Gravity from Entanglement: Observable Consequences

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Gravity from Entanglement: Observable Consequences

1. Gravity, entanglement and the holographic principle.
2. Observable quantum gravity effects in interferometers.
3. Gravity from entanglement: observed consequences.

Is quantum gravity observable?

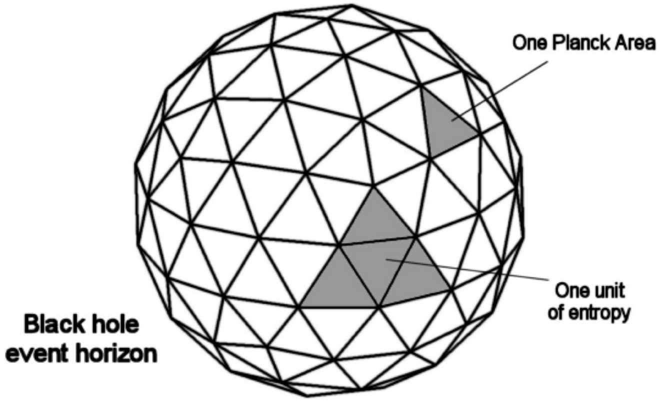
$$\ell_p = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-35} m.$$

Standard lore:

QG is only observable at the Planck scale.

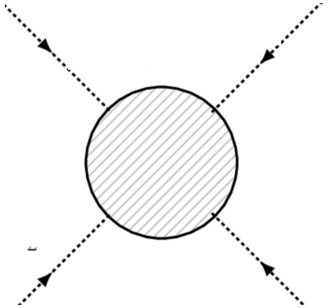
Why this may not be true:

- *QG leads to divergences*
- *Black hole information paradox*
- *Holographic principle*
- *Infrared effects*



$$S = \frac{A\, c^3}{4G\hbar}$$

Is quantum gravity observable?



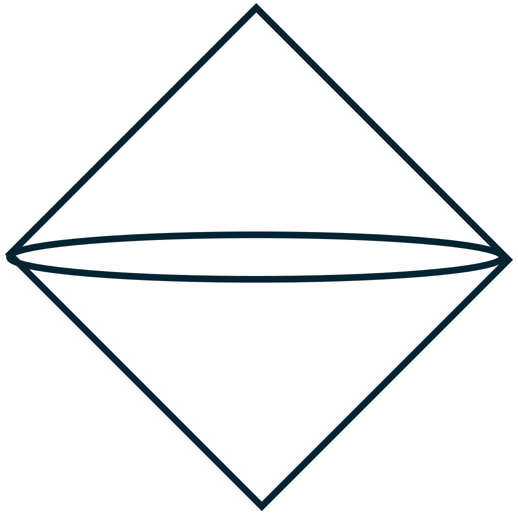
Effective field theory approach to General Relativity suggest

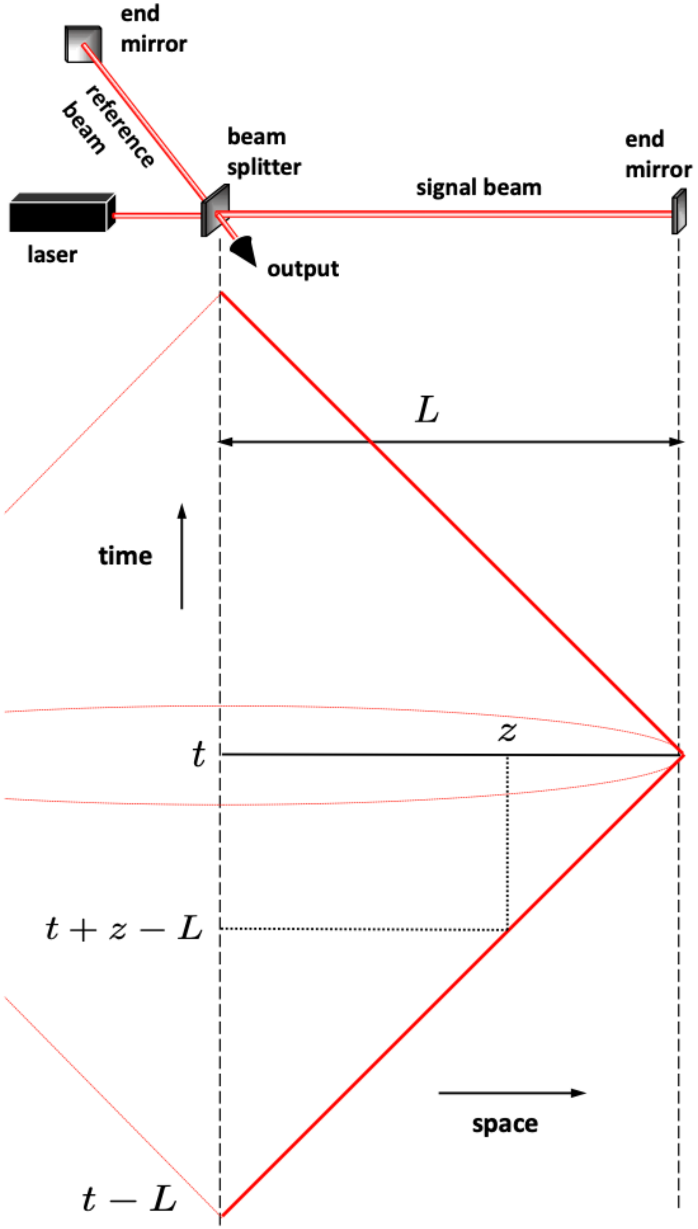
quantum gravity effects are surpressed by powers of : $\frac{E^2}{M_p^2} \sim \frac{L^2}{\ell_p^2}$

Observable signatures of QG require enhancement due to infrared effects

Theoretical arguments why QG involves situations where (naïve) EFT intuition breaks down.

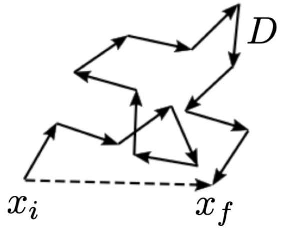
- string theory: UV/IR correspondence
- black hole information paradox
- holography
- soft physics
- Inherent non-locality



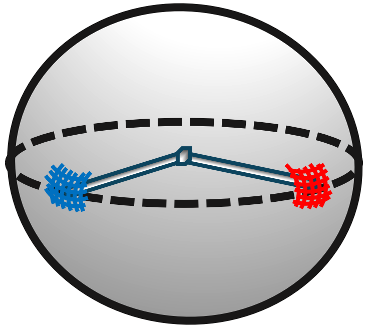


Observational signatures in interferometer experiments

with Kathryn Zurek (2019)



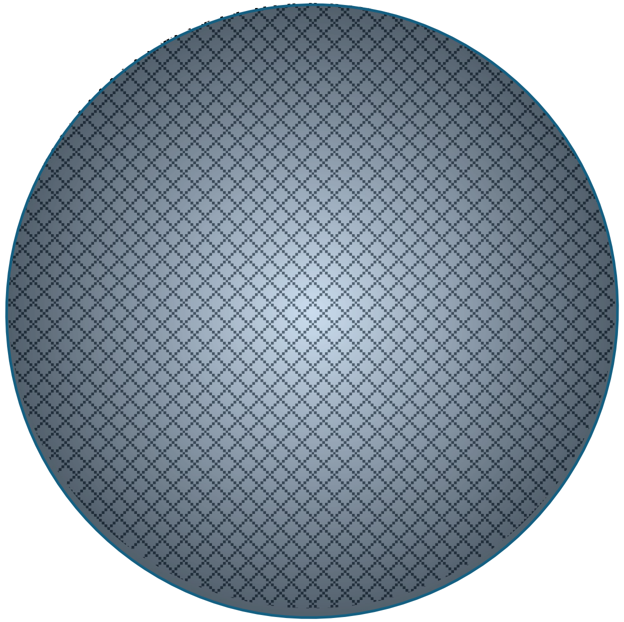
$$\langle \Delta x^2 \rangle = 2DT$$



Holographic Noise in Interferometers

Craig J. Hogan
University of Chicago and Fermilab

Bekenstein-Hawking Entropy



horizon

$$S = \frac{A}{4G\hbar}$$

Hawking temperature

$$T = \frac{\hbar\kappa}{2\pi}$$

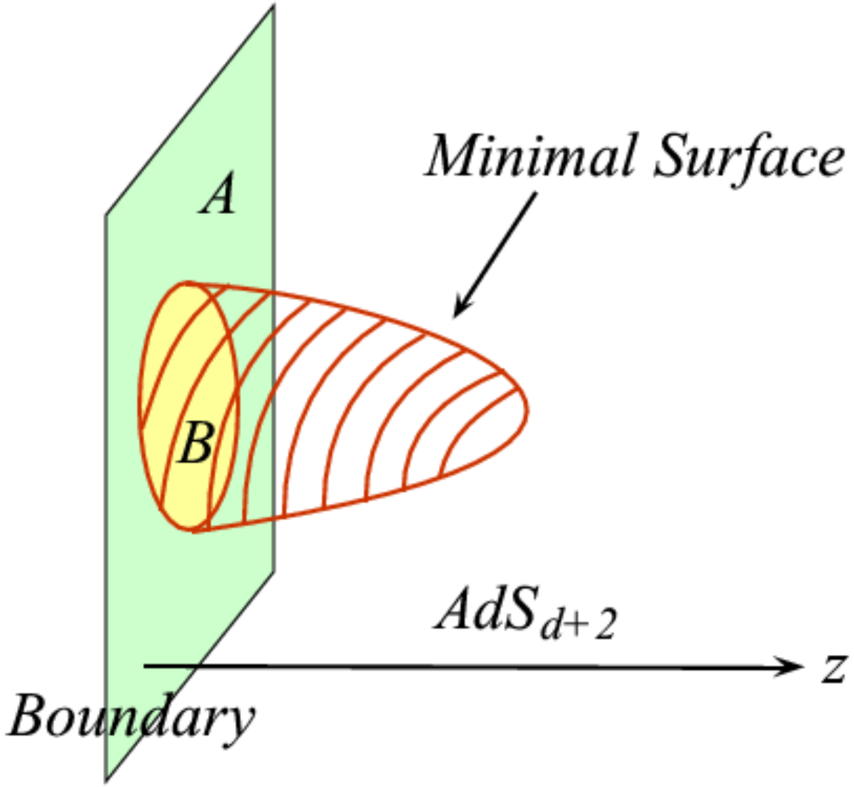
$\kappa = \text{surface gravity}$

Two possible interpretations

$\boldsymbol{S}$ = log(# black hole microstates)

$\boldsymbol{S}$ = entanglement entropy of spacetime vacuum

Ryu-Takanayagi Formula in AdS/CFT

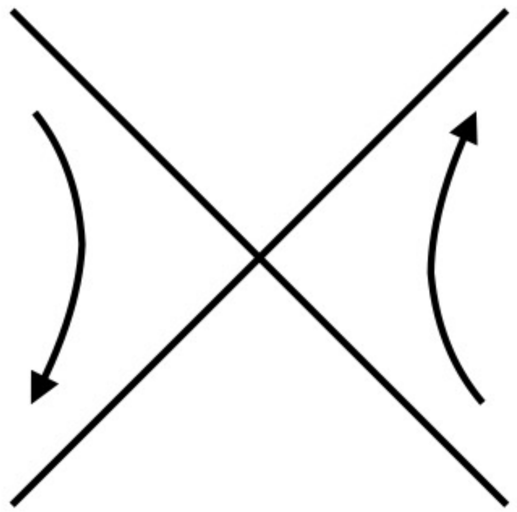


$$S_{ent} = \frac{Area}{4G\hbar}$$

The entanglement entropy of a boundary region *B* is given by the area of a minimal surface that is homologous to this region

Quantum Entanglement of the Vacuum

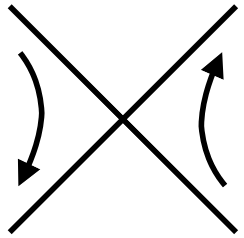
$$|\text{vac}\rangle = \frac{1}{\sqrt{Z}} \sum_i |K_i\rangle_L |K_i\rangle_R e^{-\pi K_i/\hbar}$$



The Minkowski vacuum, when viewed from the perspective of an accelerated observer, looks like a purified thermal state.

This is nowadays called the Thermo-field double state.

1st law of entanglement entropy



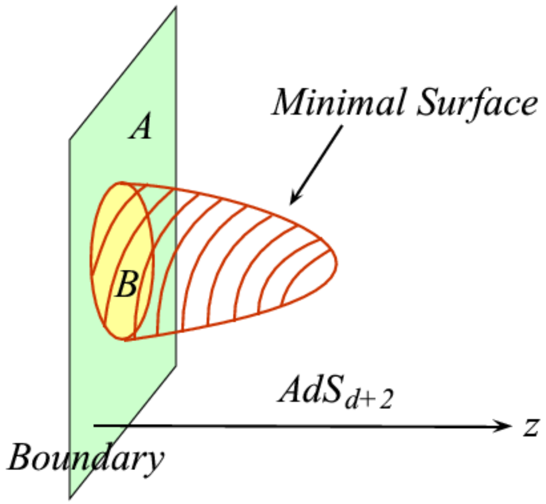
$$S = -\mathrm{tr}(\rho \log \rho) \qquad \rho = \mathrm{tr}_{\mathcal{H}'} \left(|\Psi\rangle\langle\Psi| \right)$$

$$\rho = \frac{e^{-K}}{Z} \qquad Z = \mathrm{tr}\left(e^{-K}\right)$$

Modular Hamiltonian K: defined by entanglement =>
becomes boost generator (= emergent time)

$$\delta S = \delta\langle K\rangle \qquad \delta\langle K\rangle = \mathrm{tr}\left(\delta\rho K\right)$$

Gravity from Quantum Entanglement



Gravitational equations are derived from 1st law of **entanglement entropy**

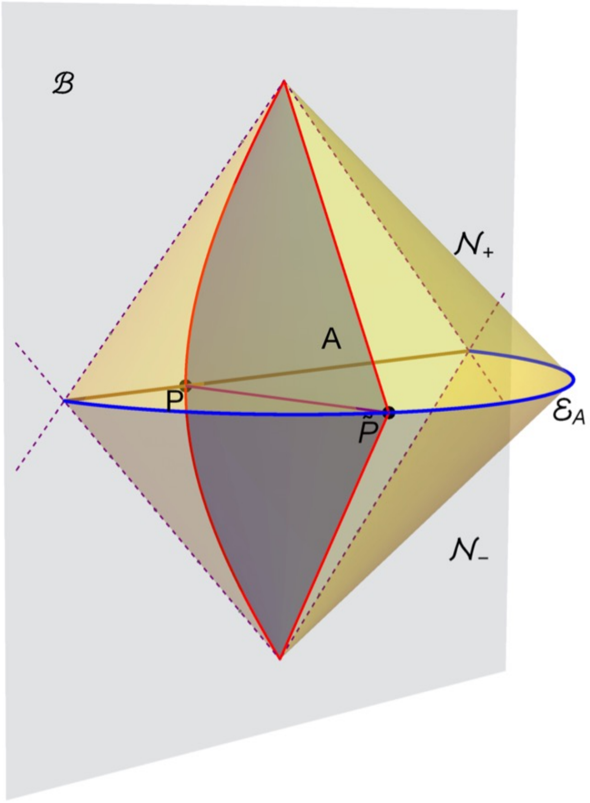
$$S_{ent} = \frac{Area}{4G\hbar}$$

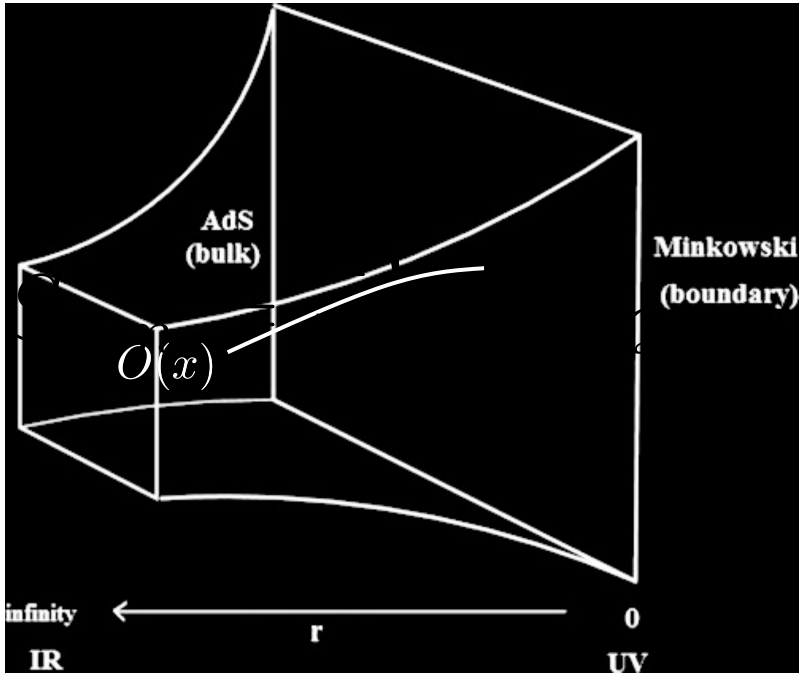
Ryu, Takanayagi
van Raamsdonk
Myers, Casini etal.

Observables in quantum gravity are inherently non-local

General covariance => observables are defined relationally.

In AdS/CFT observables are defined in relation to the boundary.



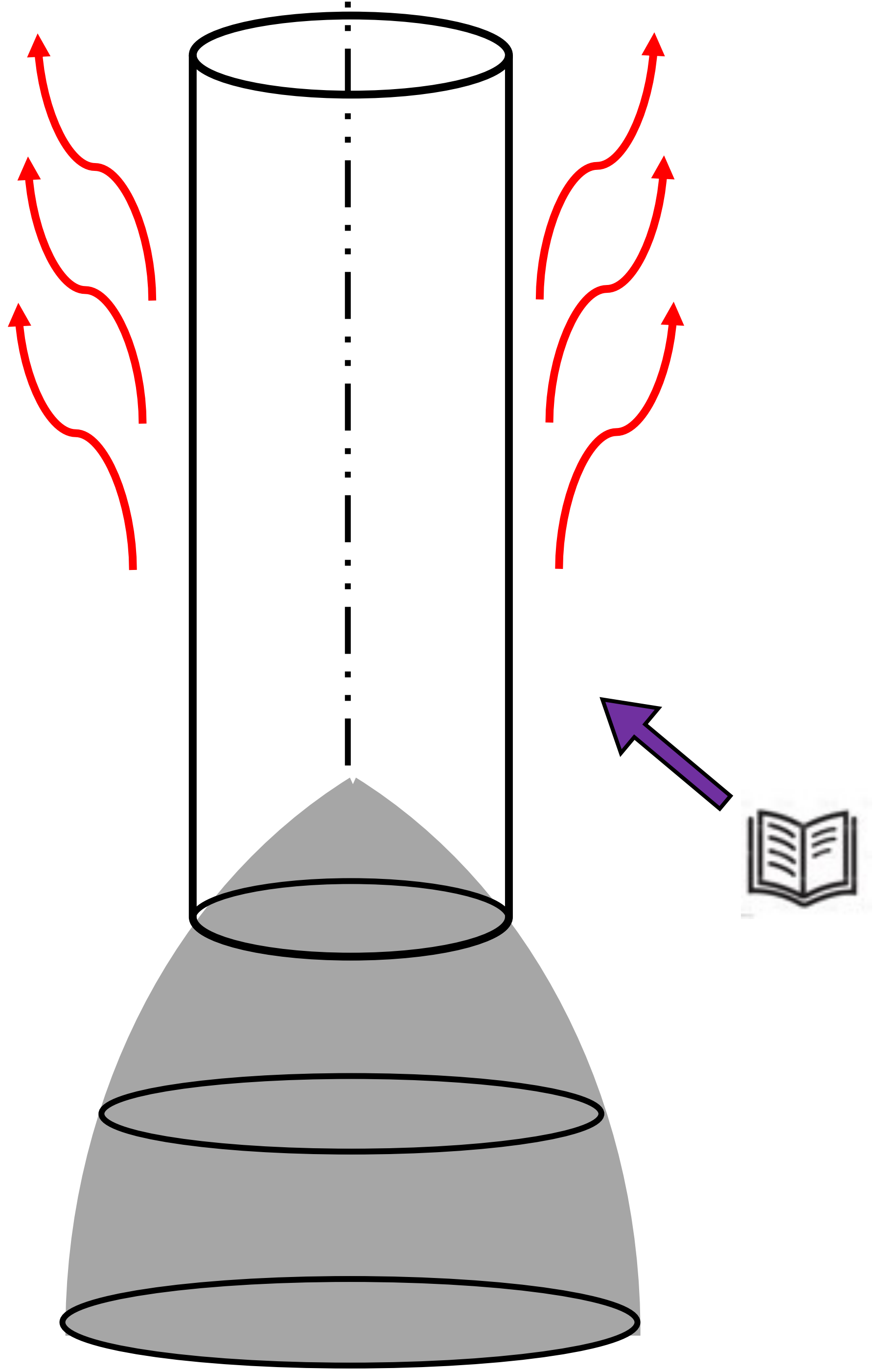


We do not live in Anti-de Sitter space... but we can learn from AdS/CFT

Black hole information paradox

Semiclassical GR + local EFT:
BH formation + evaporation
destroys (quantum) information.

Unitarity in QM: can only be restored
via non-local quantum gravity effects.

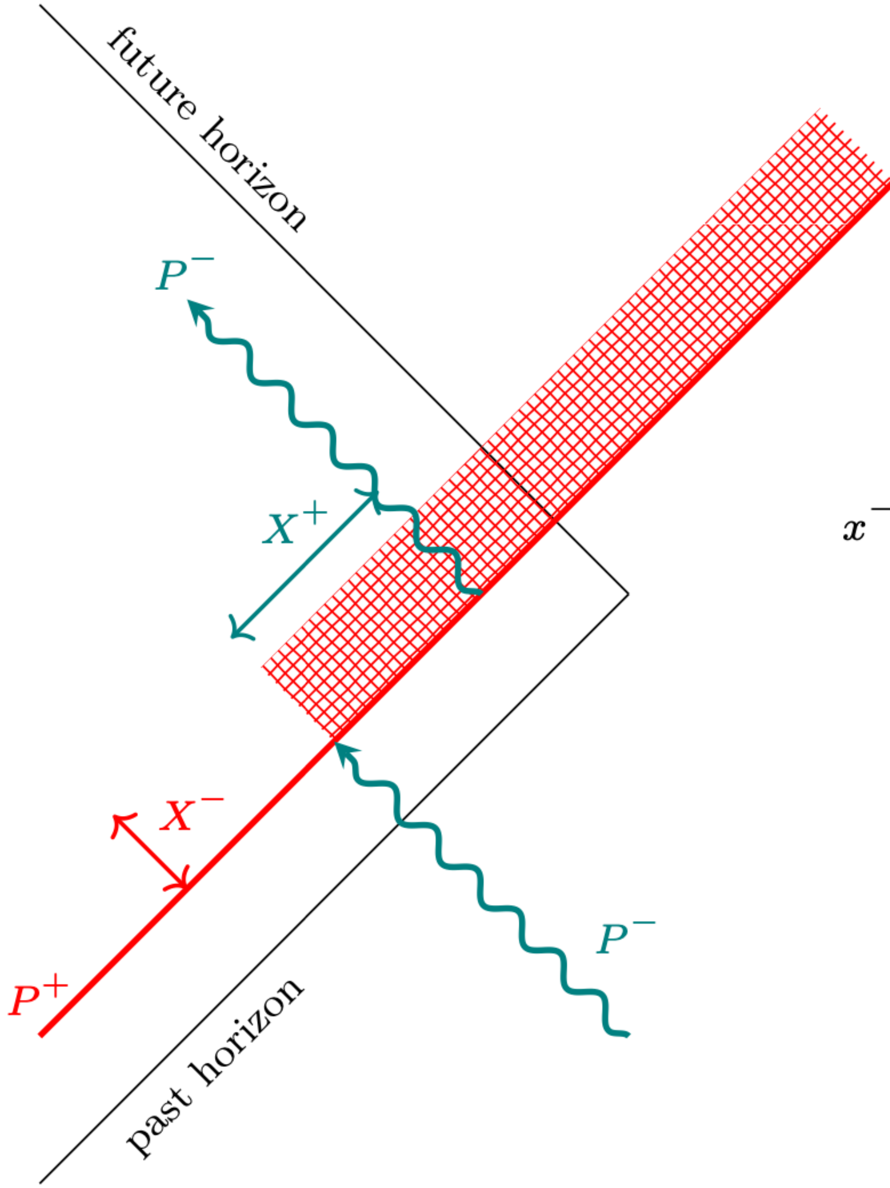


Lessons learned from the Black hole information paradox

- Horizon scale physics
- Quantum entanglement.
- Quantum chaos
- Shockwaves

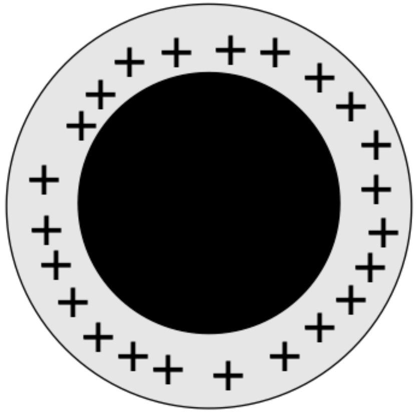
’t Hooft commutation relations

$$[X^+(z,\bar{z}), X^-(z',\bar{z}')] = 8\pi i G_N G(z - z')$$



5 Jan 2004

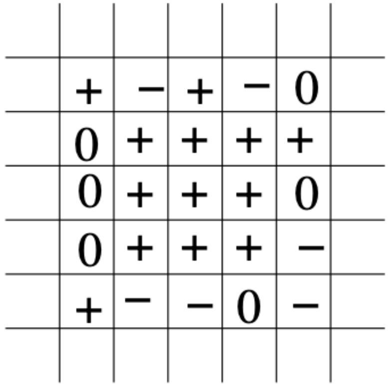
On the quantum width of a black hole horizon



Donald Marolf

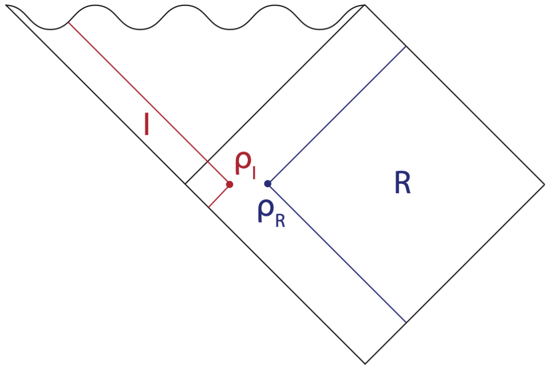
Physics Department, UCSB, Santa Barbara, CA 93106. marolf@physics.ucsb.edu

$$\delta L^2 \sim \frac{L^2}{\sqrt{S_{BH}}}, \qquad \delta L \sim \sqrt{l_p L}.$$



20 Dec 2023

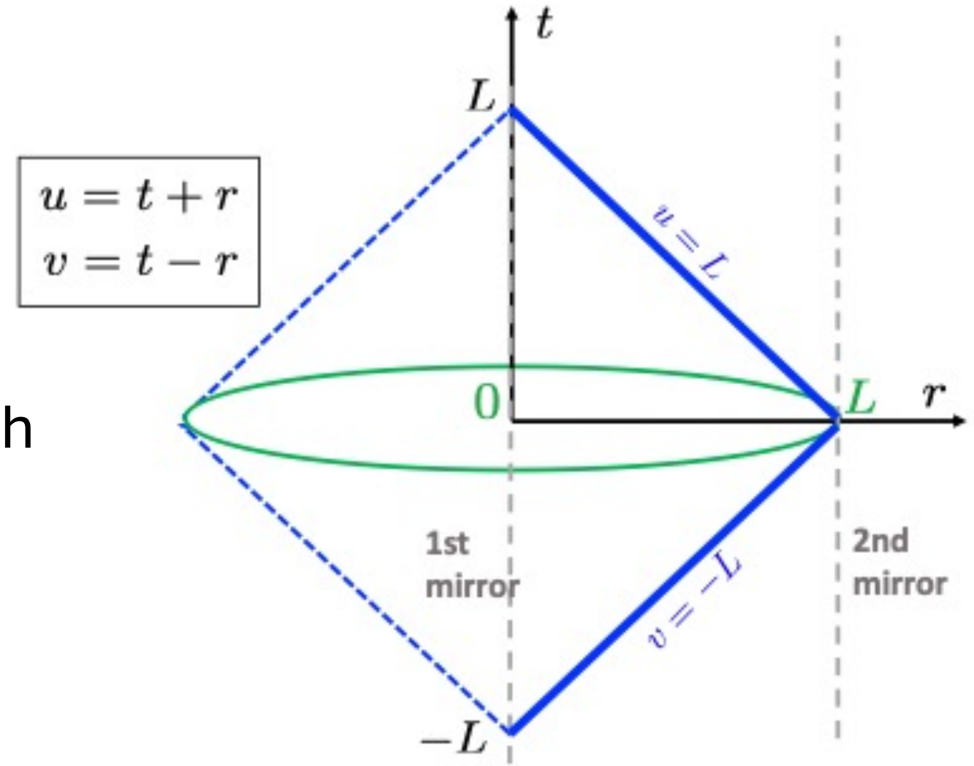
Islands Far Outside the Horizon

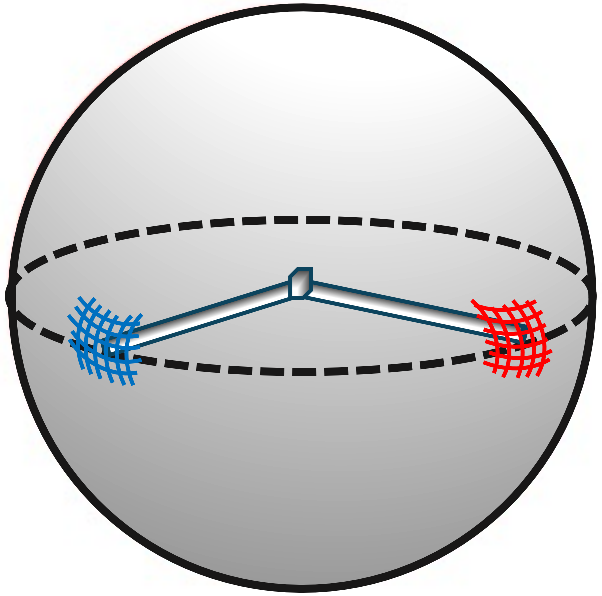


Raphael Bousso and Geoff Penington

Center for Theoretical Physics and Department of Physics,
University of California, Berkeley, California 94720, U.S.A.

Causal Diamond





contains single light path

Length fluctuations

$$\delta L = \left(\delta v(L) + \delta u(L)\right)/2.$$

$$\delta L^2 = \delta u(L)\delta v(L)$$

$$\delta v(L) = \int_{-L}^{L} du\ h_{uu}(u, L)$$

$$\delta u(L) = \int_{-L}^{L} dv\ h_{vv}(L, v).$$

Calculation follows Marolf's argument

Via the coordinate transformation

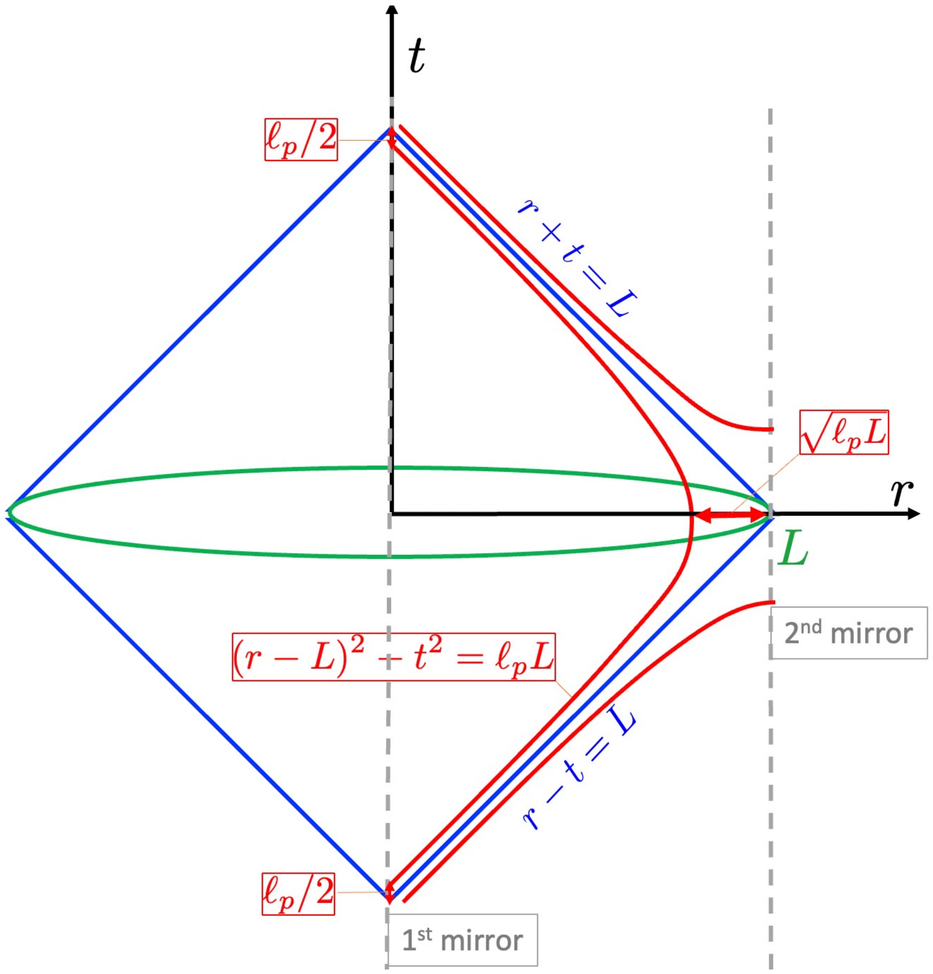
$$(u-L)(v-L) = 4L^2 f(R), \qquad \log\frac{u-L}{v-L} = \frac{T}{L}$$

The metric becomes spherical Rindler space

$$ds^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

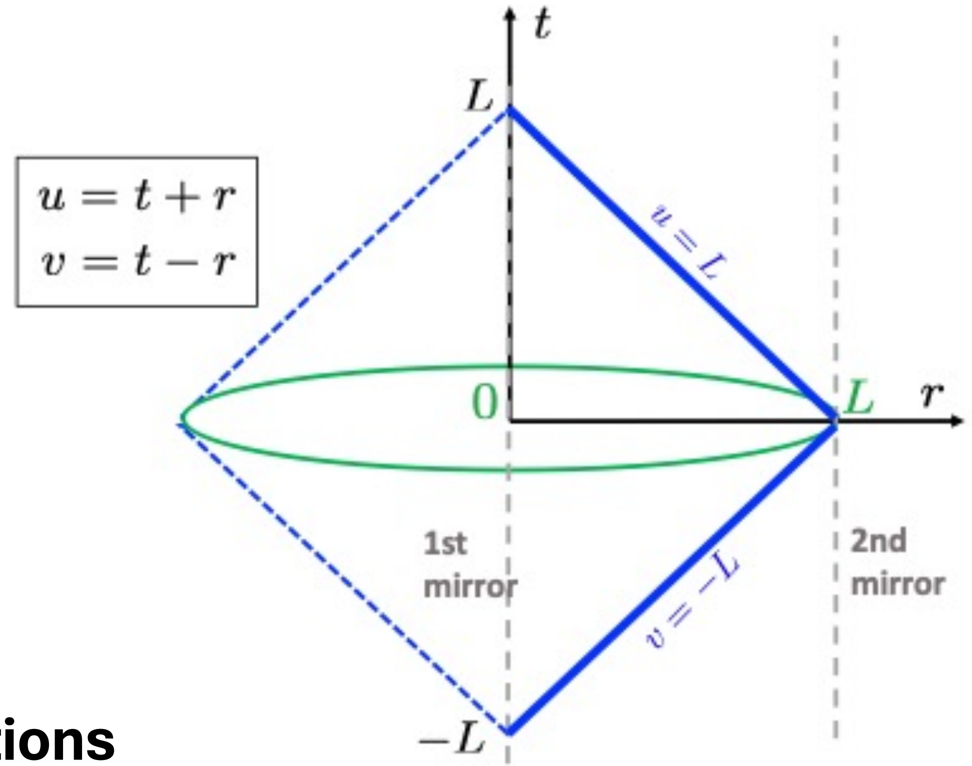
Backreaction effects are encoded in the potential Φ

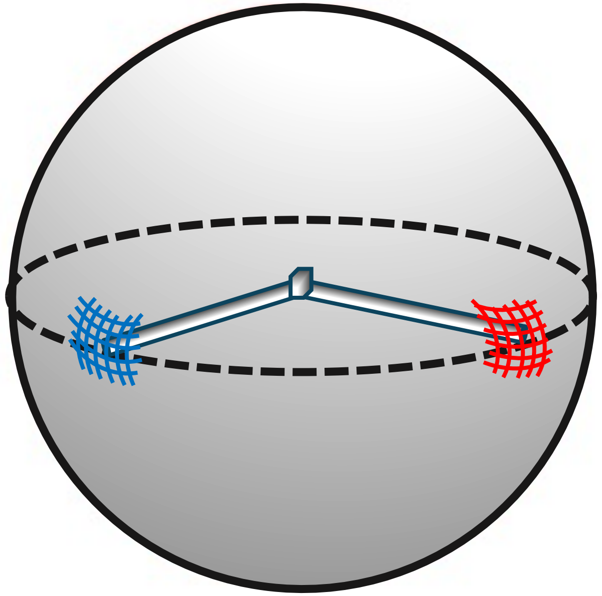
$$f(R) = 1 - \frac{R}{L} + 2\Phi.$$



Causal Diamond

boundary behaves like Rindler horizon





Modular energy fluctuations lead to variations of the Newtonian potential

$$\frac{\delta v(L)\delta u(L)}{L^2} = 2\Phi(L)$$

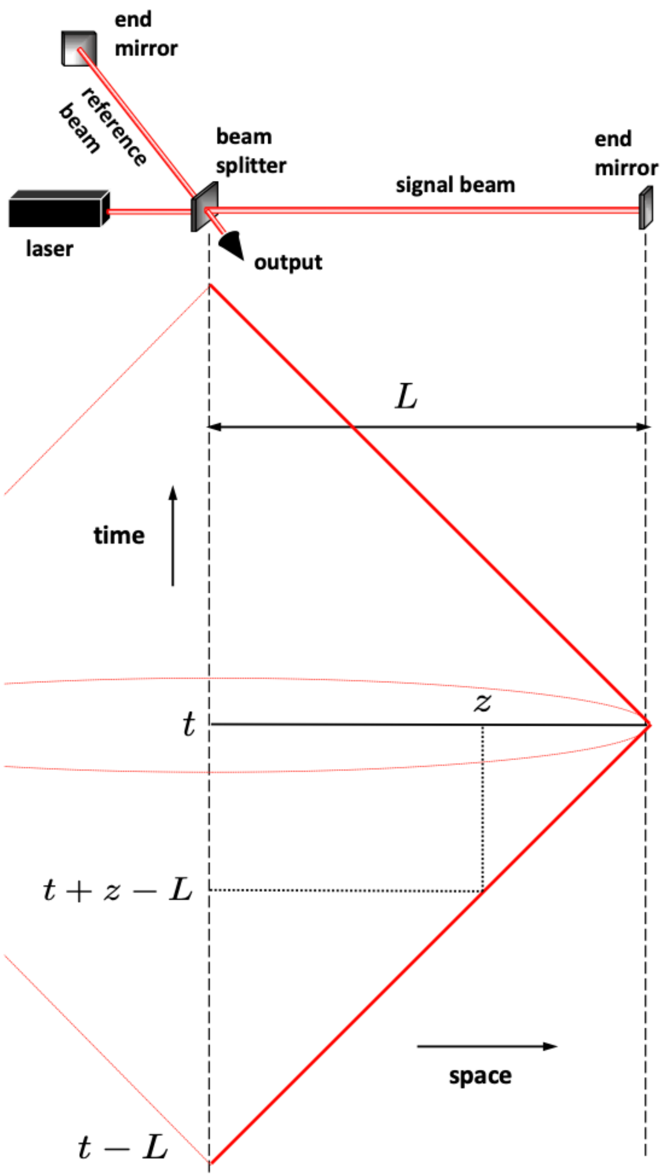
$$\langle \Delta M^2 \rangle = -\frac{\partial^2}{\partial \beta^2}(\beta F) = \frac{1}{l_p^2}.$$

$$\delta L^2 = \delta u(L)\delta v(L)$$

$$\left\langle \frac{\delta L^2}{L^2} \right\rangle = \frac{l_p^2 \Delta M}{4\pi L} = \frac{l_p}{4\pi L},$$

Observational signatures in interferometer experiments

with Kathryn Zurek (2019)



Vacuum energy fluctuations

$$\langle \Delta M^2 \rangle = -\frac{\partial^2}{\partial \beta^2}(\beta F) = \frac{1}{l_p^2}.$$

lead to length fluctuations

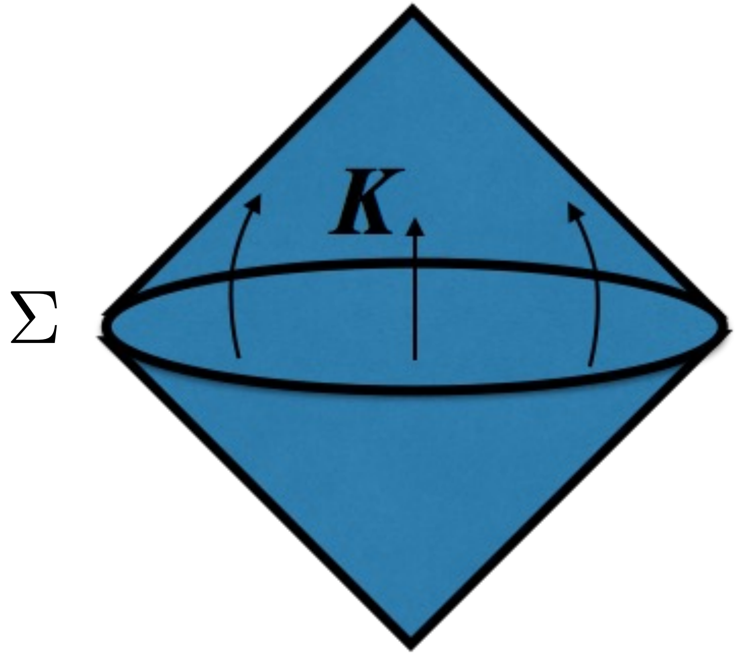
$$\left\langle \frac{\delta L^2}{L^2} \right\rangle = \frac{l_p^2 \Delta M}{4\pi L} = \frac{l_p}{4\pi L},$$

Actual observable is the power spectral density

$$S(\omega, t) = \int_{-\infty}^{\infty} d\tau \left\langle \frac{\delta L(t)}{L} \frac{\delta L(t-\tau)}{L} \right\rangle e^{-i\omega\tau}$$

Article PDF

Causal Diamonds, Entanglement and Modular Energy



Region bounded by past and future lightcones of two timelike seperated points.

Described by a density matrix:

$$\rho = \mathrm{tr}_{\mathcal{H}'}\left(|vac\rangle\langle vac|\right) = e^{-K}$$

Entanglement entropy:

$$\langle K \rangle = \frac{A(\Sigma)}{4G}.$$

Modular Energy fluctuations

$$\langle \Delta K^2 \rangle = \frac{A(\Sigma)}{4G}.$$

Spacetime Fluctuations in AdS/CFT

(*with Kathryn Zurek*)

The AdS-metric

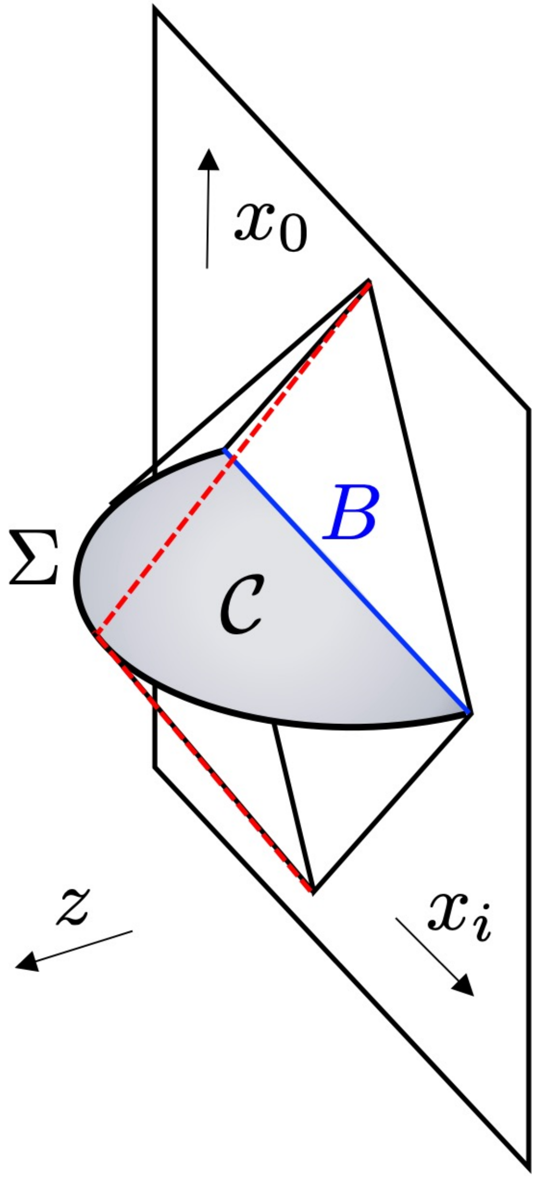
$$ds^2 = L^2 \, \frac{dz^2 + dx_i^2 - dx_0^2}{z^2}$$

can be transformed to the AdS-Rindler metric

$$ds^2 = -\left(\frac{r^2}{L^2} - 1\right) dt^2 + \left(\frac{r^2}{L^2} - 1\right)^{-1} dr^2 + r^2 d\Sigma_{d-1}^2$$

The fluctuations in the modular Hamiltonian

$$\Delta K = \int_{\mathcal{C}} \xi_K^{\mu} T_{\mu\nu}^{bulk} d\mathcal{C}^{\nu},$$



Can be computed using (a gravitational version of) the replica method

Spacetime Fluctuations in AdS/CFT

From the free energy on the n-fold cover

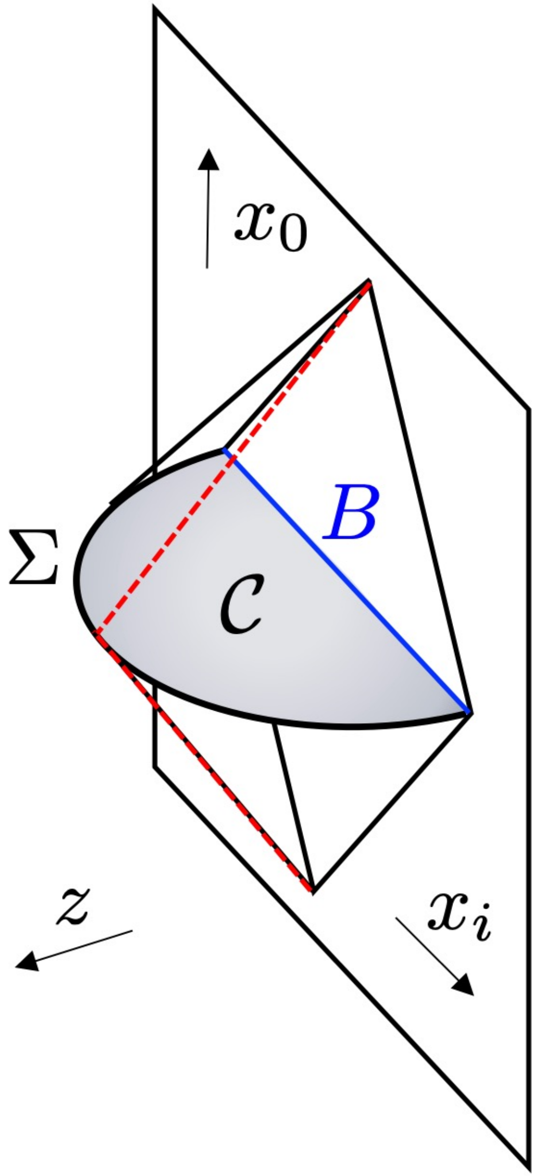
$$Z_n = \mathrm{tr}(e^{-nK}) = e^{-nF_n}$$

One derives the expectation value and fluctuations of the modular Hamiltonian

$$\langle K \rangle = \frac{d}{dn}\Big(nF_n\Big)\Big|_{n=1} \qquad \langle \Delta K^2 \rangle = -\frac{d^2}{dn^2}\Big(nF_n\Big)\Big|_{n=1}$$

One finds

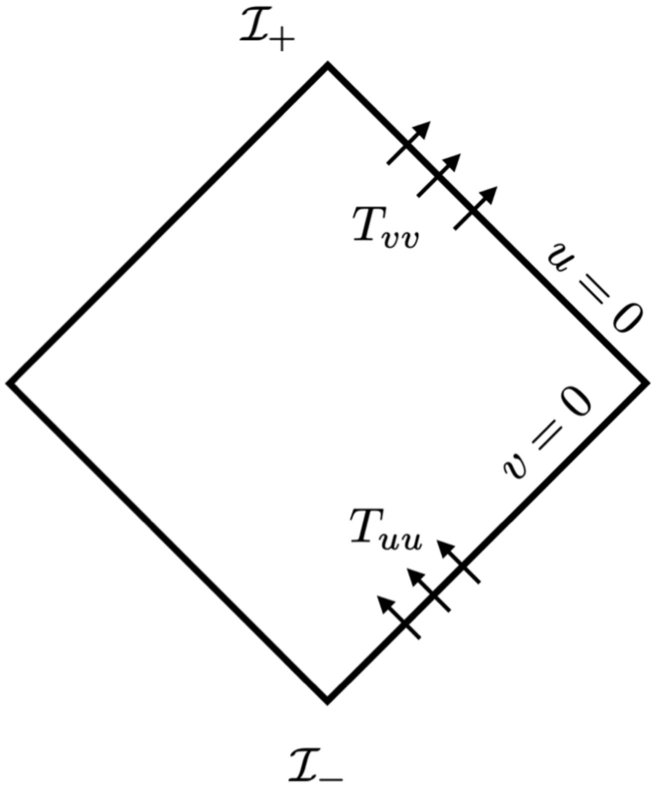
$$\langle K \rangle = \frac{A(\Sigma)}{4G} \qquad \langle \Delta K^2 \rangle = \frac{A(\Sigma)}{4G}$$



Modular Energy Fluctuations from 't Hooft commutation relations

Metric fluctuations

$$ds^2 = -dudv + \nabla_y X^v dudy + \nabla_y X^u dvdy + dy^2.$$



The modular energy equals

$$K = \frac{1}{\ell_p^{d-2}} \int d^{d-2}y \, \nabla_y X^u(y) \nabla_y X^v(y)$$

$$= \int d^{d-2}y \left[\int_{-\infty}^{0} du \, X^u T_{uu} + \int_0^{\infty} dv \, X^v T_{vv} \right]$$

As operators the coordinate shifts obey

$$\left\langle X^u(y) X^v(y') \right\rangle = \ell_p^{d-2} f(y, y').$$

Modular Energy Fluctuations from 't Hooft commutation relations

Metric fluctuations

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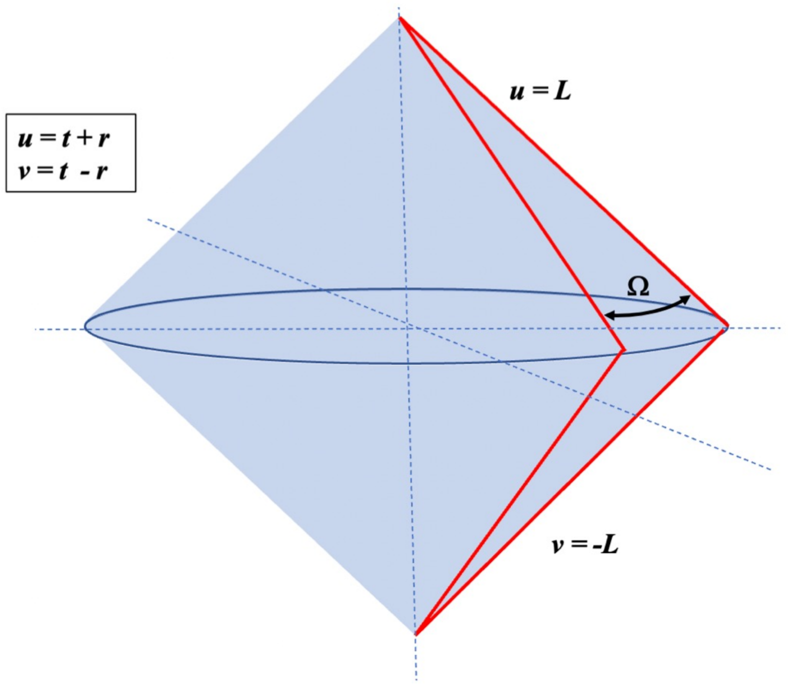


The modular energy equals

$$K = \frac{1}{\ell_p^{d-2}} \int d^{d-2}y\, \nabla_y X^u(y) \nabla_y X^v(y)$$

We derive using $\left\langle X^u(y) X^v(y') \right\rangle = \ell_p^{d-2} f(y, y').$

the following size of the metric fluctuations

$$\left\langle \left(\delta g_y{}^u \delta g_y{}^v\right)^2 \right\rangle \sim \left\langle \left[\nabla_y X^u \nabla_y X^v\right]^2_{avg} \right\rangle \sim \left(\frac{\ell_p}{L}\right)^{d-2}$$

Heising-Simons Foundation (HSF) Collaboration on Quantum Gravity and Its Observational Signatures (QuRIOS)

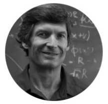
Seeking to bridge the divide between theory and observability

Kathryn Zurek (Caltech)
Particle theory, Effective field theory and models

Erik Verlinde (University of Amsterdam)
String theory, emergent gravity

Maulik Parikh (Arizona State University)
Quantum gravity, classical and quantum black holes

Cynthia Keeler (Arizona State University)
String theory, fluid gravity

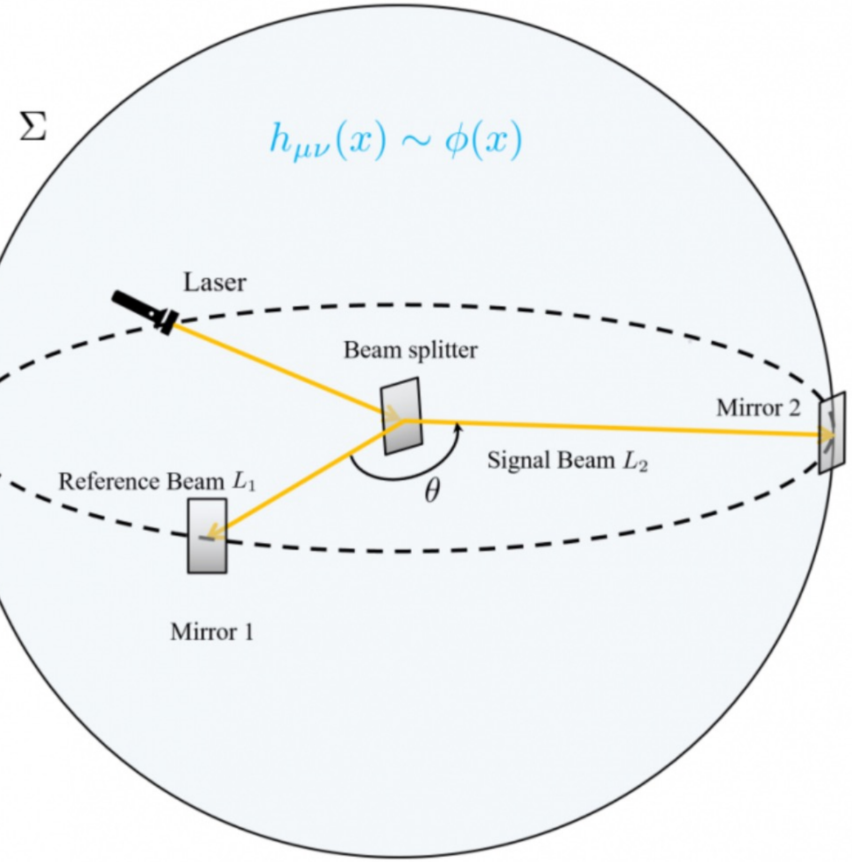


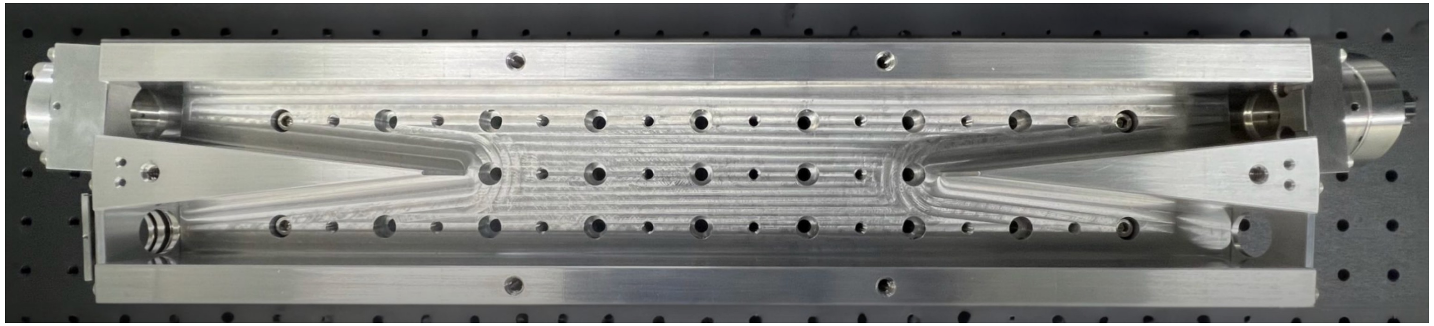
Steve Giddings (UC Santa Barbara)
Quantum gravity, black holes

Ben Freivogel (University of Amsterdam)
String theory, cosmology, early universe

GQuEST

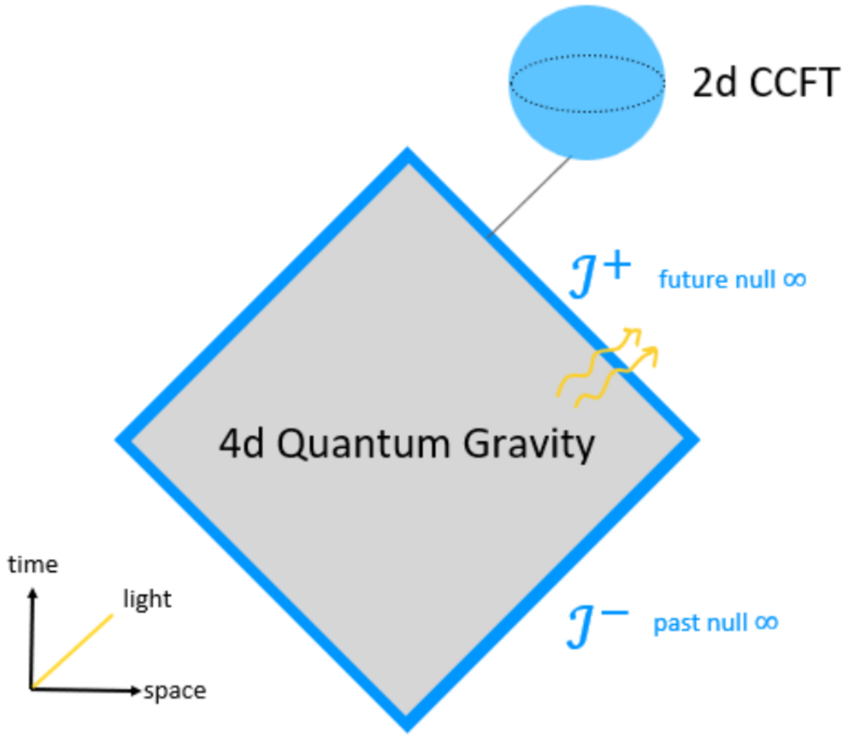
Gravity from the Quantum Entanglement of Space Time

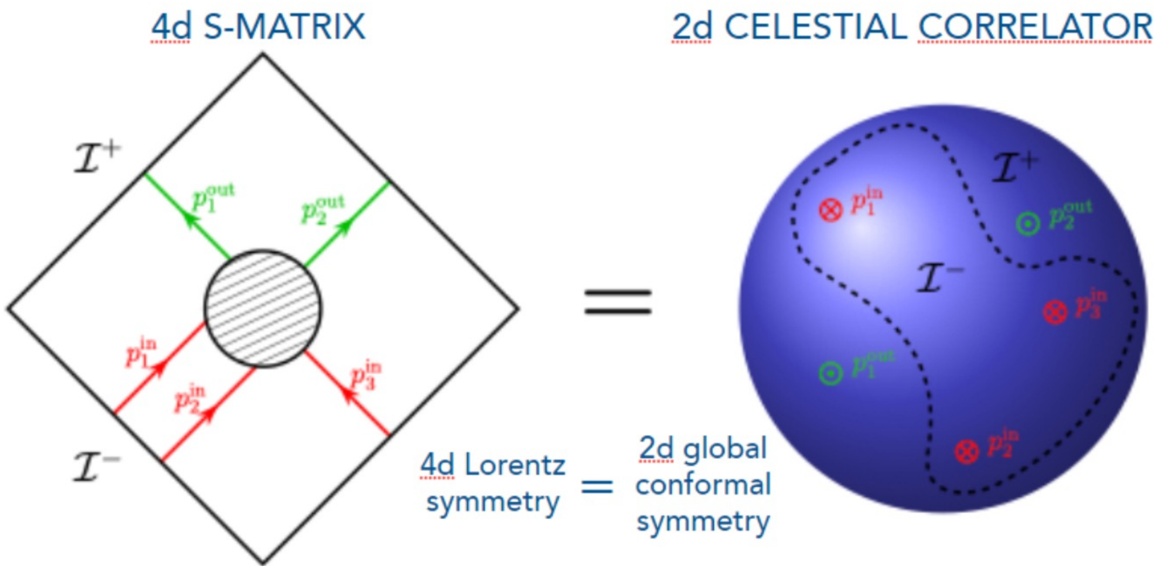




Relation with Celestial Holography

talks by Ana, Andrea, Temple and Kathryn





Relation with Celestial Holography

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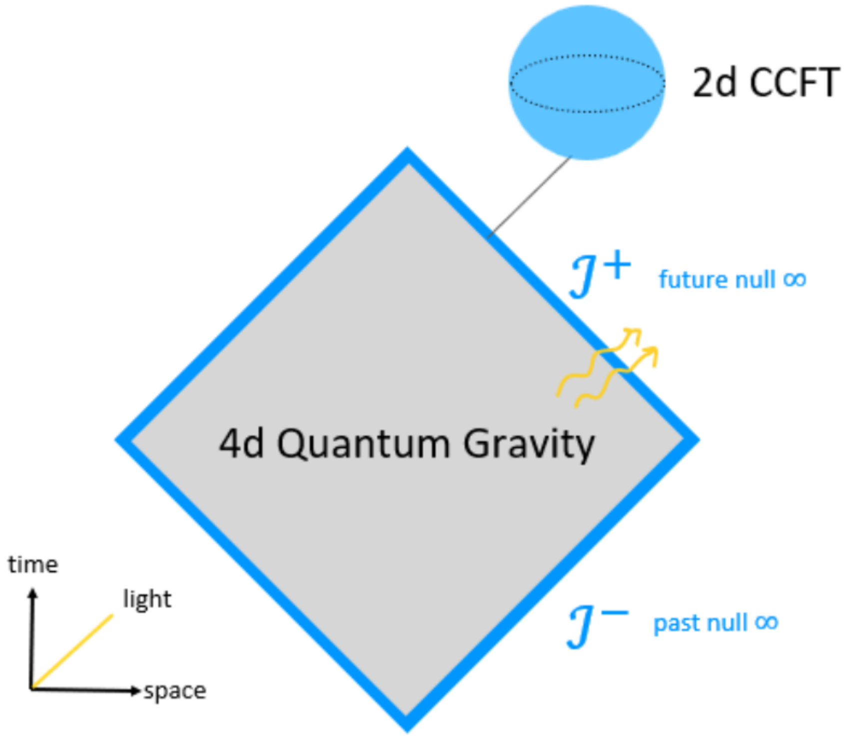
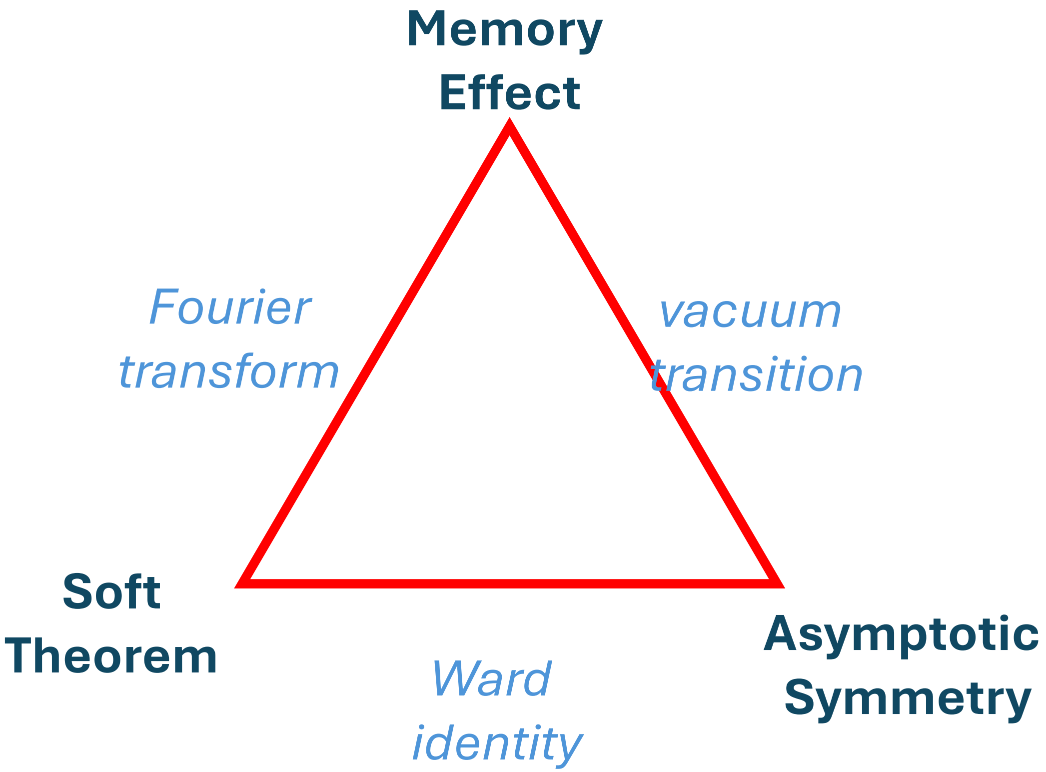
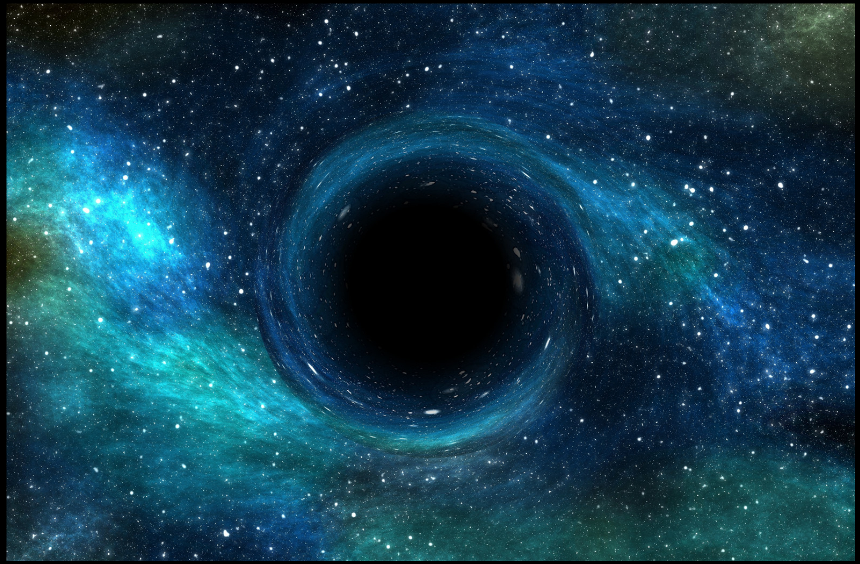
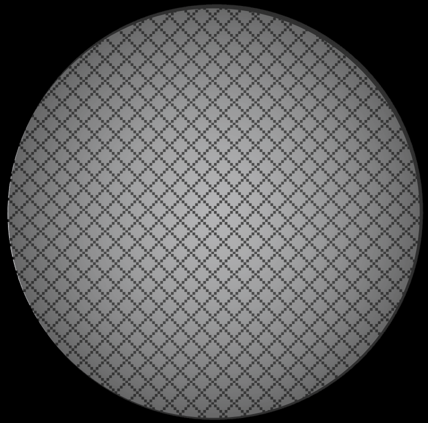




The shockwave geometries are related to the gravitational memory





$$S = k_B \frac{Ac^3}{4\pi G\hbar}$$

$$k_B T = \frac{\hbar\kappa}{2\pi c}$$

Black hole hole thermodynamics relates Einstein equations
to 1st law of thermodynamics = derivable from microscopic theory

$$dM = \frac{\kappa}{2\pi}\frac{dA}{4G}$$

$$dE = TdS$$

$$\kappa = \text{surface gravity}$$

Emergent gravity = derivable
from microscopic theory!

Gravity and spacetime from quantum entanglement.

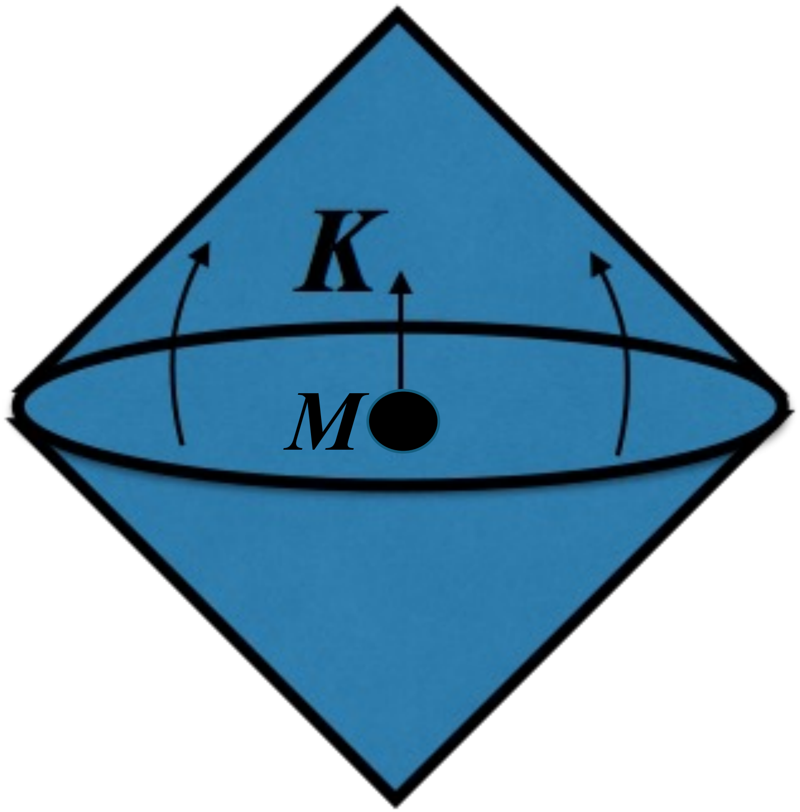
A change in the modular energy for a region of size ***R***

$$K = 2\pi \int d^n x \left(\frac{r^2 - |x|^2}{2r} \right) T_{00}(x)$$

gives a change in the entanglement entropy

$$\frac{\Delta A(R)}{4G} = -2\pi M R$$

=> the Einstein equations when the volume is kept fixed.

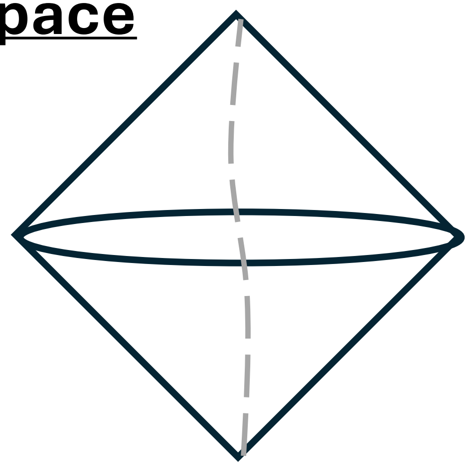


Ted Jacobson,
Manus Visser

Can gravity and spacetime be derived purely from Quantum entanglement? What are the additional assumptions?

What is the Microscopic Quantum description of de Sitter space

$$ds^2 = -\left(1 - \frac{R^2}{L^2}\right) dt^2 + \frac{dR^2}{1 - R^2/L^2} + R^2 d\Omega^2$$

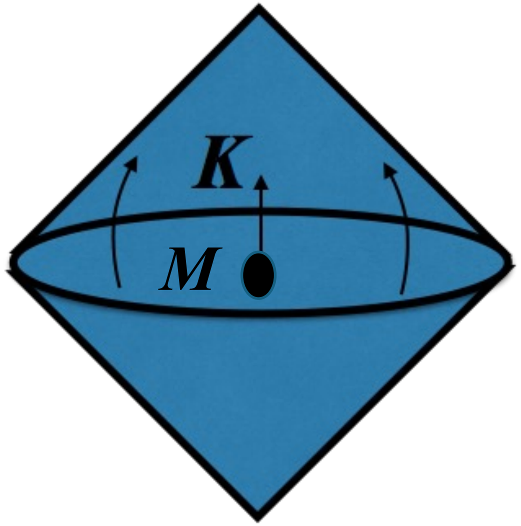


What is the interpretation of the de Sitter entropy?

$$S = \frac{A(L)}{4G}$$

What is the microscopic explanation of

$$\Delta S = -2\pi M L$$

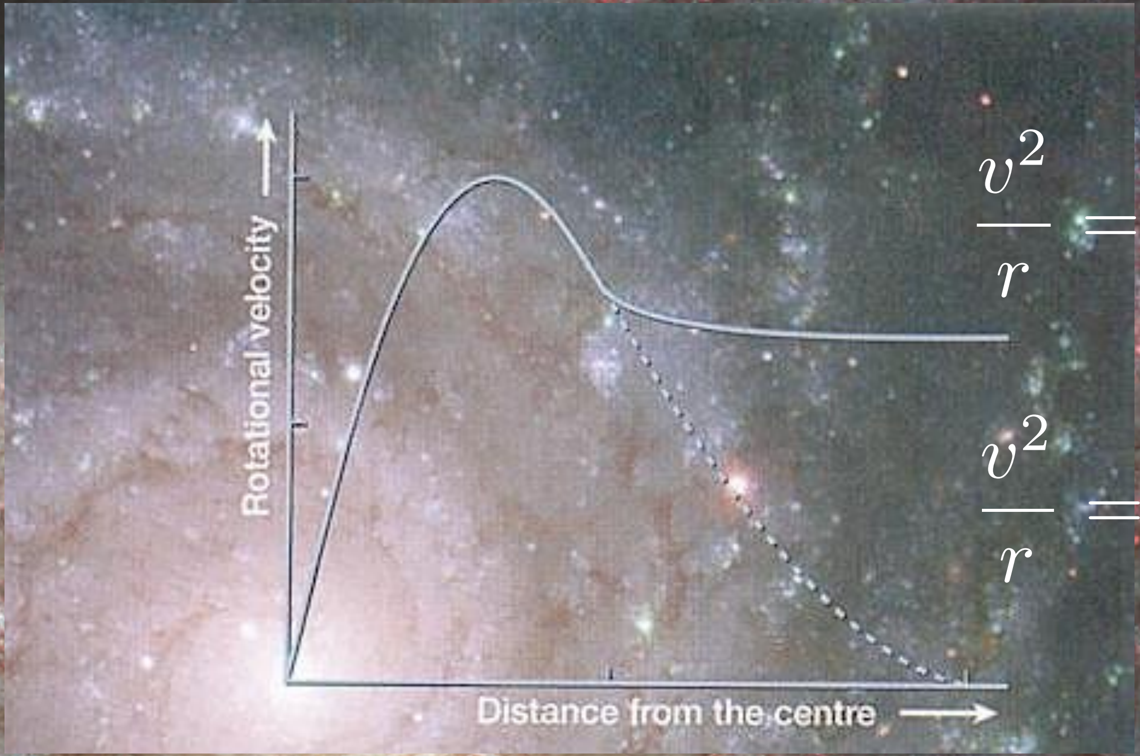


Can we again derive the gravitational equations?

Why does gravity appear to behave differently when

$$2\pi M L \leq \frac{A(R)}{4G}$$

?



$$\frac{v^2}{r} = \frac{GM_B}{r^2} + \frac{GM_D}{r^2}$$

$$\frac{v^2}{r} = \frac{GM_B}{r^2}$$

Empirically DM-effects appear when

$$\frac{GM}{r^2} < \frac{cH_0}{2}$$

$$\frac{Mc^2}{\hbar H_0/2\pi} < \frac{Ac^3}{4G\hbar}$$

THANK YOU

Matter entangles with Dark Energy

- The empirical fact

$$\frac{GM}{R^2} < \frac{cH_0}{2}$$

- implies that DM-effects appear when

$$2\pi \frac{McR}{\hbar} < \frac{A(R)c^3}{4G\hbar} \frac{R}{L}$$

- The left hand side is the entanglement entropy of matter.
- The right hand side represents the entropy contained in DE.