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A MODEL WITH COSMOLOGICAL BELL INEQUALITIES

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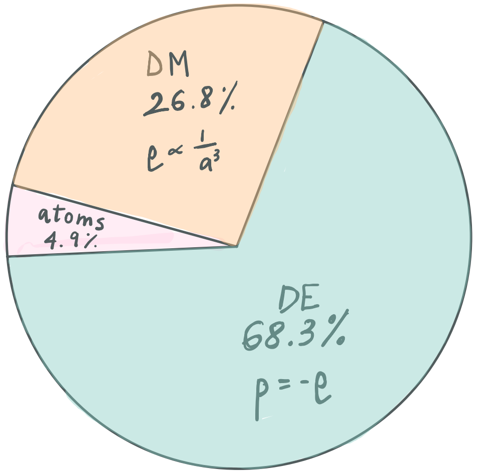
Based on: *arXiv:1508.01082*

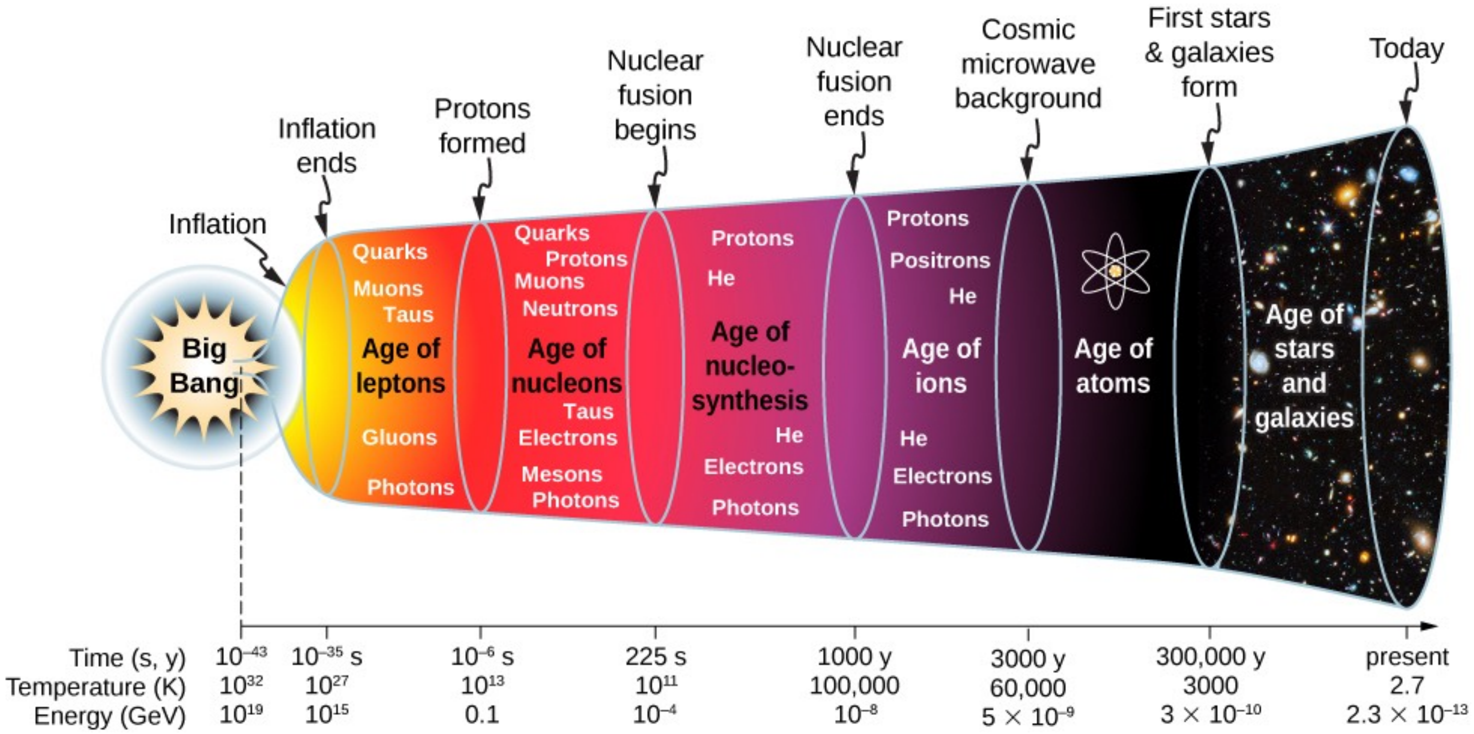
Gravity & Entanglement Workshop ~ Hamburg, October 9

INTRODUCTION:
INFLATION AND BELL EXPERIMENTS

THE OBSERVABLE UNIVERSE

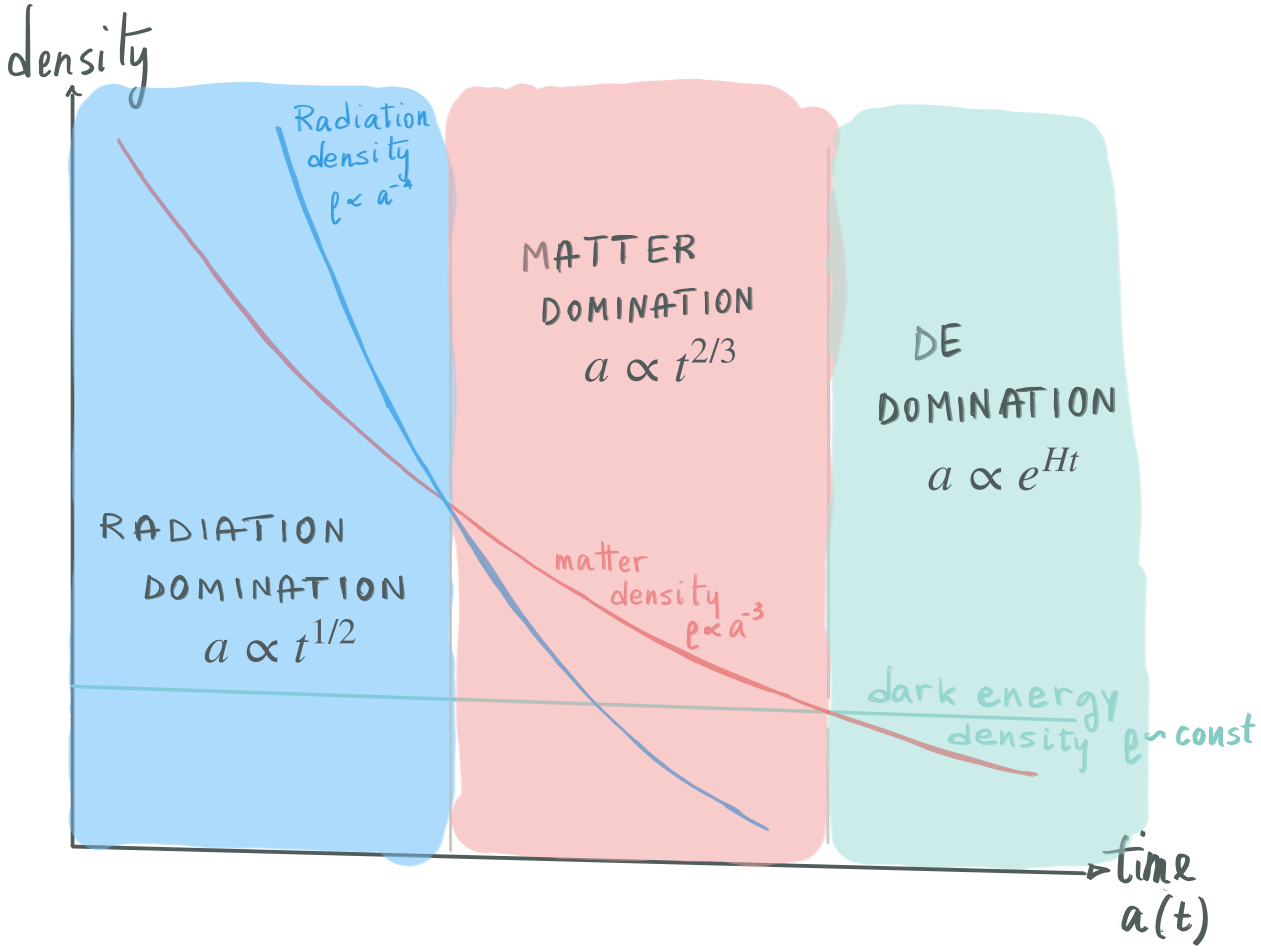
ΛCDM

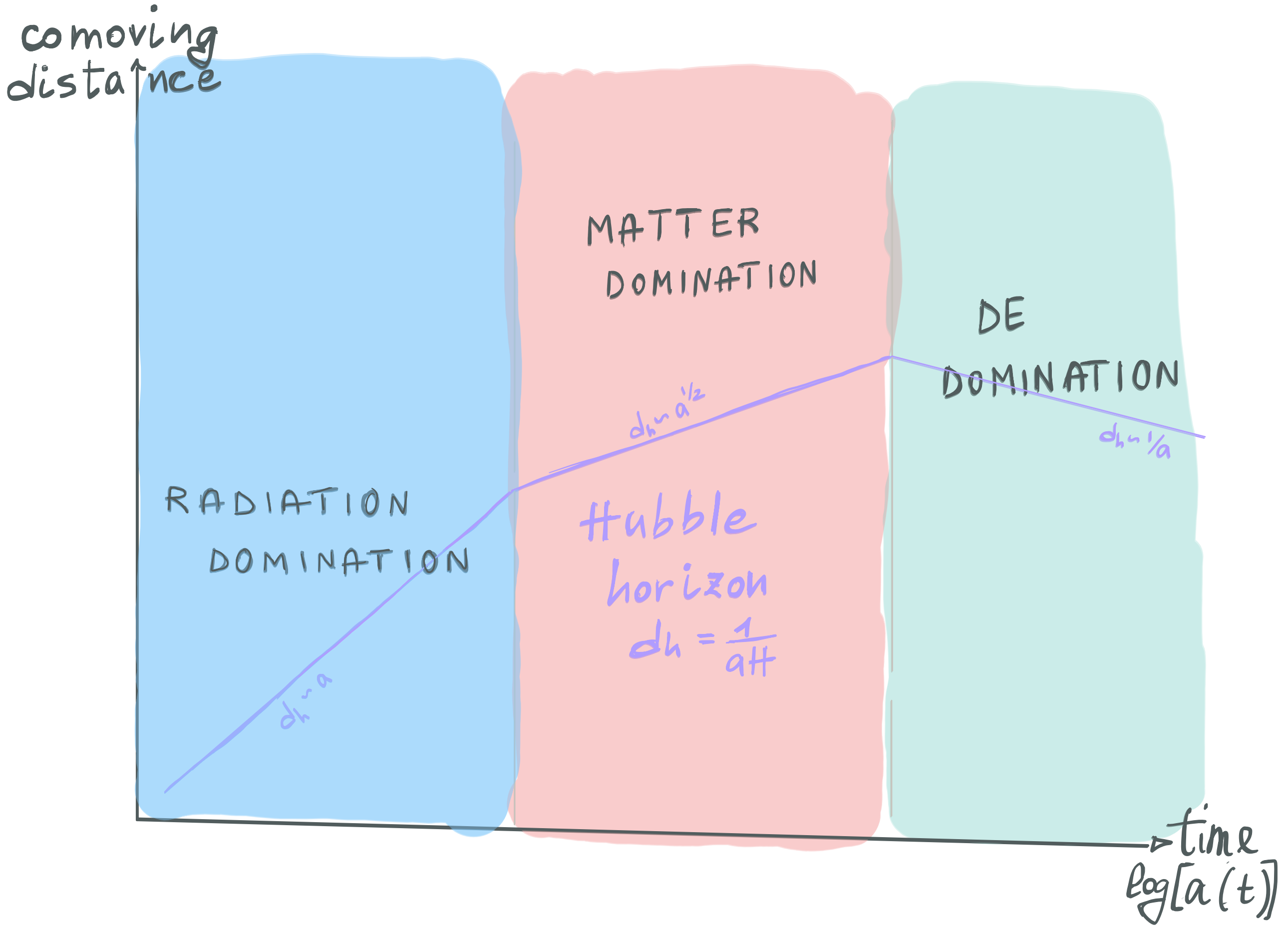




Big Bang Cosmology

THE OBSERVABLE UNIVERSE - HORIZON





$$ds^2 = dt^2 + a(t)^2 d\vec{x}^2 = a(\eta)^2(-d\eta^2 + d\vec{x}^2)$$

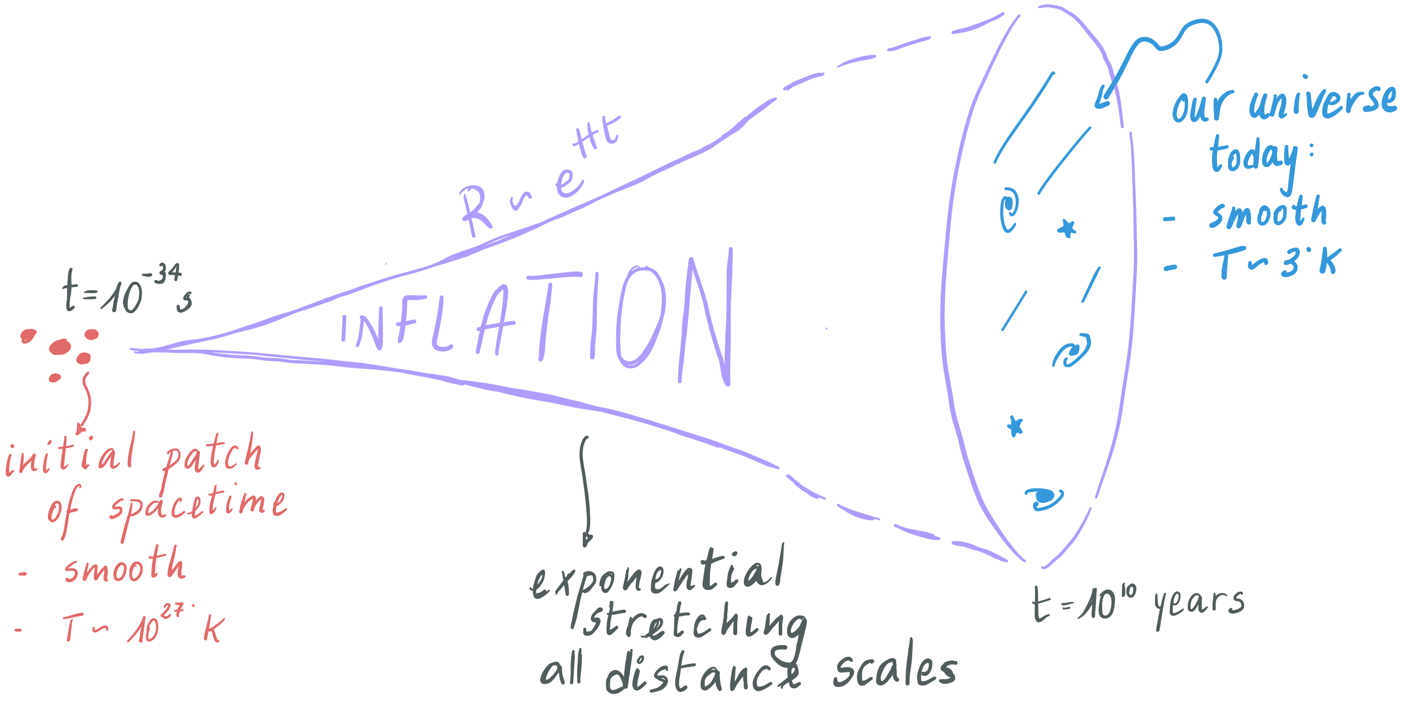
$$d_h = \frac{1}{aH} \qquad \frac{\dot{a}}{a} = H(t)$$

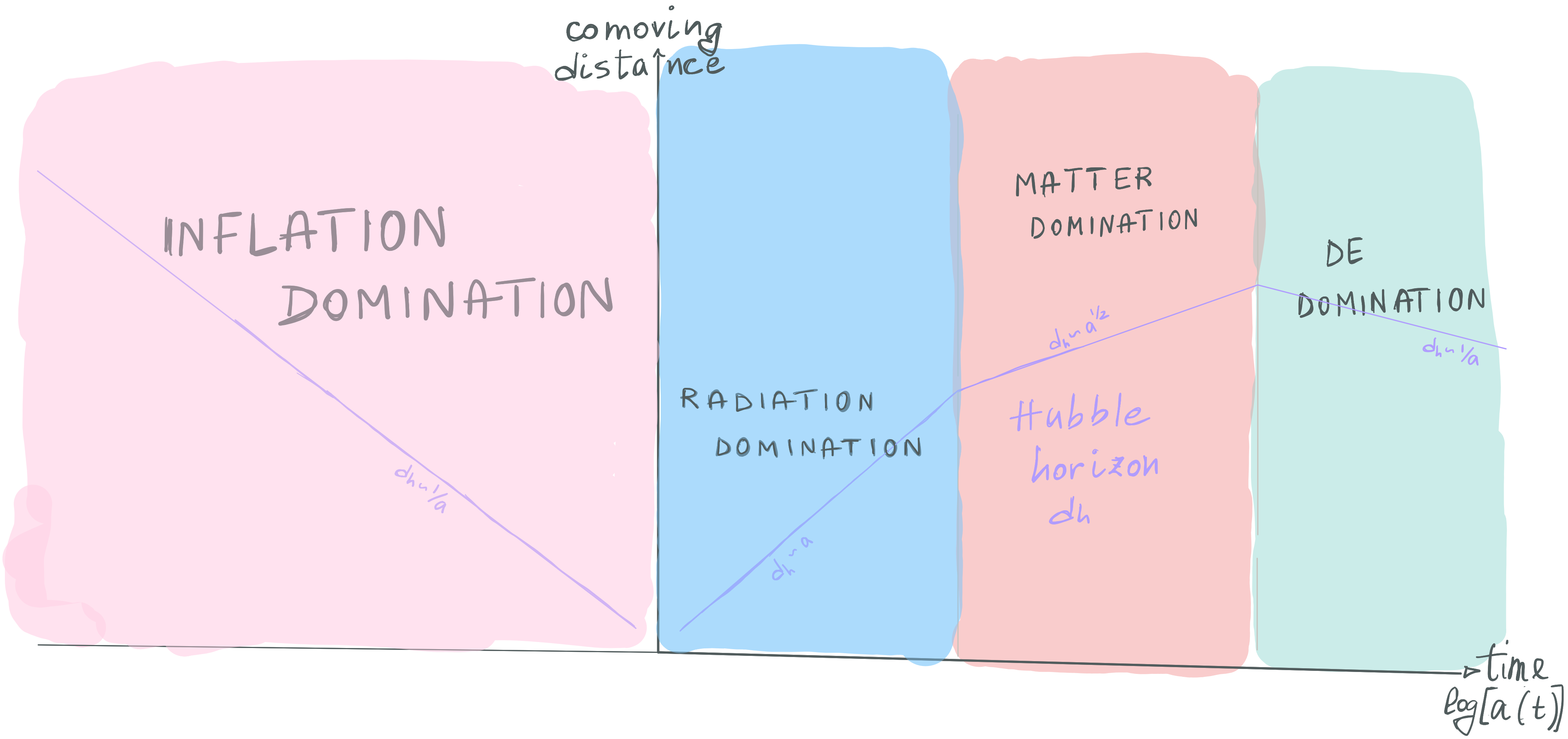
Horizon problem: why do we see homogeneity of causally disconnected regions of space ?

INFLATION

$$\frac{\dot{a}}{a} = H(t)$$

$$a \propto e^{Ht}$$

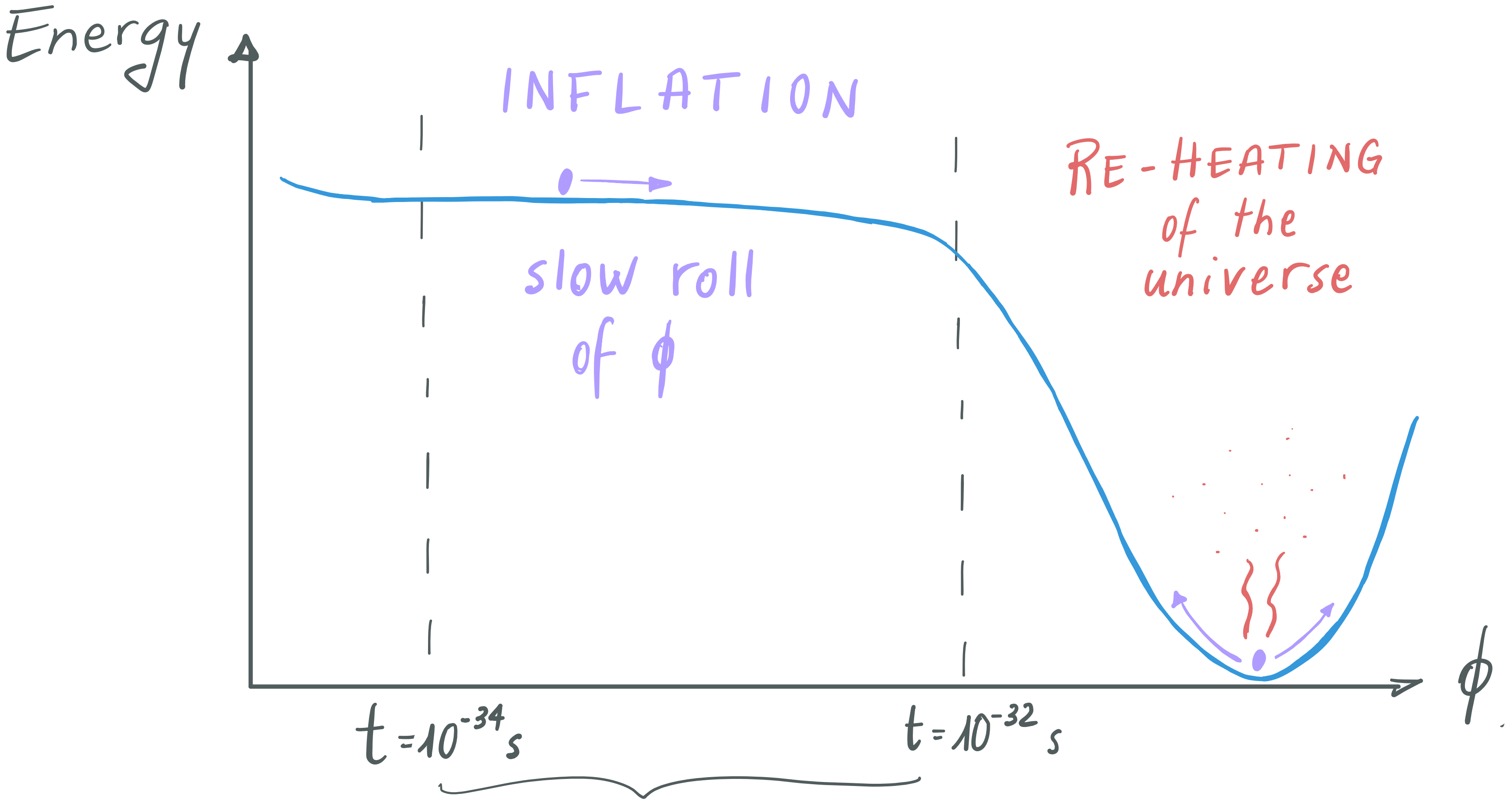




INFLATION

classical approximation $\phi = \phi_0(t)$

QM $\Longrightarrow$ $\phi_{\vec{k}}(\eta)$ quantum fluctuations

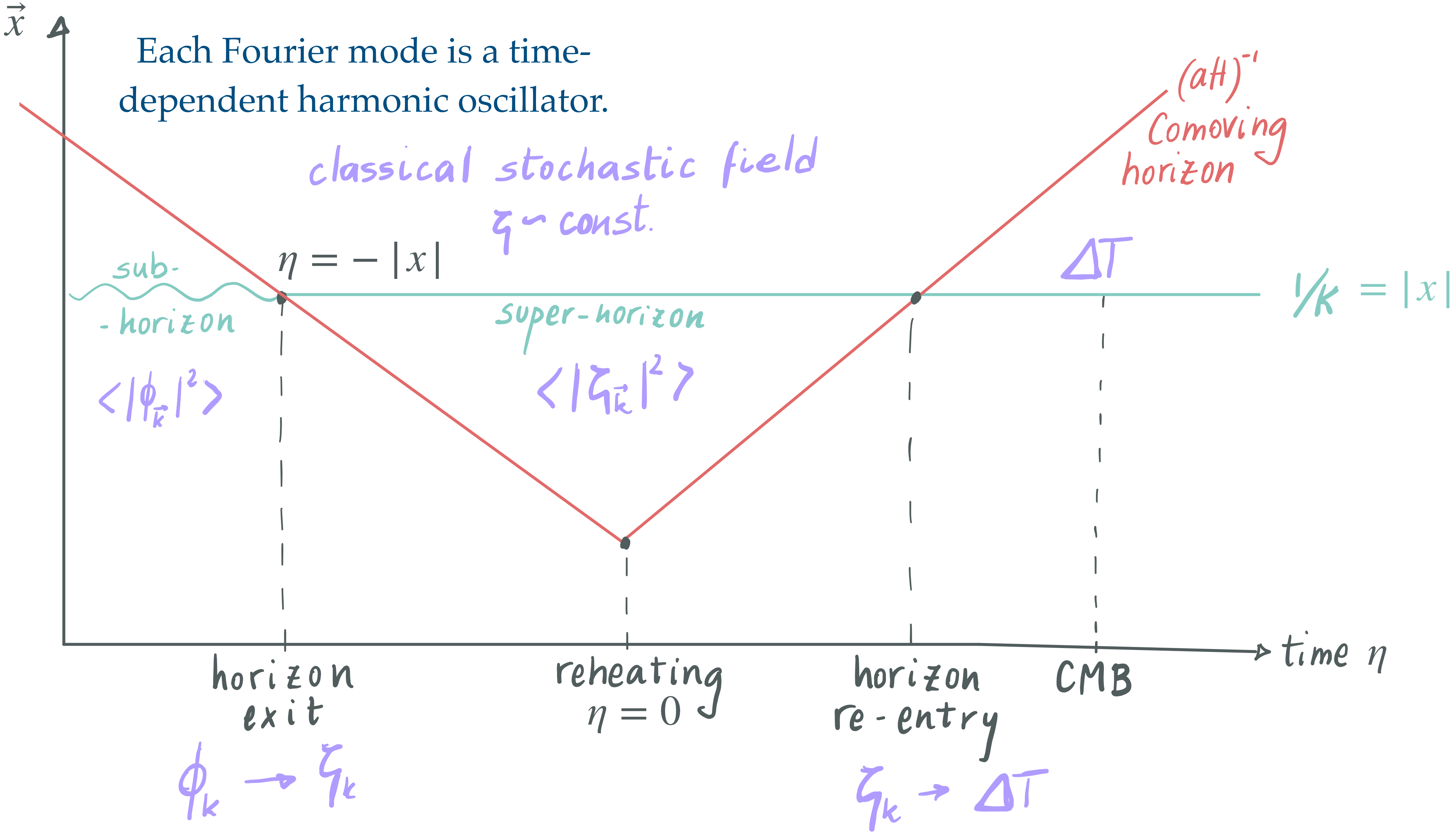


inflaton field ϕ
slowly rolls down its potential
varying slowly $H = \frac{\dot{a}}{a}$ s.t. $a \sim e^{Ht}$

INFLATION

classical approximation $\phi = \phi_0(t)$

QM $\Longrightarrow$ $\phi_{\vec{k}}(\eta)$ quantum fluctuations



Each Fourier mode is a time-dependent harmonic oscillator.

INFLATION

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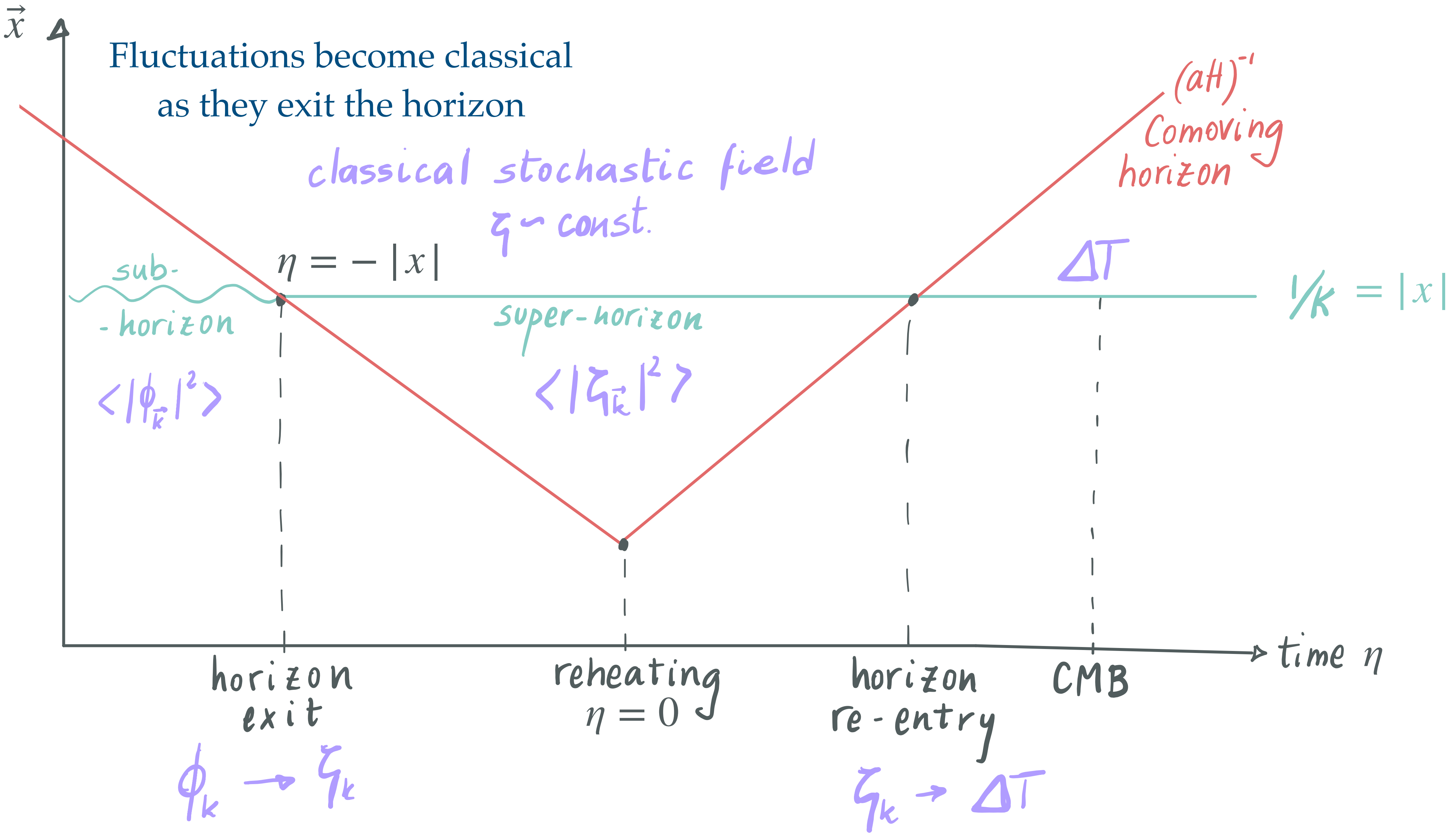
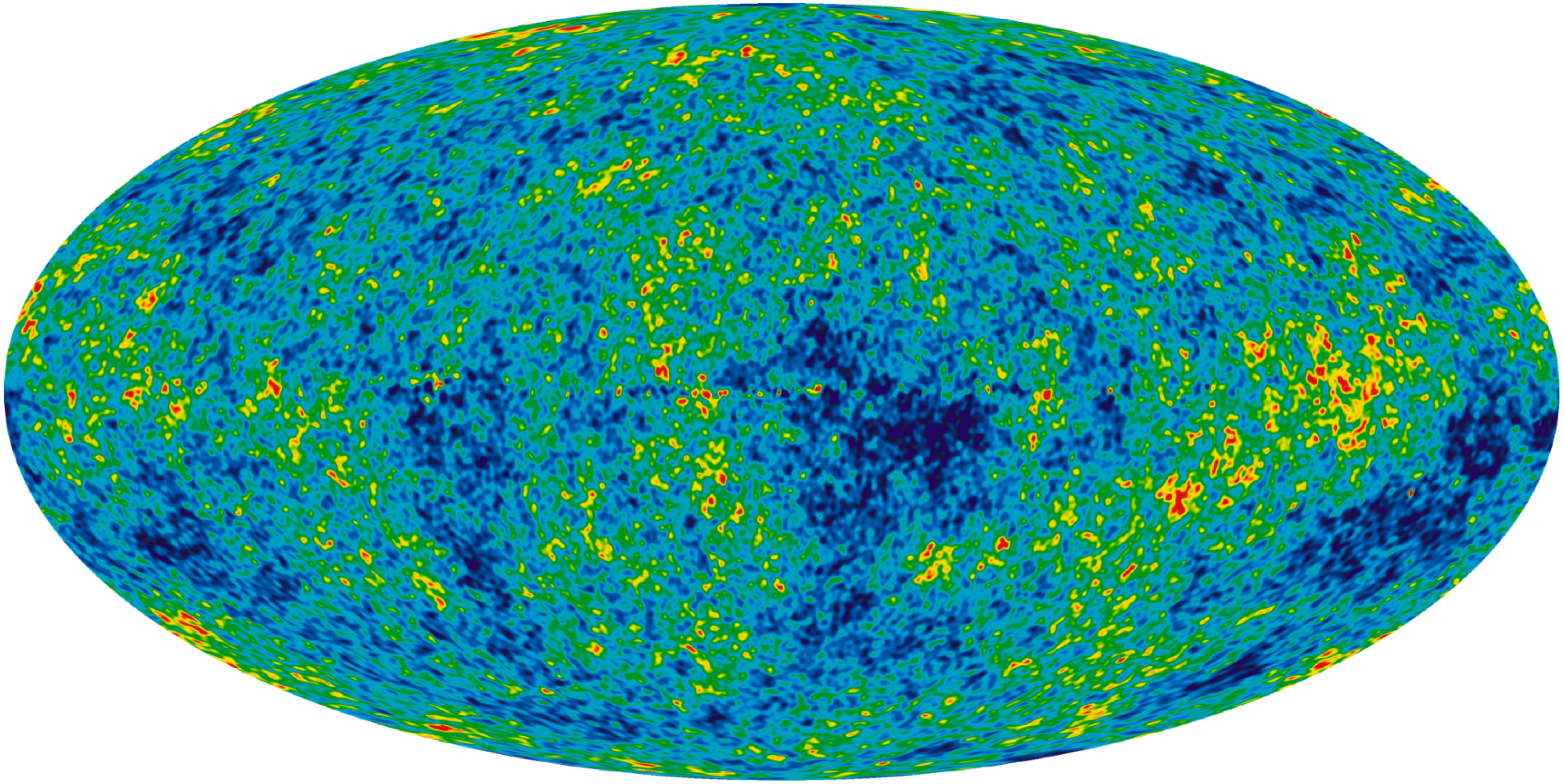


Fluctuations become classical
as they exit the horizon

THE CMB

Primordial fluctuations from QM effects in the early universe.
Current observed fluctuations in the CMB are classical.



$$\frac{\Delta T}{T} \sim 10^{-5}$$

OBSERVABLES

Primordial fluctuations from QM effects in the early universe.
Current observed fluctuations in the CMB are classical.

Each Fourier mode is a time-dependent harmonic oscillator.

$$ds^2 = \frac{-d\eta^2 + dx^2}{\eta^2} \qquad\qquad S = \int \frac{d\eta}{\eta}(|\dot{\phi}|^2 - k|\phi|^2)$$

Fluctuations become classical as they exit the horizon

$$k^3[\zeta_k, \dot{\zeta}_{-k}] \propto i(\eta k)^3 \ \to 0 \ \text{ as } \eta k \to 0$$

At reheating we have a classical measure, or probability distribution

$$\rho(\zeta(\vec{x})) = |\Psi[\zeta(\vec{x})]|^2$$

Can we distinguish this probability distribution from a purely classical one?

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$$k^3[\zeta_k, \dot{\zeta}_{-k}] \propto i(\eta k)^3 \rightarrow 0 \text{ as } \eta k \rightarrow 0$$

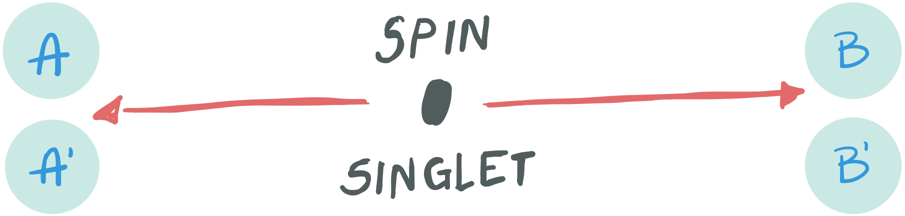
In there is an additional field we can have isocurvature perturbations
but we still find classical prob. distribution

$$\rho(\zeta(\vec{x}), \theta(\vec{x})) = |\Psi[\zeta(\vec{x}), \theta(\vec{x})]|^2$$

Can we distinguish this probability distribution from a purely classical one?

TESTING QM

Fundamental deviation from classical physics → Bell inequalities



All operators: A, A′, B, B′ have eigenvalues +1 or -1

$$A = \vec{n} \cdot \vec{\sigma}\,,\ A' = \vec{n}' \cdot \vec{\sigma}$$

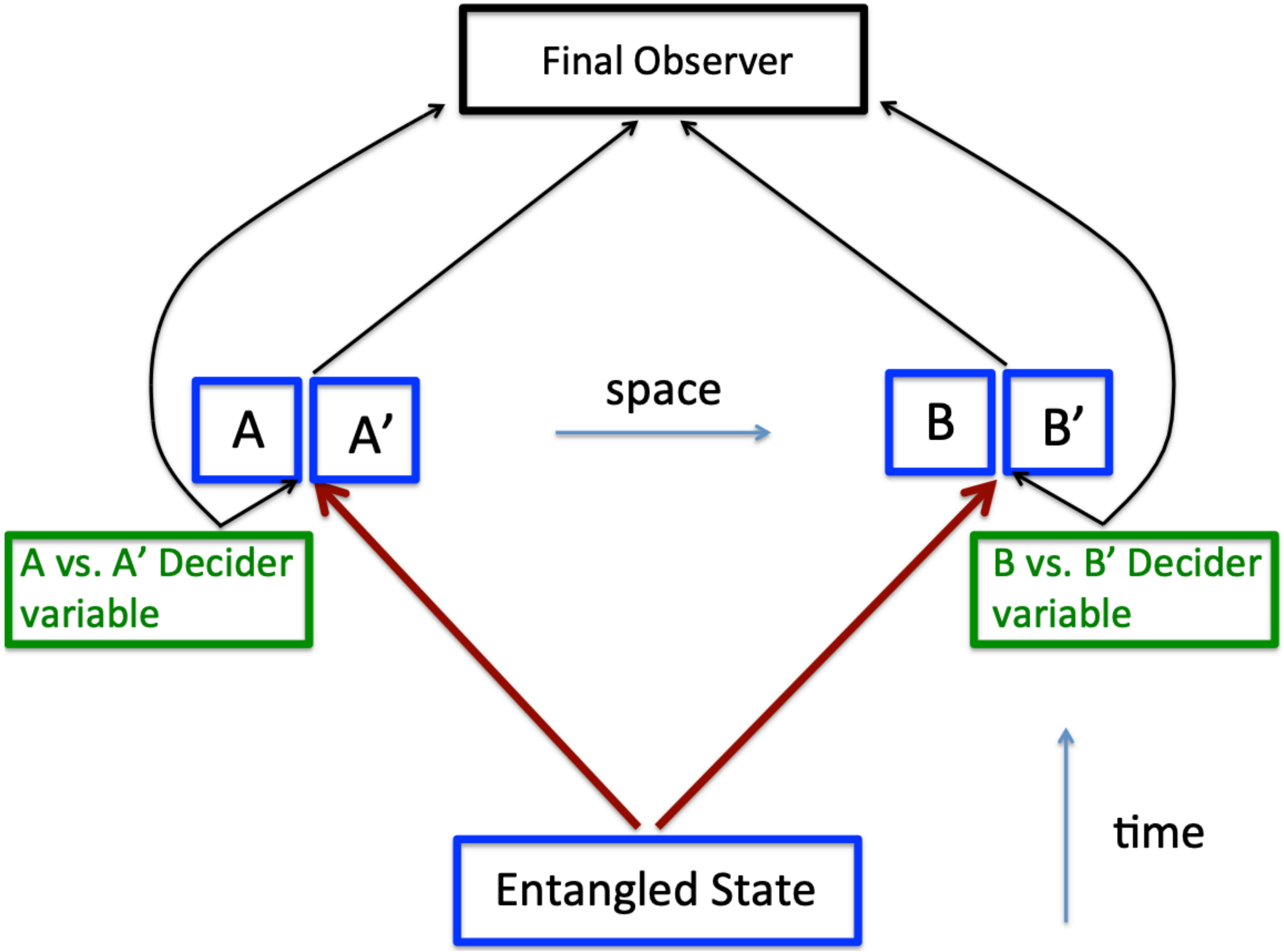
$$C = AB - AB' + A'B + A'B' = A(B - B') + A'(B + B')$$

$$C^2 = 4 + [A, A'][B, B']$$

$$|\,C\,|_{QM,max} = 2\sqrt{2} > 2 = |\,C\,|_{cl,max}$$

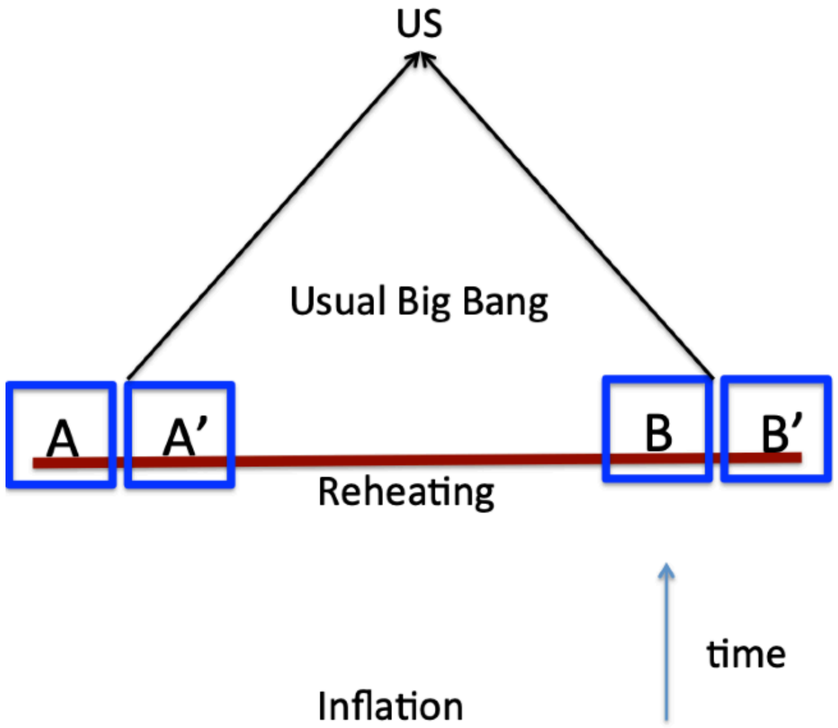
BELL EXPERIMENT

Fundamental deviation from classical physics → Bell inequalities



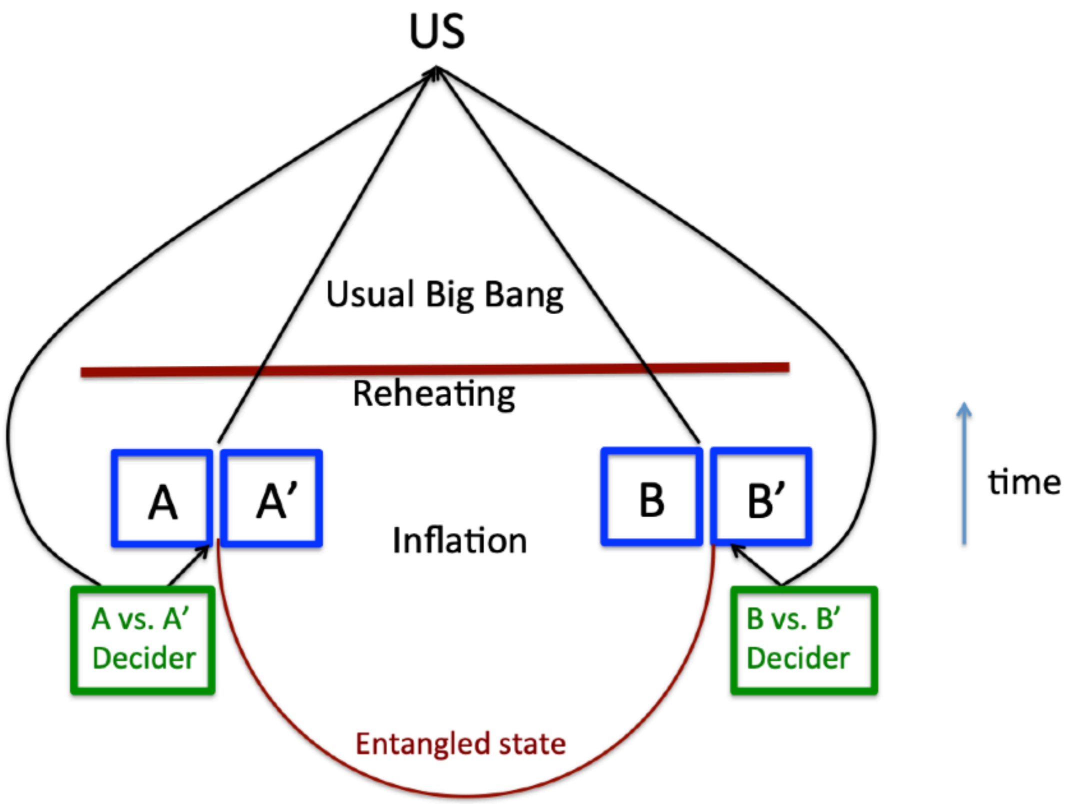
COSMOLOGICAL BELL EXPERIMENT

In cosmology there are only commuting observables



COSMOLOGICAL BELL EXPERIMENT

Use classical probability as classical message transmitting the result



Entangled state: ϕ_k . Measurement apparatus and decider variable $\phi_{k'}$.
Measurement: process that produced a big effect on the fluctuations today
E.g. massless scalar field fluctuations amplified during inflation.

Need $\frac{1}{k'} < \frac{1}{k}$ s.t. decider variable is local
→ entangled state more classical than measuring device.

A 'BAROQUE' MODEL FOR TESTING BELL INEQUALITIES

THE MAIN INGREDIENTS

Field, or particle content

- Inflaton ϕ;
- Massive particles: represented by a complex scalar field, and are created in pairs. They constitute the **entangled states**.

- Axion: a real scalar field, on compact domain. It has non-trivial interactions with the massive particles, and plays the role of **decider**.

Methodology

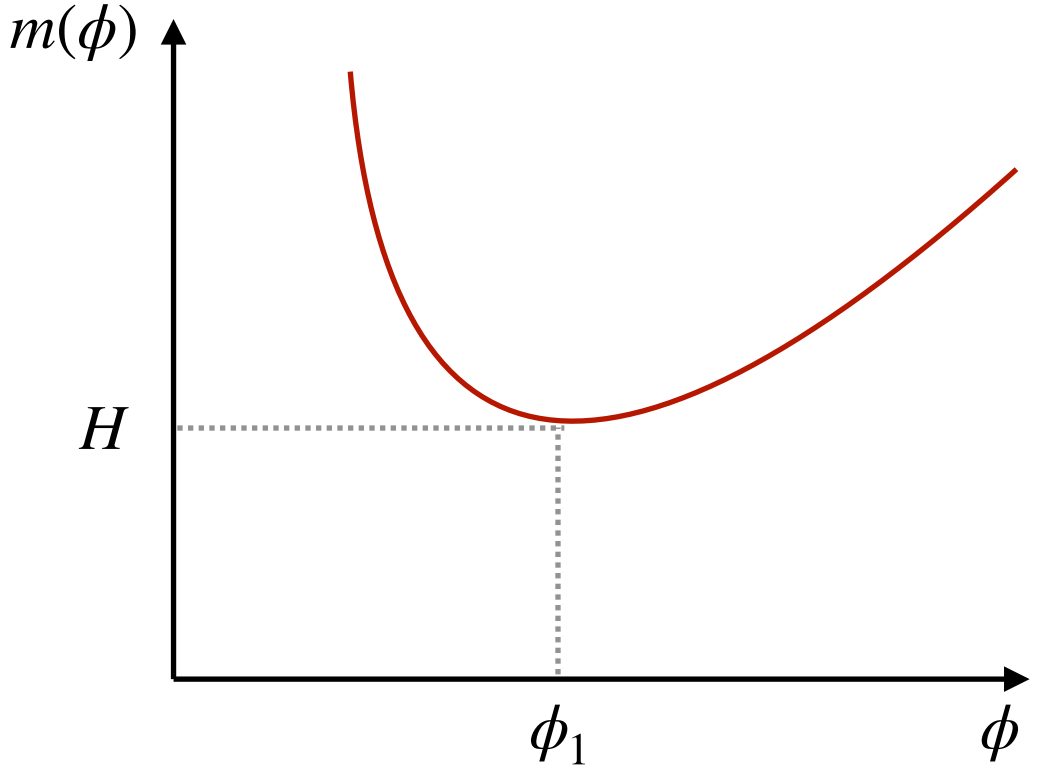
- Measurement: the quantity that is 'measured' is the isospin of the massive particles;

- Preservation to post-inflationary observers: the signatures of the massive particles are read from the CMB fluctuations.

ENTANGLED STATES: MASSIVE PARTICLE PAIRS

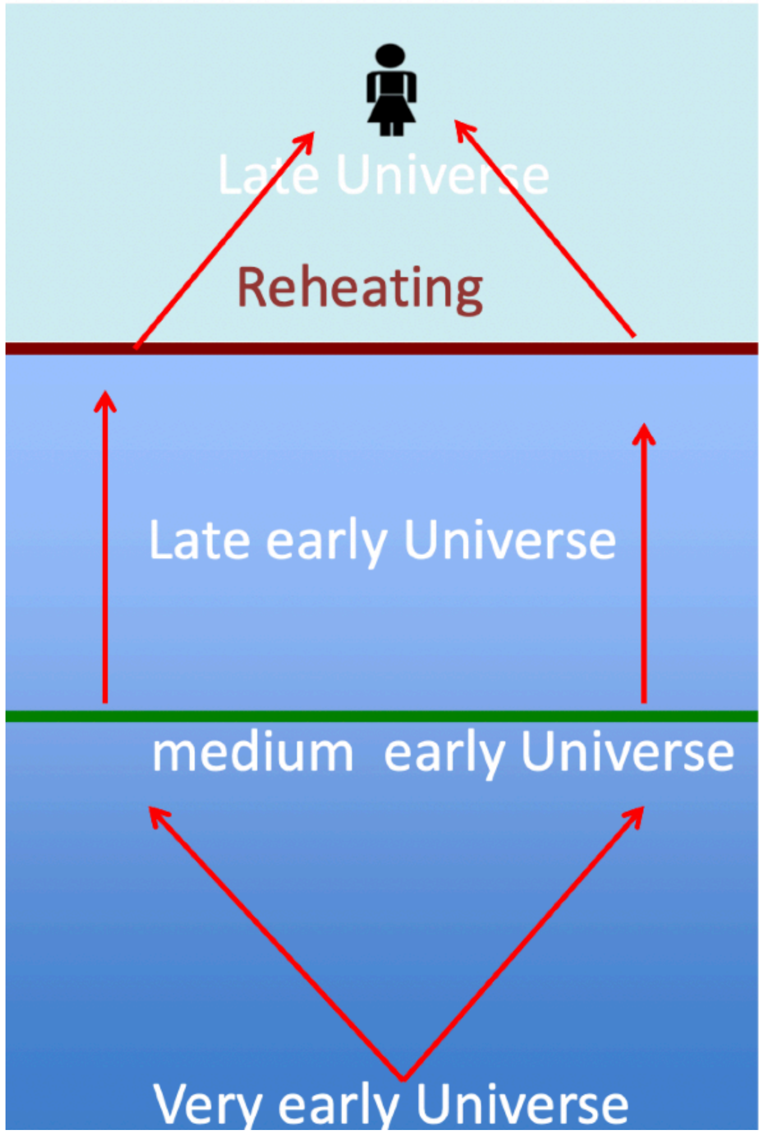
We assume that there exist particles such that:

- they are represented by a complex scalar field h;
- they have isospin;
- their masses $m(\phi)$ depend on the inflaton ϕ, and they are such that they can be created during inflation:



During inflation, they become lighter
⇒ They can be created, in pairs, by the inflaton

This should happen during inflation



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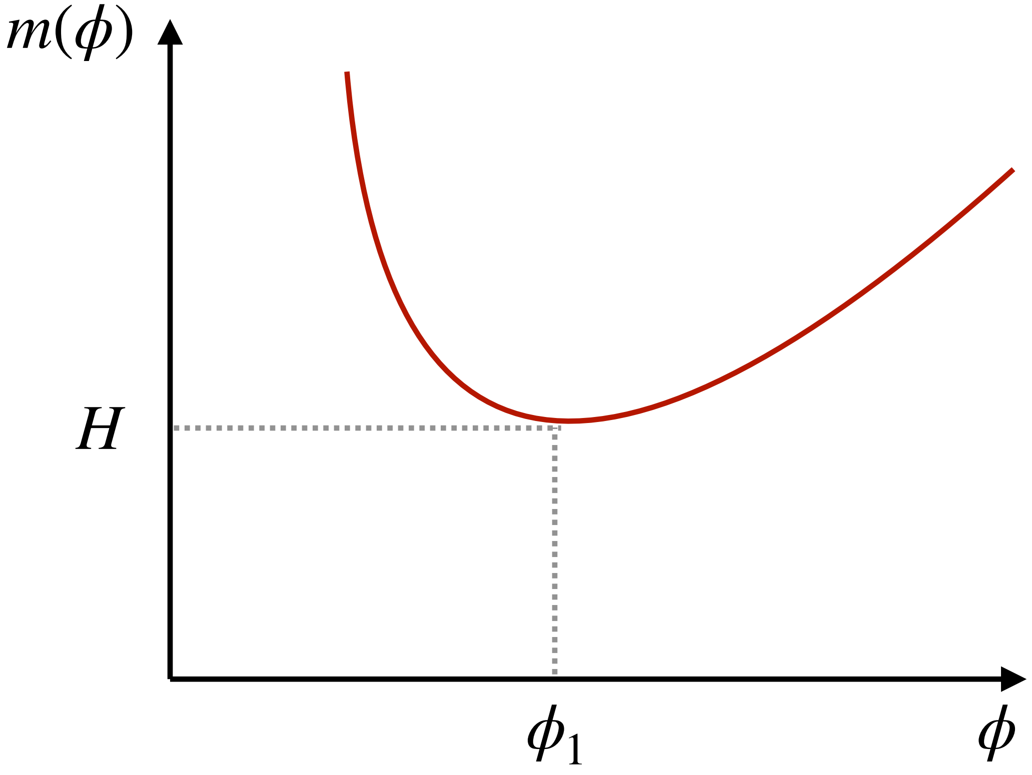

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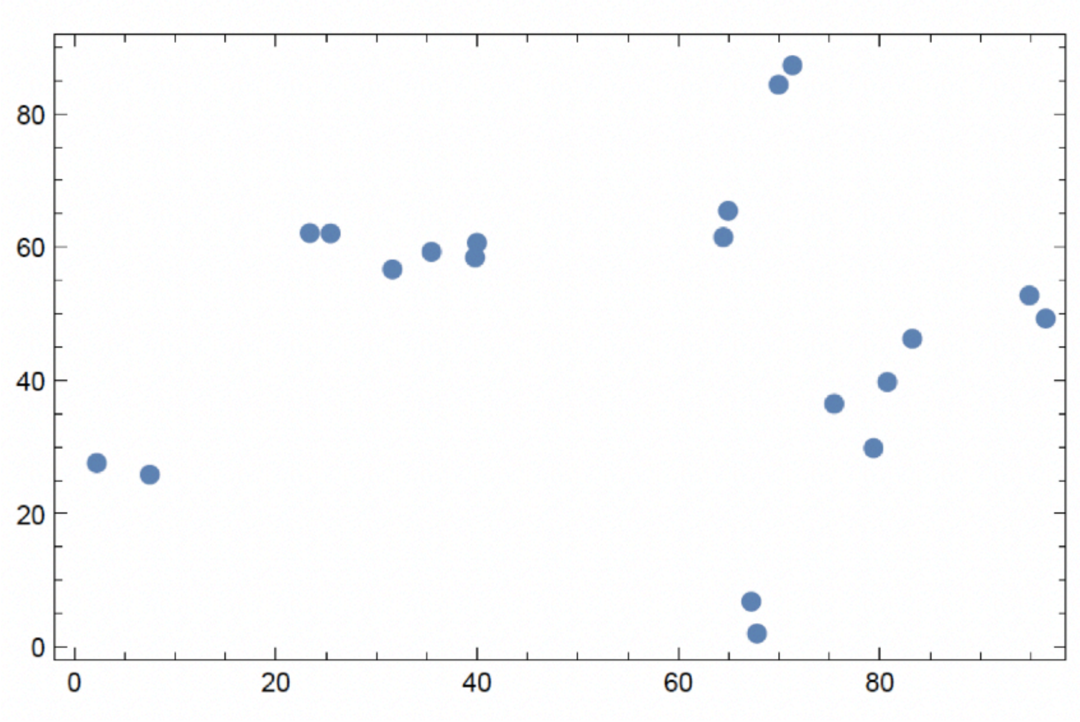


…but they must not be 'too many'!
The particles have to be well-separated.

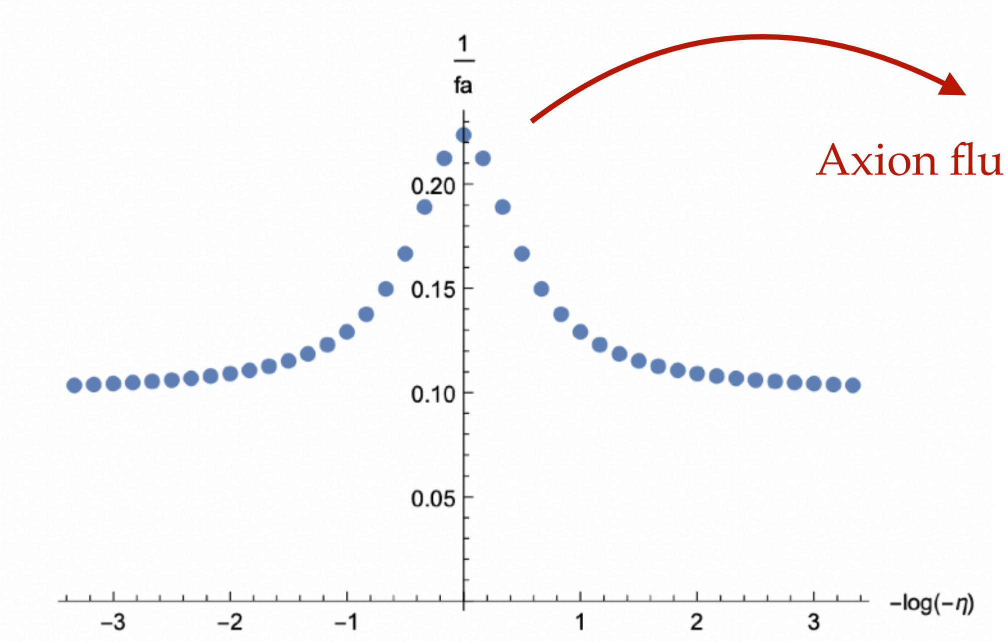
THE DECIDER: THE AXION

We assume that there exists a single axion θ:

- they span a compact domain $\theta \in [-\pi, \pi[$, with the identification $\theta \sim \theta + 2\pi$;
- they appear with the action

$$S = \int f_a^2 (\nabla\theta)^2 = \int d\eta d^3x \frac{f_a^2(\eta)}{H^2} \frac{[(\partial_\eta\theta)^2 - (\partial_i\theta)^2]}{\eta^2}$$

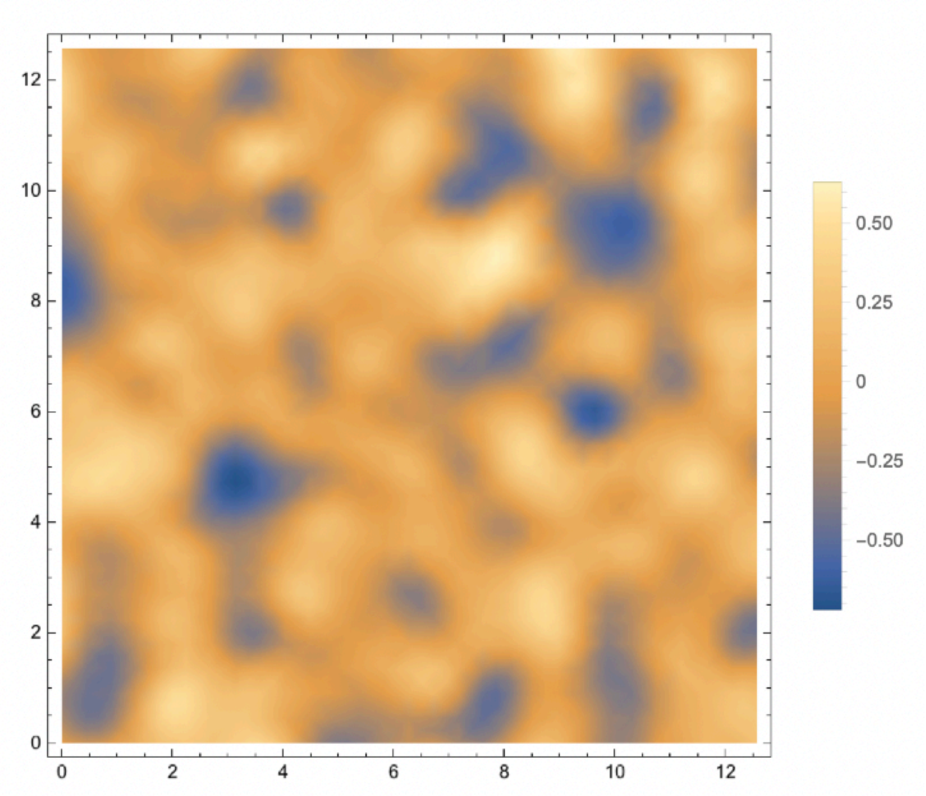
Decay constant



Axion fluctuations $\propto \frac{1}{f_a}$

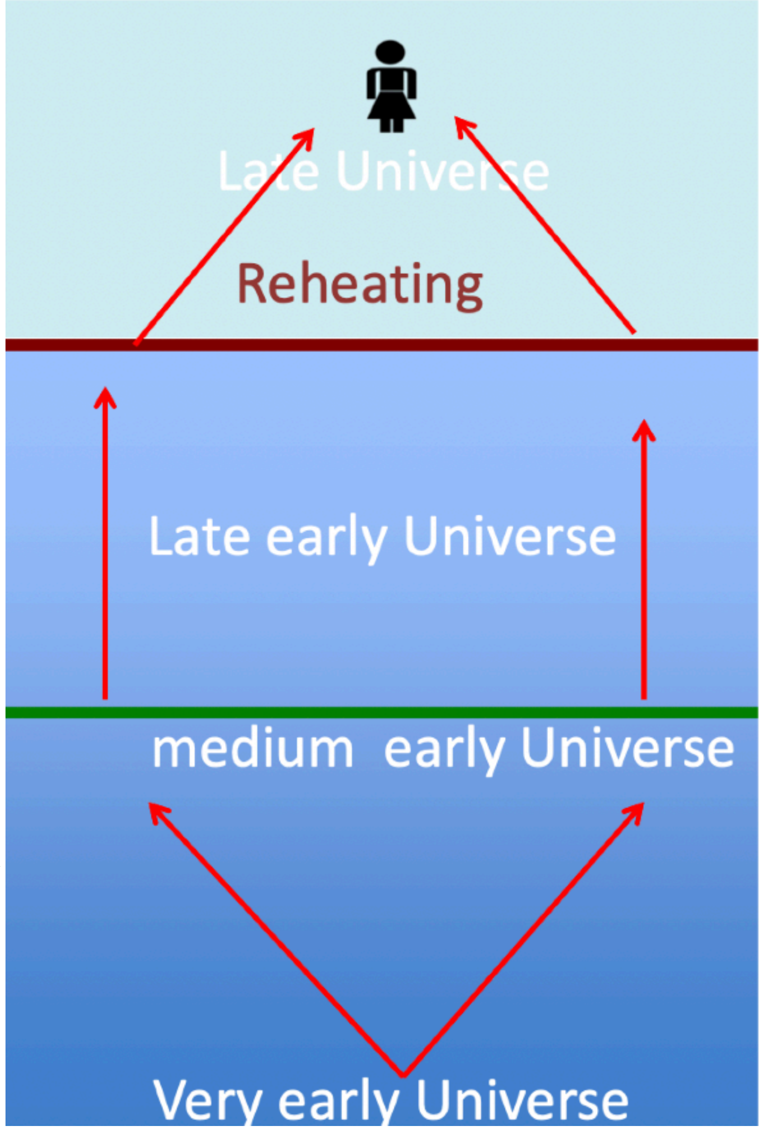
The decay constant *decreases* some time during inflation, after the creation of the massive particle

The axion has a profile in space:



THE MEASUREMENT

- Firstly, particles are created;

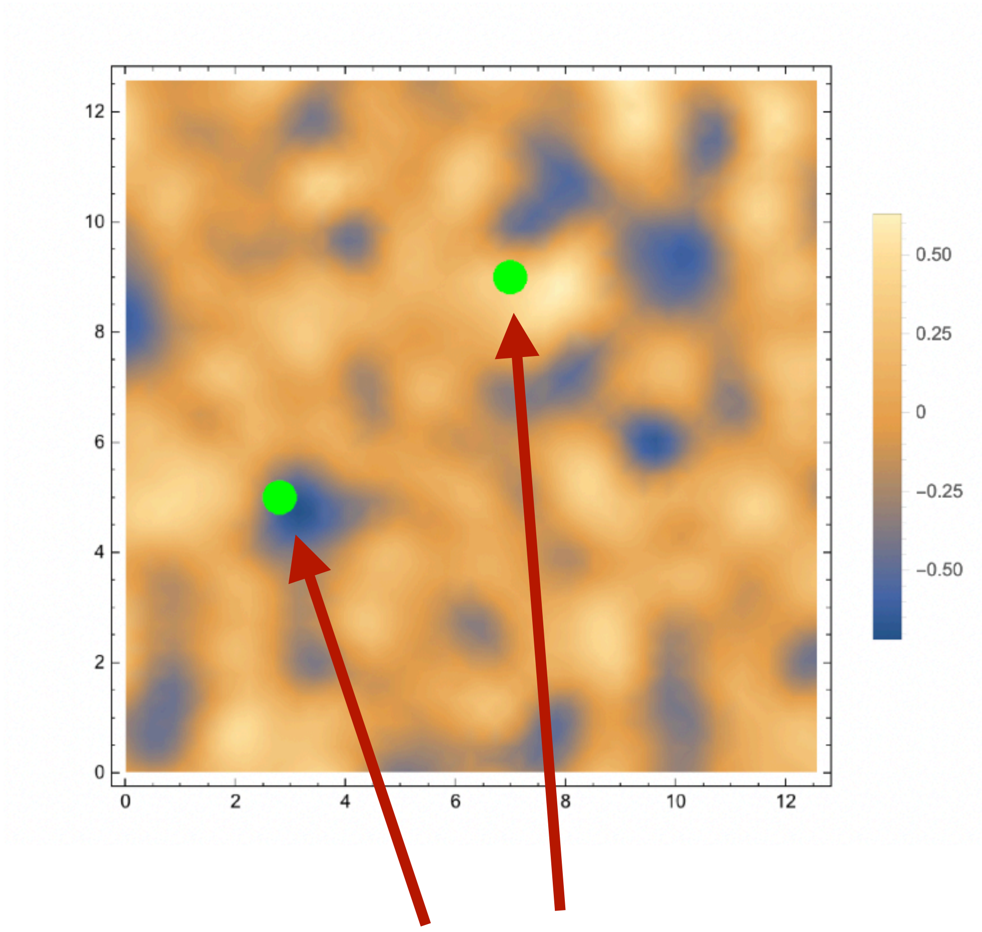


Particle creation

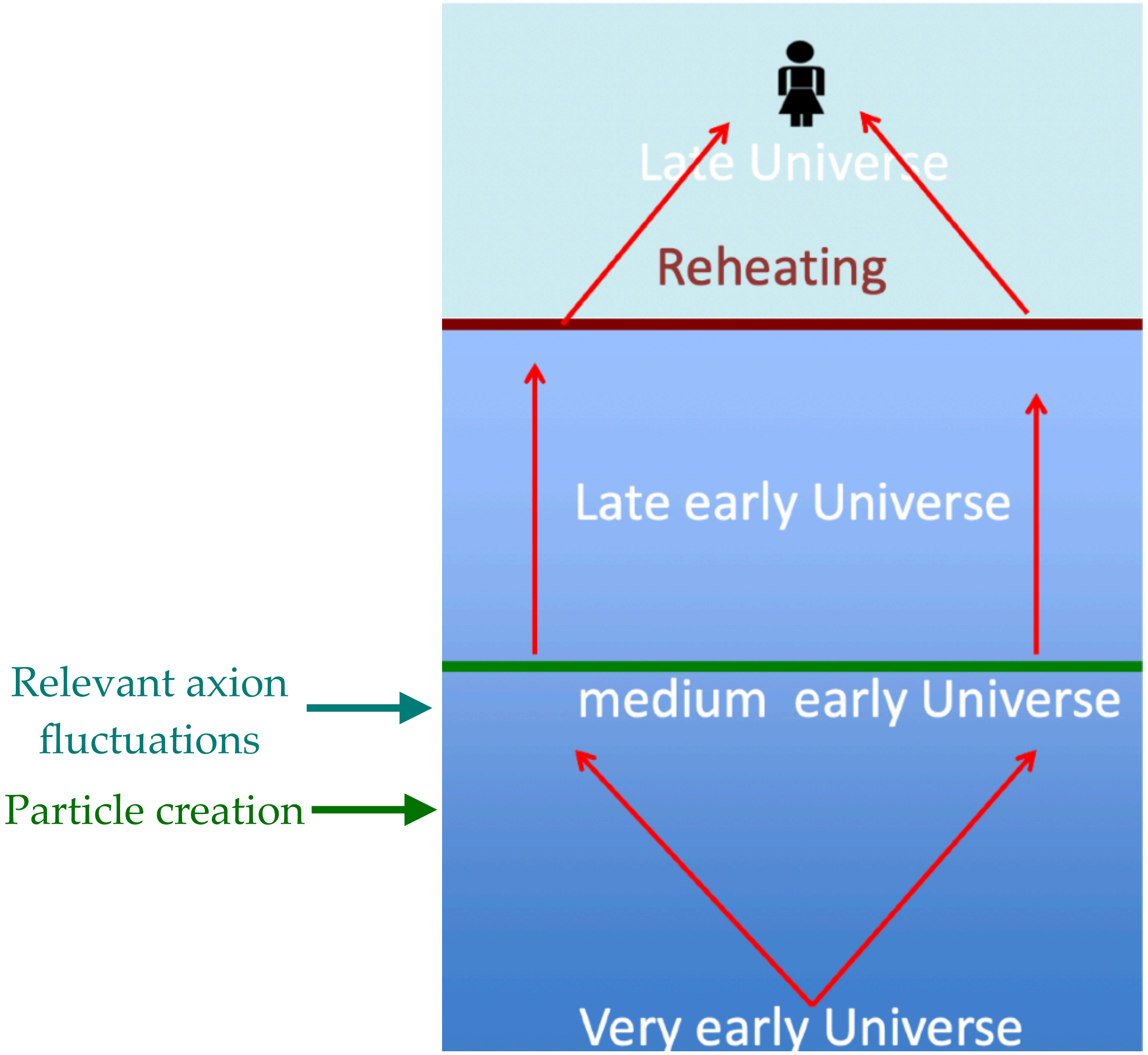
THE MEASUREMENT

- Firstly, particles are created;
- The axion fluctuations involve smaller distances than the distance between the particles of a pair

  ⇒ Each member of a pair sees a different value of the axion!



Particles of a pair



THE MEASUREMENT

- Firstly, particles are created;
- The axion fluctuations involve smaller distances than the distance between the particles of a pair

  ⇒ Each member of a pair sees a different value of the axion!
- The massive particles leave observable traces:

  they create ‘*hotspots*’ with their fluctuations

$$\langle \zeta_{part}(x) \rangle = \underbrace{\frac{m(\eta = -|x|)}{2\sqrt{2\epsilon}M_{pl}}}_{\text{Classical contribution}} \times \underbrace{\left(\frac{1}{2\pi\sqrt{2\epsilon}} \frac{H}{M_{pl}} \right)}_{\text{Quantum contribution}}$$

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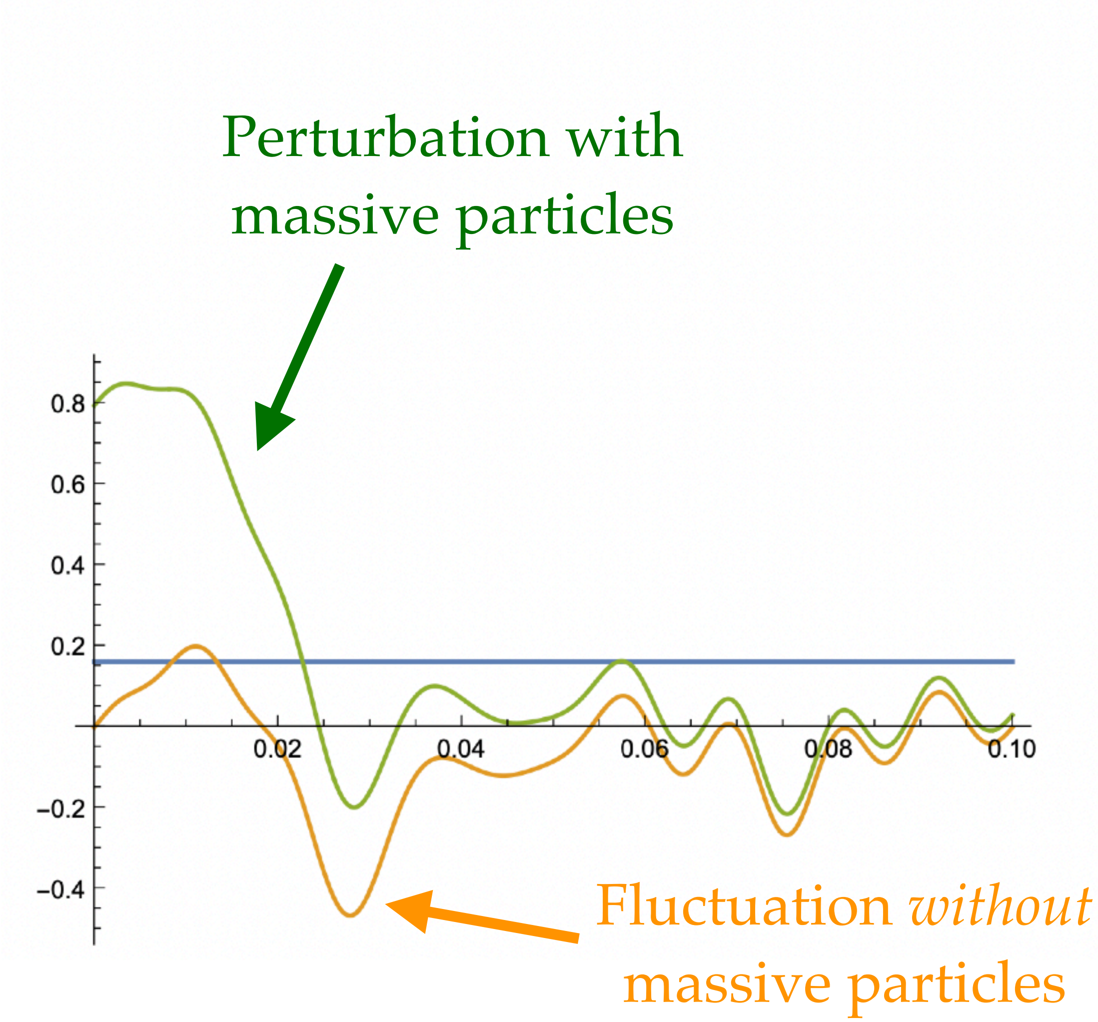
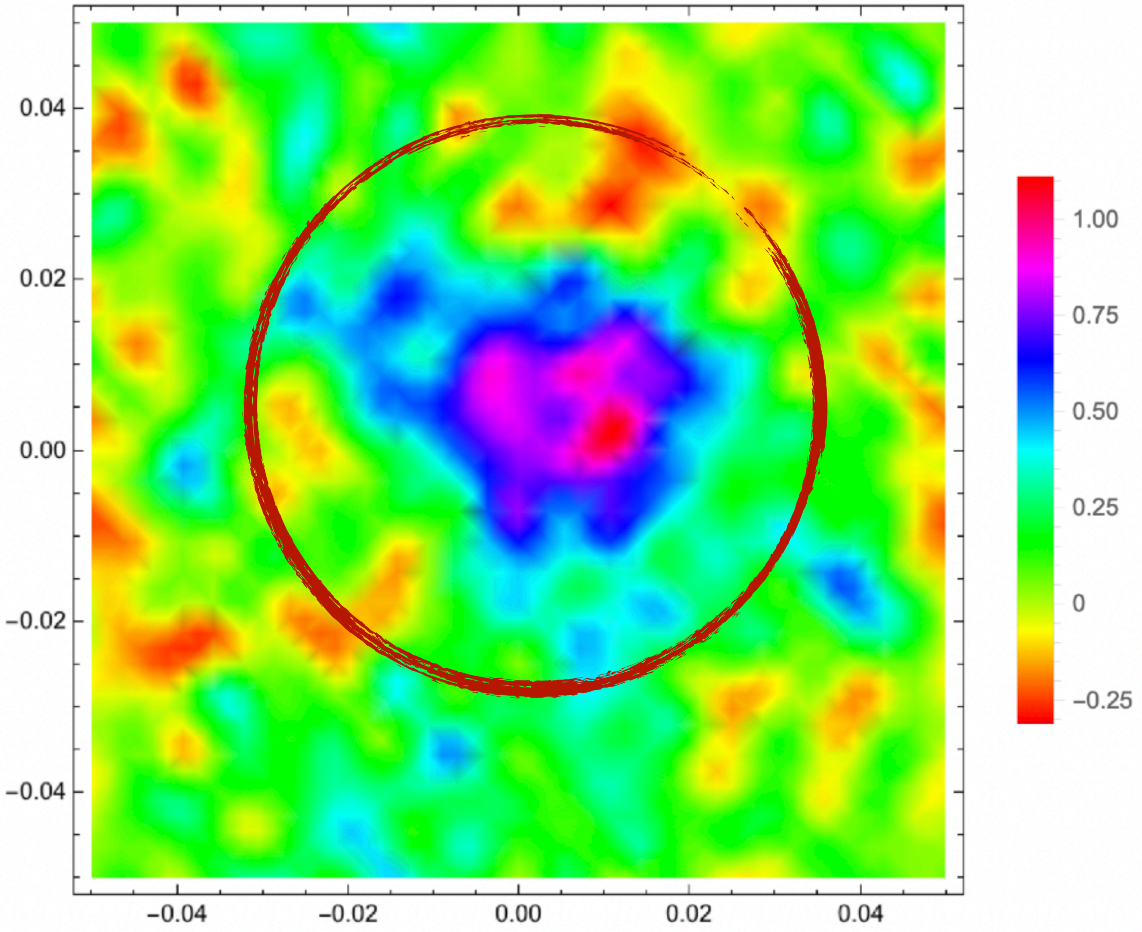


Perturbation with massive particles

Fluctuation *without* massive particles



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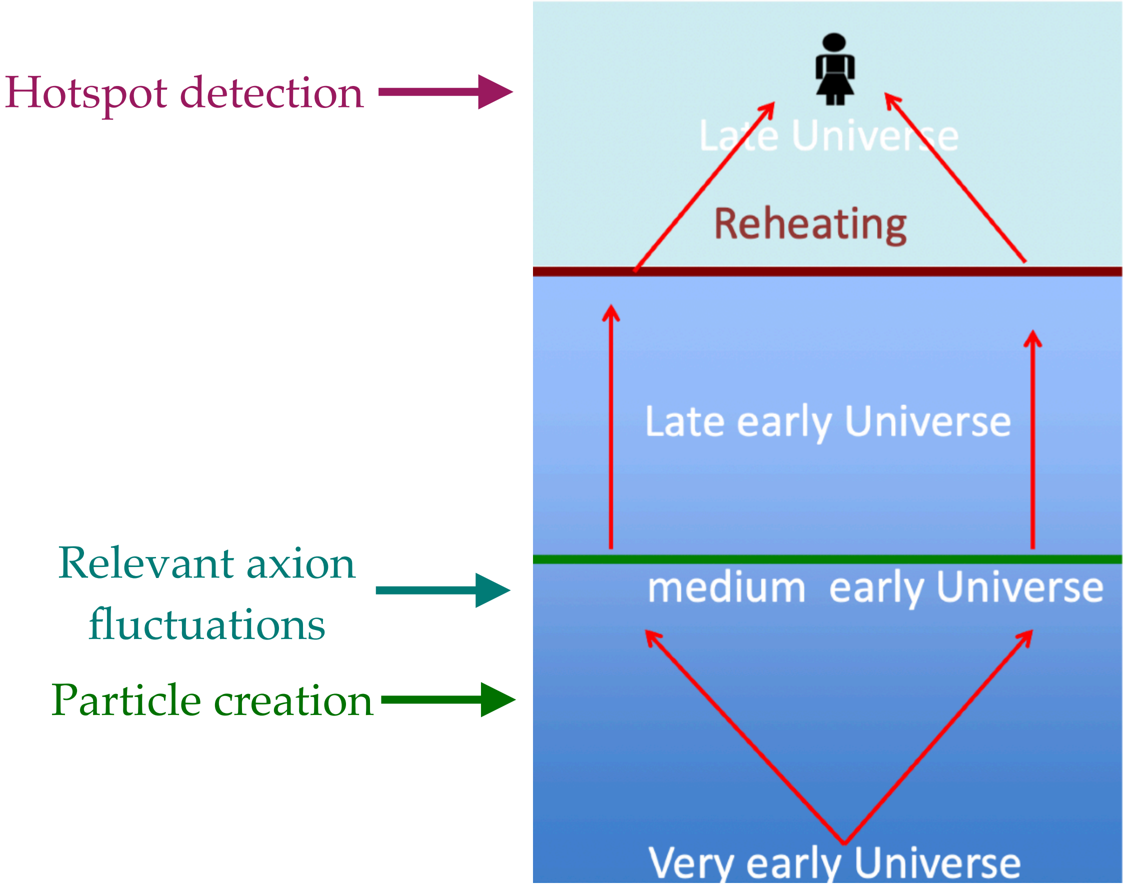
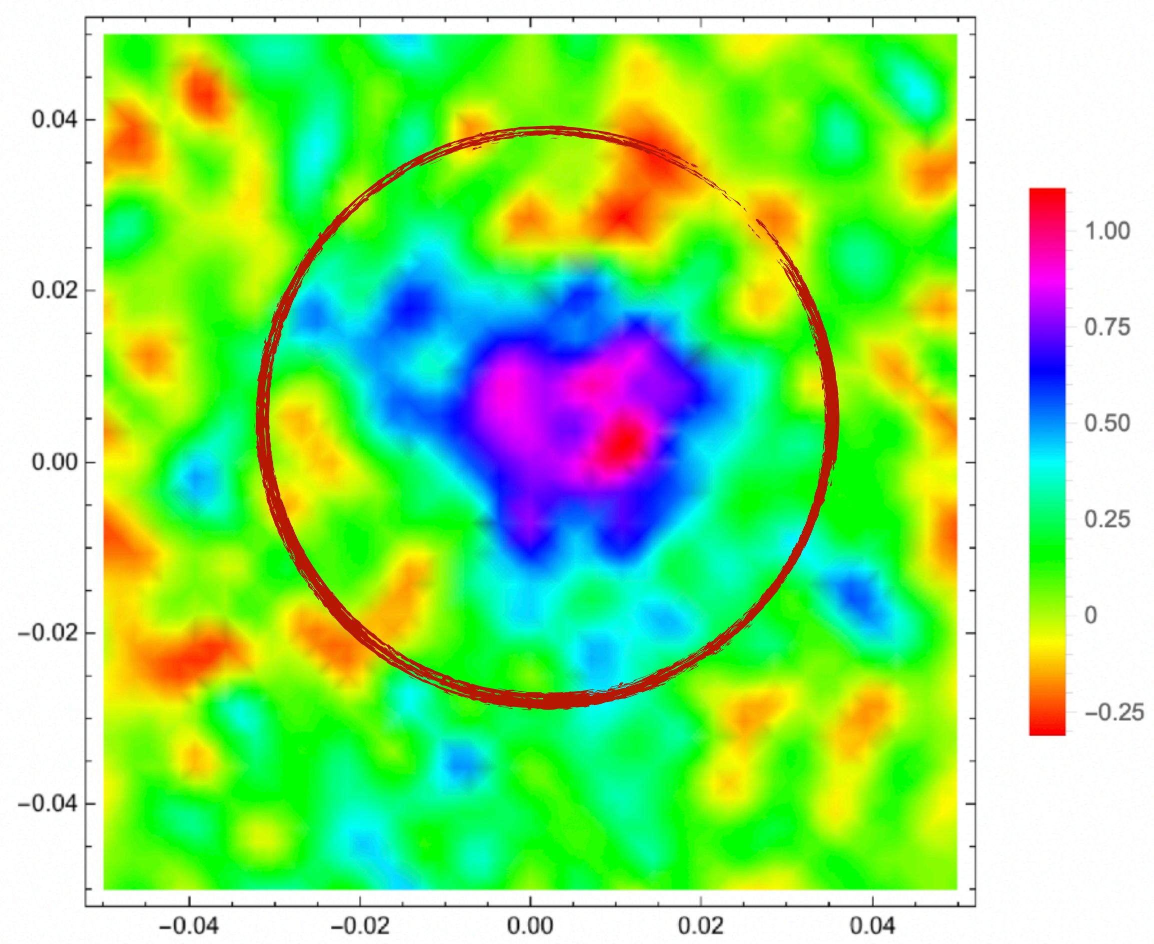


Hotspot detection

Relevant axion fluctuations

Particle creation



THE MEASUREMENT

We want to measure the isospin projections for the particles of a pair.
⇒ they are ±1 measures, and offer measures to test with Bell's inequalities.

- Assume that the massive particles and the axion are coupled via Lagrangian mass terms as follows

$$m_1^2(\phi)h^\dagger h + \lambda_2(\phi)h^\dagger(\sigma_x \cos n\theta + \sigma_y \sin n\theta)h =$$
$$= m_1^2(\phi)\left[|h_1|^2 + |h_2|^2\right] + \left[\lambda_2(\phi)e^{in\theta}h_1^* h_2 + c.c.\right]$$

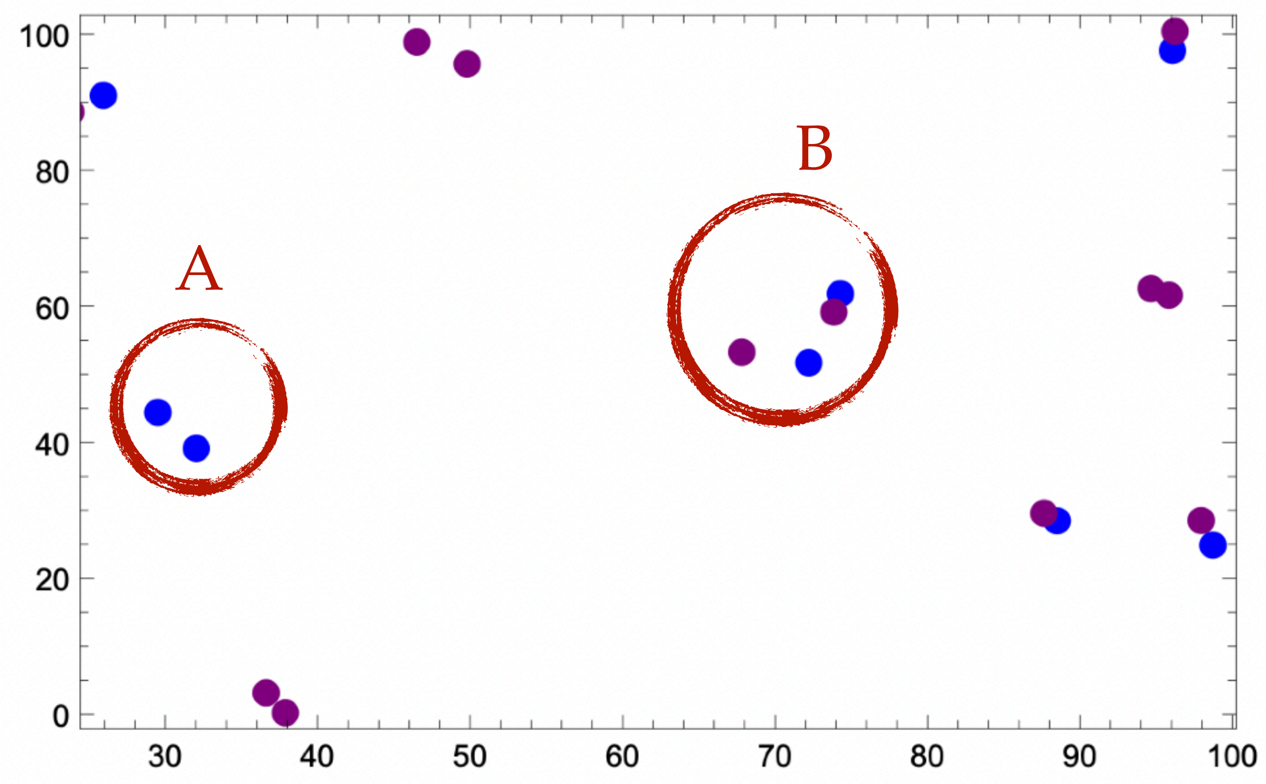
The mass matrix has eigenvalues $m_\pm = \sqrt{m_1^2(\phi) \pm |\lambda_2(\phi)|}$

- The eigenstates of the mass matrix are the same as the projection operator $\vec{\sigma} \cdot \vec{n}$ (projected along the direction with polar angle $n\theta$)

⇒ If we know the mass of a particle (*at that inflationary stage*) - i.e. whether it is $m_\pm$ - we also know the associated spin-projection eigenvalue!

THE MEASUREMENT

- We check the different particle pairs;
- We identify whether their mass is m_+ or m_-;
- In turn, this identifies the associated projection $\vec{\sigma} \cdot \vec{n}$ (dictated by the axion value);
- Each particle is associated with a ±1 measurement.



Finally, we can check the Bell's inequality:

- Define: A and B as two **locations**, such that a pair is split between them; for instance, in A there is a particle, in B the antiparticle of a pair;
- Around A and B we perform *two* measurements, for two values of the axion $\Rightarrow$ **We obtain** $(\vec{\sigma} \cdot \vec{n})_{A,A'}$ **and** $(\vec{\sigma} \cdot \vec{n})_{B,B'}$.
- Define the quantity $\langle C \rangle = \langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle$ $\Rightarrow$ **test Bell's inequality**

CONCLUSIONS

- The model offers a possible, concrete way of testing Bell's inequalities in a cosmological context;

- The proposed ingredients naturally appears in string theory effective theories.

However…

- The model requires a lot of **fine tuning**;

- It is very **hard to check** (the fluctuations employed must be subdominant).

Thanks for
your attention!