

Infrared subtraction at NNLO and beyond

Matteo Marcoli Loop Summit 2 - Cadenabbia 21/07/2025



Newton
International
Fellowship

Work in collaboration with:
Elliot Fox, Nigel Glover
Xuan Chen, Petr Jakubcik, Giovanni Stagnitto

Overview



Introduction: s.o.t.a. of fixed-order calculations in QCD

Generalized final-state antenna functions at NNLO

First steps and results at N³LO with antenna subtraction

Summary and outlook

INTRODUCTION

Antenna subtraction, NNLOJET

qT slicing, MATRIX

$$pp \rightarrow \gamma \gamma j$$

pp→γγj

PHYSICAL REVIEW LETTERS 134, 179001 (2025)

Precise Predictions for Event Shapes in Diphoton Production at the LHC

Federico Baccaro^{1,*}, Xian Chen², Wei-Jie Feng^{3,2}, Thomas Gehrmann^{3,4}, Alexander H

and Matteo Marcolli⁵

Automation of antenna subtraction in colour space:

U. N. Glover,^a A. Huss^a and M. Marcolli^a

^a Institute of Technology,

ology.

**Automation of antenna
gluonic processes**

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We present the gluonic antenna subtraction method, a
new for next-to-next-to-leading order (NNLO)
to achieve a general and process-
independent. We rely on the pro-
cess in a colorless

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pp→jjj
LC, gluons

pp→Zbb
massification (LC)

Next-to-next-to-leading order event generation for
Z-boson production in association with a bottom-quark pair

Z-boson production¹, ²Ulysses Stöckli³, ⁴Matthias
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We present the production of a Z boson decaying to leptons in association with a bottom-quark pair in hadronic collisions. For the first time, we use the full set of next-to-next-to-leading order (NNLO) QCD, and we combine this with the all-order next-to-next-to-leading order (NNLO) production simulation (NNLOPS). Our method allows us to obtain a consistent NNLO prediction for the cross section of the process $pp \rightarrow Z + b\bar{b}$ in the $b\bar{b}$ channel. This calculation is a first step towards the simulation of processes involving color singlet final states. We predict the final state leptons to be produced in the $b\bar{b}$ channel. NNLO QCD corrections lead to a small increase in the cross section, and we find that the sizeable NNLO QCD corrections lead to a small increase in the cross section. We present our predictions in the form of a plot production, achieving an overall

pp→Hbb
massification

Matteo Marcoli

Simone Devoto^a, Massimiliano Grazzini^b, Stefan Kallweit^b, Javier Mazzitelli^c
and Chiara Savoini^d

PHYSICAL REVIEW LETTERS **131**, 231901 (2023)

Precise Predictions for the Associated Production of a W Boson with a Top-Antitop Quark Pair at the LHC

Luca Buonocore¹, Simone Devoto², Massimiliano Grazzini¹, Stefan Kallweit³,
Javier Mazzitelli⁴, Luca Rottoli¹ and Chiara Savoini¹

PHYSICAL REVIEW D **107**, 074032 (2023)Associated production of a W boson and massive bottom quarks
at next-to-next-to-leading order in QCD

Luca Buonocore¹, Simone Devoto², Stefan Kallweit³, Javier Mazzitelli⁴, Luca Rottoli¹ and Chiara Savoini¹

PHYSICAL REVIEW LETTERS 130, 111902 (2023)

Higgs Boson Production in Association with a Top-Antitop Quark Pair
in Next-to-Next-to-Leading Order QCD

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Triphoton production at hadron colliders in NNLO QCD

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ARTICLE INFO ABSTRACT

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We present next-to-next-to-leading-order (NNLO) QCD corrections to the production of three isolated photons in hadronic collisions at the fully differential level. We employ g_k subtraction within $\overline{\text{MS}}$ and an efficient implementation of analytic two-loop amplitudes in the leading colour approximation to achieve the first g_k -by-the- g_k -calculation for this process at NNLO accuracy. Numerical results are presented for proton–proton collisions at energies ranging from 13 to 130 TeV. We find good agreement with the 8 TeV results of Ref. [1] and confirm that NNLO corrections are indispensable to describe ATLAS 8 TeV data. In addition, we demonstrate the significance of NNLO corrections for future precision studies of triphoton production at higher collision energies.

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Precision studies have become of major importance in the rich physics programme at the Large Hadron Collider (LHC). Many LHC reactions, in particular $2 \rightarrow 1$ and $2 \rightarrow 2$ processes, are not only measured, but also predicted at a remarkable accuracy by now. Prime examples are colour singlet processes, such as vector-boson pair production, cf. for instance the recent $Z\nu$ [2] and ZZ [3] mea-

tor bosons gives direct access to anomalous quartic gauge couplings, e.g. the $Z \rightarrow \gamma\gamma\gamma$ decay has been constrained in Ref. [38]. Furthermore, the triphoton final state is important to constrain anomalous Higgs couplings in rare Higgs boson decays [39–41] or in the rare Higgs boson production process in association with a photon [42] with the Higgs boson decaying into a pair of pho-

Isolated photon production in association with a jet pair through next-to-next-to-leading order in QCD

$pp \rightarrow \gamma jj$

pp→jjj

pp→Wbb
LC, mb=0

pp→jjj

$pp \rightarrow \gamma \gamma j$

$pp \rightarrow \gamma\gamma\gamma$

4CD
"Gyu Haranto," Ryan Moodie,"
simone Zola"
Tosino, Gold-Rege Center, Università di Torino
NNLO QCD corrections to event shapes at the LHC
"uel Alvarez," Josu Cantero," Michal Czakon"
ene Poncellet
at for Theoretische Teilchen-
Universität Gießen

Next-to-next-to-leading order QCD corrections to $Wb\bar{b}$ production at the LHC

Next-to-leading order QCD corrections to $Wb\bar{b}$ production
at the LHC
Kenshiro Bayu Hartanto,^{1,*} Rene Poncelet^{2,3}, Andrei Popescu^{2,3} and Simone Zola^{2,3}
PHYSICAL REVIEW LETTERS 127, 152001 (2021)
Next-to-Next-to-Leading-Order

NNLO QCD corrections to diphoton production with
 an additional jet at the LHC

WLO QCD corrections to three
the LHC

Chawdhry,^a Michał Czako,^b Alexander Mitov^c and Rene Poncelet^d

Chawdhry,^a Michał Czako,^b Alexander Mitov^c and Rene Poncelet^d

State of the art of NNLO calculations: formalisms and tools

Antenna subtraction

Thomas' talk

Nested soft-collinear subtraction

Local analytic sector subtraction

Sandro's talk

Towards a general subtraction formula for NNLO QCD corrections to processes at hadron colliders: final states with quarks and gluons

Federica Devoto,^a Kirill Melnikov,^b Raoul Röntsch,^c Chiara Signorile-Signorile,^d Davide Maria Tagliabue,^{b,e} Matteo Tresoldi^f

A fresh look at the nested soft-collinear subtraction scheme: NNLO QCD corrections to N -gluon final states in $q\bar{q}$ annihilation

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CoLoRful subtraction

NNLOCAL: completely local subtractions for color-singlet production in hadron collisions

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CoLoRful for hadron collisions: Integrating the counterterms^a

S. Van Thurenhout^{a,f}, V. Del Duca^{b,c,d}, C. Duhr^e, L. Fekésházy^{a,f,g}, F. Guadagni^a, P. Mukherjee^a, G. Somogyi^a and F. Tramontano^{b,h,i}

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Advances in Local Analytic Sector Subtraction: massive NLO and elements of NNLO automation

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NNLO subtraction for any massless final state: a complete analytic expression

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ABSTRACT: We use the Local Analytic Sector Subtraction scheme to construct a completely analytic set of expressions implementing a fully local infrared subtraction at NNLO for generic coloured massless final states. The cancellation of all explicit infrared poles appearing in the double-virtual contribution is explicitly verified, and all finite contributions arising from integrated local counterterms are analytically evaluated in terms of ordinary polylogarithms up to weight three. The resulting subtraction formulae can readily be implemented in any numerical framework containing the relevant matrix elements up to NNLO.

NNLO grids with STRIPPER

HighTEA: high energy theory event analyser

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State of the art of NNLO calculations

So, is NNLO **solved**? In principle yes ...

STRIPPER: given the relevant amplitudes and enough computational resources, the NNLO calculation is streamlined

But:

- prohibitive computational cost

refine IR schemes, efficiency

improve resources
(ML, GPUs, grids, ...)

- missing cross-validation

different techniques

NNLO tools

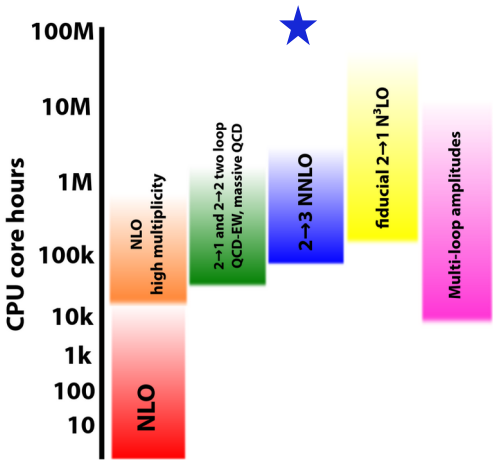
cooperation

2-loop automation

- still a long way to NNLO event generation

PS matching

pp → jjj event shapes with STRIPPER



[Febres Cordero, von Manteuffel, Neumann '22]

State of the art of N³LO calculations

Inclusive:

- $gg \rightarrow H$ [Anastasiou, Durh, Dulat, Herzog, Mistleberger '15]
[Mistleberger '18]
- $gg \rightarrow HH$ [Chen, Li, Shao, Wang '19, '20]
- VBF H [Dreyer, Karlberg '16]
- VBF HH [Dreyer, Karlberg '18]
- $pp \rightarrow \gamma/Z/W$ [Durh, Dulat, Mistleberger '20]
[Durh, Mistleberger '21]
- $bb \rightarrow H$ [Baglio, Durh, Mistleberger, Szafron '22]
- $pp \rightarrow Z/W H$

Fully differential:

- $gg \rightarrow H$ [Cieri, Chen, Ghermann, Glover, Huss '18] *
[Billis, Dehnadi, Ebert, Michel, Tackmann '21] *
[Chen, Gehrmann, Glover, Huss, Mistleberger, Pelloni '21] †
- $pp \rightarrow \gamma/Z/W$ [Chen, Gehrmann, Glover, Huss, Yang, Zhu '21, '22] *
[Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli '22] *
[Campbell, Neumann '22, '23] *
- DIS [Currie, Gehrmann, Glover, Huss, Niehues, Vogt '18] †
- $H \rightarrow bb$ [Mondini, Schiavi, Williams '19] †
- $e^+e^- \rightarrow jj$ [Chen, Jakubcik, MM, Stagnitto '25] ‡

Several phenomenologically relevant results despite the extreme complexity.

Available techniques are applicable to limited cases.

New approaches must be developed for more complicated processes.

* **qT slicing**

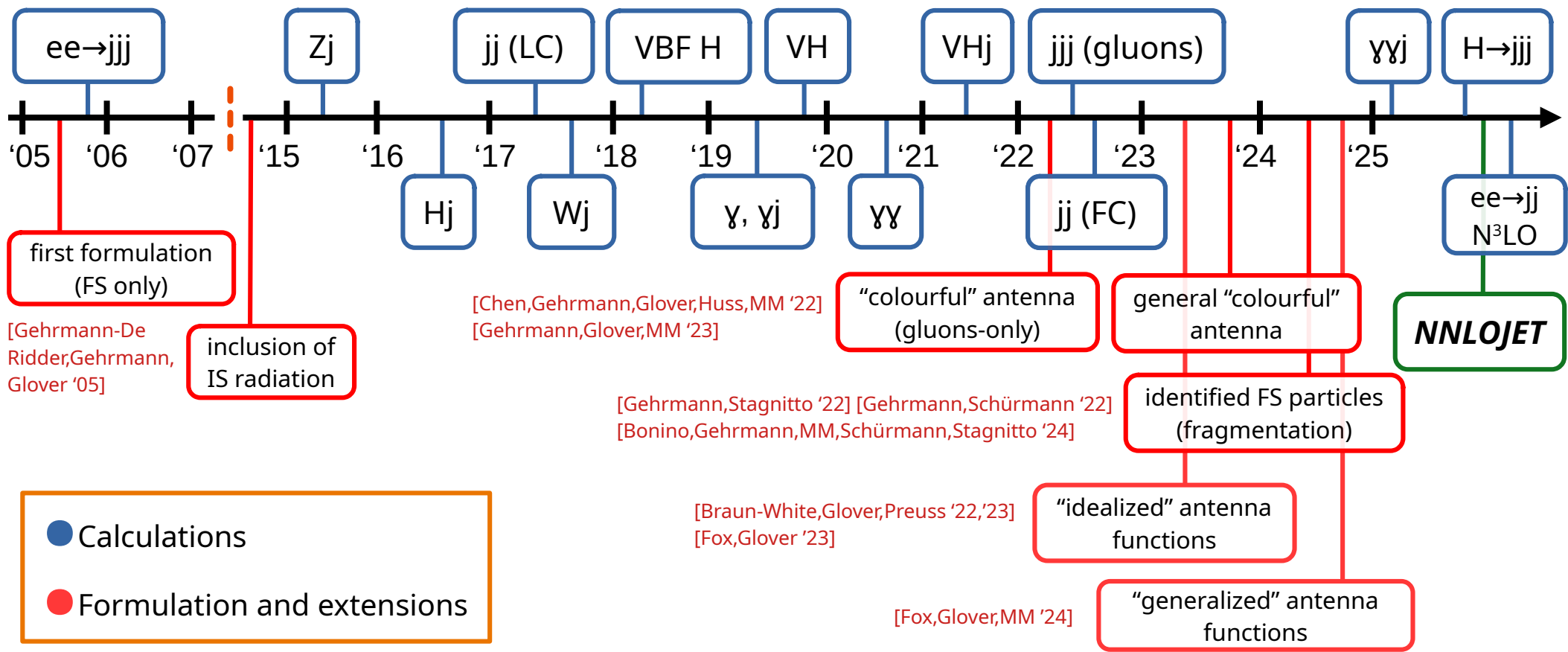
† **Projection-to-Born**

‡ **Antenna Subtraction**

Antenna subtraction

[NNLOJET collaboration: Huss *et al.* '25]

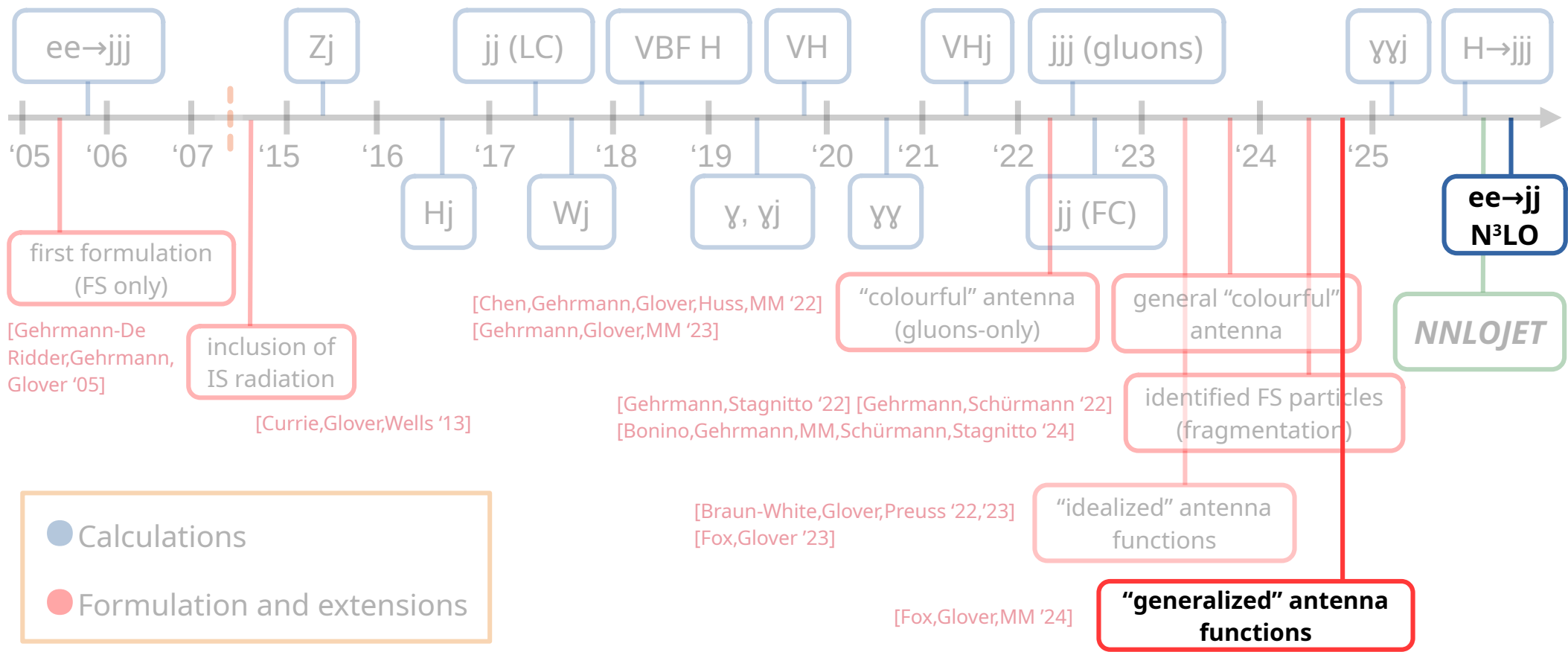
Successfully applied at NNLO to a variety of processes within the **NNLOJET** Monte Carlo framework



Antenna subtraction

[NNLOJET collaboration: Huss *et al.* '25]

Successfully applied at NNLO to a variety of processes within the **NNLOJET** Monte Carlo framework



GENERALIZED FINAL-STATE ANTENNA FUNCTIONS AT NNLO

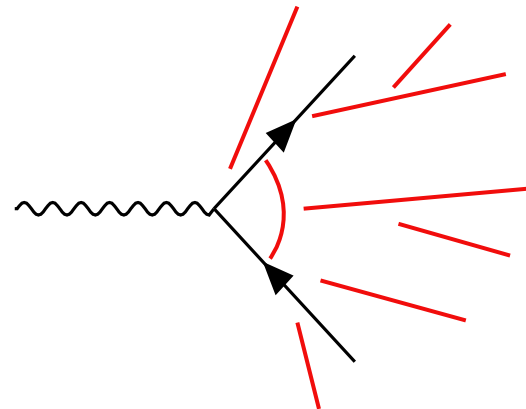
work with Elliot Fox and Nigel Glover, JHEP 12 (2024) 225

Standard antenna functions (FSR)

Describe unresolved emissions between **two hard radiators**

Traditionally extracted from colour-ordered matrix elements for colour-singlet decay

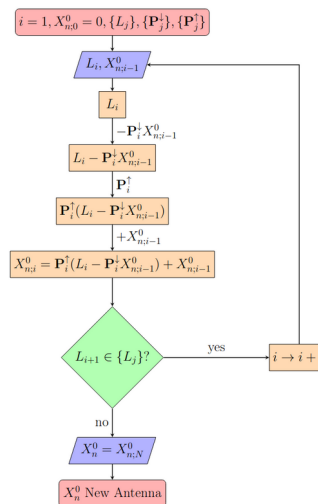
[Gehrmann-De Ridder,
Gehrmann,Glover '04,'05]



Recently: **designer antenna algorithm** to build antenna functions directly from unresolved factors

- Simpler expressions
- Better isolation of IR limits
- Removal of some unphysical singularities

[Braun-White,Nigel,Preuss '22,'23]

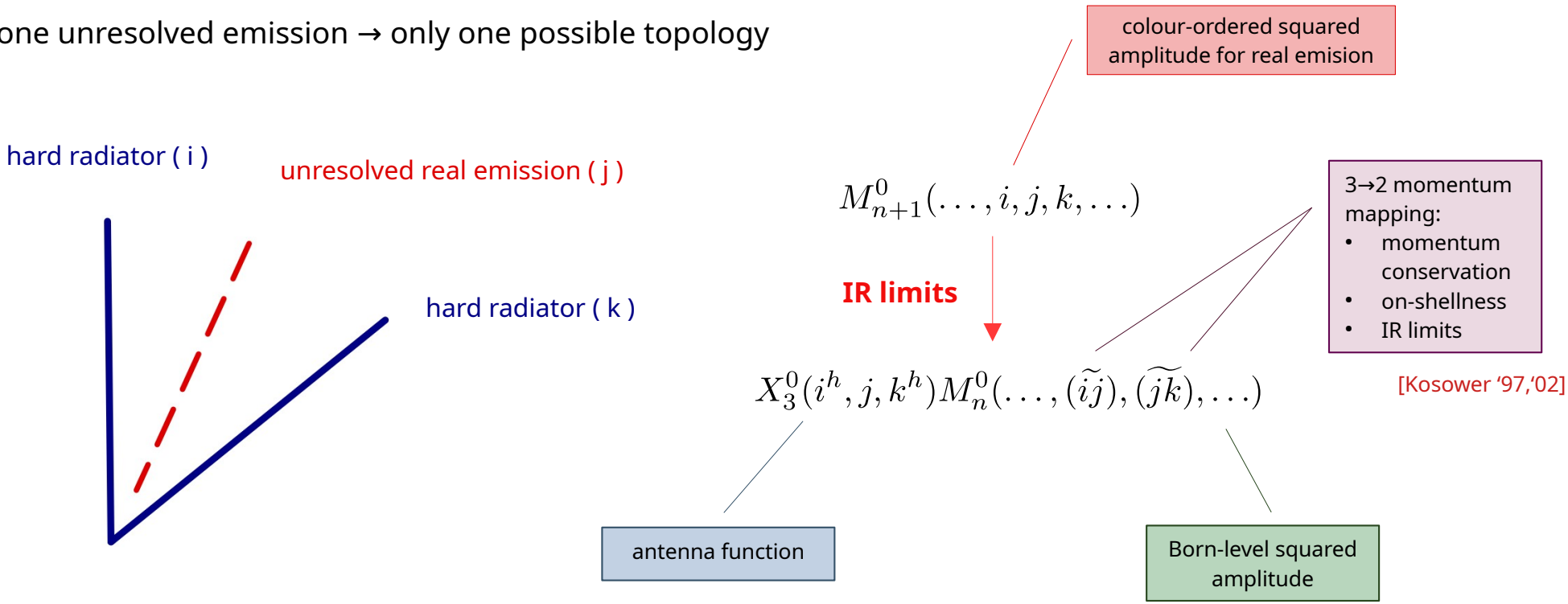


Generalized antenna functions [Fox,Glover,MM '24]

Two-hard-radiator antenna functions work very well for some configurations, **less well** for others

What **emission topologies** can they easily describe?

NLO: one unresolved emission → only one possible topology

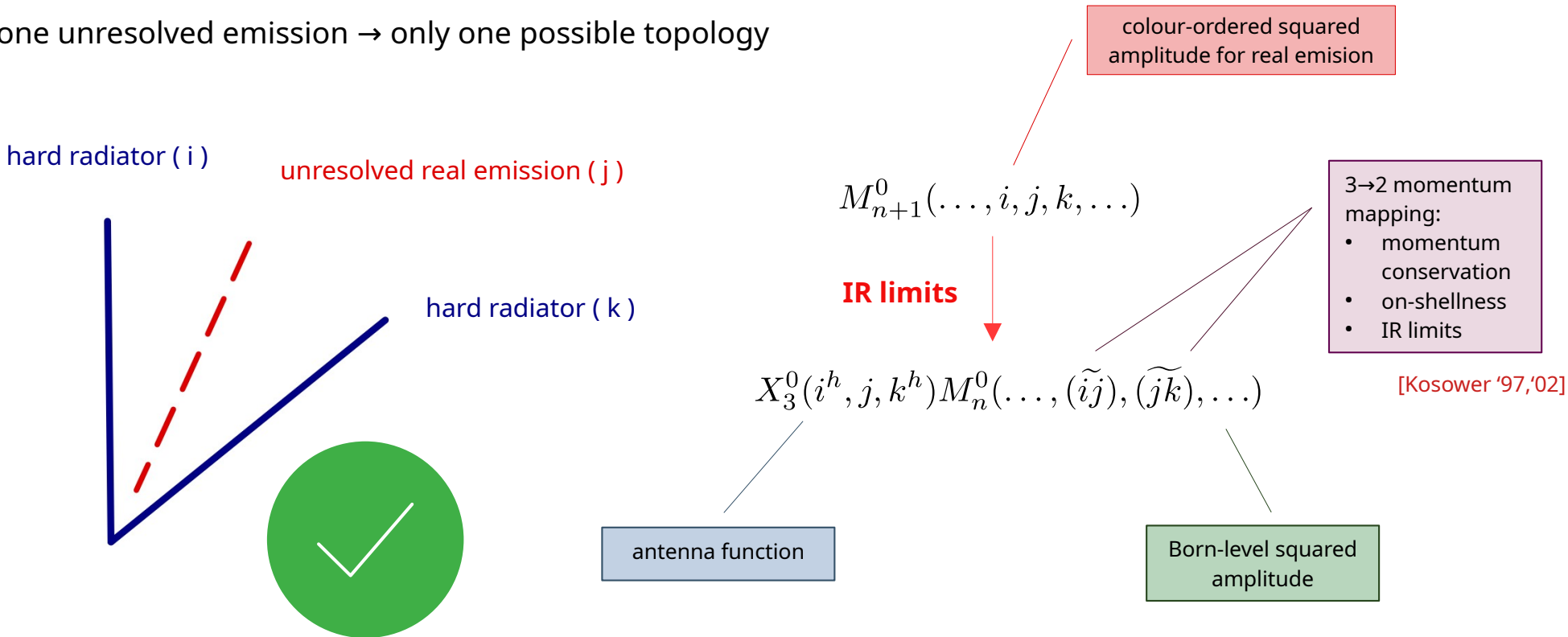


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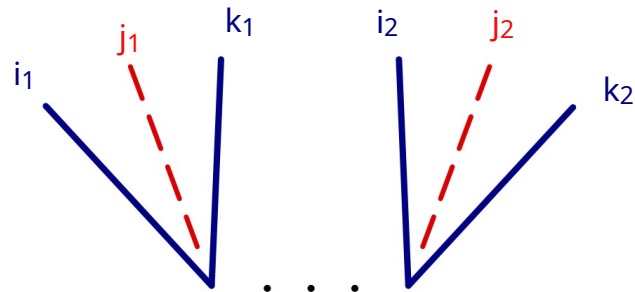
NLO: one unresolved emission → only one possible topology



Generalized antenna functions [Fox,Glover,MM '24]

NNLO: two unresolved emissions \rightarrow multiple topologies

colour-unconnected emissions: no shared hard radiator



$$M_{n+2}^0(\dots, i_1, j_1, k_1, \dots, i_2, j_2, k_2, \dots)$$

fully **iterated** structure

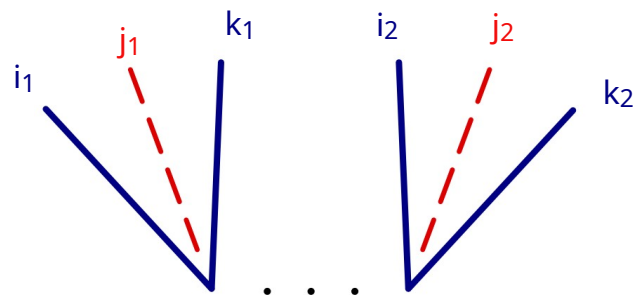


$$X_3^0(i_1^h, j_1, k_1^h) X_3^0(i_2^h, j_2, k_2^h) M_n^0(\dots, (\widetilde{i_1 j_1}), (\widetilde{j_1 k_1}), \dots, (\widetilde{i_2 j_2}), (\widetilde{j_2 k_2}), \dots)$$

Generalized antenna functions [Fox,Glover,MM '24]

NNLO: two unresolved emissions → multiple topologies

colour-unconnected emissions: no shared hard radiator



$$M_{n+2}^0(\dots, i_1, j_1, k_1, \dots, i_2, j_2, k_2, \dots)$$



$$X_3^0(i_1^h, j_1, k_1^h) X_3^0(i_2^h, j_2, k_2^h) M_n^0(\dots, (\widetilde{i_1 j_1}), (\widetilde{j_1 k_1}), \dots, (\widetilde{i_2 j_2}), (\widetilde{j_2 k_2}), \dots)$$

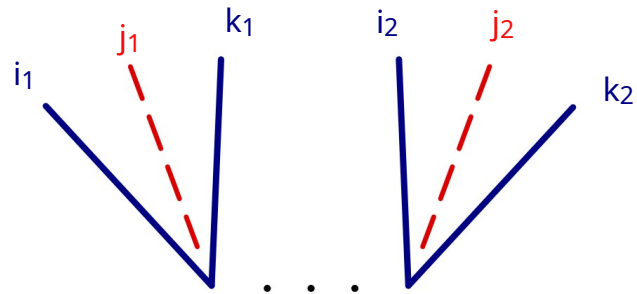


fully **iterated** structure

Generalized antenna functions [Fox,Glover,MM '24]

NNLO: two unresolved emissions \rightarrow multiple topologies

colour-unconnected emissions: no shared hard radiator



$$M_{n+2}^0(\dots, i_1, j_1, k_1, \dots, i_2, j_2, k_2, \dots)$$

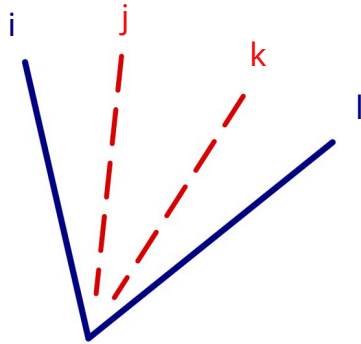


$$X_3^0(i_1^h, j_1, k_1^h) X_3^0(i_2^h, j_2, k_2^h) M_n^0(\dots, (\widetilde{i_1 j_1}), (\widetilde{j_1 k_1}), \dots, (\widetilde{i_2 j_2}), (\widetilde{j_2 k_2}), \dots)$$



fully **iterated** structure

colour-connected emissions: both hard radiators shared



$$M_{n+2}^0(\dots, i, j, k, l, \dots)$$



$$X_4^0(i^h, j, k, l^h) M_n^0(\dots, (\widetilde{ijk}), (\widetilde{jkl}), \dots)$$

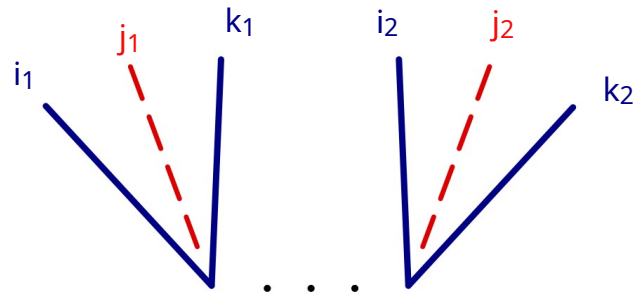


4 \rightarrow 2 momentum mapping

Generalized antenna functions [Fox,Glover,MM '24]

NNLO: two unresolved emissions \rightarrow multiple topologies

colour-unconnected emissions: no shared hard radiator



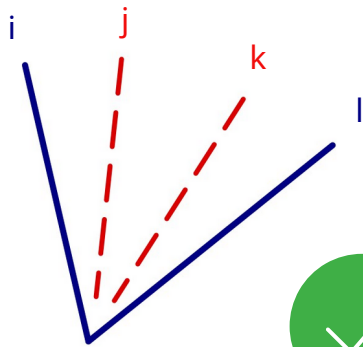
$$M_{n+2}^0(\dots, i_1, j_1, k_1, \dots, i_2, j_2, k_2, \dots)$$



fully **iterated** structure

$$X_3^0(i_1^h, j_1, k_1^h) X_3^0(i_2^h, j_2, k_2^h) M_n^0(\dots, (\widetilde{i_1 j_1}), (\widetilde{j_1 k_1}), \dots, (\widetilde{i_2 j_2}), (\widetilde{j_2 k_2}), \dots)$$

colour-connected emissions: both hard radiators shared



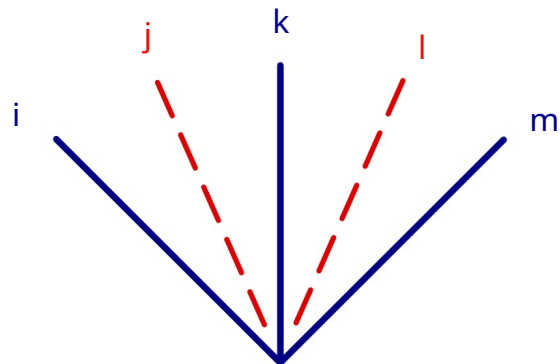
$$M_{n+2}^0(\dots, i, j, k, l, \dots)$$

4 \rightarrow 2 momentum mapping

$$X_4^0(i^h, j, k, l^h) M_n^0(\dots, (\widetilde{ijk}), (\widetilde{jkl}), \dots)$$

Generalized antenna functions [Fox,Glover,MM '24]

There is more ... **almost colour-connected** emissions: only one shared hard radiator



NOT fully iterated: the two emissions “feel” each other through the recoil on the shared radiator

traditional antenna functions can be used,
but a **very complicated** sequence of iterated
structures is needed, plus **Large-Angle-Soft-Terms**

[Gehrmann De-Ridder,Gehrmann,Glover,Heinrich '07]

[Weinzierl '08] [Currie,Glover,Wells '13]

most complicated and inefficient
sector of antenna subtraction

[illegible]

from $e^+e^- \rightarrow jjj$ @NNLO

Ideally we want:

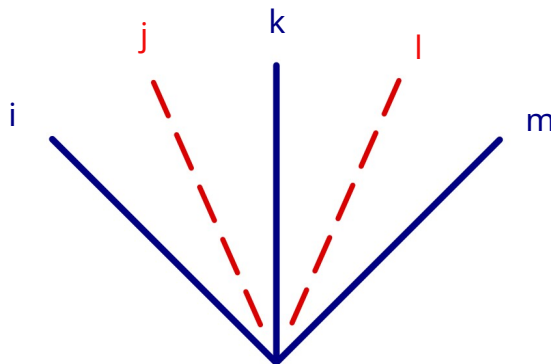
$$M_{n+2}^0(\dots, i, j, k, l, m \dots)$$

5→3 momentum mapping

$$X_{5,3}^0(i^h, j, k^h, l, m^h) M_n^0(\dots, (\widetilde{ijk}), (\widetilde{ijklm}), (\widetilde{k\ell m}), \dots)$$

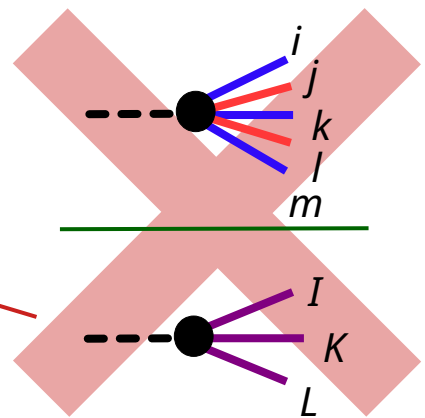
generalized three-hard-radiator antenna function

Generalized antenna functions [Fox,Glover,MM '24]



Not possible with matrix element-based antenna functions

non-trivial function of the three-particle phase space



[Braun-White,Nigel,Preuss '22,'23]

With the **designer antenna algorithm**, it is possible to construct antenna functions with **more than two hard radiators: generalized antenna functions.**

Analytical integration made particularly simple thanks to a convenient choice of **5→3 momentum mapping**.

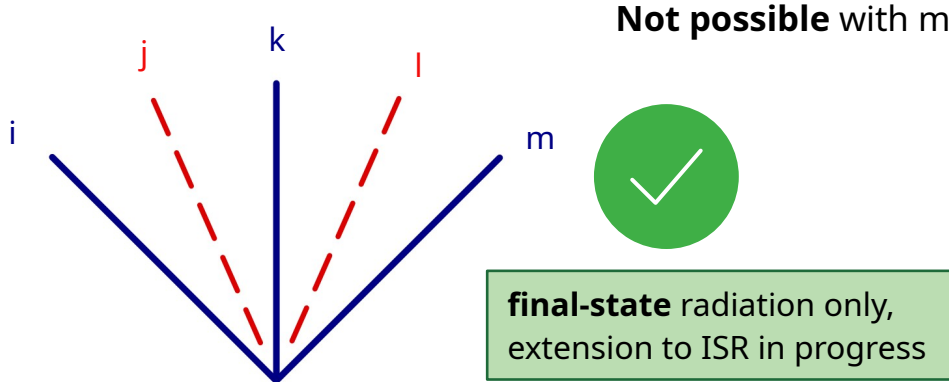
$$\text{map}_{5 \rightarrow 3} : \begin{aligned} p_I &= p_i + p_j - \frac{s_{ij}}{s_{ik} + s_{jk}} p_k \\ p_K &= \left(1 + \frac{s_{ij}}{s_{ik} + s_{jk}} + \frac{s_{lm}}{s_{lk} + s_{mk}} \right) p_k \\ p_M &= p_l + p_m - \frac{s_{lm}}{s_{lk} + s_{mk}} p_k \end{aligned}$$

iterated dipole mapping

Type	Name and parton content	Reconstructed hard partons	$X_3^0 \otimes X_3^0$
quark-antiquark-gluon	$A_{5,3}^0(i_q^h, j_{\bar{q}}^h, k_g^h, l_g, m_q^h)$	$q\bar{q}\bar{q}$	$D \otimes \bar{D}$
	$B_{5,3}^0(i_q^h, j_{\bar{q}}^h, k_Q^h, l_Q, m_q^h)$	$q\bar{q}\bar{q}$	$A \otimes \bar{E}$
	$\tilde{A}_{5,3}^0(i_q^h, j_{\bar{q}}^h, k_{\gamma}^h, l_g, m_g^h)$	$\bar{q}qg$	$A \otimes D$
	$\tilde{B}_{5,3}^0(i_q^h, j_{\bar{q}}^h, k_{\gamma}^h, l_Q, m_Q^h)$	$\bar{q}qg$	$A \otimes E$
quark-antiquark-quark	$\tilde{A}_{5,3}^0(i_q^h, j_{\bar{q}}^h, k_{\bar{q}}^h, l_g, m_q^h)$	$q\bar{q}Q$	$A \otimes A$
	$\tilde{B}_{5,3}^0(i_q^h, j_{\bar{q}}^h, k_{\bar{q}}^h, l_g, m_q^h)$	$q\bar{q}Q$	$A \otimes A$
quark-gluon-gluon	$D_{5,3}^0(i_q^h, j_g^h, k_g^h, l_g, m_q^h)$	qgg	$D \otimes F$
	$E_{5,3}^{0(a)}(i_q^h, j_{\bar{Q}}^h, k_Q^h, l_g, m_q^h)$	qgg	$E \otimes D$
	$E_{5,3}^{0(b)}(i_q^h, j_{\bar{Q}}^h, k_Q^h, l_Q, m_q^h)$	qgg	$A \otimes \bar{G}$
	$E_{5,3}^{0(c)}(i_q^h, j_g^h, k_g^h, l_Q, m_q^h)$	qgg	$D \otimes G$
	$E_{5,3}^{0(d)}(i_Q^h, j_{\bar{Q}}^h, k_Q^h, l_g, m_q^h)$	qgg	$\bar{E} \otimes D$
	$K_{5,3}^0(i_q^h, j_{\bar{Q}}^h, k_Q^h, l_R^h, m_R^h)$	qgg	$E \otimes E$
	$K_{5,3}^0(i_q^h, j_{\bar{Q}}^h, k_Q^h, l_R^h, m_R^h)$	qgg	$E \otimes E$
gluon-gluon-gluon	$F_{5,3}^0(i_g^h, j_g^h, k_g^h, l_g, m_g^h)$	ggg	$F \otimes F$
	$G_{5,3}^{0(a)}(i_g^h, j_q^h, k_q^h, l_g, m_g^h)$	ggg	$\bar{G} \otimes F$
	$G_{5,3}^{0(b)}(i_g^h, j_{\bar{q}}^h, k_q^h, l_g, m_g^h)$	ggg	$G \otimes D$
	$H_{5,3}^{0(a)}(i_Q^h, j_{\bar{Q}}^h, k_Q^h, l_q, m_q^h)$	ggg	$\bar{G} \otimes G$
	$H_{5,3}^{0(b)}(i_Q^h, j_{\bar{Q}}^h, k_Q^h, l_{\bar{q}}^h, m_q^h)$	ggg	$G \otimes E$
	$H_{5,3}^{0(b)}(i_Q^h, j_{\bar{Q}}^h, k_Q^h, l_{\bar{q}}^h, m_q^h)$	ggg	$G \otimes E$

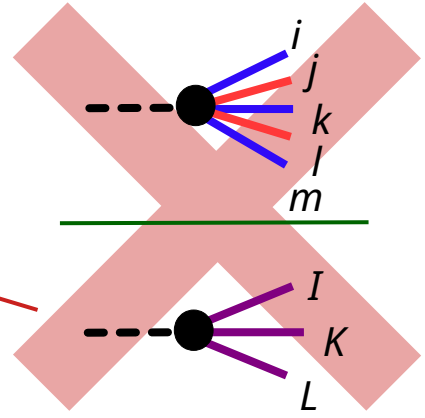
[Fox,Glover,MM '24]

Generalized antenna functions [Fox,Glover,MM '24]



Not possible with matrix element-based antenna functions

non-trivial function of the three-particle phase space



[Braun-White,Nigel,Preuss '22,'23]

With the **designer antenna algorithm**, it is possible to construct antenna functions with **more than two hard radiators: generalized antenna functions.**

Analytical integration made particularly simple thanks to a convenient choice of **5→3 momentum mapping**.

$$\text{map}_{5 \rightarrow 3} : \begin{aligned} p_I &= p_i + p_j - \frac{s_{ij}}{s_{ik} + s_{jk}} p_k \\ p_K &= \left(1 + \frac{s_{ij}}{s_{ik} + s_{jk}} + \frac{s_{lm}}{s_{lk} + s_{mk}} \right) p_k \\ p_M &= p_l + p_m - \frac{s_{lm}}{s_{lk} + s_{mk}} p_k \end{aligned}$$

iterated dipole mapping

Type	Name and parton content	Reconstructed hard partons	$X_3^0 \otimes X_3^0$
quark-antiquark-gluon	$A_{5,3}^0(l_q^h, j_g, k_q^h, l_g, m_q^h)$	$q\bar{q}\bar{q}$	$D \otimes \bar{D}$
	$B_{5,3}^0(i_q^h, j_g, k_q^h, l_Q, m_q^h)$	$q\bar{q}\bar{q}$	$A \otimes \bar{E}$
	$\tilde{A}_{5,3}^0(i_q^h, j_\gamma, k_q^h, l_g, m_g^h)$	$\bar{q}qg$	$A \otimes D$
	$\tilde{B}_{5,3}^0(i_q^h, j_\gamma, k_q^h, l_Q, m_Q^h)$	$\bar{q}qg$	$A \otimes E$
quark-antiquark-quark	$\bar{A}_{5,3}^0(i_q^h, j_\gamma, k_q^h, l_g, m_g^h)$	$q\bar{q}Q$	$A \otimes A$
	$\bar{B}_{5,3}^0(i_q^h, j_\gamma, k_q^h, l_Q, m_Q^h)$	$q\bar{q}Q$	$A \otimes A$
quark-gluon-gluon	$D_{5,3}^0(i_q^h, j_g, k_g^h, l_g, m_g^h)$	qgg	$D \otimes F$
	$E_{5,3}^{0(a)}(i_q^h, j_Q, k_Q^h, l_g, m_g^h)$	qgg	$E \otimes D$
	$E_{5,3}^{0(b)}(i_q^h, j_Q, k_Q^h, l_Q, m_Q^h)$	qgg	$A \otimes \bar{G}$
	$E_{5,3}^{0(c)}(i_q^h, j_g, k_g^h, l_Q, m_Q^h)$	qgg	$D \otimes G$
	$E_{5,3}^{0(d)}(i_Q^h, j_Q, k_Q^h, l_g, m_g^h)$	qgg	$\bar{E} \otimes D$
	$K_{5,3}^0(i_q^h, j_Q, k_Q^h, l_R, m_R^h)$	qgg	$E \otimes E$
	$\bar{K}_{5,3}^0(i_q^h, j_Q, k_Q^h, l_R, m_R^h)$	qgg	$E \otimes E$
gluon-gluon-gluon	$F_{5,3}^0(i_g^h, j_g, k_g^h, l_g, m_g^h)$	ggg	$F \otimes F$
	$G_{5,3}^{0(a)}(i_q^h, j_q, k_q^h, l_g, m_g^h)$	ggg	$\bar{G} \otimes F$
	$G_{5,3}^{0(b)}(i_g^h, j_q, k_q^h, l_g, m_g^h)$	ggg	$G \otimes D$
	$H_{5,3}^{0(a)}(i_Q^h, j_Q, k_Q^h, l_q, m_q^h)$	ggg	$\bar{G} \otimes G$
	$H_{5,3}^{0(b)}(i_g^h, j_Q, k_Q^h, l_q, m_q^h)$	ggg	$G \otimes E$
	$\bar{H}_{5,3}^{0(b)}(i_g^h, j_Q, k_Q^h, l_q, m_q^h)$	ggg	$G \otimes E$

[Fox,Glover,MM '24]

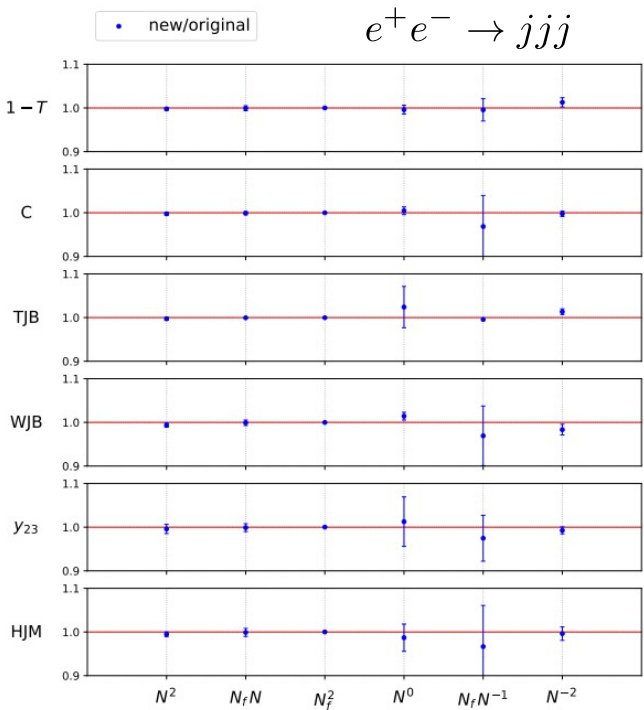
Generalized antenna functions: validation and applications

NNLO correction to event shapes in e^+e^- annihilation:

- perfect agreement with original method
- simpler subtraction terms
- up to 5-10x faster

e
v
e
n
t

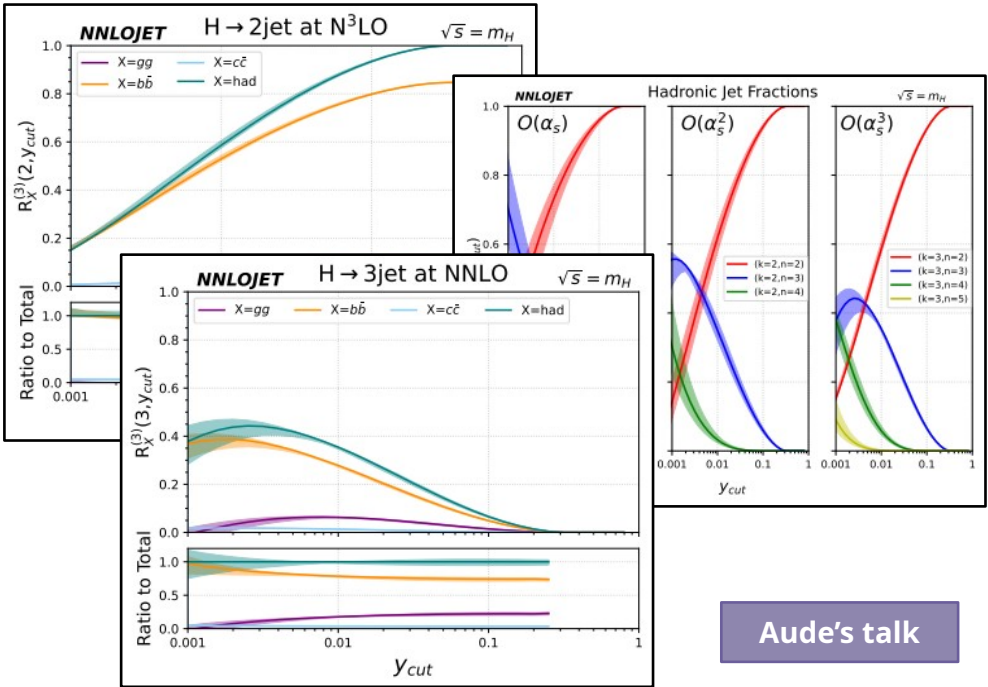
s
h
a
p
e
s



[Fox,Glover,MM '24]

Hadronic Higgs decays:

- differences between $H \rightarrow b\bar{b}$ and $H \rightarrow g\bar{g}$
- jet rates at order α_s^3 (3jet @NNLO, 2jet @N³LO)



Aude's talk

[Fox,Gehrmann-De Ridder,Gehrmann,Glover,MM,Preuss '25]

FIRST STEPS AND RESULTS AT N³LO WITH ANTENNA SUBTRACTION

work with Xuan Chen, Petr Jakubcik and Giovanni Stagnitto
2505.10618, 2507.12537

Jet production at lepton colliders: $e^+e^- \rightarrow jj$ @N³LO

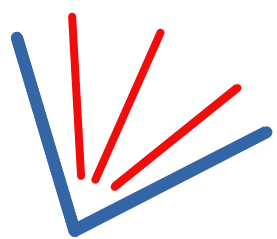
[Chen,Jakubcik,MM,Stagnitto '25]

Simple process:

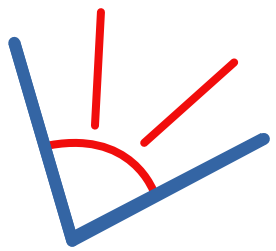
- only $q\text{-}\bar{q}$ N³LO antenna functions;
- only **dipole-like correlations** at N³LO;

Goals:

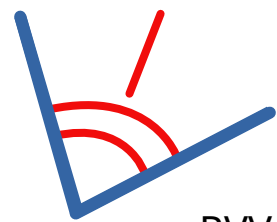
- definition of **N³LO antenna functions**;
- exploration of **numerical challenges** (IR stability of loop amplitudes);
- preparation of **computational framework** for more complicated processes;



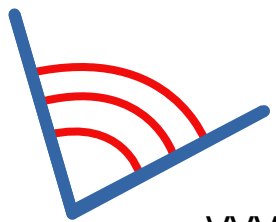
RRR



RRV



RVV



VVV

The calculation builds on top of $e^+e^- \rightarrow jjj$ @NNLO in **NNLOJET**

[Gehrmann,Glover,Huss, Nieuhes,Zhang '17]

Analytic integration of FSR
N³LO antenna functions in:

[Jakubcik,MM,Stagnitto '22]
[Chen,Jakubcik,MM,Stagnitto '23]

standard matrix-element based antenna functions

Local subtraction at N³LO

Subtraction at N³LO:

$$d\sigma_{N^3LO} = \int_n [d\sigma^{VVV} - d\sigma^W] + \int_n [d\sigma^{RVV} - d\sigma^U] + \int_{n+1} [d\sigma^{RRV} - d\sigma^T] + \int_{n+2} [d\sigma^{RRR} - d\sigma^S]$$

triple-virtual
subtraction term

double-virtual real
subtraction term

double-real-virtual
subtraction term

triple-real
subtraction term

with:

$$d\sigma^S = d\sigma^{S_1} + d\sigma^{S_2} + d\sigma^{S_3}$$

$$d\sigma^U = d\sigma^{V_2S_1} - \int_1 d\sigma^{V_1S_1} - \int_2 d\sigma^{S_2}$$

[Chen,Jakubcik,MM,Stagnitto '25]

See also [Chen,MM '25]

$$d\sigma^T = d\sigma^{V_1S_1} + d\sigma^{V_1S_2} - \int_1 d\sigma^{S_1}$$

$$d\sigma^W = - \int_1 d\sigma^{V_2S_1} - \int_2 d\sigma^{V_1S_2} - \int_3 d\sigma^{S_3}$$

one-loop double-unresolved quantities

Rescue-system to trigger
quadruple precision

Challenge: numerical stability of

two-loop single-unresolved quantities

Rescue-system to trigger
Taylor expansions of
special functions

Results

Basic checks: inclusive cross section

N³LO coefficient

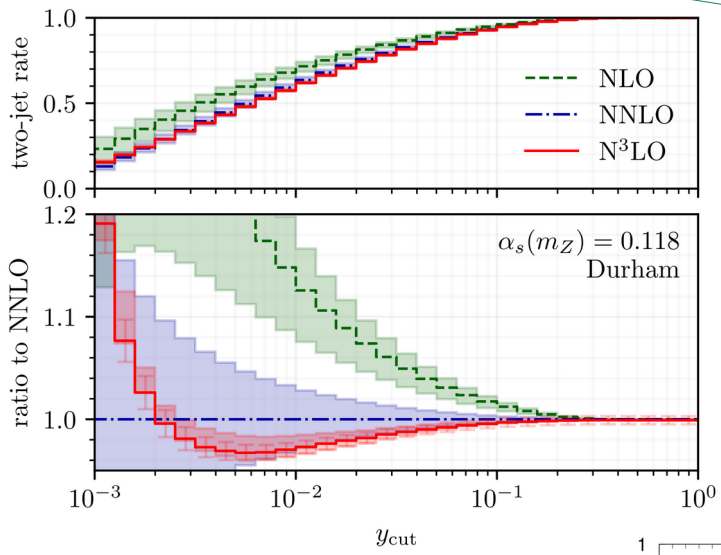
$$\sigma^{(3)} = \sigma_0 \left(\frac{\alpha_s}{2\pi} \right)^3 (-105 \pm 11)$$

Monte Carlo error.
Not so small for inclusive quantities due to large cancellations: not the *most clever* way to compute inclusive XS.

$$\sigma_{\text{exact}}^{(3)} = \sigma_0 \left(\frac{\alpha_s}{2\pi} \right)^3 (-102.14 \dots)$$

[Chetyrkin,Künn,Kwiatkowski '95]

Two-jet rate at order O(α_s³) (direct calculation)



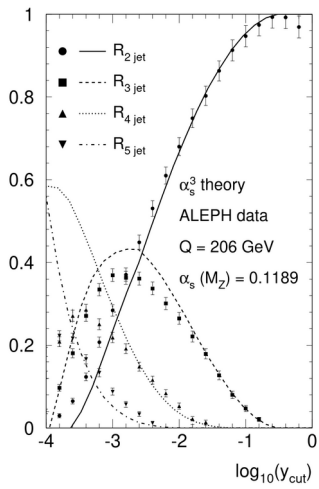
No need of previous knowledge of N³LO quantities. Fiducial cuts can be implemented too

Full agreement with indirect calculation of

[Gehrmann-De Ridder, Gehrmann,Glover,Heinrich '08]

$$R_n^{(3)}(y_{\text{cut}}) = \frac{\Gamma_{\gamma^* \rightarrow n \text{ jets}}^{(3)}(y_{\text{cut}})}{\Gamma_{\gamma^* \rightarrow \text{hadrons}}^{(3)}}$$

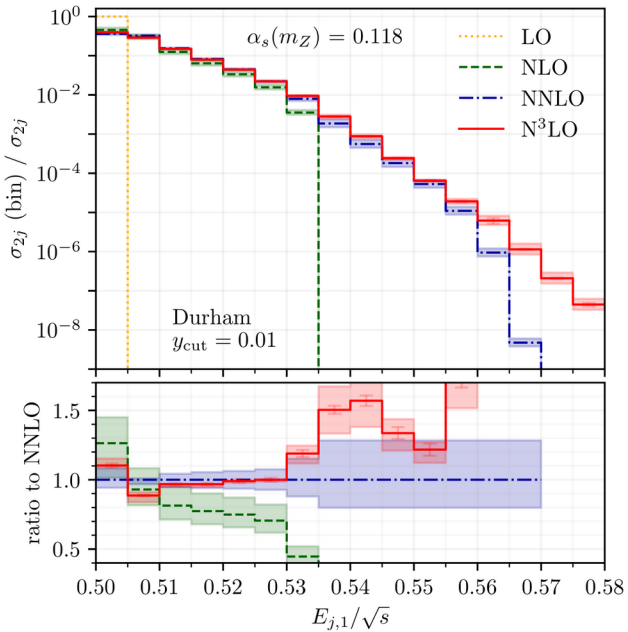
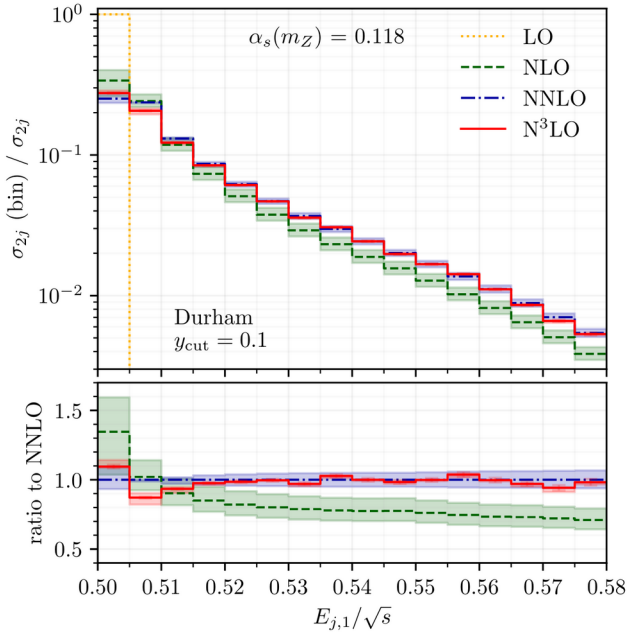
$$R_2^{(3)}(y_{\text{cut}}) = 1 - \sum_{n=3}^5 R_n^{(3)}(y_{\text{cut}})$$



Results

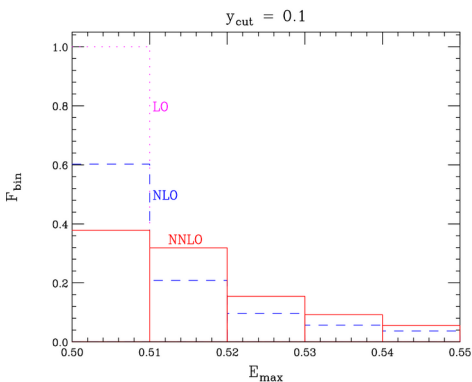
Leading-jet energy:

- defined on 2-jet events, bin-integrated;
- lower orders vanish quickly at large E_{j1} for smaller y_{cut} because energetic jets recoil against multiple emissions which are more likely to be clustered as three or more jets;
- again, the whole distribution can be obtained by combining $e^+e^- \rightarrow jjj$ @NNLO with the $N^3\text{LO}$ inclusive XS.
It's a proof-of-principle application.



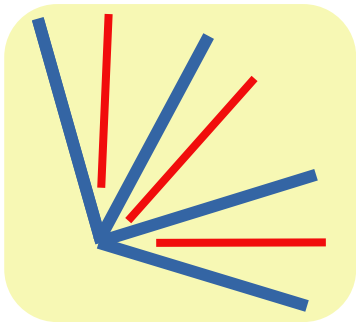
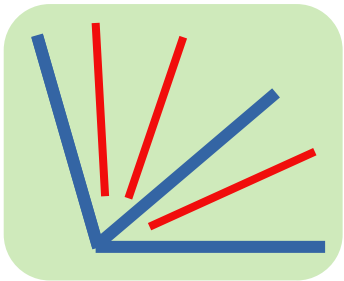
Full agreement up to NNLO with

[Anastasiou,Melnikov,Petriello '04]



How to proceed? Generalized antenna functions at N³LO

RRR:

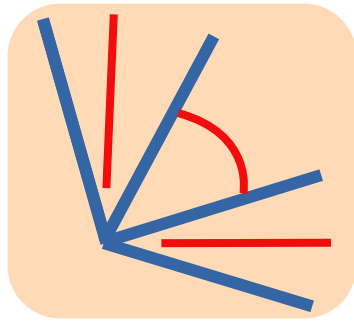
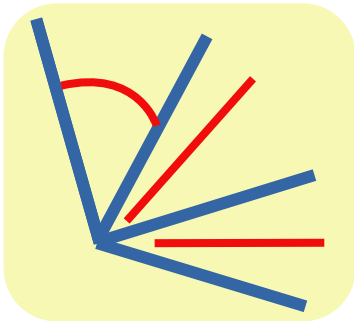
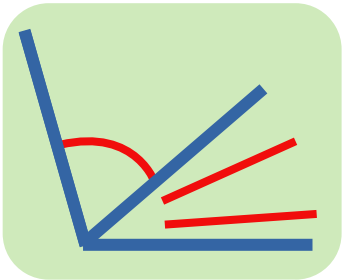
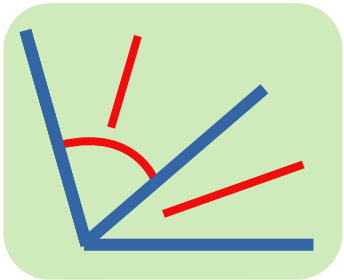


Starting from:
 $e^+e^- \rightarrow 3j$
 $pp \rightarrow V+j$

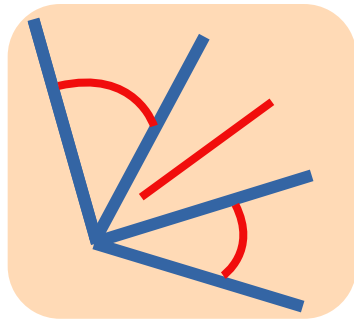
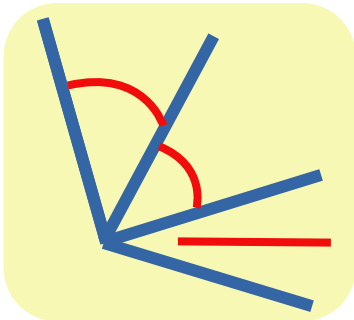
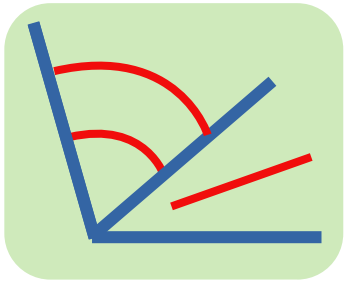
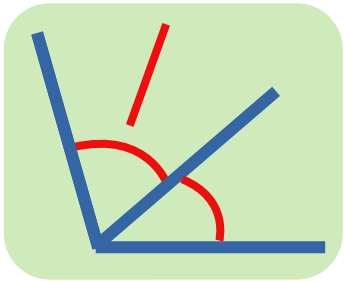
Starting from:
 $e^+e^- \rightarrow 4j$
 $pp \rightarrow 2j$

Special case for:
 $e^+e^- \rightarrow 3j$
 $pp \rightarrow V+j$
 General case for:
 $e^+e^- \rightarrow 4j$
 $pp \rightarrow 2j$

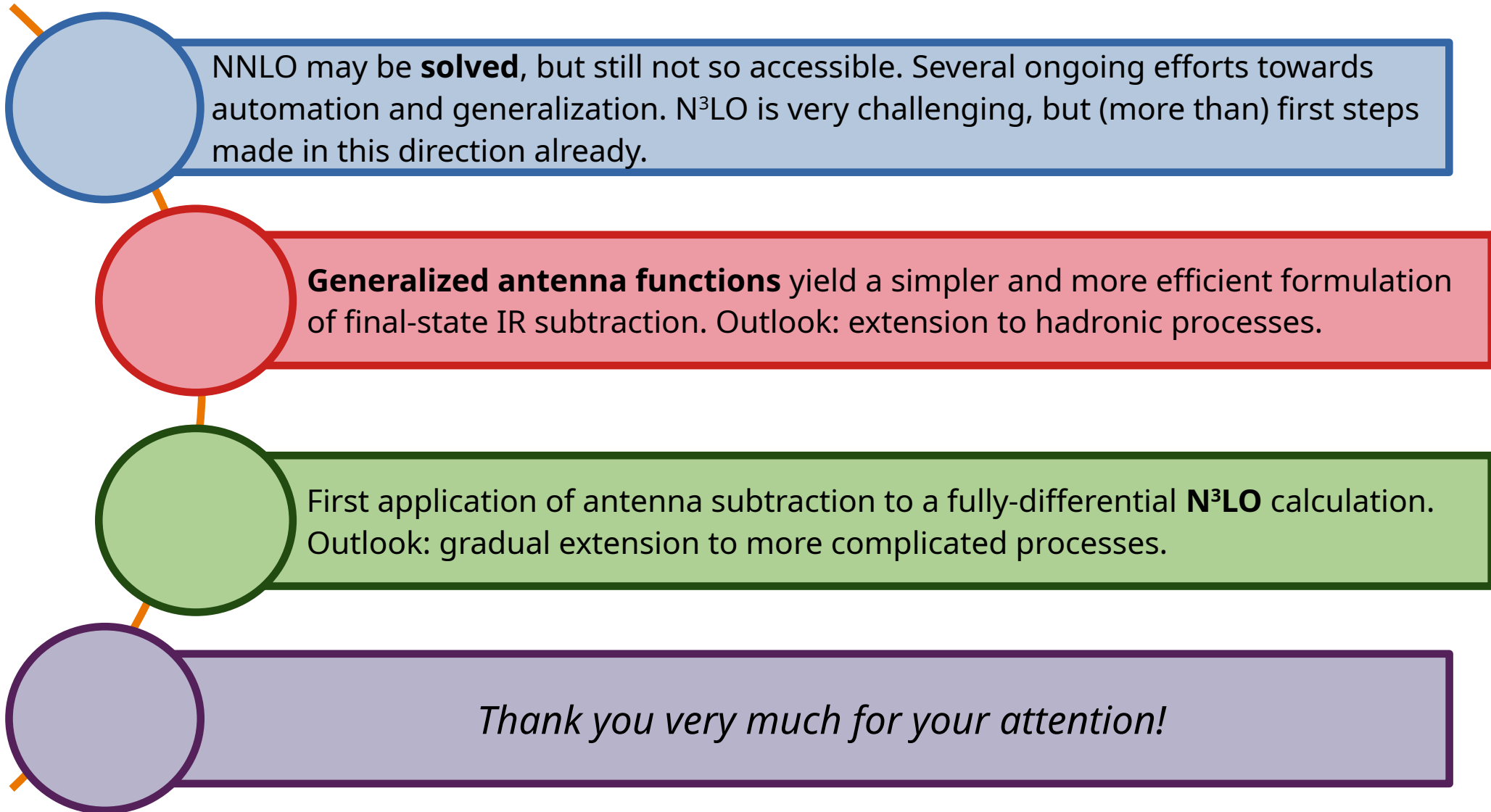
RRV:



RVV:



SUMMARY AND OUTLOOK



NNLO may be **solved**, but still not so accessible. Several ongoing efforts towards automation and generalization. $N^3\text{LO}$ is very challenging, but (more than) first steps made in this direction already.

Generalized antenna functions yield a simpler and more efficient formulation of final-state IR subtraction. Outlook: extension to hadronic processes.

First application of antenna subtraction to a fully-differential **$N^3\text{LO}$** calculation. Outlook: gradual extension to more complicated processes.

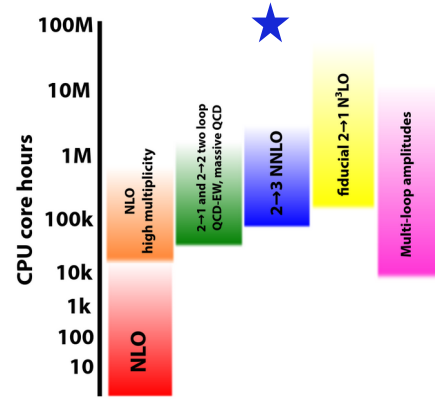
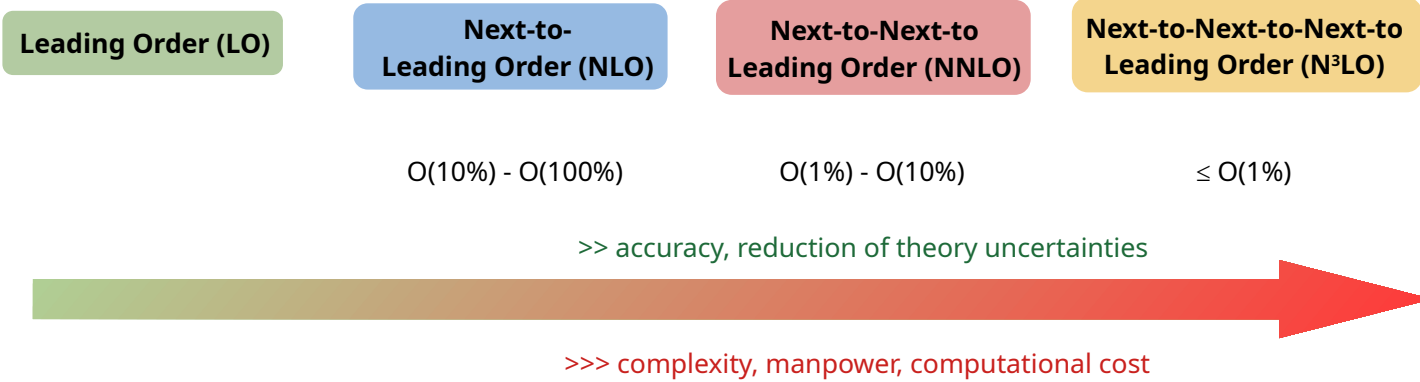
Thank you very much for your attention!

BACKUP SLIDES

Fixed-order calculations in QCD

pp → jjj event shapes with STRIPPER

$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right) \sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma_3 + \dots$$

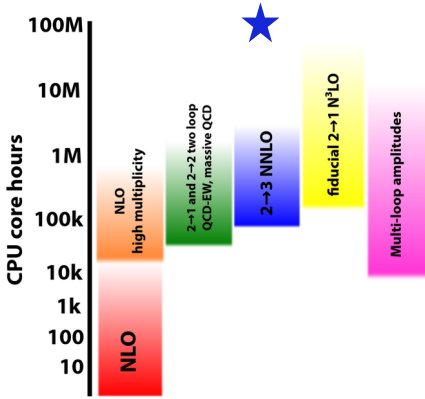
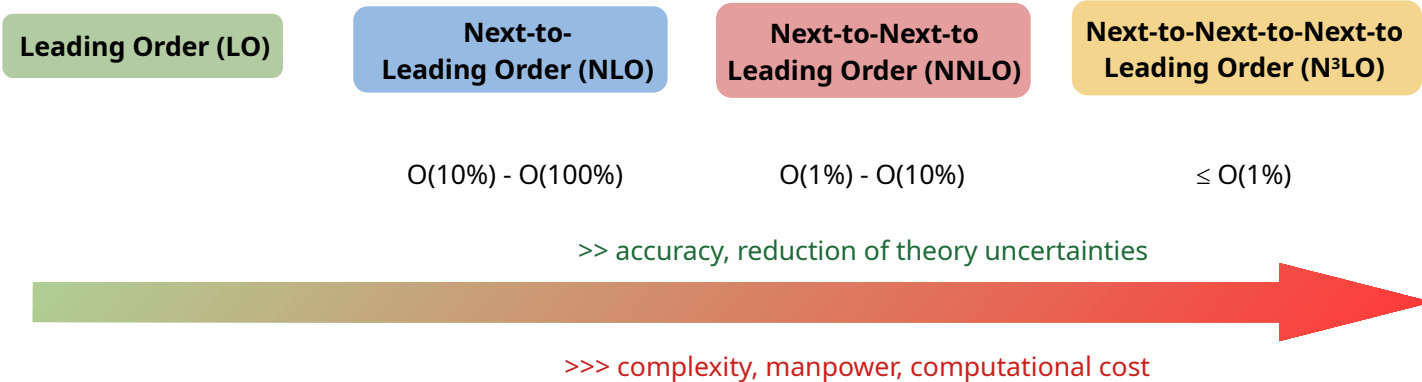


[Febres Cordero, von Manteuffel, Neumann '22]

Fixed-order calculations in QCD

pp → jjj event shapes with STRIPPER

$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right) \sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma_3 + \dots$$



[Febres Cordero, von Manteuffel, Neumann '22]

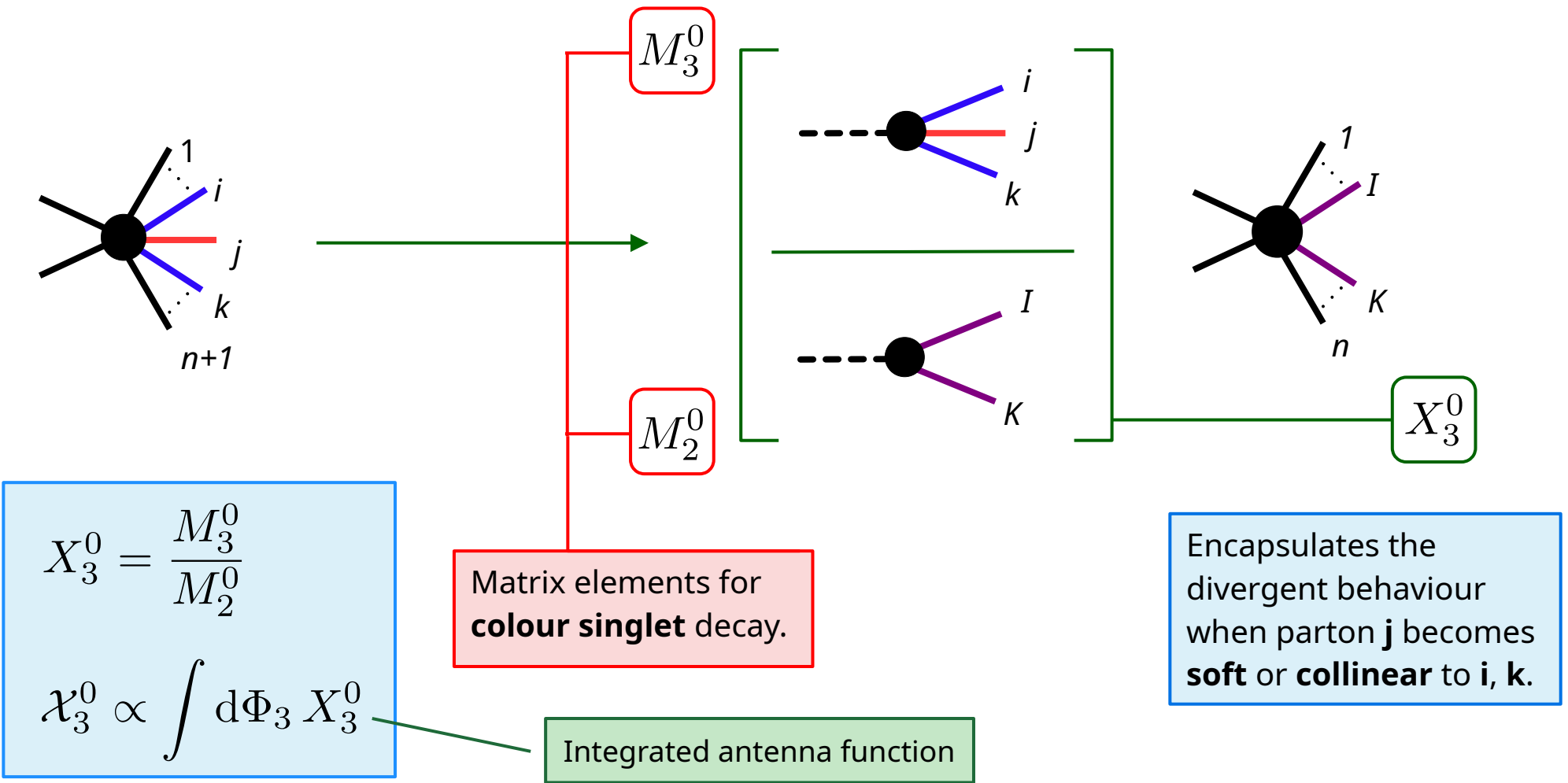
Conditions to claim that infrared subtraction is **solved** at a N^kLO?

- Given matrix elements and resources, the N^kLO correction can be computed
- New N^kLO calculations are streamlined
- Different groups/approach available (validations, efficiency, ...)
- N^kLO calculations can be easily reproduced and performed by anyone
- N^kLO event generation is possible

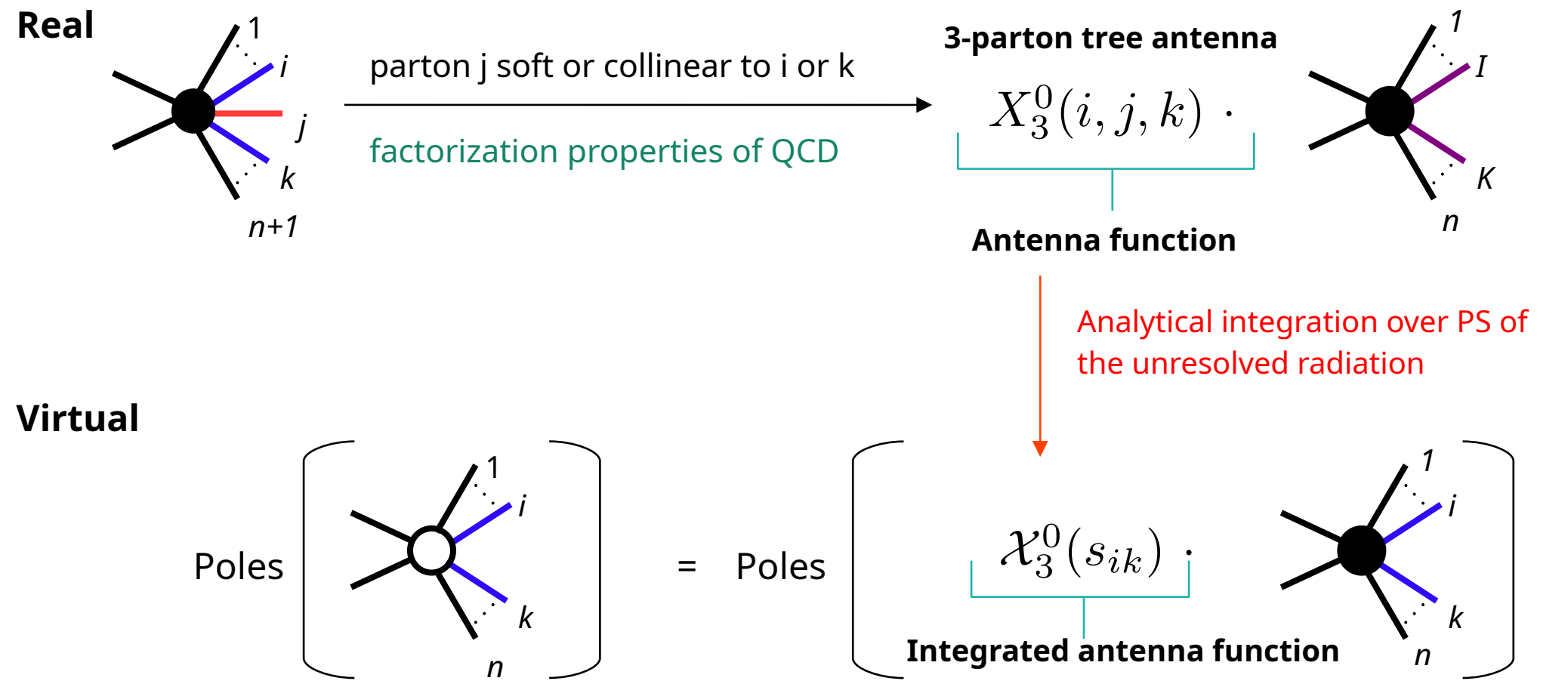
NLO	NNLO	N ³ LO
✓	✓	✗
✓	✓	✗
✓	●	✗
✓	✗	✗
✓	✗	✗

STRIPPER

NLO: three-parton tree-level antenna functions

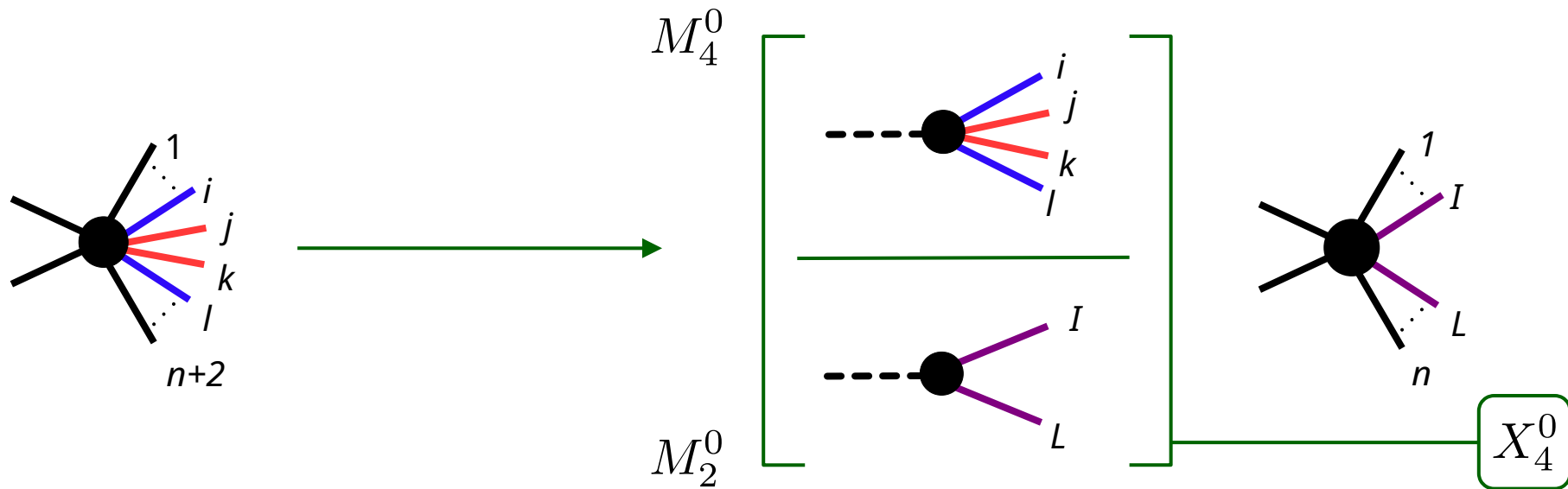


Antenna subtraction at NLO



[Gehrmann-De Ridder,Gehrmann,Glover '05] [Currie,Glover,Wells '13]

NNLO: four-parton tree-level antenna functions

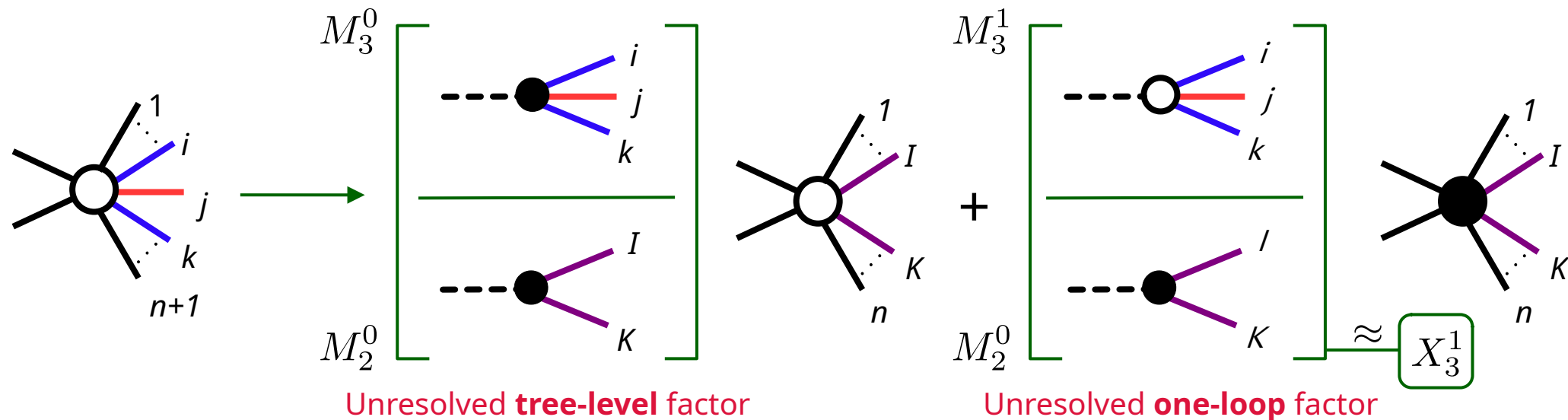


$$X_4^0 = \frac{M_4^0}{M_2^0}$$

$$\mathcal{X}_4^0 \propto \int d\Phi_4 X_4^0$$

Four-parton tree-level antenna functions are extracted analogously to the three-parton ones

NNLO: three-parton one-loop antenna functions



Three-parton one-loop antenna defined removing from the one-loop decay matrix element the unresolved tree-level configuration:

$$X_3^1 = \frac{M_3^1}{M_2^0} - X_3^0 \frac{M_1^2}{M_2^0}, \quad \mathcal{X}_3^1 \propto \int d\Phi_3 X_3^1$$

Antenna subtraction at NNLO

[Gehrmann-De Ridder,Gehrmann,Glover '05] [Currie,Glover,Wells '13]

RR:

Diagram illustrating the reduction of an $n+2$ parton process to an $n+1$ parton process via a single unresolved antenna, and its decomposition into a 4-parton tree antenna and a double unresolved n parton process.

single unresolved

$X_3^0 \cdot$

$n+2$

$n+1$

4-parton tree antenna

$X_4^0 \cdot$

$X_3^0 X_3^0 \cdot$

double unresolved

n

RV:

Diagram illustrating the reduction of an $n+1$ parton process to an $n+1$ parton process via removing ϵ -poles, and its decomposition into tree x loop and loop x tree terms.

removes ϵ -poles

$\chi_3^0 \cdot$

$n+1$

$n+1$

tree x loop

$X_3^0 \cdot$

loop x tree

$X_3^1 \cdot$

3-parton 1-loop antenna

n

n

VV:

Diagram illustrating the reduction of an n parton process to an n parton process via a vertical cut, and its decomposition into various terms including a tensor product of X_3^0 terms.

$\chi_3^0 \cdot$

n

n

$\chi_4^0 \cdot$

$\chi_3^1 \cdot$

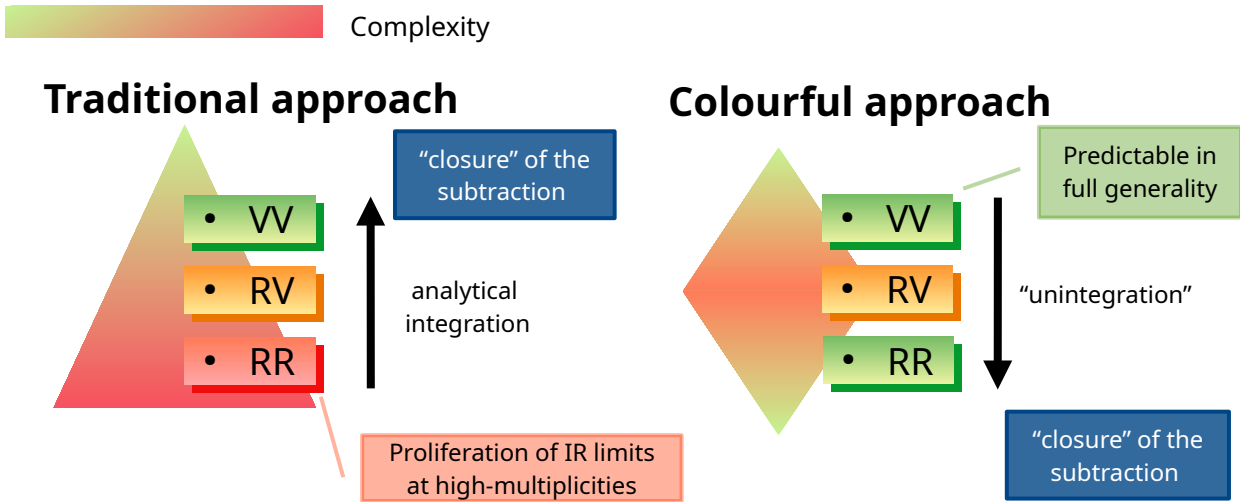
$\chi_3^0 \otimes \chi_3^0 \cdot$

n

Colourful antenna subtraction

Ultimate goal: combine **generalized antenna functions** with the **colourful antenna subtraction** method

[Chen,Gehrmann,Glover,Huss,MM '22]
[Gehrmann,Glover,MM '23]



$$d\sigma^T = \mathcal{N}_V \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d\Phi_n J_n^n(\Phi_n) \left[2 \langle A_{n+2}^0 | \mathcal{J}^{(1)} | A_{n+2}^0 \rangle \right]$$

one loop

$$d\sigma^U = \mathcal{N}_{VV} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d\Phi_n J_n^n(\Phi_n) \times 2 \left\{ \langle A_{n+2}^0 | \mathcal{J}^{(1)} | A_{n+2}^1 \rangle + \langle A_{n+2}^1 | \mathcal{J}^{(1)} | A_{n+2}^0 \rangle - \frac{\beta_0}{\epsilon} \langle A_{n+2}^0 | \mathcal{J}^{(1)} | A_{n+2}^0 \rangle - \langle A_{n+2}^0 | \mathcal{J}^{(1)} \otimes \mathcal{J}^{(1)} | A_{n+2}^0 \rangle + \langle A_{n+2}^0 | \mathcal{J}^{(2)} | A_{n+2}^0 \rangle \right\}$$

Exploit **universal** IR singularity structure in virtual corrections to **systematically** construct real-radiation counterterms

two loops

Mapping (in)dependence [Fox,Glover,MM '24]

mapping to absorb the recoil
of unresolved radiation:

$$\{p\} \rightarrow \{\tilde{p}\}$$

Let's consider:

- \mathbf{n}_p momenta $\{p\}$ involved in an unresolved configuration
- \mathbf{n}_q spectator momenta $\{q\}$

generic subtraction term

$$d\sigma^S \propto \int \underbrace{dPS_{n_p+n_q}(\{p\}, \{q\})}_{\text{full phase-space}} \underbrace{X(\{p\})}_{\text{unresolved factor (antenna function)}} \underbrace{M(\{\tilde{p}\}, \{q\})}_{\text{resolved matrix element}} \underbrace{J_{n_{jets}}^{(n_{\tilde{p}}+n_q)}(\{\tilde{p}\}, \{q\})}_{\text{measurement function}}$$

selects n_{jets} jets
applies fiducial cuts

The mapping is chosen to induce a **factorization of the phase space**

$$dPS_{n_p+n_q}(\{p\}, \{q\}) = \underbrace{dPS_X(\{p\}/\{\tilde{p}\})}_{\text{unresolved phase-space}} dPS_{n_{\tilde{p}}+n_q}(\{\tilde{p}\}, \{q\})$$

resolved phase-space:
the measurement
function acts on it

$$\begin{aligned} d\sigma^S &\propto \int dPS_{n_{\tilde{p}}+n_q}(\{\tilde{p}\}, \{q\}) \left[\int X(\{p\}) dPS_X(\{p\}/\{\tilde{p}\}) \right] M(\{\tilde{p}\}, \{q\}) J_{n_{jets}}^{(n_{\tilde{p}}+n_q)}(\{\tilde{p}\}, \{q\}) \\ &= \int dPS_{n_{\tilde{p}}+n_q}(\{\tilde{p}\}, \{q\}) \underbrace{\mathcal{X}(\{\tilde{p}\})}_{\text{integrated unresolved factor}} M(\{\tilde{p}\}, \{q\}) J_{n_{jets}}^{(n_{\tilde{p}}+n_q)}(\{\tilde{p}\}, \{q\}) \end{aligned}$$

two mappings are
equivalent if the yield
the same $\mathcal{X}(\{\tilde{p}\})$

Mapping (in)dependence [Fox,Glover,MM '24]

two hard radiators: $n_{\tilde{p}} = 2$, $\{p_1, \dots, p_{n_p}\} \rightarrow \{\tilde{p}_A, \tilde{p}_B\}$

$$s_{1\dots n_p} \equiv (p_1 + \dots + p_{n_p})^2 = (\tilde{p}_A + \tilde{p}_B)^2 \equiv s_{AB}$$

$$\mathcal{X}(\{\tilde{p}_A, \tilde{p}_B\}) = C(\epsilon)(s_{AB})^\alpha \text{ any momentum-conserving mapping gives same result}$$

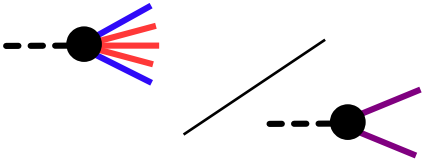
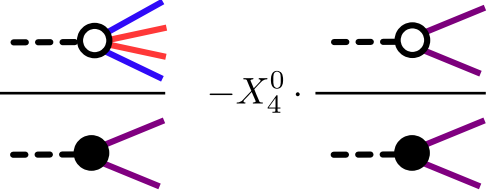
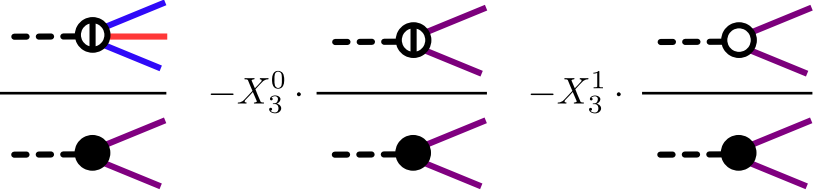
three hard radiators: $n_{\tilde{p}} = 3$, $\{p_1, \dots, p_{n_p}\} \rightarrow \{\tilde{p}_A, \tilde{p}_B, \tilde{p}_C\}$

$$s_{1\dots n_p} \equiv (p_1 + \dots + p_{n_p})^2 = (\tilde{p}_A + \tilde{p}_B + \tilde{p}_C)^2 \equiv s_{ABC}$$

$$\mathcal{X}(\{\tilde{p}_A, \tilde{p}_B, \tilde{p}_C\}) = \sum_i C_i(\epsilon)(s_{AB})^{\alpha_i}(s_{AC})^{\beta_i}(s_{BC})^{\gamma_i} + \dots$$

many “unfixed” scales, different result for different mappings

N³LO antenna functions

	Antenna	Integrated antenna
<div>RRR:</div> 	$X_5^0 = \frac{M_5^0}{M_2^0}$	$\mathcal{X}_5^0 \propto \int d\Phi_5 X_5^0$
<div>RRV:</div> 	$X_4^1 = \frac{M_4^1}{M_2^0} - X_4^0 \frac{M_2^1}{M_2^0}$	$\mathcal{X}_4^1 \propto \int d\Phi_4 X_4^1$
<div>RVV:</div> 	$X_3^2 = \frac{M_3^2}{M_2^0} - X_3^0 \frac{M_2^2}{M_2^0} - X_3^1 \frac{M_2^1}{M_2^0}$	$\mathcal{X}_3^2 = \int d\Phi_3 X_3^2$

Analytic integration of final-state N³LO antenna functions

Integration of **renormalized matrix elements** for colour-singlet decay over the **fully inclusive phase space**:

$$\int d\Phi_5 M_5^0, \quad \int d\Phi_4 M_4^1, \quad \int d\Phi_3 M_3^2, \quad \int d\Phi_2 M_2^3 \quad \searrow$$

[Jakubcik,MM,Stagnitto '22]
[Chen,Jakubcik,MM,Stagnitto '23]

Two-parton three-loop, for validation:

Master integrals from

$$\int d\Phi_5 M_5^0 + \int d\Phi_4 M_4^1 + \int d\Phi_3 M_3^2 + \int d\Phi_2 M_2^3 = \text{finite N}^3\text{LO inclusive XS}$$

[Gituliar,Magerya,Pikelner '18]
[Magerya,Pikelner '19]

Reverse unitarity:

$$2\pi i \delta^+(p^2) \rightarrow \frac{1}{p^2 - i0} - \frac{1}{p^2 + i0} \quad \begin{array}{l} \text{[Cutkosky '60]} \\ \text{[Anastasiou, Melnikov '02,'03]} \end{array}$$

- Phase space and (genuine) loop integrals addressed simultaneously;
- **Systematic treatment of all four layers** within a common framework;

