QUANTUM ALGORITHMS FOR CHARGED PARTICLE TRACK RECONSTRUCTION IN THE LUXE EXPERIMENT

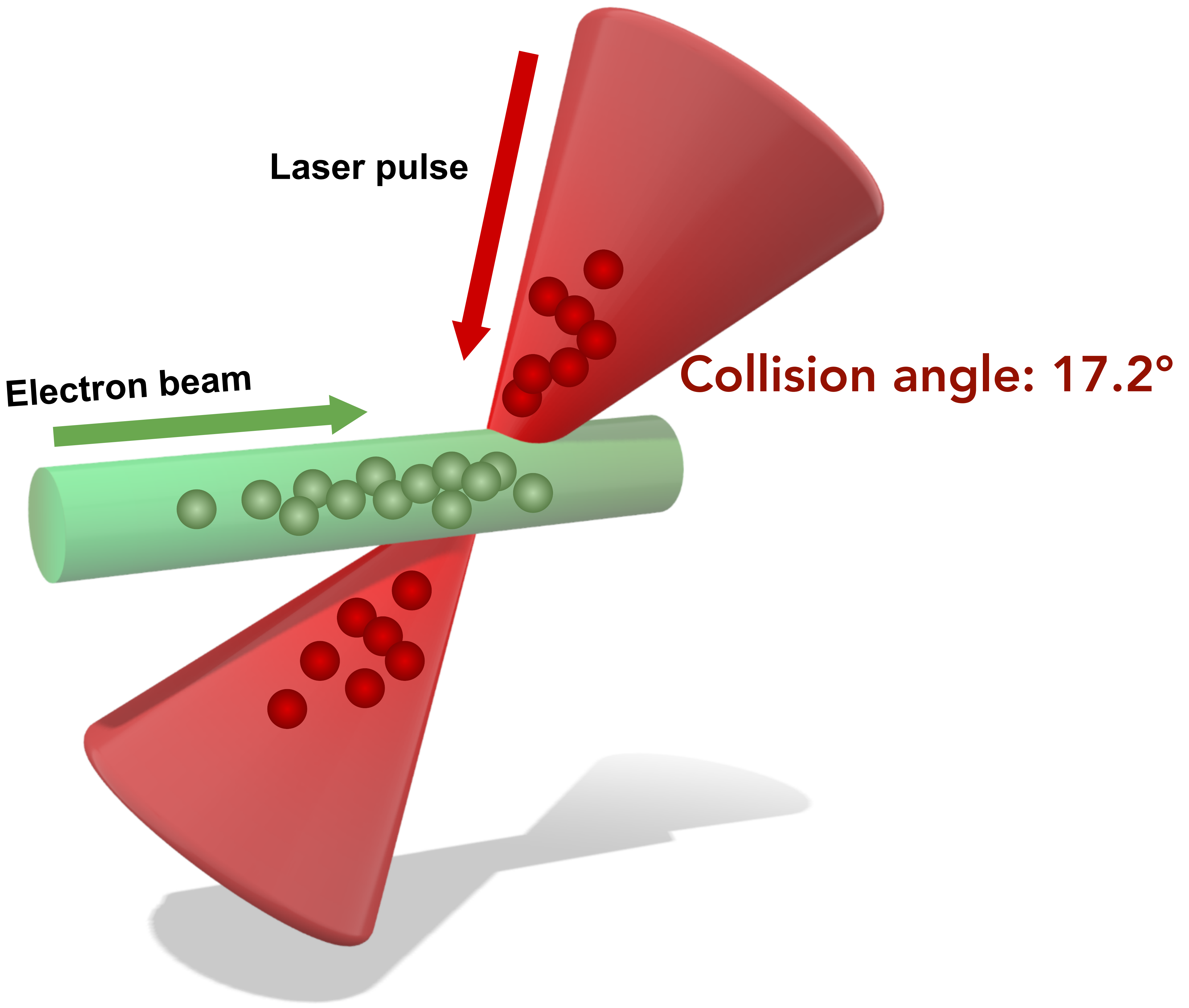
Arianna Crippa[1,2], Lena Funcke[3], Tobias Hartung[4], Beate Heinemann[1,5], Karl Jansen[1], Annabel Kropf[1,5], Stefan Kühn[1], Federico Meloni[1], David Spataro[1,5], Cenk Tüysüz[1,2], **Yee Chinn Yap**[1]

[1]Deutsches Elektronen-Synchrotron DESY
[2]Humboldt-Universität zu Berlin
[3]Rheinische Friedrich-Wilhelms-Universität Bonn
[4]Northeastern University London
[5]Albert-Ludwigs-Universität Freiburg

FH SciComp Workshop, 1st July 2024

LUXE: LASER UND XFEL EXPERIMENT

- Experiment in planning at DESY and European XFEL to study collisions of high-energy XFEL electron beam and high-power laser.



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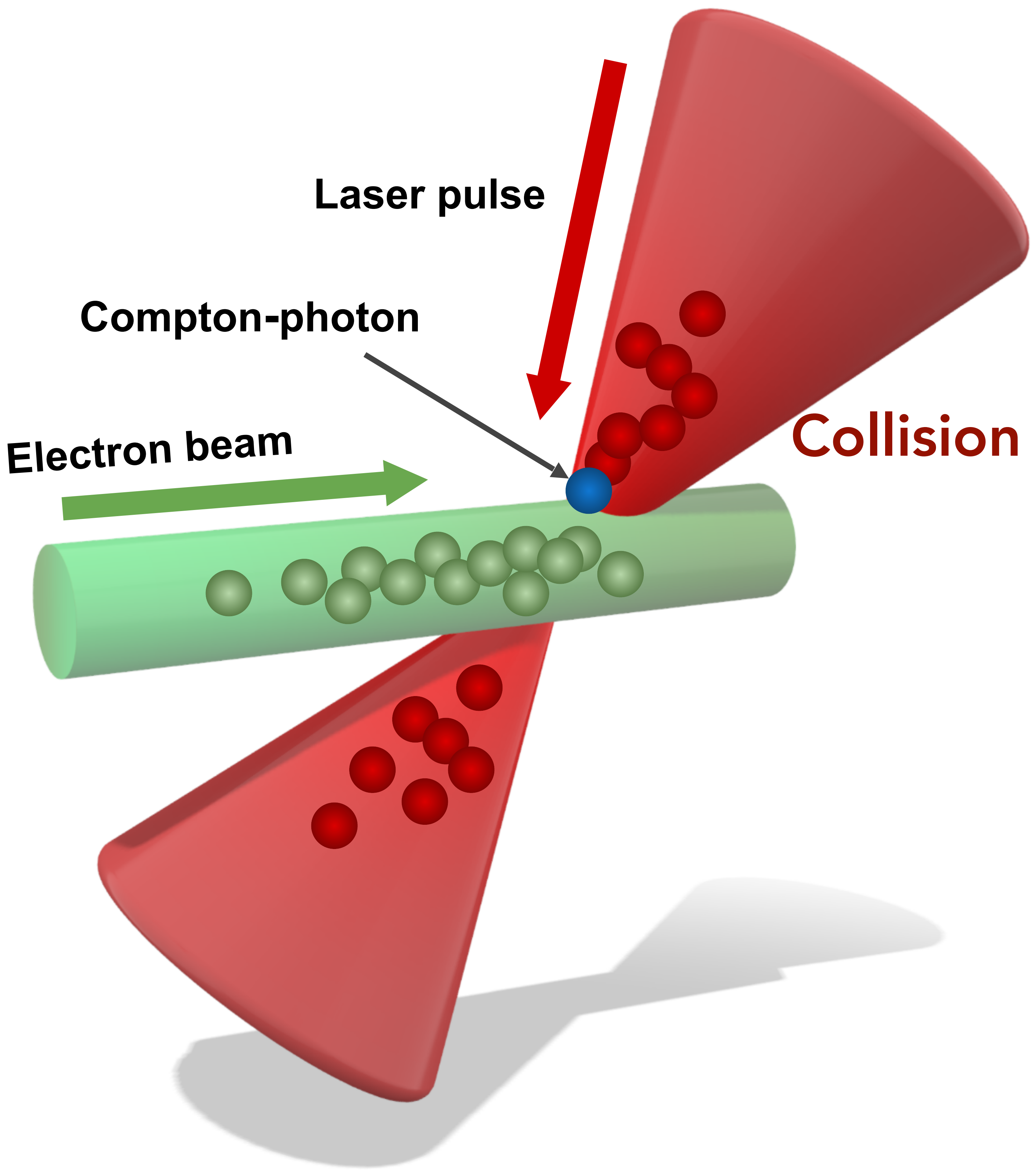
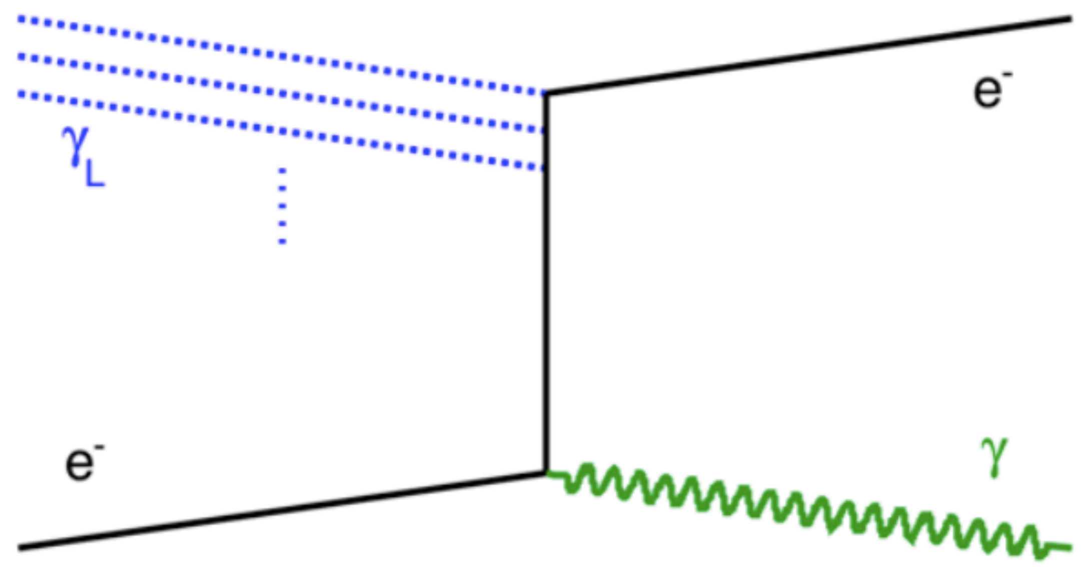


Collision angle: 17.2°

Non-linear Compton scattering:

$$e^- + n\gamma_L \rightarrow e^- + \gamma_C$$



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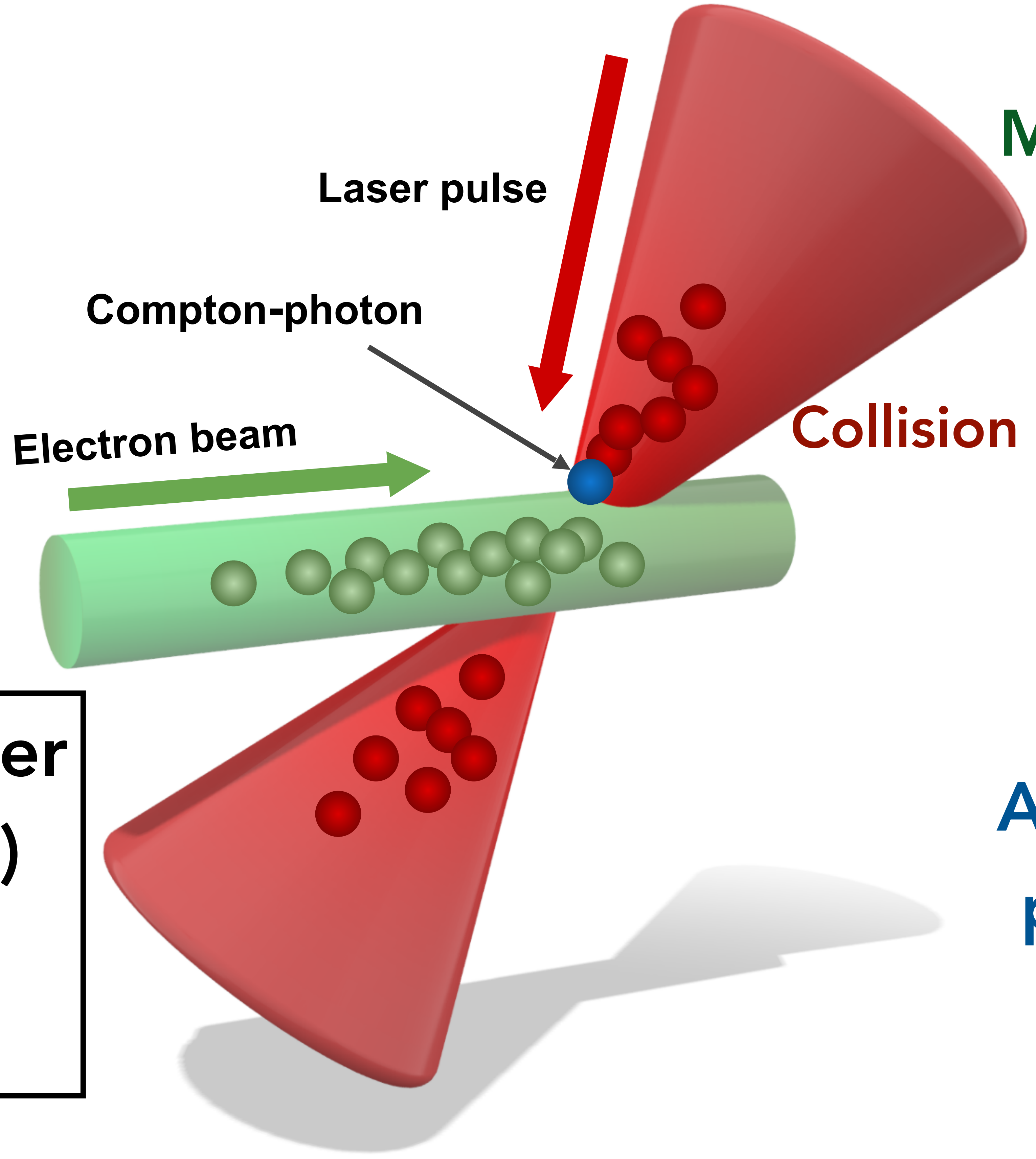

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Non-linear Compton scattering:

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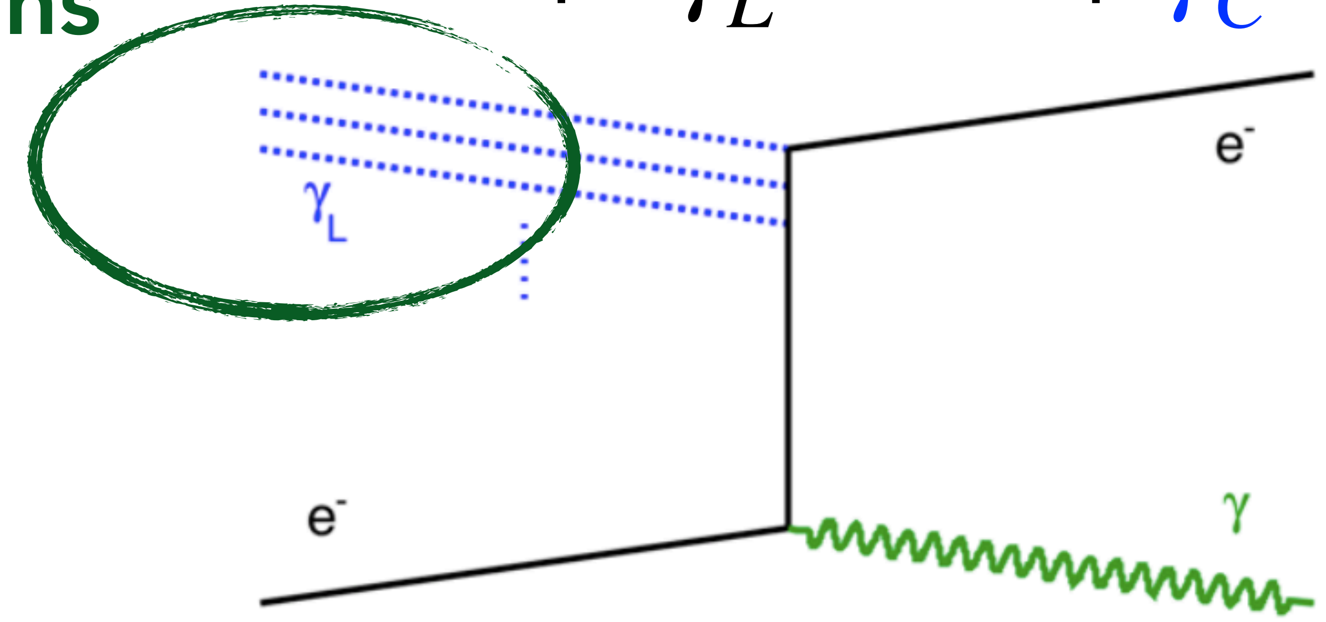
Multiple photons =non-linear



Field intensity parameter (charge-field coupling)

$$\xi = \frac{m_e}{\omega_L}\frac{E_L}{E_{cr}}$$

All-order process, i.e. non-perturbative for $\xi \sim O(1)$

$$P(n\gamma_L \rightarrow \gamma) \propto \alpha\xi^{2n}$$

LUXE: LASER UND XFEL EXPERIMENT

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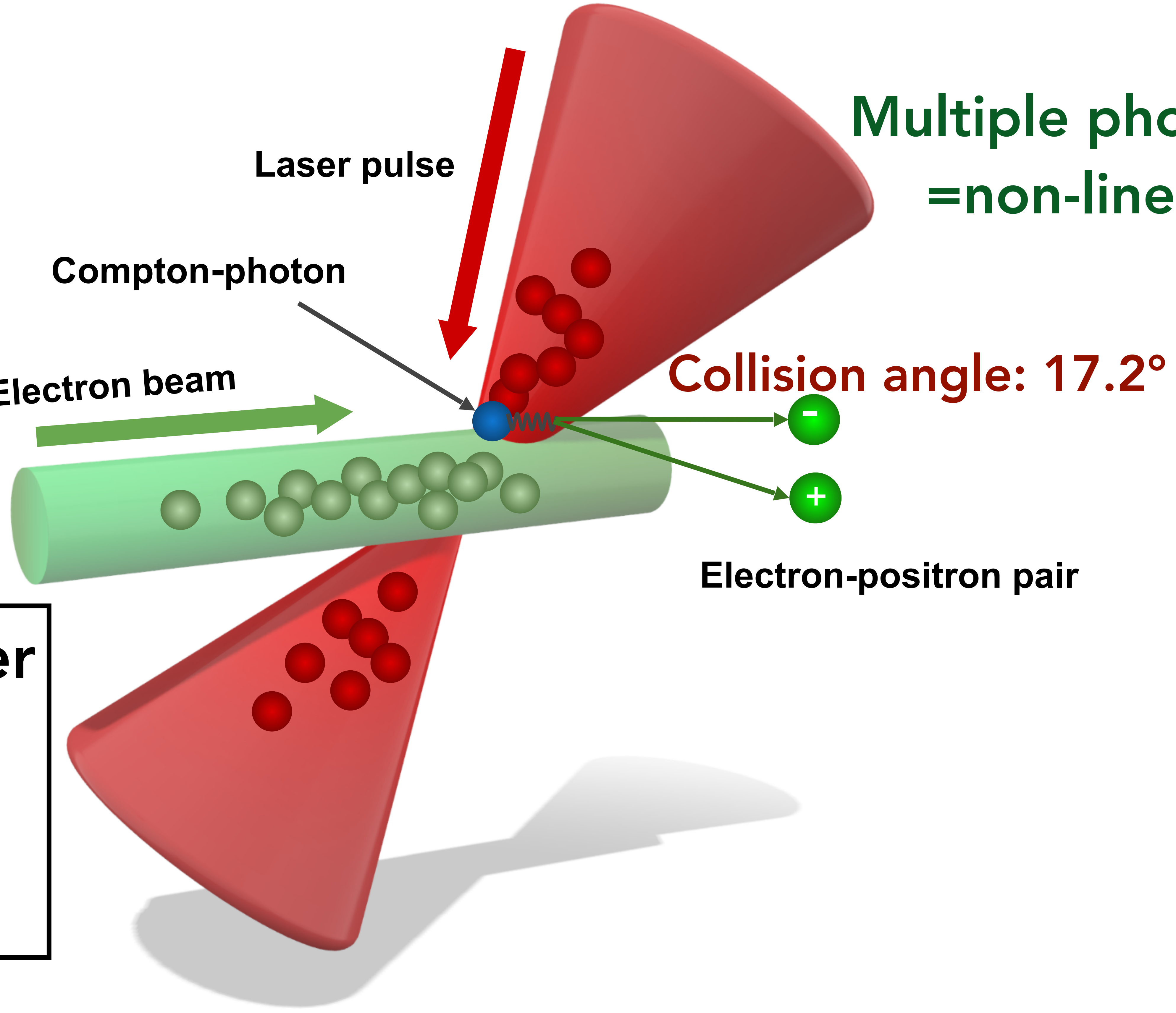

Non-linear Compton scattering:

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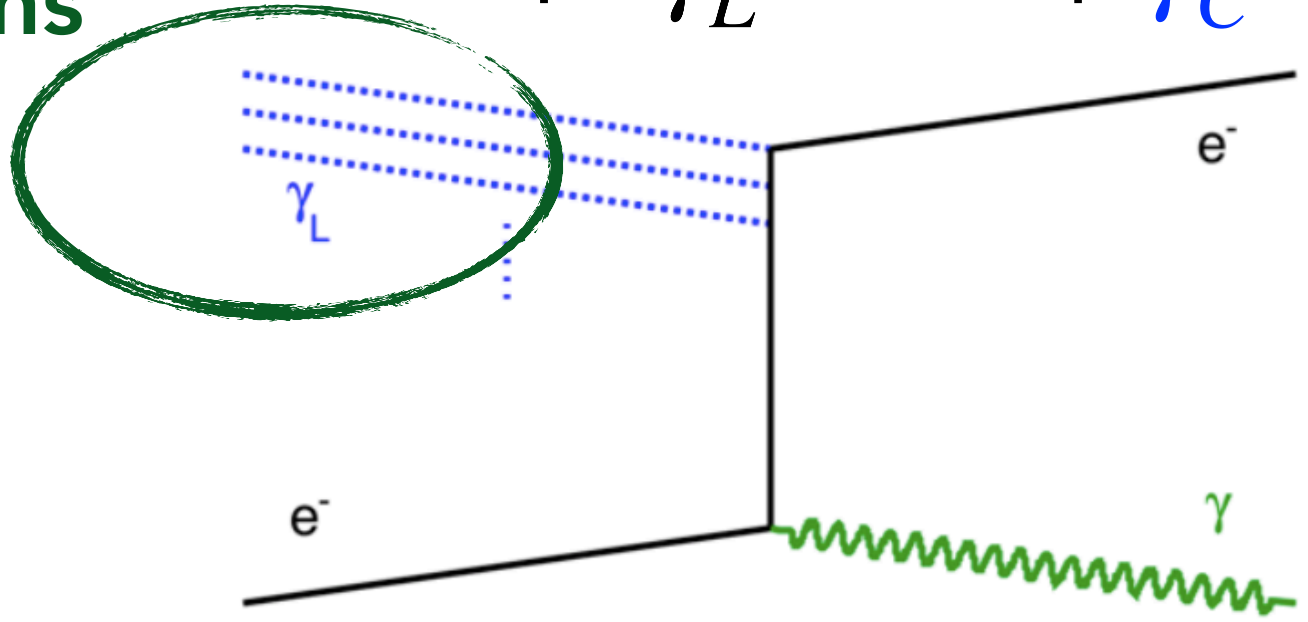
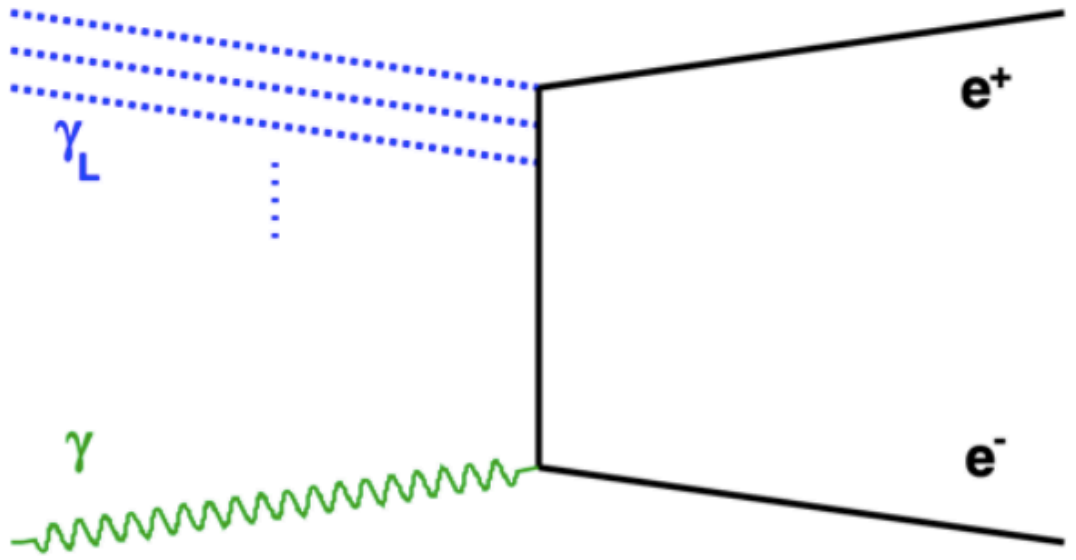
Multiple photons =non-linear



Non-linear Breit Wheeler:

$$\gamma_C + n'\gamma_L \rightarrow e^+ + e^-$$

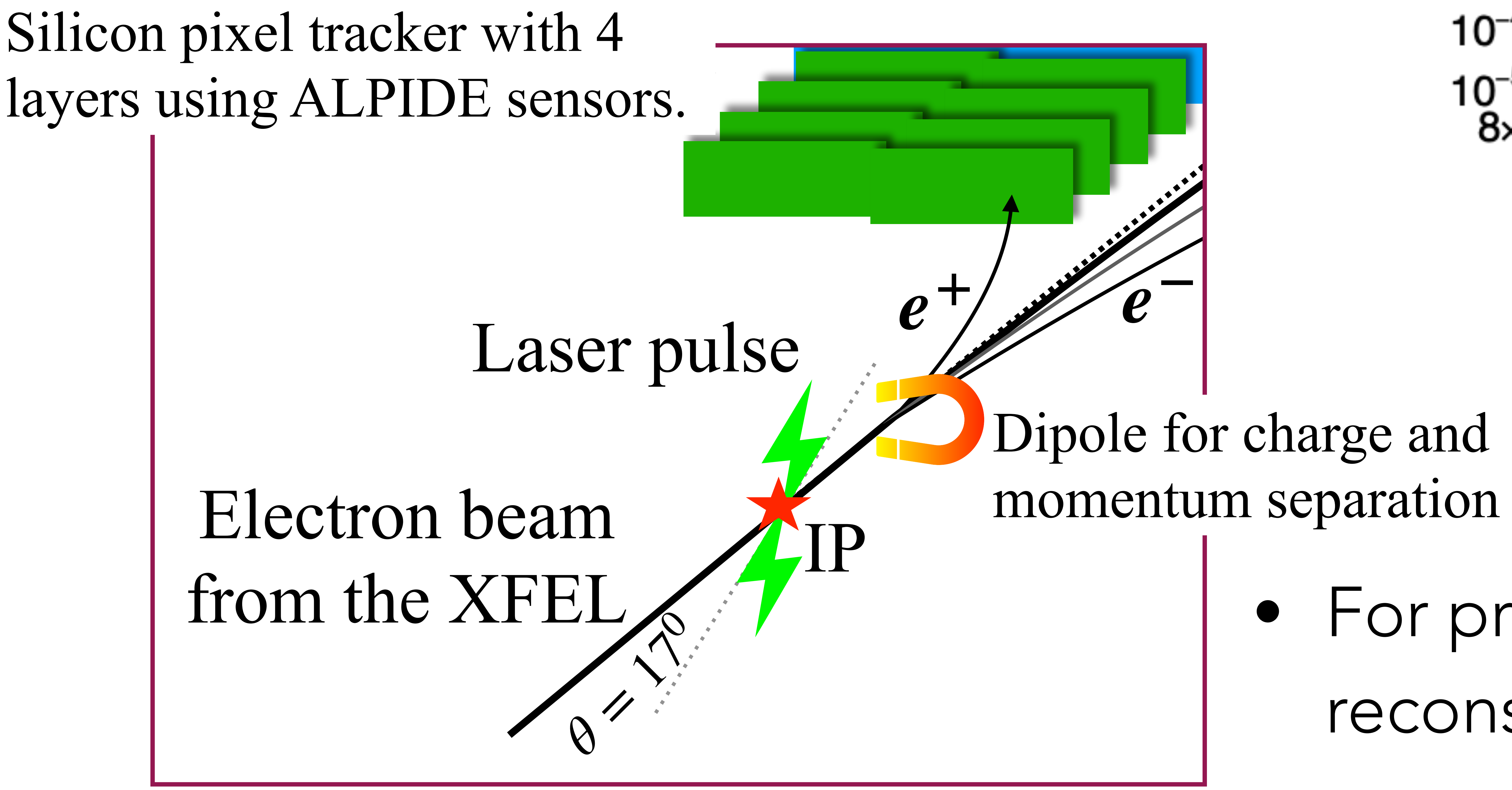


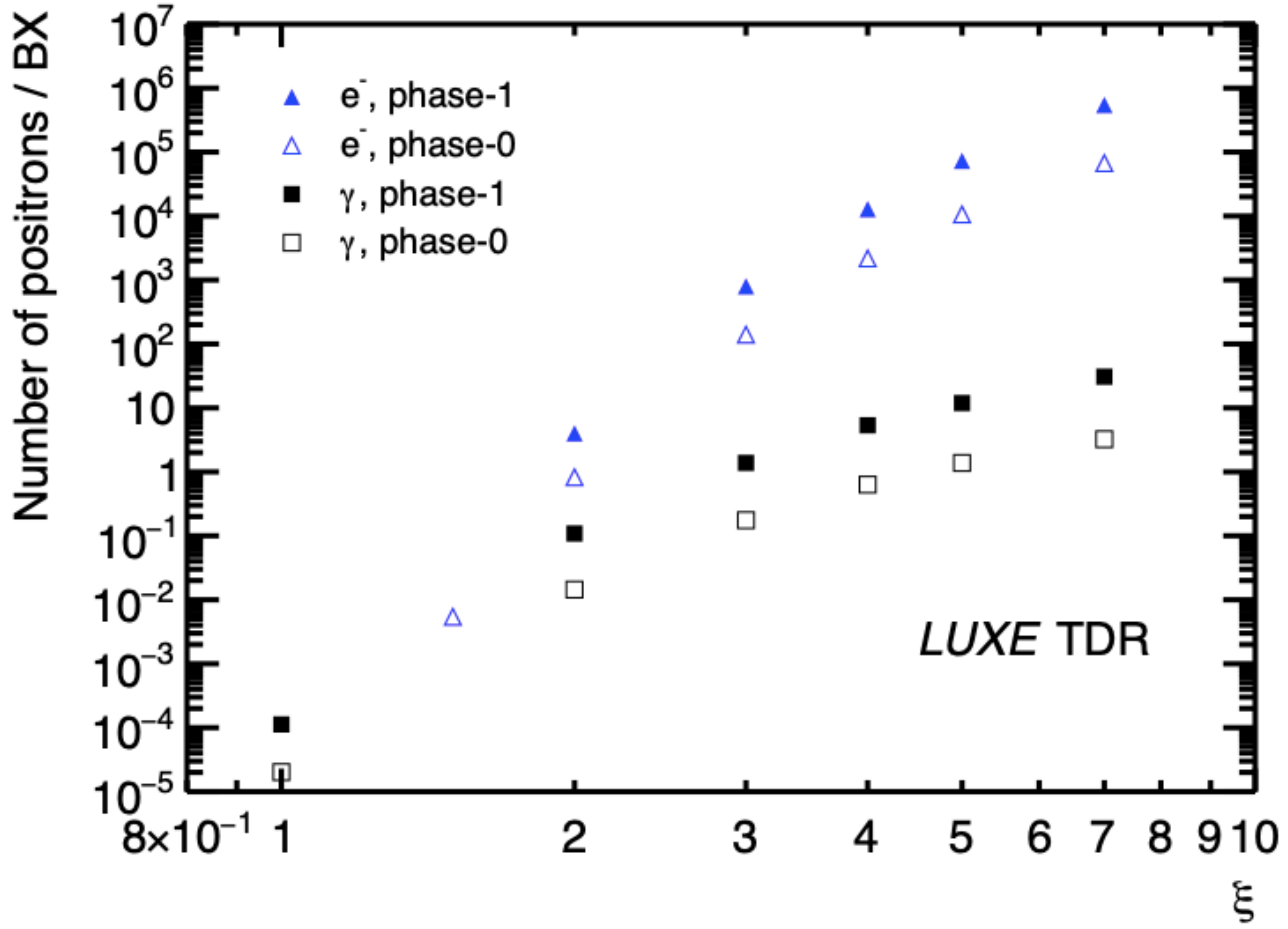
Field intensity parameter (charge-field coupling)

$$\xi = \frac{m_e}{\omega_L}\frac{E_L}{E_{cr}}$$

MEASUREMENT

- LUXE goal: **precision measurements** in a **transition** from perturbative to non-perturbative QED.



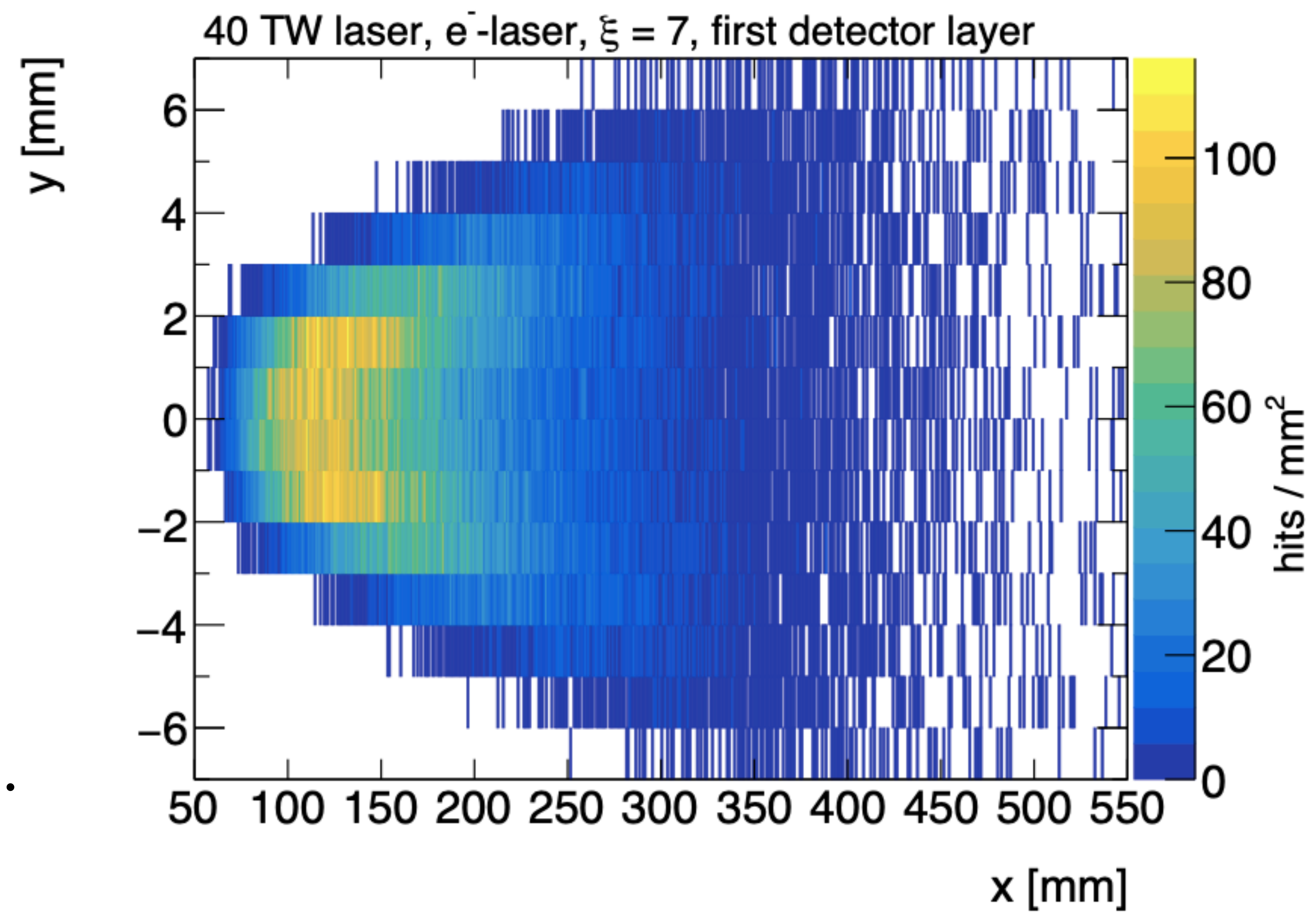


- For precise positron rate measurement, reconstruct particle path with tracking.

LUXE TRACKING CHALLENGE

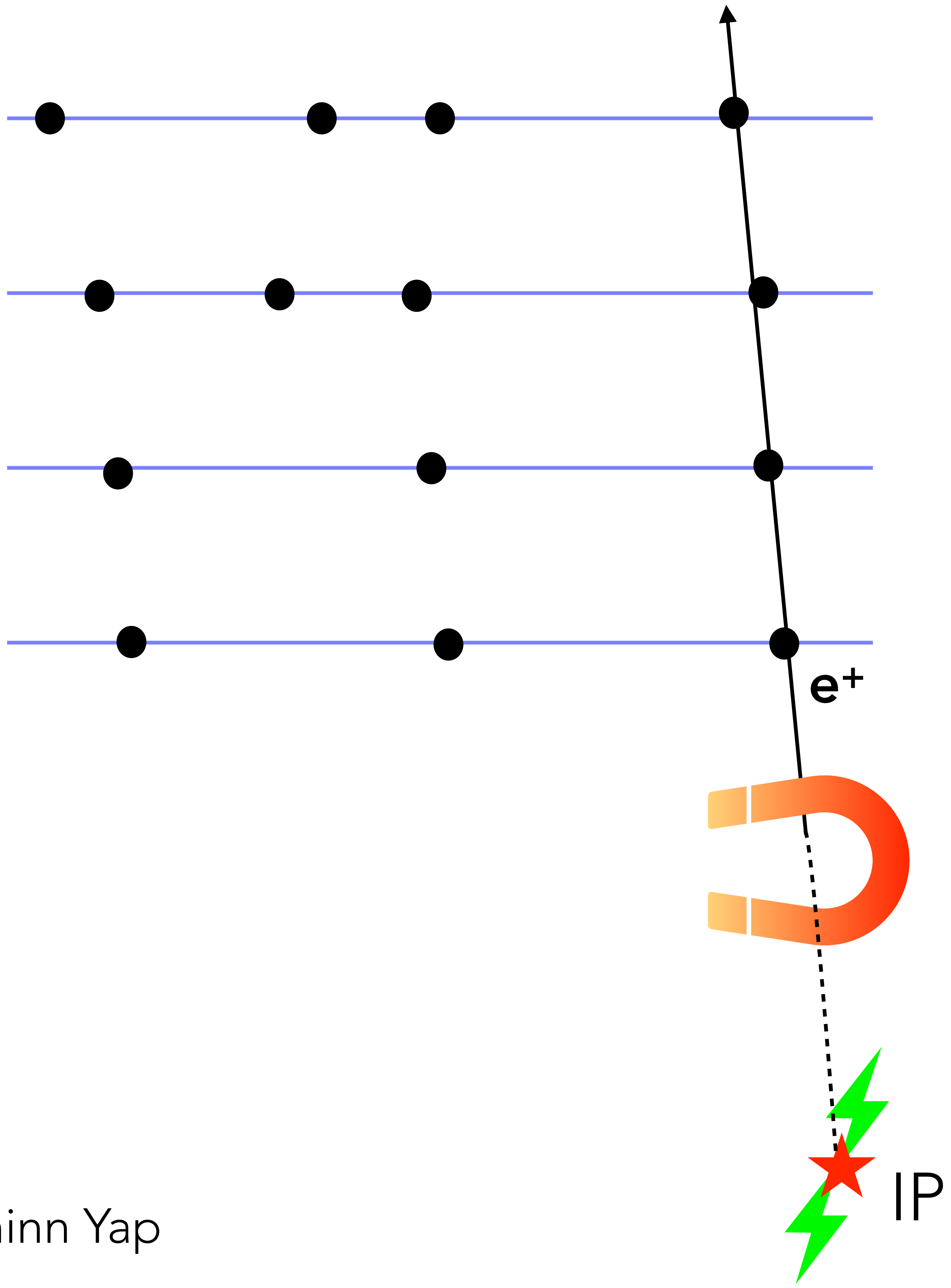
- Tracking at LUXE becomes challenging due to combinatorics at high track multiplicities.

- At phase-0 (40 TW laser), **occupancies** at the pixel detector reach 100 particles/mm^2.
  - Orders of magnitudes higher than other planned HEP experiments, e.g. HL-LHC.



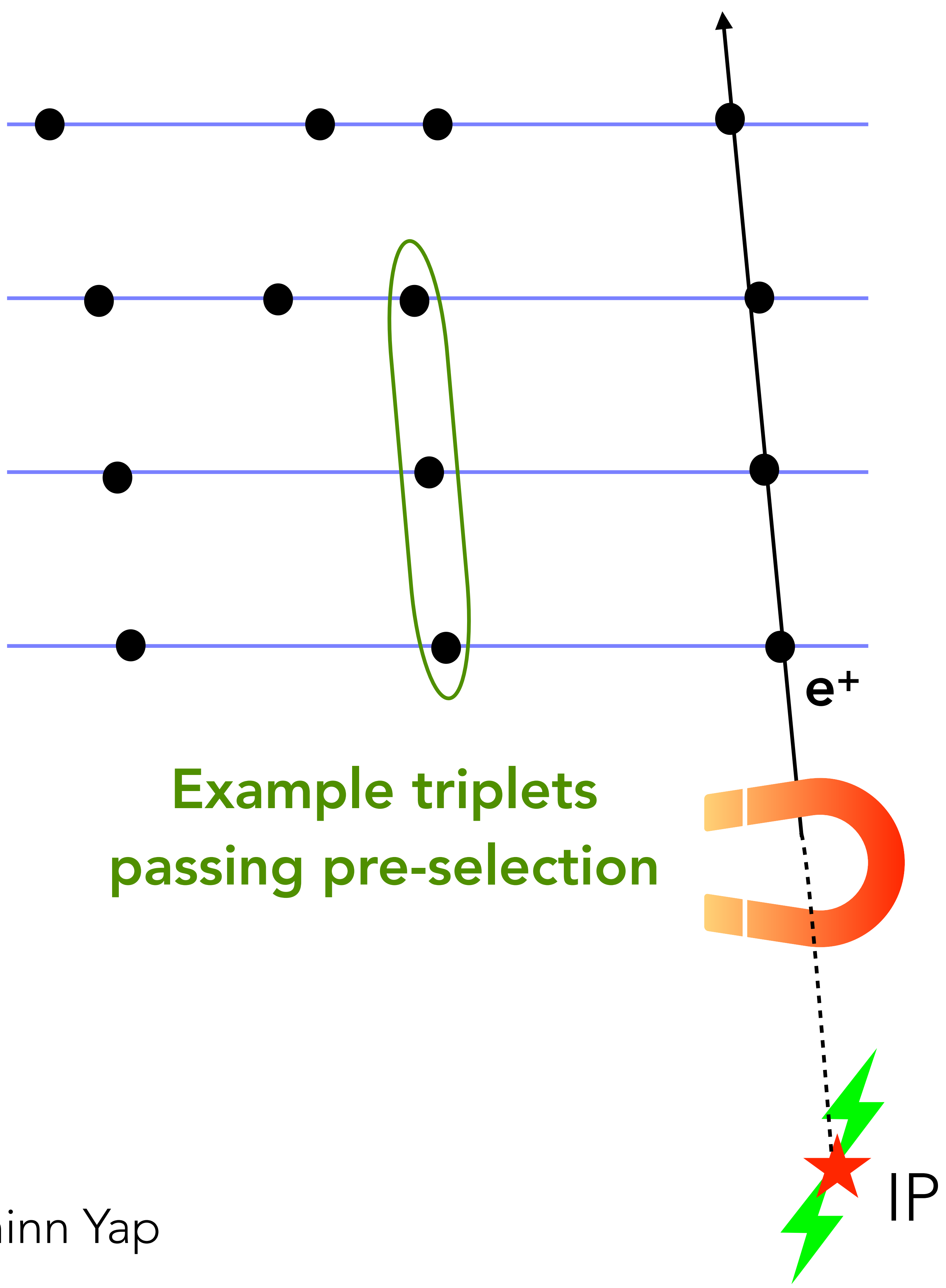
- Quantum computing may offer an advantage. In our paper (arXiv:2304.01690), we study various tracking methods using quantum and classical algorithms.

TRACKING USING QUANTUM COMPUTING



TRACKING USING QUANTUM COMPUTING

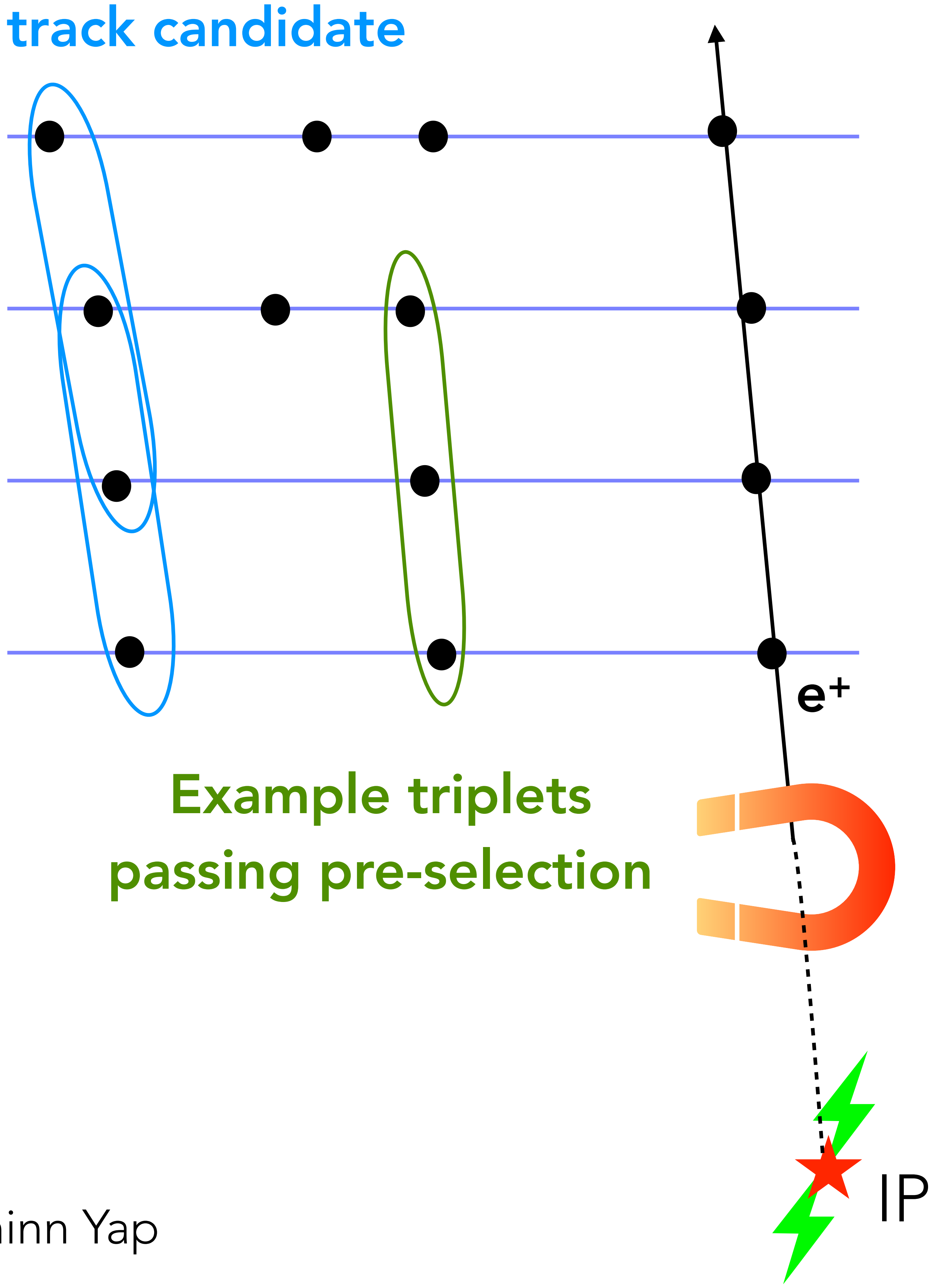
Step 1: form triplets



TRACKING USING QUANTUM COMPUTING

Step 1: form triplets

Two selected triplets form a track candidate



Example triplets passing pre-selection

Step 2: find the best sets of triplets

Quadratic
Unconstrained
Binary
Optimisation

$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j \qquad T_i, T_j \in \{0,1\}$$

Weighting triplet T_i with quality a_i

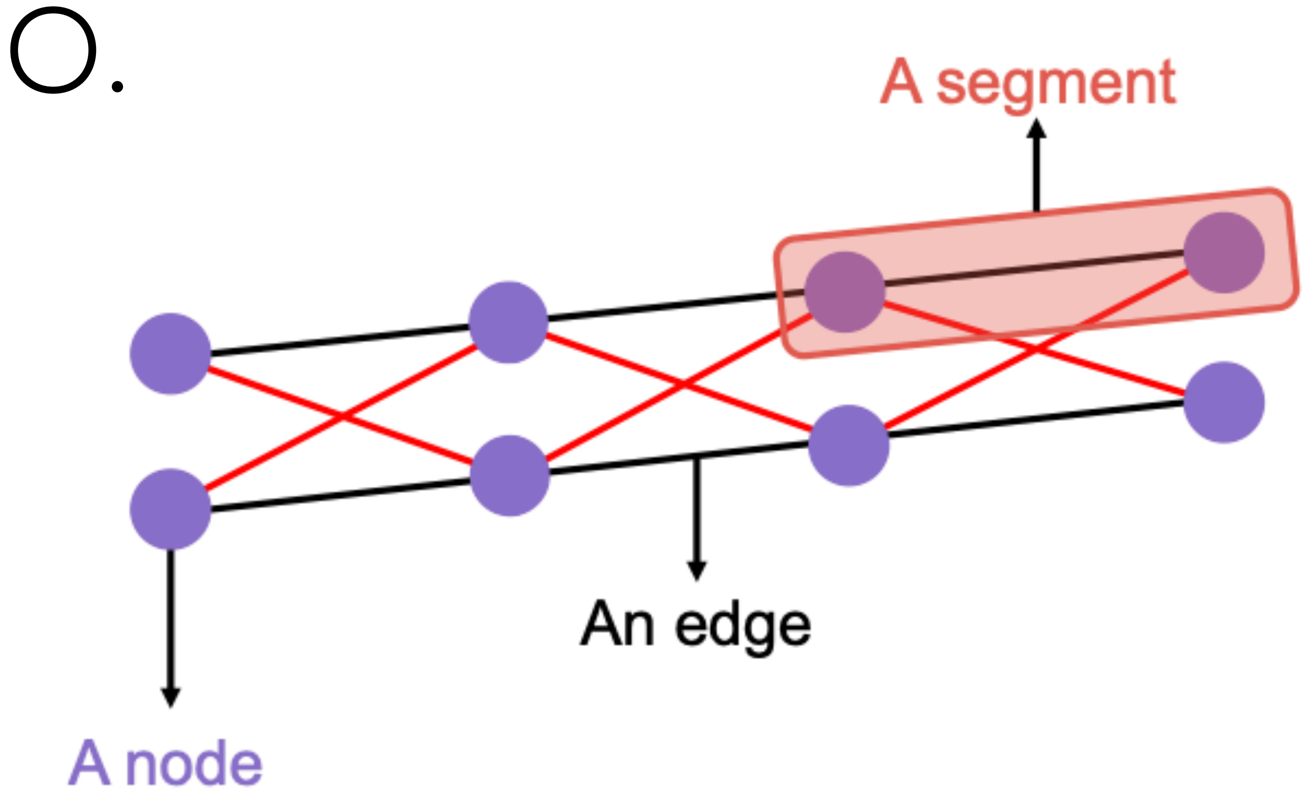
Compatibility b_{ij} between two triplets

$$b_{ij} = \begin{cases} -S(Ti, Tj), & \text{if } (T_i, T_j) \text{ form a quadruplet,} \\ \zeta & \text{if } (T_i, T_j) \text{ are in conflict,} \\ 0 & \text{otherwise.} \end{cases}$$

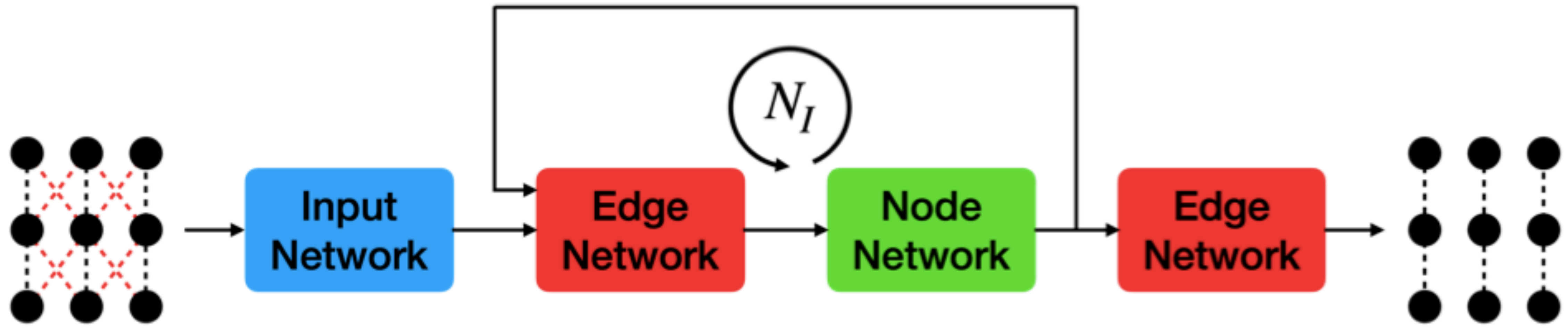
Find T_i, T_j that minimises QUBO!

OVERVIEW OF METHODS

- **Variational Quantum Eigensolver (VQE)** for minimising QUBO.
  - Exact solution with **matrix diagonalisation** as benchmark.

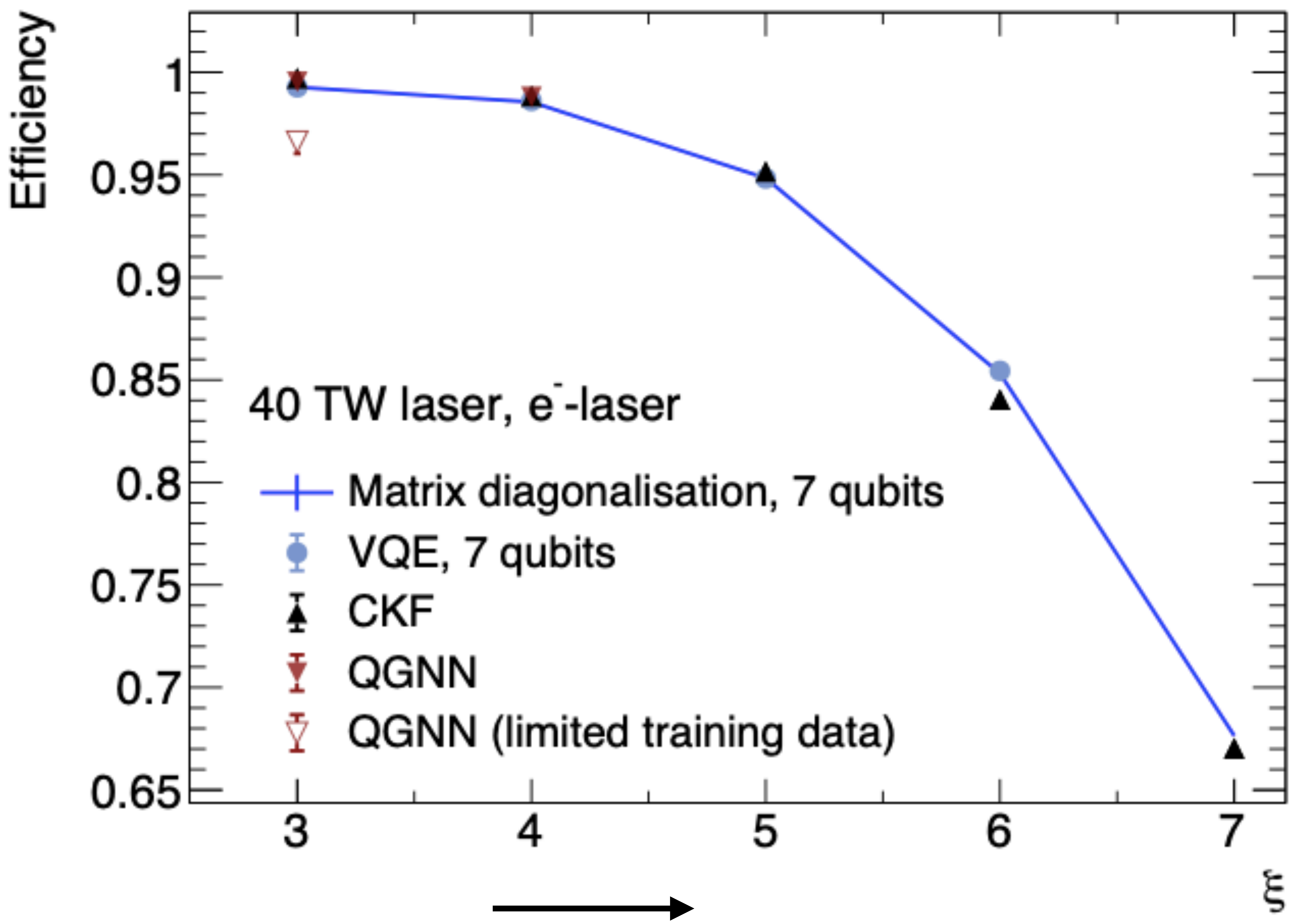


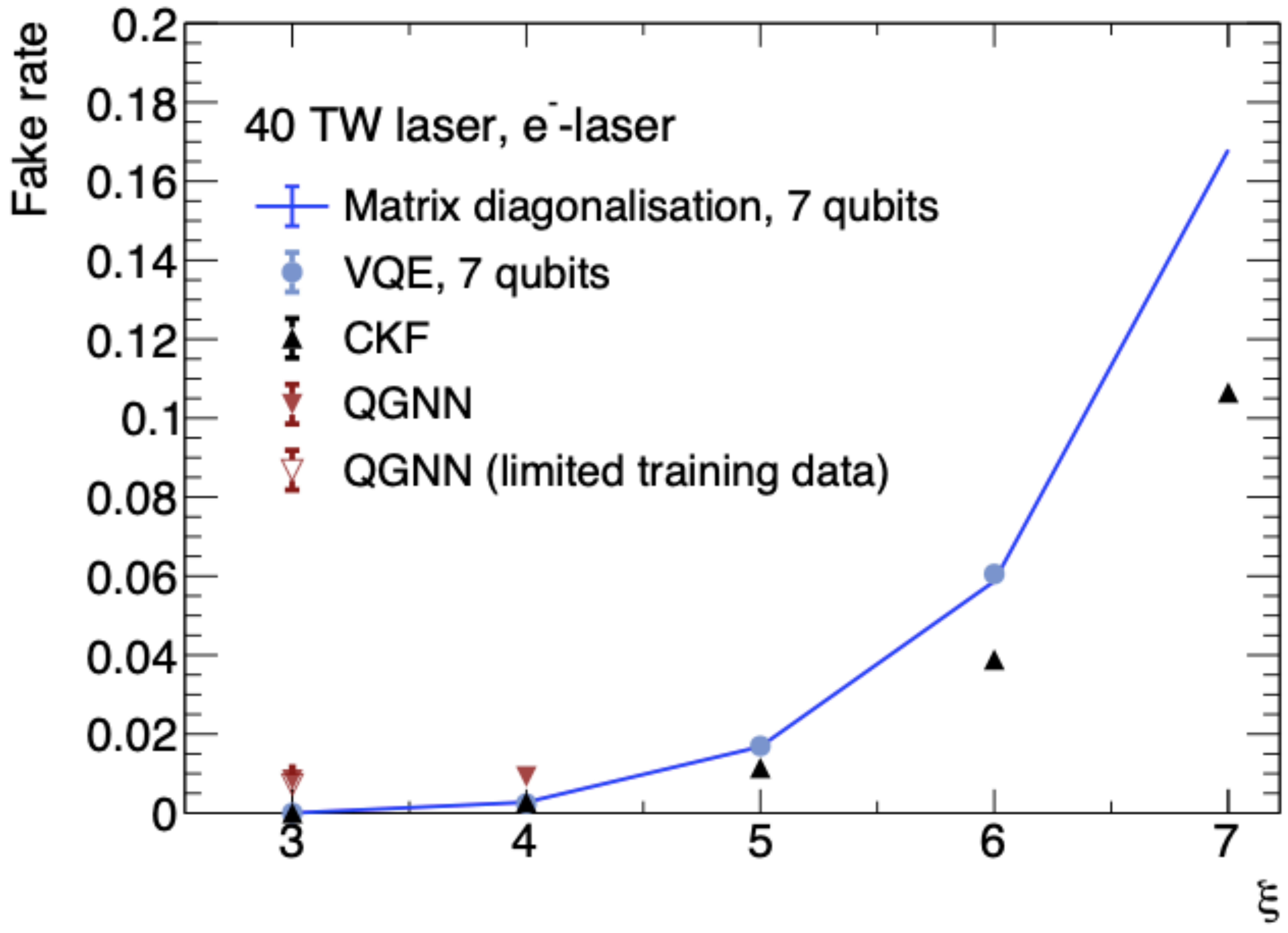
- **Quantum Graph Neural Network (QGNN)**
  - Doublet classification. Graph constructed from doublets.
  - Hybrid quantum-classical model with 10 hidden features (qubits).



- **Combinatorial Kalman Filter (CKF)**
  - CKF in a common tracking software (ACTS) used.
  - Triplets from first three layers are used as seeds to steer the tracking.

RESULTS





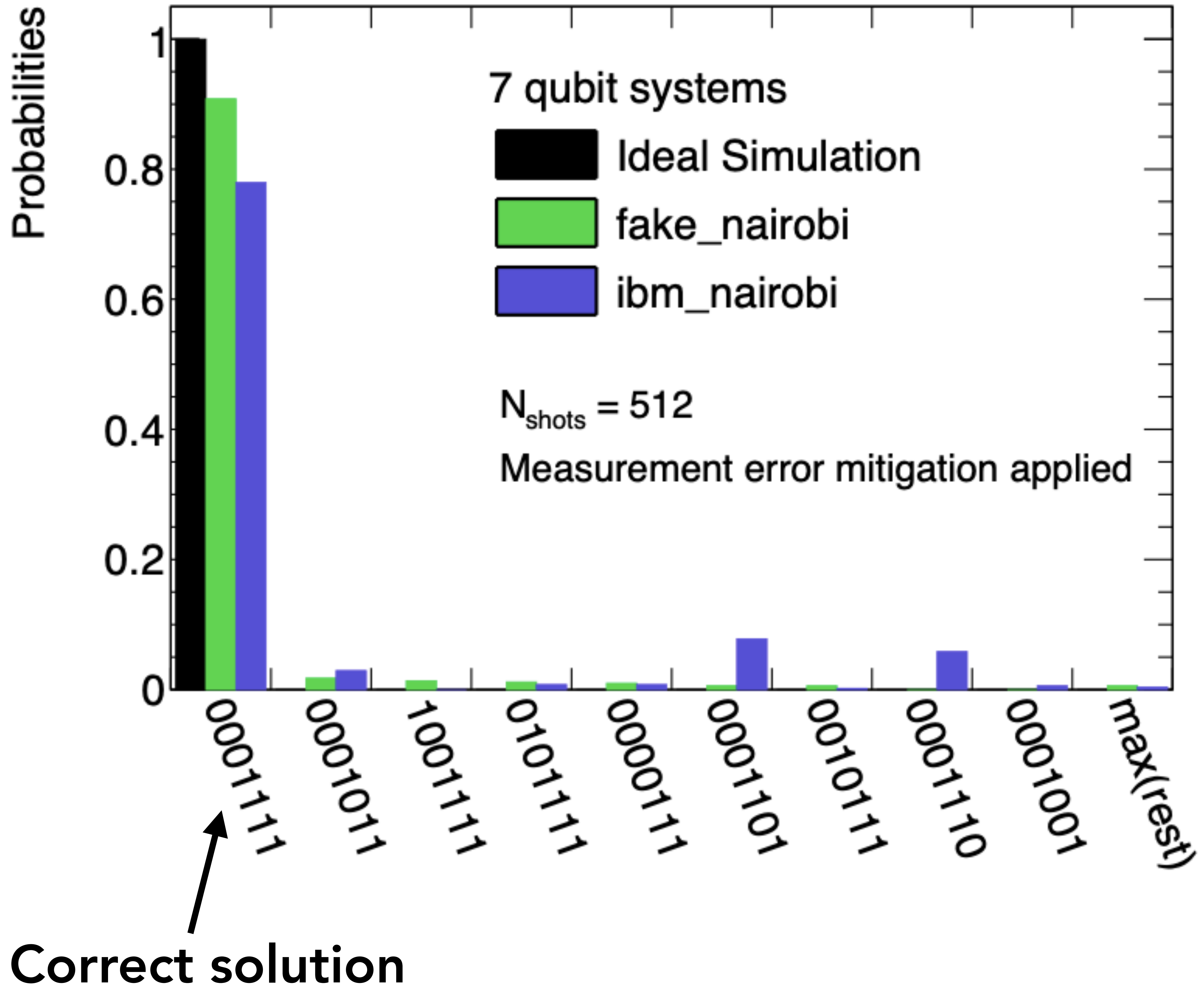
Increasing complexities/#particles

TEST ON REAL QUANTUM HARDWARE

- Results shown so far obtained using classical simulations of quantum hardware without noise.

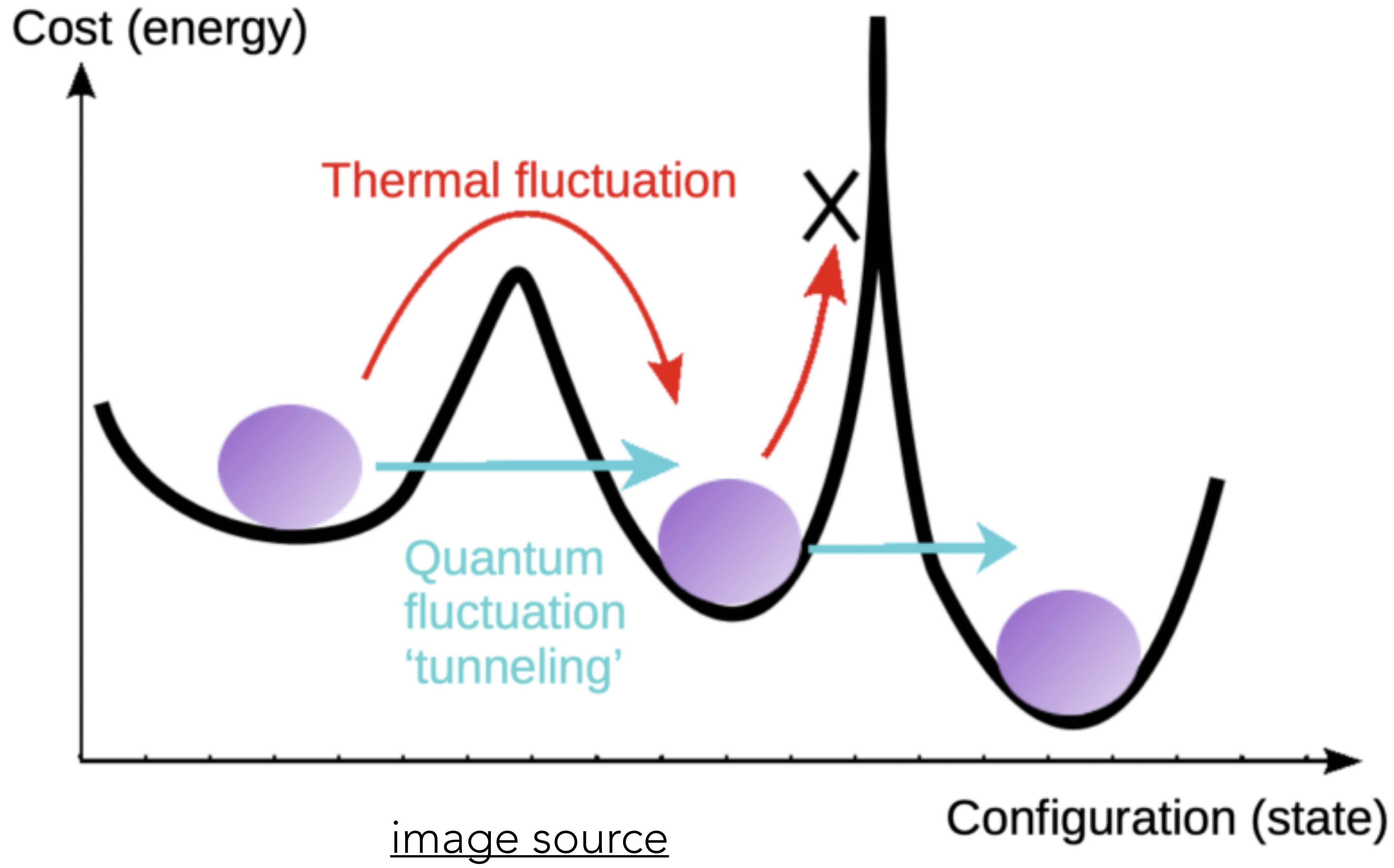
- To study how well VQE works, we study an example with 7 triplets (matching the #qubits of the device tested).

- Compare results from running on **quantum hardware (IBM Nairobi)** to **ideal simulation** as well as a **simulated device with noise**.



QUANTUM ANNEALING

- IBM offers universal gate-based quantum computers. Other types of Quantum computers exist.
- Quantum annealers specialise in solving optimisation problems.
- Advantage: > 5000 qubits in D-Wave.



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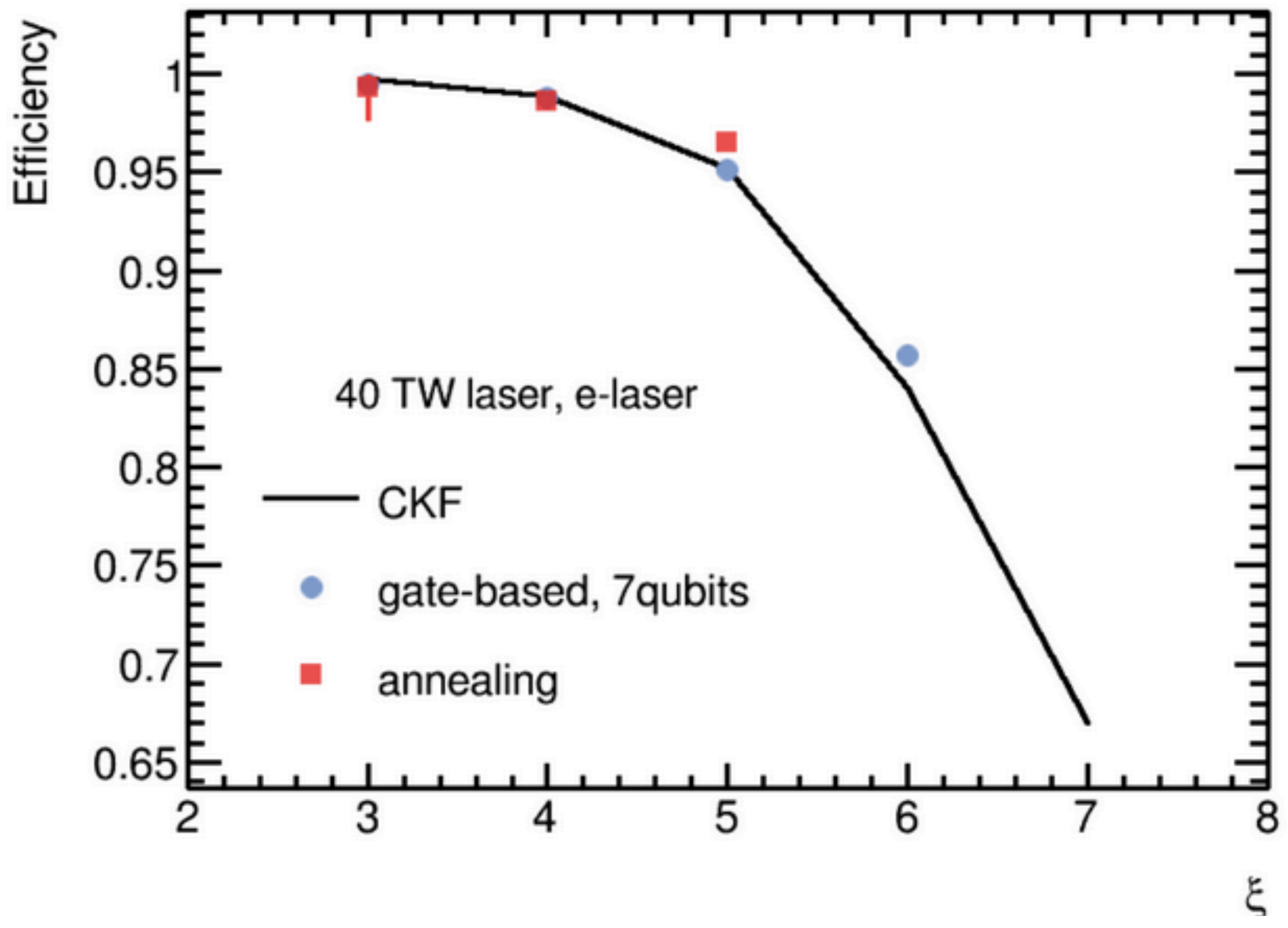
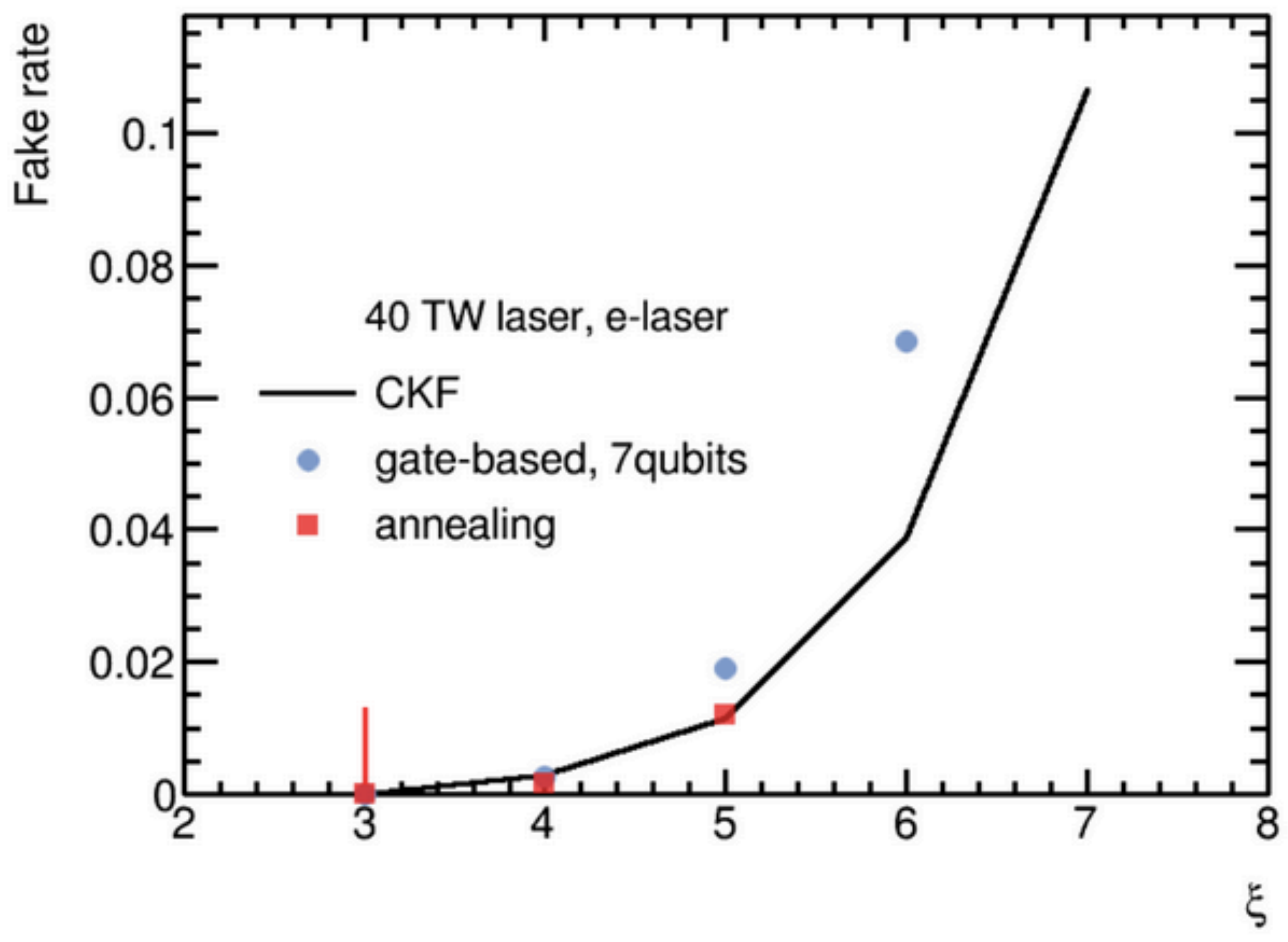
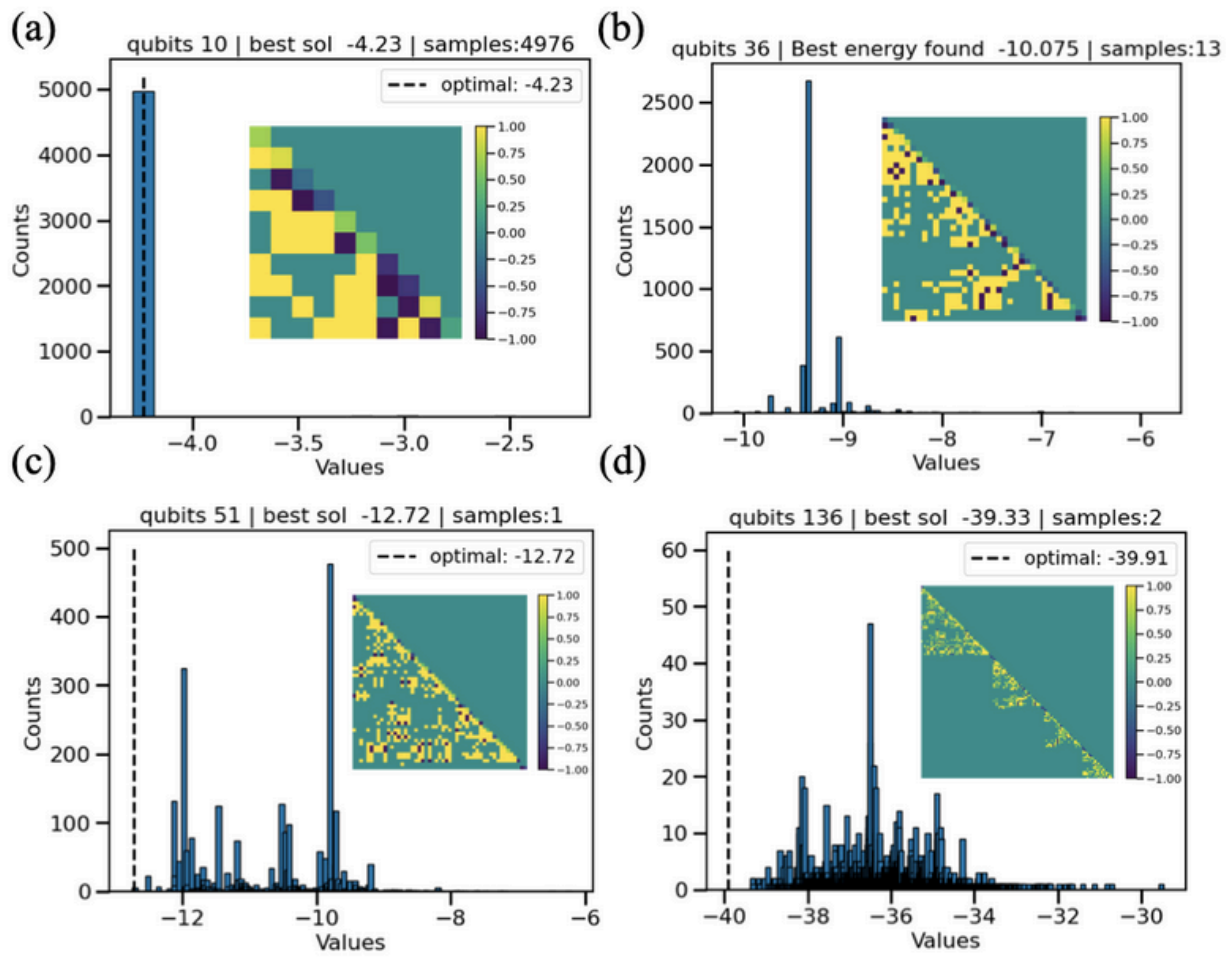
ANNEALING

- Simulated annealing
  - No partitioning





- Real D-wave annealer at Forschungszentrum Jülich
  - QUBO examples with sizes 10 — 136.
  - Ground state found for smaller QUBO sizes.



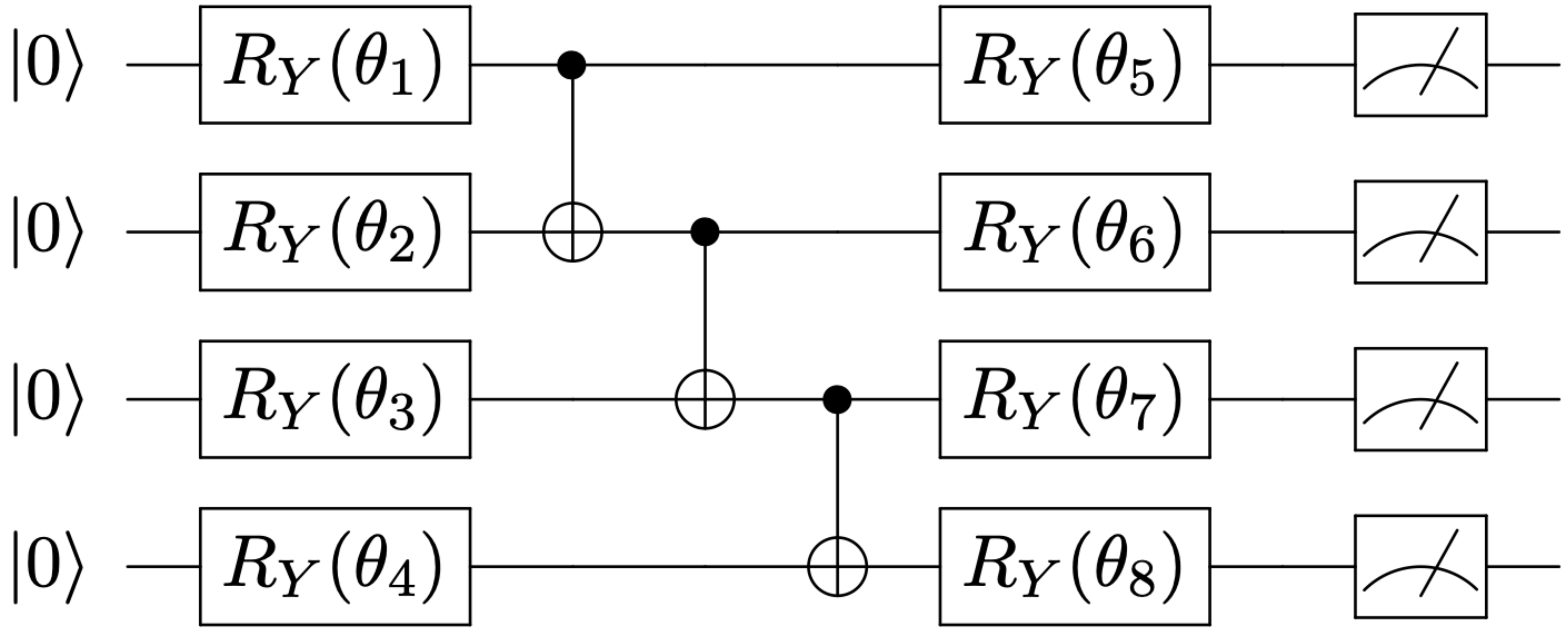
OUTLOOK

- LUXE will study strong-field QED in an unprecedented regime using high-intensity optical laser pulse and 16.5 GeV XFEL electron beam.
- Demonstrated the feasibility of tracking using a quantum approach.
  - Achieved similar performance as classical tracking.
- Outlook:
  - Detailed QUBO size scaling studies.
  - Use Machine Learning to learn a better QUBO encoding.
  - Extension to 4D tracking (including timing) for muon collider, see D. Spataro's talk tomorrow.

BACK-UP

QUBO

- The QUBO is mapped onto a quantum computer (here: simulator).
- The ground state is found using **Variational Quantum Eigensolver (VQE)**, a hybrid quantum-classical algorithm.
  - Nakanishi-Fujii-Todo (NFT) optimiser used.



- QUBO is partitioned into sub-QUBOs of the size of the quantum device (7 qubits assumed) to be solved iteratively.
- Exact solution using **matrix diagonalisation** used as benchmark.
- Another method of finding the ground state is with quantum annealing.

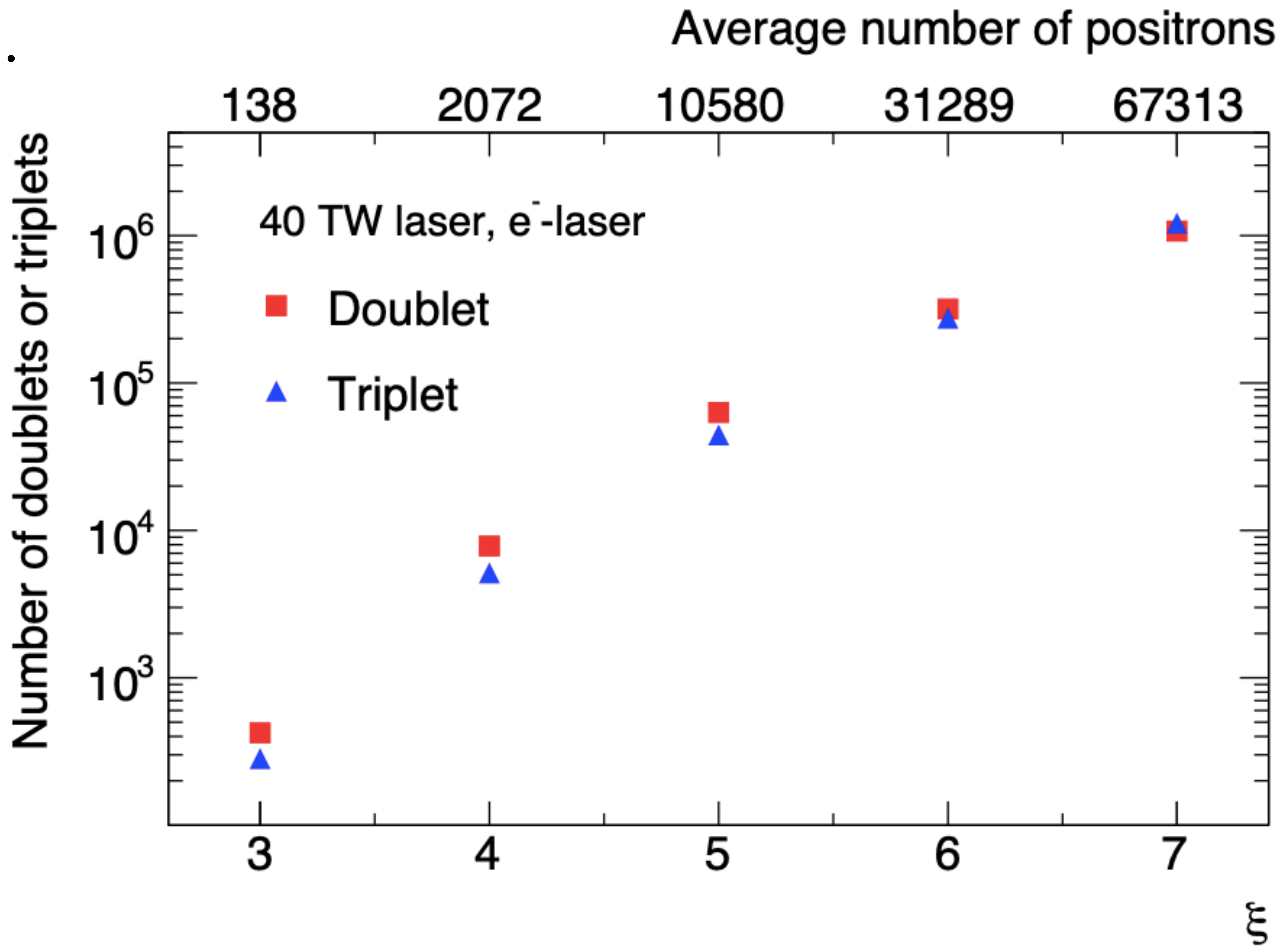
PERFORMANCE

- Apply final track selection and compare performance of these tracking methods for ξ = 3 – 7 in LUXE phase-0 e-laser interactions, where the number of positrons are between 140 and 67,000.

- Two metrics:

$$\text{Efficiency} = \frac{N_{\text{tracks}}^{\text{matched*}}}{N_{\text{tracks}}^{\text{generated}}}$$

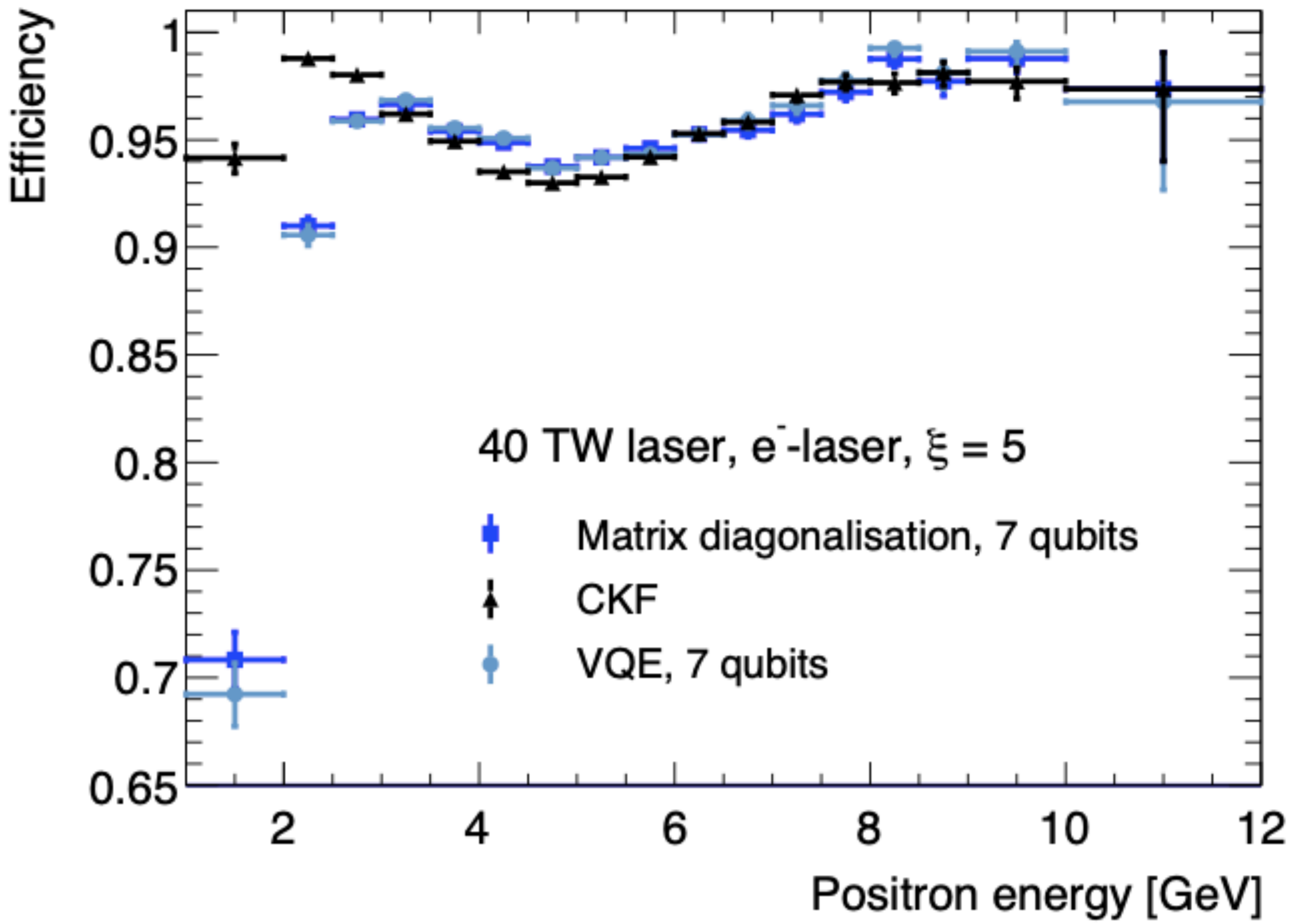
$$\text{Fake rate} = \frac{N_{\text{tracks}}^{\text{fake}}}{N_{\text{tracks}}^{\text{reconstructed}}}$$

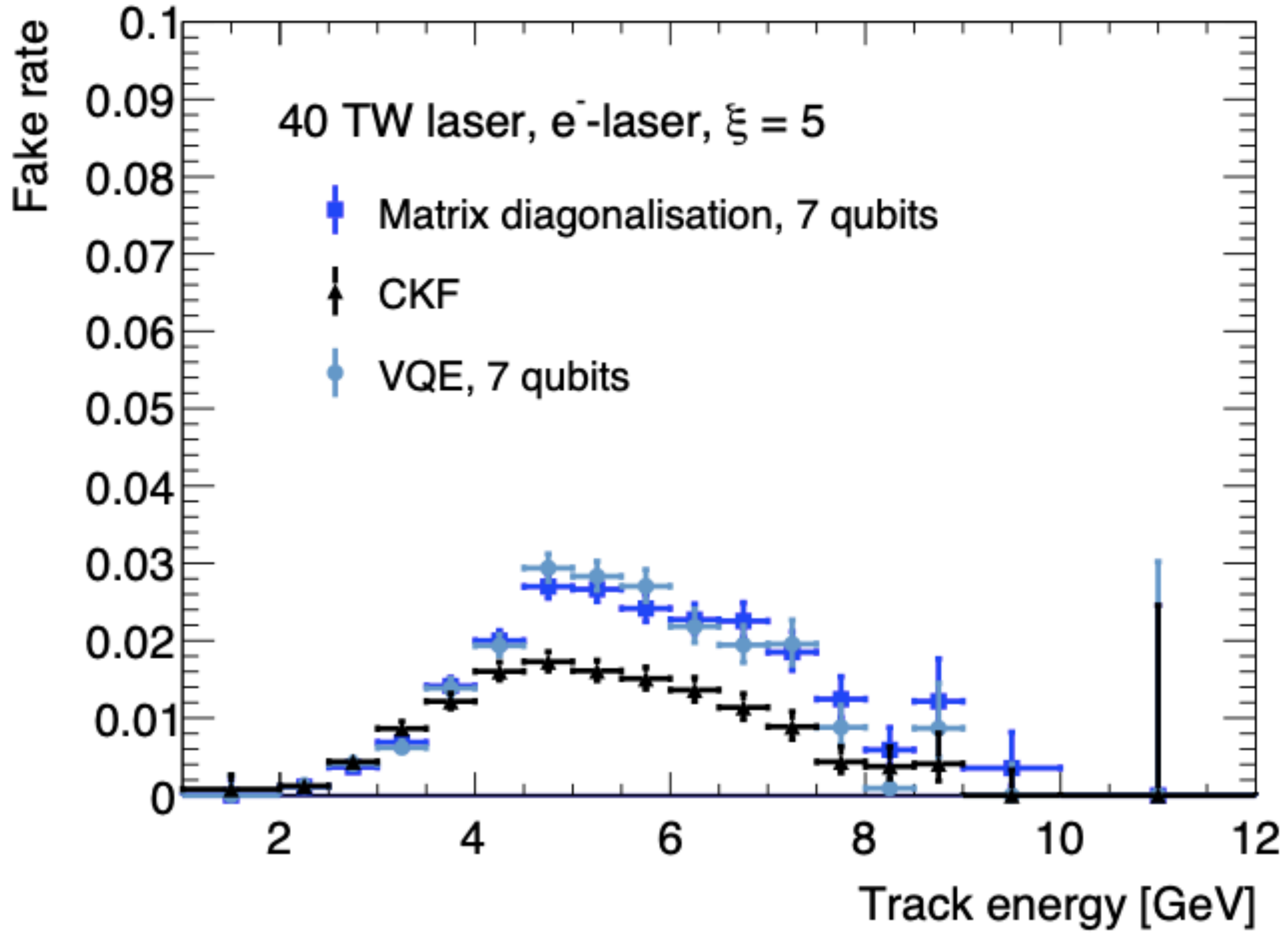


*A track is considered matched if the majority of its hits belong to the same particle (i.e. at least 3 out of 4 hits).

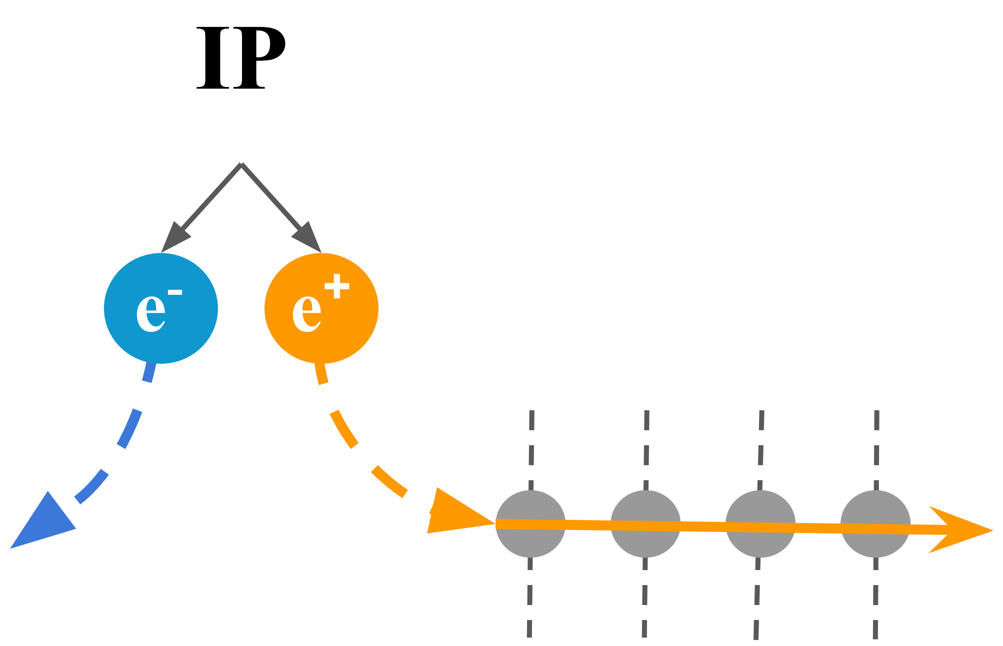
PERFORMANCE VS ENERGY

GNN results not available





WORKFLOW



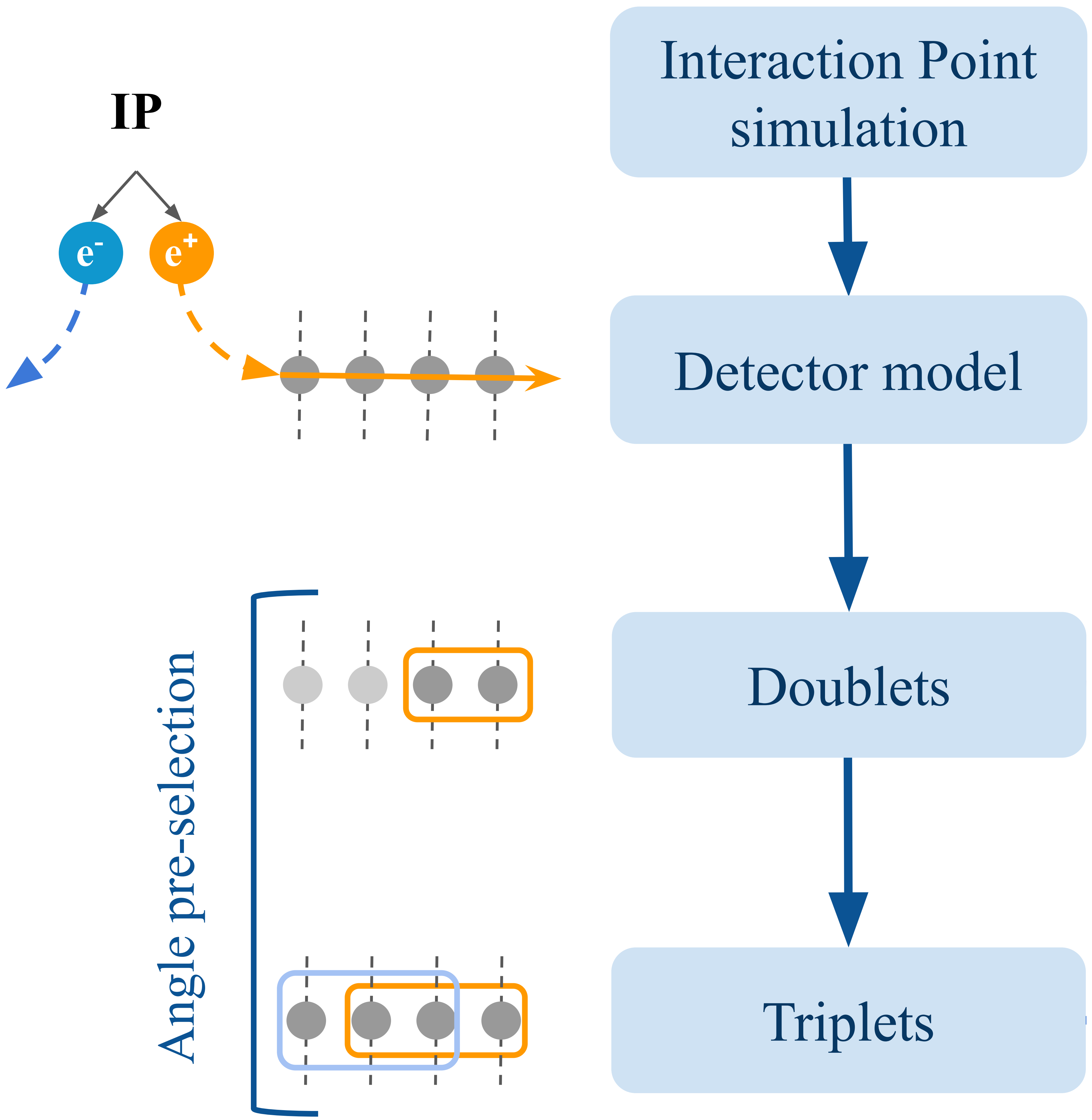
Interaction Point simulation

Detector model

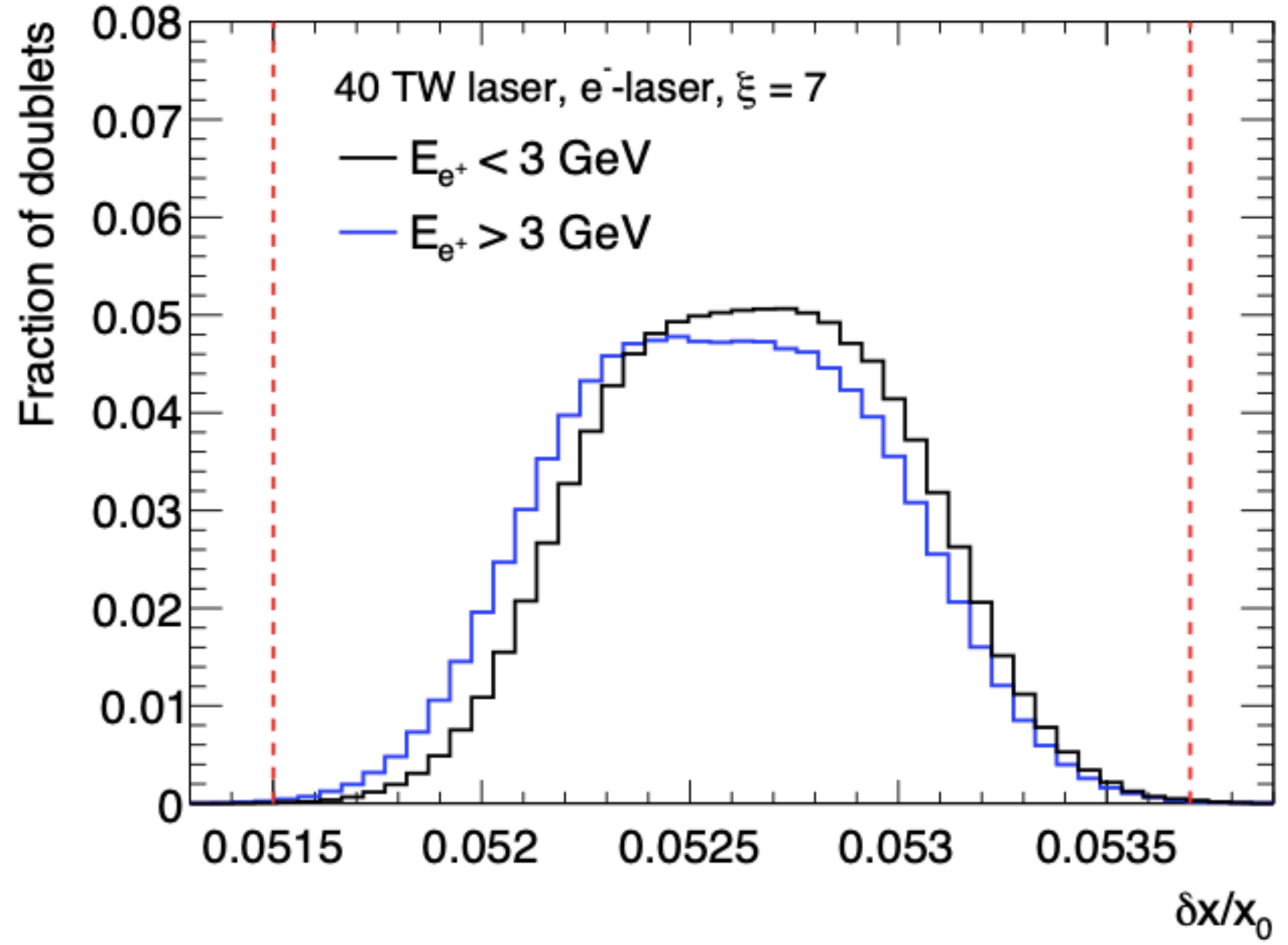
PTARMIGAN arXiv:2108.10883

Custom fast tracker simulation with simplified detector setup

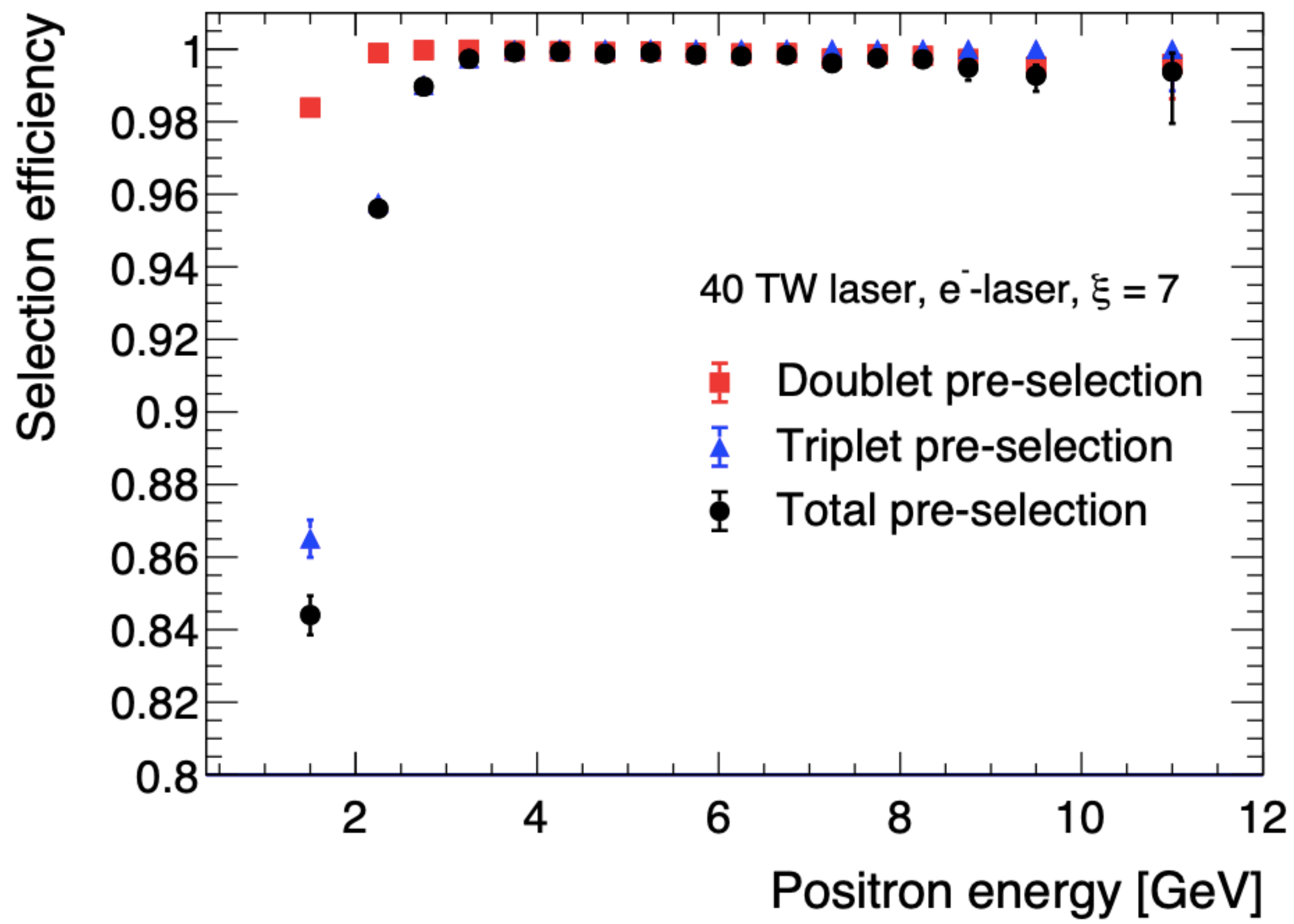
WORKFLOW



IP
e-
e+
Interaction Point simulation
Detector model
Doublets
Triplets
Angle pre-selection

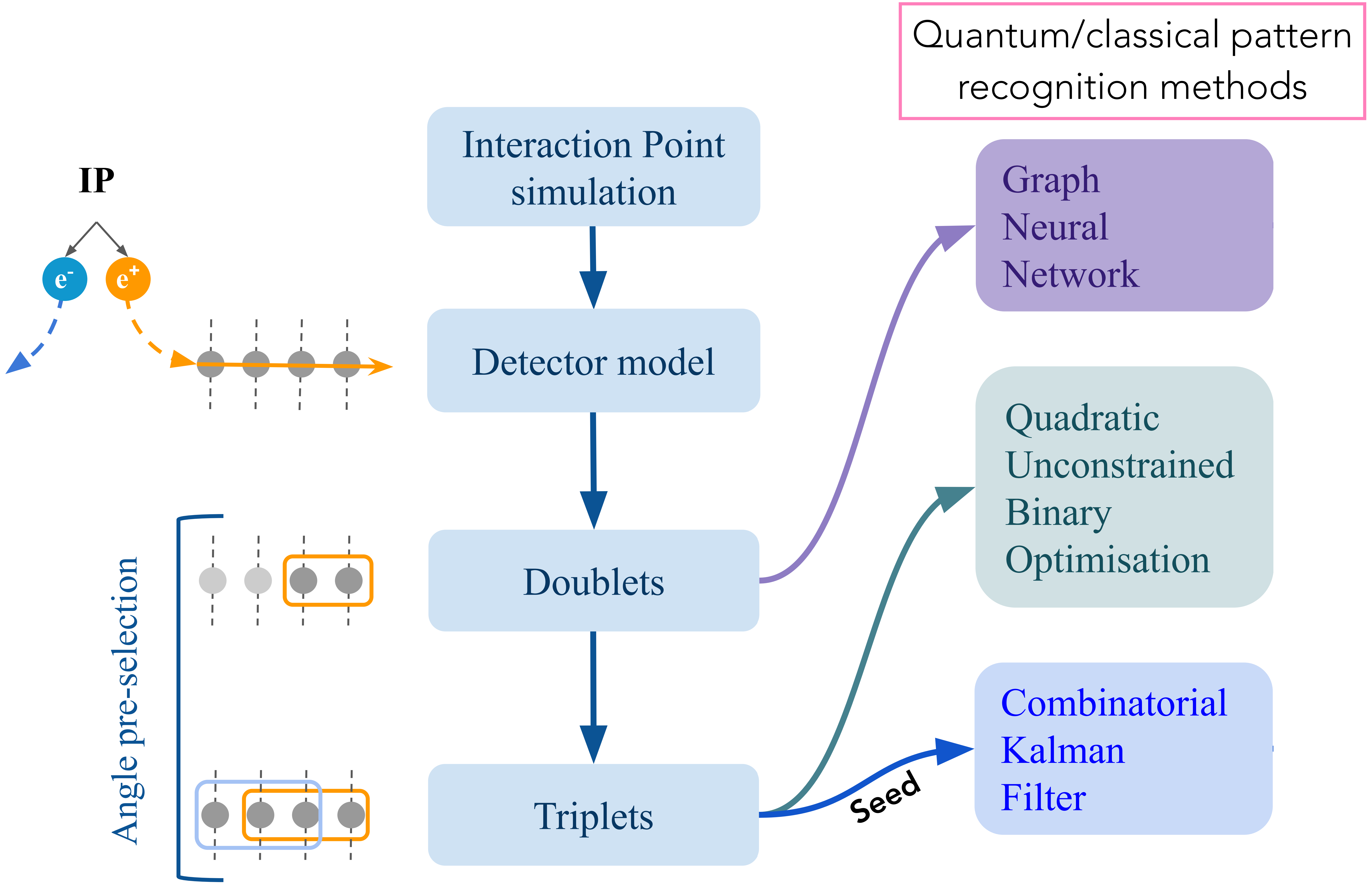


Fraction of doublets
40 TW laser, e-laser, ξ = 7
E_{e^+} < 3 GeV
E_{e^+} > 3 GeV
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.0515
0.052
0.0525
0.053
0.0535
δx/x0



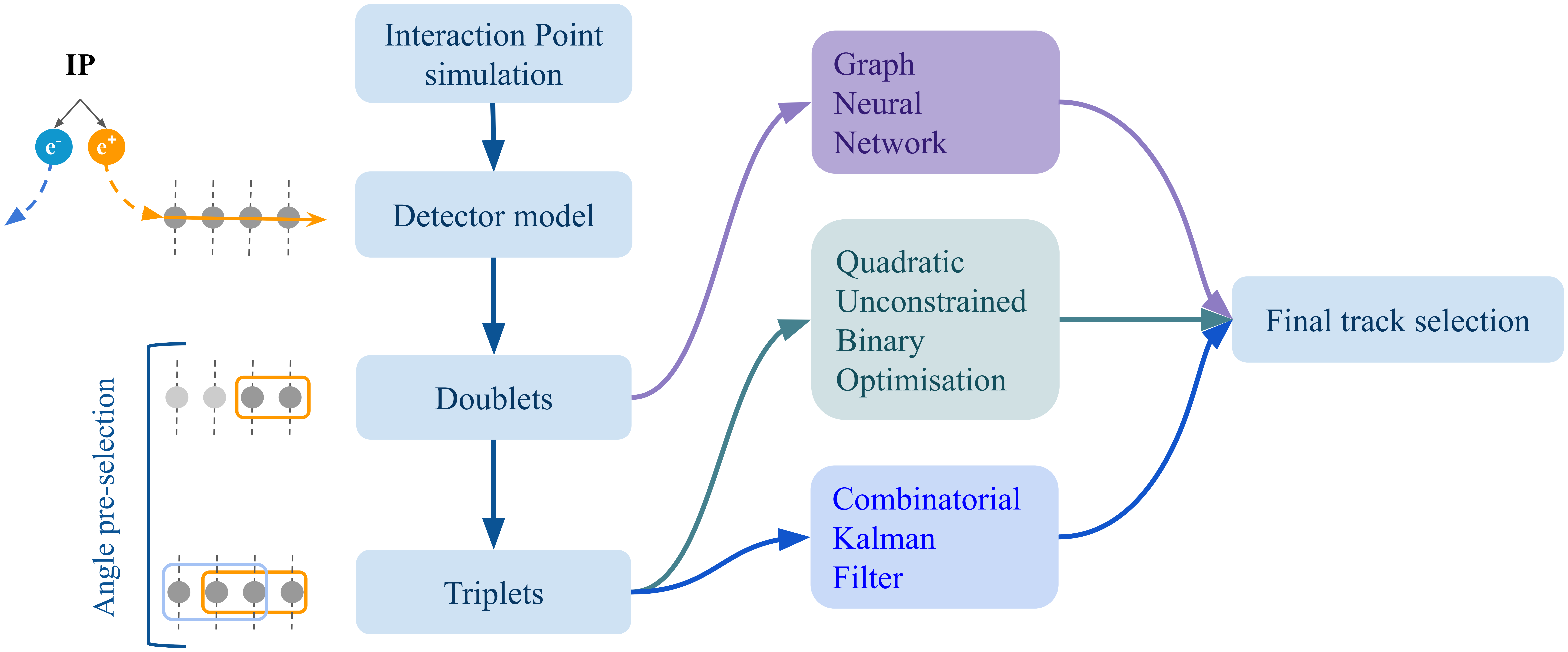
Selection efficiency
40 TW laser, e-laser, ξ = 7
Doublet pre-selection
Triplet pre-selection
Total pre-selection
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
2
4
6
8
10
12
Positron energy [GeV]

WORKFLOW



Quantum/classical pattern recognition methods
IP
e-
e+
Interaction Point simulation
Detector model
Doublets
Triplets
Angle pre-selection
Graph Neural Network
Quadratic Unconstrained Binary Optimisation
Seed
Combinatorial Kalman Filter

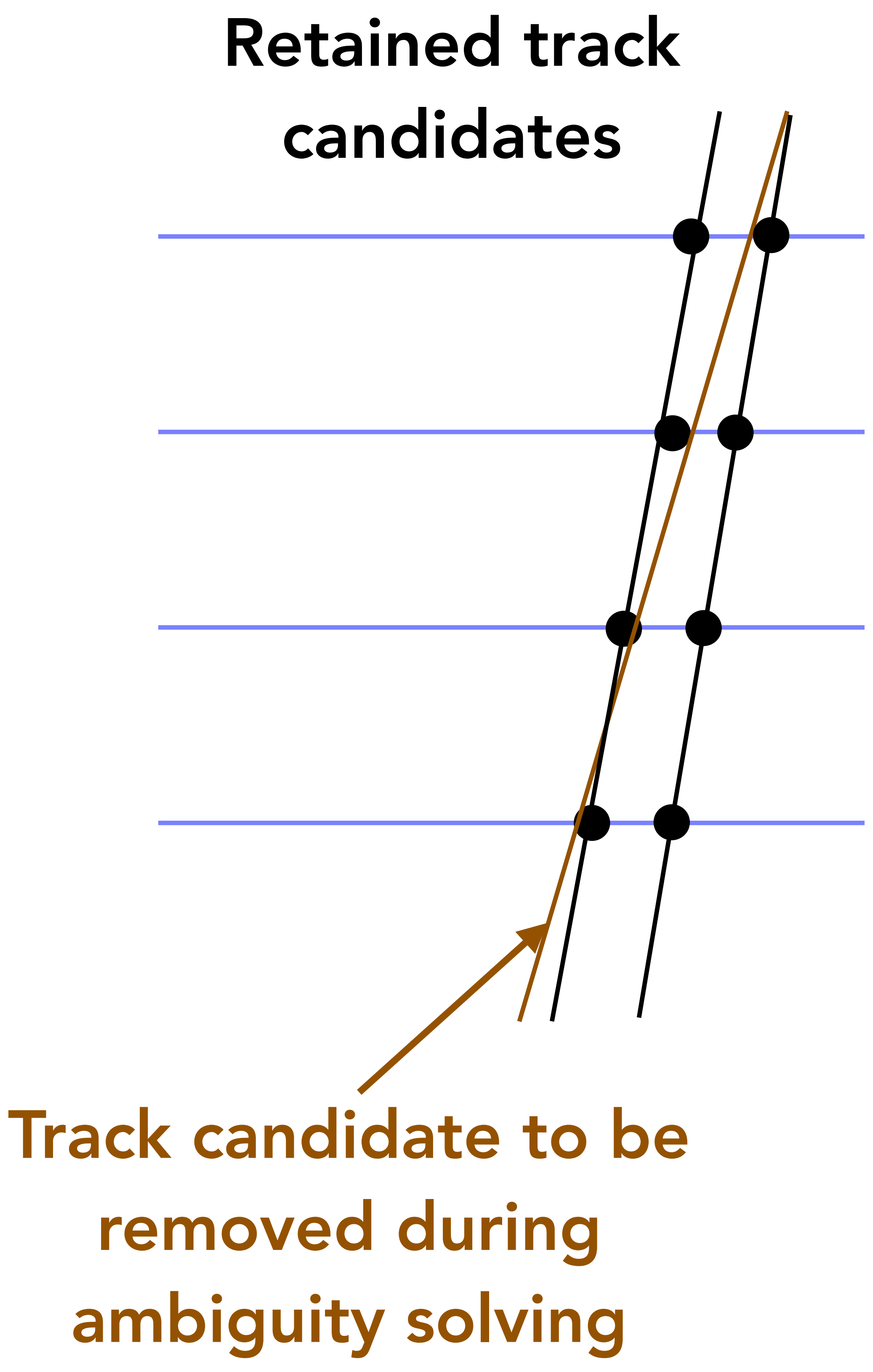
WORKFLOW

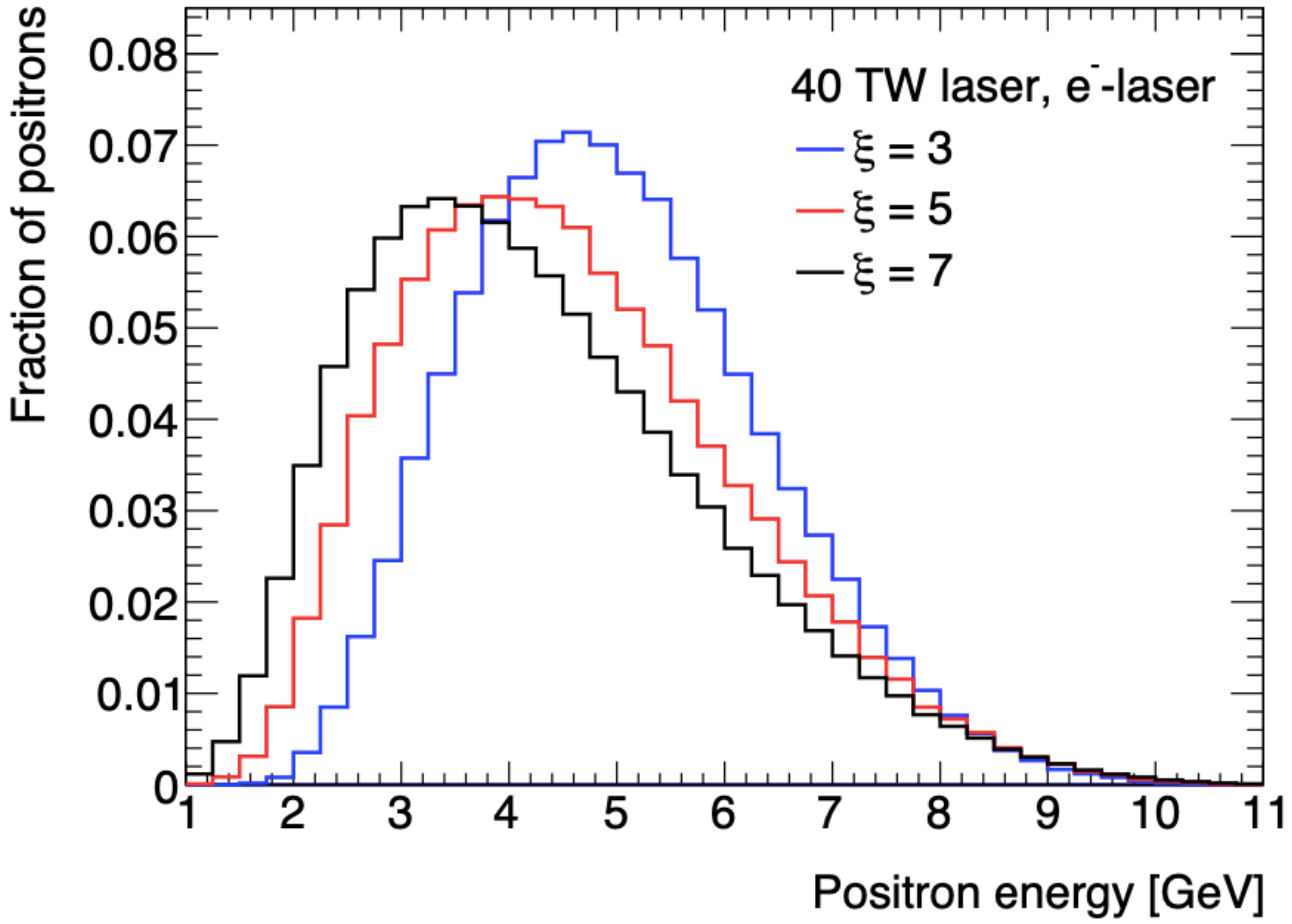


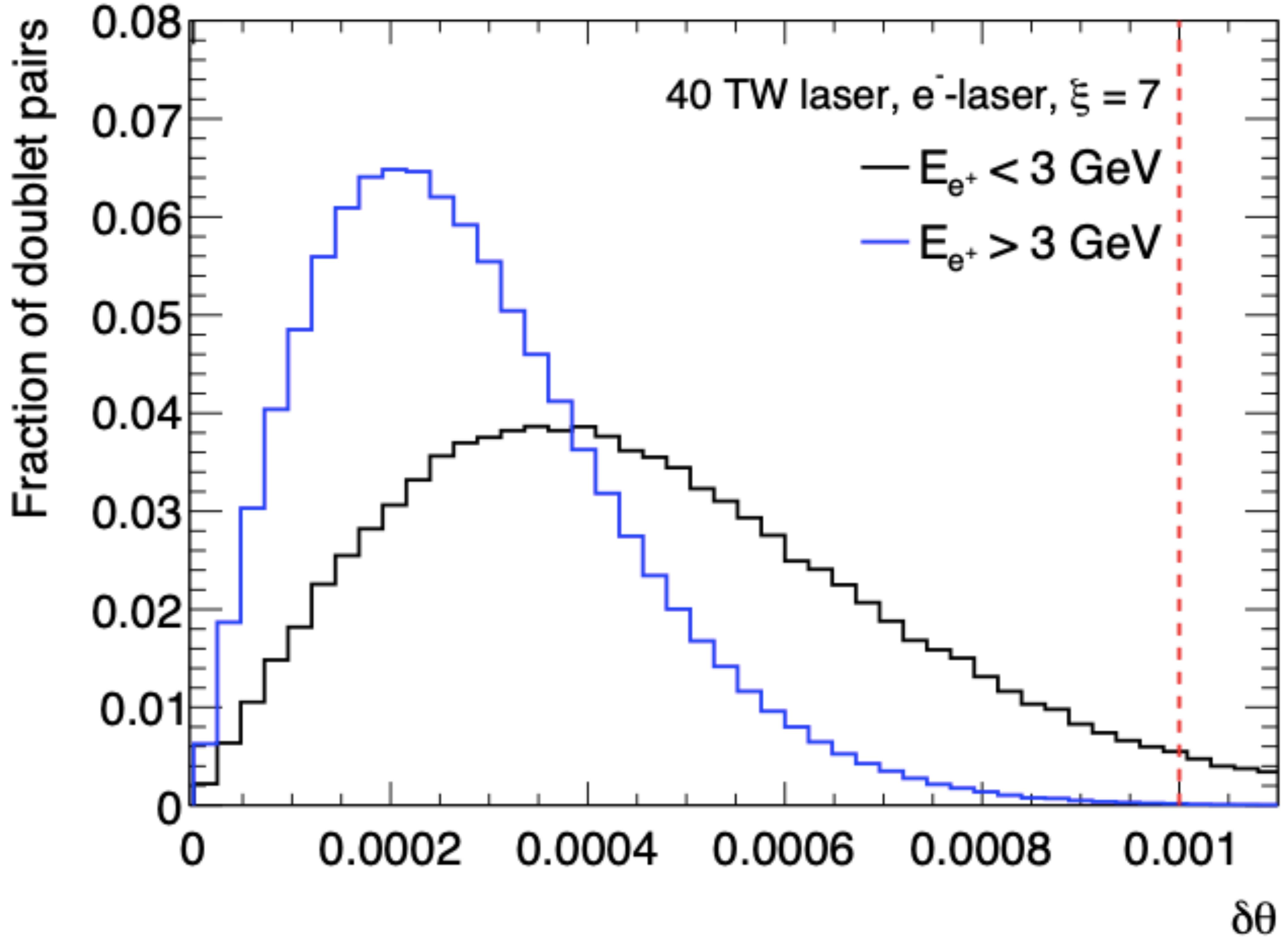
IP
e-
e+
Angle pre-selection
Interaction Point simulation
Detector model
Doublets
Triplets
Graph Neural Network
Quadratic Unconstrained Binary Optimisation
Combinatorial Kalman Filter
Final track selection

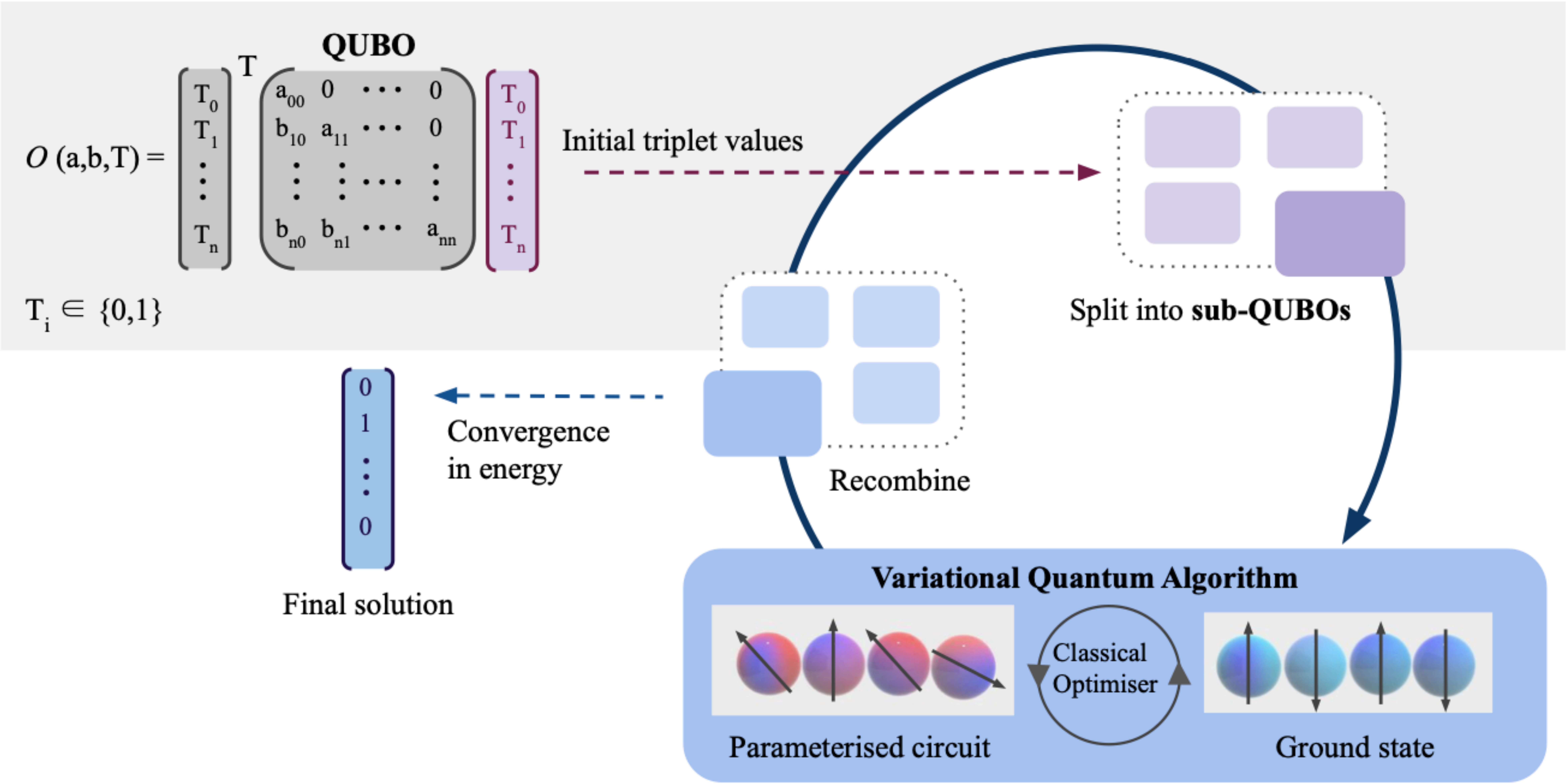
FINAL TRACK SELECTION

- Tracks are required to have 4 hits.
- Found either directly with classical CKF tracking or by combining selected doublets/triplets into quadruplets in the GNN/QUBO approaches.
- Tracks are fitted and ambiguity solving applied to remove worse quality tracks with shared hits from the track collection.
    - No track is allowed to have more than 1 shared hit.

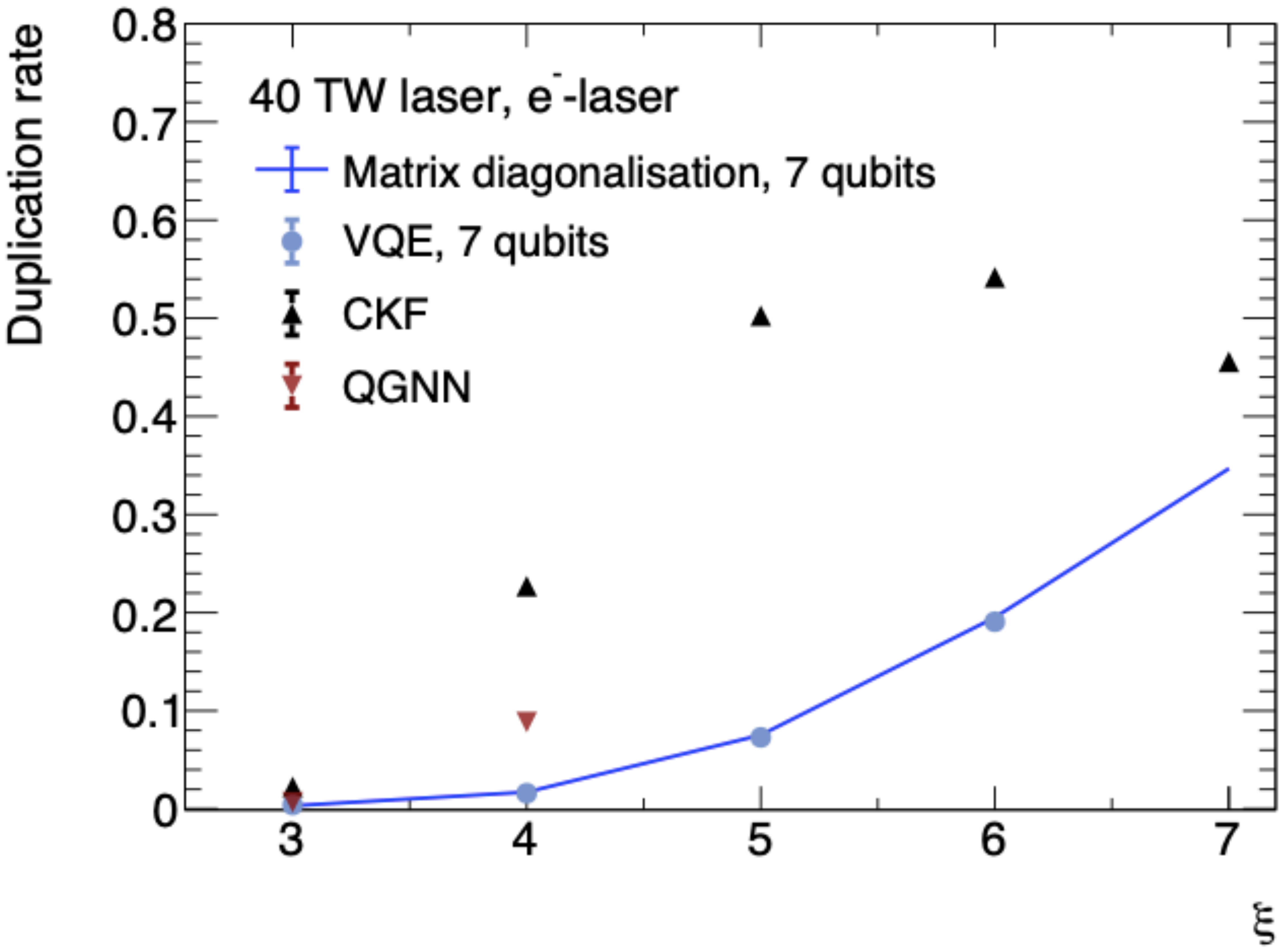


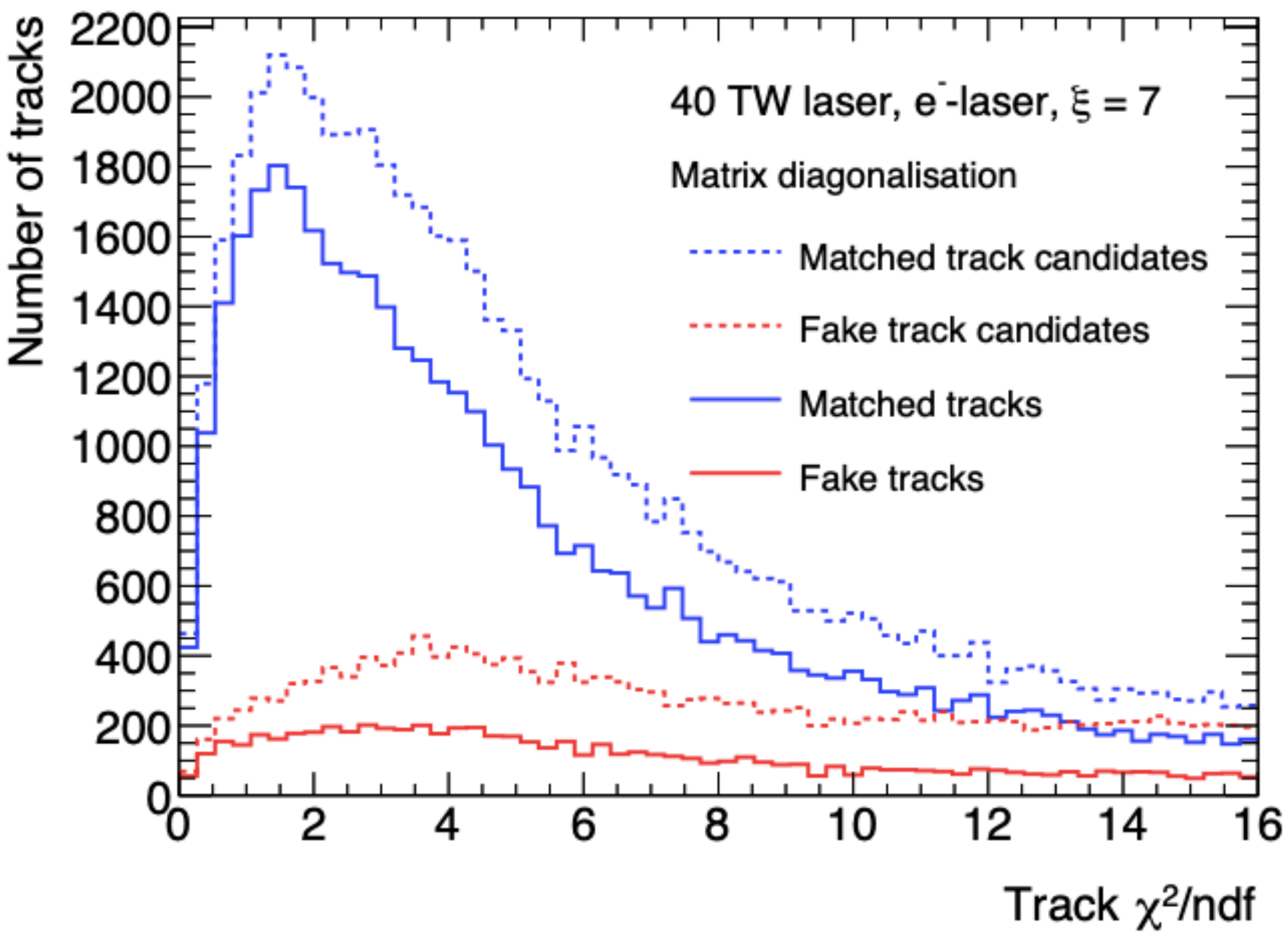

40 TW laser, e⁻-laser
ξ = 3
ξ = 5
ξ = 7
Fraction of positrons
Positron energy [GeV]
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
9
10
11


40 TW laser, e⁻-laser, ξ = 7
$E_{e^+} < 3$ GeV
$E_{e^+} > 3$ GeV
Fraction of doublet pairs
δθ
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.0002
0.0004
0.0006
0.0008
0.001


QUBO
$O\,(a,b,T) = \begin{bmatrix} T_0 \\ T_1 \\ \vdots \\ T_n \end{bmatrix}^T \begin{bmatrix} a_{00} & 0 & \cdots & 0 \\ b_{10} & a_{11} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ b_{n0} & b_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \\ \vdots \\ T_n \end{bmatrix}$
$T_i \in \{0,1\}$
Initial triplet values
Split into **sub-QUBOs**
Recombine
Convergence in energy
$\begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$
Final solution
Variational Quantum Algorithm
Classical Optimiser
Parameterised circuit
Ground state

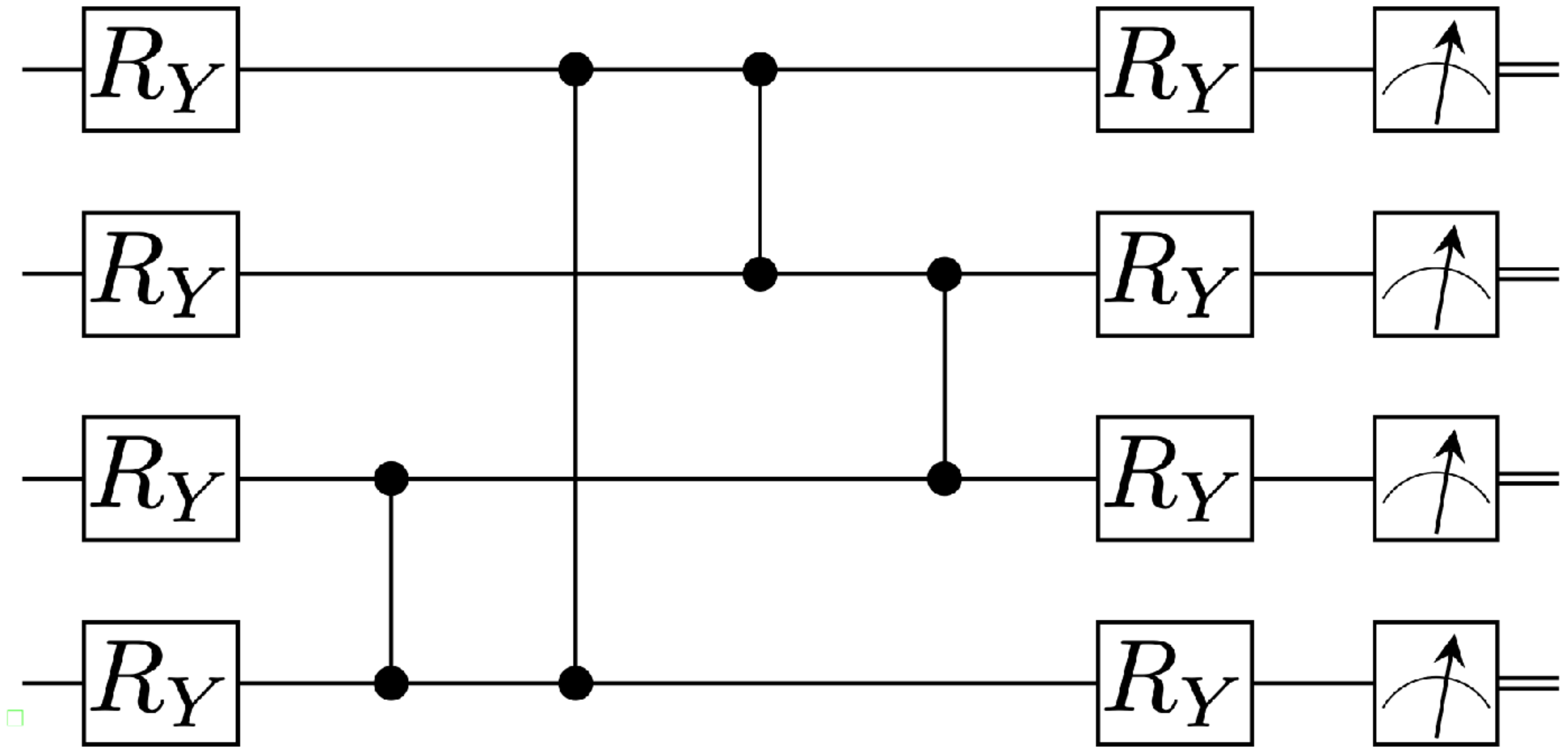
TRACK POST PROCESSING





GNN

- Circuit 10 with two layers and 10 qubits used.



(d) Circuit 10 in four qubits and single layer configuration. Adapted from Sim et al. (2019).

- EdgeNet and the NodeNet are applied alternately four times to allow the node features to be updated using farther nodes, as determined in a scan of the optimal model parameters.

GNN

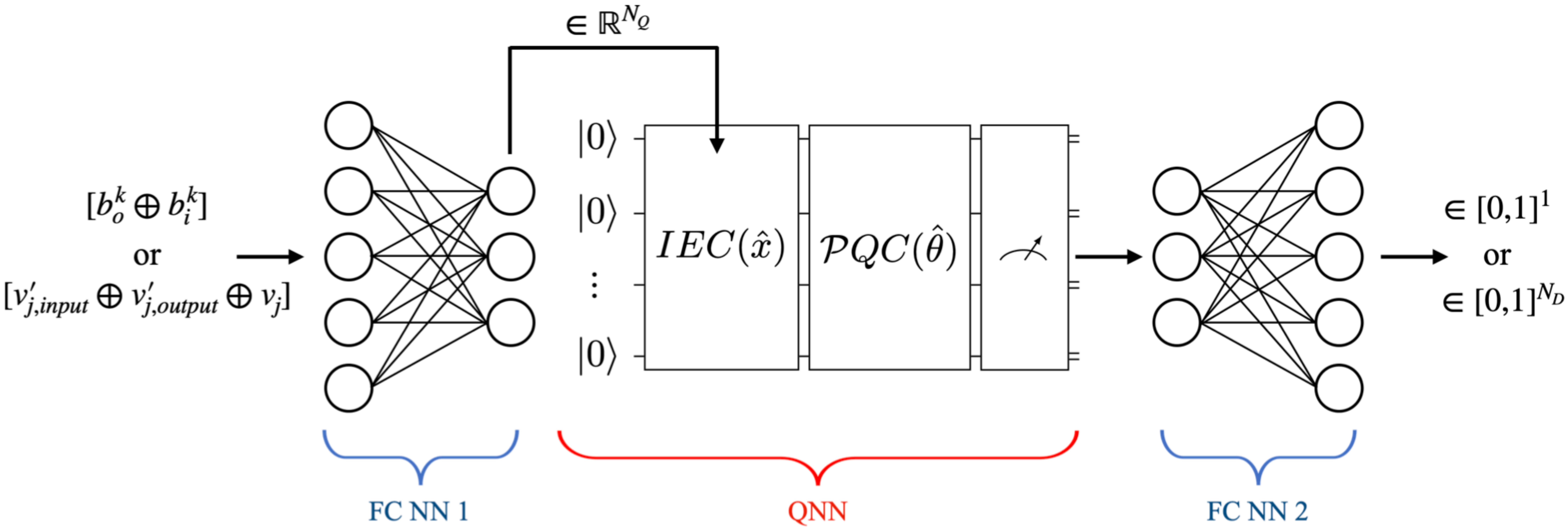
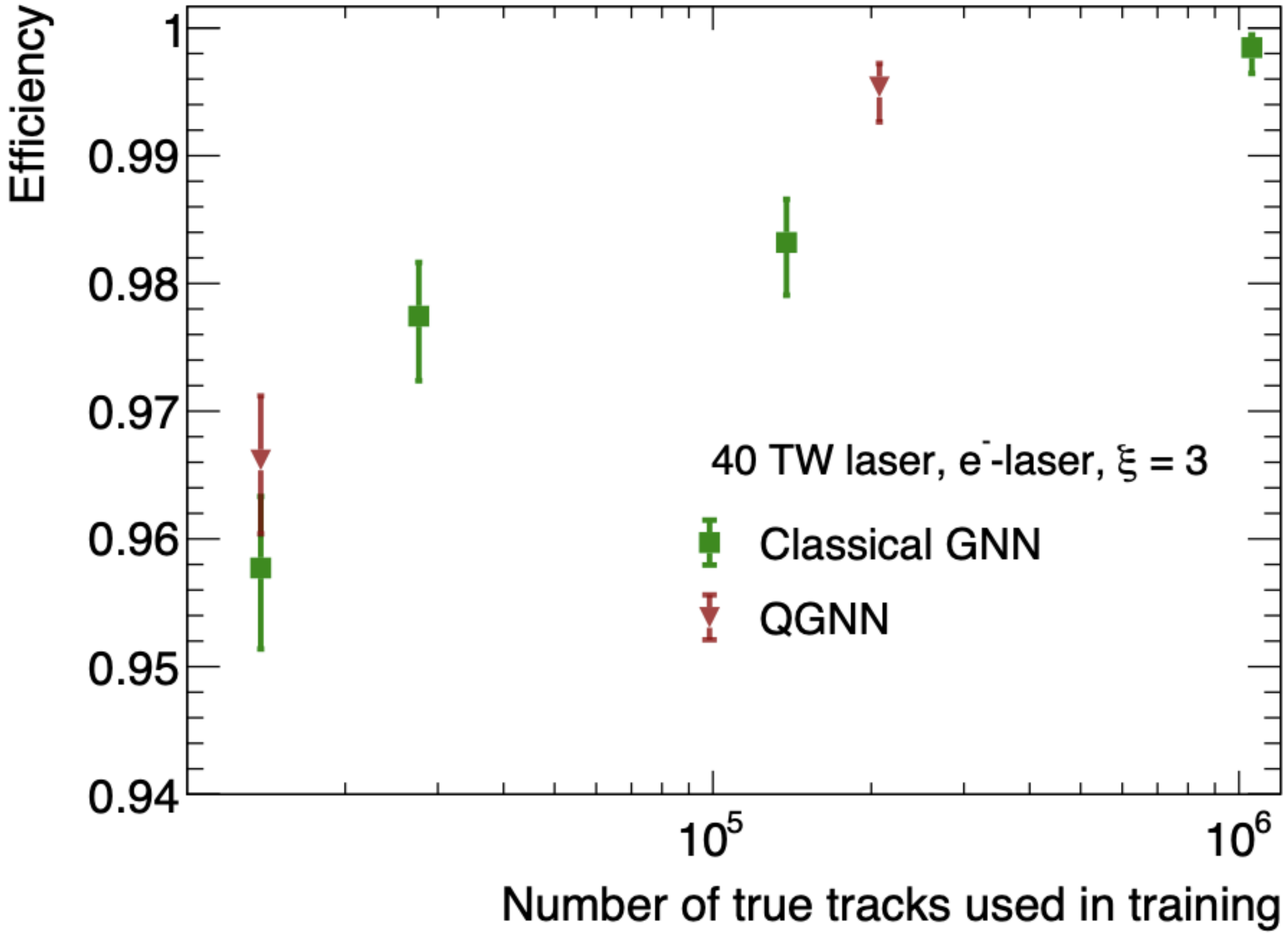


Fig. 6 The Hybrid Neural Network (HNN) architecture. The input is first fed into a classical fully connected Neural Network (FC NN) layer with sigmoid activation. Then, its output is encoded in the QNN with the Information Encoding Circuit (IEC). Next, the Parametrized Quantum Circuit (PQC) applies transformations on the encoded states. The output of QNN is obtained as expectation values for each qubit that is measured. A final FC NN layer with sigmoid activation is used to combine the results of different qubit measurements. The same HNN architecture is used in Edge (upper input and output dimension) and Node Networks (lower input and output dimension) with different parameters. The input and output dimension sizes change according to the network type. Details of the dimensions of each layer are given in Table 1.

GNN



Efficiency
1
0.99
0.98
0.97
0.96
0.95
0.94
10^5
10^6
Number of true tracks used in training
40 TW laser, e^--laser, ξ = 3
Classical GNN
QGNN

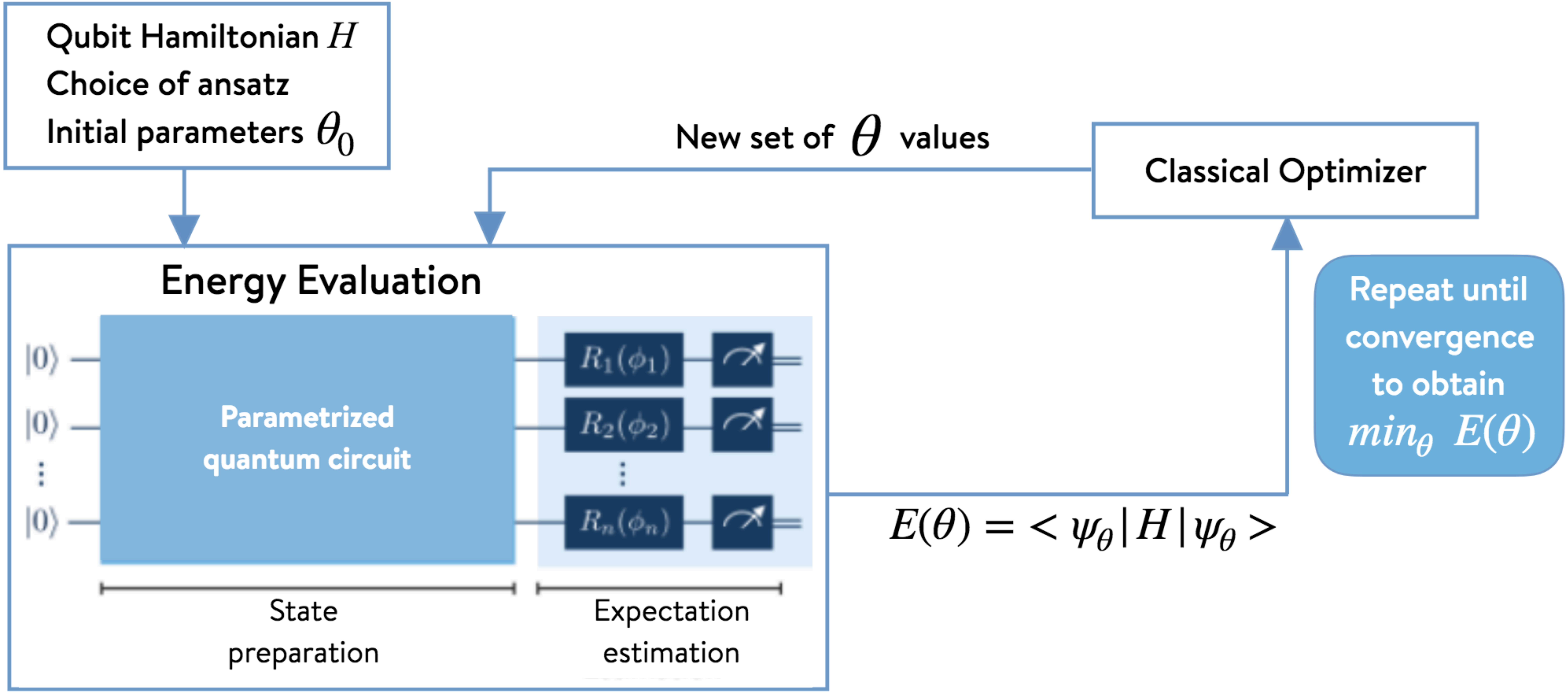


Image from http://openqemist.1qbit.com/docs/vqe_microsoft_qsharp.html

METHOD COMPARISON

Methods	GNN	QUBO	CKF
Starting point	Doublet	Triplet	Seed
Local/global	Global	Global	Local
Scope	Pattern recognition only	Pattern recognition only	Pattern recognition + track fitting
Classical benchmark	Classical GNN	Matrix diagonalisation	-