Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Precise predictions for hadronic Higgs decay observables

Aude Gehrmann-De Ridder
In collaboration with E. Fox, T. Gehrmann, N. Glover, M. Marcoli, C. Preuss

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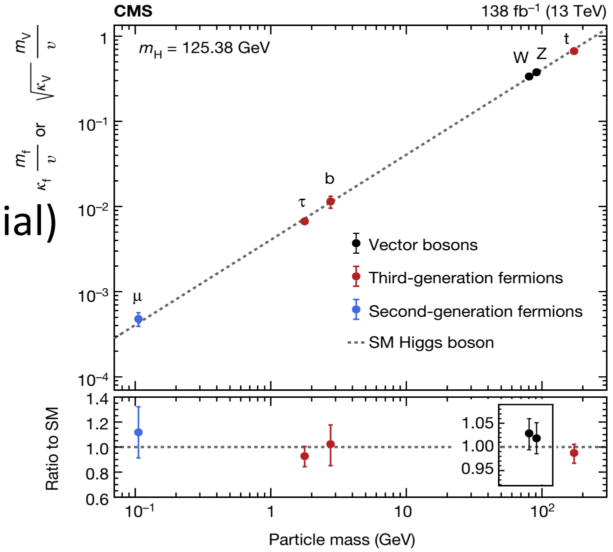
Introduction: The Higgs boson in the Standard Model

- Higgs boson H: prediction of Brout-Englert-Higgs mechanism (1964, Nobel Prize 2013) of electroweak symmetry breaking for mass generation of Standard Model (SM) particles
- Discovered in 2012 the $H \to \gamma\gamma$ channel [ATLAS: 1207.7214, CMS:1207.7235]
- Present data compatible with a scalar particle with spin 0 and even parity (as predicted by the SM) of mass m_H ~ 125.2 GeV
- Couplings
  - fermions (f): $g_{Hff} \sim m_f/v$ (largest for top-quarks)
  - EW gauge bosons (V): $g_{HVV} \sim m_V^2/v^2$ (no direct γ-coupling)
  - Higgs (H): $g_{HHH} \sim m_H^2/v$, $g_{HHHH} \sim m_H^2/v$ (Ultimate test of Higgs potential)
- Couplings probed in Higgs production or decay channels
- No deviation from SM observed so far at LHC

CMS Nature (2022)
2207.00043

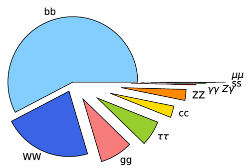


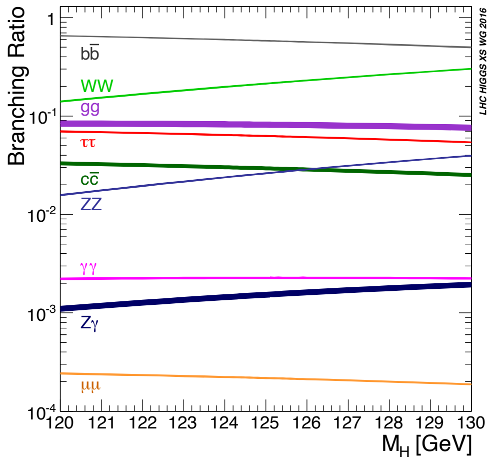
Introduction: Higgs decays at a lepton collider

- FCC_{ee}: Higgs factory at ZH production threshold: (expect: 10^6 events at $\sqrt{s}$=240 GeV)
  - Number of Higgs bosons produced: comparable to the number of Z-bosons at LEP

- $e^+e^- \to ZH$ (leptonic Z-decays): Ideal for precision studies of Higgs properties and couplings
  - clean environment : free from QCD initial state radiation
  - Expected experimental uncertainty for (most) Higgs couplings @ per-mille level

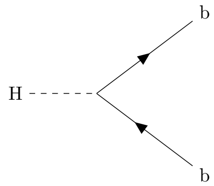
- Require precise theory
  - computations in perturbative QCD with higher order corrections
  - Focus on hadronic Higgs decay observables for dominant decay channels $H \to bb$ (or: $H \to cc$) and $H \to gg$



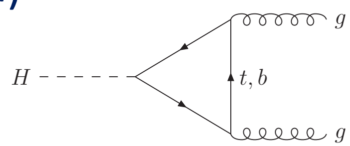


Hadronic Higgs decays: main decay modes

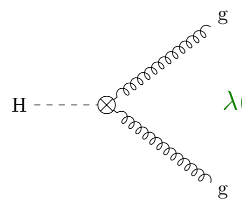
- H⟶bb: main Higgs decay channel
  - essential for precise determination of Γ_H
  - already accessible at LHC via ZH production and leptonic Z-decay
  - direct (Yukawa induced) coupling at α_s^0 : $y_b(\mu_R) = \sqrt{2} m_b(\mu_R)/\mathrm{v}$



- H⟶gg : accessible only at lepton colliders (fixed energy, low QCD background)
  - no direct coupling: loop induced coupling at α_s^1



- Both processes included using $\mathcal{L}_{\text{Higgs}} = -\frac{\lambda(M_t,\mu_R)}{4} H G^a_{\mu\nu} G^{a,\mu\nu} + \frac{y_b(\mu_R)}{\sqrt{2}} H \bar{\psi}_b \psi_b$
  - with H⟶gg decay computed in HEFT with an effective Hgg coupling:
    - top quark Wilson coefficient $C(M_t)$ known to four-loops
  - no interference between both categories for m_q=0 (except in y_b)



$$\lambda(M_t,\mu_R) = -\frac{\alpha_S(\mu_R) C(M_t,\mu_R)}{3\pi v}$$

Hadronic Higgs decay observables: theory status beyond NLO

- Inclusive branching fractions: known analytically @N^4LO
  [F.Herzog, B. Ruijl, T.Ueda, J.Vermaseren, A.Vogt, '17]
  - $H \rightarrow bb$: for $m_q=0$ (except in y_b)
  - $H \rightarrow gg$: in HEFT, i.e with infinite top mass limit)
- Exclusive observables: Jet rates: (known only for $H \longrightarrow bb$ decay mode)
  - $H \longrightarrow bb$: known @N^3LO [R.Mondini, M.Schiavi, C.Williams, '19](MCFM)
  - $H \longrightarrow bb$ +jet : known @NNLO [R.Mondini, C.Williams, '19](MCFM)
- Implicit infrared pole cancellations: dealt with N-jettiness slicing method
  [R.Boughezal, X.Liu, F.Petriello; J.Gaunt, M.Stahlhofen, F.Tackmann, J.Walsh, '15]
  - Split the phase space into singular/non-singular regions using
    τ_N : the distance from an N-jet configuration

$$\tau_3 = \sum_{j=1,m} \min_{i=1,2,3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\}$$

Subtraction at NNLO

- Parton level NNLO cross section with m-jets in the final state

$$\mathrm{d}\hat{\sigma}_{NNLO} = \int_{\mathrm{d}\phi_{m+2}} \left[\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right] + \int_{\mathrm{d}\phi_{m+1}} \left[\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right] + \int_{\mathrm{d}\phi_{m}} \left[\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right]$$

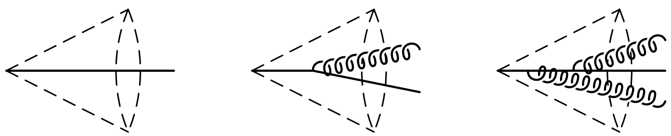
- Unintegrated subtraction terms
  - Reproduce double real (RR) and real-virtual (RV) contributions in all infrared limits
- Integrated subtraction terms in $\mathrm{d}\hat{\sigma}^{T}_{NNLO},\ \mathrm{d}\hat{\sigma}^{U}_{NNLO}$
  - Cancel explicit infrared poles in real-virtual (RV) and double virtual (VV)
- Terms in square brackets are
  - finite, well-behaved in all infrared regions
  - evaluated numerically with a parton-level event generator
- Challenges: - Construction and convergence of subtraction terms
  - Integrated subtraction terms

Hadronic Higgs decay jet observables: NNLOJET

- Focus: H⟶3-jets @NNLO, H⟶2-jets@N^3LO for both Higgs decay categories
- Matrix-elements known at all levels: for H ⟶ gg+jet , from pp ⟶ H+jet @NNLO (NNLOJET)
- IR behaviour of real emission matrix-elements: new designer antenna formalism @NNLO
  [E. Fox, N. Glover, M. Marcoli, '24] ⟶ See talk by Matteo Marcoli
- Implementation:

  H ⟶ bb+jet: identical infrared structure of QCD corrections as e^+e^- ⟶ 3-jets (used for validation of new formalism), agrees with MCFM [C.Williams, R.Mondini, '19]

  H ⟶ gg+jet: New derivation and implementation in NNLOJET
- Jets defined with the IR–safe k_T algorithm : partons i,j clustered if $y_{ij} < y_{cut}$



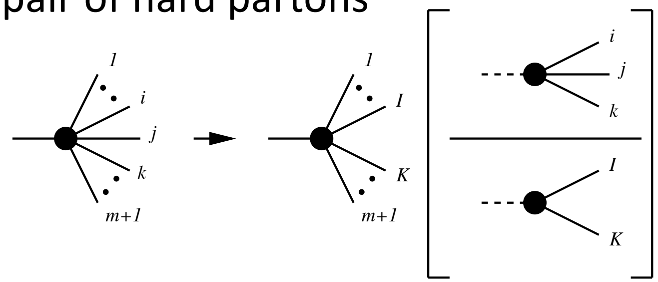
LO NLO NNLO

$$y_{ij} = \frac{2(1-\cos\theta_{ij})}{E_{\rm tot}^2}\min(E_i^2, E_j^2)$$

Antenna subtraction: Conventional formalism

[T. Gehrmann, N. Glover, AG,'05; J. Currie, N. Glover, S. Wells,'13]

- Building blocks of subtraction terms (here: final-final, @NLO, 1 unresolved parton)
  - Antenna functions: built with physical matrix-elements, capturing all unresolved radiation between a pair of hard partons



$$X^0_{ijk} = S_{ijk}\frac{|\mathcal{M}^0_{ijk}|^2}{|\mathcal{M}^0_{IK}|^2} \qquad \mathrm{d}\Phi_{X_{ijk}} = \frac{\mathrm{d}\Phi_3}{P_2}$$

  - Phase space factorization and mapping: $(i,j,k) \to (I,K)$

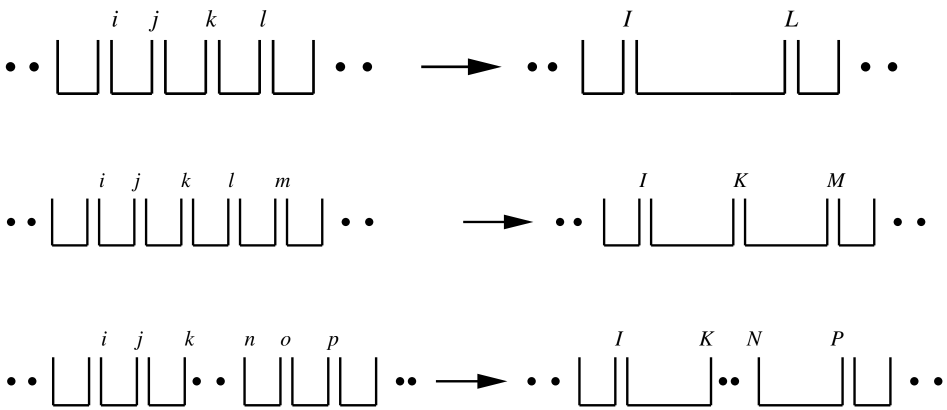
$$\mathrm{d}\Phi_{m+1}(p_1\cdots,p_i,p_j,p_k,\cdots,p_{m+1}) = \mathrm{d}\Phi_m(p_1,\cdots,p_I,p_K,\cdots,p_{m+1})\mathrm{d}\Phi_{X_{ijk}}(p_i,p_j,p_k,p_I,p_K)$$

  - Analytically integrated subtraction term involve $\mathcal{X}_{ijk} = \int d\Phi_{X_{ijk}} X_{ijk}$

- @NNLO: X^0_3 supplemented by X^0_4, X^1_3 and $X^0_3 * X^0_3$ and their integrated forms

Double Real Subtraction: $\mathrm{d}\sigma^{S}_{NNLO}$

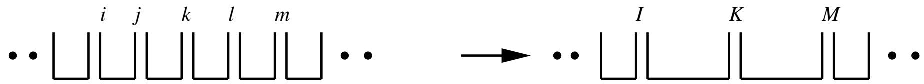
- Distinct configurations for m+2 partons → m jets : Colour connections
- one unresolved parton: (a)
  - three parton antennae: X^0_3 :(NLO-type)
- two colour-connected unresolved partons: (b)
  - four-parton antennae: X^0_4: (genuine NNLO)
- two almost colour-unconnected partons: (c)
  - product of two non-independent three-parton antennae X^0_3 (common radiator)
    - radiation shared between five partons
- two colour-unconnected unresolved partons: (d)
  - product of two independent three-parton antennae X^0_3



Designer antenna formalism: almost colour-unconnected case

[E.Fox, N.Glover, M.Marcoli, ‘24]

- Use $X^0_{5,3}$ (i^h, j, k^h, l, m^h): tree-level five parton antenna with three hard radiators:
    - with 5 → 3 mapping: [i,j,k,l,m] → [I,K,M]



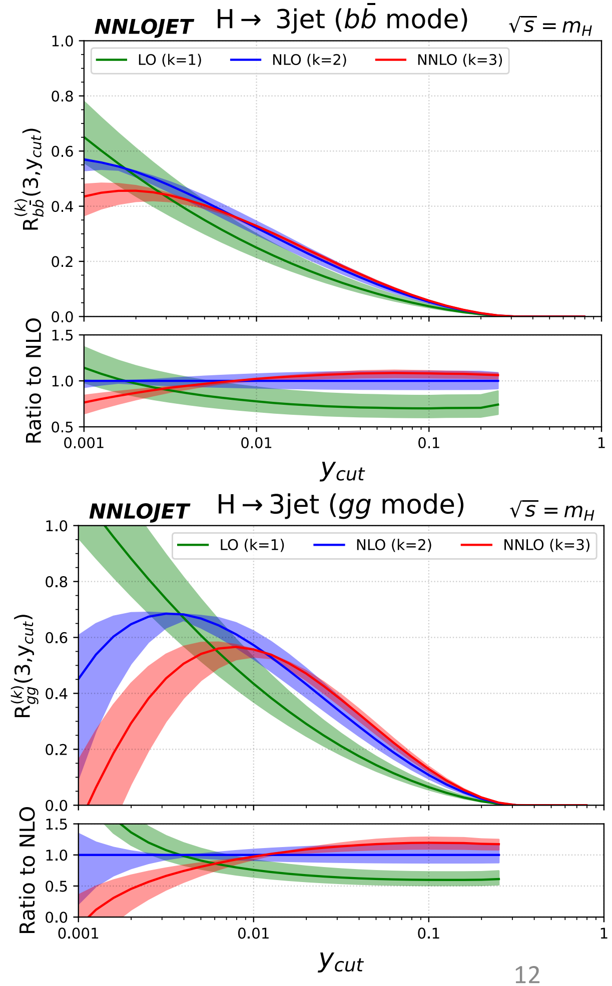
- Antennae constructed with an iterative algorithm
  [O.Braun-White, N.Glover, C.Preuss, ‘23]
    - from the desired IR limits (not from physical matrix-elements)
    - using projectors (up-down) to connect full phase space (antennae) and subspace (IR limits)
    - can be integrated analytically (as in conventional method)
- Subtraction terms: Considerably more compact
  → See talk by Matteo Marcoli
- First new (final-state radiation) application : Hadronic Higgs decay observables

Hadronic Higgs decay observables : Results

Normalised 3-jet rates in Higgs decay up to order α_s^3

- Jet rates (k=1,2,3 ; n=3) $$R_X^{(k)}(n, y_{\rm cut}) = \frac{\Gamma^{(k)}_{H\to X}(n, y_{\rm cut})}{\Gamma^{(k)}_{H\to X}}, \text{ with } X = gg, b\bar{b}$$

- Size and shape of jet rates: y_{cut} dependence
  - Size of NNLO corrections:
    - large y_{cut} : good perturbative convergence, largest corrections in Hgg mode (25 % at y_{cut} =0.1)
    - small y_{cut} : significant and negative, resummation needed
  - Differences in shape: Peak of distributions at low y_{cut} shifted @NNLO
    - $H \to bb$ (peak at y_{cut} =0.002), $H \to gg$ (peak at y_{cut}=0.007)

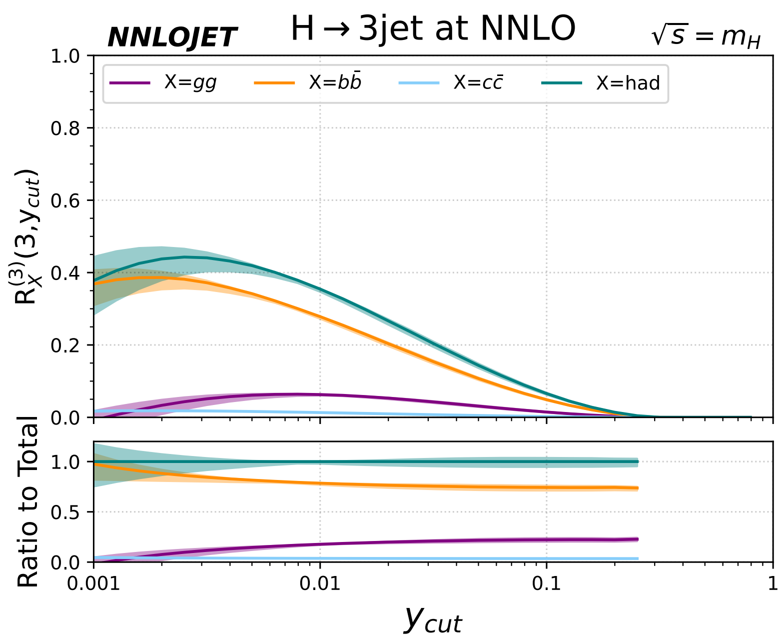


Normalised 3-jet rates @NNLO

- Total hadronic 3-jet rate @α_s^3: $R_X^{(3)}(n, y_{\rm cut})$ with $X = gg,\, b\bar{b}, c\bar{c}, had$
- Normalisation and ratio : Total decay rate to hadrons

$$\Gamma^{(3)}_{H\to \rm had} = \Gamma^{(3)}_{H\to b\bar{b}} + \Gamma^{(3)}_{H\to c\bar{c}} + \Gamma^{(3)}_{H\to gg}$$

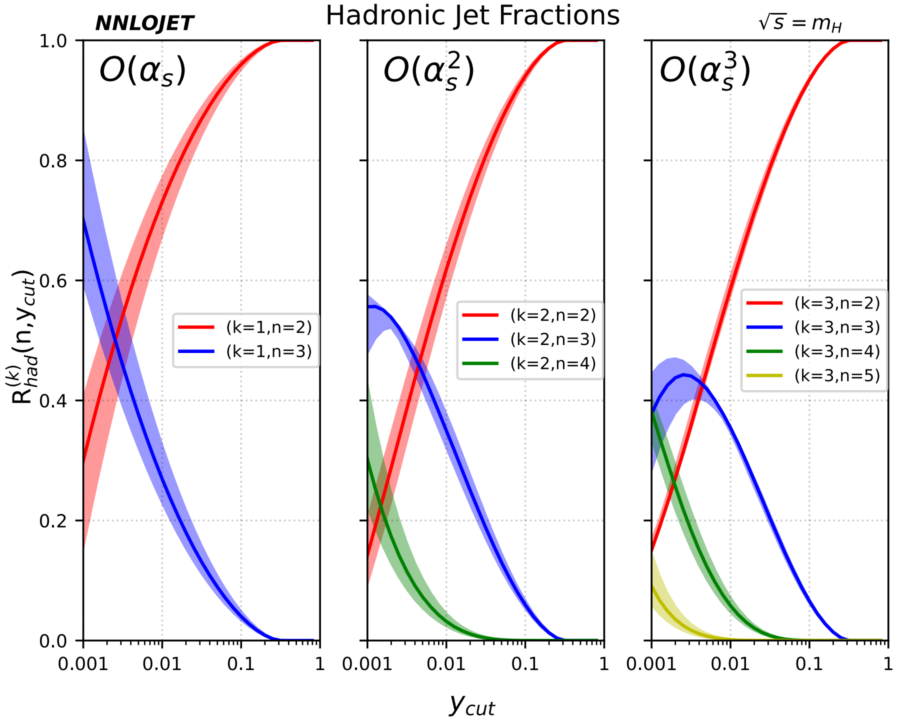
- Shape and size: dominated by $H \longrightarrow bb$:
  - in accordance with the highest inclusive branching ratio
- Highest sensitivity to the decay mode $H \longrightarrow gg$:
  - in hard 3-jet final state kinematical region
    - for large $y_{\rm cut}$ values



Fractional jet rates up to order α_s^3

- Hadronic jet fractions: Perturbative QCD at work
  - 5-jets@LO, 4jets@NLO, 3-jets@NNLO, 2-jets@N^{3}LO

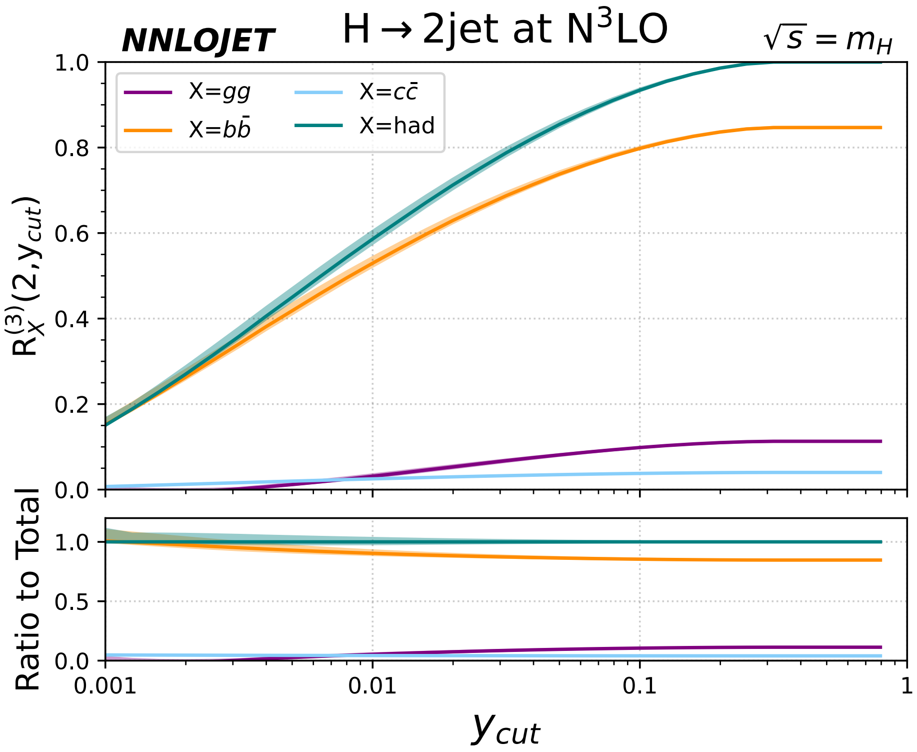
- dependence on y_{cut}: Inclusion of higher orders
  - Lowering y_{cut}: 0pening of higher mutiplicity channels
    with -visible shape changes for n-jet rates (n ≥ 3)
    -need for resummation (at small y_{cut})
- Shape dominated by behaviour of H → bb mode:
  - Similar as in Z-decay:
    colour singlet decay to a fermion pair



H ⟶ 2-jets @ N^{3}LO: Individual hadronic contributions

- Computational ingredients:
  - Higgs total decay rate (known @N^{4}LO)
  - n-jet fractional rates (with n=3,4,5) @ O(α_s^3)

- Size of individual contributions (inclusive):
  - 85 %: H → bb, 11.5 %: H → gg , 4%: H → cc
  - 2-jet rates yield inclusive values for y_{cut} >1/3



- Event shapes for H $\longrightarrow$ 3 particles at NNLO (Preliminary results)

Event shapes for H $\to$ 3 particles @ NNLO

- Classical QCD observables as testing ground for QCD
  - perturbative theory, power corrections and resummation
- Thrust variable (T) : Measure of isotropy of multi-particle final states

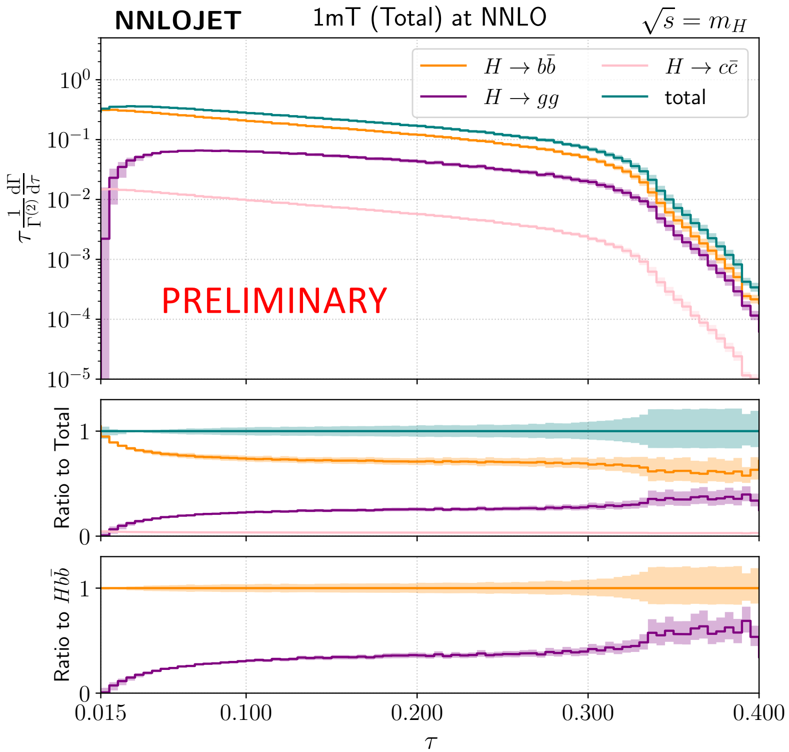
$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \right)$$

$T \to 1$: 2-particle limit, 2-jet (back-to-back configuration)
$T=1/2$: Spherical event

- Observables widely used at LEP : Precise determination of α_s [G. Dissertori@al '09]
  - with Z-decay event shape computation at NNLO [T. Gehrmann, N. Glover, G. Heinrich, AG, '09]
- Hadronic Higgs decays: 3-jet like event-shape observables known at NLO [J. Gao, Y.Gong, W-.Ju, L.L. Yang, '19 (Thrust); G. Coloretti, C. Preuss, AG, '22]
- Used as discriminators between both Higgs decay modes

Event shapes in hadronic Higgs decay: 1-T at NNLO

- Observable: $\tau \frac{1}{\Gamma^{(2)}} \sum_X \frac{\mathrm{d}\Gamma(s,\mu_R)_{H\to X}}{\mathrm{d}\tau}, \quad \text{with } X = b\bar{b}, c\bar{c}, gg, \text{total}$ (1-T=τ)
  - Behaviour dominated by 2-jet region (and $H \longrightarrow bb$ mode)
  - Observable $\longrightarrow 0$ as $\tau \longrightarrow 0$ ($H \rightarrow gg$ negative below $\tau_{min}= 0.015$)
  - Forbidden region at LO : $\tau > 1/3$



- Shape (τ dependence):
  - Small τ : Sharp decrease for $H \longrightarrow gg$ mode (Peak shift)
  - Large τ : (above τ =1/3): Sizeable NNLO corrections
    - phase space restrictions lifted
    - Scale uncertainty band: NLO-like

Conclusions and Outlook

- Predictions for hadronic Higgs decay observables related to H→n-jets
  - NNLO for 3-jet rates, N^3LO for 2-jet rates
  - first application of the new designer antenna formalism @NNLO to a decay process
- Distinct signatures
  - Rates largest for the H→bb decay mode
  - Corrections largest for H→gg (HEFT)
  - Highest sensitivity for gluonic Higgs decay mode: by selecting hard three jet final states
- Work in progress : 3-jet like Higgs decay event-shape observables @ NNLO

- Inclusion of higher order QCD corrections crucial for precise Higgs phenomenology at lepton colliders

Thank you !