High-precision theory predictions for LHC and EIC processes

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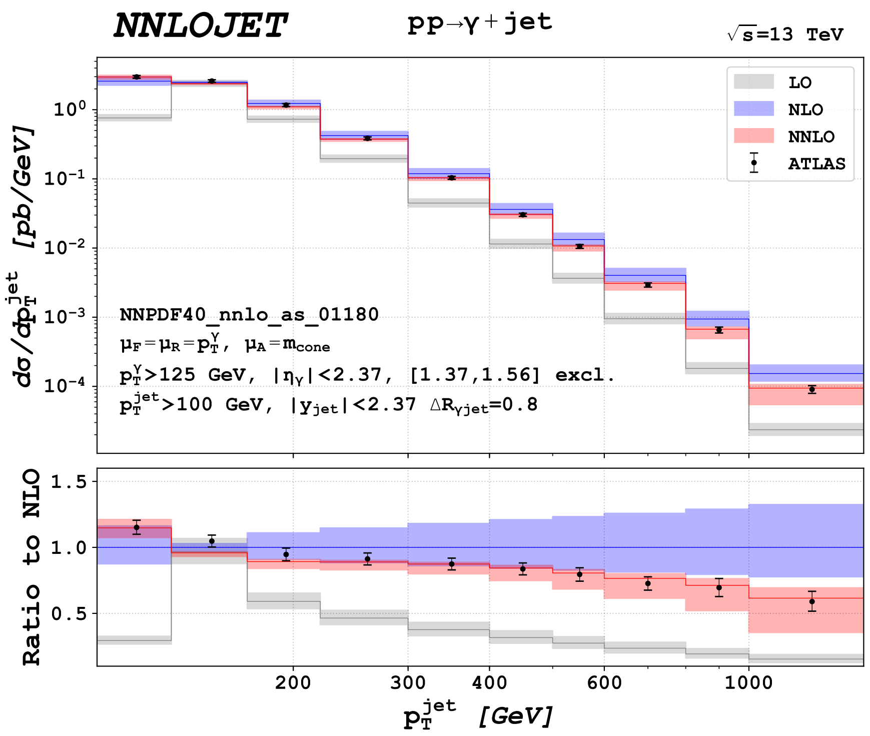
Precision Predictions: State-of-the-art

- NNLO QCD predictions for $2 \longrightarrow 2$ processes (NNLO revolution, 2015 $\longrightarrow$)
  - accomplished during past 10 years on case-by-case basis
  - as parton-level event generators (full final state information)
  - computationally expensive
  - current frontier at NNLO: $2 \longrightarrow 3$



- Typical size of corrections and uncertainty
  - NLO corrections: 10..100%, uncertainty: 10..30%
  - NNLO corrections: 2..15%, uncertainty: 3..8%
  - expect N3LO to yield uncertainty at level of 1%.

Ingredients to fixed order calculations

- Matrix elements with extra real (R) or virtual (V) partons

	Matrix elements	Parton evolution
LO	Born	1-loop
NLO	R, V	2-loop
NNLO	RR, RV, VV	3-loop
N3LO	RRR, RRV, RVV, VVV	4-loop

- Infrared singularities in all R-type and V-type subprocesses
  - sum of all subprocesses finite
  - require subtraction procedure to arrange IR cancellations between subprocesses
- Incoming hadrons: parton distributions
  - mass factorization of initial-state radiation and parton evolution

NNLOJET code

- NNLO parton level event generator
  - Based on antenna subtraction
- Provides infrastructure
  - Process management
  - Phase space, histogram routines
  - Validation and testing
  - Parallel computing (MPI) support for warm-up and production
  - ApplGrid/fastNLO interfaces
- Processes implemented at NNLO
  - Z+(0,1)jet, γ+1 jet, H+(0,1)jet, W+(0,1)jet
  - DIS-2j, LHC-2j
  - Typical runtimes: 60’000-250’000 core-hours

NNLOJET project:
A.Huss, L.Bonino, O.Braun-White, S.Caletti, X.Chen, J.Cruz-Martinez, J.Currie, W.Feng, G.Fontana, E.Fox, R.Gauld, A.Gehrmann-De Ridder, E.W.N.Glover, M.Höfer, P.Jakubcik, M.Jaquier, M.Löchner, F.Lorkowski, I.Majer, M.Marcoli, P.Meinzinger, J.Mo, T. Morgan, J.Niehues, J.Pires, C.Preuss, A.Rodriguez-Gracia, K.Schönwald, R.Schürmann, V.Sotnikov, G.Stagnitto, D.Walker, J.Whitehead, T.Z.Yang, H.Zhang, TG

NNLOJET code

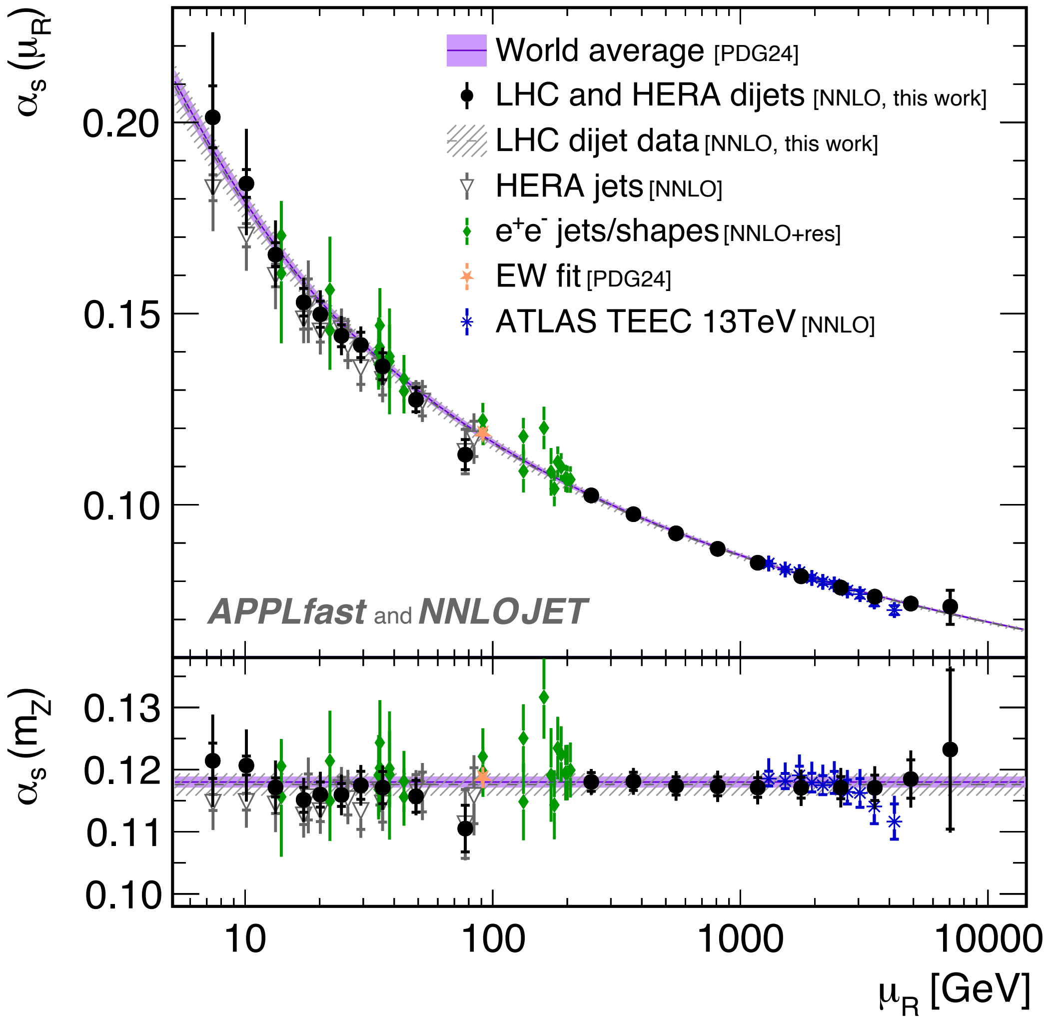
- Open-source code release: NNLOJET v1.0.2
  - download from nnlojet.hepforge.org
- Runcard options
  - process/sub-process selection
  - generic histogramming
  - multi-run features: e.g. jet radius
  - example runcards for many processes
- Cluster workflow management: DOKAN
  - automated resource allocation
  - works with slurm and htcondor (lxplus)
  - combination of results, quality control

#	LO	R	V	RR	RV	VV
1	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	WRM A[0] D[4]	PRD A[1] D[0]	PRD A[0] D[1]
2	PRD A[0] D[1]	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]	WRM A[1] D[3]	PRD A[0] D[1]
3	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
4	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]
5	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
6	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]
7	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
8	-	-	-	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
9	-	-	-	PRD A[0] D[0]	PRD A[1] D[0]	PRD A[0] D[1]
10	-	-	-	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
11	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
12	-	-	-	PRD A[0] D[1]	-	-
13	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
14	-	-	-	PRD A[0] D[0]	-	-
15	-	-	-	PRD A[0] D[1]	WRM A[1] D[3]	PRD A[0] D[1]
16	-	-	-	PRD A[1] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
17	-	-	-	PRD A[0] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
18	-	-	-	PRD A[1] D[0]	-	-
19	-	-	-	PRD A[0] D[1]	-	-
20	-	-	-	PRD A[0] D[1]	-	-
21	-	-	-	WRM A[1] D[3]	-	PRD A[0] D[1]
22	-	-	-	PRD A[0] D[1]	-	PRD A[0] D[1]
23	-	-	-	PRD A[1] D[0]	-	PRD A[0] D[1]
24	-	-	-	WRM A[1] D[3]	-	PRD A[0] D[1]
25	-	-	-	WRM A[1] D[3]	-	-
26	-	-	-	PRD A[1] D[0]	-	-
27	-	-	-	PRD A[0] D[1]	-	-

Two-jet production

- 2-jet production at LHC
  - most basic hard QCD process
  - large cross section: high-statistics measurement
  - combine ATLAS/CMS data sets
  - group data according to $\mu = M_{2j}$
- correlation of α_s with PDFs
  - vary PDF through APPLgrid interface
  - combine with DIS 2-jet (HERA)
- reach: 7.4 GeV ... 7.0 TeV

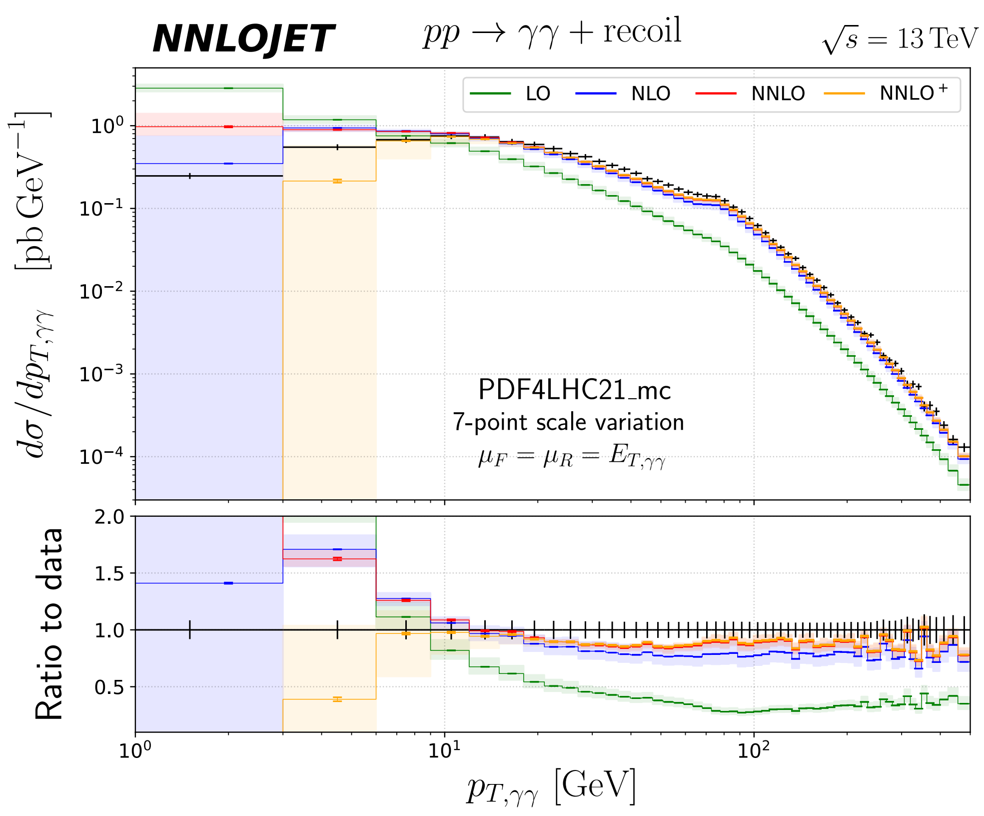
$$\alpha_s (M_Z) = 0.1178 \pm 0.0014_{exp,pdf} \pm 0.0017_{th}$$



(APPLfast+NNLOJET: F. Ahamdova et al.)

Di-photon+jet production at NNLO

- Di-photon+jet production: 2⟶3 process
  - multi-differential precision measurements
  - derived observables
    - transverse momentum distribution
    - event shape distributions
  - interplay of fiducial cuts
  - agreement with earlier NNLO results
    (H.Chawdhry, M.Czakon, A.Mitov, R.Poncelet)
- Photon isolation
  - use dynamical cone: no fragmentation
- Testing ground
  - treatment of loop-induced processes
  - towards $\gamma\gamma$ at N3LO



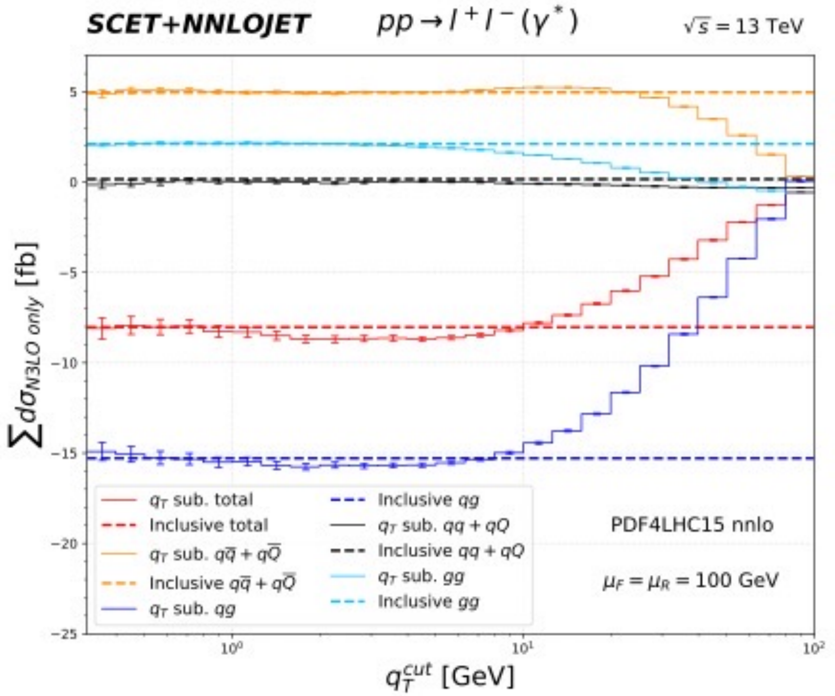
(F.Buccioni, X.Chen, W.J.Feng, A.Huss, M.Marcoli, TG)

N3LO for Drell-Yan observables

Slicing parameter: transverse momentum (q_T slicing) [S.Catani, M.Grazzini]

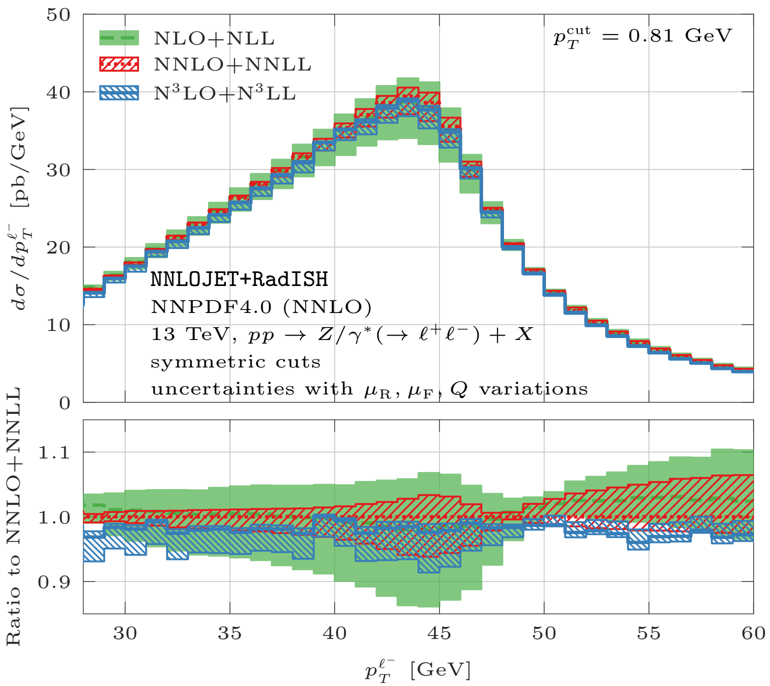
$$\frac{d\sigma_X^{N3LO}}{dO} = \mathcal{H}_{N3LO} \otimes \frac{d\sigma_X^{LO}}{dO} + \left[\int_{q_{T,X}} \frac{d\sigma_{X+j}^{NNLO}}{dO} - \frac{d\sigma_{X,CT}^{NNLO}}{dO}(q_T) \right]$$

- below-cut contribution from expansion of N3LL q_T resummation to $O(\alpha_s^3)$ [W.Bizon, P.Monni, E.Re, P.Torrielli; S.Camrada, L.Cieri, G.Ferrera;T.Becher, T.Neumann; W.L.Ju, M.Schönherr]
- ingredients: three-loop soft and beam functions [Y.Li, H.X.Zhu; M.Ebert, B.Mistlberger, G.Vita; M.X.Luo, T.Z.Yang, Y.J.Zhu]
- check: independence on $q_{T,cut}$ slicing parameter
- check: reproduce inclusive coefficient functions (no ingredients or methodology in common!) [X.Chen, E.W.N.Glover, A.Huss, T.Z.Yang, H.X.Zhu, TG]



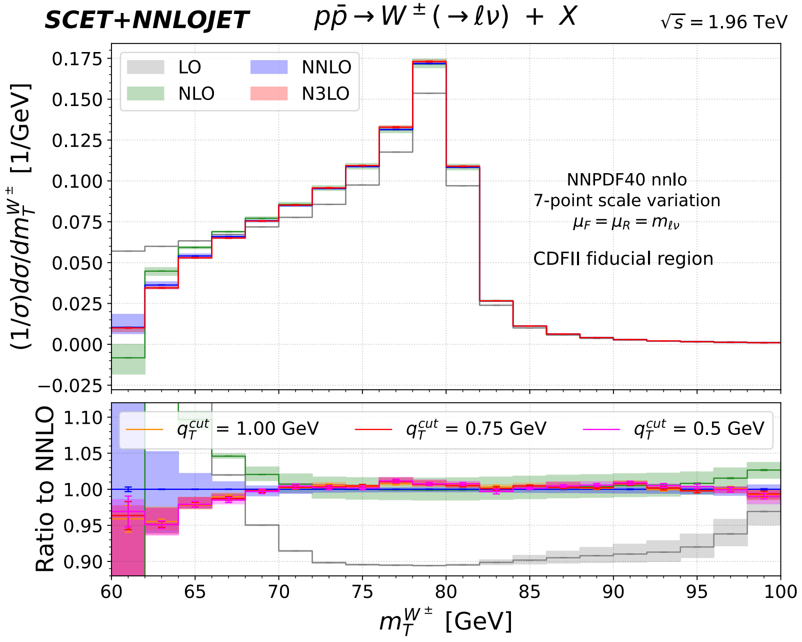
N3LO for Drell-Yan observables

Results: fiducial distributions



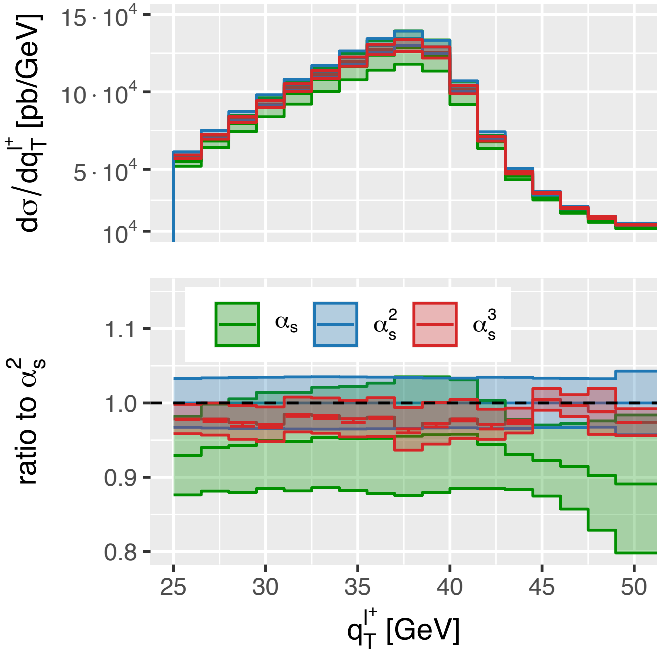
single lepton distribution in NC Drell-Yan, matched to N3LL resummation (RadISH)

[X.Chen, E.W.N.Glover, A.Huss, P.F.Monni, E.Re, L.Rottoli, P.Torrielli, TG]



transverse mass distribution in W boson production (CDF II cuts)

[X.Chen, E.W.N.Glover, A.Huss, T.Z.Yang, H.X.Zhu, TG]

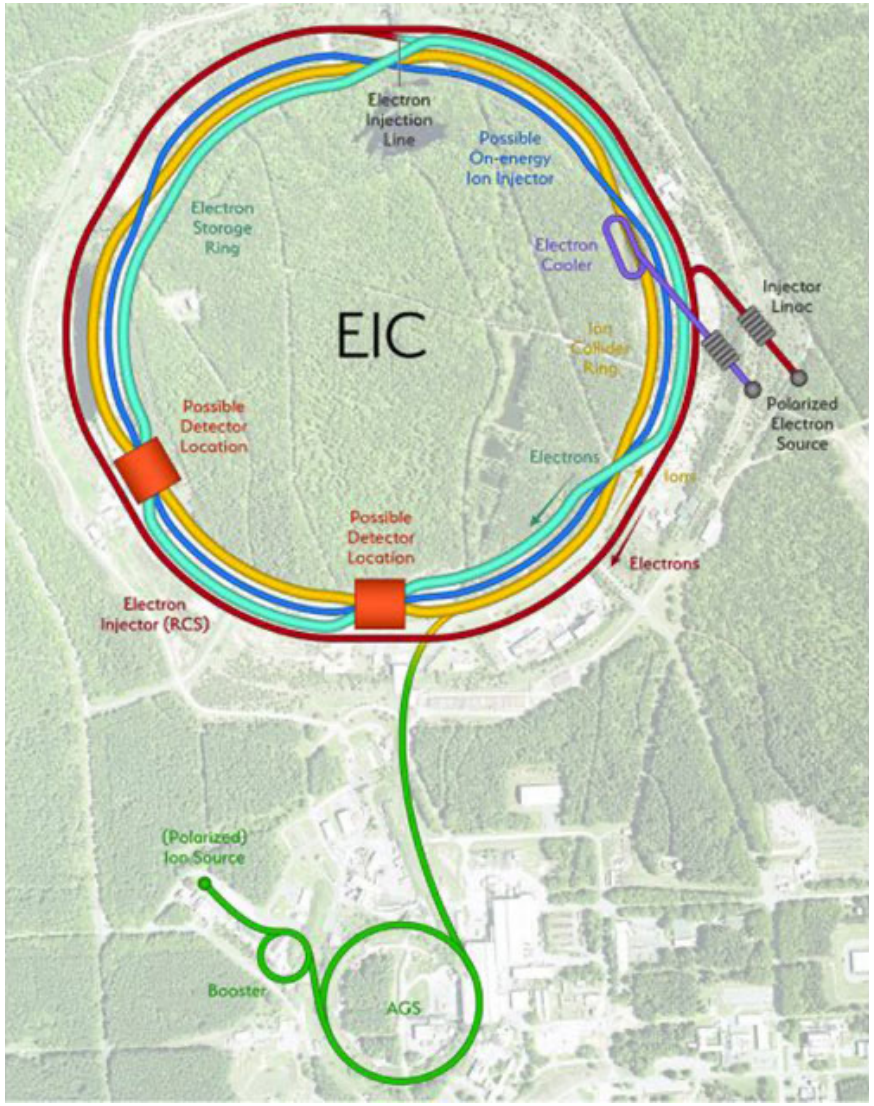


charged lepton distribution in W boson production (ATLAS 5.02 TeV)

[J.Campbell, T.Neumann]

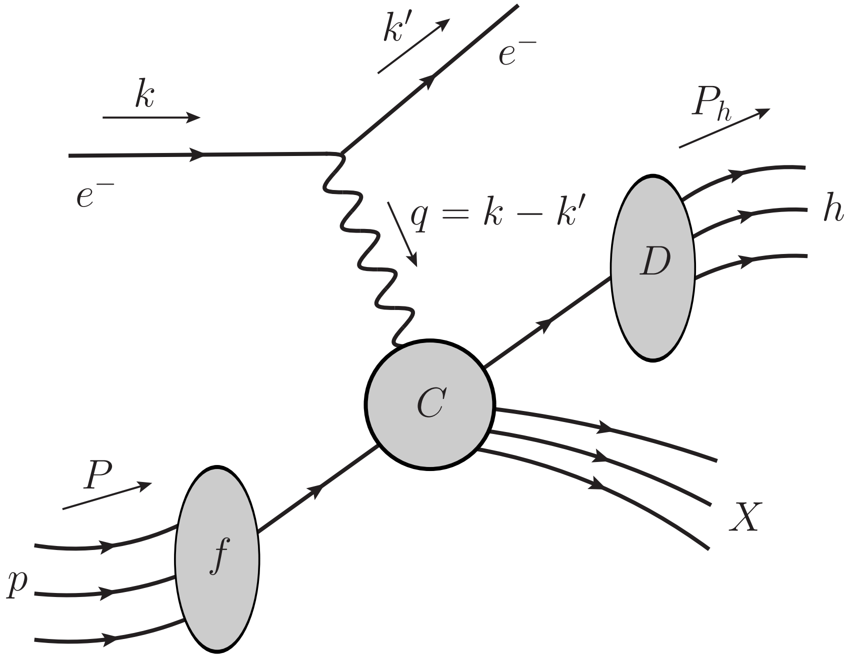
Beyond LHC: EIC

- Electron-Ion Collider at BNL
  - High-luminosity: $10^{33}...10^{34}$ $cm^{-2}s^{-1}$
  - Centre-of-mass energy range: 40..140 GeV
  - Full identification of hadronic final state
  - precision QCD: hadron structure, fragmentation,....
  - Polarized collisions
- Theory challenges
  - NNLO precision for benchmark processes
  - identified hadrons, fully exclusive final states
  - novel types of observables: TMD, GPD, ...



Semi-inclusive deep inelastic scattering

$l(k) + p(P) \to l(k') + h(P_h) + X$



variables

$$Q^2 = -q^2 \qquad x = \frac{Q^2}{2P\cdot q} \qquad z = \frac{P\cdot P_h}{P\cdot q} \qquad y = \frac{P\cdot q}{P\cdot k}$$

cross sections

$$\frac{\mathrm{d}^3\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y} F_T^h(x,z,Q^2) + \frac{1-y}{y} F_L^h(x,z,Q^2)\right]$$

$$\frac{\mathrm{d}^3\Delta\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} = \frac{4\pi\alpha^2}{Q^2}(2-y)\, g_1^h(x,z,Q^2)$$

Observables: multiplicity

$$\frac{\mathrm{d}M_h}{\mathrm{d}z} = \frac{\mathrm{d}^3\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} \Big/ \frac{\mathrm{d}^2\sigma^{\mathrm{DIS}}}{\mathrm{d}x\mathrm{d}y}$$

spin asymmetry

$$A_1^h(x,z,Q^2) = \frac{g_1^h(x,z,Q^2)}{F_1^h(x,z,Q^2)}$$

SIDIS coefficient functions at NNLO

Partonic channels

- $$C_{qq}^{i,(2)} = C_{\bar{q}\bar{q}}^{i,(2)} = e_q^2 C_{qq}^{i,\mathrm{NS}} + \left(\sum_j e_{q_j}^2\right) C_{qq}^{i,\mathrm{PS}} \,,$$

$$C_{\bar{q}q}^{i,(2)} = C_{q\bar{q}}^{i,(2)} = e_q^2 C_{\bar{q}q}^{i} \,,$$

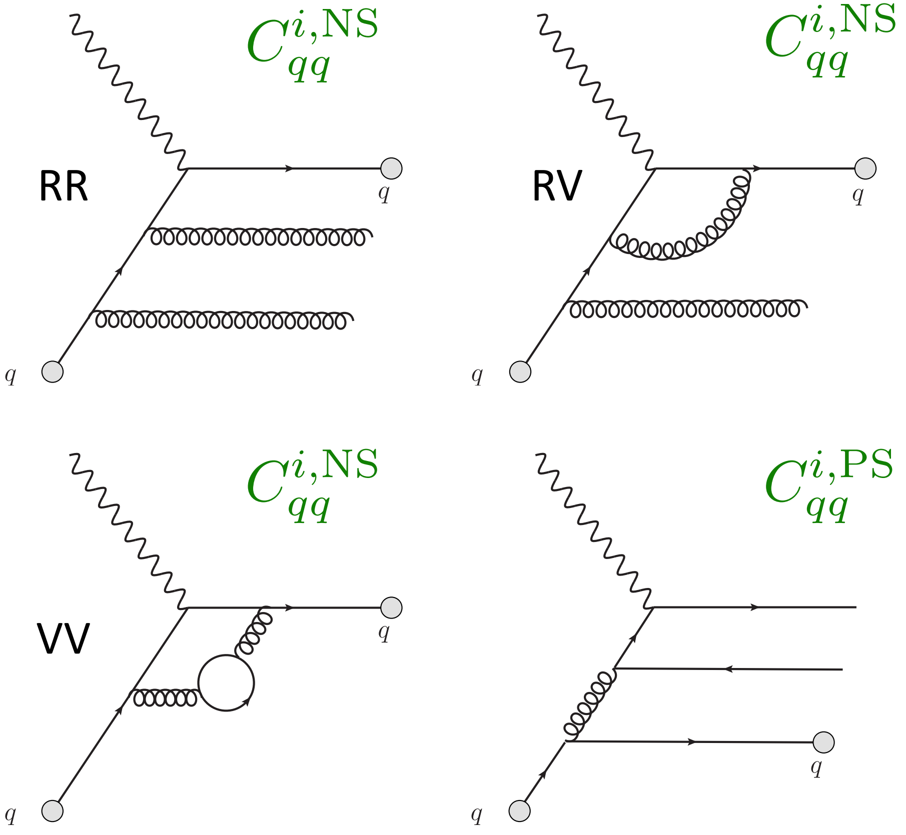
$$C_{q'q}^{i,(2)} = C_{\bar{q}'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} + e_q e_{q'} C_{q'q}^{i,3} \,,$$

$$C_{\bar{q}'q}^{i,(2)} = C_{q'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} - e_q e_{q'} C_{q'q}^{i,3} \,,$$

$$C_{gq}^{i,(2)} = C_{g\bar{q}}^{i,(2)} = e_q^2 C_{gq}^{i} \,,$$

$$C_{qg}^{i,(2)} = C_{\bar{q}g}^{i,(2)} = e_q^2 C_{qg}^{i} \,,$$

$$C_{gg}^{i,(2)} = \left(\sum_j e_{q_j}^2\right) C_{gg}^{i} \,,$$



not a forward scattering amplitude:
evaluate all contributions separately

SIDIS: NNLO corrections

RR: tree-level double-real matrix elements

$$C_{\mathrm{RR}}^{(2)} \sim \int \mathrm{d}\phi_3(k_p, k_j, k_k; q, k_i)\, |\mathcal{M}|^2_{\mathrm{RR}}(\{s_{ab}\})\, \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip}+s_{ij}+s_{ik}}\right)$$

- correspond to cut two-loop integrals = antenna functions for fragmentation processes

$$C_{\mathrm{RR}}^{(2)} \sim \int \mathrm{d}^{4-2\epsilon}k_p \mathrm{d}^{4-2\epsilon}k_j\, |\mathcal{M}|^2_{\mathrm{RR}}(\{s_{ab}\})\, \delta^+(k_p^2)\, \delta^+(k_j^2)\, \delta^+((q+k_i-k_p-k_j)^2)\, \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip}+s_{ij}+s_{ik}}\right)$$

- use Cutkosky rule to arrive at Standard integral form

$$I_{t,r,s}(p_1,\ldots,p_n) = \int \frac{d^dk}{(2\pi)^d} \frac{d^dl}{(2\pi)^d} \frac{1}{D_1^{m_1}\ldots D_t^{m_t}} S_1^{n_1}\ldots S_q^{n_q}$$

SIDIS: NNLO corrections

Integrals [L.Bonino, M.Marcoli, R.Schürmann, G.Stagnitto, TG]

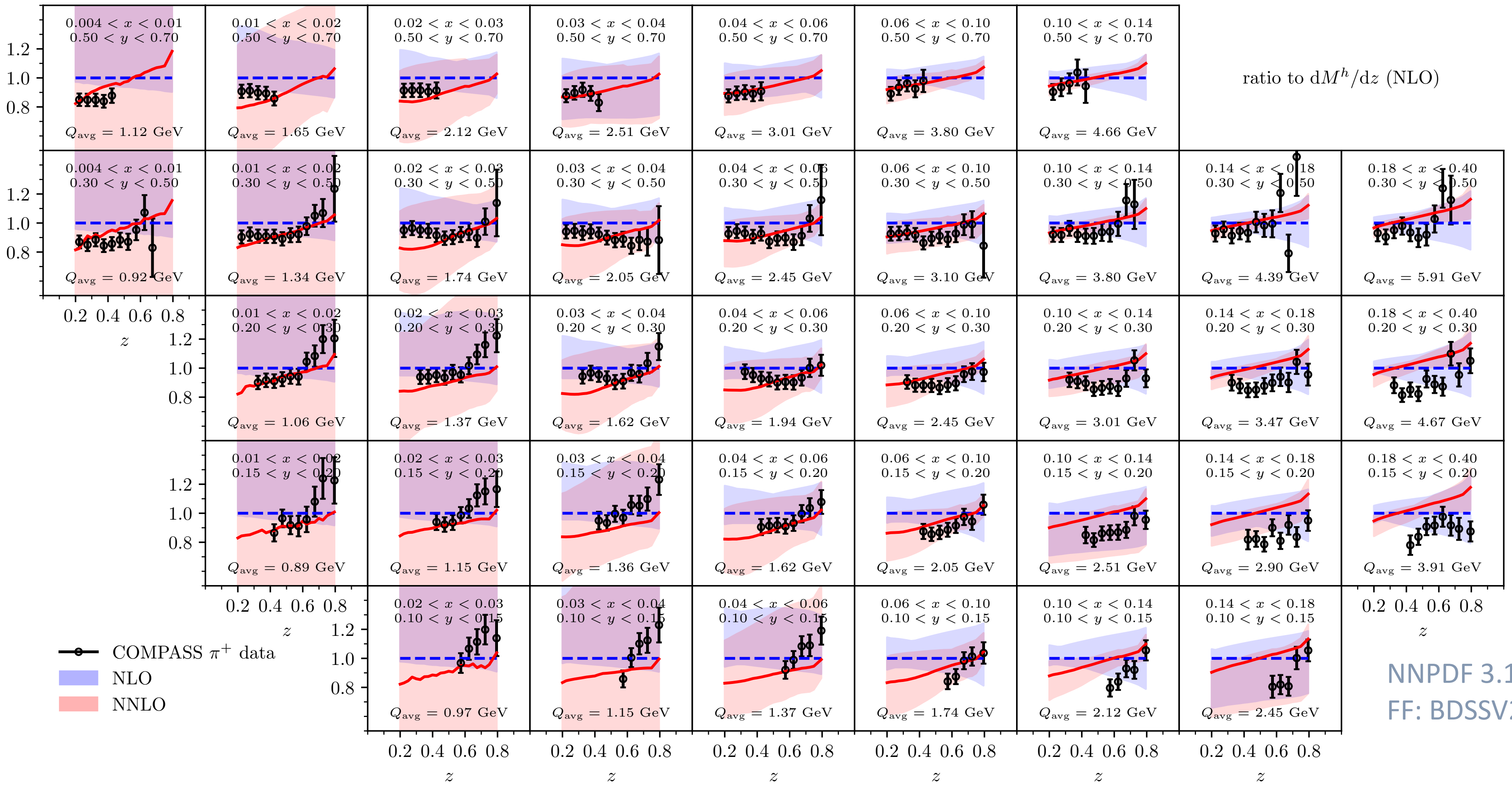
- 12 propagators (4 cut), 7 of them linearly independent
- 13 integral families with total 21 master integrals
- analytical results throughout
- family A,B,C previously computed for photon fragmentation [R.Schürmann, TG]

$$\begin{aligned}
D_1 &= (q-k_p)^2\,,\\
D_2 &= (p_i+q-k_p)^2\,,\\
D_3 &= (p_i-k_l)^2\,,\\
D_4 &= (q-k_l)^2\,,\\
D_5 &= (p_i+q-k_l)^2\,,\\
D_6 &= (q-k_p-k_l)^2\,,\\
D_7 &= (p_i-k_p-k_l)^2\,,\\
D_8 &= (k_p+k_l)^2\,,\\
D_9 &= k_p^2\,,\\
D_{10} &= k_l^2\,,\\
D_{11} &= (q+p_i-k_p-k_l)^2\,,\\
D_{12} &= (p_i-k_p)^2+Q^2\frac{z}{x}\,,
\end{aligned}$$

family	master	deepest pole	at $x=1$	at $z=1$
	$I[0]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
A	$I[5]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
B	$I[7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-2,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-3,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
C	$I[5,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[3,5,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
D	$I[1]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,4]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,3,4]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
E	$I[1,3,5]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
G	$I[1,3,8]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
H	$I[1,4,5]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
I	$I[2,4,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
J	$I[4,7]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[3,4,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
K	$I[3,5,8]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
L	$I[4,5,7]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
M	$I[4,5,8]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$

Table 1. Summary of the double real radiation master integrals.

Numerical results: π^+ multiplicity



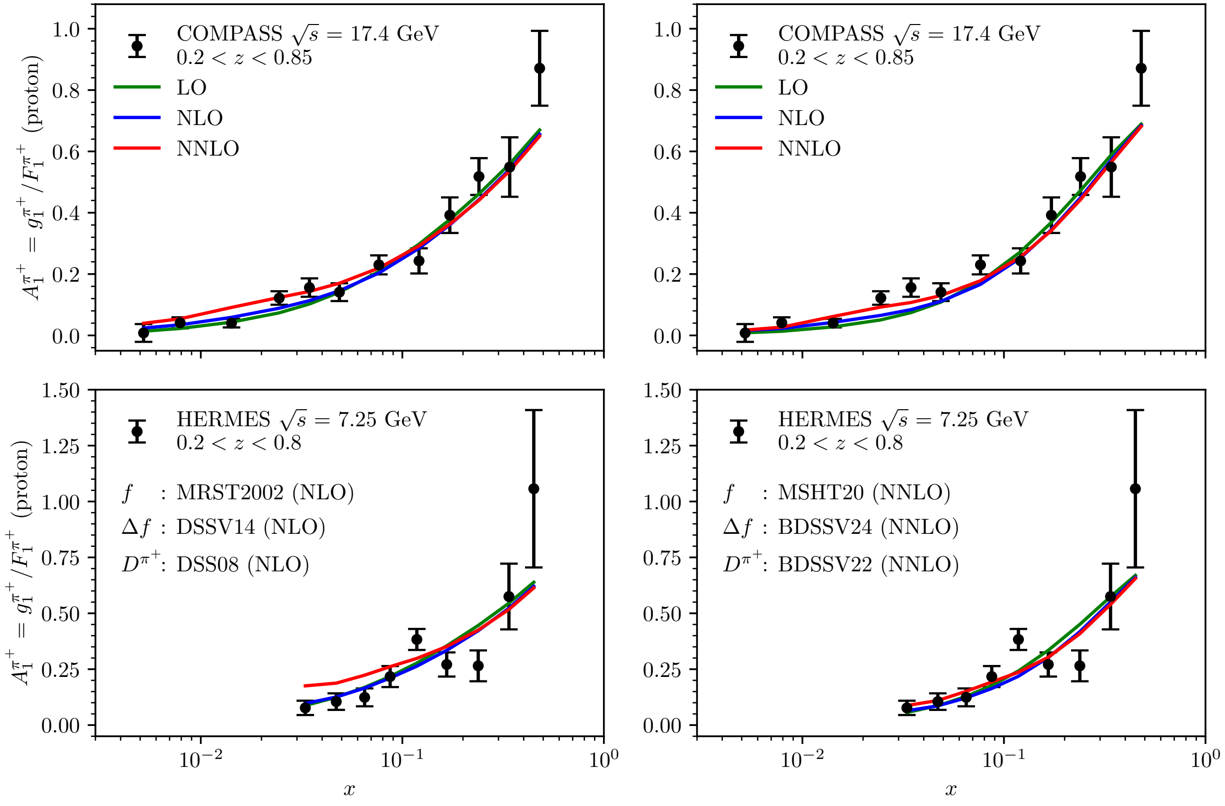
Experiment	# data	NLO	Approx. NNLO	NNLO
SIA	288	1.05	0.96	0.85
COMPASS	510	0.98	1.14	0.96
HERMES	224	2.24	2.27	2.52
TOTAL	1022	1.27	1.33	1.26

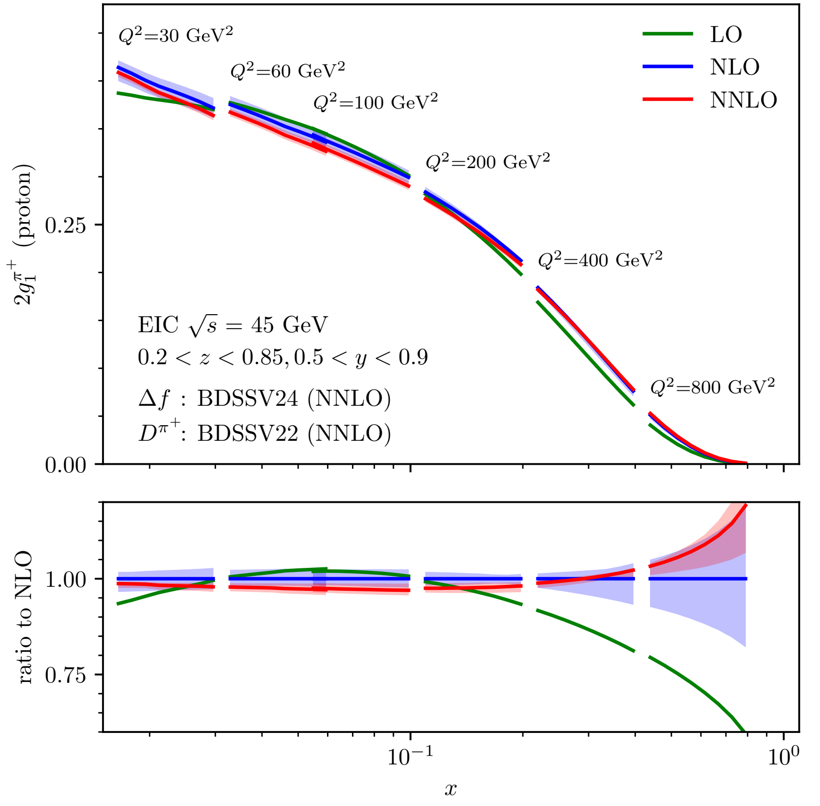
[I.Borsa: BDSSV22 revisited]

NNPDF 3.1
FF: BDSSV22

Numerical results

Polarized SIDIS [L.Bonino, M.Löchner, K.Schönwald, G.Stagnitto, TG]





- analytical agreement with independent calculations [S.Goyal, R.Lee, S.Moch, V.Pathak, N.Rana, V.Ravindran]

Summary

- LHC and EIC are ultimate precision challenge for QCD
  - predictions for complex final states at per-cent level accuracy
  - enable precise determination of fundamental parameters
- Theory ready to face the LHC challenge
  - NNLO predictions becoming the new standard
  - first N3LO results
- Precision theory for EIC only just starting
  - semi-inclusive and exclusive final states
  - require new frameworks beyond collinear factorization