QFT beyond particle physics

Anomalous dimensions and critical exponents for condensed matter systems

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in collaboration with

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A Loop Summit 2
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- Calculate anomalous dimensions/critial exponents of the ren. group equations in 2d and/or 4d and extrapolate to 3d
- Calculation of anom dim very well established field in particle physics
  $\hookrightarrow$ apply these methods here

Models

- Gross-Neveu-Yukawa [Zerf,Mihaila,PM,Herbut,Scherer '17]
  - chiral Ising model
  - chiral XY model
  - chiral Heisenberg model
- QED_3-Gross-Neveu-Yukawa [Zerf,PM,Boyack,Maciejko '18]
- Néel algebraic spin liquid [Zerf,Boyack,PM,Gracey,Maciejko '19]
- Abelian Higgs model [Ihrig,Zerf,PM,Herbut,Scherer '19]
- lattice quantum electrodynamics [Zerf,Boyack,PM,Gracey,Maciejko '20]

In short: Models with interactions between scalars, fermions and photons

Problem

Model: Gross-Neveu-Yukawa

renormalised Lagrangian

$$\mathcal{L} = Z_\psi \bar{\psi} \not{\partial} \psi + Z_{\phi\bar{\psi}\psi} y \mu^{\frac{\epsilon}{2}} \phi \bar{\psi} \psi + \frac{1}{2} \phi (Z_{\phi^2} m^2 - Z_\phi \partial_i^2) \phi + Z_{\phi^4} \lambda \mu^\epsilon \phi^4$$

with $O(N)$ symmetry

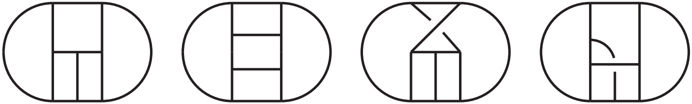
1. calculate corresponding β functions and anomalous dimensions in 4 dimensions ($d = 4 - \epsilon$)
2. extrapolate to 3 dimensions

Setup

Use computational setup similar to calculation of QCD β function @ five loops

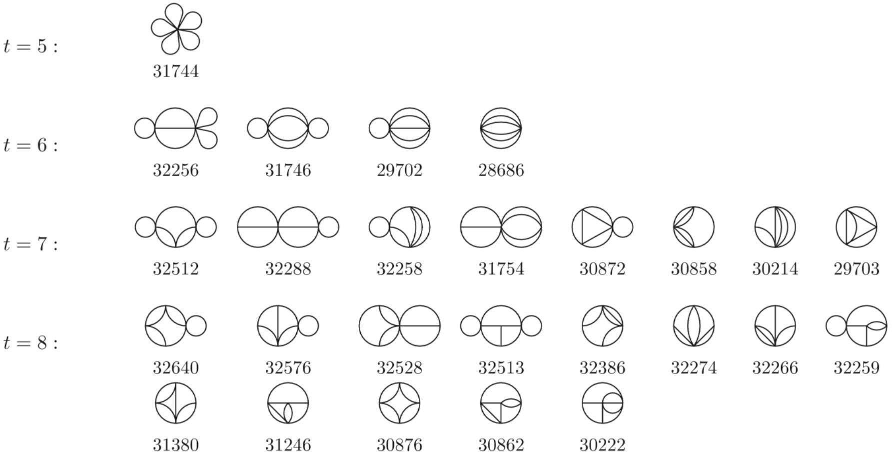
[Luthe,Maier,Marquard,Schröder]

- isolate ultraviolet divergences by introducing an auxiliary mass
- expand required n-point functions in external momenta
- reduce to master integrals NEW using finite fields
- 5-loop fully massive tadpoles need to be calculated
  $\Rightarrow$ 4 top-level families, high-precision numerical results [Luthe '15]



- use PSQL at the end to obtain result in terms of zetas

Massive Tadpoles – 5 loop



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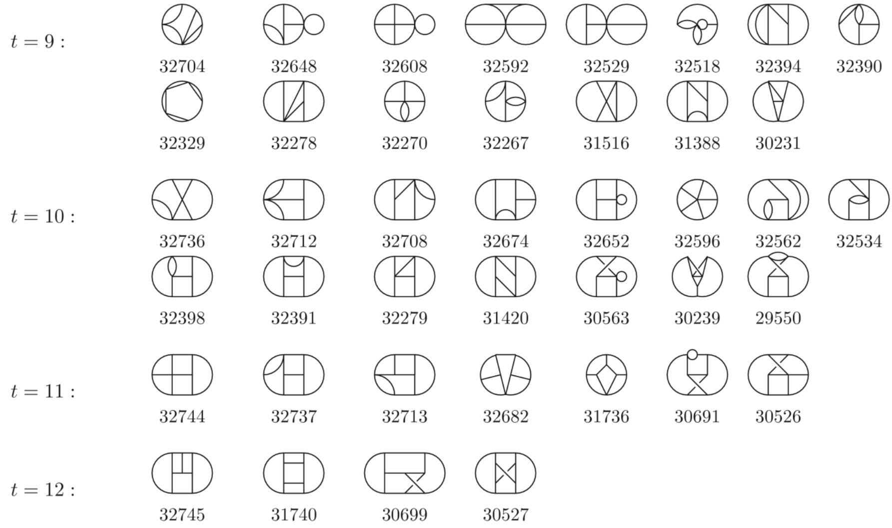

[Luthe '15]

Results

$$\beta_\lambda(\lambda, y) = \frac{d\lambda}{d\ln\mu}, \quad \beta_y(\lambda, y) = \frac{dy}{d\ln\mu}$$

$$\beta_y = -\epsilon y + \sum_{L=1}^{5} \beta_y^{(L)}, \quad \beta_\lambda = -\epsilon\lambda + \sum_{L=1}^{5} \beta_\lambda^{(L)}$$

with one-loop coefficients

$$\beta_\lambda^{(1)} = 36\lambda^2 + 4\lambda N y - N y^2$$

$$\beta_y^{(1)} = 2Ny^2 + 3y^2$$

value of N corresponds to different physical scenarios

- $N = 2$: graphene on a hexagonal lattice
- $N = 1$: spin-less fermions on a honeycomb lattice
- $N = 5$: for comparison with other methods

Fixed points: $N = 2$

Solve

$$\beta_y(y^*, \lambda^*) = \beta_\lambda(y^*, \lambda^*) = 0$$

$$\begin{aligned} y^* =& \left(-\frac{56967\zeta_3{}^2}{13176688} - \frac{9370620999331\zeta_3}{755093162831616} + \frac{19976417\zeta_4}{1481167296} - \frac{408151805\zeta_5}{5489031744} + \frac{9525\zeta_6}{537824} - \frac{46467\zeta_7}{2151296} - \frac{2263630018850239}{736498993696887456}\right)\epsilon^5 \\ &+ \left(\frac{53897407\zeta_3}{2221750944} - \frac{165\zeta_4}{19208} + \frac{635\zeta_5}{33614} + \frac{355798279177}{92460387285504}\right)\epsilon^4 + \left(\frac{2274385}{634785984} - \frac{165\zeta_3}{9604}\right)\epsilon^3 + \frac{533\epsilon^2}{10584} + \frac{\epsilon}{7} \\ \lambda^* =& \left(-\frac{291071171435\zeta_3{}^2}{15731107558992} - \frac{701069774301193\zeta_3}{81833221521876384} + \frac{4358213827\zeta_4}{1926258068448} - \frac{46314065257\zeta_5}{793165087008} + \frac{2651375\zeta_6}{130691232} - \frac{9304531\zeta_7}{82287072}\right. \\ &\left. - \frac{20192282387582749855}{20433428081126445579264}\right)\epsilon^5 + \left(\frac{2067645667\zeta_3}{240782258556} - \frac{45139\zeta_4}{13224708} + \frac{306275\zeta_5}{8168202} + \frac{17286322925167}{10020394472066496}\right)\epsilon^4 \\ &+ \left(-\frac{47437\zeta_3}{3306177} - \frac{24550427}{91726574688}\right)\epsilon^3 + \frac{1691\epsilon^2}{134946} + \frac{2\epsilon}{63} \end{aligned}$$

insert into anomalous dimensions to get critical exponents

Critical exponents: $N = 2$

$$\eta_\psi = \frac{1}{14}\epsilon - \frac{71}{10584}\epsilon^2 + \left[-\frac{2432695}{158696496} - \frac{18}{2401}\zeta_3\right]\epsilon^3 + \left[\frac{150}{16807}\zeta_5 - \frac{27}{4802}\zeta_4 + \frac{11109323}{555437736}\zeta_3 - \frac{111266497289}{11557548410688}\right]\epsilon^4$$
$$+ \left[-\frac{89279362217932877}{117839838991501992 96} - \frac{1905116200933}{377546581415808}\zeta_3 - \frac{136650473}{2744515872}\zeta_5 - \frac{5643}{537824}\zeta_7 - \frac{2004}{823543}\zeta_3^2 + \frac{375}{33614}\zeta_6 + \frac{11109323}{740583648}\zeta_4\right]\epsilon^5 + \mathcal{O}(\epsilon^6)$$

$$\eta_\phi = \frac{4}{7}\epsilon + \frac{109}{882}\epsilon^2 + \left[\frac{1170245}{26449416} - \frac{144}{2401}\zeta_3\right]\epsilon^3 + \left[\frac{20491307339}{481564517112} - \frac{108}{2401}\zeta_4 + \frac{1200}{16807}\zeta_5 + \frac{1563532}{23143239}\zeta_3\right]\epsilon^4$$
$$+ \left[\frac{32079891787774525}{98199865826251 6608} - \frac{1237285035017}{31462215117984}\zeta_3 - \frac{69575957}{228709656}\zeta_5 - \frac{15885}{823543}\zeta_3^2 - \frac{5643}{67228}\zeta_7 + \frac{1500}{16807}\zeta_6 + \frac{390883}{7714413}\zeta_4\right]\epsilon^5 + \mathcal{O}(\epsilon^6)\ .$$

$$\eta_{\phi^2} = -\frac{8}{21}\epsilon + \frac{2942}{22491}\epsilon^2 + \left[\frac{88644929}{2547960408} - \frac{144404}{1102059}\zeta_3\right]\epsilon^3$$
$$+ \left[\frac{18303692723219}{835032872672208} - \frac{36101}{367353}\zeta_4 + \frac{155735}{1361367}\zeta_5 + \frac{1707335624}{20065188213}\zeta_3\right]\epsilon^4$$
$$+ \left[\frac{3531904209162553649}{21284820917840047 84} - \frac{6505866583}{16524272646}\zeta_5 + \frac{24305439575}{327731407479}\zeta_3^2 + \frac{44217078119105}{681943512682303 2}\zeta_3 + \frac{778675}{5445468}\zeta_6 + \frac{8001521}{13714512}\zeta_7 + \frac{426833906}{6688396071}\zeta_4\right]\epsilon^5 + \mathcal{O}(\epsilon^6).$$

Critical exponents: $N = 2$ cont'd

$$\frac{1}{\nu} = 2 - \frac{20}{21}\epsilon + \frac{325}{44982}\epsilon^2 + \left[-\frac{36133009}{3821940612} - \frac{78308}{1102059}\zeta_3 \right]\epsilon^3$$
$$+ \left[\frac{58535}{1361367}\zeta_5 - \frac{19577}{367353}\zeta_4 + \frac{351753380}{20065188213}\zeta_3 - \frac{17228234202607}{835032872672208} \right]\epsilon^4$$
$$+ \left[-\frac{1368564934335029857 9}{851392836713601899136} - \frac{5916014759}{66097090584}\zeta_5 + \frac{30626922980}{327731407479}\zeta_3^2 + \frac{1249594437836159}{272777405072921 28}\zeta_3 \right.$$
$$\left. + \frac{292675}{5445468}\zeta_6 + \frac{9152693}{13714512}\zeta_7 + \frac{87938345}{6688396071}\zeta_4 \right]\epsilon^5 + \mathcal{O}(\epsilon^6)$$

$$\eta_\psi = 0.071429\epsilon - 0.006708\epsilon^2 - 0.024341\epsilon^3 + 0.017584\epsilon^4 - 0.051782\epsilon^5 + \mathcal{O}(\epsilon^6)$$
$$\eta_\phi = 0.571429\epsilon + 0.123583\epsilon^2 - 0.027849\epsilon^3 + 0.149112\epsilon^4 - 0.296922\epsilon^5 + \mathcal{O}(\epsilon^6)$$
$$\frac{1}{\nu} = 2 - 0.952381\epsilon + 0.007225\epsilon^2 - 0.094868\epsilon^3 - 0.012653\epsilon^4 + 0.823067\epsilon^5 + \mathcal{O}(\epsilon^6)$$

Going to 3 dimensions

$d = 3 \rightarrow \epsilon = 1$

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even better: use two-sided Padé approximation including 2d data

2 dimensional Gross-Neveu model

$$\mathcal{L}^{\text{GN}} = \bar{\psi}\partial\!\!\!/\psi + h\phi\bar{\psi}\psi + \frac{1}{2}\phi^2$$

known up to 4-loop order, five loop still to do

4d GNY and 2d GN models belong to the same universality class

Interpolation

two-sided Padé

$$\mathcal{P}_{[m/n]}(d) = \frac{\sum_{p=0}^{m} a_p d^p}{1 + \sum_{q=1}^{n} b_q d^q}$$

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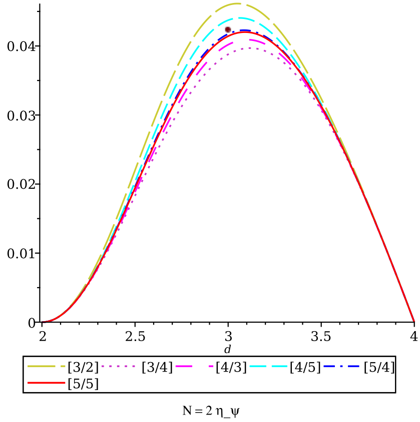
Interpolating polynomial

$$\mathcal{I}_{[i,j]}(d) = \sum_{m=0}^{i} \eta_m^{(2)} (d-2)^m + \sum_{n=i+1}^{i+j+1} c_n (d-2)^n$$

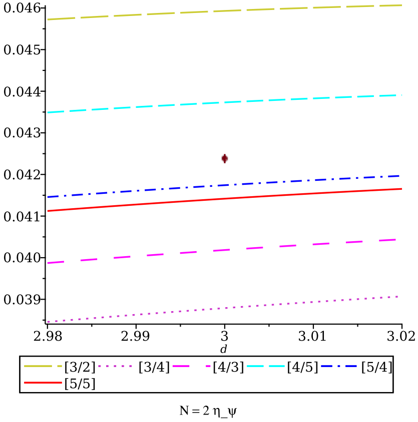
Previous results

Method and source	η_ψ	η_ϕ	$1/\nu$
Large N [17, 28–33]	0.044	0.743	0.952
Monte Carlo [17]	——	0.754(8)	1.00(4)
Monte Carlo [18]	0.38(1)	0.62(1)	1.20(1)
Functional renormalization group [23]	0.032	0.760	0.982
Functional renormalization group [23]	0.033	0.767	0.978
Functional renormalization group [23]	0.032	0.756	0.982
Functional renormalization group [24]	0.0276	0.7765	0.994(2)
Four loop $d = 2$ naive Padé [34]	0.082	0.745	0.931
Three loop $d = 4$ naive Padé [35]	0.0740	0.672	1.048
Conformal bootstrap [25]	0.044	0.742	0.880
Four loop $d = 4$ naive Padé [36]	0.0539	0.7079	0.931
Four loop $d = 4$ naive Padé [36]	0.0506	0.6906	0.945
Four loop two-sided Padé [37]	0.042	0.735	1.004
Four loop interpolating polynomial [37]	0.043	0.731	0.982
Four loop Padé-Borel [37]	0.043(12)	0.704(15)	0.993(27)
Monte Carlo [21]	0.05(2)	0.59(2)	1.01(1)
Conformal bootstrap [26]	0.04238(11)	0.7329(27)	0.998(12)
Conformal bootstrap [27]	——	0.7339(26)	0.998(12)

η_ψ: Padé

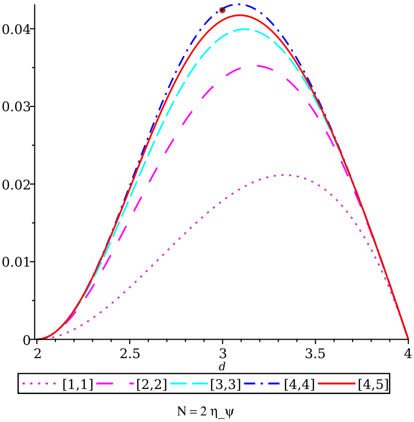


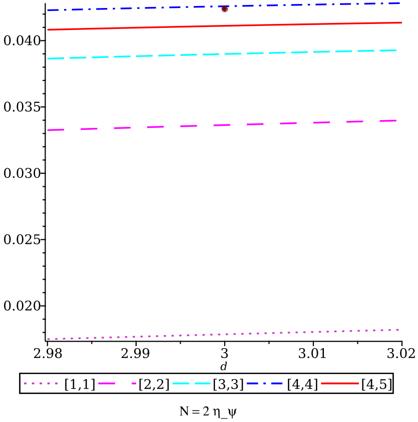
0.04
0.03
0.02
0.01
0
2
2.5
3
3.5
4
d
[3/2]
[3/4]
[4/3]
[4/5]
[5/4]
[5/5]
N = 2 η_ψ



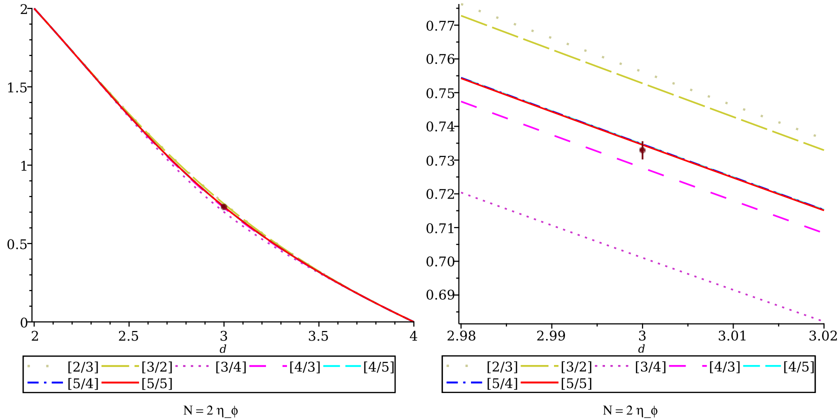
0.046
0.045
0.044
0.043
0.042
0.041
0.040
0.039
2.98
2.99
3
3.01
3.02
d
[3/2]
[3/4]
[4/3]
[4/5]
[5/4]
[5/5]
N = 2 η_ψ

η_ψ: interpolating polynomial

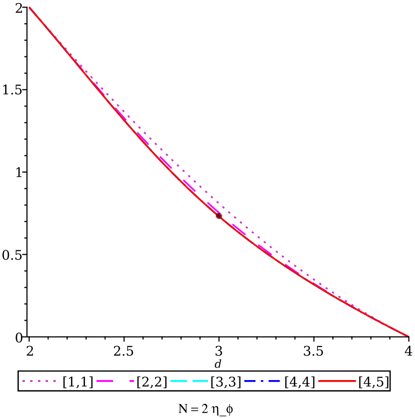


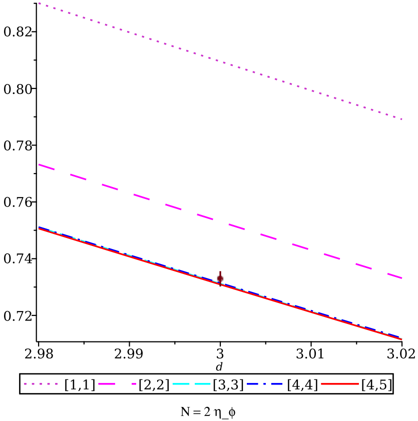


η_ϕ: Padé

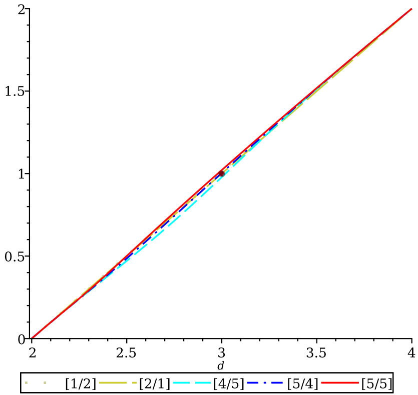

2
1.5
1
0.5
0
2
2.5
3
3.5
4
d
[2/3]
[3/2]
[3/4]
[4/3]
[4/5]
[5/4]
[5/5]
N = 2 η_ϕ
0.77
0.76
0.75
0.74
0.73
0.72
0.71
0.70
0.69
2.98
2.99
3
3.01
3.02
d
[2/3]
[3/2]
[3/4]
[4/3]
[4/5]
[5/4]
[5/5]
N = 2 η_ϕ

η_ϕ: interpolating polynomial

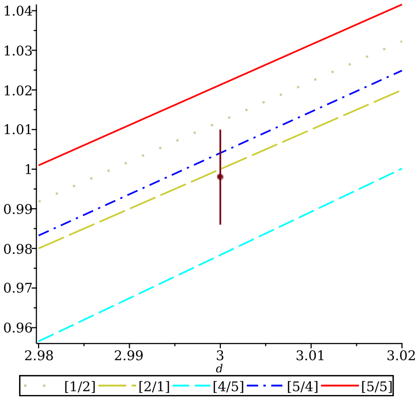




ν^{-1}: Padé

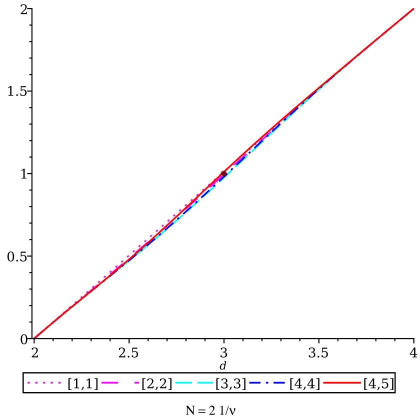


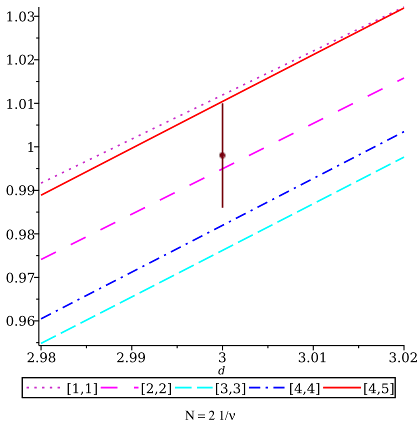
N = 2 1/ν



N = 2 1/ν

ν^{-1}: interpolating polynomial





Conclusions

- Calculated the five-loop corrections to the renormalisation constants of the GNY model
- reasonable agreement with other methods
- need 5-loop 2d GN results to check for convergence and to ascertain the uncertainty

LOOPS AND LEGS IN QUANTUM FIELD THEORY

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