4D track reconstruction with quantum algorithms

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HELMHOLTZ

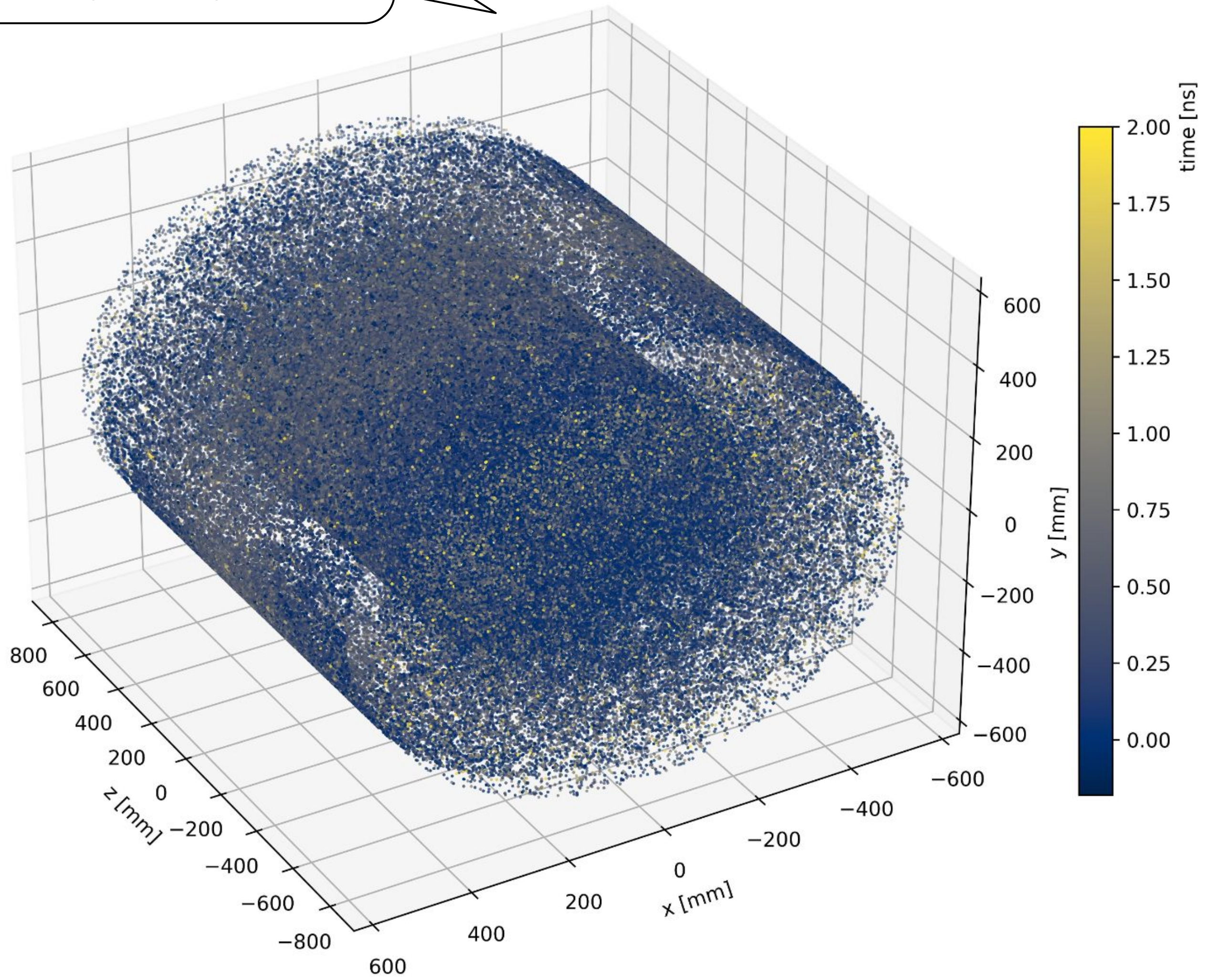
Motivation

Muon Collider:
Find single signal tracks in a large background

Track reconstruction:

- Fundamental for extracting meaningful information about particle collisions
- As a combinatorial task, it can be computationally expensive
- Quantum computing algorithms might yield an advantage over classical algorithms

This talk: 4D quantum algorithm for tracking in future collider facilities (muon collider as an example)



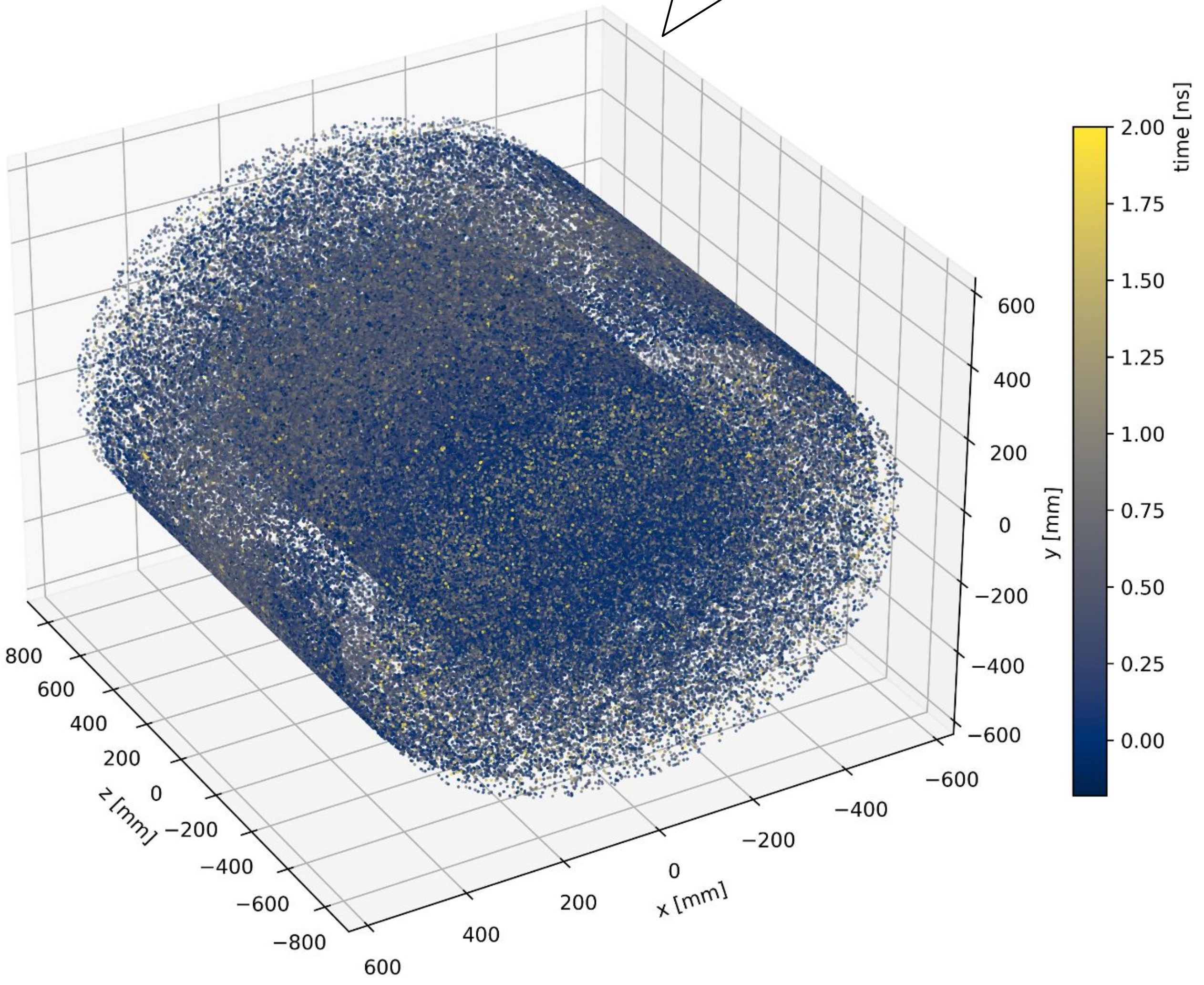
Muon Collider

- Higgs-factory colliding $\mu^+\mu^-$ - beams with a center-of-mass energy of 10 TeV
- Direct searches, high rate measurements, high-energy probes, muon physics

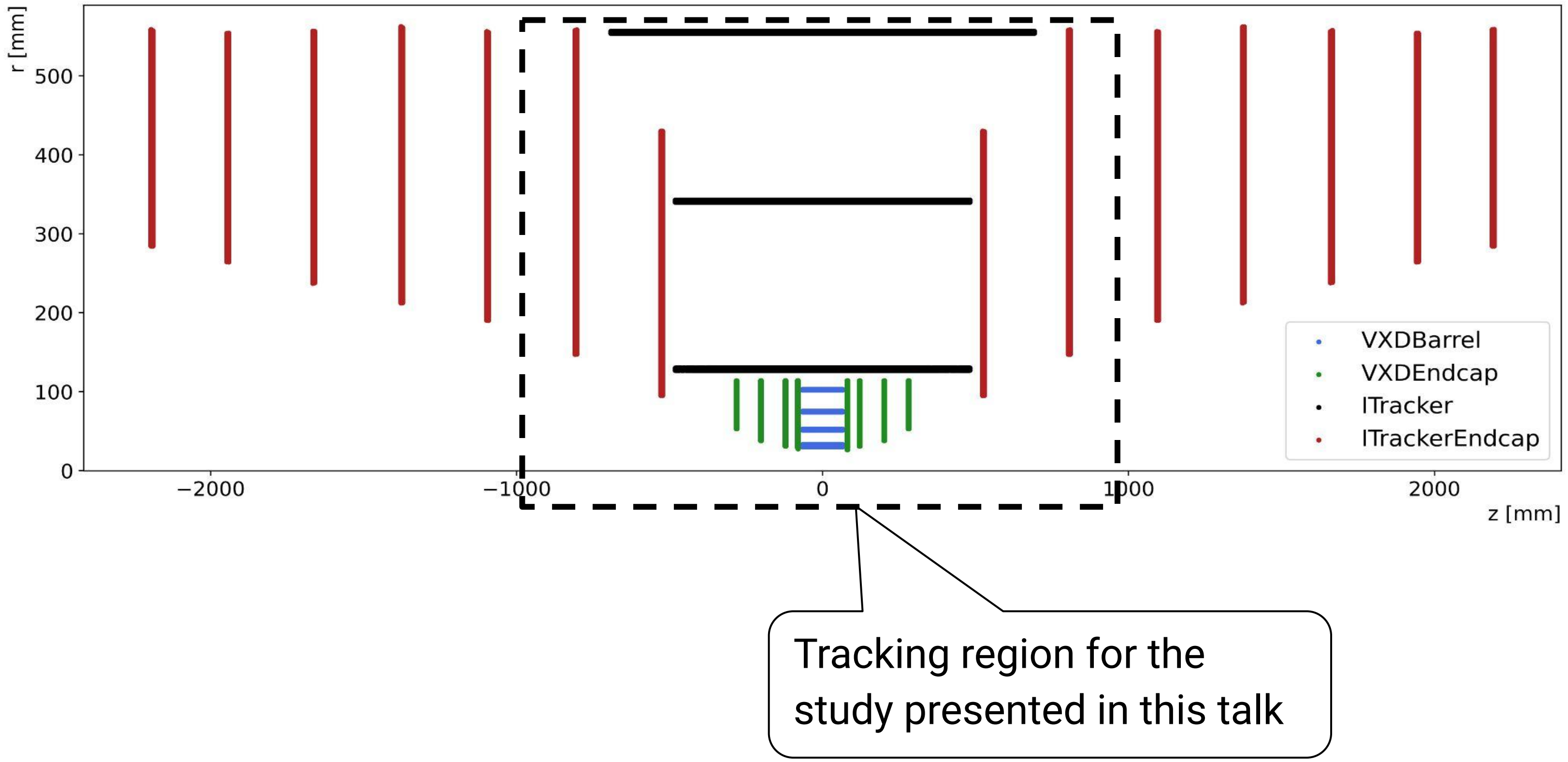
Challenges: Find single signal, reject combinatorial fake tracks originating from **B**eam-**I**nduced-**B**ackground (BIB)

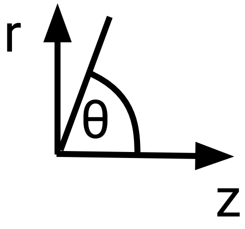
Time information as a crucial part to reduce BIB
→ available in the complete tracker region

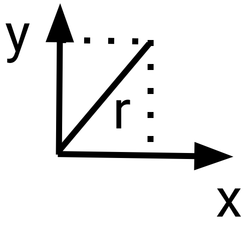
5 TeV BIB overlay simulated with FLUKA



Muon Collider detector







Time resolution:

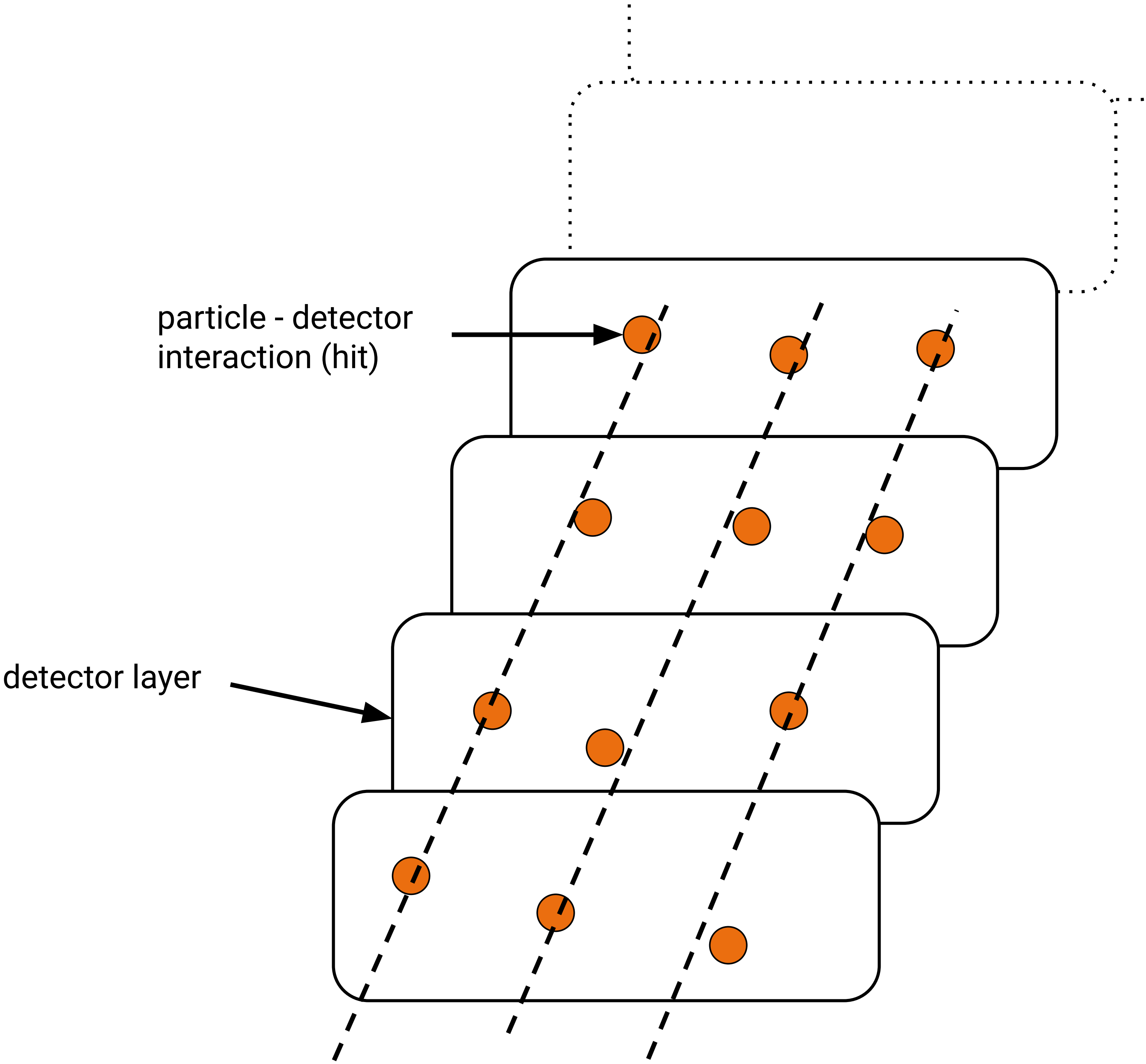
- VXD: 30 ps
- ITracker: 60 ps

Spatial resolution:

- VXD: 5 x 5 μm²
- ITracker: 7 x 90 μm²

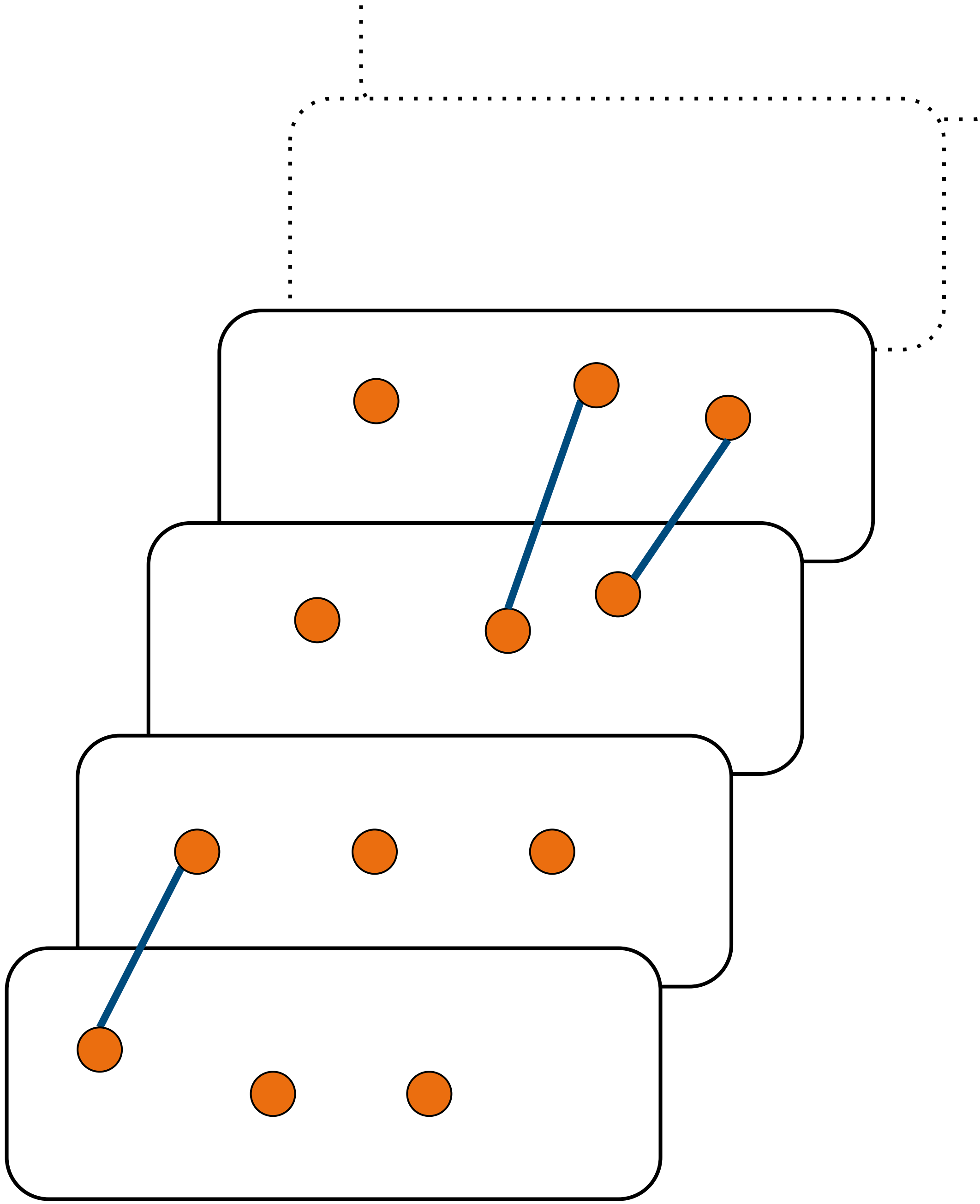
Pattern Recognition

Procedure: Connect hits from different detector layers and fit the space points to reconstruct particle tracks



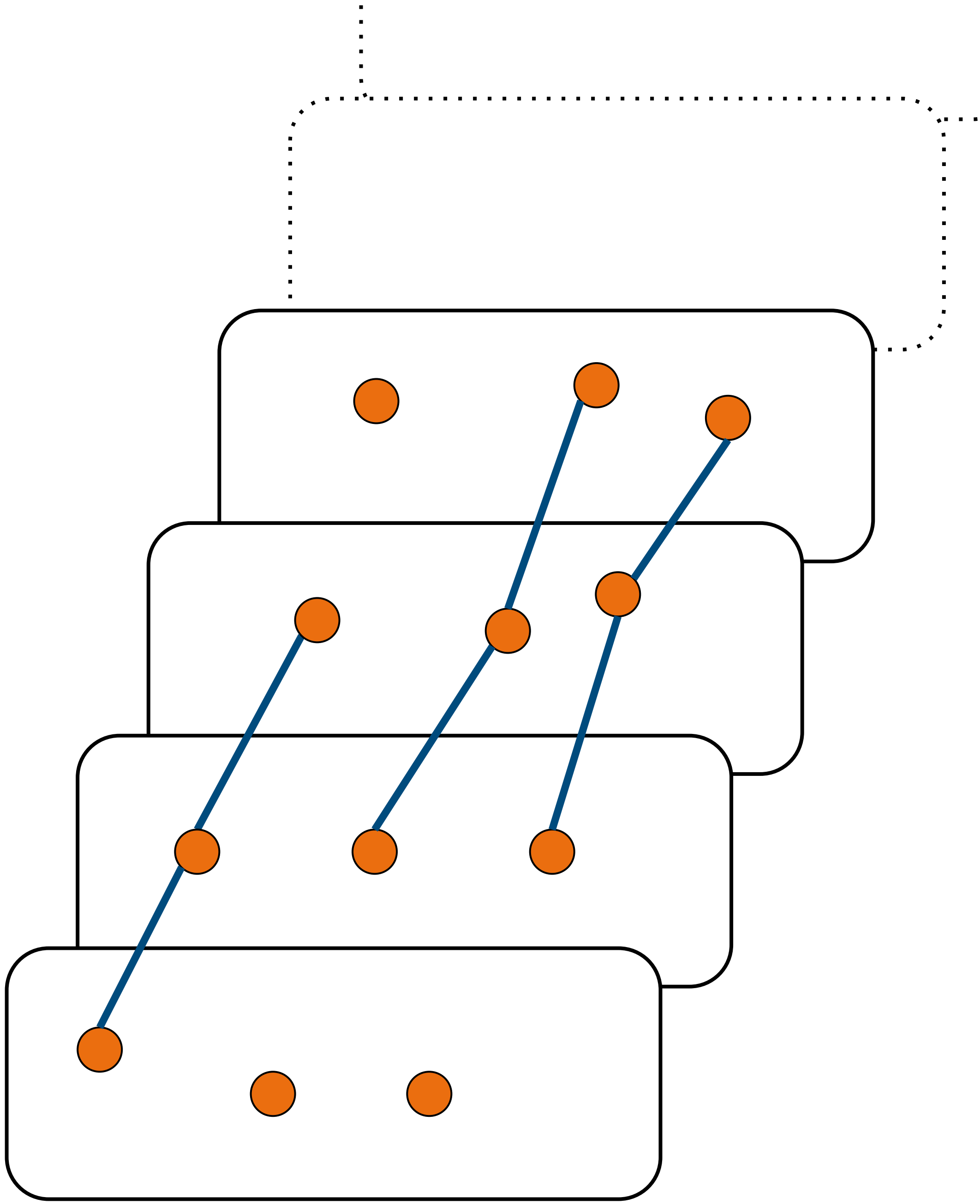
Pattern Recognition

- **Doublets** are created by combining hits from successive layers



Pattern Recognition

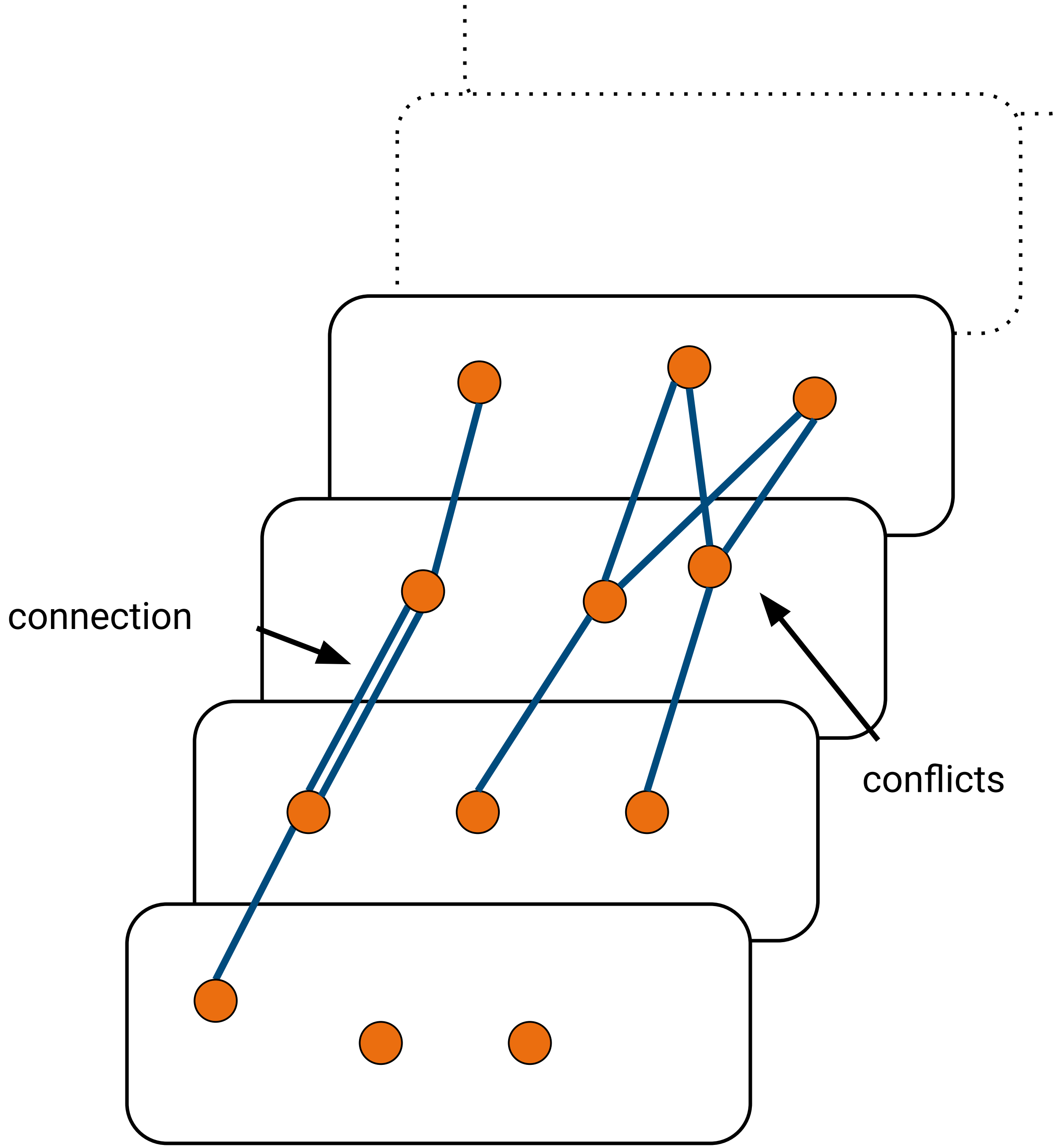
- **Doublets** are created by combining hits from successive layers
- **Triplets** are created by combining doublets



Pattern Recognition

- **Doublets** are created by combining hits from successive layers
- **Triplets** are created by combining doublets
- **Relations** of triplets as key feature

Goal: Identify triplets stemming from a single particle ("matched") and combine them to tracks



QUBO

Quadratic Unconstrained Binary Optimisation

- T_i are binary representations of triplets

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j + \sum_{i=1}^{N} a_i T_i \qquad \text{(QUBO)}$$

QUBO

Quadratic Unconstrained Binary Optimisation

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- T_i are binary representations of triplets

- Description of a system using two types of parameters:
  - **Conflicts**: $b_{ij} > 0$
  - **Connections**: $b_{ij} < 0$
  - **Individual term**: a_i

QUBO

Quadratic Unconstrained Binary Optimisation

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- T_i are binary representations of triplets
- Description of a system using two types of parameters:
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  - **Individual term**: $\mathbf{a_i}$

New: Combining *spatial and temporal* information in a_i and b_{ij} → *"4D parameters"*

Goal: Minimising the QUBO cost function (Hamiltonian) of the system

QUBO

Quadratic Unconstrained Binary Optimisation

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j + \sum_{i=1}^{N} a_i T_i \qquad \text{(QUBO)}$$

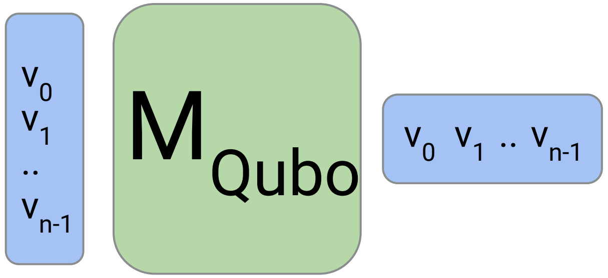
Minimise QUBO cost function

- Ground state → best set of triplets to keep
- $\mathbf{v}_{\mathbf{binary}}$: $[T_1, T_2, T_3, \ldots, T_N] \rightarrow [0, 1, 1, \ldots, 0]$
- Large QUBOs need to be split into sub-QUBOs

  → sequentially minimised and combined to global solution

Computation:

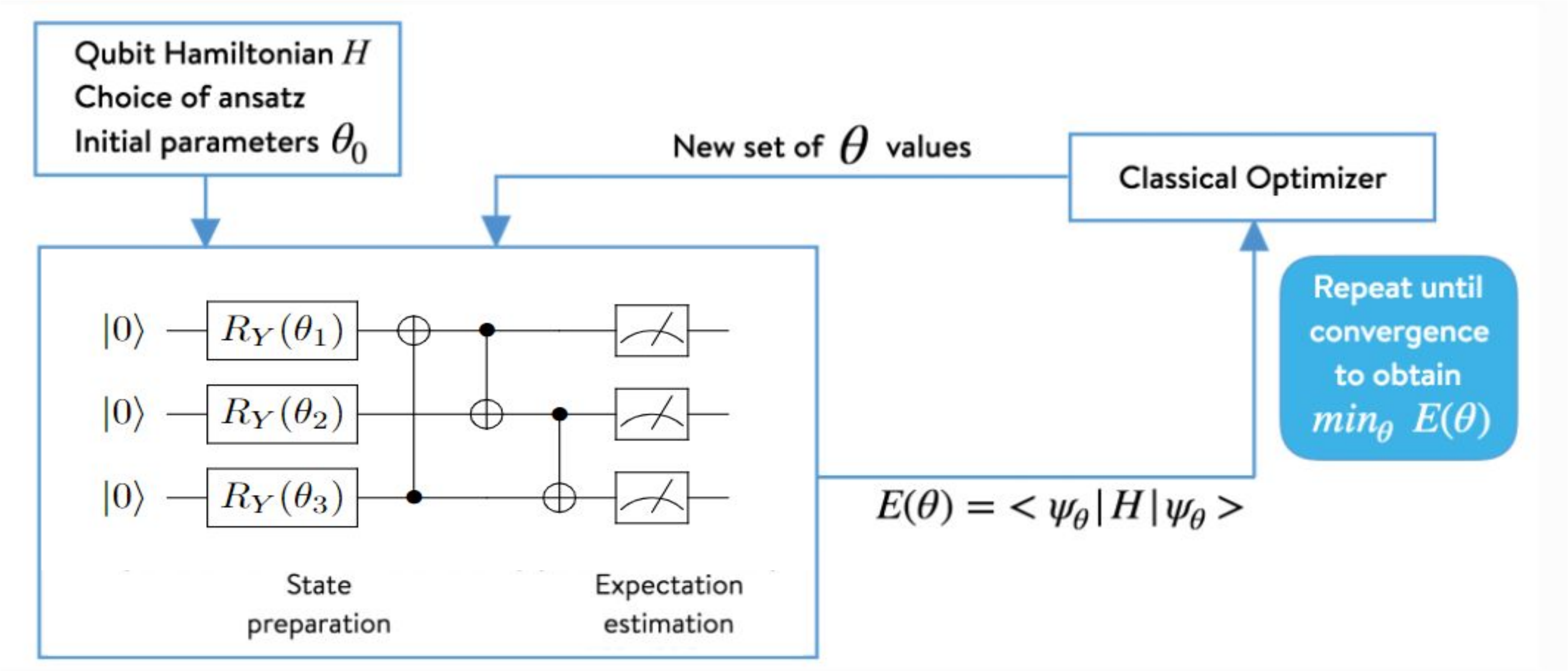
- Matrix diagonalisation (analytic solution)
- Hybrid quantum-classical algorithm (VQE)



$$\min(\hat{H}) = \min_{v_i \in v}(v^T M v)$$

VQE

Variational Quantum Eigensolver



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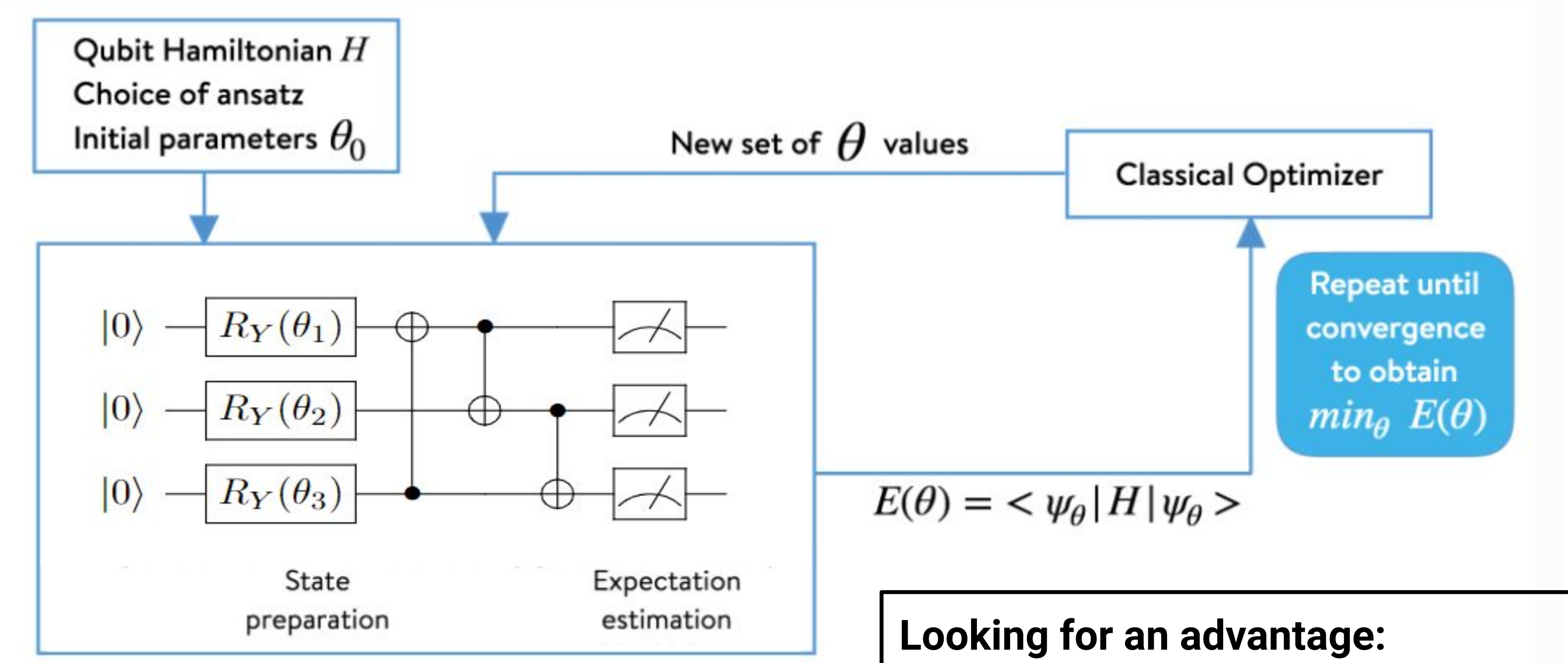
VQE

Variational Quantum Eigensolver



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$$H = \sum_{a=1}^{\mathcal{O}(Poly(N))} w_a P_a$$

$$P_a \in \{I, X, Y, Z\}^{\otimes N}$$

Looking for an advantage:

dim(H) = 2^n x 2^n, then <ψ|H|ψ> needs:

- $O(2^{2n})$ operations classically
- Possibly only O(Poly(n)) operations with a quantum computer

Track reconstruction study

Physics case study

- Search for long-lived charged-particle tracks, including slow and heavy particles (BSM candidate?)

Setup:

- Probe signal particle with m = 1.1TeV, $\beta \in [0.2, 0.7]$
- Preselection applied to doublets and triplets with > 99% selection efficiency
- Track selection:
  - At least **6** hits,
  - **Matched** if majority of hits from signal

Results

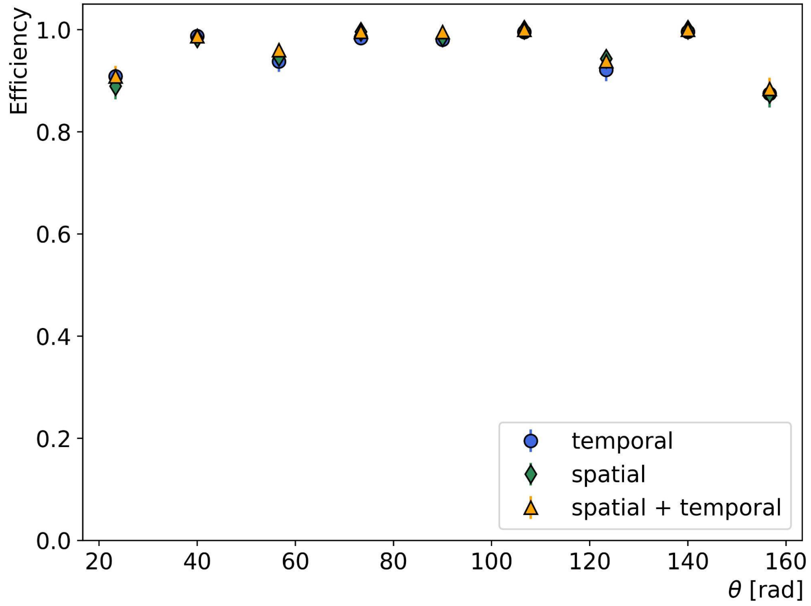
Analytical solution

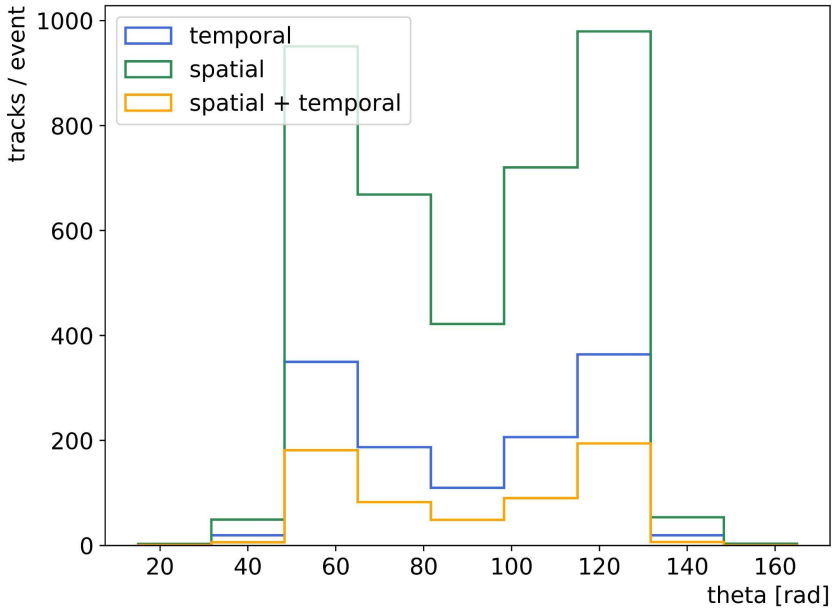
Integrated efficiency (%)

- temporal: $95.44^{+0.56}_{-0.52}$
- spatial: $95.84^{+0.44}_{-0.50}$
- combined: $96.39^{+0.41}_{-0.47}$

Fake tracks / background

- Fake tracks per event with one signal track
- 4D parameters reduce fake tracks greatly
- Future: Apply χ^2 cut to reduce fake rate further





Summary and outlook

- Presented an application of a 4D quantum algorithm which can be applied in principle to any future collider facility (not restricted to a muon collider!)
- 4D parameters help to reduce fake tracks, influence on reconstruction efficiency will be further investigated
- Optimisation of 4D parameter composition necessary

Thank You!

VQE result example

- Calculations on a quantum device are noisy:
  →Error mitigation and error correction
- 10 shots (number of circuit evaluations) sufficient for 99% success rate