ENTANGLED PHOTON PAIRS, QUANTUM EVENTS AND THE EQUIVALENCE PRINCIPLE

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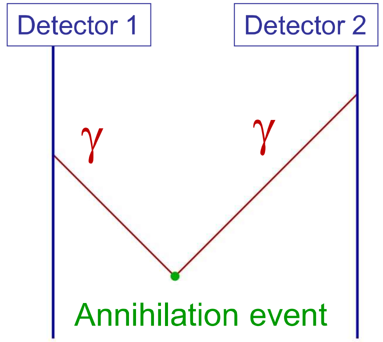
Thanks to Marc-Thierry Jaekel (LPTENS) and the many colleagues with whom I discussed in physics and metrology labs or conferences …

Entangled photon pair associated with a quantum event

An example to fix ideas :
Annihilation of an electron and a positron leading to the emission of a pair of entangled γ photons

$$e^- + e^+ \rightarrow \gamma + \gamma$$

Detections of the two photons give access to the positions in space and time of the annihilation event



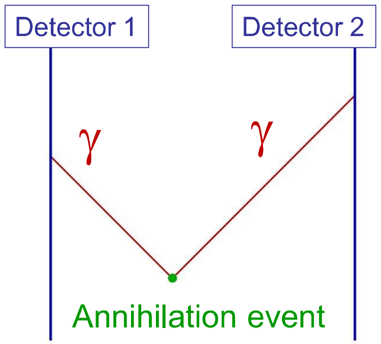
Entanglement associated with conservation laws

Linear and angular momenta of the outgoing two-photon state are sums of contributions of each photon

$$e^- + e^+ \rightarrow \gamma + \gamma$$

As linear and angular momenta are conserved in the annihilation event, they are also those of the ingoing e^- + e^+ state

Orbital or spin variables of the two photons are entangled



Quantum position in space

In Heisenberg quantum mechanics, position in space is an observable conjugated to momentum

$$[P, X] = -\imath\hbar$$

$$[A, B] \equiv AB - BA$$

Heisenberg inequality is deduced from this basic rule

$$\Delta X^2 = \langle X^2 \rangle - \langle X \rangle^2$$
$$\Delta P^2 = \langle P^2 \rangle - \langle P \rangle^2$$

$$\Delta X \Delta P \geq \frac{\hbar}{2}$$

It is commonly admitted that it is impossible to define a quantum observable for the position in time, which damages the connection between quantum physics and relativity

Quantum position in time ?

" In quantum mechanics, time is a quite different thing than space coordinates. It is a parameter the value of which is supposed to be exactly known : it is in fact the old good time of Newton…

… it seems to me doubtless that we will have to give up this too classical notion of time…

… This notion of time is seriously insufficient in quantum mechanics (or in its current understanding)…

… The knowledge of the variable t is obtained by observing a physical system... Time is an observable and must be treated as an observable. "

E. Schrödinger : Annales Institut Henri Poincaré 1932

Quantum space-time ?

If one disregards the quantum structure, one can justify the introduction of the metric "operationally" by pointing to the fact that one can hardly doubt the physical reality of the elementary light-cone attached to a point

In doing so, one implicitly makes use of the existence of an arbitrary sharp signal. Such a signal however, as regards the quantum facts, involves infinitely high frequencies and energies

This kind of a physical justification for the introduction of the metric falls by the wayside, unless one limits ourselves to the "macroscopic"

A. Einstein : Reply to criticisms in
« *Albert Einstein Philosopher-Scientist* » P. A. Schilpp ed 1949

Outline of the talk

Positions in space and time can be built up on transfers of propagating massless fields

Simple model first in 1d space (2d space-time) :
- Synchronization observables along a propagating field
- Localization observables defined from the incidence of fields propagating in different directions

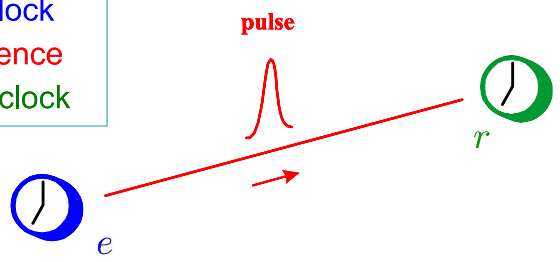
General solution in 3d space (4d space-time):
- Electromagnetic field in 3d space with spin
- Quantum relativistic observables, and their properties

Products of this construction :
- Equivalence principle (for constant gravity and acceleration)
- Quantum expressions obtained for the metric

Einstein synchronization

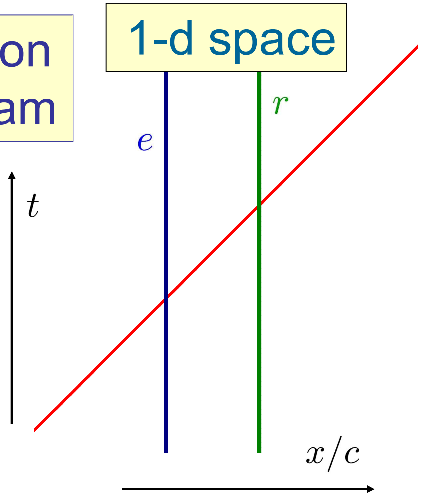
1-d space

An emitter e with a clock transfers a time reference to a receiver r with a clock



The time reference may be defined as the "center of energy" of the field pulse. It is a quantity encoded on the field and conserved by propagation.

Einstein synchronization on a space-time diagram

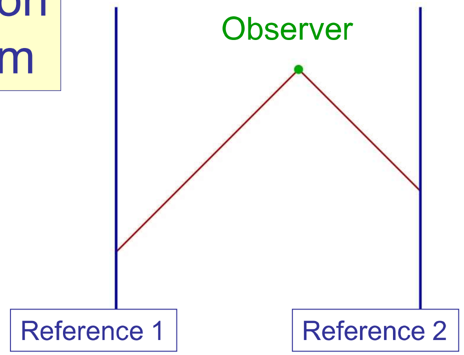


In classical physics the time reference is simply a light-cone variable and the meaning of time transfer is clear on a space-time diagram

The emitter e
and receiver r
share a common value
of the light-cone variable

$$u_+ \equiv t - \frac{x}{c}$$

Einstein localization on a space-time diagram

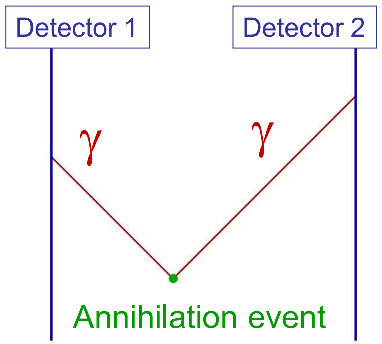


Localization techniques also have a clear meaning on such a diagram. An observer measures time references coming from two different directions

The observer deduces its positions in space **and** time from the two light-cone variables

$$t = \frac{u_+ + u_-}{2} \quad , \quad \frac{x}{c} = \frac{u_- - u_+}{2}$$

Localization of a quantum event

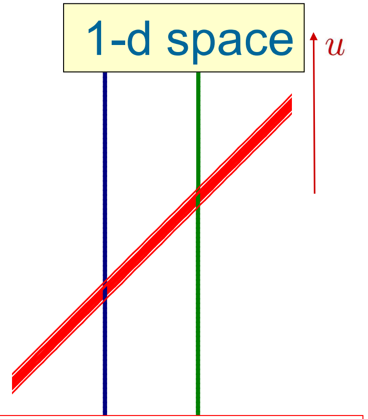


Two detectors measure the light-cone variables for light rays emitted in two different directions at a given event (say an annihilation event)

Collecting measurements of the two detectors allows one to deduce the positions in space **and** time of the event

$$t = \frac{u_+ + u_-}{2} \quad , \quad \frac{x}{c} = \frac{u_- - u_+}{2}$$

Synchronization with quantum fields



The reference shared by the two observers is a quantum observable encoded on the field and conserved by propagation

We use the barycenter of the energy density

$$U_\pm = \frac{1}{E_\pm} \cdot \int u_\pm e_\pm(u_\pm)\, du_\pm$$

$$E_\pm = \int e_\pm(u_\pm)\, du_\pm$$

$E_\pm$ and $U_\pm$ are conjugated

$$[E_+, U_+] = [E_-, U_-] = \imath\hbar$$
$$[E_+, U_-] = [E_-, U_+] = 0$$

$U_\pm$ is not defined in vacuum

$$A \cdot B \equiv \frac{AB + BA}{2}$$

Localization in space-time with quantum fields

1-d space

Quantum positions in space and time may be defined from two light-cone observables

$$T = \frac{U_+ + U_-}{2} \quad , \quad \frac{X}{c} = \frac{U_- - U_+}{2}$$

Positions in time and space are quantum observables and they are conjugated to energy and momentum

$$E = E_+ + E_- \ , \quad cP = E_+ - E_- \ ,$$

$$[E, T] = \imath\hbar \ , \quad [P, X] = -\imath\hbar$$

Positions are defined only when there is energy (at least one photon) in each of two different directions

Heisenberg rules and relativity

When writing Lorentz transformations

$$cT' = \gamma\,(cT - \beta X) \quad , \quad X' = \gamma\,(X - \beta cT)$$

$$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \beta P\right) \quad , \quad P' = \gamma\left(P - \beta\frac{E}{c}\right)$$

$$\beta \equiv \frac{v}{c} \qquad \gamma = \sqrt{1-\beta^2}$$

we need time to be conjugated to energy to ensure that the commutator between momentum and space position is preserved

$$[E, T] = -[P, X] = \imath\hbar$$

$$[P', X'] = [P, X]$$

If the time operator did not exist, or was not conjugated to energy, then the commutator between momentum and position could not be preserved by Lorentz transformations (the Heisenberg rules would be incompatible with relativity)

Electromagnetic fields in 3-d space

Same strategy : we will define positions in space-time from the algebra describing the symmetries of the field theory

This strategy is often used by starting from Galileo or Poincaré groups describing the space-time symmetries associated with Galilean and Einsteinian relativities

Here we use the larger conformal group of symmetries of electromagnetic field theory to go further

Quantum observables will be defined from the symmetry generators (translations, rotations, boosts)

Relativistic shifts under symmetry transformations and quantum commutators will be deduced from algebraic calculations in the "enveloping algebra" built up on the conformal algebra

Symmetry generators

New notation for commutators $\quad (A, B) \equiv \dfrac{AB - BA}{\imath\hbar}$

Special relativity (Einstein 1905)

Poincaré generators $\qquad \mu,\, \nu = 0,\, 1,\, 2,\, 3$

$$(P_\mu, P_\nu) = 0$$

4 translations in space-time

$$(J_{\mu\nu}, P_\rho) = \eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu$$

6 rotations in space-time

$$(J_{\mu\nu}, J_{\rho\sigma}) = \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\nu\sigma} J_{\mu\rho}$$

Dilatation generator

$$(D, P_\mu) = P_\mu$$

$$(D, J_{\mu\nu}) = 0$$

Symmetry also noticed but not used by Einstein in 1905

Decomposition into orbital and spin angular momenta

$$J_{\mu\nu} = P_\mu \cdot X_\nu - P_\nu \cdot X_\mu + S_{\mu\nu}$$

$$D = P^\mu \cdot X_\mu$$

Spin is transverse $\quad P^\mu \cdot S_{\mu\nu} = 0$

Localization observables

For states containing at least two photons propagating in two different directions, mass is not vanishing,

$$M^2c^2 = P^2 = P^\mu P_\mu \neq 0$$

which allows one to define positions in space-time

$$X_\mu = \frac{P_\mu \cdot D + P^\nu \cdot J_{\nu\mu}}{P^2}$$

The four position components are conjugated to the four momenta

$$(P_\mu, X_\nu) = -\eta_{\mu\nu}$$

They have non null commutators which reproduce the spin

$$(X_\mu, X_\nu) = \frac{S_{\mu\nu}}{P^2}$$

These results go beyond classical relativistic conceptions

Perfectly localized signal is impossible for electromagnetic fields which bear an intrinsic spin

Spin and orbital properties are coupled

Relativistic shifts of observables

Shifts of observables under frame transformations are given by commutators with symmetry generators

Shifts of positions under translations, rotations, boosts and dilatation have classically-looking forms

$$(P_\mu, X_\rho) = -\eta_{\mu\rho}$$
$$(J_{\mu\nu}, X_\rho) = \eta_{\nu\rho}X_\mu - \eta_{\mu\rho}X_\nu$$
$$(D, X_\rho) = -X_\rho$$

Shifts of momenta under translations, rotations, boosts and dilatation have classically-looking forms

$$(P_\mu, P_\rho) = 0$$
$$(J_{\mu\nu}, P_\rho) = \eta_{\nu\rho}P_\mu - \eta_{\mu\rho}P_\nu$$
$$(D, P_\rho) = P_\rho$$

If you want more information about the calculations, send me a mail asking for references

Conformal symmetry of electromagnetism

Electromagnetic field theory is invariant under the larger group of conformal transformations with 15 generators (4d space-time) :

4 translations + 6 rotations + 1 dilatation

+ 4 transformations to uniformly accelerated frames

H. Bateman *Proc London Math Soc* 1909
E. Cunningham *ibidem* 1909
E. Bessel-Hagen, *Math. Annalen* 1921

The conformal symmetry algebra determines the redshifts, that are the shifts under transformations to accelerated frames

$$(D, C_\mu) = -C_\mu$$
$$(J_{\mu\nu}, C_\rho) = \eta_{\nu\rho}C_\mu - \eta_{\mu\rho}C_\nu$$
$$(C_\mu, C_\nu) = 0$$
$$(P_\mu, C_\nu) = -2\eta_{\mu\nu}D - 2J_{\mu\nu}$$

Redshift laws

$$\Delta_a O \equiv \frac{1}{2}\left(a^\rho C_\rho, O\right)$$

Redshifts of momenta contain classical terms related to positions and extra terms related to spin

$$\Delta_a P^\nu = a^\nu P \cdot X - a^\rho \left(P_\rho \cdot X_\nu - P_\nu \cdot X_\rho + S_{\rho\nu}\right)$$

Redshifts for positions contain classical terms and non classical ones. The extra terms do not break the symmetry properties, they are consequences of the conformal symmetry

Canonical commutators keep the same form after conformal transformations to accelerated frames ($\hbar$ invariant)

$$\Delta_a \left(P_\mu, X_\nu\right) = 0$$

Metric relations are obtained with classical expressions in terms of the gravity potential arising from the acceleration

$$\left(P_\mu, \Delta_a X_\nu\right) + \left(P_\nu, \Delta_a X_\mu\right) = 2\eta_{\mu\nu}\Phi_a(X)$$

$$\Phi_a(X) = a^\rho X_\rho$$

Quantum mass

Mass observable is invariant under translations, rotations and boosts, $(P_\mu, M) = 0\ , \quad (J_{\mu\nu}, M) = 0$
but not under dilatation $(D, M) = M$

It has the same dimension as momenta (*c* invariant under conformal transformations)

Redshift of mass is determined by the gravity potential $\Delta_a M = M \cdot \Phi_a(X)$

This amounts to include the gravitational energy in the mass *M* in accordance with the equivalence principle

Equivalence between acceleration and gravity brought into the domain of quantum observables

Acknowledgement : Results proven only for special relativity + dilatation + uniform acceleration

Quantum proper time

A proper time observable can be defined from the dilatation generator and the mass $D \equiv M \cdot \tau$

It has the same dimension as positions $(D, \tau) = -\tau$

The proper time observable is conjugated to the mass observable $(\tau, M) = 1$

As their commutator is a number, variations of the two observables are necessarily directly related to each other $\Delta_a(\tau, M) = 0$

Gravity force tends to pull clocks
to places where they tick
at a slower rate

Pound-Rebka experiment

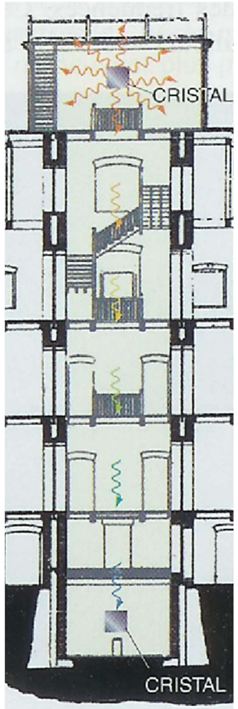
The first precise quantum test (1960) of the Einstein redshift law

- Comparison of γ emission lines of Fe-57 nuclei by using Mössbauer resonance spectroscopy and compensating the redshift by a Doppler shift
- Fe-57 samples in the basement and on the roof of Jefferson laboratory (Harvard)

Einstein redshift measured with an accuracy of a few % (for an altitude difference ≈ 22,5 m)

$$\frac{\Delta\nu}{\nu} \simeq \frac{\Delta\Phi}{c^2} \simeq \frac{g\Delta z}{c^2}$$

$$\frac{g}{c^2} \simeq 10^{-16}\mathrm{m}^{-1}$$

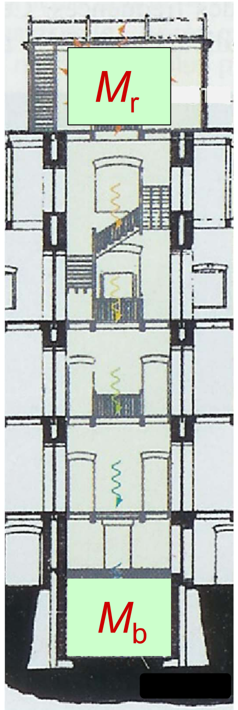


Pound-Rebka experiment

The two physicists performing (together) the Pound-Rebka experiment at the basement and on the roof have the same right to claim that they have the correct value for frequency

When comparing their values, they have to take into account the Einstein redshift

They have to treat their results consistently for frequencies and masses, which amounts to include the gravitational energy in the mass observable according to the equivalence principle



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The Pauli no-go theorem

There exists a theorem, attributed to Pauli, stating that it is impossible to define a quantum observable for time

Precise form of the Pauli theorem :
there is no operator ***T*** with the three properties

1. ***T*** is conjugated to energy ***E***
2. ***E*** is bounded (for example $E \geq 0$)
3. ***T*** is self-adjoint

The theorem is a valid formal statement but its premises cannot be applied to the observable defined above

The observable X_0 is not self-adjoint as it is not defined in all states (photons in two different directions are needed)

Positions and spin

$$P^2 = P^\mu P_\mu \neq 0$$

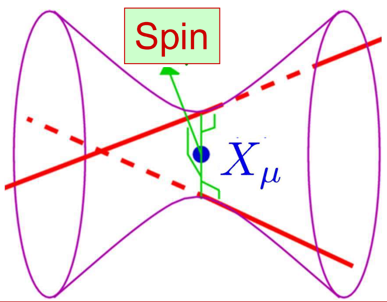
Relativistic spin :

Pauli-Lubanski vector $$W_\mu = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma$$

Spin tensor $$S_{\mu\nu} = \frac{(W_\mu, W_\nu)}{P^2} = \epsilon_{\mu\nu\rho\sigma}\frac{W^\rho P^\sigma}{P^2}$$

Position components have a non null commutator related to the spin

$$(X_\mu, X_\nu) = \frac{S_{\mu\nu}}{P^2}$$



Spin determines the quantum dispersion of positions
→ Einstein's "sharp signal" is impossible for the EM field

GNSS localization

Positions in space and time are defined from incidence of electromagnetic time references in Global Navigation Satellite Systems (GNSS)

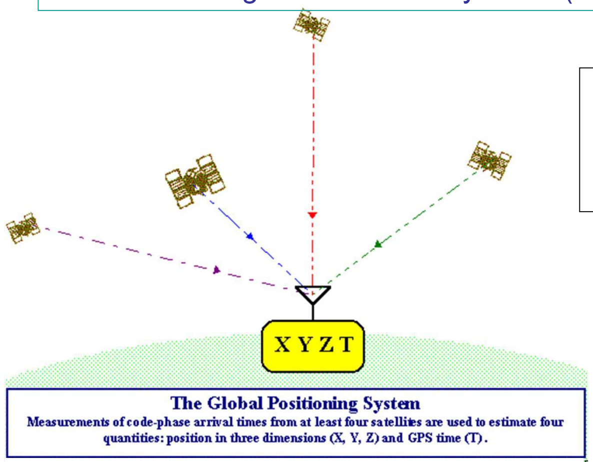


Image Peter H. Dana, The University of Colorado

A receiver deduces its TXYZ-positions from radio signals emitted by at least four satellites

Radio signals are phase fronts generated onboard the satellites and stamped there from atomic clocks