Bayesian Approach in TA5-WP1

Common TA3-WP1 — TA5-WP1 meeting

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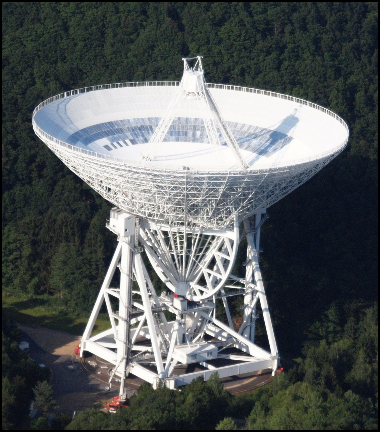
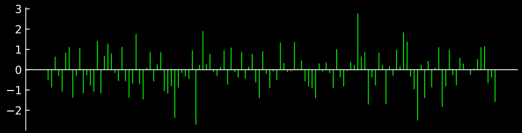
Single Radio Telescope

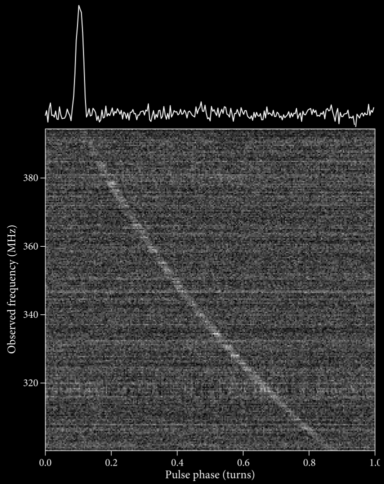


Radio telescope Effelsberg



- Stream is a time series
- Relatively wide band
- Time trace can contain narrow and wide band signals

Single Radio Telescope



- dispersed signals
- non-trivial structure of pulses

Problems for the Individual Antenna Level

- Detection of transient signals and measuring their parameters, e.g. pulsars and FRB
- Rejection of the known telecommunication signals, e.g. satellites
- Identification of radio-interference signals and rejecting them

→ There is a need to have a general view at signal detection and a need to identify its fundamental properties and limitations.

Bayes' Approach to the Signal Detection — I

Data stream: A_1 — signal is **in** the data, A_0 — signal is **not in** the data.

Decision: A_1^* — signal is **in** the data, A_0^* — signal is **not in** the data.

Possible situations:

- $A_0^* A_0$ — correct non-detection, $\hat{F} = P(A_0^* | A_0)$,
- $A_1^* A_0$ — "false alarm" or "false positive" detection, $F = P(A_1^* | A_0)$,
- $A_0^* A_1$ — missing the signal, $\hat{D} = P(A_0^* | A_1)$,
- $A_1^* A_1$ — correct detection of the signal, $D = P(A_1^* | A_1)$.

Bayes' Approach to the Signal Detection — II

Stream: A_1 — signal, A_0 — no signal
Decision: A_1^* — signal, A_0^* — no signal

$$\hat{F} = P(A_0^*|A_0) \quad F = P(A_1^*|A_0)$$
$$\hat{D} = P(A_0^*|A_1) \quad D = P(A_1^*|A_1)$$
$$\hat{F} + F = 1, \quad \hat{D} + D = 1$$

$$P(A_0^*, A_0) + P(A_1^*, A_0) + P(A_0^*, A_1) + P(A_1^*, A_1) = 1$$

Mean risk: $\bar{r} = \sum_i r_i P_i =$

$$= r_1 P(A_0^*, A_0) + r_2 P(A_1^*, A_0) + r_3 P(A_0^*, A_1) + r_4 P(A_1^*, A_1) = r_F P(A_1^*, A_0) + r_{\hat{D}} P(A_0^*, A_1)$$

$$P(A_1^*, A_0) = P(A_0) P(A_1^*|A_0) = P(A_0) F, \quad P(A_0^*, A_1) = P(A_1) P(A_0^*|A_1) = P(A_1) \hat{D}$$

$$\bar{r} = r_F F P(A_0) + r_{\hat{D}} \hat{D} P(A_1)$$

Bayes' Approach to the Signal Detection — III

Mean risk: $\bar{r} = r_F F P(A_0) + r_{\hat{D}} \hat{D} P(A_1)$

Bayes' criterion: $\operatorname{argmin} \bar{r}$

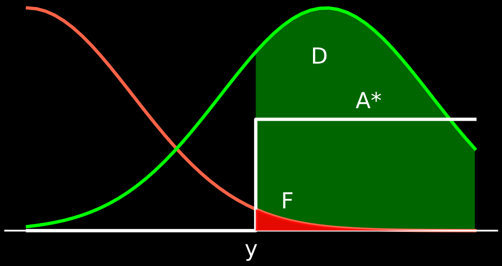
Criterion of ideal observer: $\bar{r} = FP(A_0) + \hat{D}P(A_1) \qquad (r_F = 1, r_{\hat{D}} = 1)$

Neyman–Pearson criterion: $\operatorname{argmax}(D)$ while F is fixed (fixed "false alarm" rate)

$$\bar{r} = r_{\hat{D}} P(A_1) - [D - l_0 F] r_{\hat{D}} P(A_1), \qquad l_0 = r_F P(A_0) / r_{\hat{D}} P(A_1)$$

Weighting criterion: $\operatorname{argmax}\ (D - l_0 F)$

Statistical Signal Detection



Example 1. $y = Ax + n$, $(A = 0, 1)$, $p(y|A_0) = p_n(y)$, $p(y|A_1) = p_{sn}(y)$.

$$D = \int_{-\infty}^{\infty} A^*(y) p_{sn}(y)\, dy \qquad F = \int_{-\infty}^{\infty} A^*(y) p_n(y)\, dy$$

Statistical Signal Detection

$$y = Ax + n,\ (A = 0, 1), \qquad p(y|A_0) = p_{\mathrm{n}}(y),\ p(y|A_1) = p_{\mathrm{sn}}(y).$$

$$D = \int_{-\infty}^{\infty} A^*(y) p_{\mathrm{sn}}(y)\,\mathrm{d}y \qquad F = \int_{-\infty}^{\infty} A^*(y) p_{\mathrm{n}}(y)\,\mathrm{d}y$$

$$D - l_0 F = \int_{-\infty}^{\infty} A^*(y) p_{\mathrm{n}}(y)[l(y) - l_0]\,\mathrm{d}y \qquad l(y) = \frac{p_{\mathrm{sn}}(y)}{p_{\mathrm{n}}(y)} \text{ — likelihood ratio}$$

If the “noise” is Gaussian:

$$l(y) = \mathbf{exp}\left(-\frac{x^2}{2n_0^2}\right)\mathbf{exp}\left(\frac{xy}{n_0^2}\right) \quad \text{or} \quad l(y) = \mathbf{exp}\left(-\frac{1}{2n_0^2}\sum_i x_i^2 \Delta t\right)\mathbf{exp}\left(\frac{1}{n_0^2}\sum_i x_i y_i \Delta t\right)$$

$\sum_i xy\Delta t$ — correlation sum (or correlation integral in continuous case)

Correlation sum/integral is a linear filter (optimal linear filter $= \mathbf{max}\, l(y) = \mathbf{min}\, \bar{r}$).

Summary and Plans

What is identified:

- the internal logical connection between the Bayes' approach to signal detection, likelihood ratio method, correlation-type methods, etc

Nearby plans:

- study the time-frequency formulation of the optimal filtration,
- theory of information as an additional component on top of the statistical signal processing,