM models with extended Higgs sectors
- Constraints in our parameter scan:

experimental
  - 125-GeV Higgs measurements with `HiggsSignals`
  - Direct searches for BSM scalars with `HiggsBounds`
  - b-physics constraints, using results from [Gfitter group 1803.01853]

  (Checked with `ScannerS` [Mühlleitner et al. 2007.02985])
  - EW precision observables, computed at two loops with `THDM_EWPOS` [Hessenberger, Hollik '16, '22]

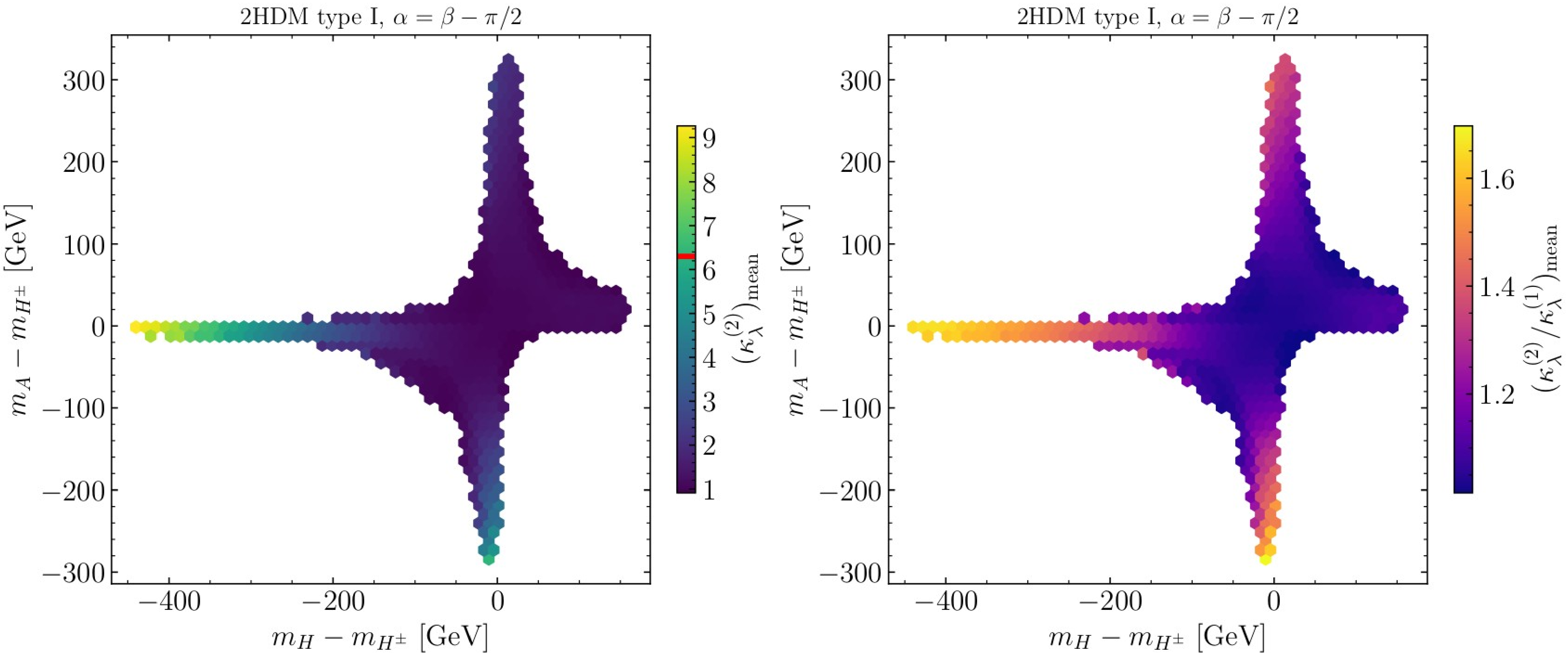
theoretical
  - Vacuum stability
  - Boundedness-from-below of the potential

  (Checked with `ScannerS`)
  - NLO perturbative unitarity, using results from [Grinstein et al. 1512.04567], [Cacchio et al. 1609.01290]
- For points passing these constraints, we **compute κ_λ at 1L and 2L**, using results from [JB, Kanemura '19]

Parameter scan results

[Bahl, JB, Weiglein PRL '22]

<u>Mean value</u> for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM} / (\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM} / (\lambda_{hhh}^{(1)})^{2HDM}$ [right] in (m_H-$m_{H\pm}$, m_A-$m_{H\pm}$) plane



NB: all previously mentioned constraints are fulfilled by the points shown here

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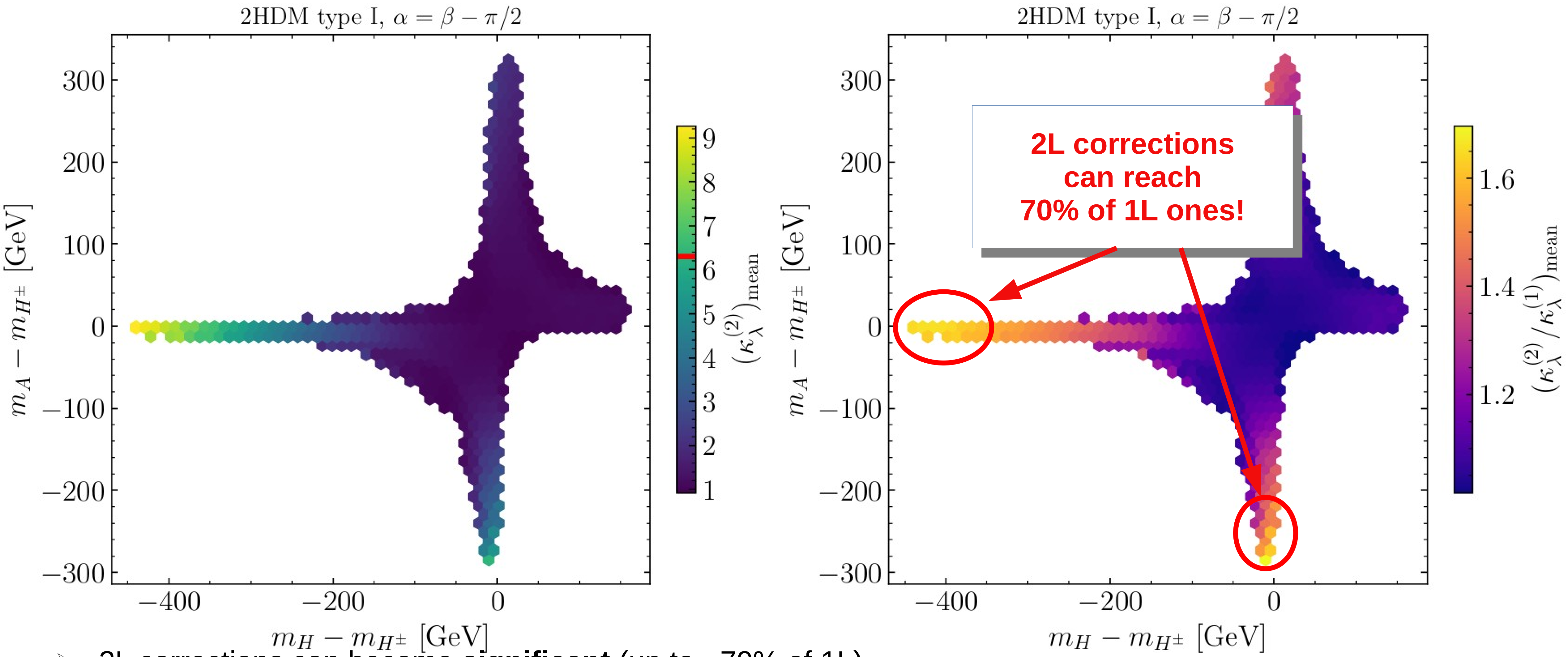
[Bahl, JB, Weiglein PRL '22]

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(1)})^{2HDM}$ [right] in $(m_H - m_{H\pm}, m_A - m_{H\pm})$ plane

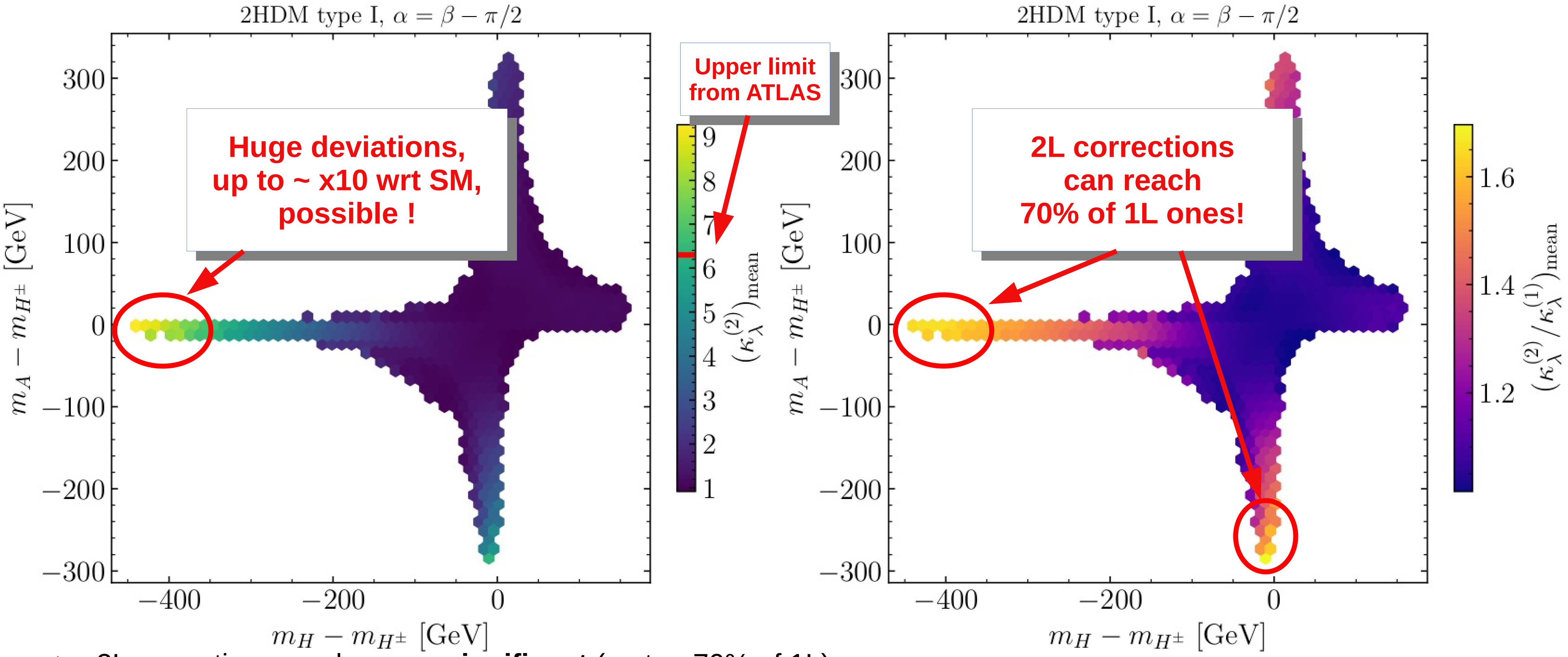


- 2L corrections can become **significant** (up to ~70% of 1L)

Parameter scan results

[Bahl, JB, Weiglein PRL '22]

<u>Mean value</u> for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(1)})^{2HDM}$ [right] in $(m_H - m_{H\pm}, m_A - m_{H\pm})$ plane



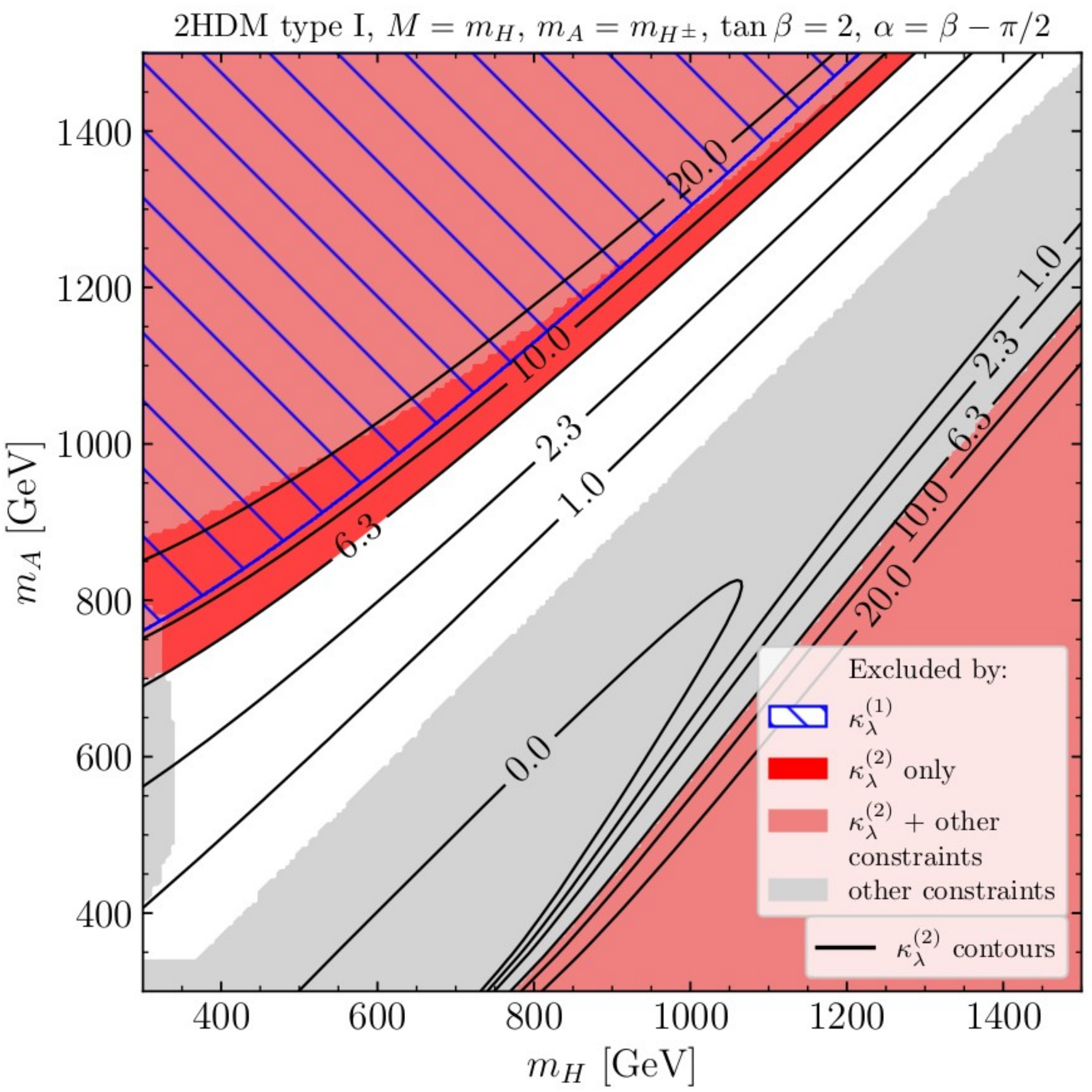
- 2L corrections can become **significant** (up to ~70% of 1L)
- **Huge enhancements** (by a factor ~10) of λ_{hhh} possible for $m_A \sim m_{H\pm}$ and $m_H \sim M$

A benchmark scenario in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

Results shown for aligned 2HDM of type-I, similar for other types (*available in backup*)

We take $m_A = m_{H\pm}$, $M = m_H$, $\tan\beta = 2$



- ***Grey area:*** area excluded by other constraints, in particular BSM Higgs searches, boundedness-from-below (BFB), perturbative unitarity
- ***Light red area:*** area excluded both by other constraints (BFB, perturbative unitarity) and by $\kappa_\lambda^{(2)} > 6.3$ [in region where $\kappa_\lambda^{(2)} < -0.4$ the calculation isn't reliable]
- ***Dark red area:*** new area that is **excluded ONLY by $\kappa_\lambda^{(2)} > 6.3$**. Would otherwise not be excluded!
- ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.3$ → impact of including 2L corrections is significant!

A benchmark scenario in the aligned 2HDM

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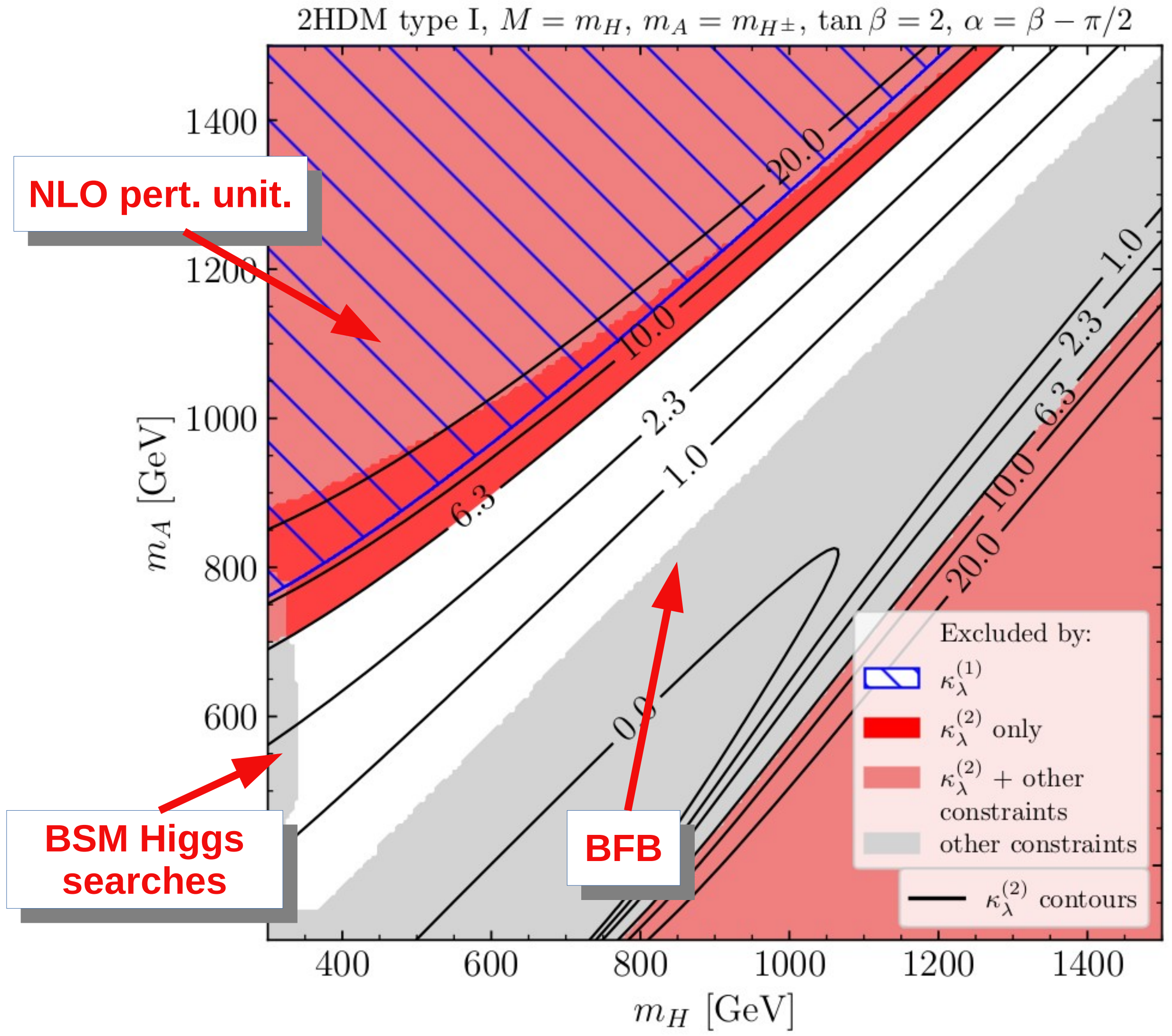

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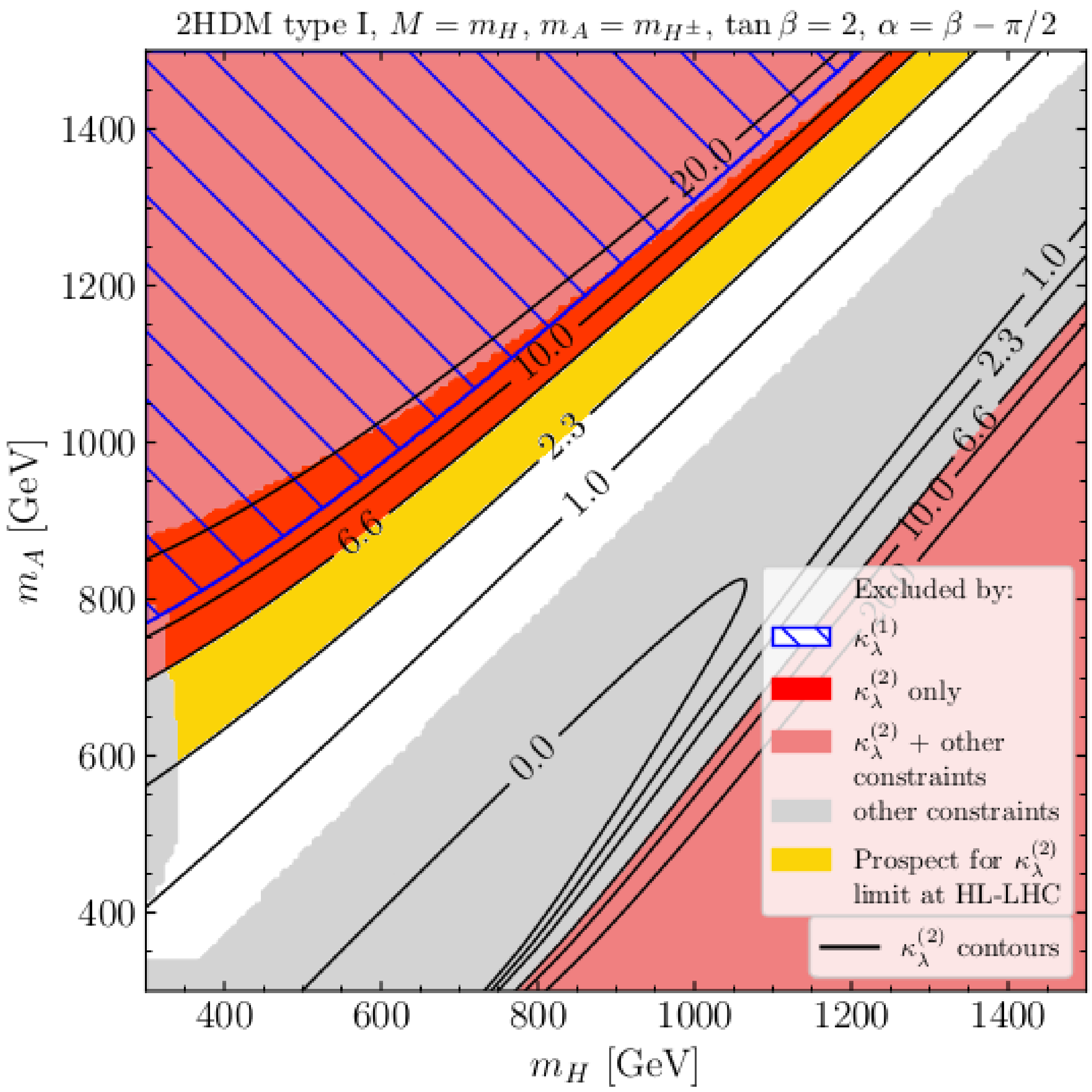
- ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.3$ → impact of including 2L corrections is significant!

A benchmark scenario in the aligned 2HDM – future prospects

Suppose for instance the upper bound on κ_λ becomes $\kappa_\lambda < 2.3$



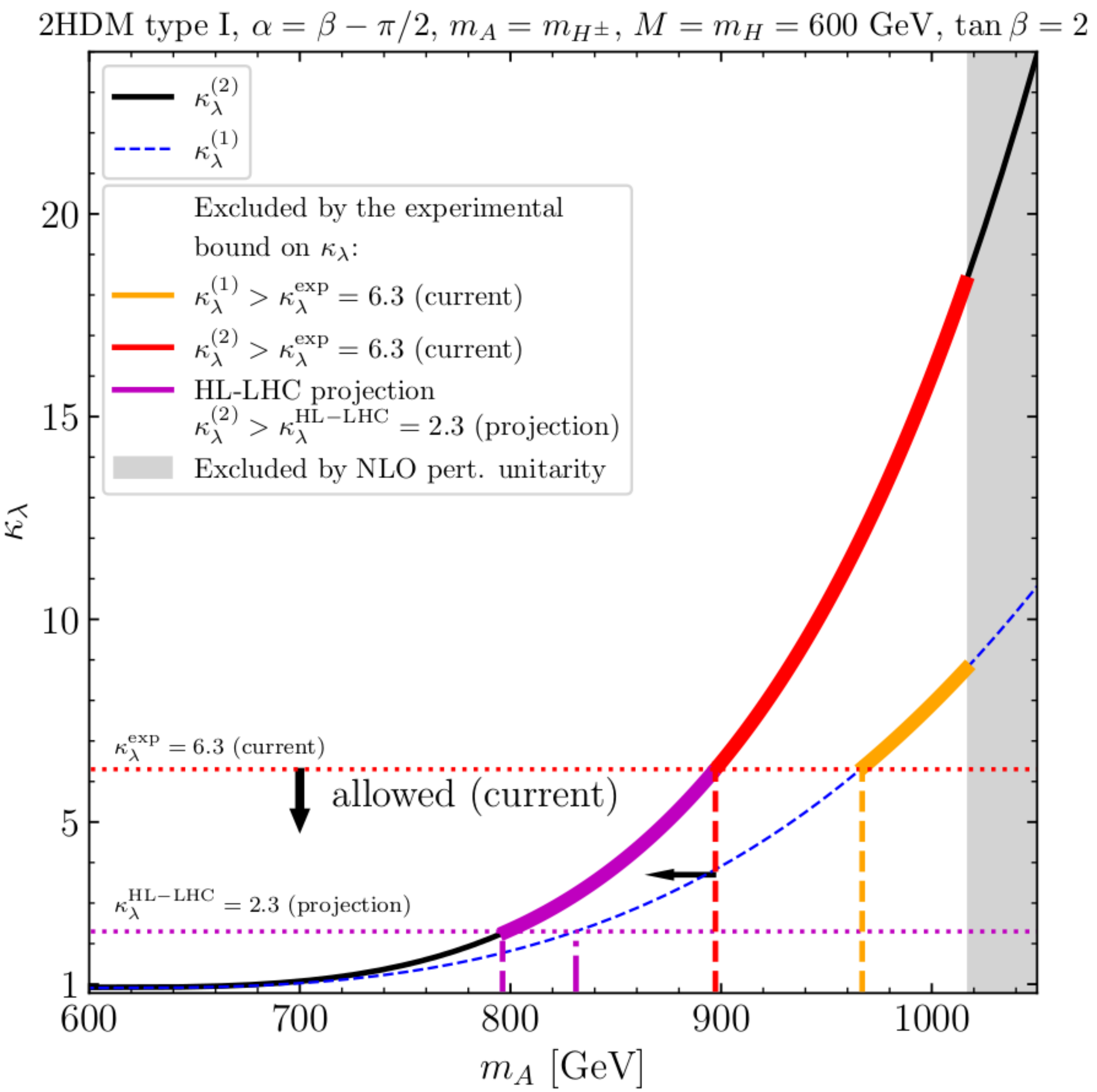
[Bahl, JB, Weiglein '23]

- ***Golden area:*** additional exclusion if the limit on κ_λ becomes $\kappa_\lambda^{(2)} < 2.3$ (achievable at HL-LHC)
- Of course, **prospects even better with an e^+e^- collider!**
- Experimental constraints, such as Higgs physics, may also become more stringent, however **not** theoretical constraints (like BFB or perturbative unitarity)

A benchmark scenario in the aligned 2HDM – 1D scan

Within the previously shown plane, we fix M=m_H=600 GeV, and vary m_A=$m_{H\pm}$

[Bahl, JB, Weiglein PRL ‘22]



- Illustrates the significantly improved reach of the experimental limit when including **2L corrections** in calculation of κ_λ
- A stricter choice for the perturbative unitarity constraint (grey) does not significantly change the region excluded by $\kappa_\lambda^{(2)}$

Constraining scalar DM models with λ_{hhh} and $\Gamma(h\to\gamma\gamma)$

The Inert Doublet Model

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge ½

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG) \end{pmatrix} \quad \text{and} \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H+iA) \end{pmatrix}$$

- **Unbroken Z_2 symmetry $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$**

$$V^{(0)}_{\text{IDM}} = \mu_1^2|\Phi_1|^2 + \mu_2^2|\Phi_2|^2 + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

- **Model parameters**:
  3 BSM masses m_H, m_A, $m_{H\pm}$, BSM mass scale μ_2, inert doublet quartic self-coupling λ_2

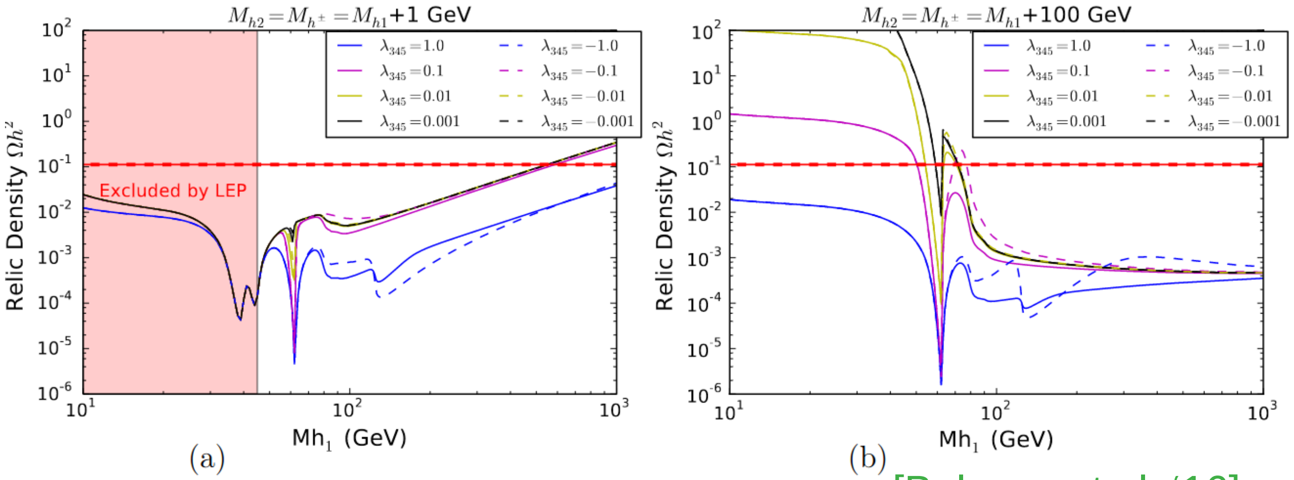
- **BSM-scalar masses** take form

$$m_H^2 = \mu_2^2 + \frac{1}{2}\lambda_H v^2\,, \quad m_A^2 = \mu_2^2 + \frac{1}{2}\lambda_A v^2\,, \quad m_{H\pm}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2\,,$$

$$\text{with } \lambda_{H,A} = \lambda_3 + \lambda_4 \pm \lambda_5$$

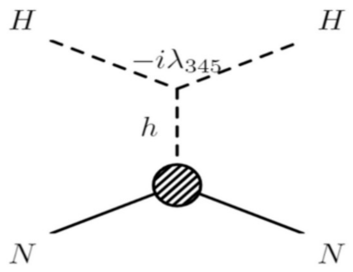
Dark Matter in the Inert Doublet Model

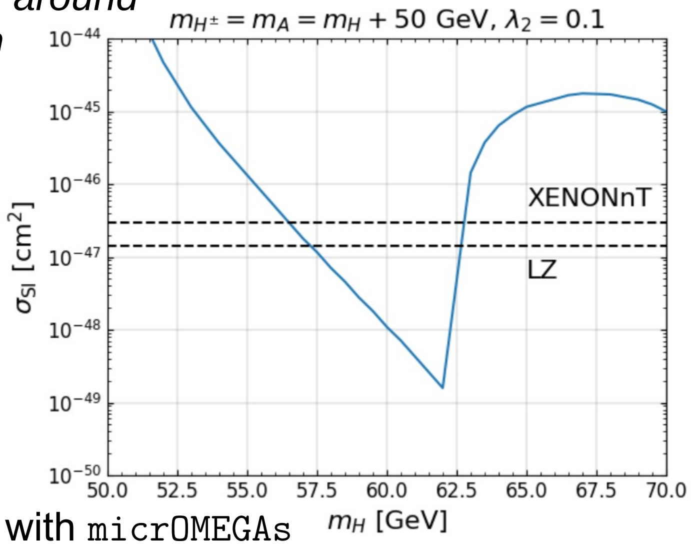
- Inert scalars: charged under Z_2 symmetry (Z_2-odd)
- Lightest inert scalar = **Dark Matter candidate**
  → *assume H in this talk*
- DM relic density obtained via freeze-out mechanism, while evading current detection bounds
- 2 main scenarios:
  → **"Higgs resonance scenario" $m_H \sim m_h/2$**
  → **"Heavy Higgs scenario" $m_H \geq 500$ GeV**
- IDM testable at current and future experiments via
  - DM direct and indirect searches
  - direct searches at colliders
  - *precision/indirect tests*
    → properties of 125-GeV Higgs boson



[Belyaev et al. '16]

Direct detection bounds around Higgs resonance region

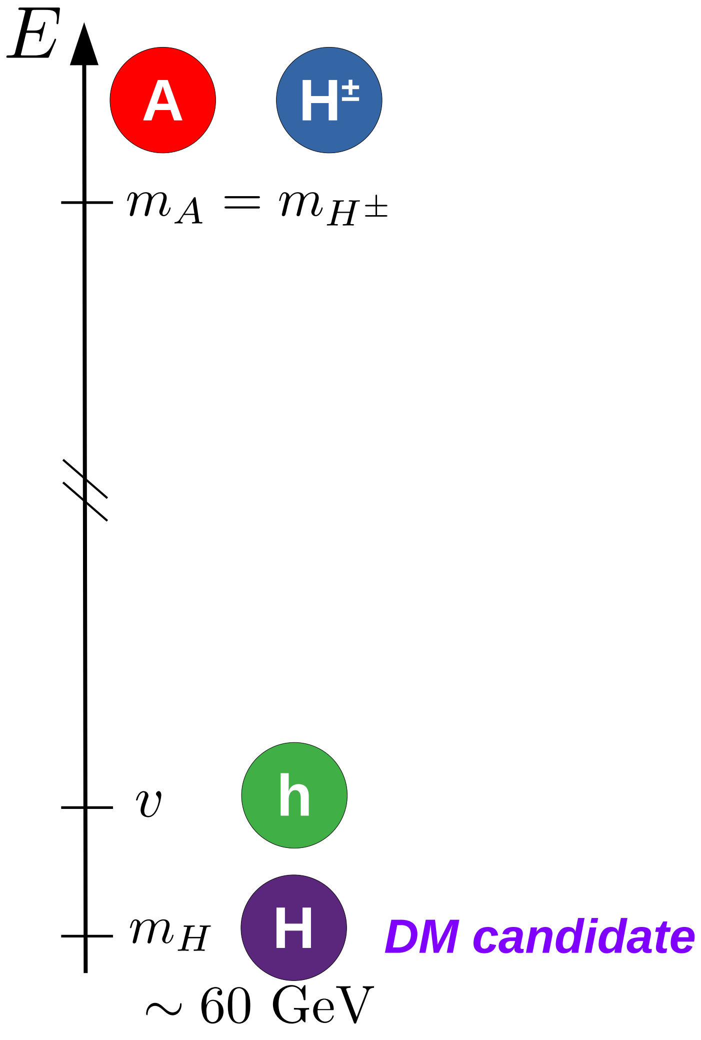


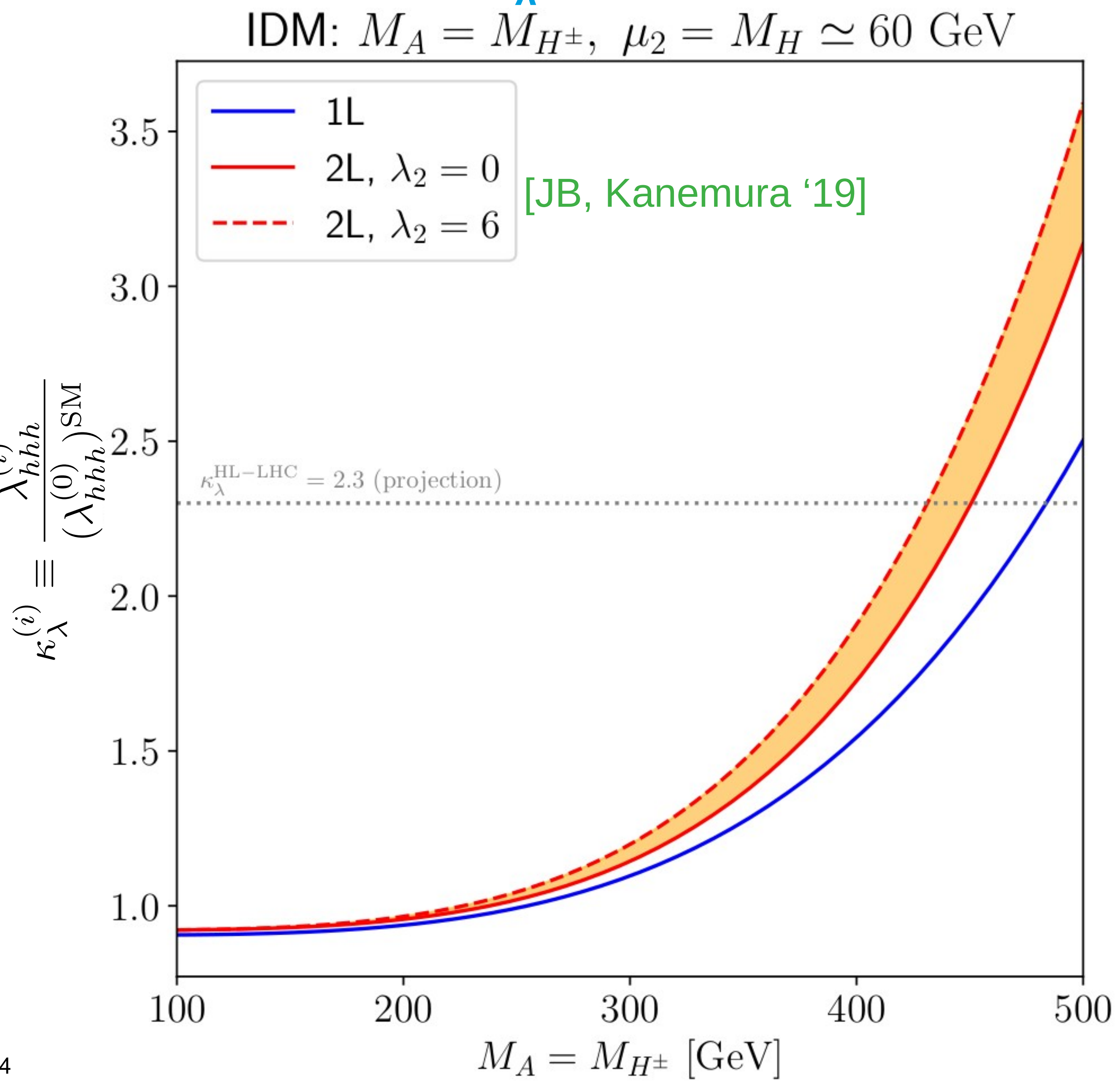


Plot made with `micrOMEGAs`
[Bélanger et al. '18]

Can we probe scalar dark matter with κ_λ ?

Inert Doublet Model in DM-inspired "Higgs resonance" scenario





[JB, Kanemura '19]

Higgs decay to two photons: existing one-loop results

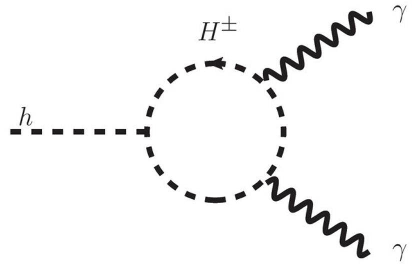
- DM scenarios of IDM investigated via Higgs properties at one loop (1L) in [Kanemura, Kikuchi, Sakurai '16]

- Additional charged inert Higgs → Higgs decay to 2 photons especially important!

$$\Gamma[h \to \gamma\gamma] \underset{m_h \ll m_{H^\pm}}{\simeq} \frac{\sqrt{2} G_F \alpha_{\text{EM}}^2 m_h^3}{64\pi^3} \left| -\frac{1}{6}\left(1 - \frac{\mu_2^2}{m_{H^\pm}^2}\right) + \sum_f Q_f^2 N_c^f I_f[m_h^2] + I_W[m_h^2] \right|^2$$

I_f, I_W: fermion/W-boson loops (SM-like)

- Charged Higgs contribution:
  Compensation between mass dependence of coupling ($\lambda_3=2(m_{H\pm}^2-\mu_2^2)/v^2$) and of loop function ($C_0\sim 1/m_{H\pm}^2$)
  → does not decouple!



- h→γγ is a loop-induced decay, i.e. ***1L is only leading order (LO)***
  → **What happens at 2L (NLO) ?**

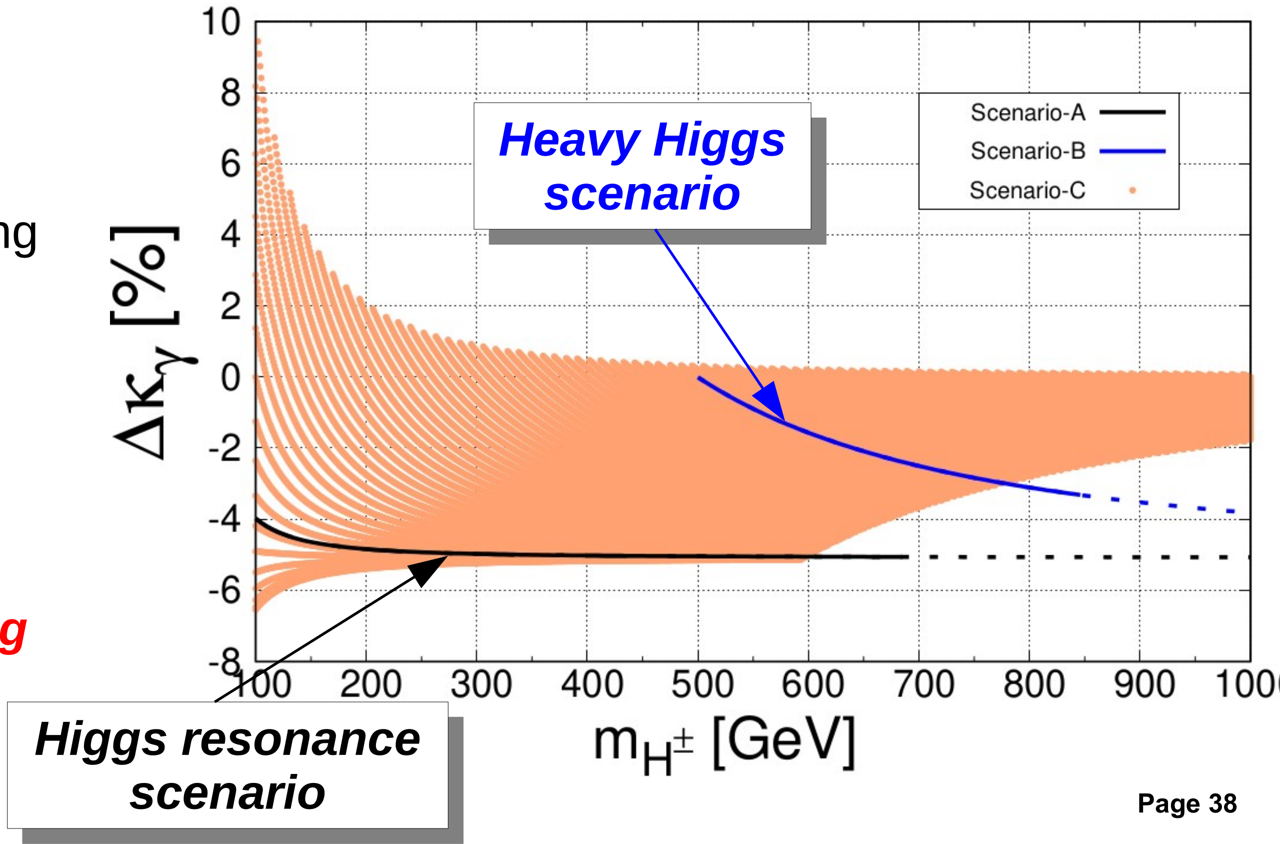


Fig. from [Kanemura, Kikuchi, Sakurai '16]

Leading two-loop corrections to $\Gamma(h\to\gamma\gamma)$

Calculation of leading two-loop effects from diagrams with inert BSM scalars, using Higgs Low-Energy Theorem *(see e.g.* [Kniehl, Spira ‘95]; *details in backup)*

Constraints for numerical scans

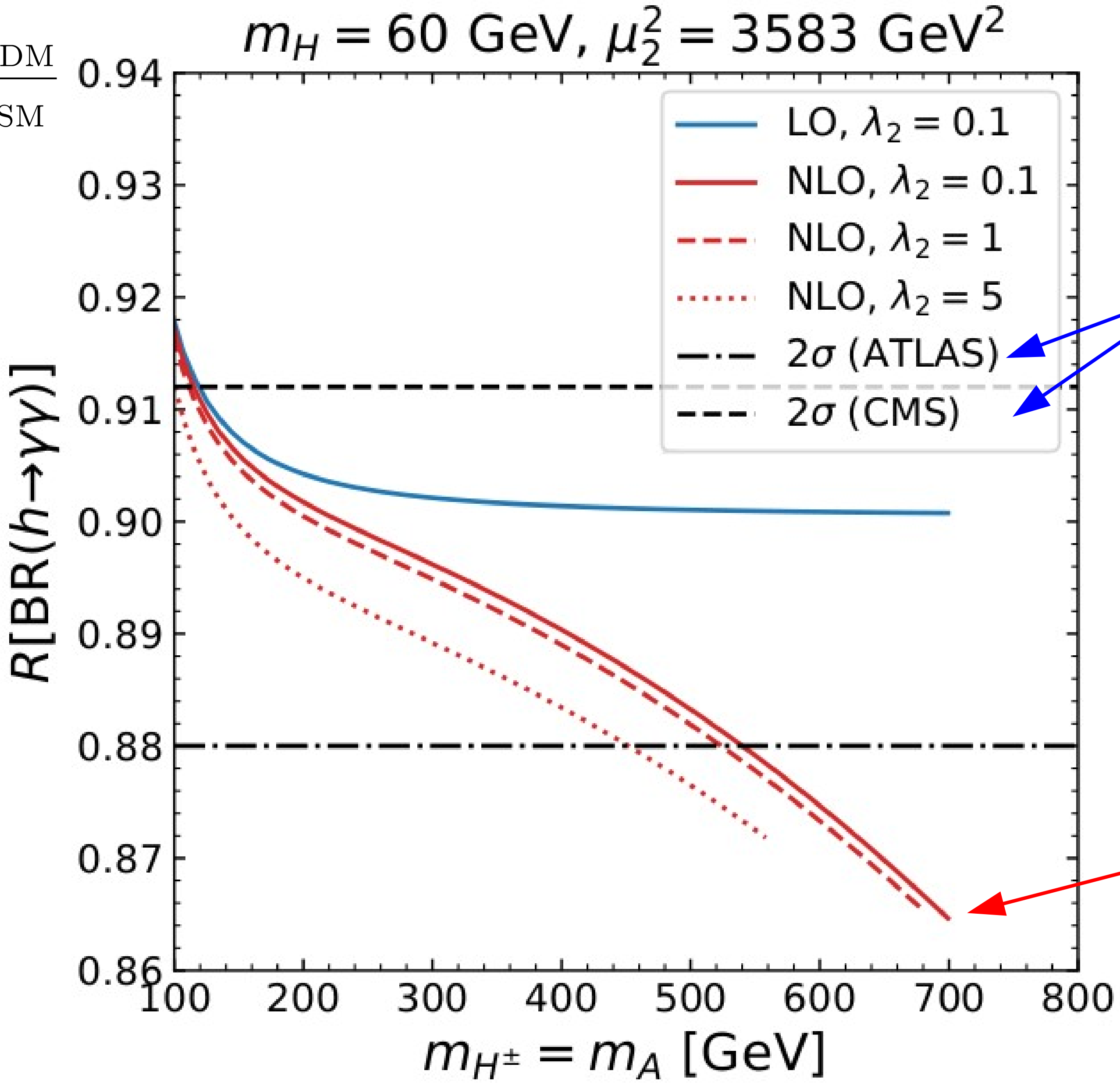
- perturbative unitarity
- vacuum stability
- inert vacuum condition
- μ_2 fixed in order to reproduce correct relic abundance (using `micrOMEGAs`)
- electroweak precision observables
- direct searches for inert scalars

Results for the Higgs resonance scenario

[Aiko, JB, Kanemura ‘23]

$$R[\mathrm{BR}(h \to \gamma\gamma)] \equiv \frac{\mathrm{BR}(h \to \gamma\gamma)_{\mathrm{IDM}}}{\mathrm{BR}(h \to \gamma\gamma)_{\mathrm{SM}}}$$

BRs computed with `H-COUP v2`
[Kanemura et al. ‘19]



[λ_2 : inert doublet self-coupling]

Expected 2σ bounds at HL-LHC

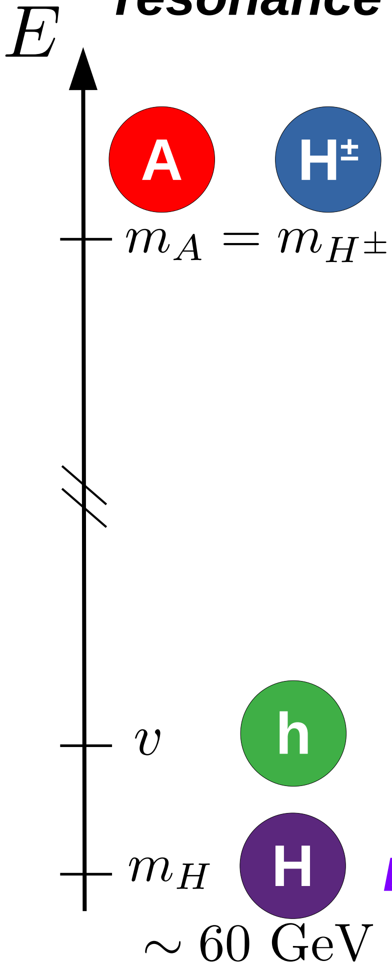
Range of $m_{H\pm}=m_A$ limited by pert. unit.

Correlation between κ_λ and BR($h \to \gamma\gamma$) at one and two loops

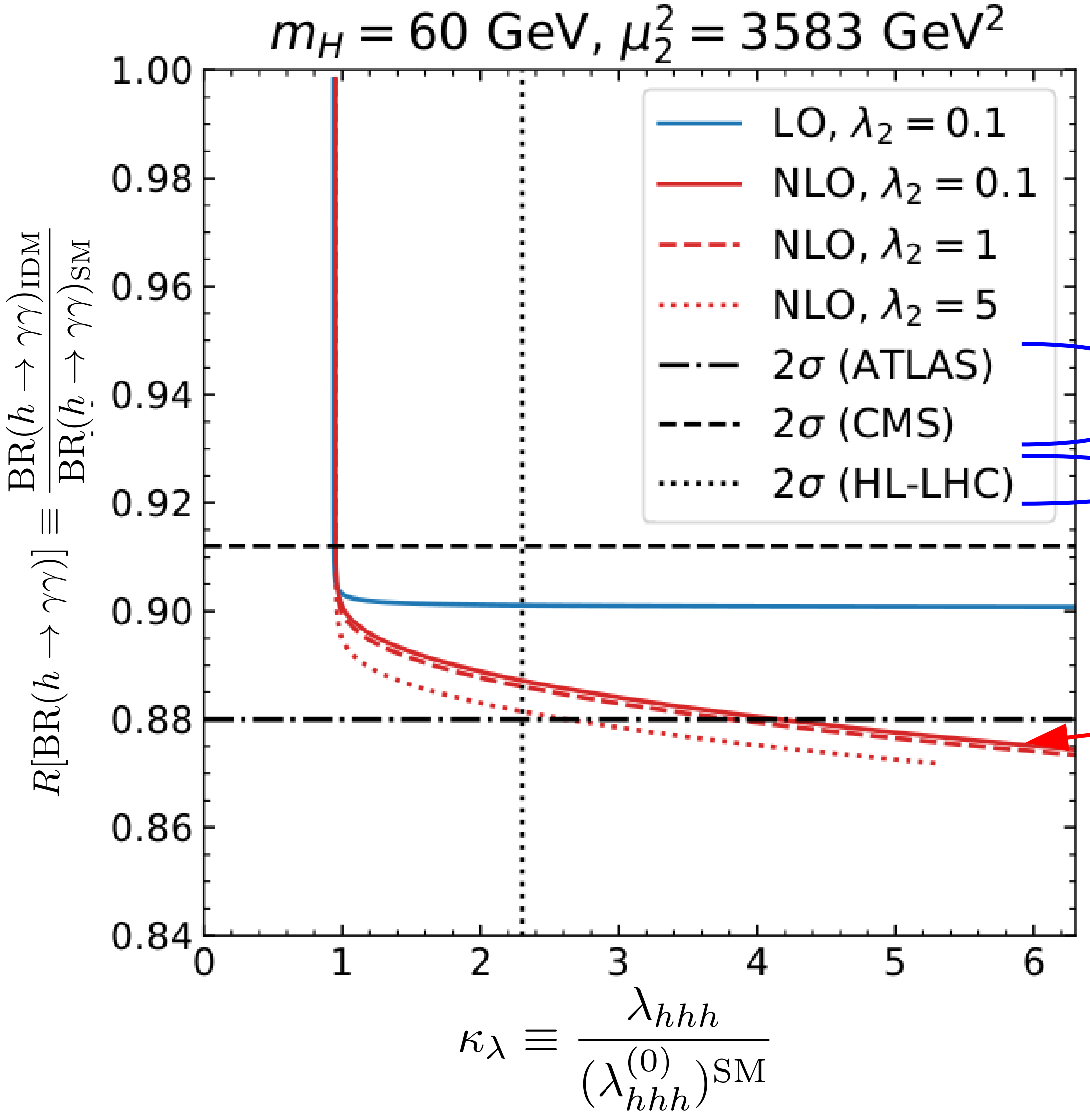
Could BSM Physics be found first in κ_λ ?

[Aiko, JB, Kanemura '23 + WIP] + [JB, Kanemura '19]

Inert Doublet Model in DM-inspired "Higgs resonance" scenario



DM candidate



[λ_2 : inert doublet self-coupling]

Expected bounds on R[BR($h \to \gamma\gamma$)] at HL-LHC

Expected bound on κ_λ at HL-LHC

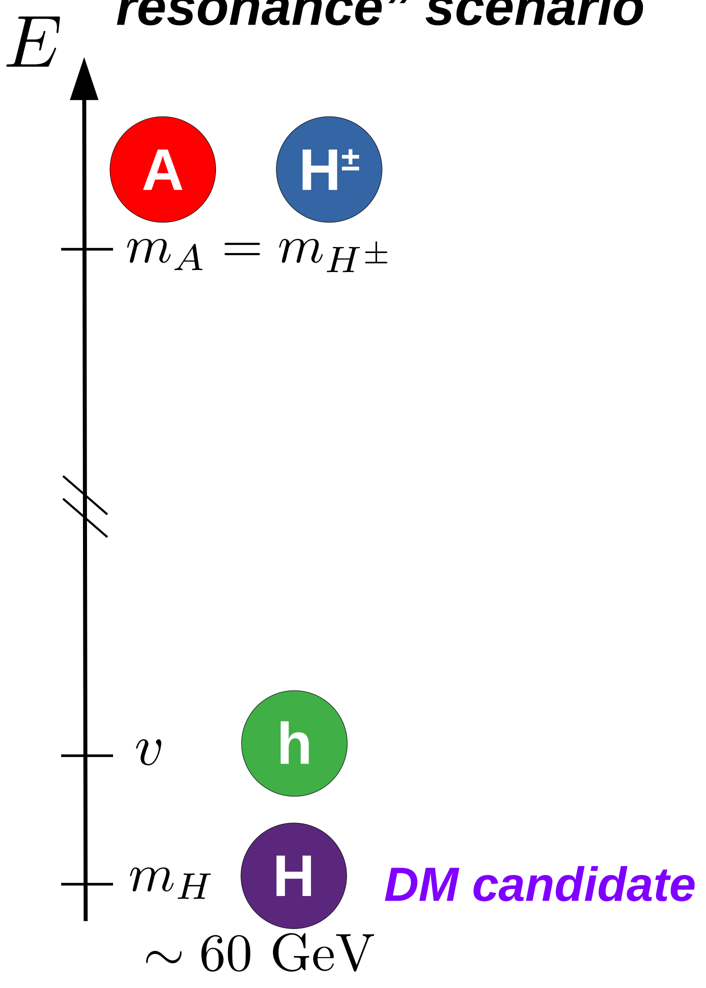
$m_{H\pm} = m_A$ varied along the curves (until limit from pert. unit.)

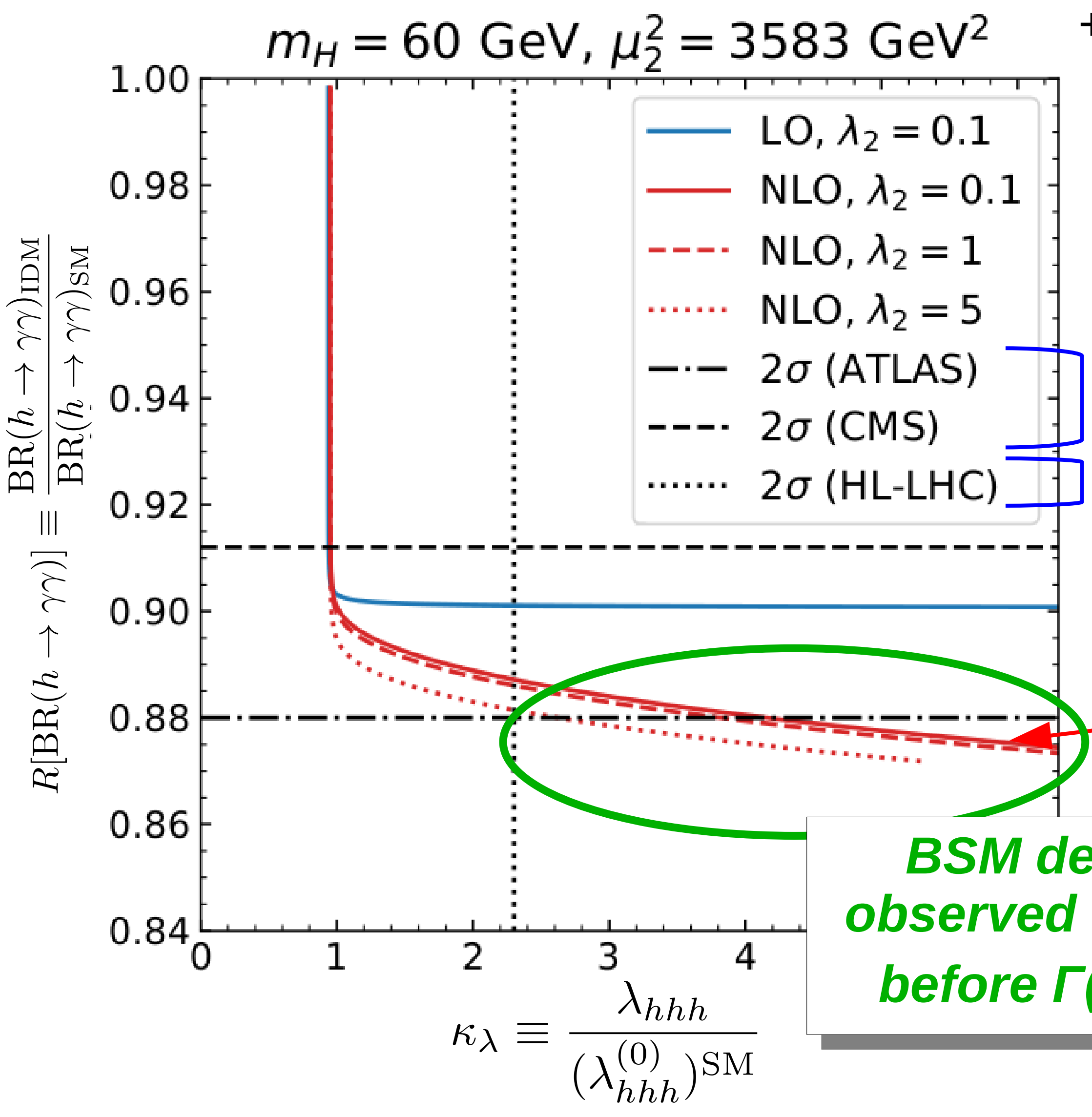
Correlation between κ_λ and BR($h\to\gamma\gamma$) at one and two loops

Could BSM Physics be found first in κ_λ ?

[Aiko, JB, Kanemura '23 + WIP] + [JB, Kanemura '19]

Inert Doublet Model in DM-inspired "Higgs resonance" scenario





[λ_2 : inert doublet self-coupling]

Expected bounds on R[BR($h\to\gamma\gamma$)] at HL-LHC

Expected bound on κ_λ at HL-LHC

$m_{H\pm} = m_A$ varied along the curves (until limit from pert. unit.)

BSM deviation observed first in κ_λ, before $\Gamma(h\to\gamma\gamma)$!

Summary of Part 1

- λ_{hhh} plays a crucial role to understand the shape of the Higgs potential, and probe indirectly signs of New Physics
- λ_{hhh} can **deviate significantly from SM** prediction (by up to a **factor ~10**), for otherwise theoretically and experimentally **allowed points**, due to mass-splitting effects in radiative corrections involving BSM scalars
- Current experimental bounds on λ_{hhh} can **already exclude significant parts of otherwise unconstrained BSM parameter space**, and future prospects even better!
- Other Higgs couplings, like $\Gamma(h\to\gamma\gamma)$, offer important additional information; where BSM would be seen first can depend on scenario
- Here, 2HDM and IDM taken as *examples*, but similar results are expected for a wider range of BSM models with extended scalar sectors
  → motivates **automating calculations of λ_{hhh}** → **Part 2**

Generic predictions for λ_{hhh}

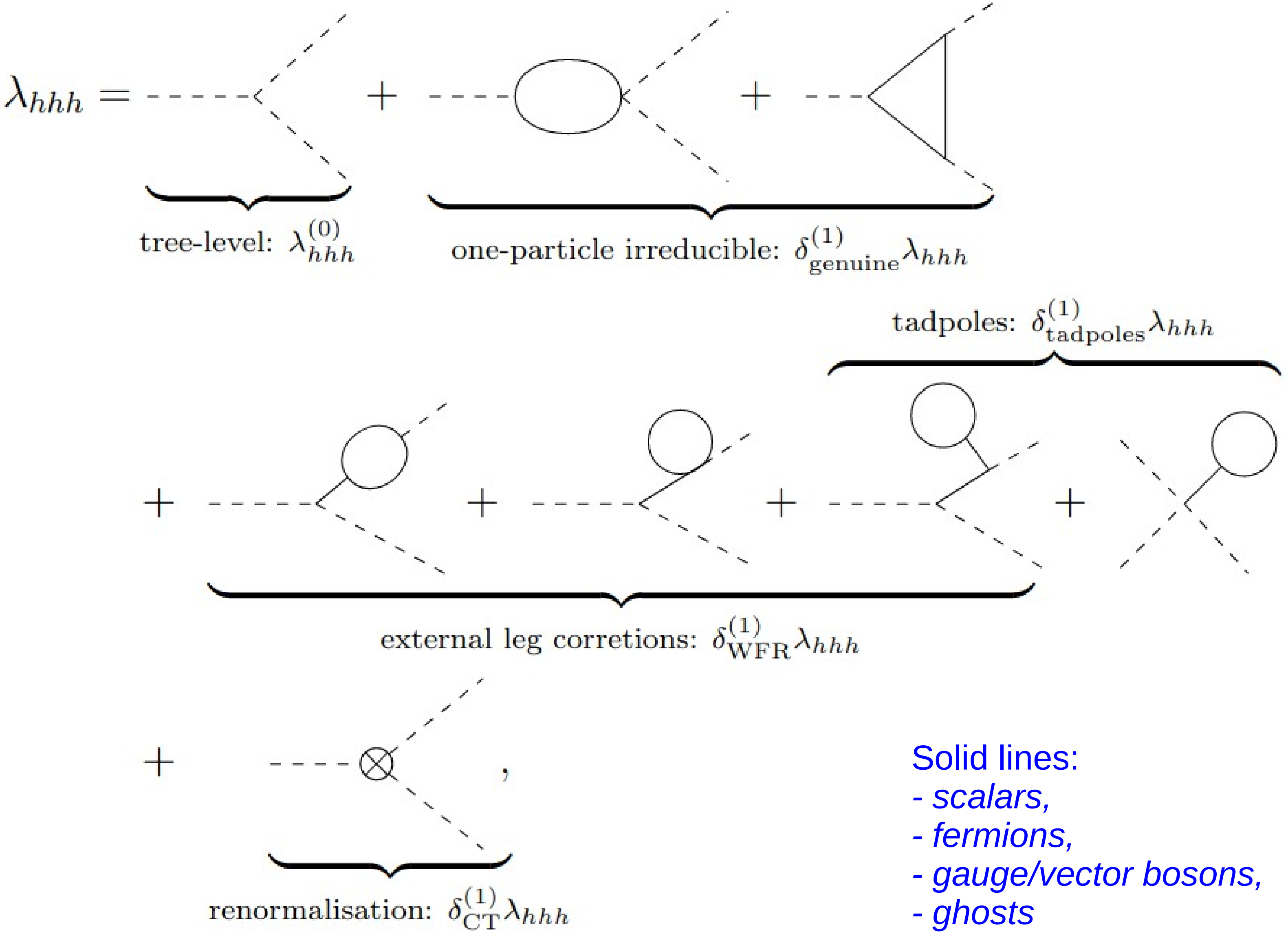
Based on

`arXiv:2305.03015` (EPJC) + WIP

in collaboration with Henning Bahl, Martin Gabelmann, Kateryna Radchenko Serdula and Georg Weiglein

Full one-loop calculation of λ_{hhh} with `anyH3`: how does it work?

- Generic results applied to concrete (B)SM model, using inputs in `UFO` format [Degrande et al., '11], [Darmé et al. '23]
- Loop functions evaluated via `COLLIER` [Denner et al '16] interface, `pyCollier`
- Restrictions on **particles** and/or **topologies** possible
- **Renormalisation performed automatically** (*more in backup*)



Solid lines:
- *scalars,*
- *fermions,*
- *gauge/vector bosons,*
- *ghosts*

New results I: mass-splitting effects in various BSM models

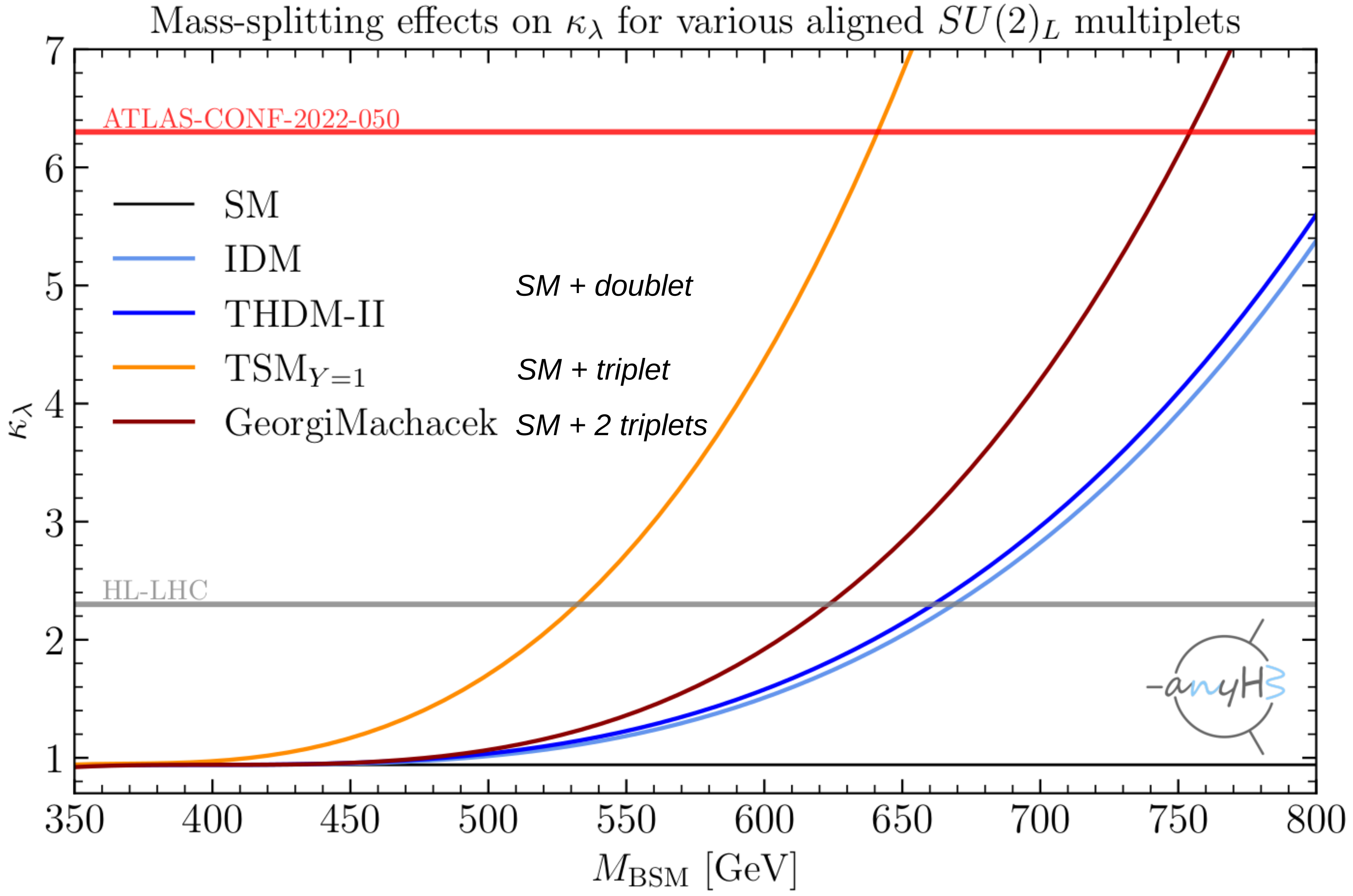
- Consider the non-decoupling limit in several BSM models

$$M^2_{\text{BSM}} = \mathcal{M}^2 + \tilde{\lambda} v^2$$

- Increase M_{BSM}, keeping $\mathcal{M}$ fixed
  → large mass splittings
  → **large BSM effects!**

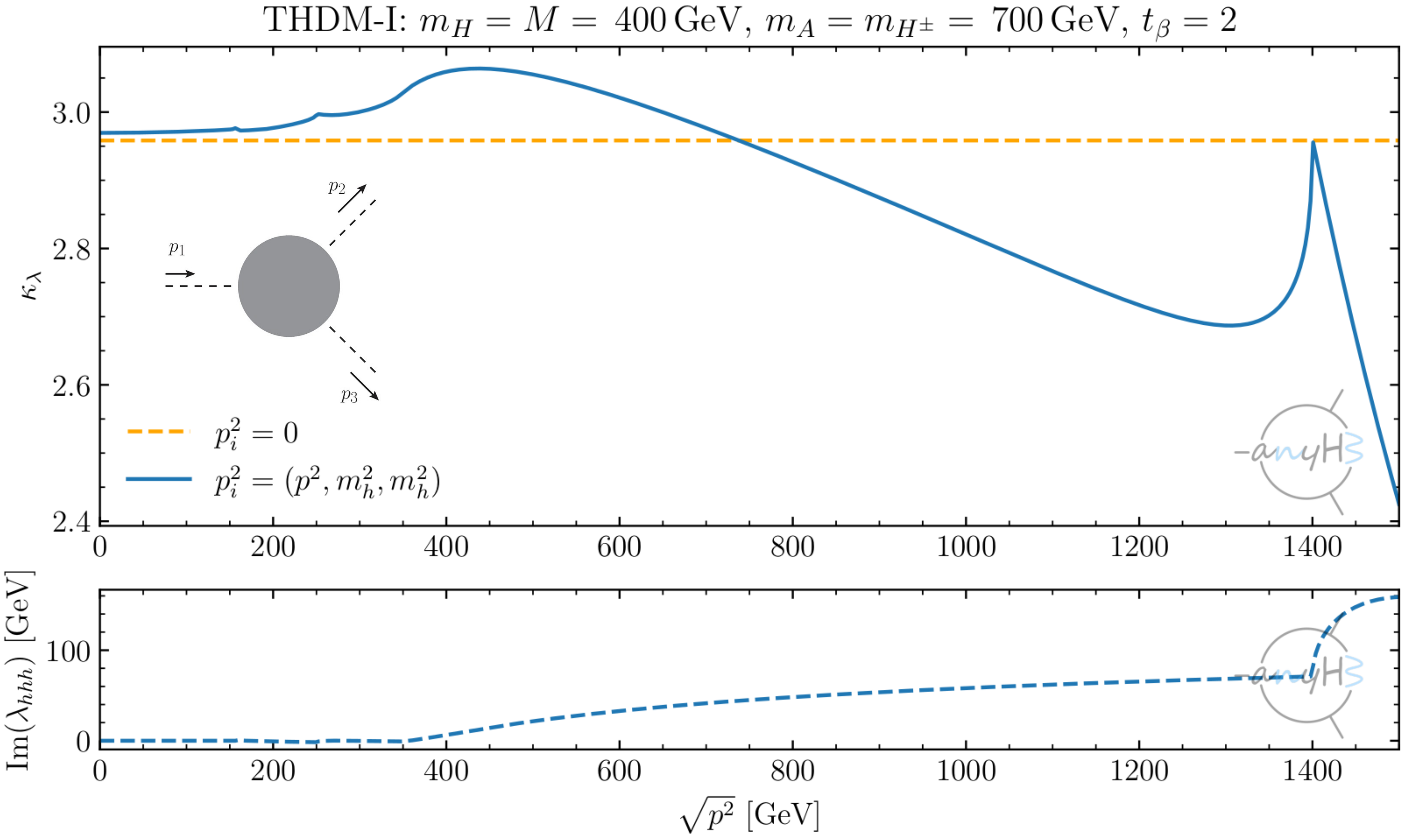
- Perturbative unitarity checked with `anyPerturbativeUnitarity`

- **Constraints on BSM parameter space!**

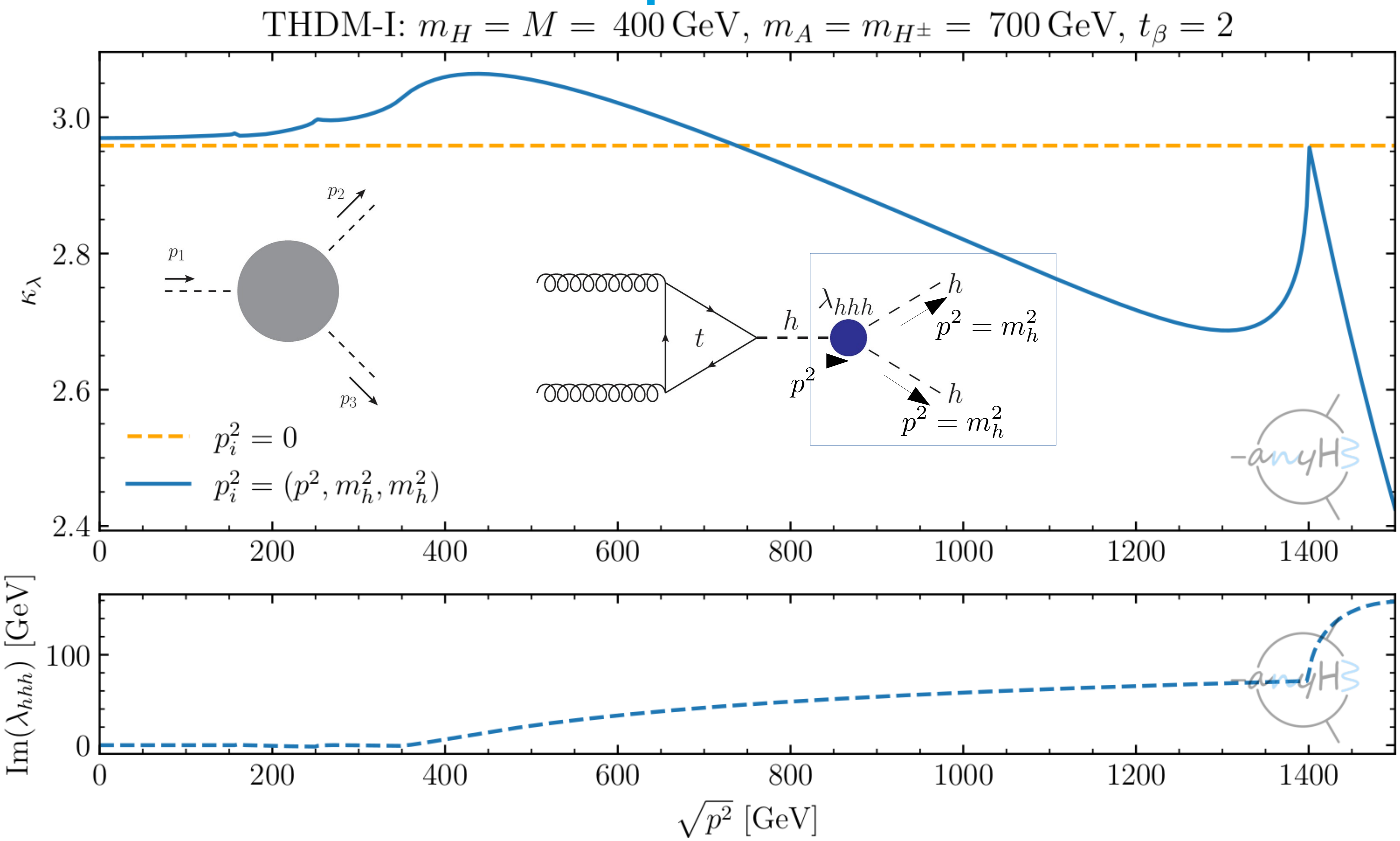


Here: scenarios with lightest BSM scalar mass & BSM mass param. at 400 GeV; other BSM scalar masses = M_{BSM}

New results II: momentum dependence in the 2HDM



New results II: momentum dependence in the 2HDM



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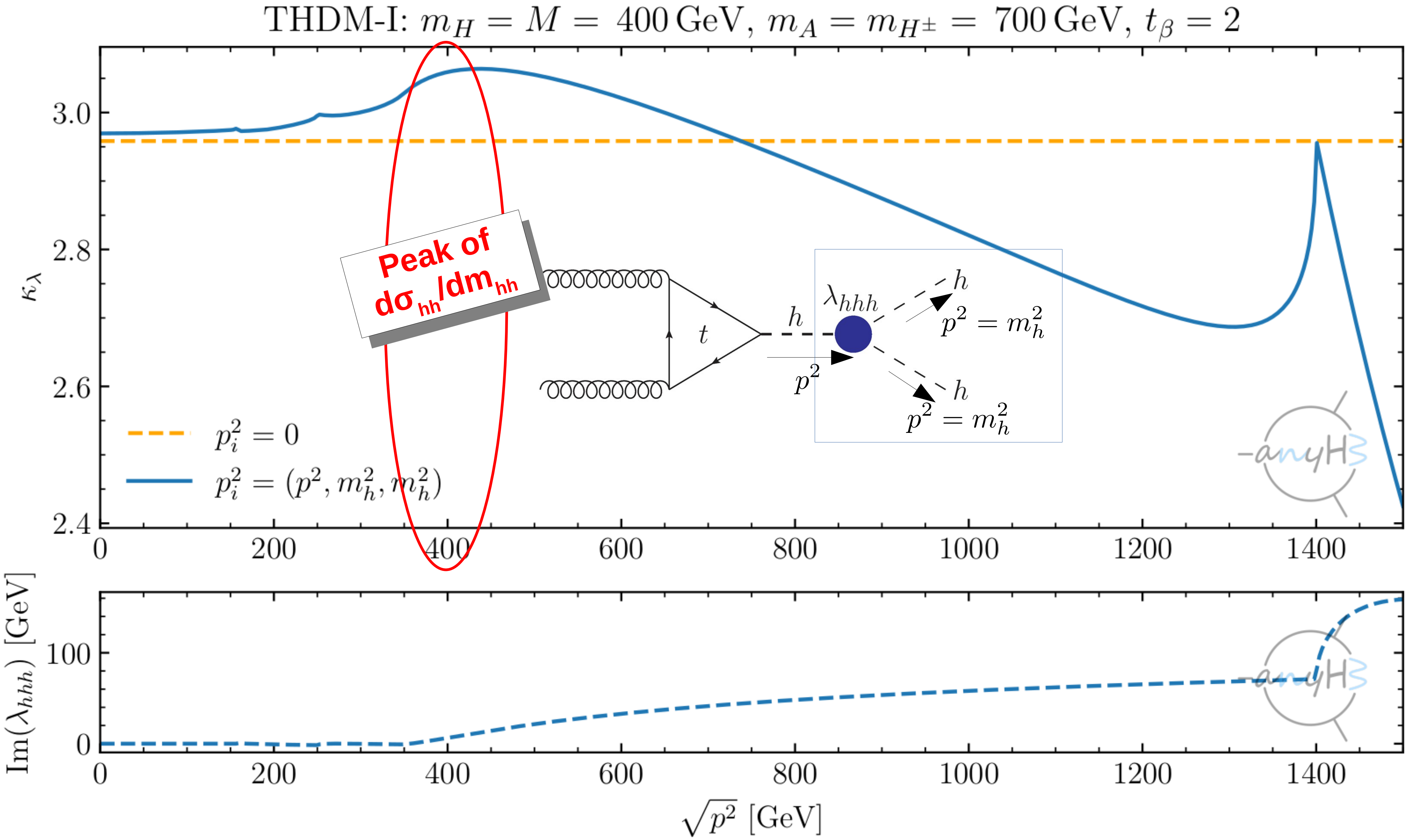
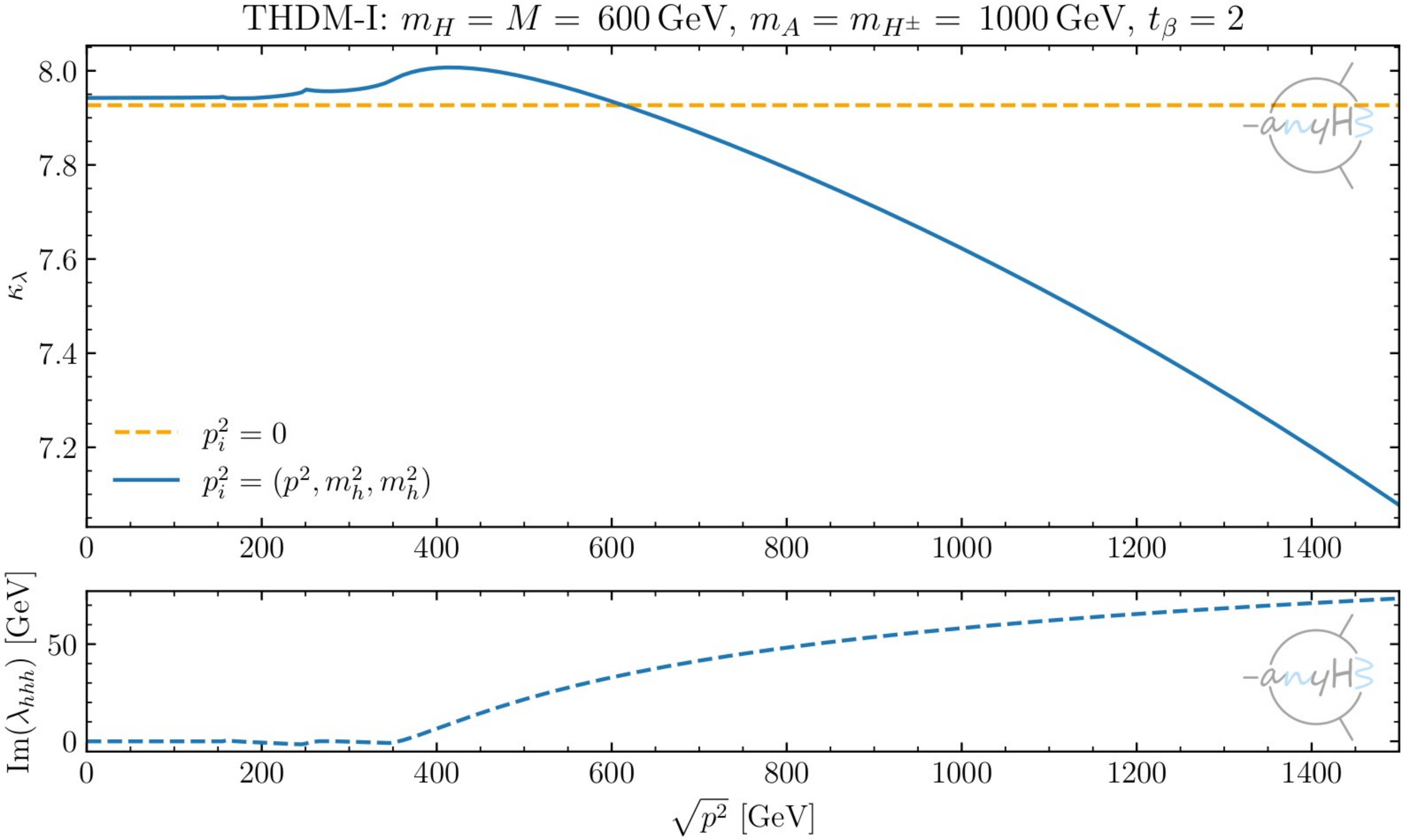
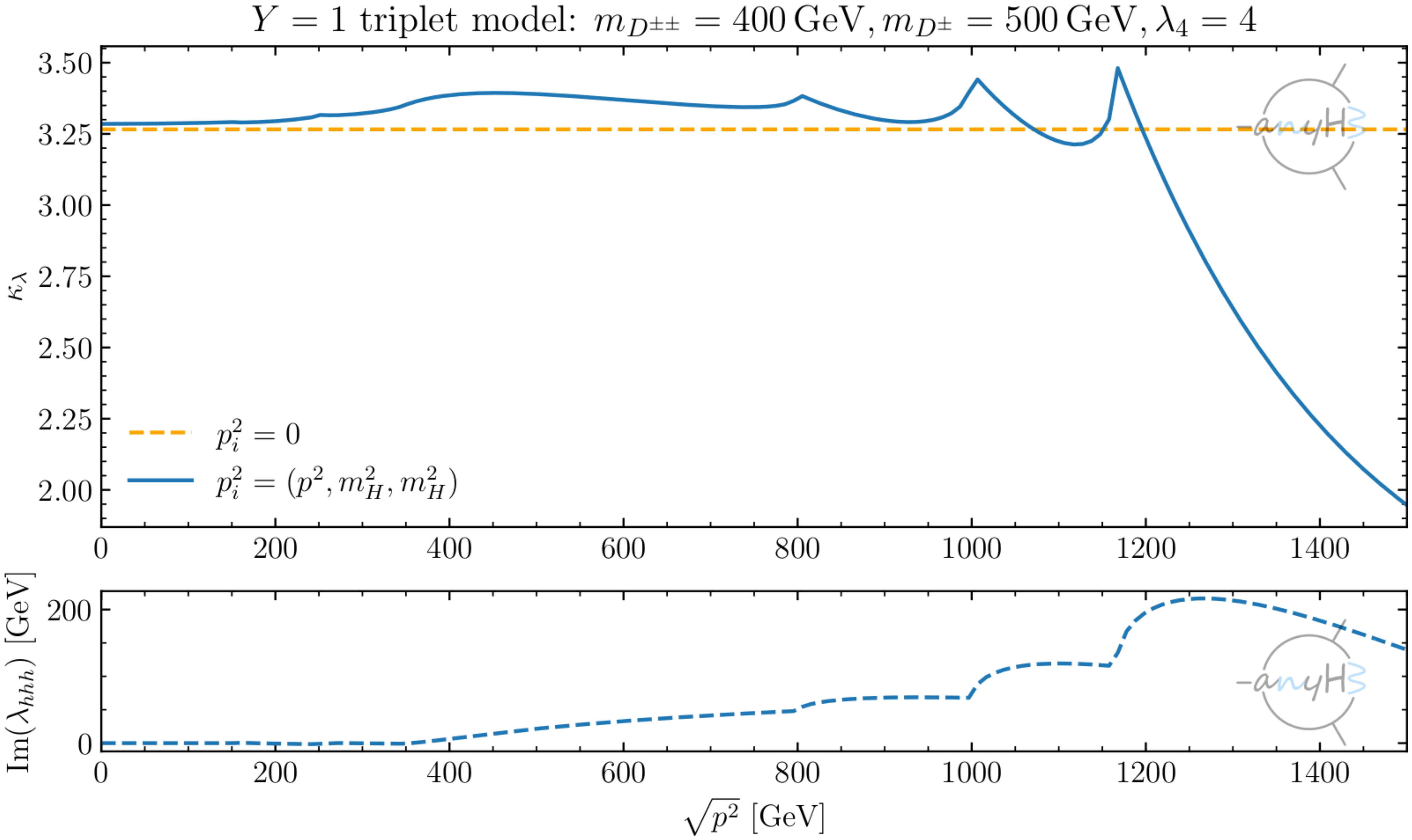


New results II’: momentum dependence in the 2HDM



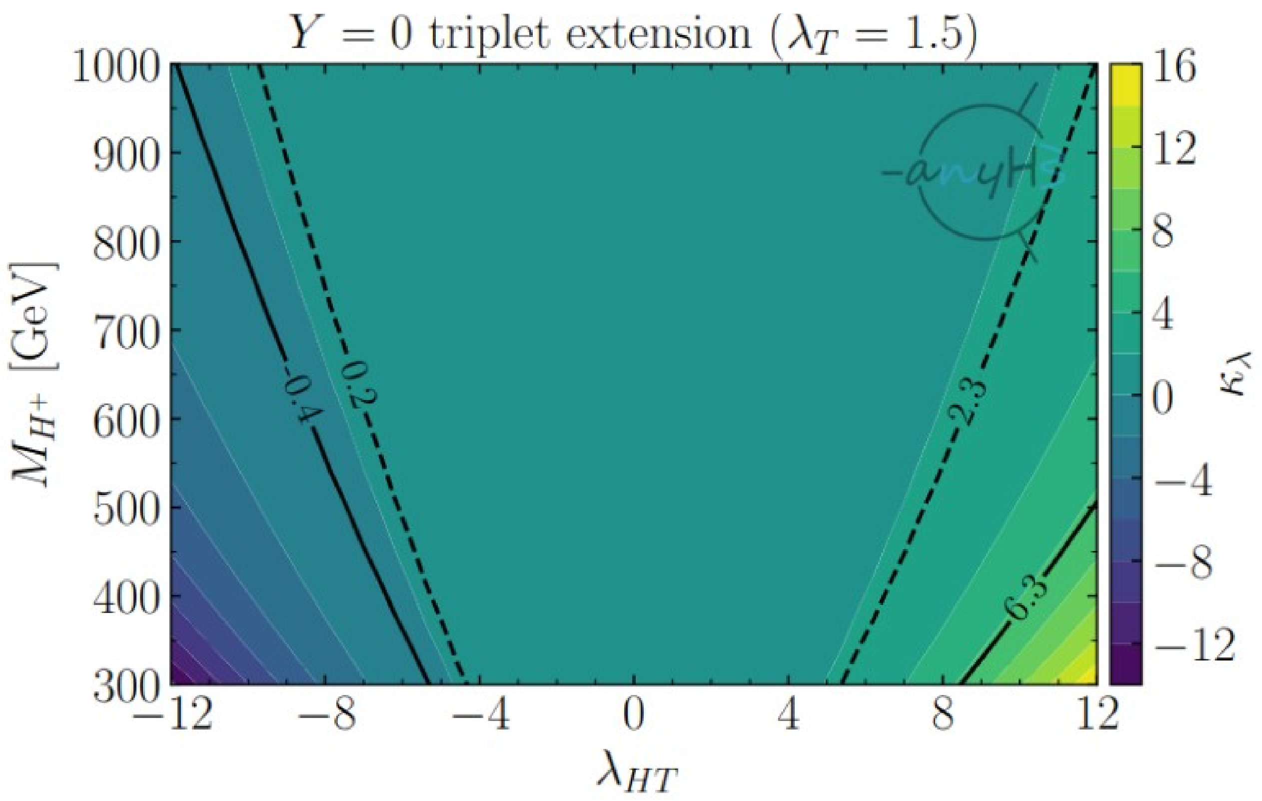
New results III: momentum dependence in a Y=1 triplet extension

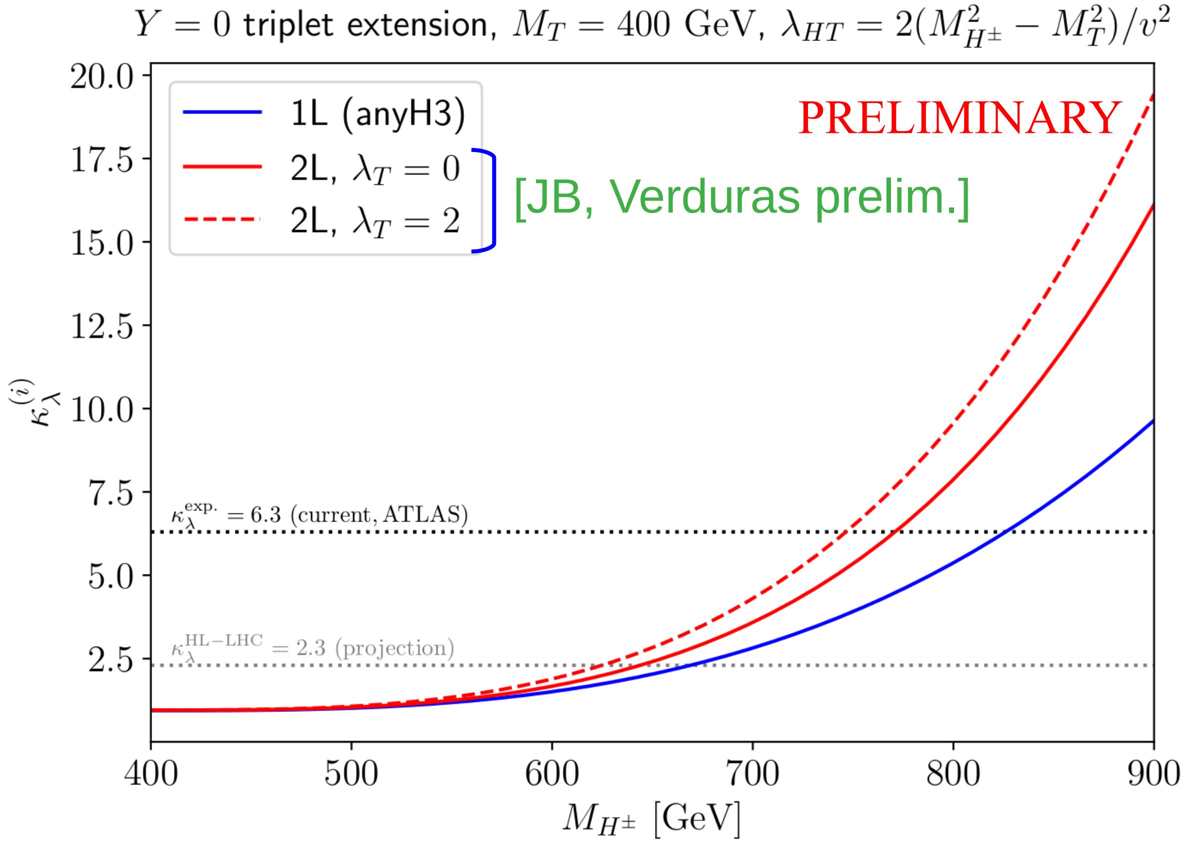


New results IV: probing scalar DM models with κ_λ

Real VEV-less triplet model:

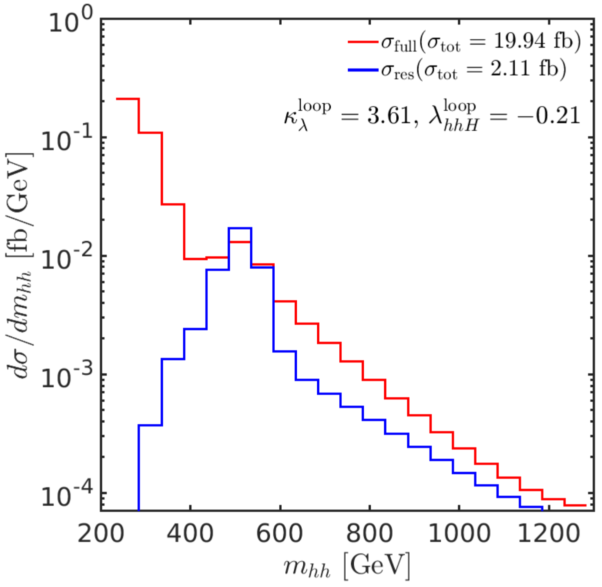
$$V(\Phi, T) = \mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4 + \frac{M_T^2}{2}|T|^2 + \frac{\lambda_T}{2}|T|^4 + \frac{\lambda_{HT}}{2}|T|^2|\Phi|^2,\ \ \langle T\rangle = 0,\ \langle\Phi\rangle = v_{SM}$$





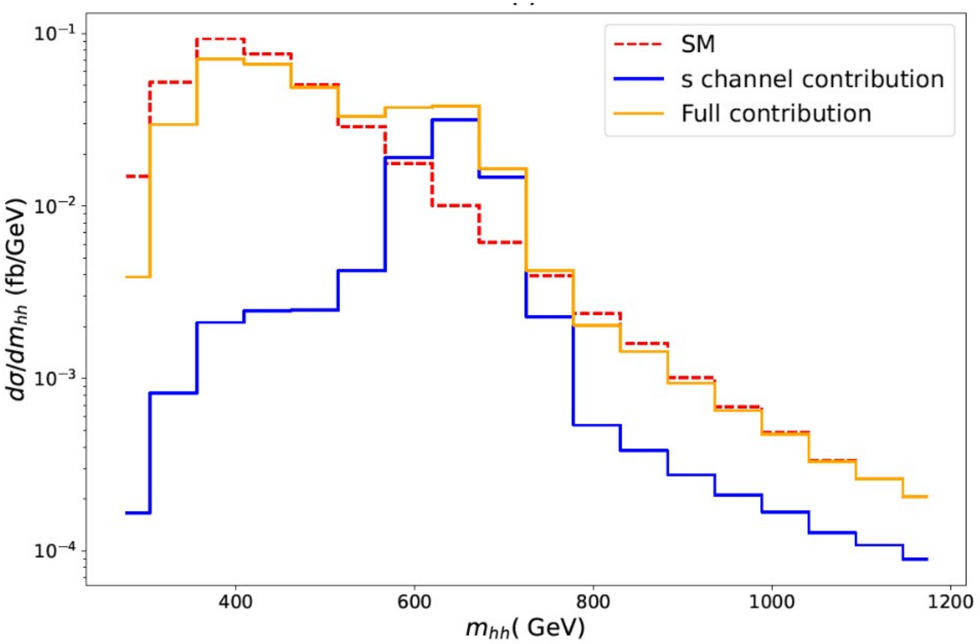
- *Left*: κ_λ @ 1L in plane of $M_{H\pm}$ and λ_{HT} (portal coupling) with anyH3
- *Right*: κ_λ @ 2L, with results from [JB, Verduras *WIP*]

Ongoing developments in anyBSM

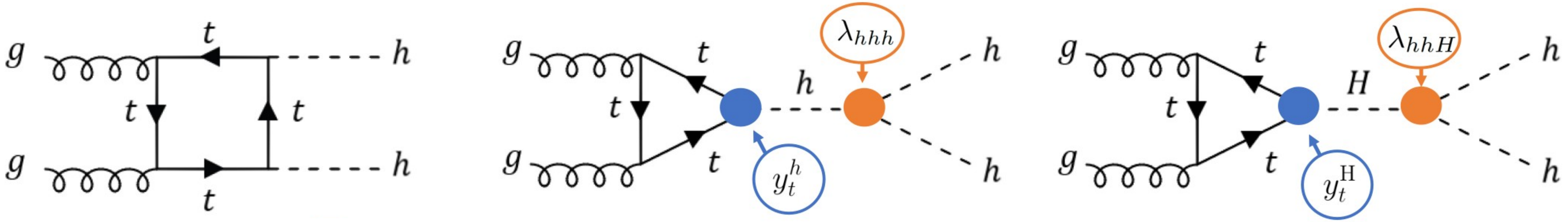


Left: 2HDM
[Heinemeyer, Mühlleitner, Radchenko Serdula, Weiglein '24]
plot from talk of K. Radchenko Serdula at 20th LHC Higgs WG workshop

Right: singlet extension
[Arco, Heinemeyer, Mühlleitner, Rivero, Verduras *WIP*]

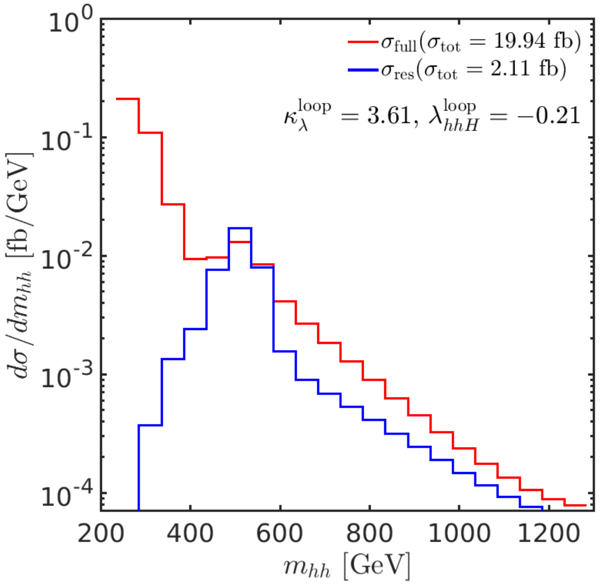


Example leading-order contributions:



[Figure by A. Verduras]

Ongoing developments in anyBSM



Left: 2HDM
[Heinemeyer, Mühlleitner, Radchenko Serdula, Weiglein '24]
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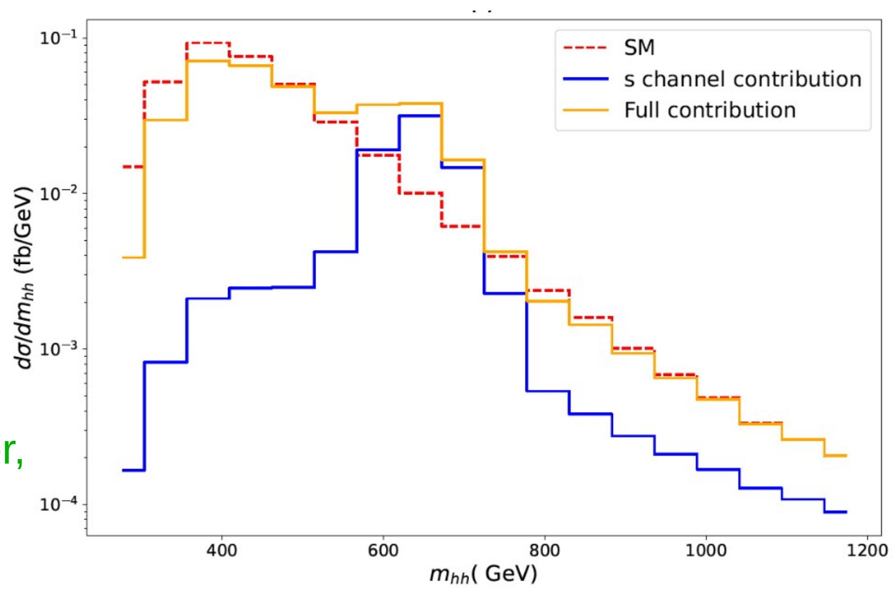
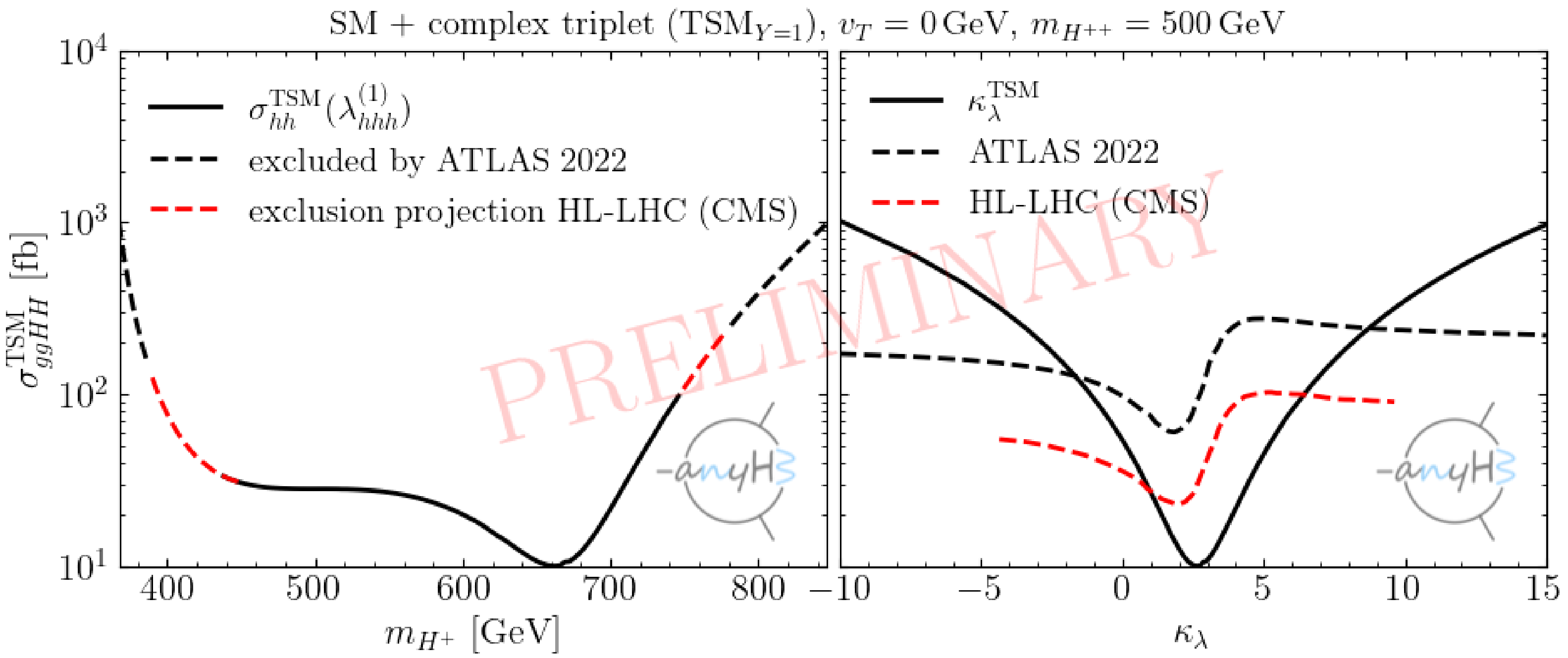


Right: singlet extension
[Arco, Heinemeyer, Mühlleitner, Rivero, Verduras *WIP*]

Having predictions for di-Higgs production, including **all (i.e. resonant + non-resonant) contributions + 1L corrections to trilinear scalar couplings** in **arbitrary models** would be highly desirable

→ new module in anyBSM

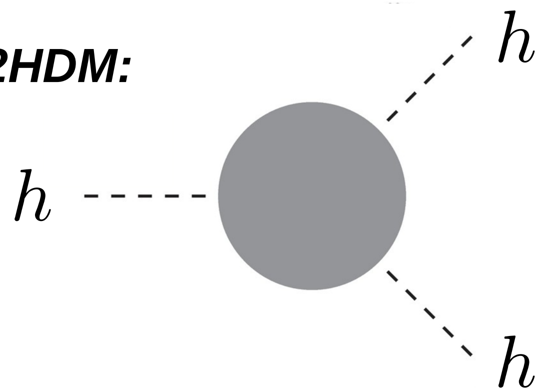
[Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein *WIP*]

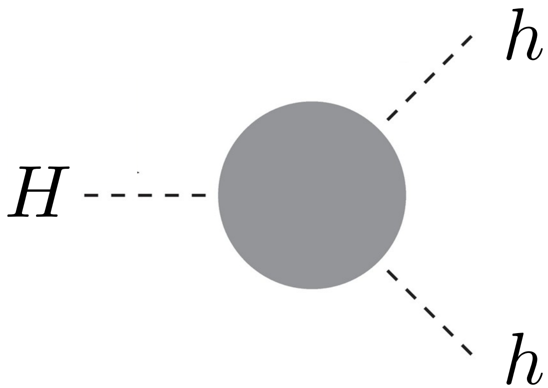


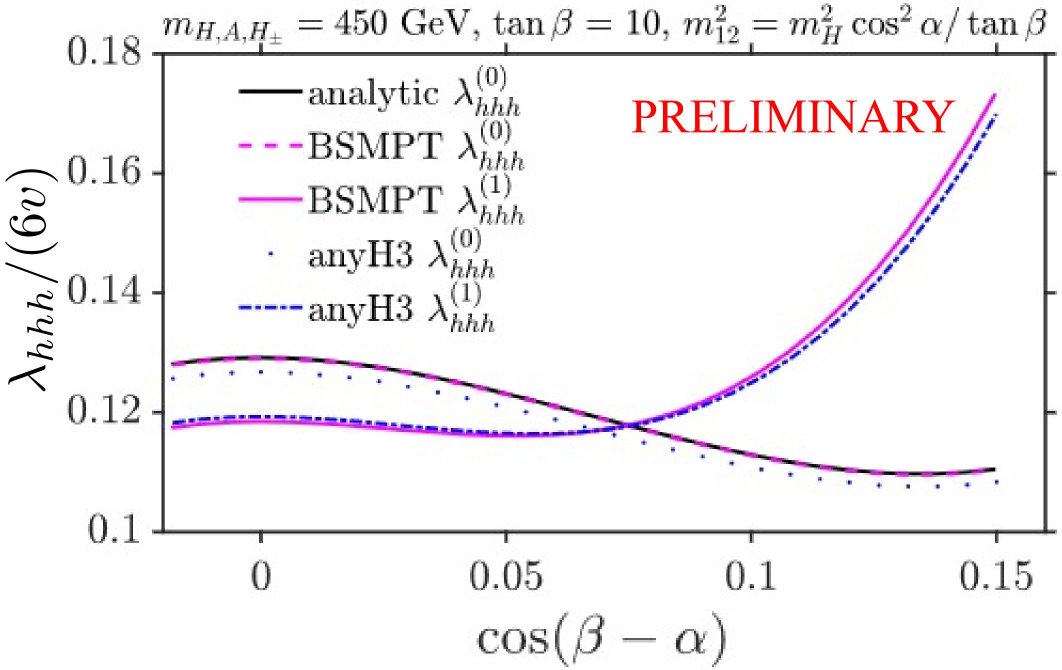
Ongoing developments: `anyLamijk`

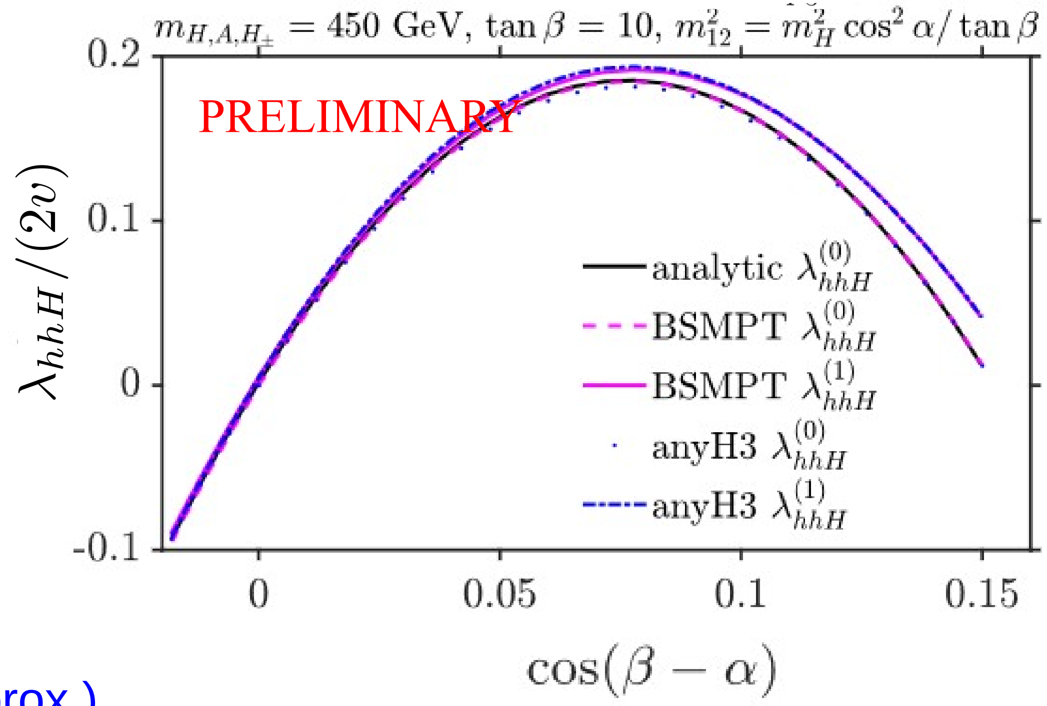
[Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein *WIP*]

Example in a 2HDM:









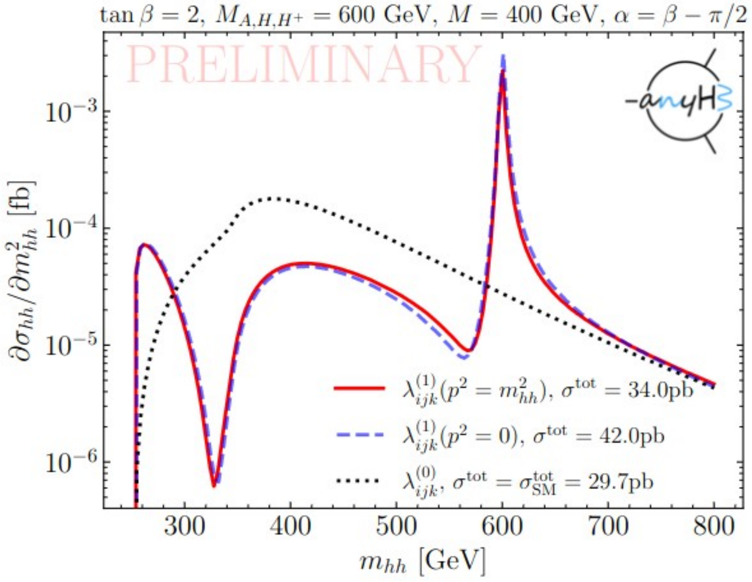
→ excellent agreement with BSMPT results (in eff. pot. approx.)

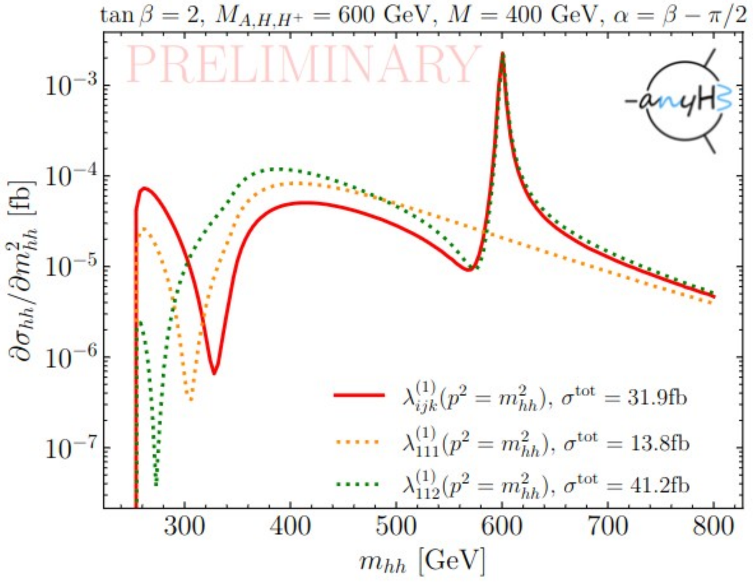
→ full OS schemes for λ_{hhh} and λ_{hhH} couplings worked out in 2HDM [Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein], SSM [JB, Heinemeyer, Verduras], and more [Bosse, JB, Gabelmann, Hannig, Weiglein]!

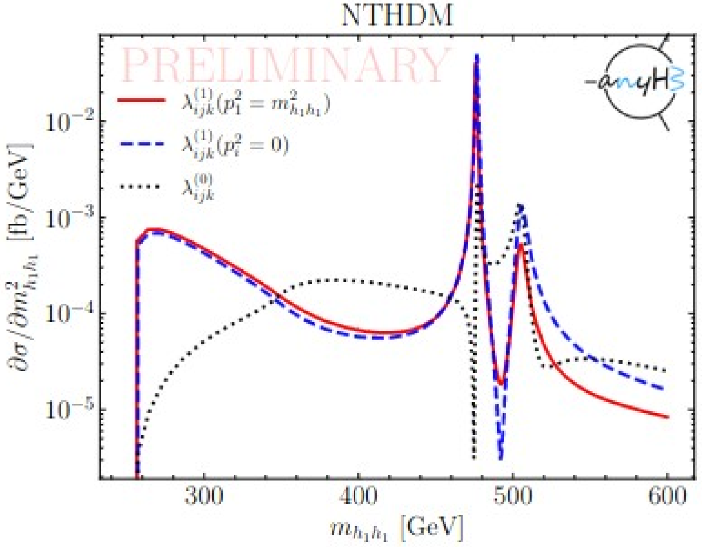
Ongoing developments: `anyHH`

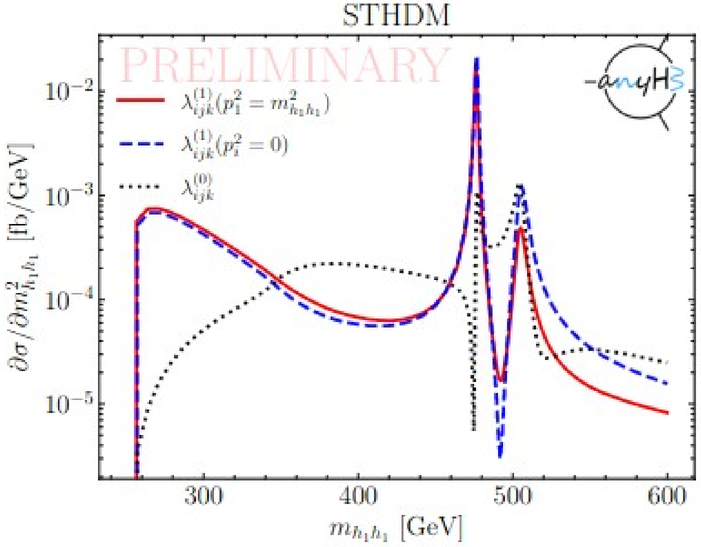
[Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein *WIP*]

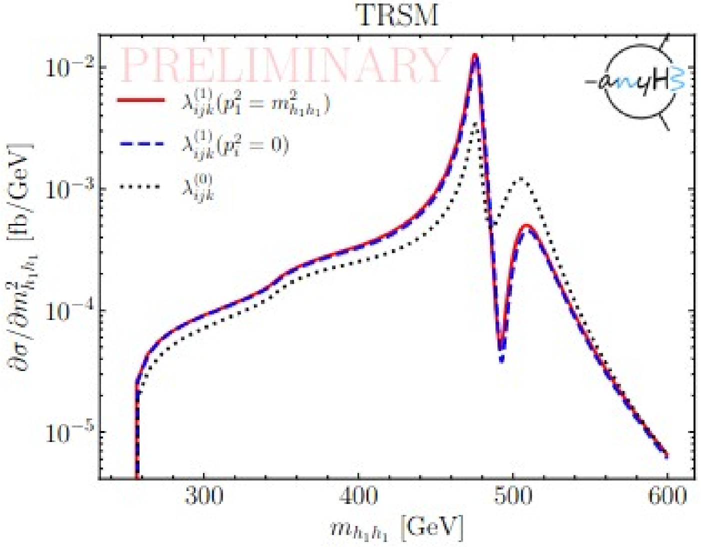
- **Total and differential cross-sections** for gg → hh including **1L corrections to λ_{ijk}** and **BSM contributions in s-channel**
- Good agreement with existing results (e.g. HPair)
- Results available in ***various new models*** for the 1st time!





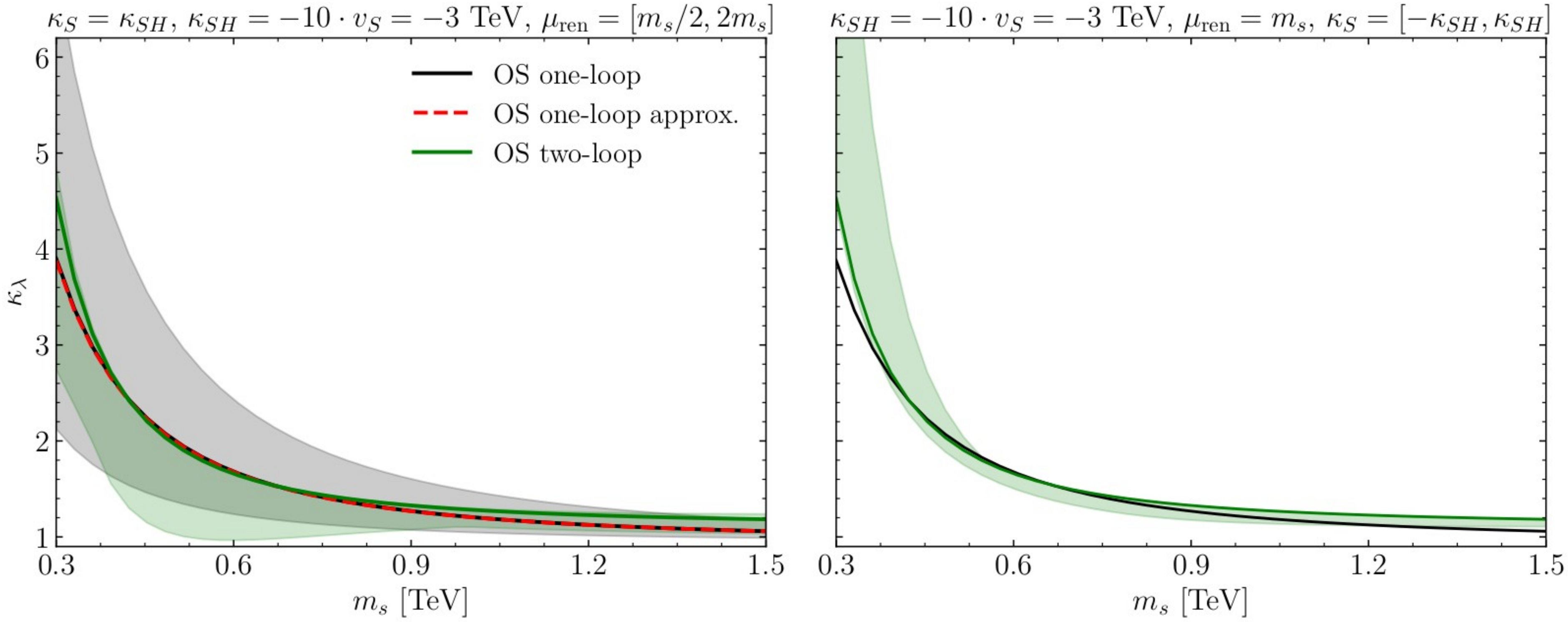






Automated calculations of λ_{hhh} beyond one loop

- All 2L contributions to Higgs/scalar self-energies computed in [Goodsell, Paßehr '19]
  → in [Bahl, JB, Gabelmann, Paßehr to appear], we generalise this to λ_{hhh} (and λ_{hhhh})
- Diagrams generated with FeynArts, computed with TwoCalc and OneCalc
- Results then mapped to specific models via private routines (via FeynArts model file)
- Example result for real-singlet extension of SM:



Summary for anyH3 / anyBSM

- **Python package anyH3 allows calculation of λ_{hhh} for arbitrary renormalisable theories** with
  - Full 1L effects including p^2 dependence
  - Highly flexible choices of renormalisation schemes → predefined or by user
- Uses **UFO** model inputs (generated with SARAH, FeynRules or using custom ones)
- Analytical results (Python, Mathematica); fast numerical results (with caching): SM → O(0.2s); MSSM → O(0.5s); handles inputs for numerical evaluation in SLHA format (example in backup)
- Currently 14 models included, easy inclusion of further models → **suggestions welcome!**
- Part of wider **anyBSM framework**, under development

  - extensions to general trilinear scalar couplings λ_{ijk} (later to other Higgs couplings)
  - complete treatment of di-Higgs production at hadron colliders
  - generic two-loop predictions for trilinear couplings

  etc.

Thank you very much for your attention!

Contact

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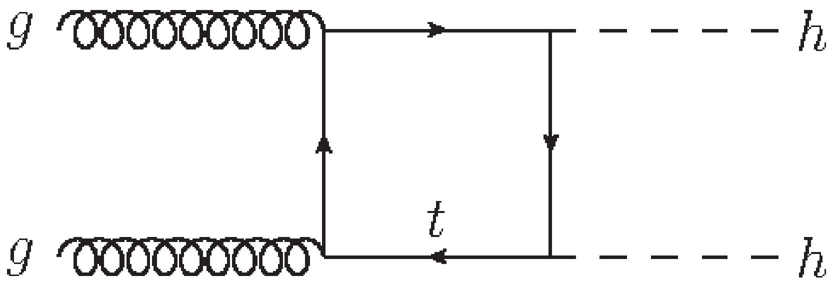
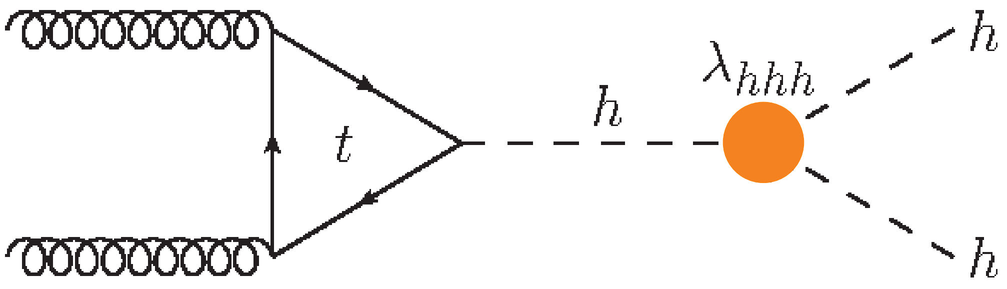
Backup

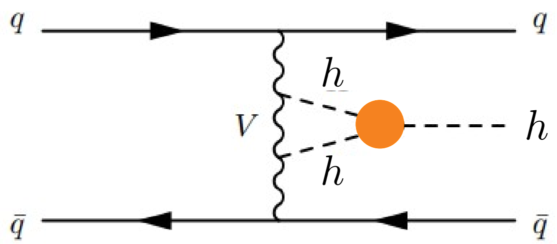
Experimental probes of λ_{hhh}

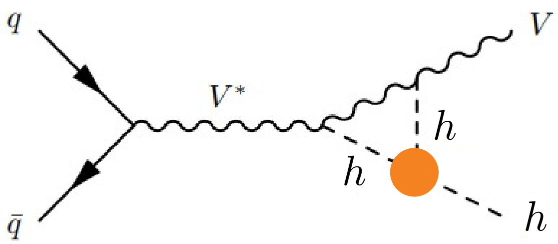
- **Double-Higgs production** → λ_{hhh} enters at leading order (LO) → **most direct probe!**

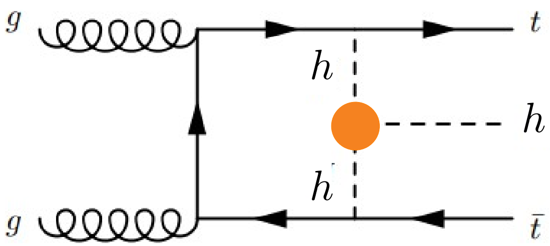


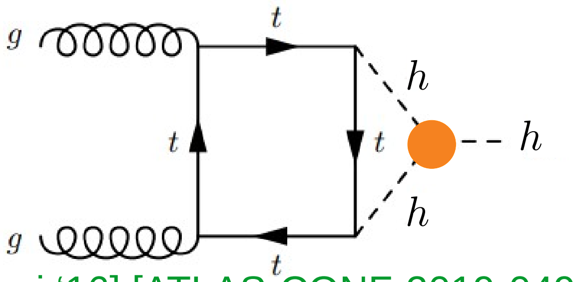


- **Single-Higgs production** → λ_{hhh} enters at NLO



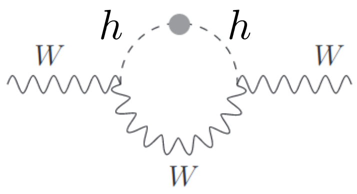


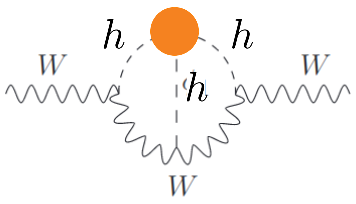


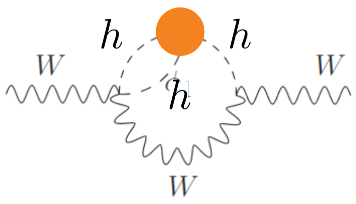


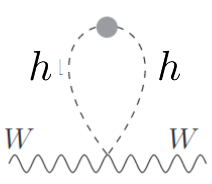
[Degrassi, Giardino, Maltoni, Pagani '16] [ATLAS-CONF-2019-049]

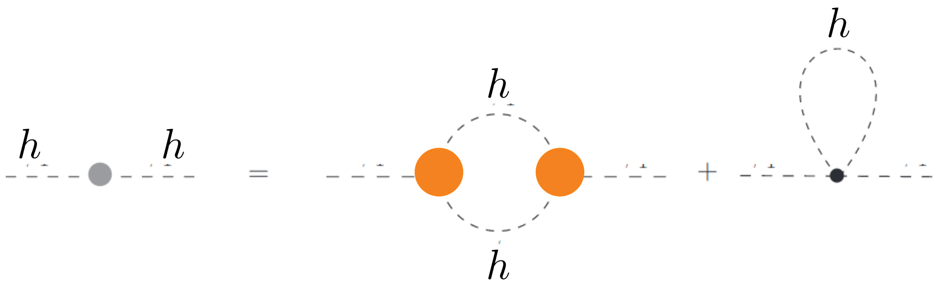
- **Electroweak Precision Observables** (EWPOs) → λ_{hhh} enters at NNLO









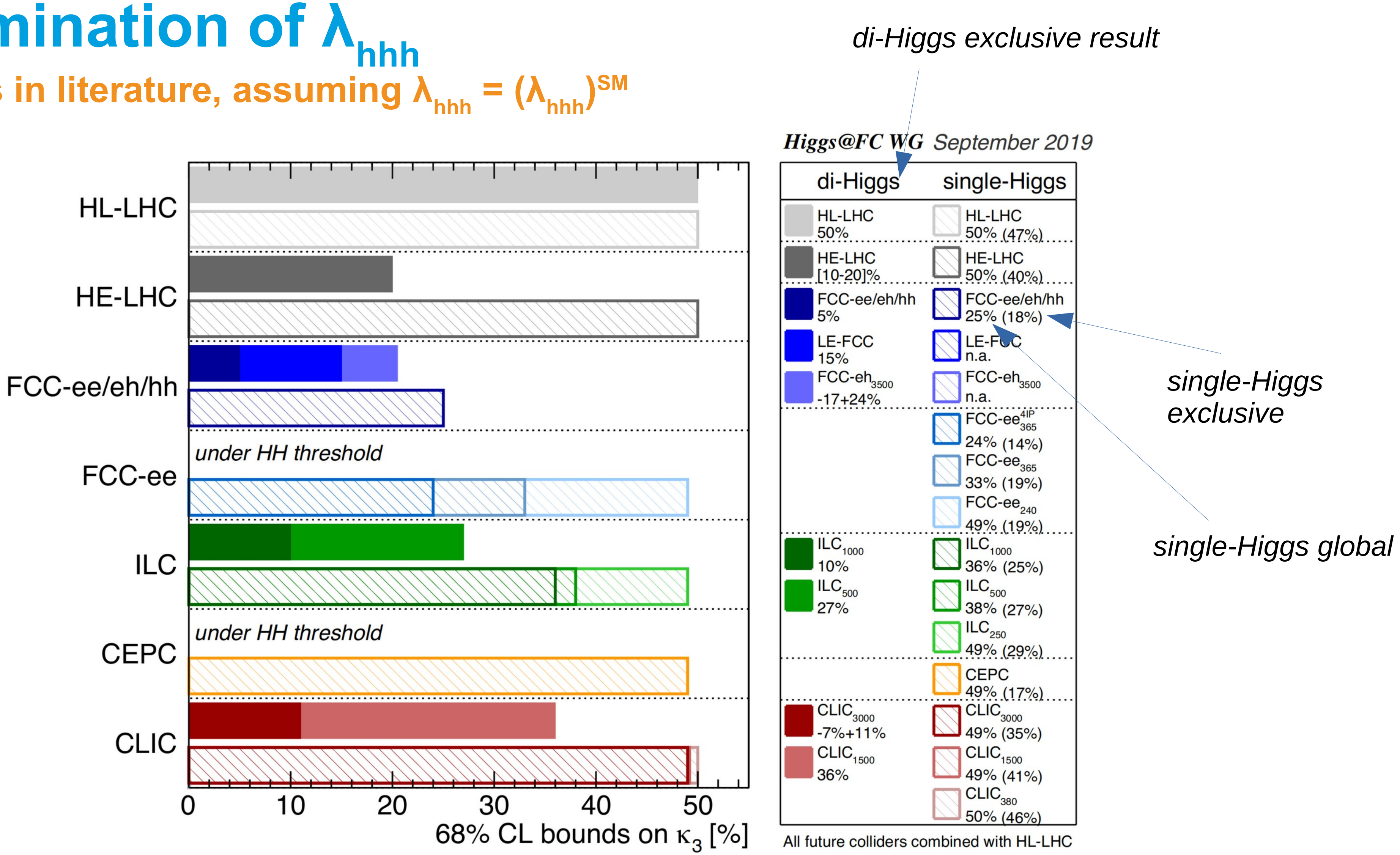


[Degrassi, Fedele, Giardino '17]

Future determination of λ_{hhh}

Expected sensitivities in literature, assuming $\lambda_{hhh} = (\lambda_{hhh})^{SM}$

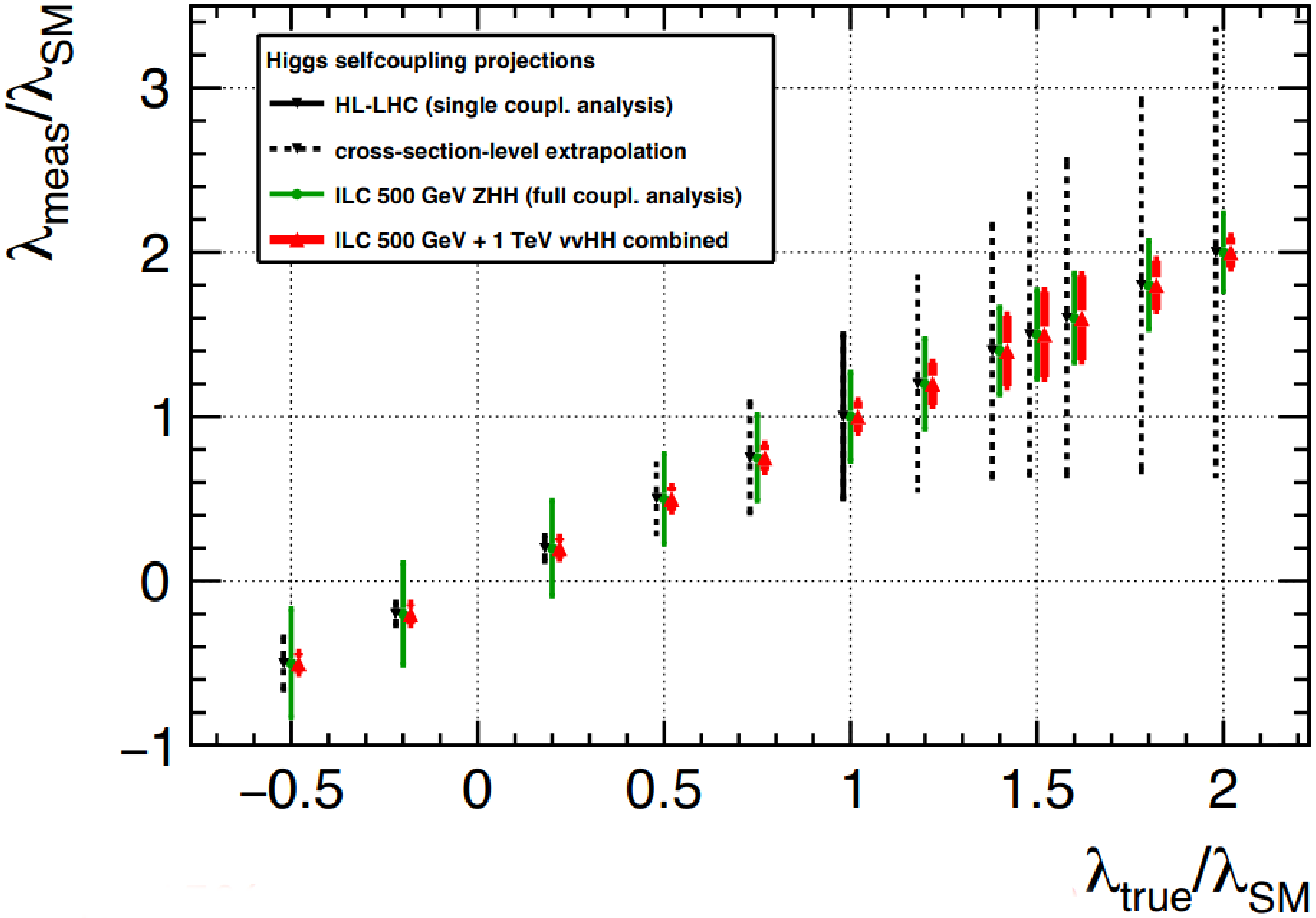
Plot taken from
[de Blas et al., 1905.03764]



see also [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], *etc.*

Future determination of λ_{hhh}

Achieved accuracy actually depends on the value of λ_{hhh}



[J. List et al. '21]

See also [Dürig, DESY-THESIS-2016-027]

Future determination of λ_{hhh}

Higgs production cross-sections (here double Higgs production) depend on λ_{hhh}

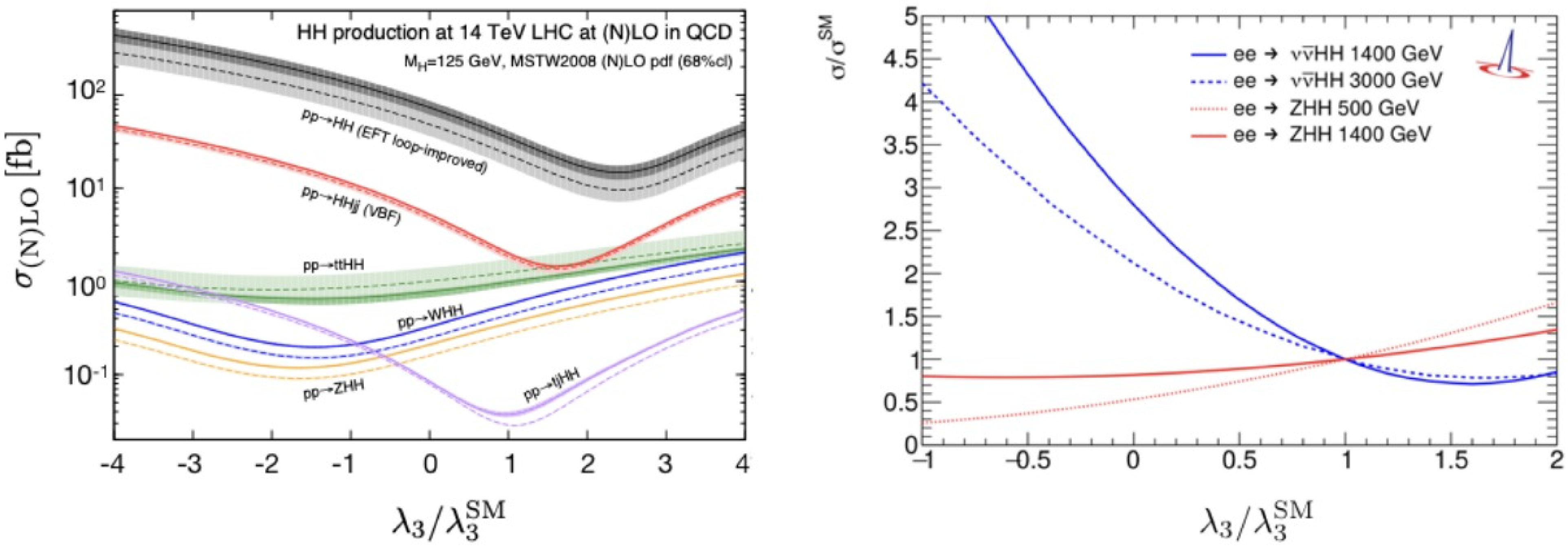


Figure 10. Double Higgs production at hadron (left) [65] and lepton (right) [66] colliders as a function of the modified Higgs cubic self-coupling. See Table 18 for the SM rates. At lepton colliders, the production cross sections do depend on the polarisation but this dependence drops out in the ratios to the SM rates (beam spectrum and QED ISR effects have been included).

[Frederix et al., 1401.7340]

Plots taken from
[de Blas et al., 1905.03764]

Baryogenesis

- **Observed Baryon Asymmetry of the Universe (BAU)**

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq 6.1 \times 10^{-10}$$ [Planck '18]

n_b: baryon no. density
$n_{\bar{b}}$: antibaryon no. density
n_γ: photon no. density

- **Sakharov conditions** [Sakharov '67] for a theory to explain BAU:
  1) Baryon number violation
  2) C and CP violation
  3) Loss of thermal equilibrium

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- **Sakharov conditions** [Sakharov '67] for a theory to explain BAU:
  1) Baryon number violation
  2) C and CP violation
  3) Loss of thermal equilibrium

In the SM
→ Sphaleron transitions (break B+L)
→ C violation (SM is chiral), but not enough CP violation
→ No loss of th. eq. → in SM, the EWPT is a crossover

SM phase diagram

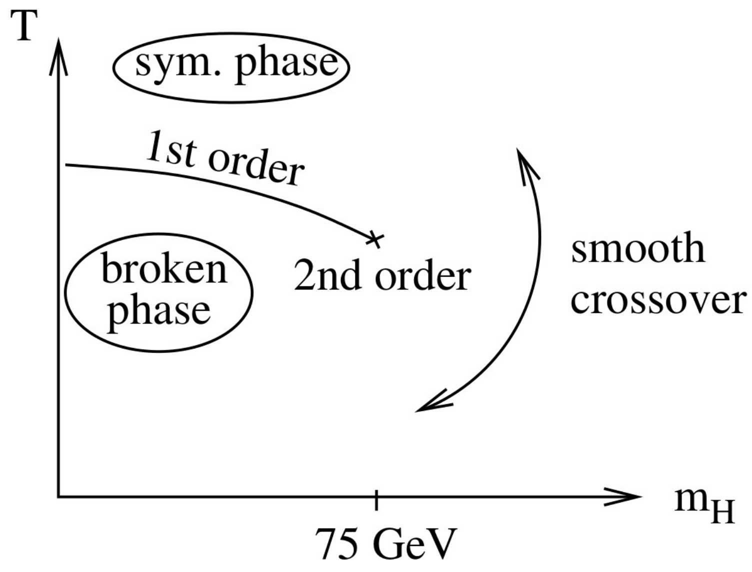


Figure from [Cline '06]

- SM cannot reproduce the BAU → **BSM physics needed!**

Electroweak Baryogenesis

- Many scenarios proposed, including:
  - Grand Unified Theories
  - Leptogenesis
  - **Electroweak Baryogenesis** (EWBG) [Kuzmin, Rubakov, Shaposhnikov, '85], [Cohen, Kaplan, Nelson '93]

- **Sakharov conditions** in EWBG

1) Baryon number violation → Sphaleron transitions (break B+L)

2) C and CP violation → C violation + CP violation in extended Higgs sector

3) Loss of thermal equilibrium → Loss of th. eq. via a strong 1st order EWPT

The Higgs potential and the Electroweak Phase Transition

Possible thermal history of the Higgs potential:

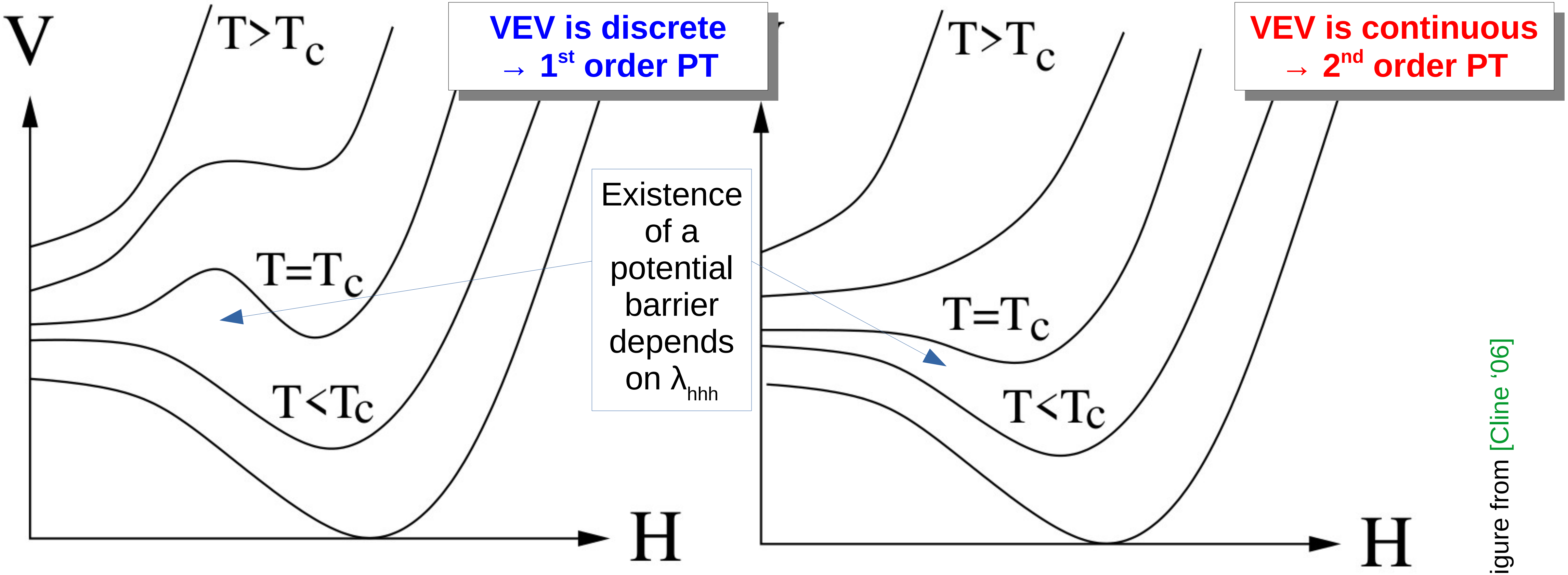


Figure from [Cline '06]

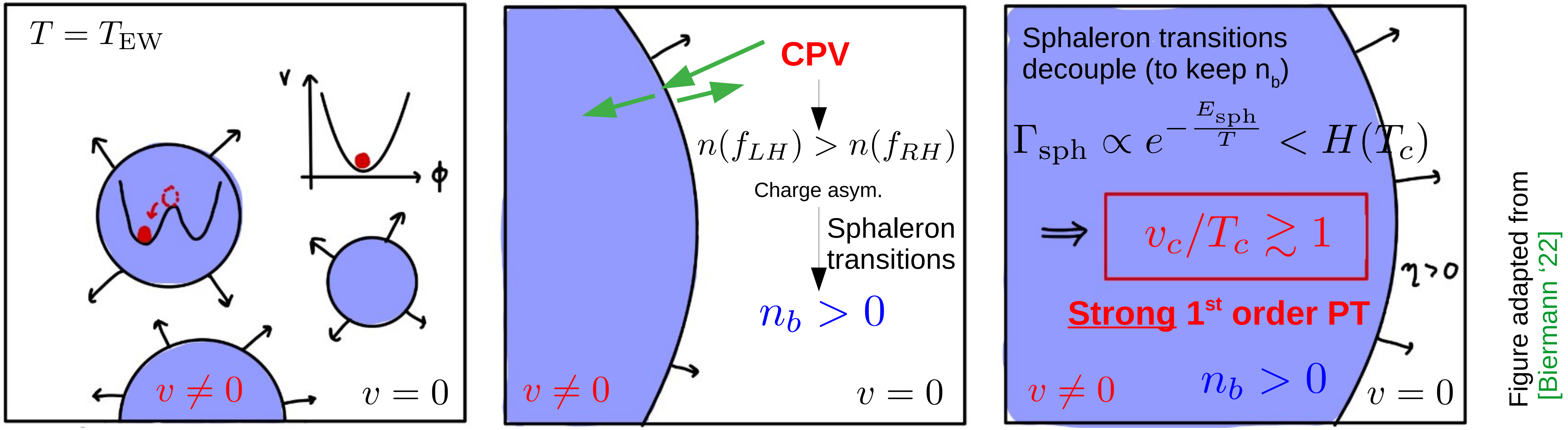
- **λ_{hhh} determines the nature of the EWPT**!

  ⇒ deviation of λ_{hhh} from its SM prediction typically needed to have a strongly first-order EWPT [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]

  ⇒ required for **electroweak baryogenesis** scenario

Electroweak Baryogenesis – a brief sketch

- **Sakharov conditions** in EWBG
  1) Baryon number violation → Sphaleron transitions (break B+L)
  2) C and CP violation → C violation + CP violation in extended Higgs sector
  3) Loss of thermal equilibrium → Loss of th. eq. via a strong 1st order EWPT



1) Bubble nucleation *2) Baryon number generation* *3) Baryon number conservation*

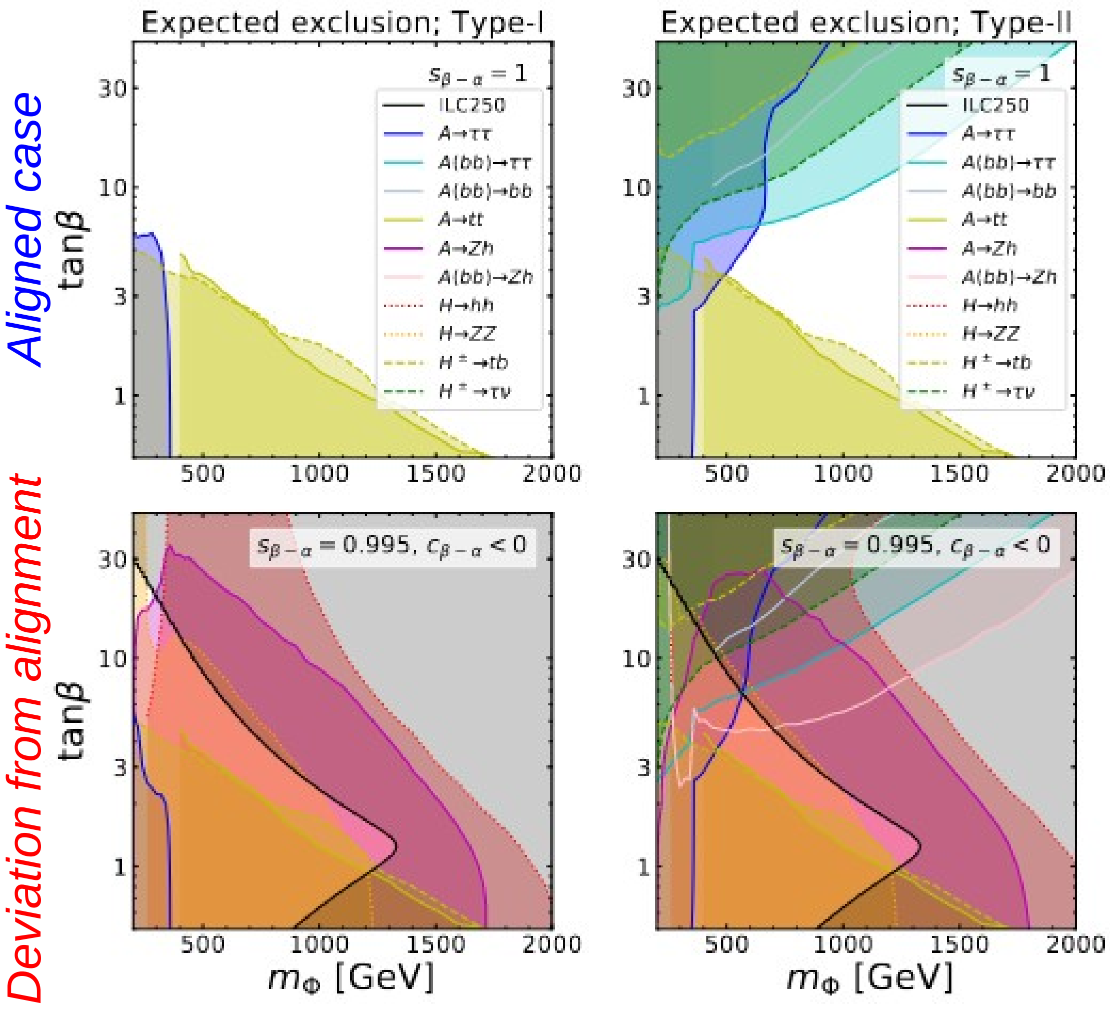
- EWBG only involves phenomena around the EW scale → **testable in the foreseeable future** via λ_{hhh}, collider searches, gravitational waves or primordial black holes (sourced by 1st order EWPT)

Distinguishing aligned scenarios with or without decoupling

e.g. for Two-Higgs-Doublet Model (2HDM) variants

- No concrete sign of BSM Physics so far + Higgs couplings are SM-like
  → favours ***aligned scenarios***, i.e. scenarios where Higgs couplings are *SM-like at tree-level*

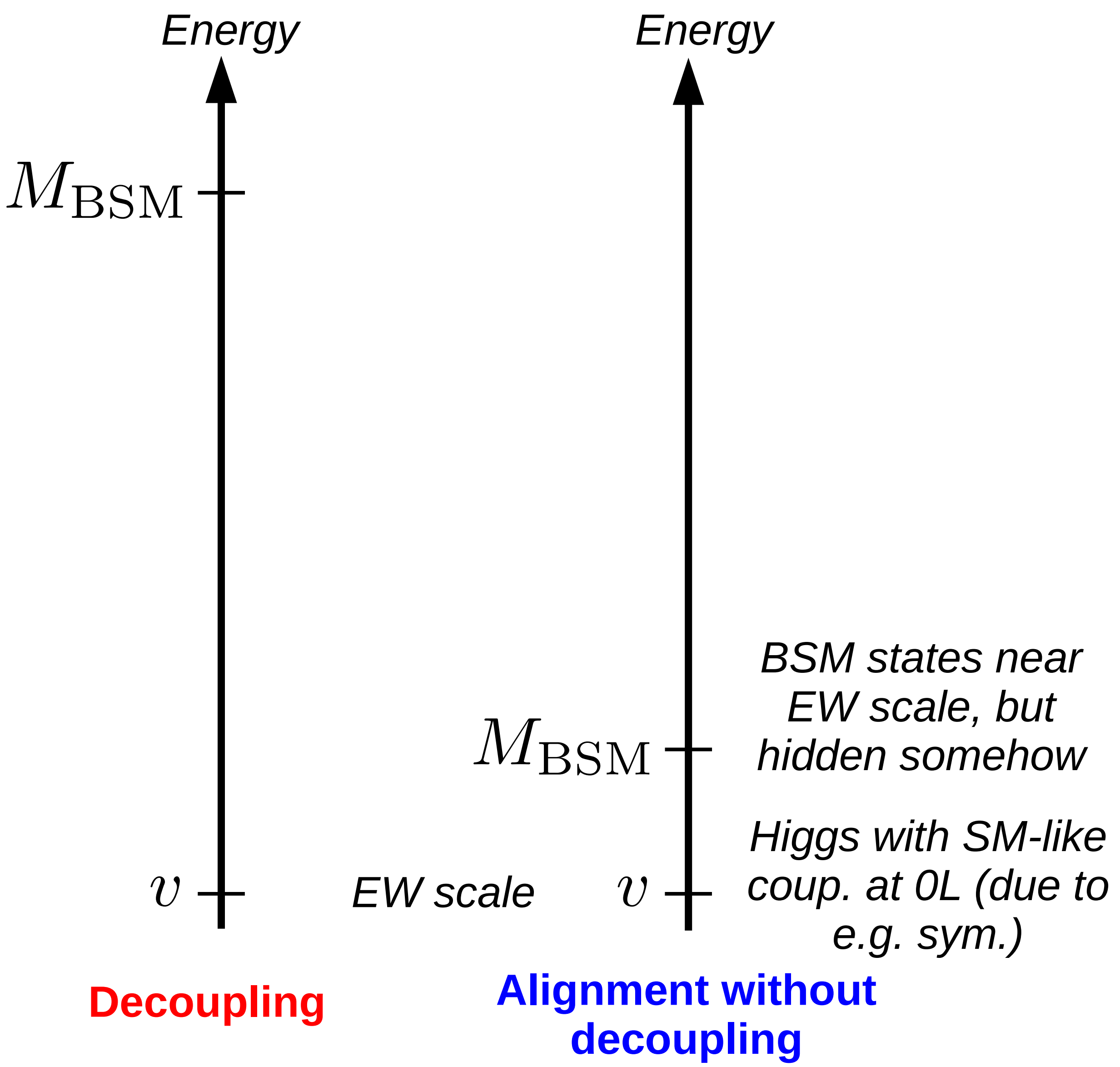
- **Synergy** of **direct searches** (LHC, HL-LHC) and **indirect searches** (→ ILC) strongly constrain non-aligned scenarios (see e.g. for MSSM [Bagnaschi et al. '18], for 2HDM [Aiko et al. '20])
  → In some models, aligned scenarios could be almost entirely excluded in near future!



[Aiko et al. 2010.15057]

Distinguishing aligned scenarios with or without decoupling

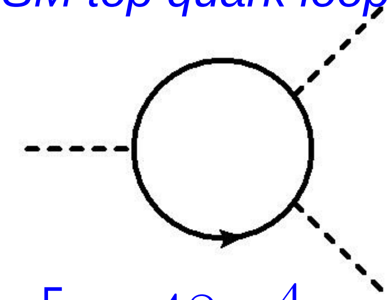
- If alignment is favoured, how does it occur? → **Alignment through decoupling**? or **alignment without decoupling**?
- If *alignment without decoupling*, Higgs couplings **like λ_{hhh}** can still exhibit large deviations from SM predictions because of **non-decoupling effects from BSM loops**
- λ_{hhh} could be a **prime target**: not very well measured yet but with prospects for drastical improvements in the future!



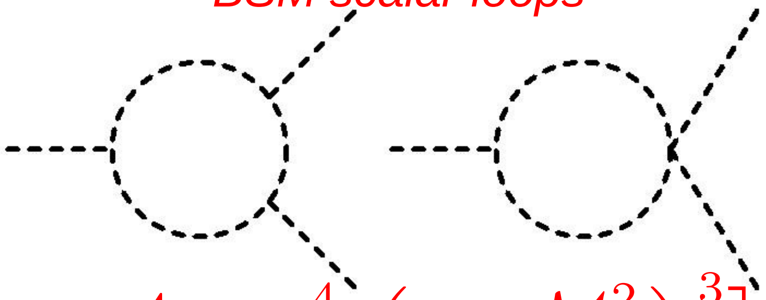
One-loop mass-splitting effects

- **Leading one-loop** corrections to λ_{hhh} in models with extended sectors (like 2HDM):

SM top quark loop



BSM scalar loops



$$\delta^{(1)}\lambda_{hhh} \supset \frac{1}{16\pi^2}\left[-\frac{48m_t^4}{v^3} + \sum_\Phi \frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3\right]$$

First found in 2HDM:
[Kanemura, Kiyoura, Okada, Senaha, Yuan '02]

$\mathcal{M}$: **BSM mass scale**, e.g. soft breaking scale M of Z_2 symmetry in 2HDM
n_Φ : # of d.o.f of field Φ

- Size of new effects depends on how the BSM scalars acquire their mass: $m_\Phi^2 \sim \mathcal{M}^2 + \tilde{\lambda}v^2$

$$\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3 \longrightarrow \begin{cases} 0, \text{ for } \mathcal{M}^2 \gg \tilde{\lambda}v^2 \\ 1, \text{ for } \mathcal{M}^2 \ll \tilde{\lambda}v^2 \end{cases}$$

→ **Huge BSM effects possible!**

One-loop mass-splitting effects

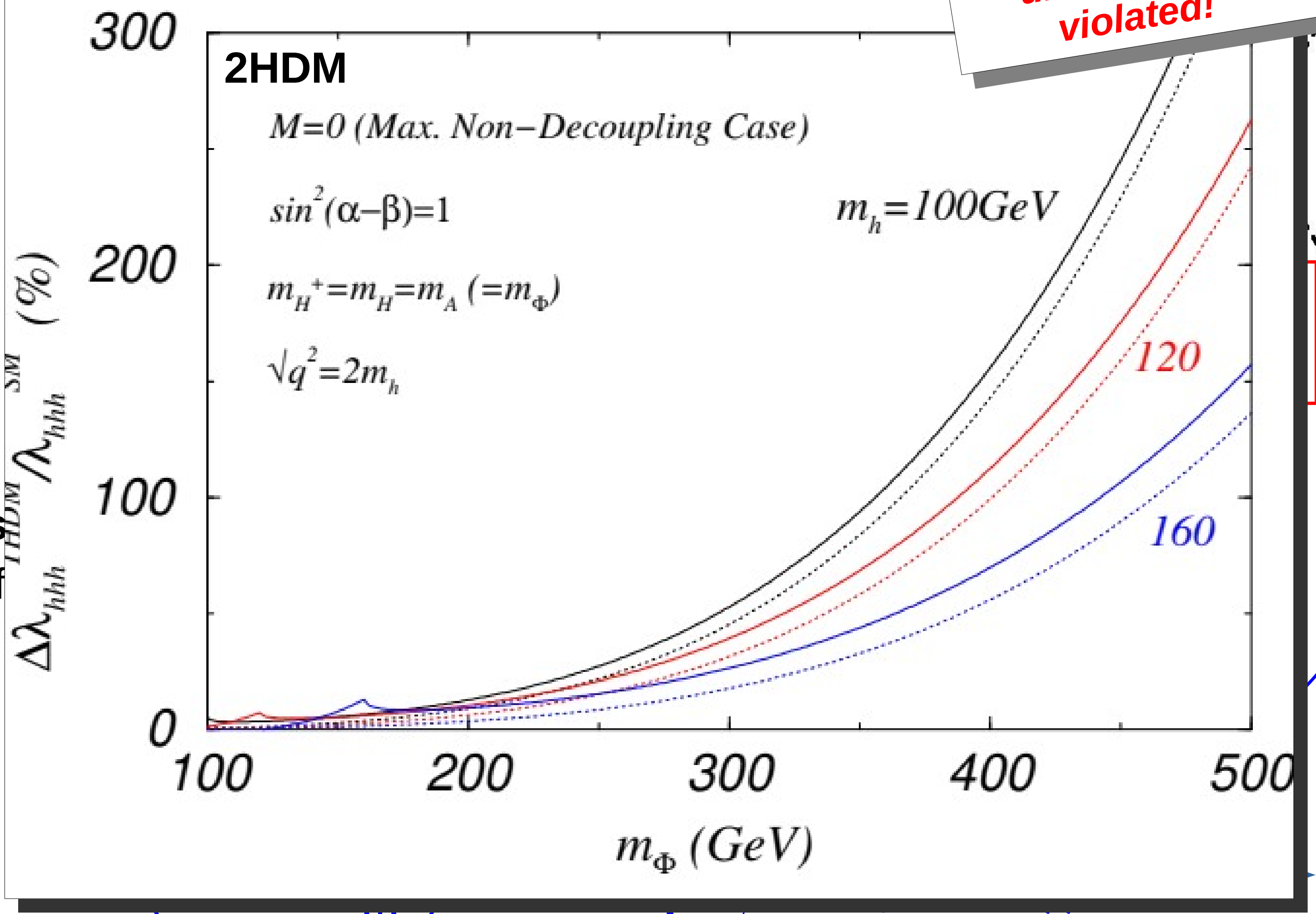
- Leading one-loop c

$\delta^{(1)}\lambda_{hhh} \supset$

$\mathcal{M}$: **BSM mass**
n_Φ : # of d.o.f of

- Size of new effects

Plot from [Kanemura, Okada, Senaha, Yuan '04]



***NB*: perturbative unitarity not violated!**

First found in 2HDM: [Kanemura, Kiyoura, Okada, Senaha, Yuan '02]

$\mathcal{M}^2 + \tilde{\lambda}v^2$

Huge BSM effects possible!

Two-loop calculation of λ_{hhh}

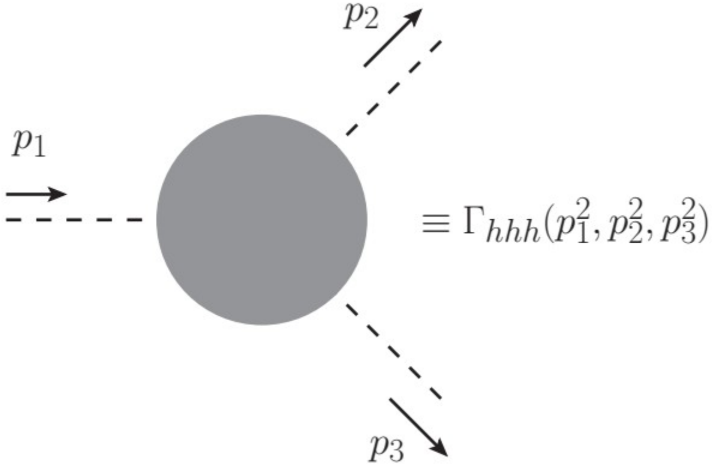
Goal: How large can the two-loop corrections to λ_{hhh} become?

Based on

arXiv:1903.05417 **(PLB) and** arXiv:1911.11507 **(EPJC) in collaboration with Shinya Kanemura**

An effective Higgs trilinear coupling

- In principle: consider 3-point function Γ_{hhh}
  but this is momentum dependent → very difficult beyond one loop



- Instead, consider an **effective trilinear coupling**

$$\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min}}$$

entering the coupling modifier

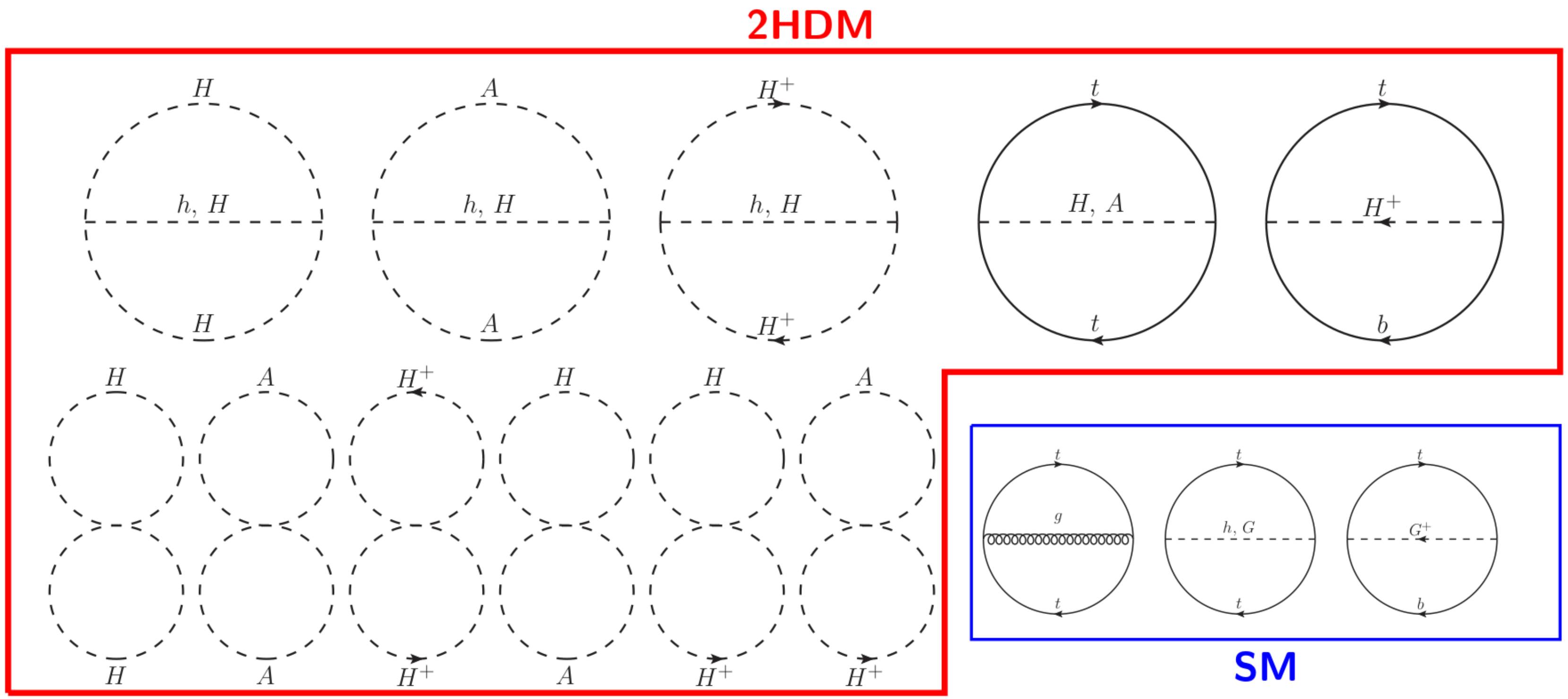
$$\kappa_\lambda = \frac{\lambda_{hhh}}{(\lambda_{hhh}^{(0)})^{\text{SM}}} \qquad \text{with } (\lambda_{hhh}^{(0)})^{\text{SM}} = \frac{3m_h^2}{v}$$

constrained by experiments (*applicability of this assumption discussed later*)

Our effective-potential calculation

[JB, Kanemura '19]

- **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)}$ ($\overline{\text{MS}}$ result)
  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to* $V^{(2)}$ = diagrams involving **heavy BSM scalars and top quark**
  - **Neglect masses of light states** (SM-like Higgs, light fermions, ...)



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  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to $V^{(2)}$* = diagrams involving **heavy BSM scalars and top quark**

- **Step 2**: derive an effective trilinear coupling

$$\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \left[\frac{\partial^3}{\partial h^3} - \frac{3}{v}\left(\frac{\partial^2}{\partial h^2} - \frac{1}{v}\frac{\partial}{\partial h}\right)\right]\Delta V\bigg|_{\text{min.}}$$

($\overline{\text{MS}}$ result too)

Express tree-level result in terms of effective-potential Higgs mass

Our effective-potential calculation

[JB, Kanemura '19]

- **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2} V^{(1)} + \frac{1}{(16\pi^2)^2} V^{(2)}$ ($\overline{\text{MS}}$ result)
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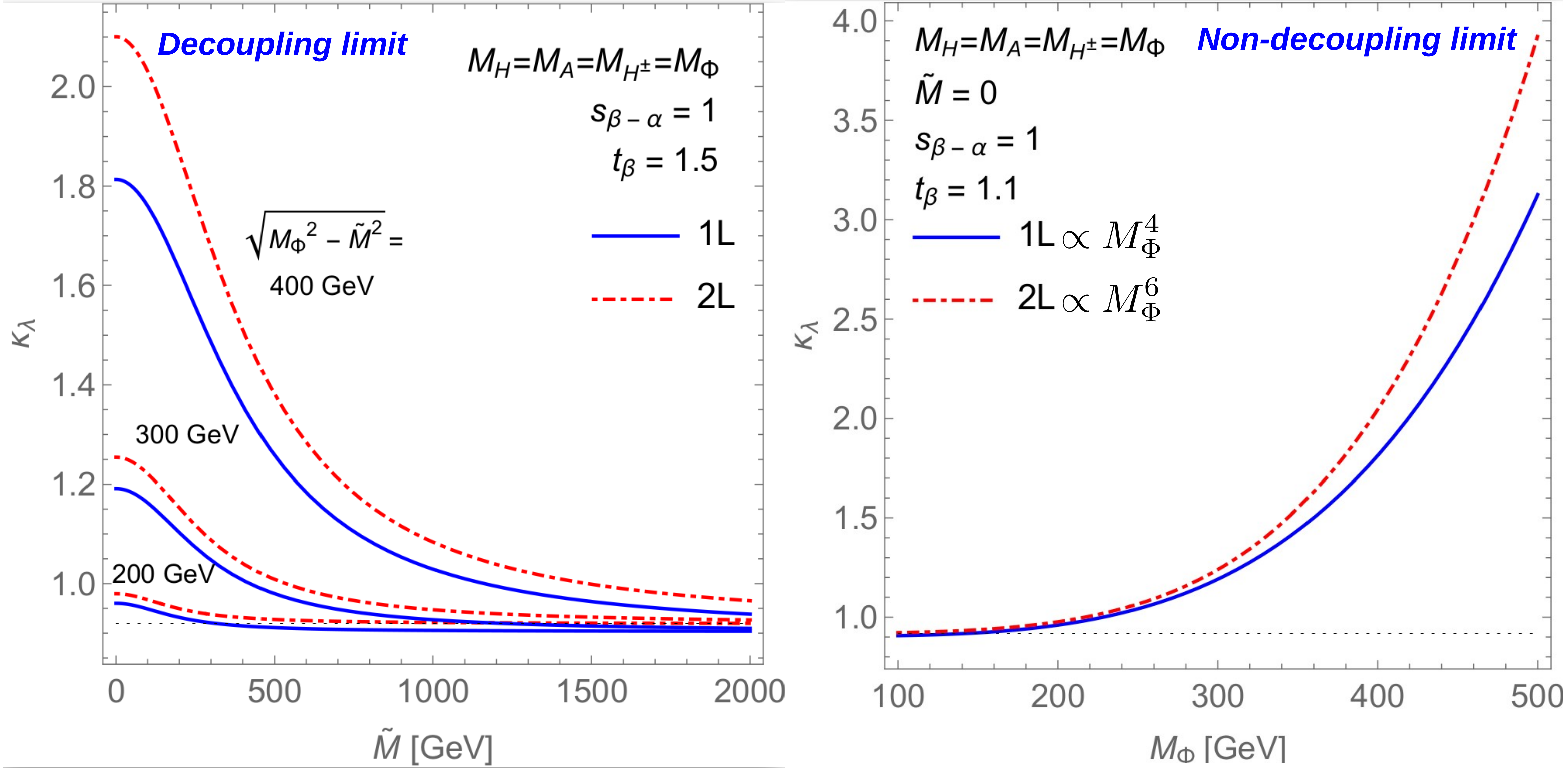
  ($\overline{\text{MS}}$ result too)

- **Step 3**: conversion from $\overline{\text{MS}}$ to OS scheme
  - Express result in terms of **pole masses**: M_t, M_h, M_Φ (Φ=H,A,H$^\pm$); OS Higgs VEV $v_{\text{phys}} = \frac{1}{\sqrt{\sqrt{2}G_F}}$
  - Include **finite WFR**: $\hat{\lambda}_{hhh} = (Z_h^{\text{OS}}/Z_h^{\overline{\text{MS}}})^{3/2}\lambda_{hhh}$
  - Prescription for M to ensure **proper decoupling** with $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$

Our results in the aligned 2HDM

[JB, Kanemura '19]

Taking degenerate BSM scalar masses: $M_\Phi = M_H = M_A = M_{H^\pm}$



$\overline{\text{MS}}$ to OS scheme conversion

- V_{eff}: we use expressions in MS scheme hence results for λ_{hhh} also in $\overline{\text{MS}}$ scheme

- We include finite counterterms to express the Higgs trilinear coupling in terms of physical quantities

$$\underbrace{m_X^2}_{\overline{\text{MS}}} = \underbrace{M_X^2}_{\text{pole}} - \mathfrak{Re}[\Pi_{XX}^{\text{fin.}}(p^2 = M_X^2)], \qquad v^2 = \underbrace{(\sqrt{2}G_F)^{-1}}_{\equiv v_{\text{OS}}^2} + \frac{3M_t^2}{16\pi^2}\left(2\log\frac{M_t^2}{Q^2} - 1\right) + \cdots$$

- Also we include finite WFR effects → OS scheme

$$\underbrace{\hat{\lambda}_{hhh}}_{\text{OS}} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{finite WFR}} \underbrace{\lambda_{hhh}}_{\overline{\text{MS}}} = -\underbrace{\Gamma_{hhh}(0,0,0)}_{\text{3-pt. func.}}$$

$\overline{\text{MS}}$ to OS scheme conversion

- OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\text{MS}}\text{ parameters} \\ \text{translated to OS ones}}}$$

- Let's suppose (for simplicity) that λ_{hhh} only depends on one parameter x, as

$$\lambda_{hhh} = f^{(0)}(x^{\overline{\text{MS}}}) + \kappa f^{(1)}(x^{\overline{\text{MS}}}) + \kappa^2 f^{(2)}(x^{\overline{\text{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2}\right)$$

and

$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\text{OS}}) + \kappa\left[f^{(1)}(X^{\text{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x\right]$$
$$+ \kappa^2\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2\right]$$

$\overline{\text{MS}}$ to OS scheme conversion

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and

$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

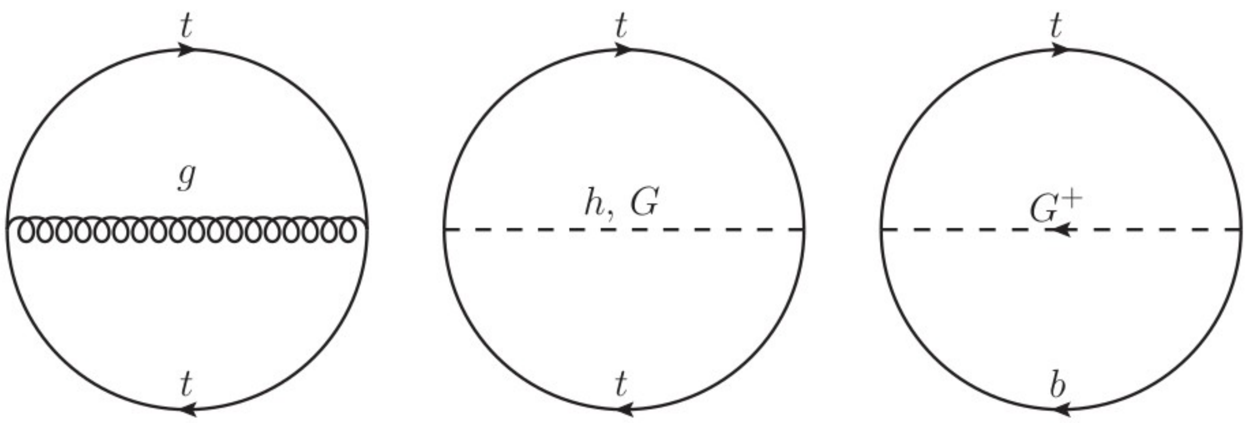
then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\text{OS}}) + \kappa\left[f^{(1)}(X^{\text{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x\right]$$
$$+ \kappa^2\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2\right]$$

because we neglect m_h in the loop corrections and $\lambda^{(0)}_{hhh} = 3m_h^2/v$ (in absence of mixing)

SM result at two loops

$$\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}}$$



- In the SM, 4 diagrams contribute to V_{eff} at order $\mathcal{O}(g_3^2 m_t^4)$ and $\mathcal{O}(m_t^6/v^2)$
- In the limit $m_t \gg m_h, m_G, \cdots$, their expression reads

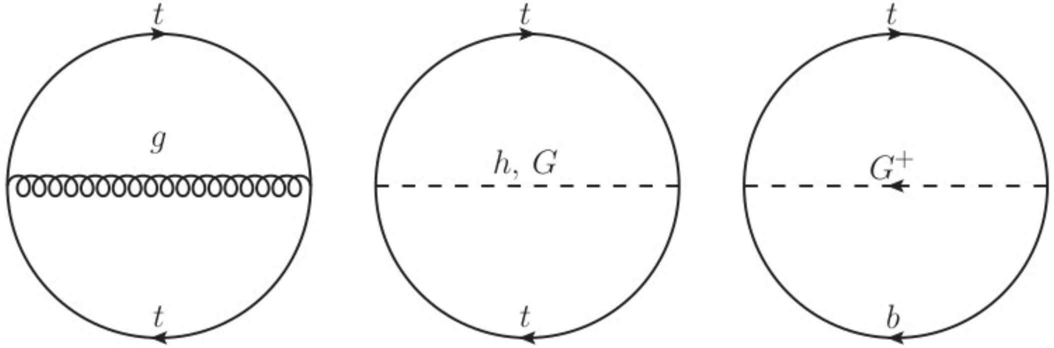
$$V^{(2)} = -4g_3^2 m_t^2 \left[4A(m_t^2) - 8m_t^2 - \frac{6A(m_t^2)^2}{m_t^2}\right] + 3y_t^2\left[2m_t^2 I(m_t^2, m_t^2, 0) + m_t^2 I(m_t^2, 0, 0) + A(m_t^2)^2\right]$$

where $A(x) \equiv x(\log(x/Q^2) - 1)$, I: two-loop sunrise integral

- Then we find in the $\overline{\text{MS}}$ scheme

$$\delta^{(2)}\lambda_{hhh} = \frac{128 g_3^2 m_t^4 (1 + 6\,\overline{\log}\, m_t^2)}{v^3} - \frac{24 m_t^4 y_t^2(-7 + 6\,\overline{\log}\, m_t^2)}{v^3} \qquad (\overline{\log}\, x \equiv \log x/Q^2)$$

SM result at two loops



- $\overline{\text{MS}}$ expression

$$\delta^{(2)}\lambda_{hhh} = \frac{128 g_3^2 m_t^4 (1 + 6\,\overline{\log}\, m_t^2)}{v^3} - \frac{24 m_t^4 y_t^2 (-7 + 6\,\overline{\log}\, m_t^2)}{v^3} \qquad (\overline{\log}\, x \equiv \log x/Q^2)$$

- Translate top quark mass and Higgs VEV from $\overline{\text{MS}}$ to OS scheme in $\delta^{(1)}\lambda_{hhh} = -\frac{48 m_t^4}{v^3}$

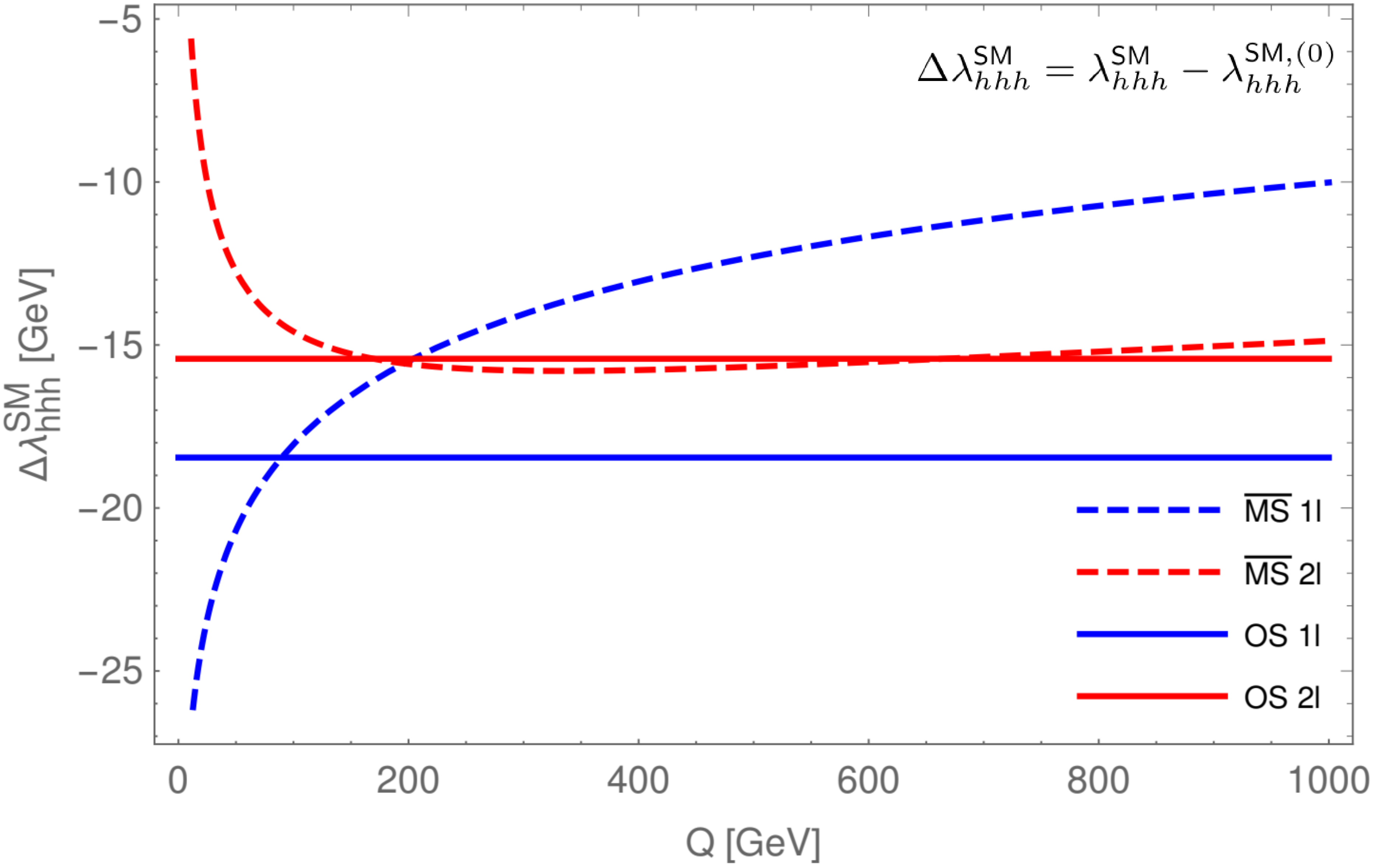
$$m_t^2 \to M_t^2 - \Pi_{tt}(p^2 = M_t^2) \qquad v \to \frac{1}{\sqrt{\sqrt{2}G_F}} - \delta v = v_{\text{phys}} - \delta v$$

$+$ include wave-function renormalisation
$\to$ OS-scheme result

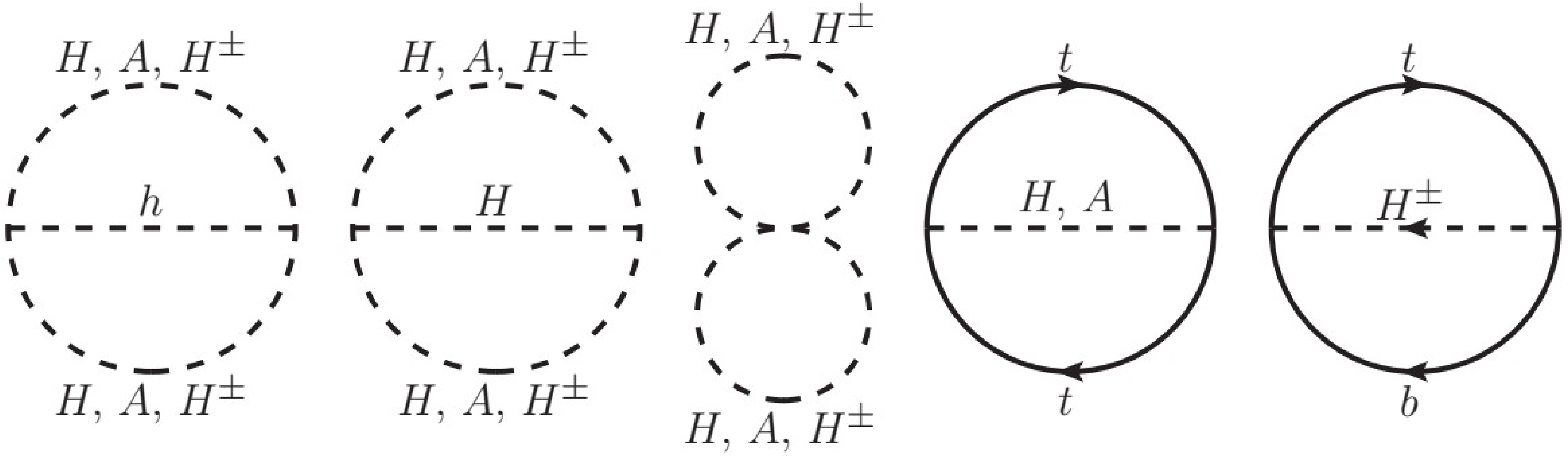
$$\delta^{(2)}\hat{\lambda}_{hhh} = \frac{72 M_t^4}{v_{\text{phys}}^3}\left(16 g_3^2 - \frac{13 M_t^2}{v_{\text{phys}}^2}\right)$$

SM result at two loops

[JB, Kanemura '19]



$\overline{\text{MS}}$ result



- Taking BSM scalars to be degenerate $M_\Phi = M_H = M_A = M_{H^\pm}$ we obtain in the $\overline{\text{MS}}$ scheme:
  (expressions for non-degenerate masses → see [JB, Kanemura 1911.11507])

$$\delta^{(2)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^5}\left(4+9\cot^2 2\beta\right)\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2-m_\Phi^2+(M^2+2m_\Phi^2)\,\overline{\log}\, m_\Phi^2\right]$$
$$+\frac{192m_\Phi^6\cot^2 2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[1+2\,\overline{\log}\, m_\Phi^2\right]$$
$$+\frac{96m_\Phi^4 m_t^2\cot^2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^3\left[-1+2\,\overline{\log}\, m_\Phi^2\right]+\mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

Decoupling property in $\overline{\text{MS}}$ scheme

- Seeing whether corrections from additional BSM states decouple if said state is taken to be very massive is a good way to check the consistency of the calculation

$$\delta^{(1)}\lambda_{hhh} = \frac{16 m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3$$

$$\delta^{(2)}\lambda_{hhh} = \frac{16 m_\Phi^4}{v^5}\left(4+9\cot^2 2\beta\right)\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2 - m_\Phi^2 + (M^2+2m_\Phi^2)\,\overline{\log}\, m_\Phi^2\right]$$
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where $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$

- To have $m_\Phi \to \infty$, then we must take $M \to \infty$, otherwise the quartic couplings grow out of control
- Fortunately all of these terms go like

$$(m_\Phi^2)^{n-1}\left(1-\frac{M^2}{m_\Phi^2}\right)^n \underset{m_\Phi^2 = M^2+\tilde{\lambda}_\Phi v^2}{=} \frac{(\tilde{\lambda}_\Phi v^2)^n}{M^2+\tilde{\lambda}_\Phi v^2} \xrightarrow[\tilde{\lambda}_\Phi v^2 \text{ fixed}]{M\to\infty} 0$$

$\overline{\text{MS}} \to$ OS scheme conversion

- To express $\delta^{(2)}\lambda_{hhh}$ in terms of physical parameters (v_{phys}, M_t, $M_A = M_H = M_{H^\pm} = M_\Phi$), we replace

$$m_A^2 \to M_A^2 - \Pi_{AA}(M_A^2), \quad m_H^2 \to M_H^2 - \Pi_{HH}(M_H^2), \quad m_{H^\pm}^2 \to M_{H^\pm}^2 - \Pi_{H^+H^-}(M_{H^\pm}^2),$$

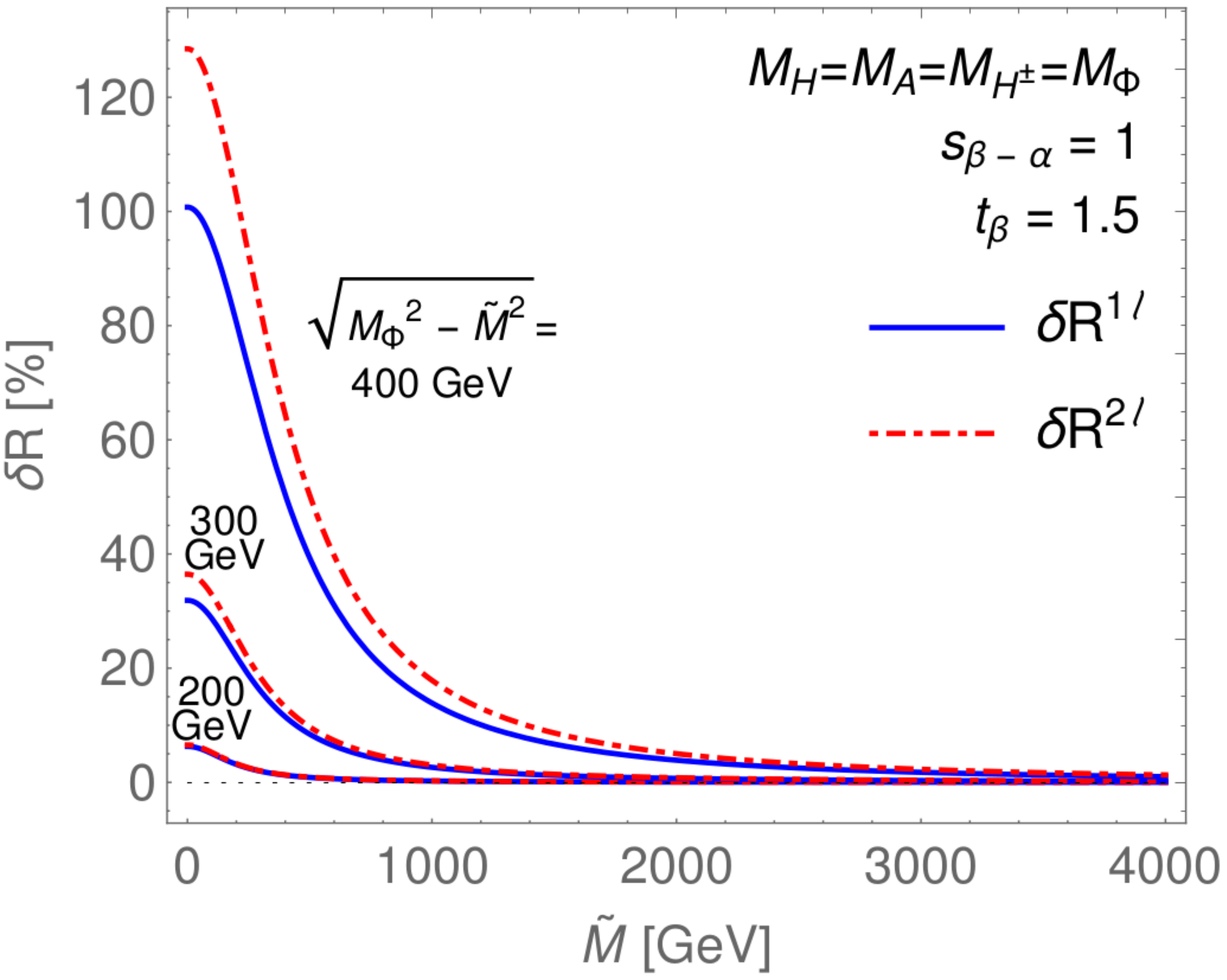
$$v \to v_{\text{phys}} - \delta v, \quad m_t^2 \to M_t^2 - \Pi_{tt}(M_t^2)$$

- A priori, M is still renormalised in $\overline{\text{MS}}$ scheme, because it is difficult to relate to physical observable

  … but then, expressions do not decouple for $M_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ and $M \to \infty$!
- This is because we should relate M_Φ, renormalised in OS scheme, and M, renormalised in $\overline{\text{MS}}$ scheme, with a **one-loop relation** $\to$ then the two-loop corrections decouple properly
- We give a new "OS" prescription for the finite part of the counterterm for M be requiring that
  1. the decoupling of $\delta^{(2)}\hat{\lambda}_{hhh}$ (in OS scheme) is apparent using a relation $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$
  2. all the log terms in $\delta^{(2)}\hat{\lambda}_{hhh}$ are absorbed in δM^2

$$\delta^{(2)}\hat{\lambda}_{hhh} = \frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^4\left\{4+3\cot^2 2\beta\left[3-\frac{\pi}{\sqrt{3}}\left(\frac{\tilde{M}^2}{M_\Phi^2}+2\right)\right]\right\}+\frac{576M_\Phi^6\cot^2 2\beta}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^4$$
$$+\frac{288M_\Phi^4M_t^2\cot^2\beta}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^3+\frac{168M_\Phi^4M_t^2}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^3-\frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^5+\mathcal{O}\left(\frac{M_\Phi^2M_t^4}{v_{\text{phys}}^5}\right)$$

Decoupling behaviour



- δR size of BSM contributions to λ_{hhh}:

$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

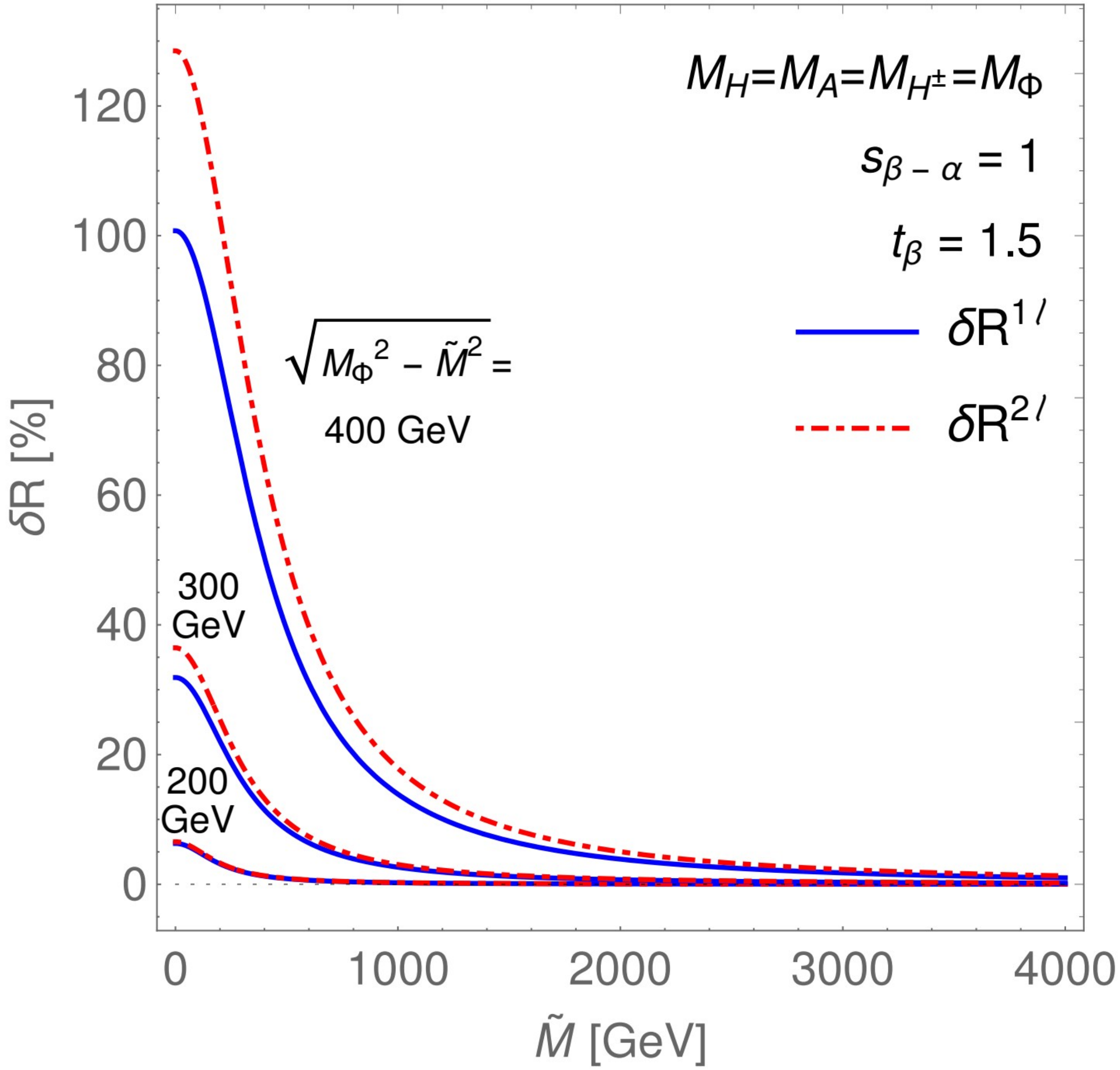
- $\tilde{M}$: "OS" version of M, defined so as to ensure proper decoupling for $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$

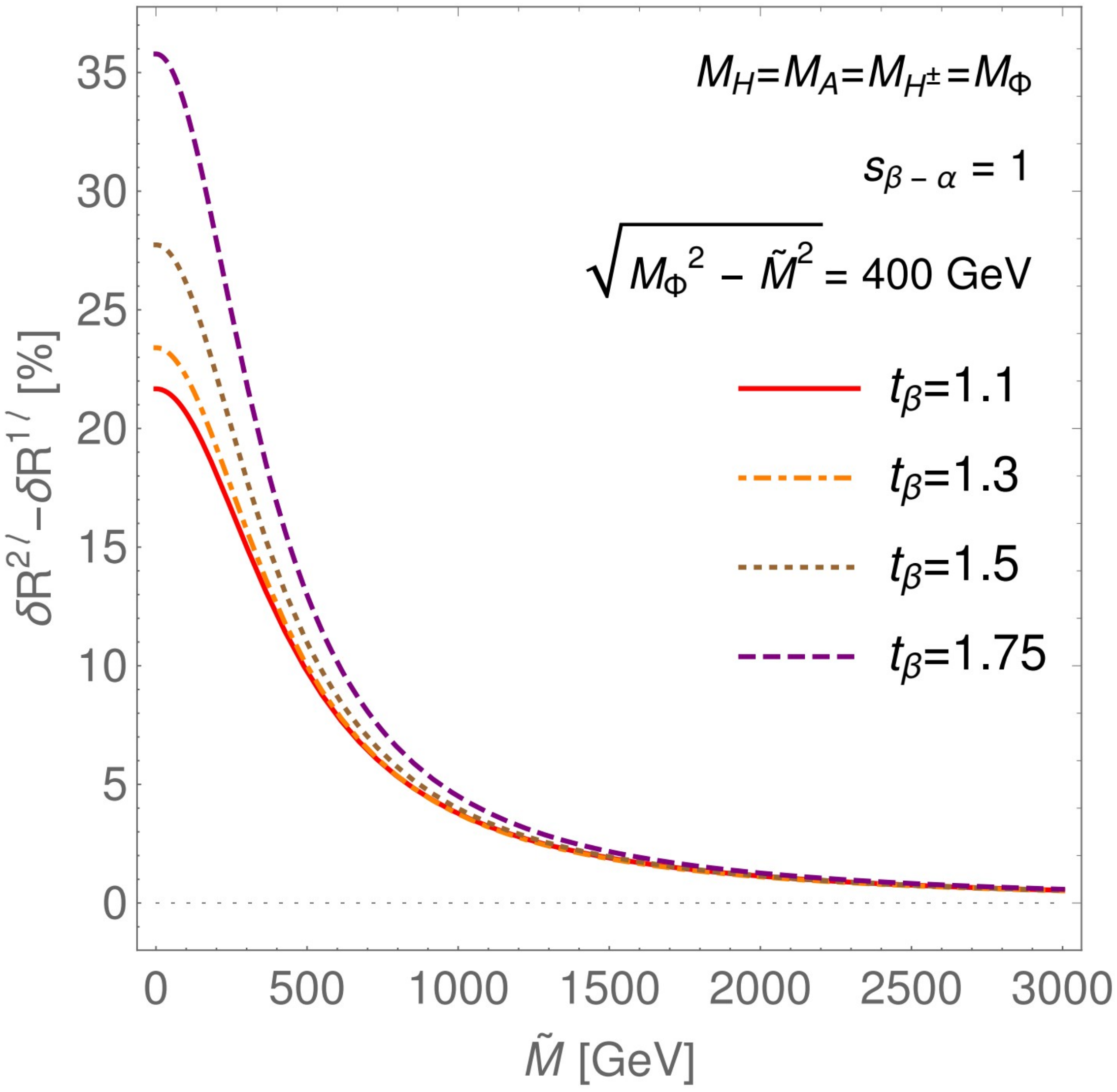
- Radiative corrections from additional scalars + top quark indeed decouple properly for $\tilde{M} \to \infty$

Decoupling of BSM effects

$\tilde{M}$: modified “OS” version of Z_2 breaking scale

[JB, Kanemura ‘19]





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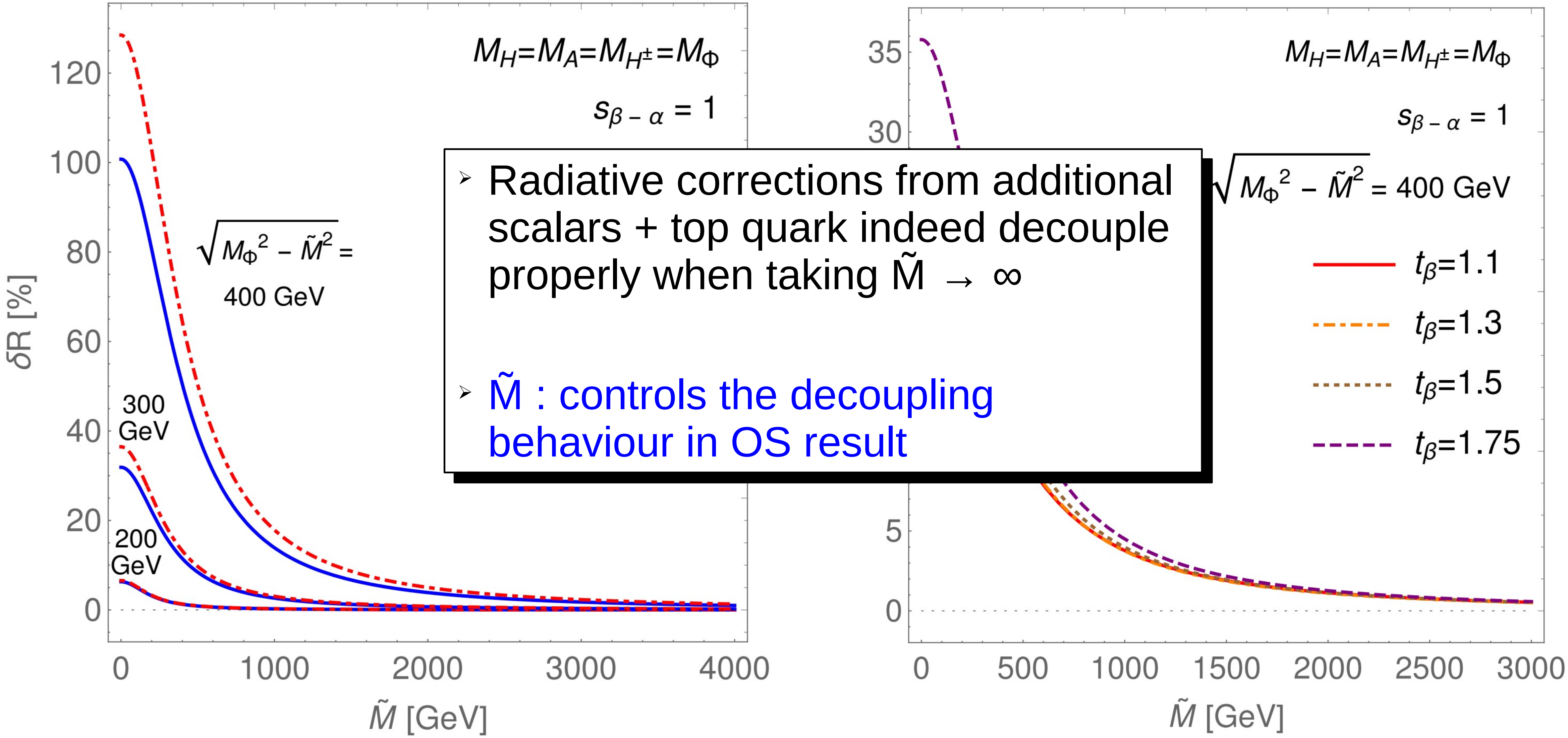
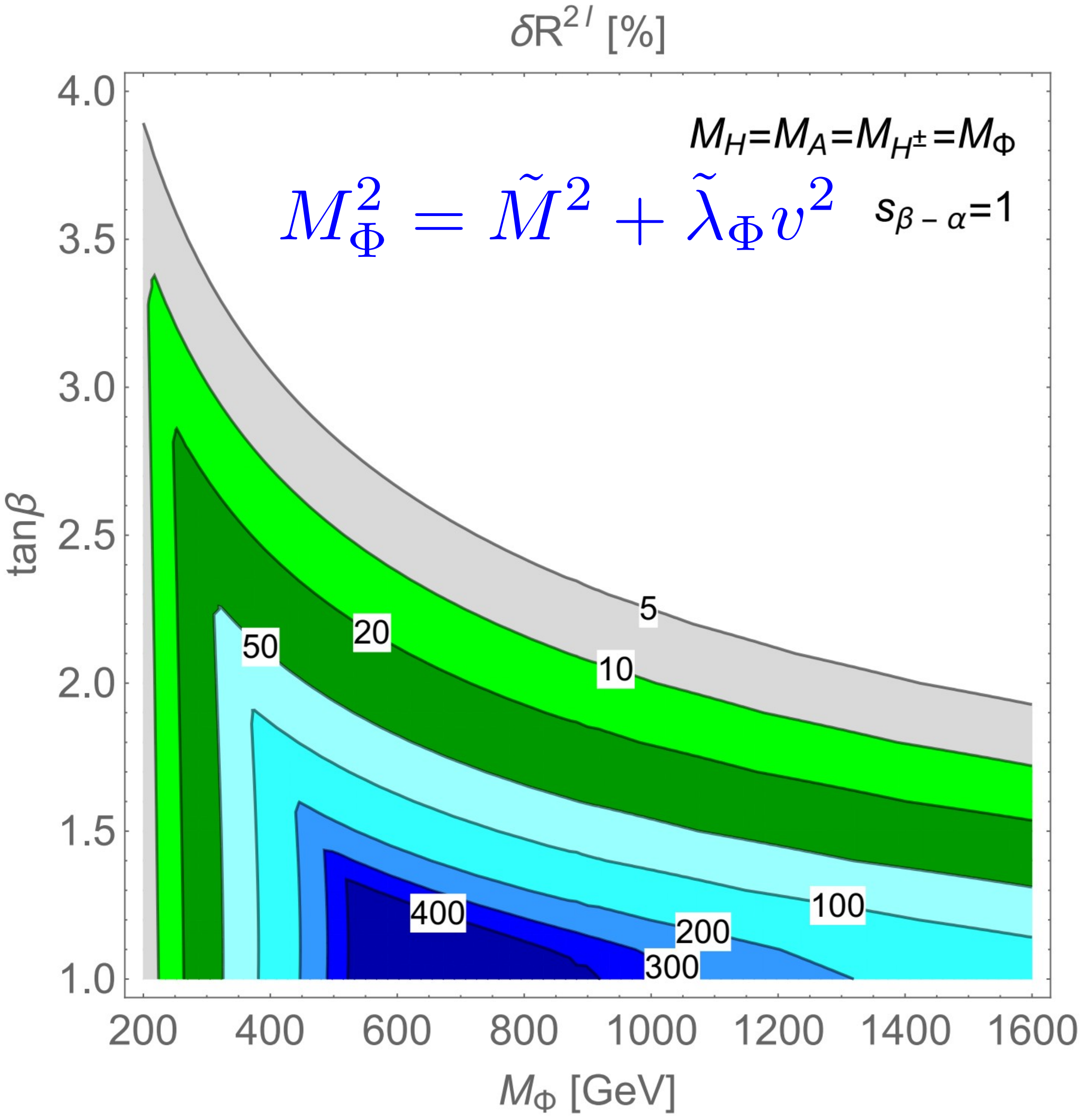


- Radiative corrections from additional scalars + top quark indeed decouple properly when taking $\tilde{M} \to \infty$
- $\tilde{M}$: controls the decoupling behaviour in OS result

Maximal BSM deviation in an aligned 2HDM scenario

[JB, Kanemura 1911.11507]



- Maximal δR (1l+2l) allowed while fulfilling perturbative unitarity [Kanemura, Kubota, Takasugi '93]

- Max. deviations for low tanβ and M_Φ~600-800 GeV →heavy BSM scalars acquiring their mass from Higgs VEV **only**
  - 1 loop: up to ~300% deviation at most
  - 2 loops: additional 100% (for same points)
- For increasing tanβ, unitarity constraints become more stringent → smaller δR

- **Blue region:** probed at **HL-LHC** (50% accuracy on λ_{hhh})
- **Green region:** probed at lepton colliders, e.g. **ILC** (50% accuracy at 250 GeV; 27% at 500 GeV; 10% at 1 TeV)

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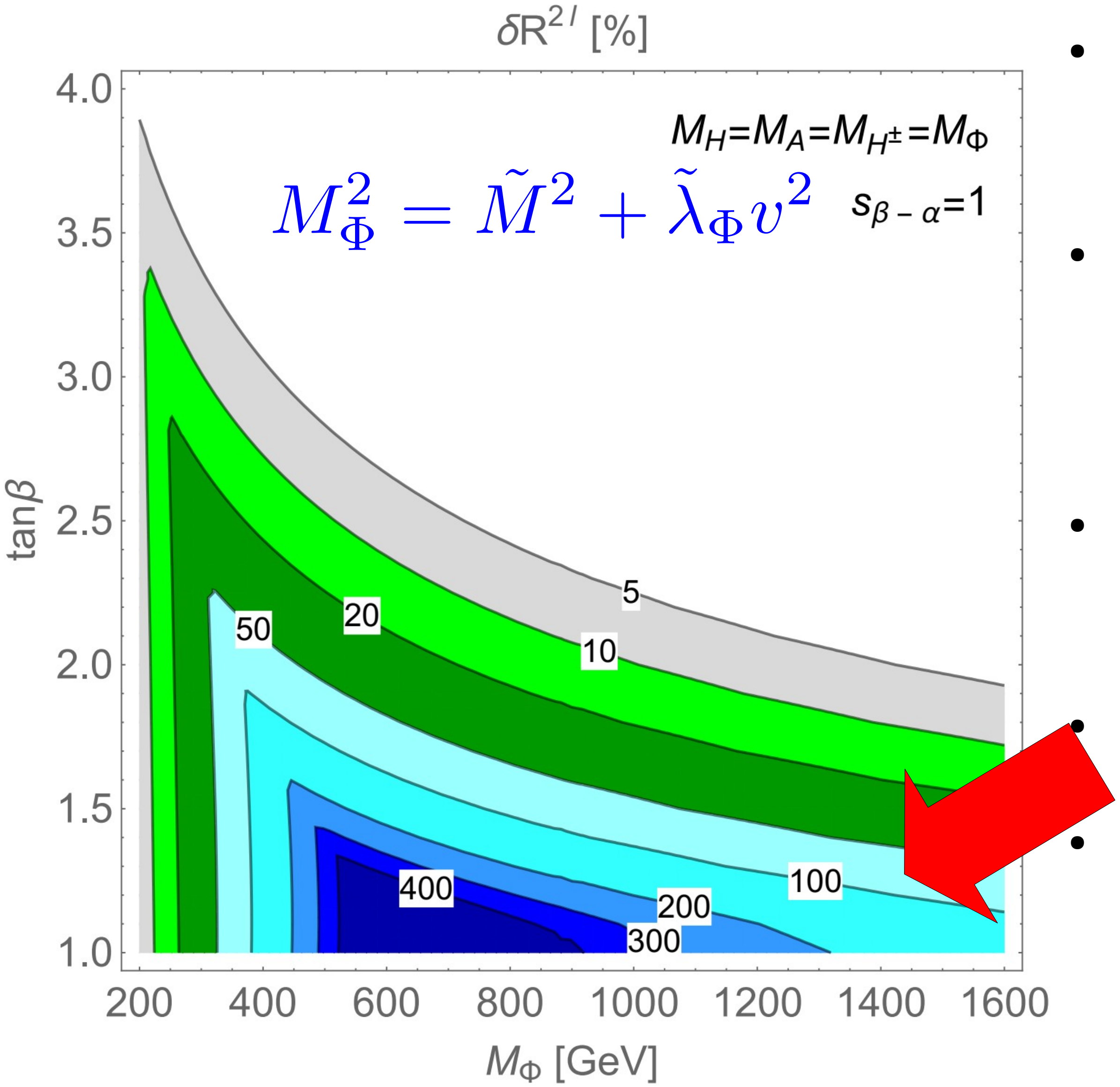

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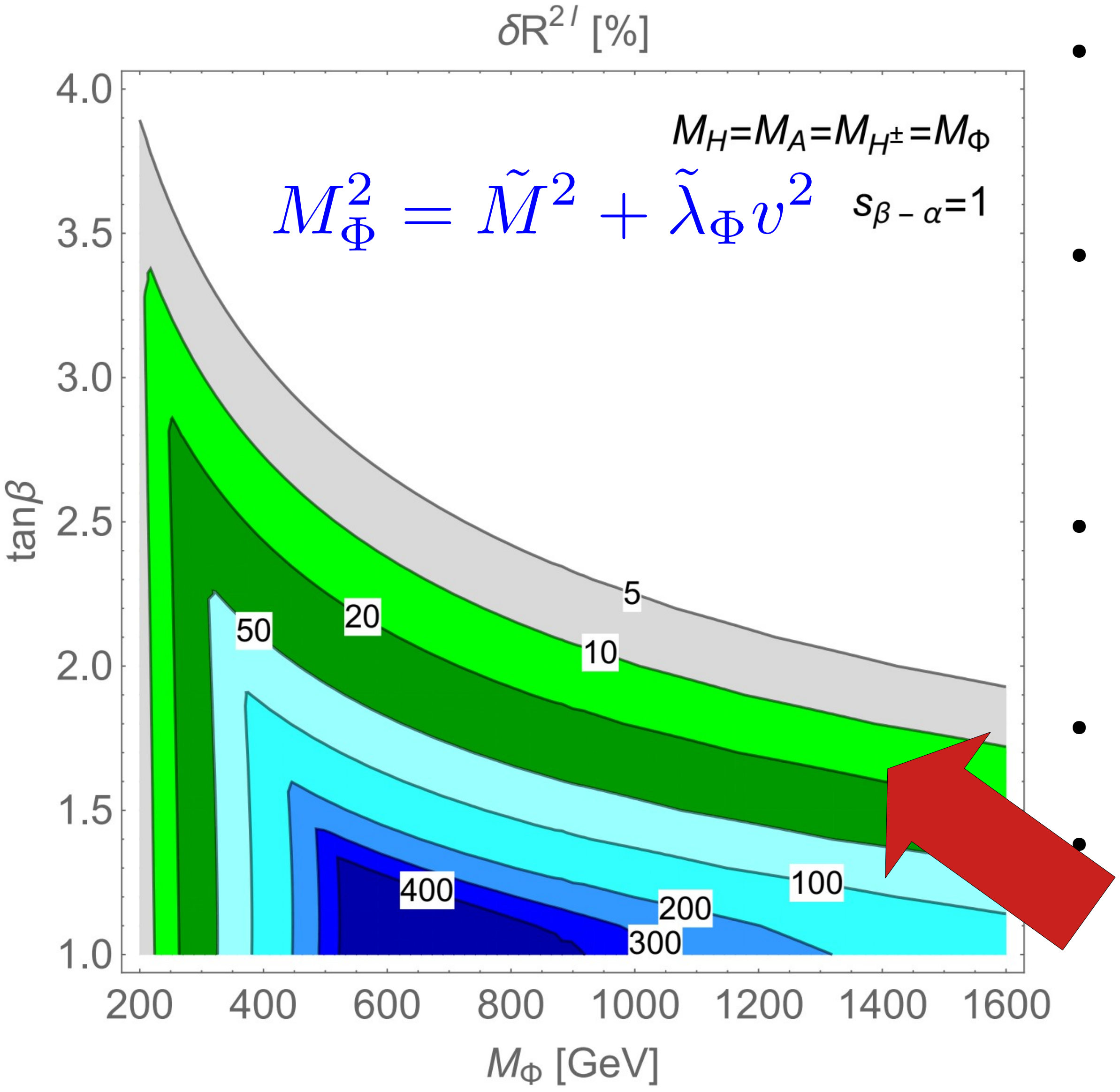

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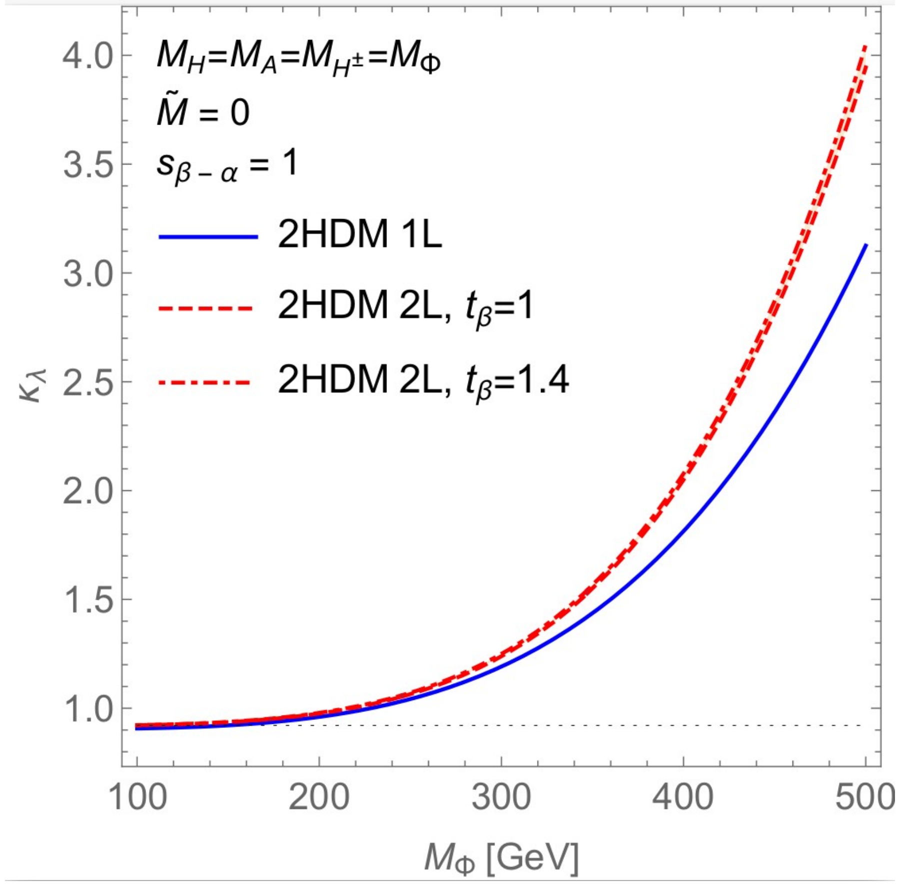
λ_{hhh} at two loops in more models

[JB, Kanemura '19]

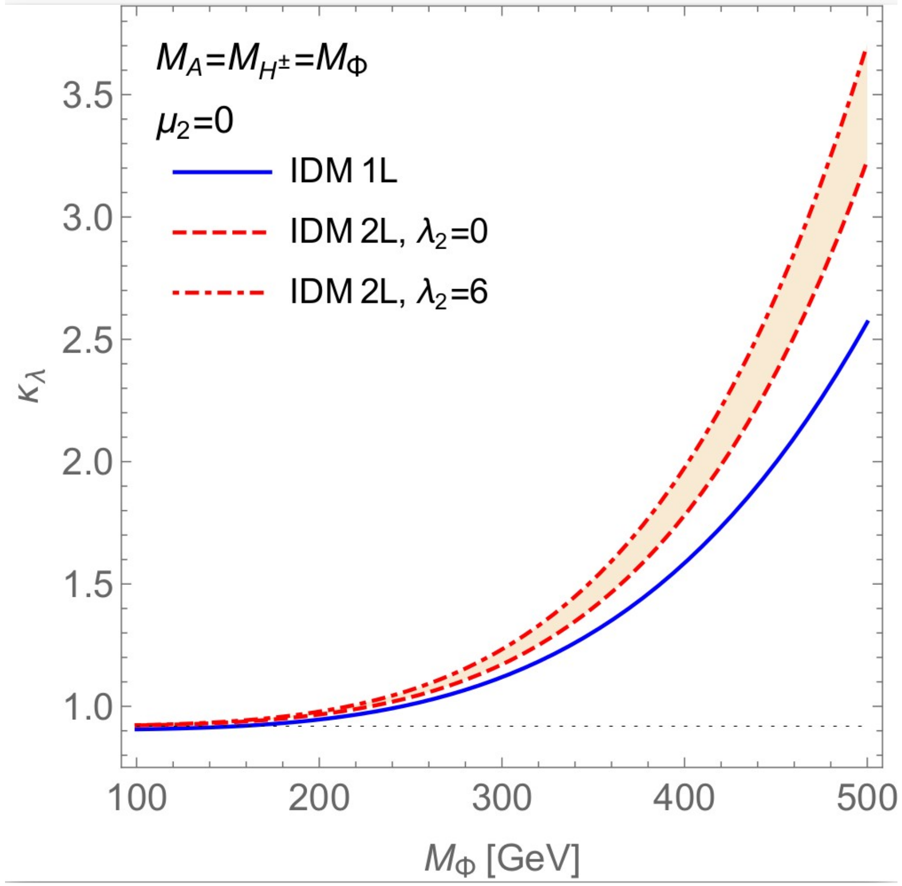
- Calculations in several other models: *Inert Doublet Model (IDM)*, *singlet extension of SM*
- Each model contains a **new parameter appearing from two loops**:

Aligned 2HDM → tanβ



tanβ constrained by perturbative unitarity → only small effects

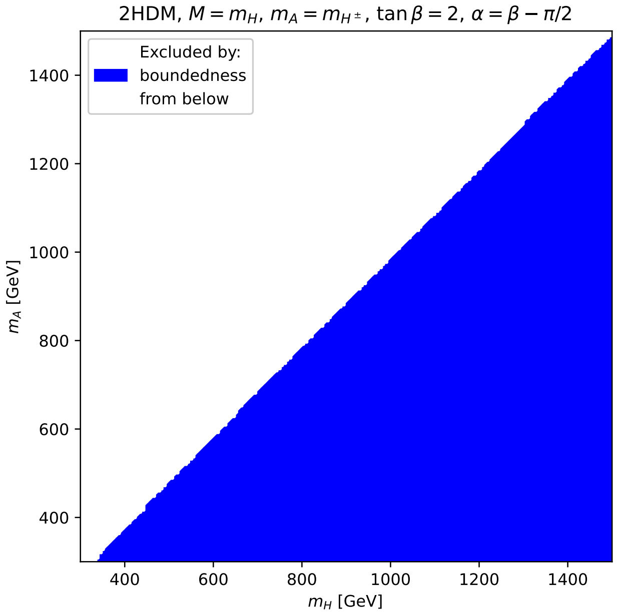
IDM → λ_2 (quartic coupling of inert doublet)



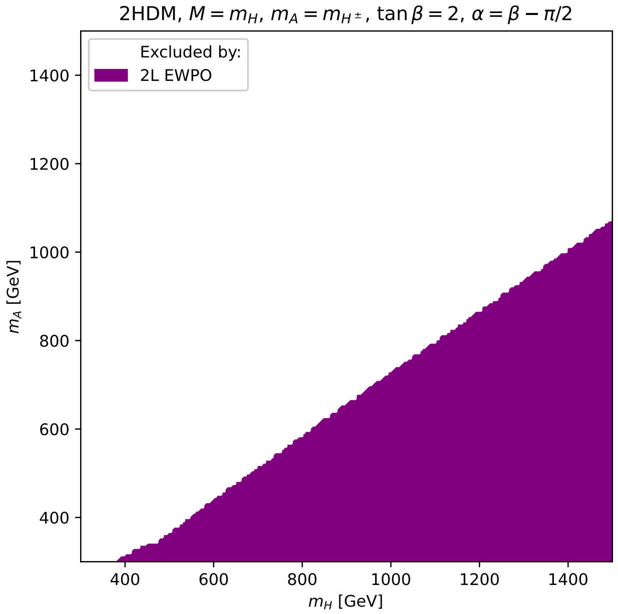
λ_2 is less contrained → **enhancement is possible**
(but 2L effects remain <u>well smaller</u> than 1L ones)

2HDM benchmark plane – individual theoretical constraints

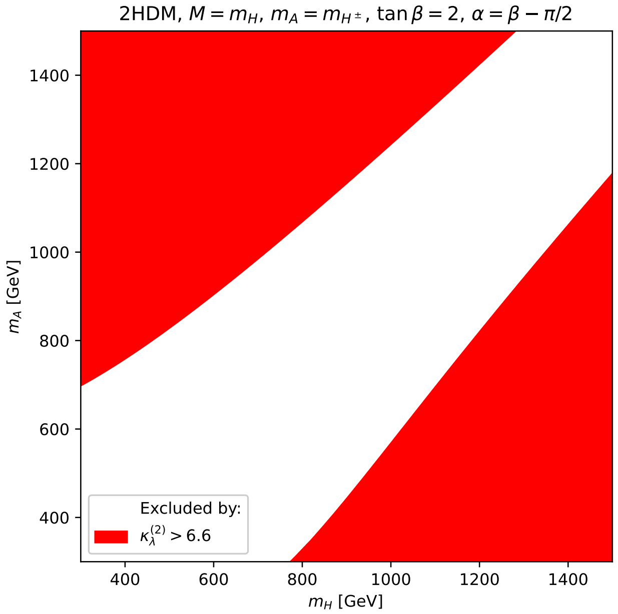
Constraints shown below are independent of 2HDM type



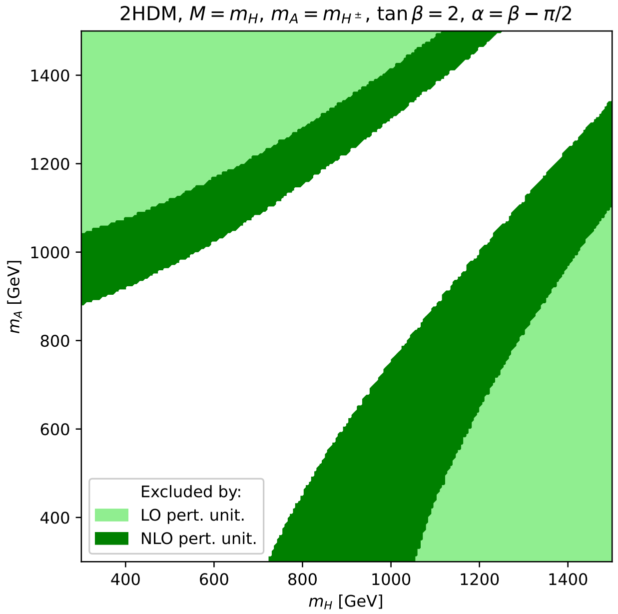
Boundedness from below



EW precision observables computed at 2L



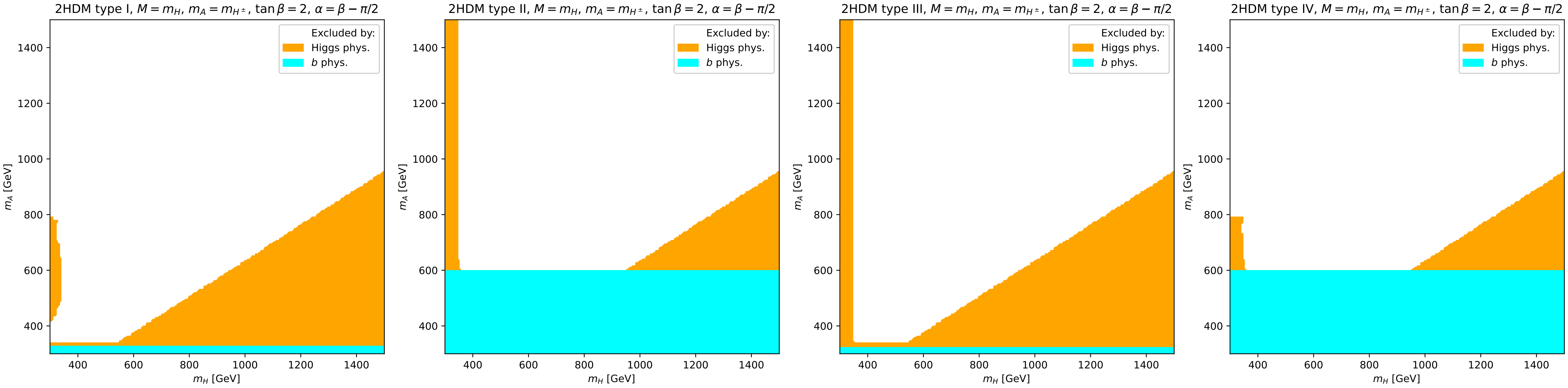
$\kappa_\lambda^{(2)} > 6.6$



Perturbative unitarity at (N)LO

2HDM benchmark plane – experimental constraints

i.e. Higgs physics (via HiggsBounds and HiggsSignals) and *b* physics (from [Gfitter group 1803.01853])



Type-I   Type-II   Type-III (LS)   Type-IV (flipped)

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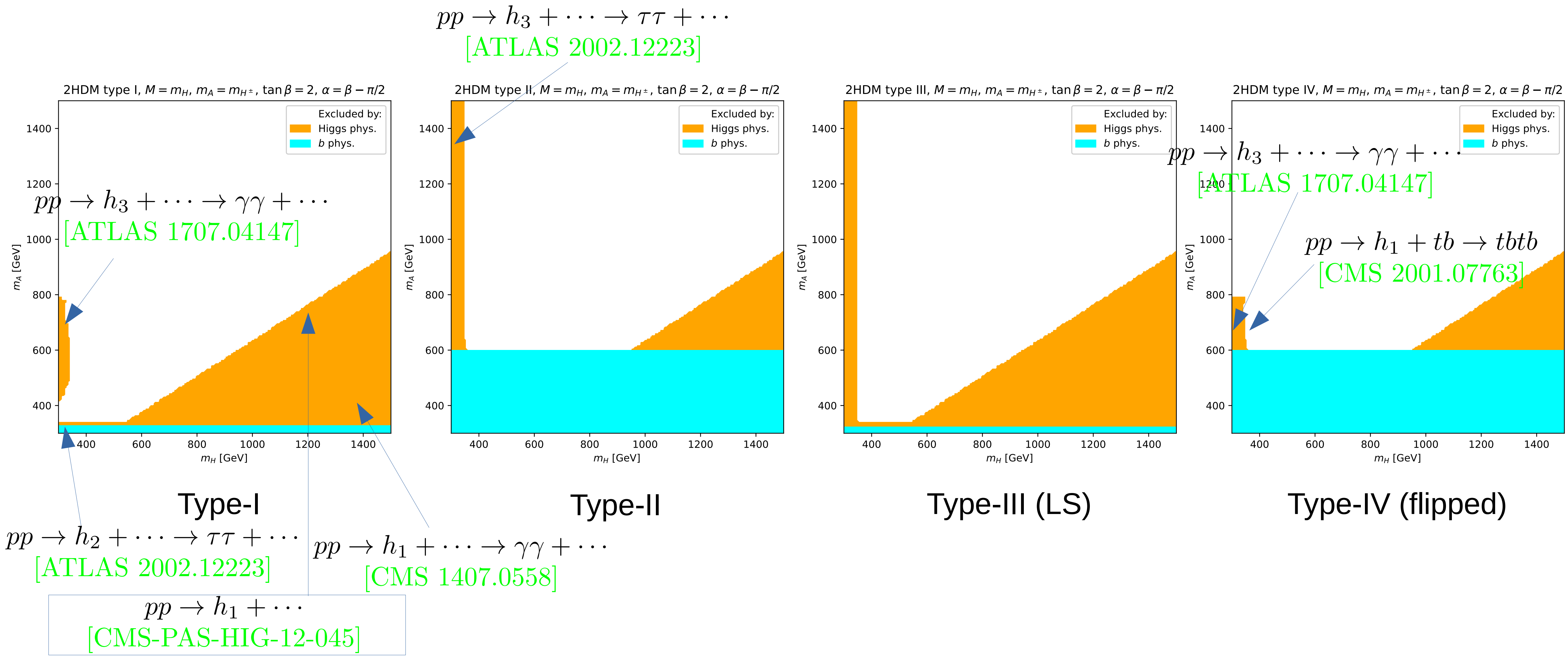
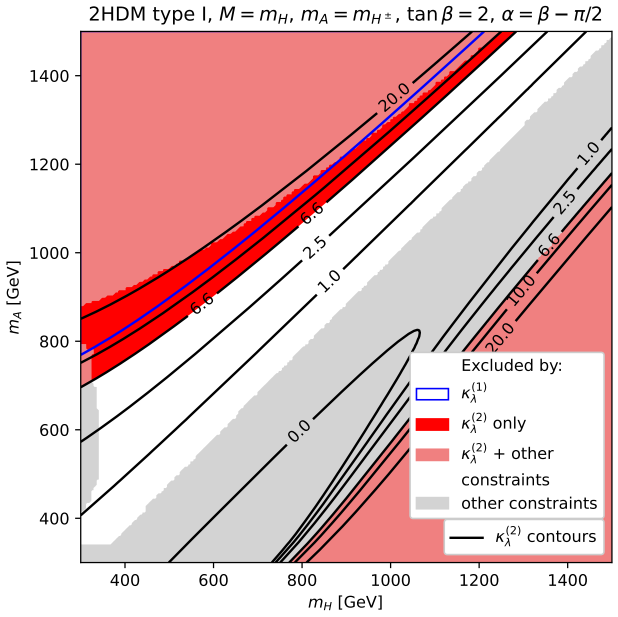
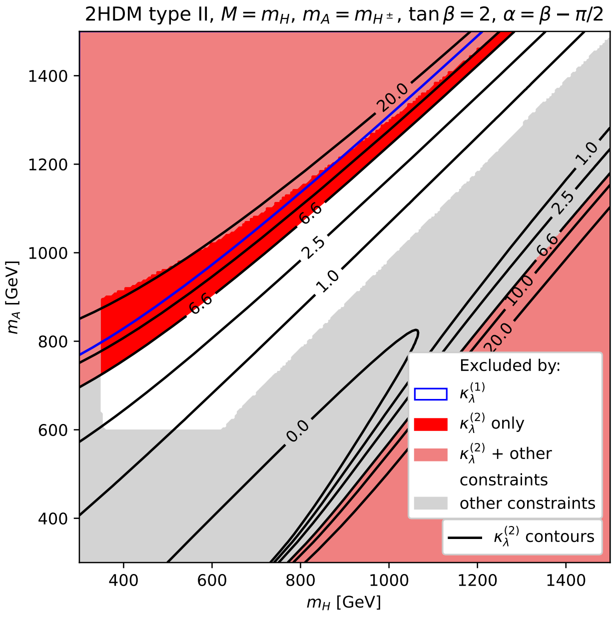


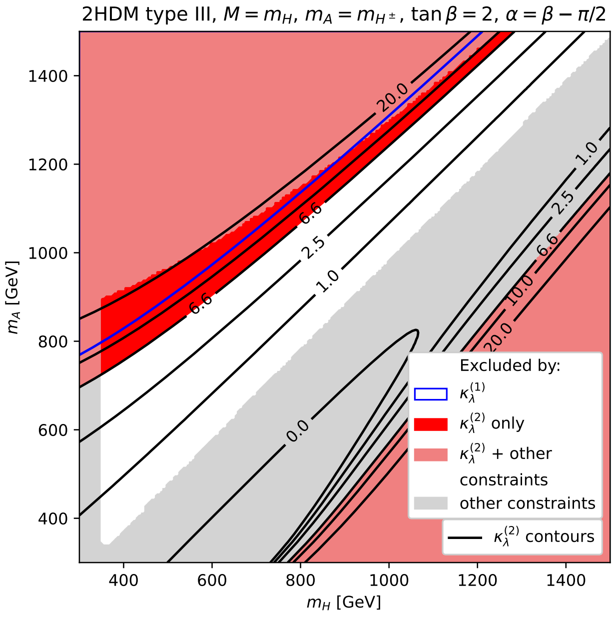
2HDM benchmark plane – results for all types



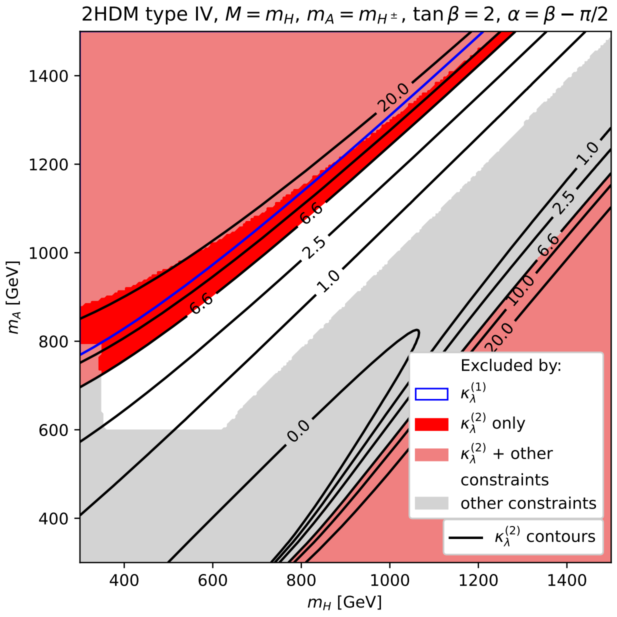
Type-I



Type-II



Type-III (LS)

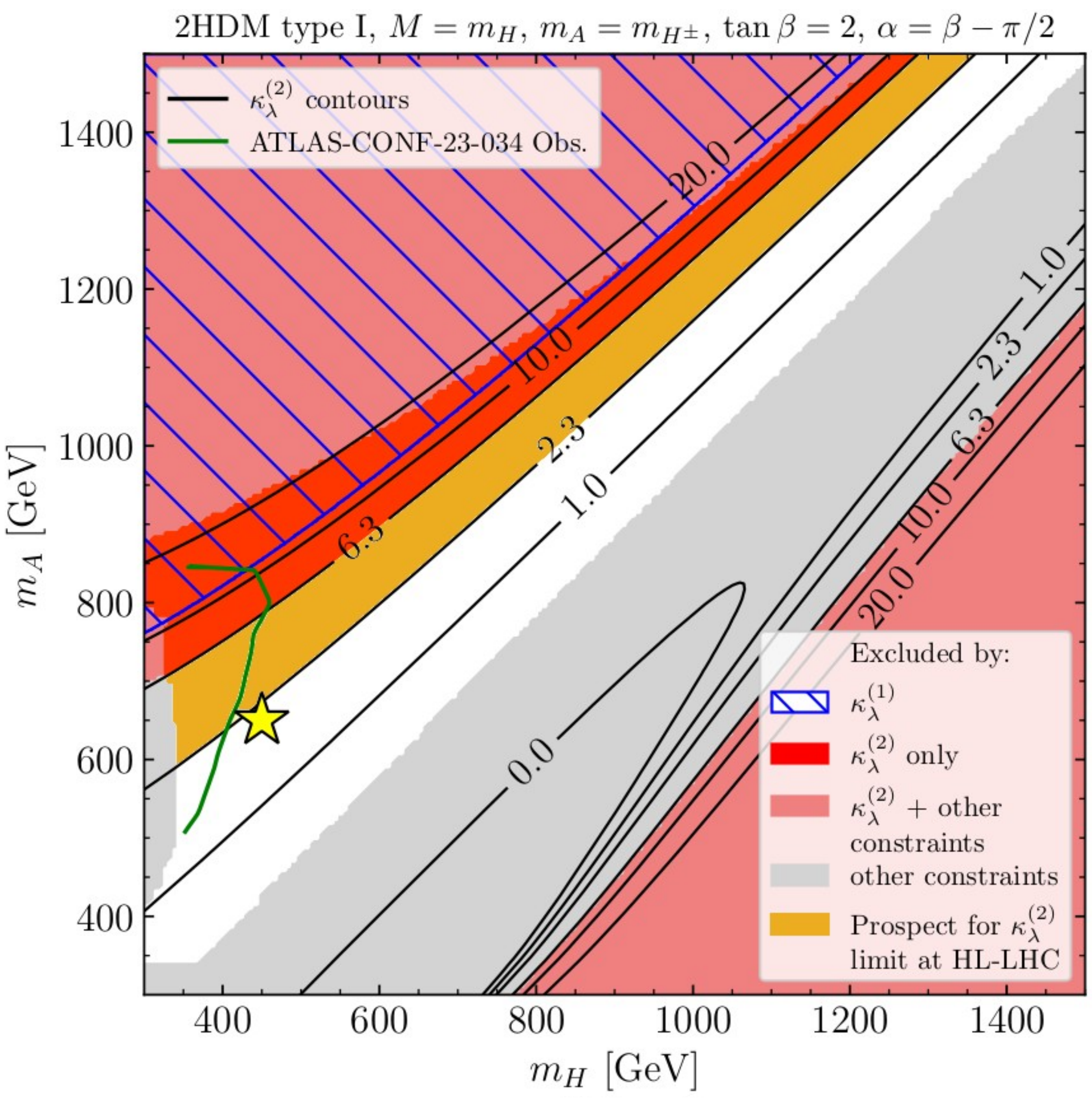


Type-IV (flipped)

A benchmark scenario in the aligned 2HDM

[Bahl, JB, Weiglein '23]

In view of recent ATLAS-CONF-23-034



- ***Green line:*** additional exclusion from direct searches for heavy Higgs bosons, via
  A → Z H
  with full LHC-Run2 data
  [ATLAS-CONF-23-034]

- ***Small excess*** *(2.9 σ)* for $\mathbf{m_H}$ **~ 450 GeV** and $\mathbf{m_A}$ **~ 650 GeV**
  → near region probed by κ_λ at HL-LHC
  → complementarity between direct and indirect searches!

λ_{hhh} in relation to thermal history of the EWPT

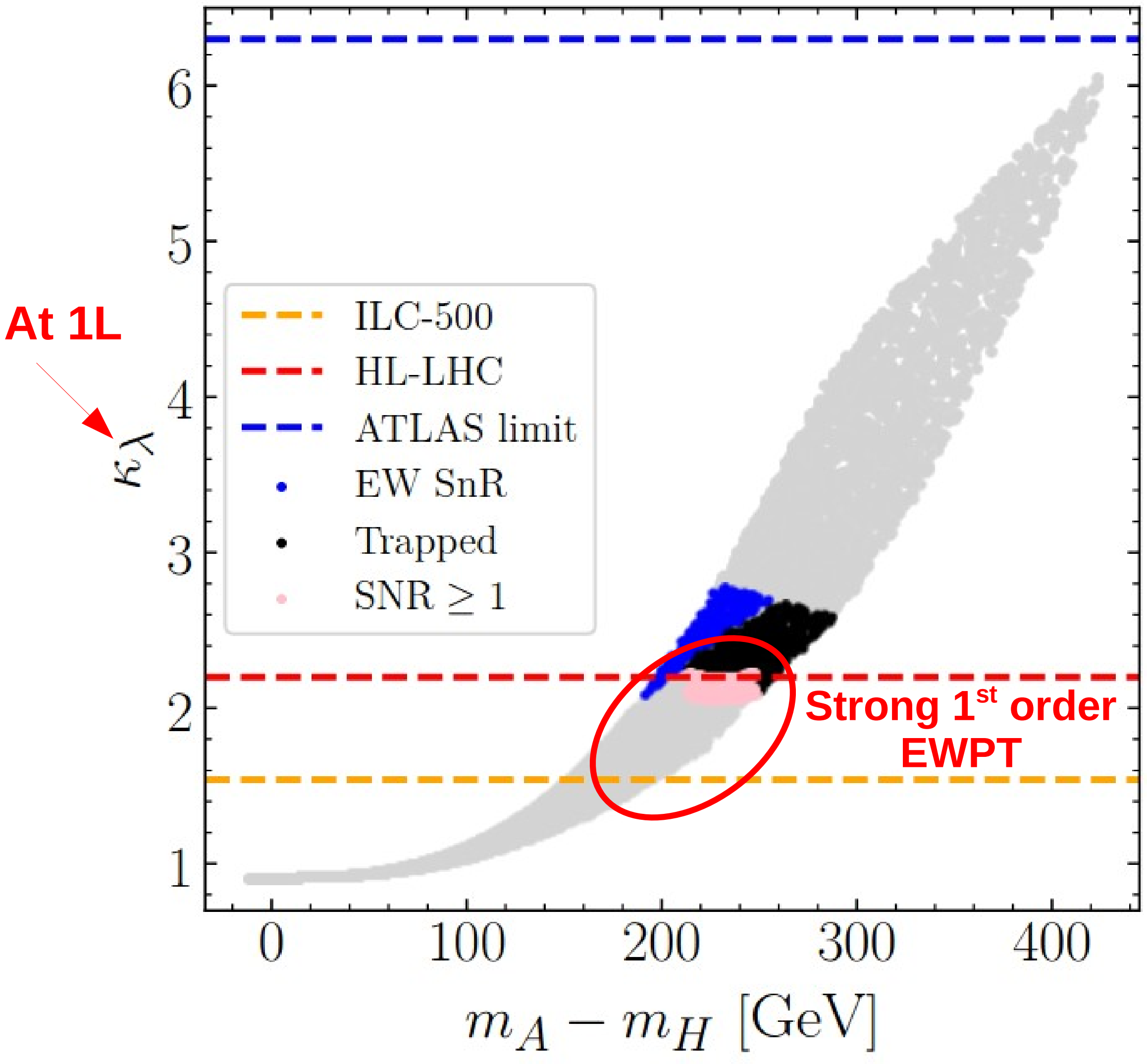


Figure from [Biekötter et al., 2208.14466]

- Corrections to λ_{hhh} correlate with the thermal history of the EWPT
  - If potential barrier is too high, the EWPT cannot occur → **vacuum trapping** (black region)
  - Conversely, it can occur that the **EW symmetry is not restored at high T** (blue region)
  - Strong 1st order EWPT, with gravitational waves (produced by bubble collisions) observable at LISA in pink
  - Impact of 2L corrections likely **strong** → works in progress with S. Kanemura and with H. Bahl, T. Biekötter, S. Heinemeyer, G. Weiglein

Sphaleron decoupling condition

$$\frac{v_c}{T_c} \gtrsim 1$$

All receive corrections!

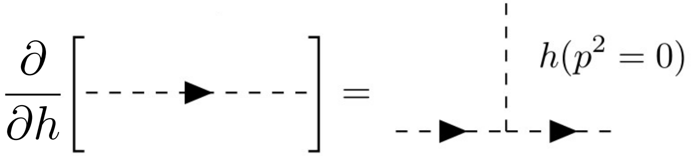
Higgs decay to two photons: Higgs Low-Energy Theorem

- Calculation of 2L 3-point functions with external momenta not possible in general
  (private results for integrals contributing to Γ(h→γγ) exist, but not available publicly)
- Assuming m_h << heavy BSM scalar masses, we can employ a **Higgs Low-Energy Theorem** (see e.g. [Kniehl, Spira '95])
- Compute **effective Higgs-photon coupling** $C_{h\gamma\gamma}$ of the form

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} C_{h\gamma\gamma} h F^{\mu\nu} F_{\mu\nu}$$

  by taking derivative of (unrenormalised) photon self-energy w.r.t Higgs field

$$C_{h\gamma\gamma} = \frac{\partial}{\partial h} \Pi_{\gamma\gamma}(p^2 = 0)\Big|_{h=0} \qquad \text{where } \Sigma^{\mu\nu}_{\gamma\gamma}(p^2) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\Pi_{\gamma\gamma}(p^2)$$

- Schematically: 

- Neglects incoming momentum on Higgs leg, but fine for m_h << $m_{H,A,H\pm}$
- Similar to approach of effective-potential calculations of Higgs mass or trilinear Higgs coupling

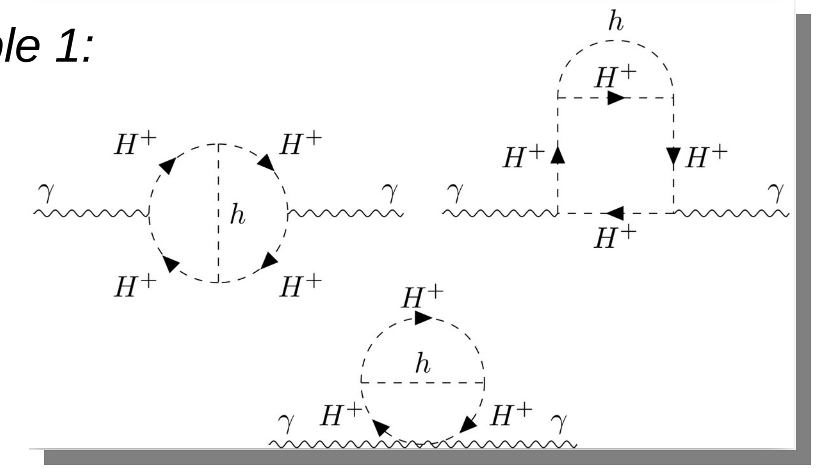
Higgs decay to two photons: what we include in our calculation

- **All known SM contributions**:
  - QCD up to 3L [Djouadi '08] (+ refs. therein)
  - EW SM-like to full 2L [Degrassi, Maltoni '05], [Actis et al. '09]

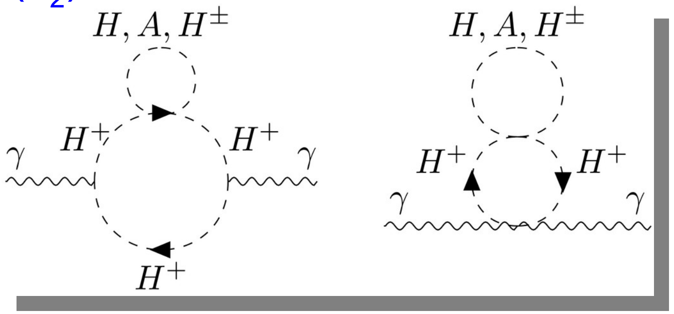
- Our new calculation: **leading two-loop BSM contributions**
  - genuine, dominant, 2L contributions involving inert scalars
  - purely scalar and fermion-scalar contributions to (1L)^2 terms from external-leg and VEV renormalisation

$$C_{h\gamma\gamma}^{(2),\text{ IDM}} = C_{h\gamma\gamma}^{\mathcal{O}(\lambda_3^2)} + C_{h\gamma\gamma}^{\mathcal{O}((\lambda_4+\lambda_5)^2)} + C_{h\gamma\gamma}^{\mathcal{O}((\lambda_4-\lambda_5)^2)} + C_{h\gamma\gamma}^{\mathcal{O}(\lambda_2)} + C_{h\gamma\gamma}^{\text{ext.-leg.+VEV}}$$

Example 1:
$O(\lambda_3^2)$



Example 2:
$O(\lambda_2)$



- Photon self-energy diagrams generated with `FeynArts`, computed with `FeynCalc` and `Tarcer`, reduced to (limits of) integrals known analytically; then derivative w.r.t. h taken

Higgs decay to two photons: renormalisation schemes and checks

- Calculation performed with
  - on-shell (OS) renormalisation of masses and VEV
  - for μ_2, we applied the "OS" prescription of [JB, Kanemura '19] (devised for calculation of λ_{hhh}) to ensure renormalisation scale independence + apparent/proper decoupling of BSM contributions
  - gauge-less limit g_2, $g_Y \to 0$

- Checks of our calculation:
  - ***Ward-Takahashi identity*** for photon self-energy contributions at 2L

$$\Sigma^{\mu\nu}_{\gamma\gamma}(p^2 = 0) = 0$$

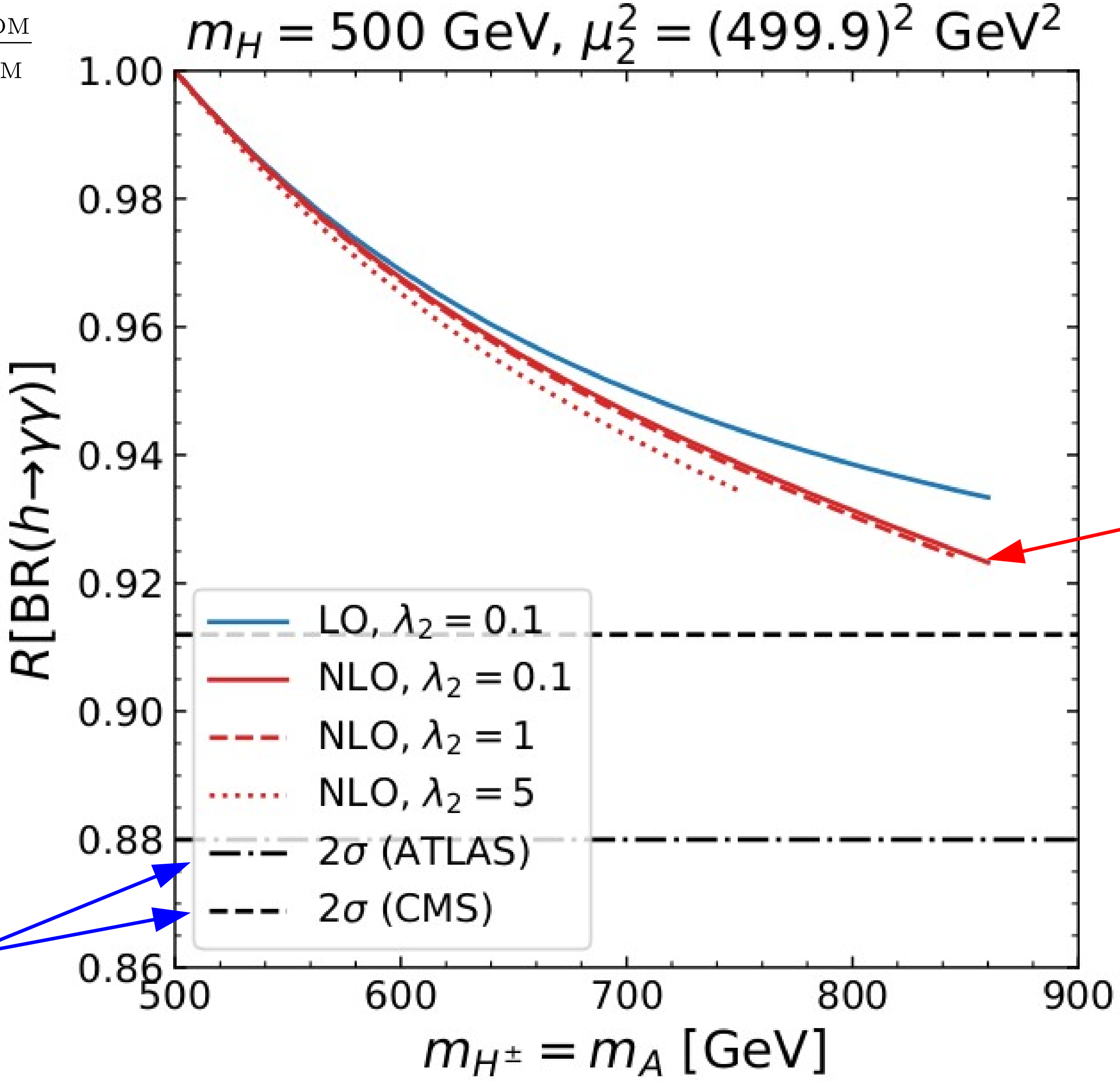
  - ***UV finiteness***: cancellation of double- and single-UV poles
  - ***IR finiteness***: individual diagrams are IR divergent in limit m_G, $m_h \to 0$ (*Goldstone Boson Catastrophe*), but divergences must cancel in total result. m_G, m_h kept as IR regulators in individual diagrams, and we verify that all IR divergences (power-like, log-like, UV-IR mixed) cancel for each contribution

Higgs decay to two photons: results in heavy Higgs scenario

$$R[\mathrm{BR}(h \to \gamma\gamma)] \equiv \frac{\mathrm{BR}(h \to \gamma\gamma)_{\mathrm{IDM}}}{\mathrm{BR}(h \to \gamma\gamma)_{\mathrm{SM}}}$$

λ_2 : inert doublet self-coupling

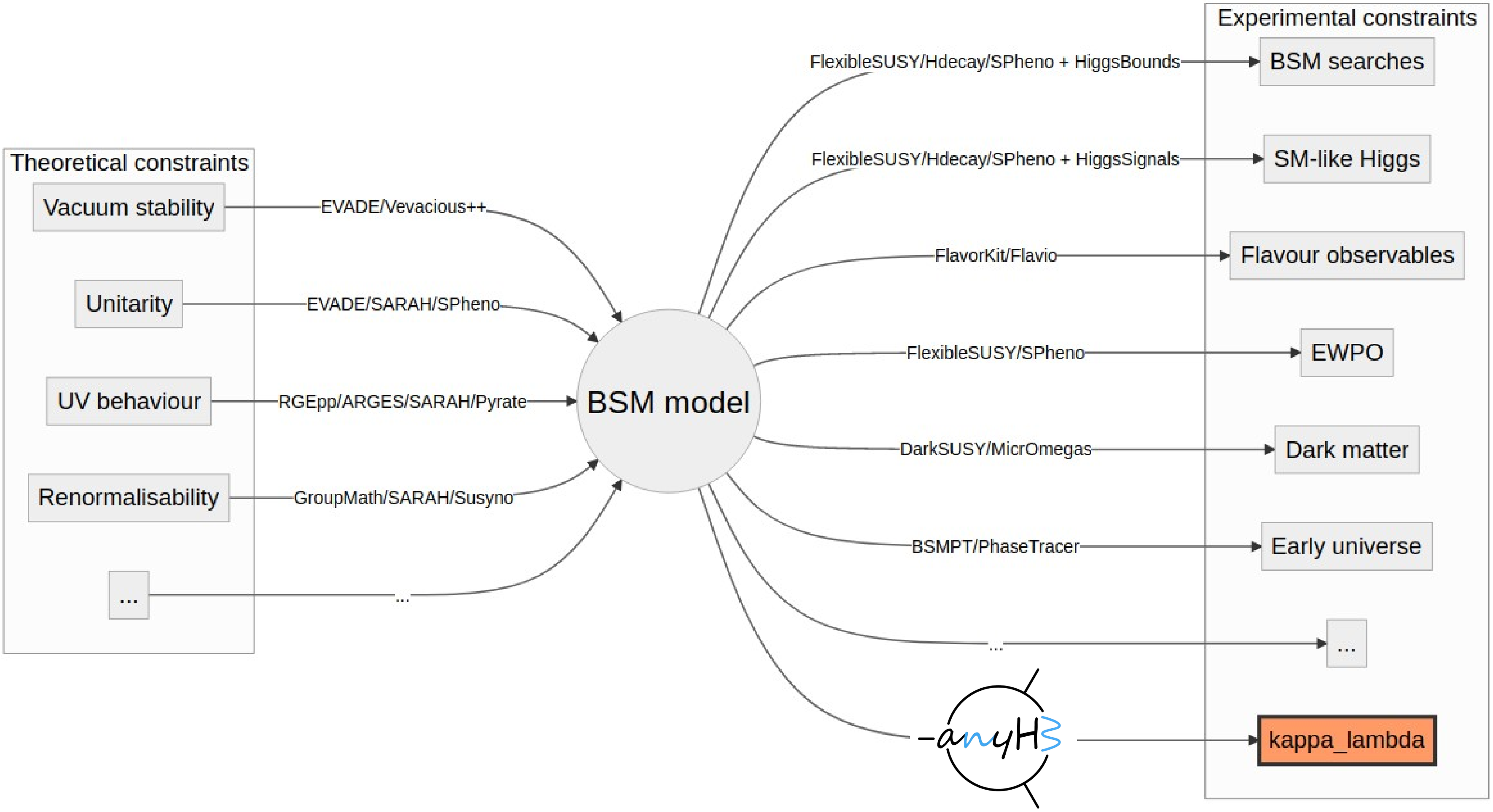
[Aiko, JB, Kanemura '23]



Range of $m_{H\pm}=m_A$ limited by pert. unit.

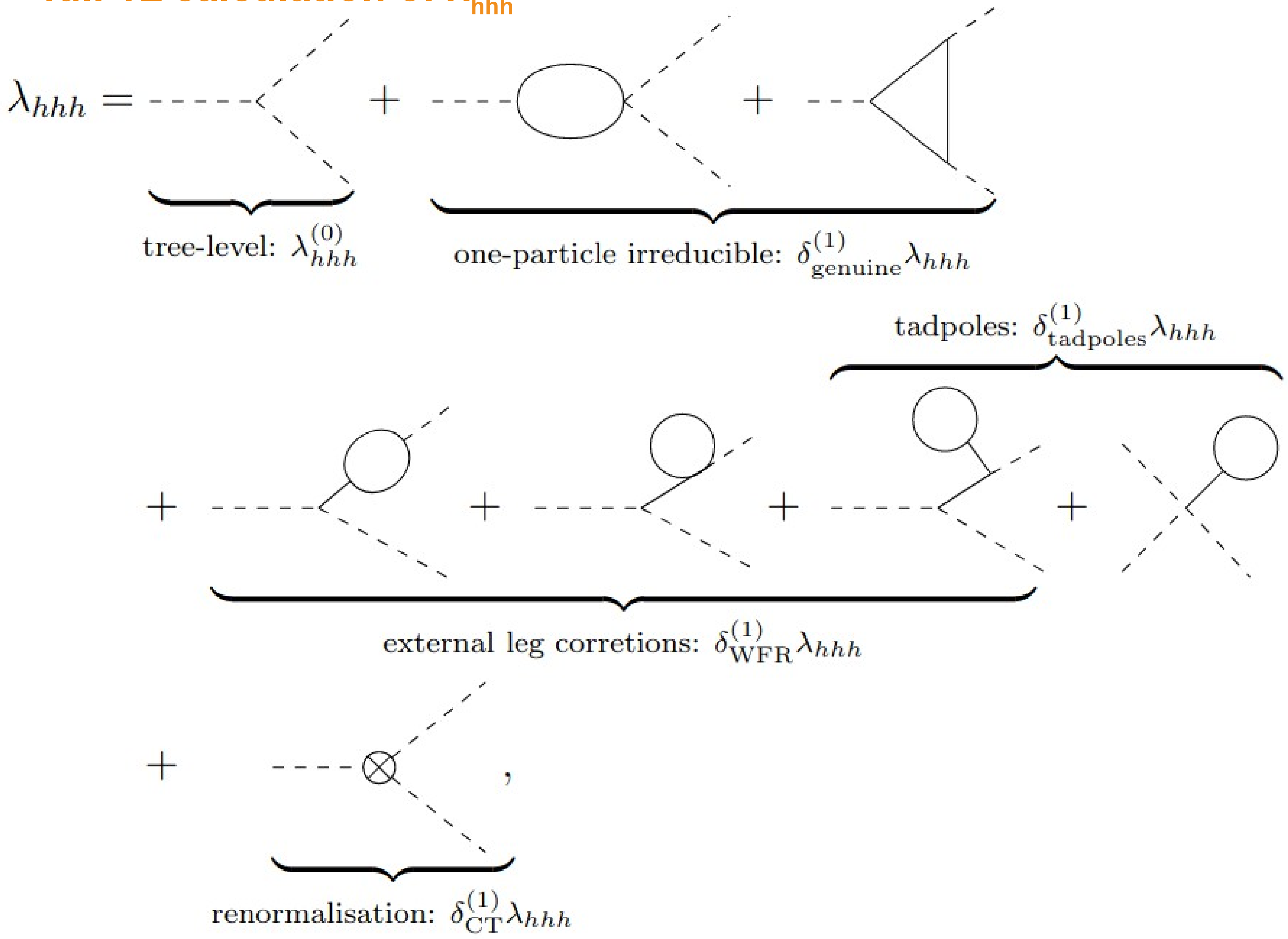
Expected 2σ bounds at HL-LHC

λ_{hhh} within the landscape of automated tools



Computing λ_{hhh} in general renormalisable theories: ingredients

anyH3 → full 1L calculation of λ_{hhh}

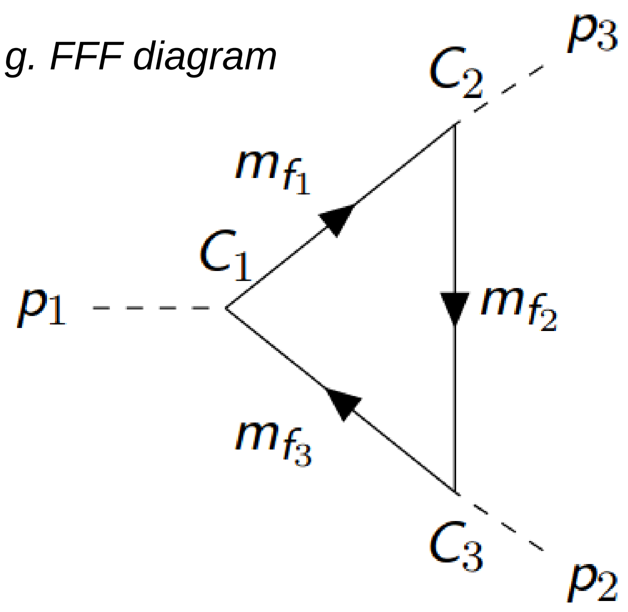


- Solid lines:
  - *scalars,*
  - *fermions,*
  - *gauge/vector bosons,*
  - *ghosts*
- Restrictions on **particles** and/or **topologies** possible
- **Renormalisation performed automatically** (*more in following*)

Computing λ_{hhh} in general renormalisable theories: method

Our method: we derive and implement analytic results for **generic diagrams**, i.e. assuming generic

e.g. FFF diagram



- Couplings $C_i = C_i^L P_L + C_i^R P_R, \text{ where } P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$
- Masses on the internal lines m_{fi}, i=1,2,3
- External momenta p_i, i=1,2,3

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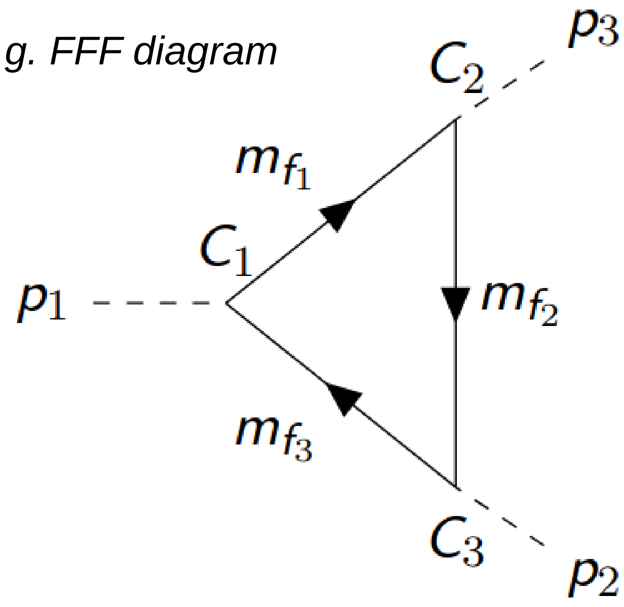

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- Masses on the internal lines m_{f_i}, i=1,2,3
- External momenta p_i, i=1,2,3

$$= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1}\mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + 2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3}))$$

(**B0**, **C0**, **C1**, **C2**: loop functions)

Computing λ_{hhh} in general renormalisable theories: method

Our method: we derive and implement analytic results for **generic diagrams**, i.e. assuming generic

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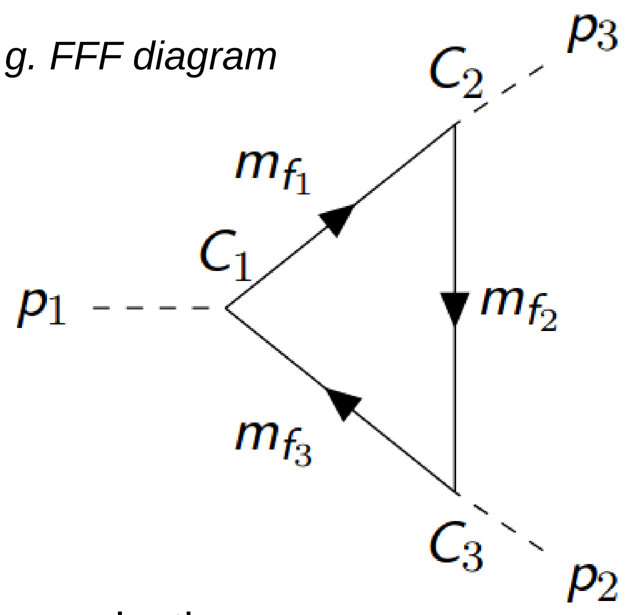

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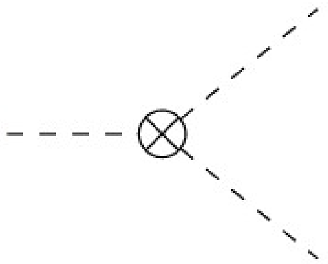
(**B0**, **C0**, **C1**, **C2**: loop functions)

For evaluation:

- Apply to concrete (B)SM model, using inputs in `UFO` format [Degrande et al., '11], [Darmé et al. '23]
- Evaluate loop functions via `COLLIER` [Denner et al '16] interface, `pyCollier`
- All included in public tool `anyH3` [Bahl, JB, Gabelmann, Weiglein '23]

Flexible choice of renormalisation schemes

$\delta^{(1)}_{\mathrm{CT}}\lambda_{hhh} =$  $= \; ?$

- **1L calculation** → renormalisation of all parameters entering λ_{hhh} at tree-level
- In general:

$$(\lambda^{(0)}_{hhh})^{\mathrm{BSM}} = (\lambda^{(0)}_{hhh})^{\mathrm{BSM}}(\underbrace{m_h \simeq 125\ \mathrm{GeV}, v \simeq 246\ \mathrm{GeV}}_{\text{SM sector}}, \underbrace{m_{\Phi_i}}_{\substack{\text{BSM}\\ \text{masses}}}, \underbrace{\alpha_i}_{\substack{\text{BSM}\\ \text{mixing angles}}}, \underbrace{v_i}_{\substack{\text{BSM}\\ \text{VEVs}}}, \underbrace{g_i}_{\substack{\text{indep.}\\ \text{BSM coups.}}})$$

- Most automated codes: $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$ only
- **anyH3**: much more flexibility, following **user choice**:
  - **SM sector** (m_h, v): fully OS or $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$
  - **BSM masses**: OS or $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$
  - **Additional couplings/vevs/mixings**: by default $\overline{\mathrm{MS}}$, but **user-defined ren. conditions** also possible!

$$\delta^{(1)}_{\mathrm{CT}}\lambda_{hhh} = \sum_x \left(\frac{\partial}{\partial x}(\lambda^{(0)}_{hhh})^{\mathrm{BSM}}\right)\delta^{\mathrm{CT}}x\,, \qquad \text{with } x \in \{m_h, v, m_{\Phi_i}, v_i, \alpha_i, g_i, \text{etc.}\}$$

Renormalised in $\overline{\mathrm{MS}}$, OS, in custom schemes, etc.

(Default) Renormalization choice of $(v^{\text{SM}})^{\text{OS}}$ and $(m_i^2)^{\text{OS}}$

- $v^{\text{OS}} \equiv \frac{2M_W^{\text{OS}}}{e}\sqrt{1-\frac{M_W^{2\,\text{OS}}}{M_Z^{2\,\text{OS}}}}$ with
  - $\delta^{(1)}M_V^{2\,\text{OS}} = \frac{\Pi_V^{(1),T}}{M_V^{2\,OS}}(p^2 = M_V^{2\,OS})\,,\, V = W, Z$
  - $\delta^{(1)}e^{\text{OS}} = \frac{1}{2}\dot{\Pi}_\gamma(p^2=0) + \text{sign}(\sin\theta_W)\frac{\sin\theta_W}{M_Z^2\cos\theta_W}\Pi_{\gamma Z}(p^2=0)$
- attention *(i)*: $\rho^{\text{tree-level}} \neq 1 \rightarrow$ further CTs needed (depends on the model)

  $\longrightarrow$ ability to define *custom* renormalisation conditions
- scalar masses: $m_i^{\text{OS}} = m_i^{\text{pole}}$
  - $\delta^{\text{OS}}m_i^2 = -\widetilde{\text{Re}}\Sigma_{h_i}^{(1)}|_{p^2=m_i^2}$
  - $\delta^{\text{OS}}Z_i = \widetilde{\text{Re}}\frac{\partial}{\partial p^2}\Sigma_{h_i}^{(1)}|_{p^2=m_i^2}$
- attention *(ii)*: scalar mixing may also require further CTs/tree-level relations

All bosonic one- & two-point functions and their derivatives for general QFTs are required for flexible OS renormalisation.

Features of `anyH3`, so far

- Import/conversion of any UFO model
- Definition of renormalisation schemes

```
# schemes.yml
```

```
renormalization_schemes:
  MS:
    SM_names:
      Higgs-Boson: h1
    VEV_counterterm: MS
    mass_counterterms:
      h1: MS
      h2: MS
  OS:
    SM_names:
      Higgs-Boson: h1
    VEV_counterterm: OS
    custom_CT_hhh: 'dbetaH =
f"({Sigma(''Hm1'',''Hm2'',momentum=''0'')} +
{Sigma(''Hm1'',''Hm2'',momentum=''MHm2**2'')})/-
(2*MHm2**2)"

      dTanBeta = f"({dbetaH})/cos(betaH)**2"
```

(extract from schemes.yml for 2HDM)

...

- Analytical / numerical / LaTeX outputs
- **3 user interfaces**:
  - Python library

```
from anyBSM import anyH3
myfancymodel = anyH3('path/to/UFO/model')
result = myfancymodel.lambdahhh()
```

  - Command line
  - Mathematica interface
- **Perturbative unitarity checks** available (at tree level and in high-energy limit for now)
- Can be used together with a spectrum generator and **handles SLHA format**
- Efficient **caching** available
- Lots more!

Example results from anyH3

A cross-check: the decoupling limit

- Consider the decoupling limit in several BSM models

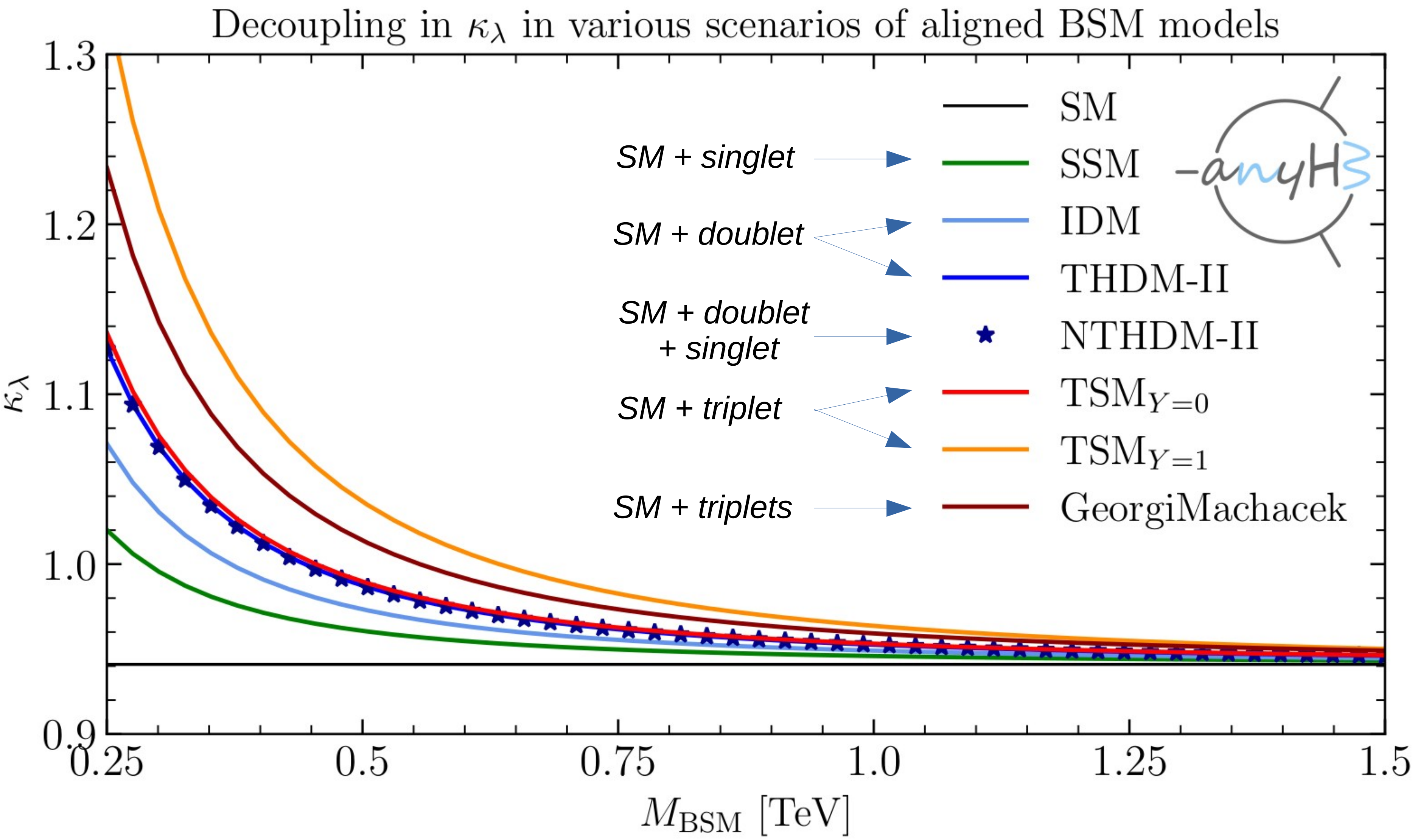
$$M_{\mathrm{BSM}}^2 = \mathcal{M}^2 + \tilde{\lambda} v^2$$

$\mathcal{M}$: BSM mass scale
$\tilde{\lambda}$: Quartic couplings

- Increase BSM mass scale

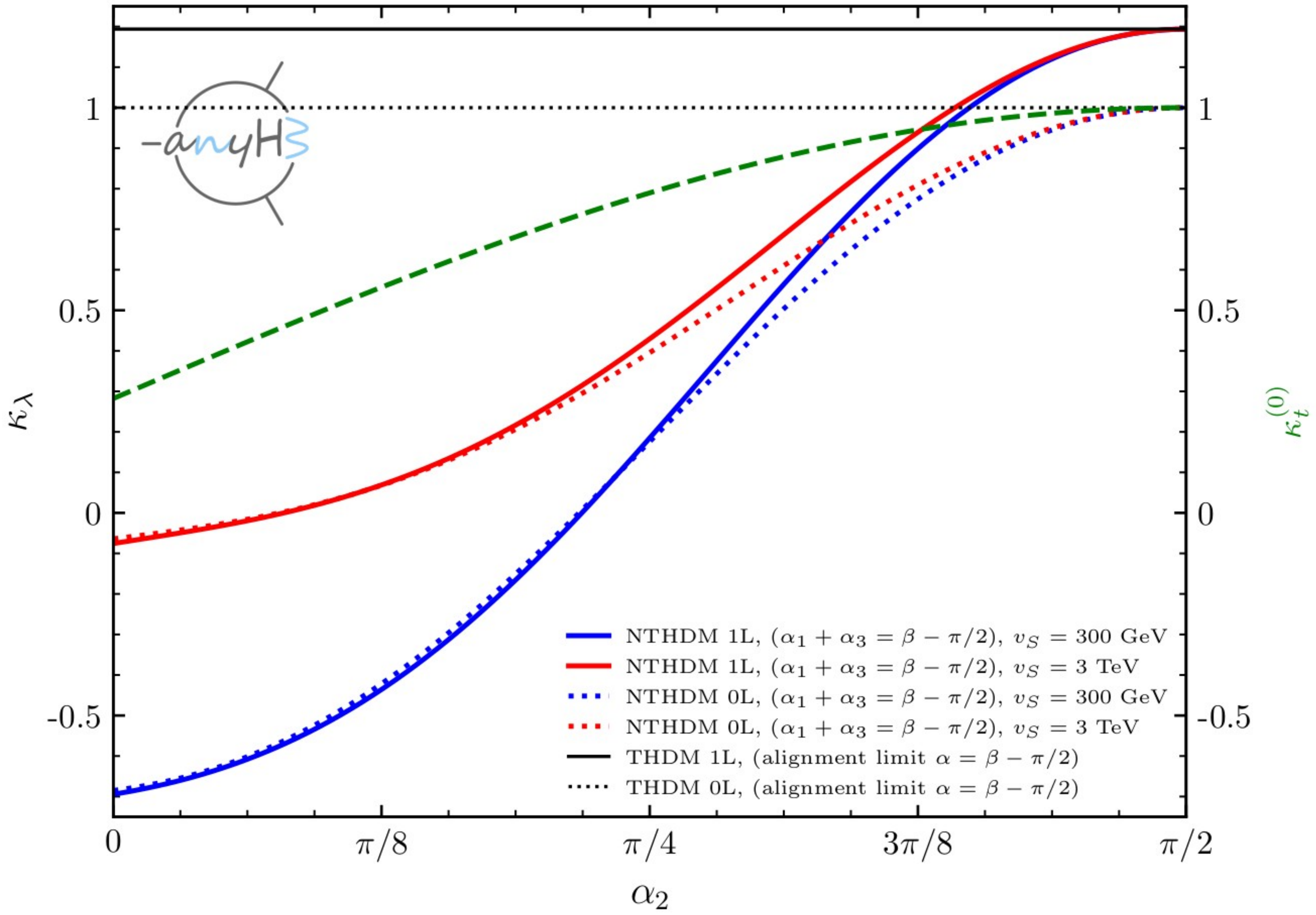
$$\mathcal{M} \to \infty$$

- BSM corrections to should vanish (c.f. decoupling theorem [Appelquist, Carrazone '75])



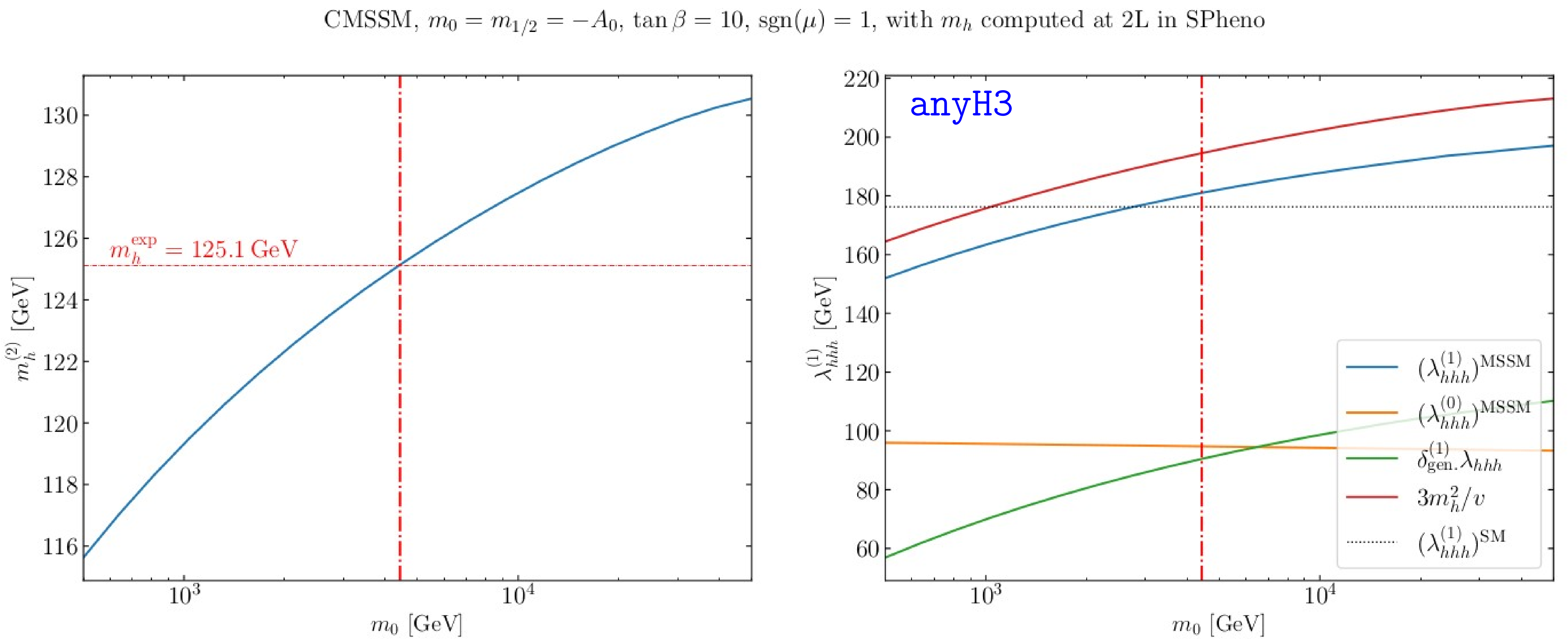
More new results with `anyH3`: an example in the N2HDM

NTHDM: $m_{h_2} = 125.1$ GeV, $m_{h_1} = m_{h_3} = m_A = m_{H^\pm} = 300$ GeV, $\tilde{\mu} = 100$ GeV, $t_\beta = 2$



- **N2HDM = 2HDM + real singlet**
- CP-even sector: 3 states h_1, h_2, h_3, with 3 mixing angles α_1, α_2, α_3
- Here $\alpha_2 \to \pi/2$ → recover 2HDM (itself in alignment limit)
- We can study e.g. the relative sign of κ_λ and κ_t → affects double-Higgs production
- κ_t too far away from 1 excluded

Full one-loop calculation of λ_{hhh} in the MSSM



- Example for a very simple version of the constrained MSSM → BSM parameters m_0, $m_{1/2}$, A_0, sgn(μ), tanβ
- For each point, M_h computed at 2L with SPheno, and SLHA output of SPheno used as input of anyH3