Event Generators for Future Colliders













CLUSTER OF EXCELLENCE QUANTUM UNIVERSE







Monte Carlo Simulation at colliders



LHC, 13 TeV, $pp \rightarrow Z \rightarrow \mu^+ \mu^-$, with pile-up



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ILC, 1 TeV, $e^+e^- \rightarrow t\bar{t}h \rightarrow jjjjjbb$



Simulation vs. Reconstruction





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"Inverse simulation": Reconstruction

Simulation vs. Reconstruction





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Simulating physics at a (lepton) future collider

Disclaimer: focus here mostly on future lepton colliders (e^+e^- and $\mu^+\mu^-$ colliders)





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Why are event generators important? Why are event generators non-trivial?





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Because all our forward simulation chain depends on them! Because they contain *all* our knowledge of particle physics!



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Why Monte Carlo integration/sampling?

Final State X	part.	$\dim(e^+e^- \to X)$
$\mu^+\mu^-$	2	4
jjj	3	7
$\ell^+\ell^-bb$	4	10
$\ell\ell bbj$	5	13
$\ell u \ell u b b$	6	16
$\dots \ \ell u \ell u b b j j j j$	 10	$\frac{1}{28}$

$$\left(\prod_{i=1}^{n} \widetilde{dq_i}\right) (2\pi)^4 \delta^4(p_1 + p_2 - \sum_{i=1}^{n} q_i) \equiv \left(\prod_{i=1}^{n} \frac{d^3 q_i}{(2\pi)^3 2q_i^0}\right) (2\pi)^4 \delta^4(p_1 + p_2 - \sum_{i=1}^{n} q_i)$$

$$\sigma = \int \frac{|\mathcal{M}|^2 \,\Theta(\text{cuts})}{F} d\Phi_n$$





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Dimensionality of phase space (PS) integration for *n* final state particles: dim = $3 \cdot n - 4$ (+ 2 beam parameters)



Why Monte Carlo integration/sampling?

$\left(\begin{array}{c}n\\ \end{array}\right)$	$ 1 \cdot (+ - \cdot \nabla)$	I ,	
dq_i ($\dim(e \cdot e \to X)$	part.	Final State X
i=1	4	2	$\mu^+\mu^-$
	$\overline{7}$	3	jjj
Dimensional	10	4	$\ell^+\ell^-bb$
Diffensional	13	5	$\ell\ell bbj$
	16	6	$\ell u \ell u b b$
Monto	•••	•••	• • •
wonte	28	10	$\ell u \ell u b b j j j j$
	-		

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Carlo integration is the only choice !

Numerical event sampling from probability distribution as predictions for experiments



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$\left(\begin{array}{c} 1 \\ i=1 \end{array}^{aq_i} \right) $	$\frac{4}{4}$	2	$\frac{\mu^+\mu^-}{\mu^+\mu^-}$
	7	3	jjj
Dimensional	10	4	$\ell^+\ell^-bb$
Dimonorona	13	5	$\ell\ell bbj$
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Carlo integration is the only choice !

Numerical event sampling from probability distribution as predictions for experiments

• Generate weighted events, *i.e.* pairs (\vec{x}_i, w_i)

• Unweighted events: event generation with same probability as

[accept events with probability $P_i = w_i / w_{max}$] nature

Suitably choose maximal weight *w_{max}*

• Avoid wildly fluctuating weights \Rightarrow clever choice of mappings



$$I = \int f dV = \int \frac{f}{g} g dV = \left\langle \frac{f}{g} \right\rangle \pm \frac{1}{\sqrt{N}} \left\langle \left\langle \left(\frac{f}{g}\right)^2 \right\rangle - \left\langle \frac{f}{g} \right\rangle^2 \right\rangle$$

Need to know inverse and Jacobian



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Importance Sampling — Mapping phase space channels

divide out singular structures of function(s)

Sampling flat in $\int g \, dV$



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Typical application in particle physics: Breit-Wigner resonance of unstable particles:



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$$\overline{\mathcal{F}^2\Gamma^2}$$
 $F(s) = \arctan \frac{s - m^2}{m\Gamma} =: \xi$





divide out singular structures of function(s) Reduce crude MC error by **importance sampling**:

$$I = \int f dV = \int \frac{f}{g} g \, dV = \left\langle \frac{f}{g} \right\rangle \pm \frac{1}{\sqrt{N}} \sqrt{\left\langle \left(\frac{f}{g}\right)^2 \right\rangle - \left\langle \frac{f}{g} \right\rangle^2}$$

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- Ş Just use binned step function and minimize the differences to true function
- Ş 1-dim. binned distributions work very well:
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$$g(x) = g(x_1)g(x_2)\dots g(x_n)$$

- Factorize singularities from resonances
- VEGAS algorithm [Lepage, 1978]
- Works for factorizable singularities





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$$\int_{s_0}^{s_1} f(s) ds =$$

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$$\sum_{i} \alpha_i \int_{s_0}^{s_1} \frac{f(s)}{g(s)} g_i(s) ds$$

Multi-channel adaptive MC integration:

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9 / 30





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Different algorithms of phase-space construction / mappings

- flat [RAMBO]
- simplistic heuristics [ALPGEN],
- diagram-based [MadEvent],
- [QCD-]radiation driven [SAGE, Comix/Sherpa],



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resonance/singularity importance-ordered [WHIZARD]



Machine Learning: MC for integration and simulation

- Ş Phase space integration / adaptation by Invertible Neural Networks (INNs) / normalizing flows
- Ş Define divergence-based loss function
- Ş Use of buffered losses and training
- Ş

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Hoeche ea., 2001.10028, Heimel/ Winterhalder ea., 2212.06172









Parallel MC integration: MPI and GPUs

Preliminary: Matrix element evaluation

Process	$t^{CPU}[s]$	$t^{GPU}[s]$
$e^+e^- \rightarrow t\bar{t}$	0.98	4.28
$e^+e^- ightarrow bW^+ \overline{b}W^-$	28.8	23.1
$e^+e^- \rightarrow bW^+\bar{b}W^-H$	57.5	37.8
$e^+e^- \rightarrow b\bar{b}\bar{\nu}_e e^-\bar{\nu}_\mu\mu^+$	154	124
$e^+e^- ightarrow 2j$	1.9	5.4
$e^+e^- ightarrow 3j$	45	65
$e^+e^- \rightarrow 4j$	870	608
$e^+e^- ightarrow 5j$	4106	978
$pp \rightarrow jj$	42	86
$\mid pp \rightarrow W^+W^-W^+W^-$	670	192

- Parallelization of integration: OMP multi-threading for different helicities MPI parallelization (using OpenMPI or MPICH)
- Ş Ş
- Ş Distributes workers over multiple cores, grid adaption needs non-trivial communication (Load balancer / non-blocking communication)

- Ş Speedups 10 to 30, saturation at O(100) [also parallel event generation]
- Ş Integration of $(2 \rightarrow 8)$ leading order (LO) processes: \mathcal{O} (week) $\longrightarrow \mathcal{O}$ (hour)
- Ş Becomes a must for higher-order perturbative processes (NLO, NNLO)

Braß/Kilian/JRR, arXiv:1811.09711



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- Off-loading from CPU to GPU Sherpa, '20; MG5 `22; Whizard, `24
- Ş Semi-automatized ME generation for GPU in MG5 and Whizard
- Ş Matrix-element evaluation vs. phase-space integration on GPU \rightarrow Data transfer between CPU and GPU costly!
- So far no revolutionary breakthroughs

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Still a lot of work needed to make it fully competitive

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Beam simulations





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Beam structure for $\ell^+\ell^-$ colliders



Beam-induced background for the machine-detector interface (MDI)

- $^{\bigcirc}$ Dense beams \Rightarrow strong EM fields: deflect particles in other bunch (beamstrahlung)
- Depends on damping rings, final focus magnet, crossing angle, beam optics, etc.
- Effects: beam energy spread, long power-law dominated tail



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- Simulation tool for beam spectrum: GuineaPig [D. Schulte, 1998+]
- Very limited statistics: O(100k) vs. MC simulations need O(many G)
- Gaussian shape with specific spreads
- Parameterized (delta peak \oplus power law) 2.
- Avail.: [√] Generator for 2D histogrammed fit 3.





Avail.: (✓)



$$D_{\ell_1 \ell_2}(x_1, x_2) = D_{\ell_1}(x_1) \cdot D_{\ell_2}(x_2)$$

$$D_{\ell_i}(x_i) = \delta(1 - x_i) + \gamma_i x_i^{\alpha_i} \cdot (1 - x_i)^{\beta_i}$$

ILC/CLIC/C³: Beams not factorizable, no simple power law







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CLIC: Dalena/Esbjerg/Schulte [LCWS 2011]

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Sampling of beam spectral

based on Lumilinker T. Barklow, 2001; CIRCE2 algorithm T. Ohl, 1996, 2005

- Adapt 2D factorized variable width histogram to steep part of distribution
- Smooth correlated fluctuations with moderate Gaussian filter \bigcirc [suppresses artifacts from limited GuineaPig statistics
- Smooth continuum/boundary bins separately \bigcirc

[avoid artificial beam energy spread]

Future work: 3D structure of spectrum, does machine learning help? \bigcirc



Parameterized spectra still be useful: fast evaluation, unfolding



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(171,306 GuineaPig events in 10,000 bins)

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Hard processes, shower, hadronization and all that





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 \cong Perturbative amplitudes for $2 \rightarrow n$ scattering grows factorially with n

[Parke/Taylor, '86; Berends/Giele, '88; Caravaglios et al., 1998; Ohl/JRR, 2000/2023; Papadopoulos, 2001]

Directed Acyclical Graphs (DAGs) [O'Mega]







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		MCSANO	Cee[3	7]						
\sqrt{s} [G	$ar{s} \; [ext{GeV}] \; ig \; \; \sigma_{ ext{LO}}^{ ext{tot}} \; [ext{fb}] \; ig \; \sigma_{ ext{NI}}^{ ext{tot}}$		$_{\rm O}^{\rm t}$ [fb]	$\sigma_{ m LO}^{ m tot}~[{ m fb}]$		$\sigma_{ m NLO}^{ m tot}~[{ m fb}]$	δ_{EV}	v [%]	$\sigma^{ m sig}~(m LO/N$	
250		225.59(1)	206	.77(1)	225.60(1)		207.0(1)	-	-8.25	0.4/2.1
500		53.74(1)	62	.42(1)	53.74(3)		62.41(2)	+	16.14	0.2/0.3
1000		12.05(1)	14	.56(1)	12.0549(6)		14.57(1)) +20.84		0.5/0.5
	Pro	ocess		WHI $\sigma_{\rm LO}$	HIZARD+OpenLoops Lo [fb] σ_{NLO} [fb]				ee @)1TeV, N
	e^+	$e^- \rightarrow jj$	622.737(8)		639.39(5)					
	e^+	$+e^- \rightarrow jjj$		340.6(5)		317.8(5)				
	e^+	$e^- \rightarrow j j j j$	105.0(3)		10	4.2(4)				
	e^+	$e^- \rightarrow j j j j j j$		22.33	3(5) 24		24.57(7)			
	e^+	$e^+e^- \rightarrow jjjjjjj$		3.583	B(17)	4.4	16(4)			



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The "Exclusive" Frontier — fN(N)LO, Automation in MCs

NLO)





		MCSANO	Cee[3	7]						
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The "Exclusive" Frontier — fN(N)LO, Automation in MCs

μμ @ 3 TeV, NLO EW

NLO)



$\mu^+\mu^- o X, \sqrt{s}$ =	$\sigma_{ m LO}^{ m incl} \; [{ m fb}]$	$\sigma_{ m NLO}^{ m incl}$ [fb]	$\delta_{ m EW}~[\%]$
W^+W^-	$4.6591(2)\cdot 10^2$	$4.847(7)\cdot 10^2$	+4.0(2)
ZZ	$2.5988(1)\cdot 10^{1}$	$2.656(2)\cdot 10^{1}$	+2.19(6)
HZ	$1.3719(1)\cdot 10^{0}$	$1.3512(5)\cdot 10^{0}$	-1.51(4)
HH	$1.60216(7) \cdot 10^{-7}$	$5.66(1) \cdot 10^{-7}$ *	
W^+W^-Z	$3.330(2)\cdot 10^{1}$	$2.568(8) \cdot 10^{1}$	-22.9(2)
W^+W^-H	$1.1253(5)\cdot 10^{0}$	$0.895(2)\cdot 10^{0}$	-20.5(2)
ZZZ	$3.598(2)\cdot 10^{-1}$	$2.68(1)\cdot 10^{-1}$	-25.5(3)
HZZ	$8.199(4)\cdot 10^{-2}$	$6.60(3)\cdot 10^{-2}$	-19.6(3)
HHZ	$3.277(1)\cdot 10^{-2}$	$2.451(5)\cdot 10^{-2}$	-25.2(1)
HHH	$2.9699(6) \cdot 10^{-8}$	$0.86(7)\cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1)\cdot 10^{0}$	$0.993(6)\cdot 10^{0}$	-33.1(4)
W^+W^-ZZ	$1.209(1)\cdot 10^{0}$	$0.699(7)\cdot 10^{0}$	-42.2(6)
W^+W^-HZ	$8.754(8)\cdot 10^{-2}$	$6.05(4)\cdot 10^{-2}$	-30.9(5)
W^+W^-HH	$1.058(1)\cdot 10^{-2}$	$0.655(5)\cdot 10^{-2}$	-38.1(4)
ZZZZ	$3.114(2)\cdot 10^{-3}$	$1.799(7)\cdot 10^{-3}$	-42.2(2)
HZZZ	$2.693(2)\cdot 10^{-3}$	$1.766(6)\cdot 10^{-3}$	-34.4(2)
HHZZ	$9.828(7) \cdot 10^{-4}$	$6.24(2) \cdot 10^{-4}$	-36.5(2)
HHHZ	$1.568(1) \cdot 10^{-4}$	$1.165(4) \cdot 10^{-4}$	-25.7(2)



		MCSANO	Cee[3	7]						
\sqrt{s} [G	$ar{s} \; [ext{GeV}] \; ig \; \; \sigma_{ ext{LO}}^{ ext{tot}} \; [ext{fb}] \; ig \; \sigma_{ ext{NI}}^{ ext{tot}}$		$_{\rm O}^{\rm t}$ [fb]	$\sigma_{ m LO}^{ m tot}~[{ m fb}]$		$\sigma_{ m NLO}^{ m tot}~[{ m fb}]$	δ_{EV}	v [%]	$\sigma^{ m sig}~(m LO/N$	
250		225.59(1)	206	.77(1)	225.60(1)		207.0(1)	-	-8.25	0.4/2.1
500		53.74(1)	62	.42(1)	53.74(3)		62.41(2)	+	16.14	0.2/0.3
1000		12.05(1)	14	.56(1)	12.0549(6)		14.57(1)) +20.84		0.5/0.5
	Pro	ocess		WHI $\sigma_{\rm LO}$	HIZARD+OpenLoops Lo [fb] σ_{NLO} [fb]				ee @)1TeV, N
	e^+	$e^- \rightarrow jj$	622.737(8)		639.39(5)					
	e^+	$+e^- \rightarrow jjj$		340.6(5)		317.8(5)				
	e^+	$e^- \rightarrow j j j j$	105.0(3)		10	4.2(4)				
	e^+	$e^- \rightarrow j j j j j j$		22.33	3(5) 24		24.57(7)			
	e^+	$e^+e^- \rightarrow jjjjjjj$		3.583	B(17)	4.4	16(4)			



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The "Exclusive" Frontier — fN(N)LO, Automation in MCs

μμ @ 3 TeV, NLO EW









		MCSANO	7]							
\sqrt{s} [G	$\sqrt{s} \; [ext{GeV}] \; ig \; \sigma_{ ext{LO}}^{ ext{tot}} \; [ext{fb}] \; ig \; \sigma_{ ext{NI}}^{ ext{tot}}$		$\sigma_{ m NL}^{ m tot}$	_O [fb]	$\sigma_{ m LO}^{ m tot}~[{ m fb}]$		$\sigma_{ m NLO}^{ m tot}~[{ m fb}] \mid \delta_{ m E}$		v [%]	$\sigma^{ m sig}~(m LO/N$
250		225.59(1)	206	.77(1)	225.60	0(1)	207.0(1)	(1) -8		0.4/2.1
500		53.74(1)	62	.42(1)	53.74(3)		62.41(2)	+	16.14	0.2/0.3
1000		12.05(1)	14.56(1)		12.0549(6)		14.57(1)	4.57(1) +20.84		0.5/0.5
	Pro	ocess		$_{\sigma_{\rm LO}}^{\rm WHI}$	ZARD+C [fb])pen $\sigma_{\sf N}$	Loops LO [fb]		ee @)1TeV, N
	$\begin{array}{c} e^+e^- \rightarrow jj \\ e^+e^- \rightarrow jjj \\ e^+e^- \rightarrow jjjj \\ e^+e^- \rightarrow jjjjj \\ e^+e^- \rightarrow jjjjjj \end{array}$			622.7 340.6 105.6 22.33 3.583	737(8) 5(5) 5(3) 5(5) 5(17)	$632 \\ 312 \\ 104 \\ 242 \\ 4.42 $	$\begin{array}{c c} 9.39(5) \\ 7.8(5) \\ 4.2(4) \\ .57(7) \\ 6(4) \\ \end{array}$			arXiv



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		MCSANO	7]							
\sqrt{s} [G	$[\text{GeV}] \mid \sigma_{ ext{LO}}^{ ext{tot}} [ext{fb}] \mid \sigma_{ ext{NI}}^{ ext{tot}}$		$_{\rm O}^{\rm c}$ [fb]	$\sigma_{ m LO}^{ m tot}~[{ m fb}]$		$\sigma_{ m NLO}^{ m tot}~[{ m fb}]$	$ \delta_{\rm EV}$	v [%]	$\sigma^{ m sig}~(m LO/N$	
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	Pro	ocess		$_{\sigma_{\rm LO}}^{\rm WHI}$	ZARD+OpenLoops [fb] σ_{NLO} [fb]				ee @	91TeV, N
	e^+ e^+ e^+ e^+	$e^- ightarrow jj$ $e^- ightarrow jjjj$ $e^- ightarrow jjjjj$ $e^- ightarrow jjjjj$	622.7 340.6 105.0 22.33 3.583	737(8) 5(5) 5(3) 3(5) 3(17)	$63 \\ 31 \\ 10 \\ 24 \\ 4.4$	$9.39(5) \\7.8(5) \\4.2(4) \\.57(7) \\46(4)$			arXiv	

Three major bottlenecks to go to NNLO

- Virtual integrals with many mass scales / off-shell legs
- Process-independent automated NNLO subtraction
- Negative weights in NLO simulations deteriorate at NNLO



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The "Exclusive" Frontier — fN(N)LO, Automation in MCs

μμ @ 3 TeV, NLO EW







Parton Showers, Matching, Merging, Hadronization

- 0
- 0
- 0
- 0





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Parton showers resums large logarithms; provide exclusive multi-jet events A lot of progress driven by LHC: final-state showers already accurate at NLL "Interleaved" showers: QCD and QED emissions $\alpha_s/\alpha \sim 15$ (sampled with veto algorithm) Matching: consistently combine fixed-order emissions with resummed shower emissions







Parton Showers, Matching, Merging, Hadronization



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Exclusive Photon Simulation



Exclusive photon distribution important for detector optimization / mono-photon searches etc. Different algorithms: QED shower, soft/eikonal resummation (YFS), recursive algorithms



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Initial State Radiation – Lepton PDFs





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□ For QCD: non-perturbative bound-state PDFs need to be fitted from data **G** For QED / EW: calculable from first principle (collinear factorization)



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□ For QCD: non-perturbative bound-state PDFs need to be fitted from data **□** For QED / EW: calculable from first principle (collinear factorization)



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DESY-FH Future Collider + Scientific Computing Workshop, 1.7.2024





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Integrable power-like singularity 1/(1-z) for $z \rightarrow 1$



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NLL, $\mu_0 = m_e$, $\mu = 100 \text{ GeV}$



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Integrable power-like singularity 1/(1-z) for $z \rightarrow 1$

QED PDFs = electron/ISR structure functions, ISR structure functions

- Gives most precise normalization of total cross section
- Very intricate numerical behavior at peak, especially at NLO
- "Photon PDF" (a.k.a. EPA, Weizsäcker-Williams) Γ_{ν} , peaked at small z



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NLL, $\mu_0 = m_e$, $\mu = 100 \text{ GeV}$



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- **□** Fully inclusive in collinear/forward/beam direction
- At very (, very) high energies lepton colliders become $\gamma\gamma/VV$ colliders (like LHC is gg)
- Work in progress in Krakow, DESY, Pittsburgh
- □ Has to be accompanied by EW fragmentation functions (event selection!)



Collinear factorization not in QED, but in full SM Han/Ma/Xie, 2007.14300, 2103.09844; Garosi/Marzocca/Trifinopoulos, 2303.16941



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Lepton vs. Hadron Colliders





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Lepton vs. Hadron Colliders





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Thresholds and "special processes"

- Luminometry: Special treatment for Bhabha scattering ($\ell^+\ell^- \to \ell^+\ell^-$) and diphotons ($\ell^+\ell^- \to \gamma\gamma$) [$10^{-4} 10^{-5}$ precision] t and W mass measurements with precisions at $10^{-4} - 10^{-5}$ precision
- Exclusive Monte Carlo need to take into account QED and QCD threshold effects



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Beyond SM (BSM) Modelling in Simulation





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BSM Models: UFO magic avoids hard-coding

- Old school: hard-coding by hand



MuC example for SMEFT/HEFT UFO, from: T. Han et al. arXiv:2108.05362



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Challenges of Monte Carlo Event Generators







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Challenges of Monte Carlo Event Generators







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- Typical MC generator $\gtrsim 0.5$ M lines of code
- Many physics parts: necessity of a team/collaboration
- No tool implements all physics (and probably never will)
- Modularity and interchangeability is a must
- e.g. typically interfaces to ca. 15 external libraries
- Unit testing & Continuous integration







Challenges of Monte Carlo Event Generators







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- 3—5 major MC event generators
- Most of these MC members will retire around 2040-45
- Need for ca. MC 8–10 staff positions world-wide in the next ca. 20 years
- Already many example of "zombie codes" in experiments











Conclusions & Outlook



- Monte-Carlo generators almighty workhorses of particle physics!! 0
- MCs implement all necessary SM and BSM physics 9
- Tedious work for MC collaboration members 9
- Tremendous progress on QCD corrections driven by 15 years of LHC running 6
- NLO QCD+EW for SM and NLO QCD BSM (mostly) under control, attempts for NNLO automation 9
- Precision in initial-state QED radiation resummation and exclusive photons crucial 9
- Parton Showers for QCD and QED radiation much matured (now up to NLL for FSR) 6
- Hadronization will be probed with much enhanced precision at future e^+e^- collider 6
- Computing bottlenecks: parallelization & optimization of phase space integration, negative weights 6
- Quite extensive activities at DESY: many opportunities to participate 0











Event generators: Accuracy vs. Precision





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Accuracy and Precision



Accurate **Not Precise**



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Quark and gluon jets hadronize at low energy scales (fragmentation)

Non-perturbative physics: has to be extracted from experiment [mainly $e^+e^- \rightarrow$ hadrons, DIS, LHC]

Old models [1970s]: flux tubes, independent fragmentation [Feynman/Field, 1970; Isajet: Paige et al.]



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Lund string fragmentation model [Pythia]

- based on old string model of strong interactions
- Strong physical motivation, but: invented without parton shower in mind
- Universal description of data (ee fit \rightarrow hadrons)
- \bigcirc Plethora of parameters: ~ O(1) per hadron
- Baryon production difficult [string junctions, popcorn]











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Cluster fragmentation model [Herwig]

- Parton shower orders partons in color space
- Large N_C limit: planar graphs dominate
- Cluster: continuum of high-mass resonances, decay to hadrons
- No spin info, just plain phase space
- Cluster spectrum determined by PS (perturbation theory)











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A hadronic decay chain of typical complexity:

 $B^{*0} \rightarrow \gamma B^0$



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Radiative electromagnetic decay





A hadronic decay chain of typical complexity:

$$B^{*0} \to \gamma B^0$$
$$\hookrightarrow \overline{B}^0$$



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Radiative electromagnetic decay Weak mixing





A hadronic decay chain of typical complexity:

 $B^{*0}
ightarrow \gamma B^0$ $\hookrightarrow \overline{B}^0$ $\hookrightarrow e^-\overline{\nu}_e D^{*+}$



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Radiative electromagnetic decay Weak mixing Weak decay





A hadronic decay chain of typical complexity:





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Radiative electromagnetic decay Weak mixing Weak decay Strong decay





A hadronic decay chain of typical complexity:





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Radiative electromagnetic decay Weak mixing Weak decay Strong decay Weak decay, p mass smeared





A hadronic decay chain of typical complexity:





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Radiative electromagnetic decay Weak mixing Weak decay Strong decay Weak decay, p mass smeared ρ⁺ polarized, angular correlations





A hadronic decay chain of typical complexity:

 $B^{*0} \to \gamma B^0$ $\hookrightarrow \overline{B}^0$ $\hookrightarrow e^- \overline{\nu}_e D^{*+}$ $\hookrightarrow \pi^+ D^0$ $\hookrightarrow K^- \rho^+$ $\hookrightarrow \pi^+ \pi^0$



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Radiative electromagnetic decay Weak mixing Weak decay Strong decay Weak decay, p mass smeared p⁺ polarized, angular correlations Dalitz decay, *m*_{ee} peaked

 $\hookrightarrow e^+ e^- \gamma$





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 $\hookrightarrow \pi^+ D^0$

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Final-state hadronic QED radiation for shower shapes and correct distributions

20 pb⁻¹ (13 TeV) 10 Events / GeV **Trigger paths** CMS 10^{7} Preliminary **J/**ψ 4**0**⁶ ow mass double muon + tracl 10⁵ Bs double muon inclusive **10**⁴ 10³ 10² 10 ¹⁰ $\mu^+\mu^-$ invariant mass [GeV]

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Radiative electromagnetic decay Weak mixing Weak decay Strong decay Weak decay, p mass smeared ρ⁺ polarized, angular correlations Dalitz decay, *m*_{ee} peaked

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 $\hookrightarrow e^+e^-\gamma$

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PDG: 100s of particles, 1000s of decay modes, form factors, peak shapes, special cases, "PDG unitarity violation"



Radiative electromagnetic decay Weak mixing Weak decay Strong decay Weak decay, p mass smeared ρ^+ polarized, angular correlations Dalitz decay, *m*_{ee} peaked







Example: Search for low-mass sbottoms at a 800 GeV e⁺e⁻ collider





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- **G** Fully inclusive in collinear/forward/beam direction
- Exclusive photons contained in NLL + NLO calculation
- Approximations exist for photons with pT for LL PDFs







Example: Search for low-mass sbottoms at a 800 GeV e⁺e⁻ collider





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800

Photons from lepton PDF collinear & non-observable **G** Fully inclusive in collinear/forward/beam direction Exclusive photons contained in NLL + NLO calculation Approximations exist for photons with pT for LL PDFs

Also soft factorization / exponentiation

Yennie/Frautschi/Suura, 1961; YFS:

Presumably best description for thresholds: soft effects

Collinear corrections can be added in principle

$$d\sigma = \sum_{n_{\gamma}}^{\infty} \frac{\exp[Y_{res.}]}{n_{\gamma}!} \prod_{j=1}^{n_{\gamma}} \left[d\text{LIPS}_{j}^{\gamma} S_{res.}(k_{j}) \right]$$
$$[\sigma_{0} + \text{correction}]$$









