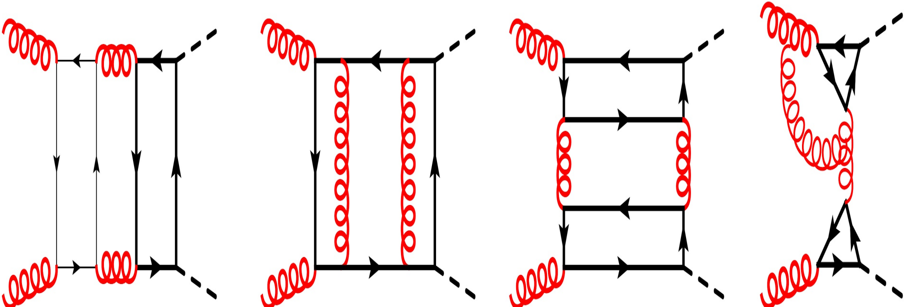
Higher order corrections to double Higgs production

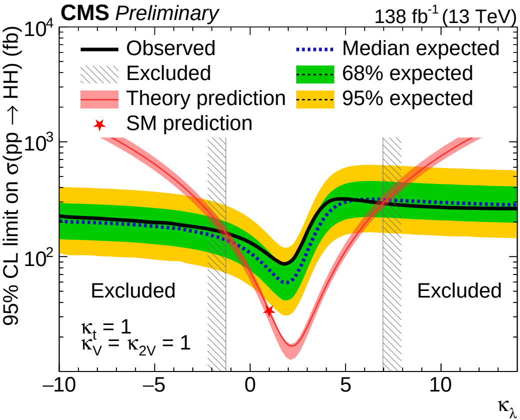
Loop Summit 2 — Cadenabbia, July 20-25, 2025

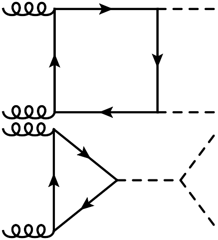
Matthias Steinhauser | in collaboration with J. Davies, K. Schönwald, D. Stremmer, M. Vitti

TTP KIT



Higgs boson self coupling

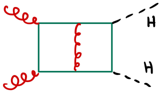




$$\kappa_\lambda = \lambda/\lambda_{\mathrm{SM}}$$

- $-1.2 < \kappa_\lambda < 7.2$ [ATLAS] $\quad -1.39 < \kappa_\lambda < 7.02$ [CMS]
- *gg* $\to$ *HH* is most important channel
- Precise theory predictions important; NLO prediction $\sim$ 100%

Speed and flexibility



- NLO corrections known since almost 10 years

[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]
[Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira'20]

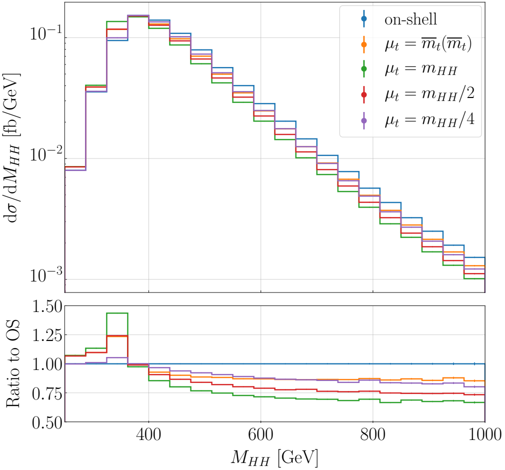
- But: slow!
- Unless: Generate grids (`hhgrid`)

[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]

➪ fast but not flexible (cannot modify $m_t, m_H, \mu_t, \ldots$)

- Wanted: Compute hardronic cross sections in less than 1 hour for any value for $m_t, m_H, \lambda, \mu_t, \mu_s, \ldots$

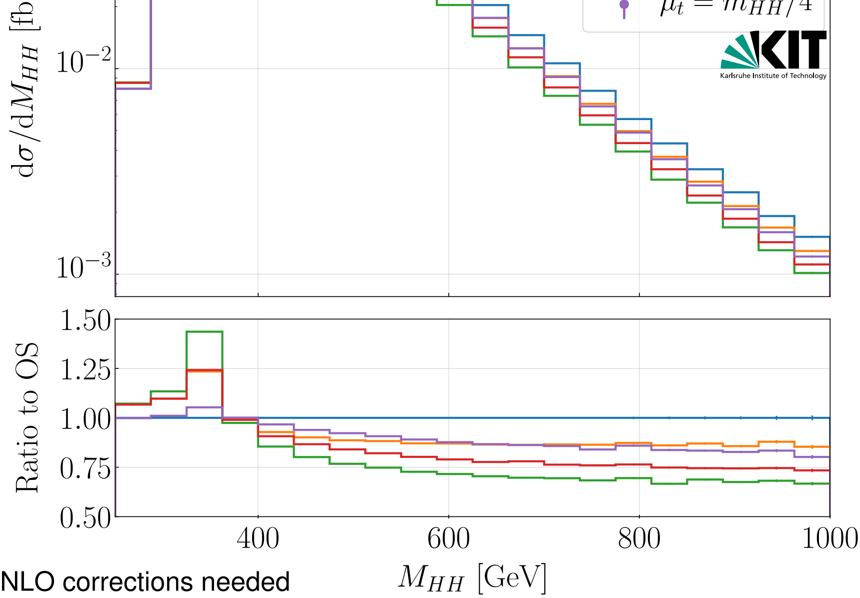
NLO: Large dependence on m_t renormalization scheme



[Baglio,Campanario,Glaus,Mühlleitner, Ronca,Spira'20]
[Bagnaschi,Degrassi,Gröber'23]
[Davies,Schönwald,Steinhauser,Stremmer'25]

➪ NNLO corrections needed

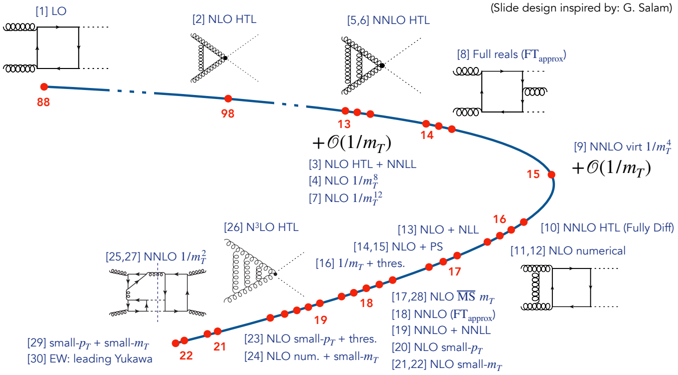
N
r



➪ NNLO corrections needed

[slide from Stephen Jones]

HH: Theory History



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22;

Davies,Schönwald,Steinhauser,Zhang'24'25; Heinrich,Jones,Kerner,Stone,Vestner'24; Jaskiewicz,Jones,Szafron,Ulrich'24; Davies,Schönwald,Steinhauser'24,25; Davies,Schönwald,Steinhauser,Stremmer'25

High-energy expansion and Padé improvement

Karlsruhe Institute of Technology

$$\mathcal{A} = \sum_j^N \sum_{i,k} c_{ijk} \epsilon^i \left(\frac{m_t}{\sqrt{s}}\right)^j \log^k \left(\frac{m_t^2}{s}\right)$$

$N = \mathcal{O}(100)$

$$\sum_{k=0}^{N} c_k m_t^k \quad \to \quad \frac{a_0 + \cdots + a_r {m_t}^r}{1 + b_1 + \cdots + b_s m_t^s} \qquad r + s = N$$

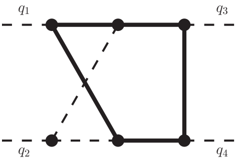
➪ For each phase-space point $(\sqrt{s}, p_T)$:
prediction of central value and corresponding uncertainty

$p_T^2 = (tu - m_H^4)/s, s + t + u = 2m_H^2$

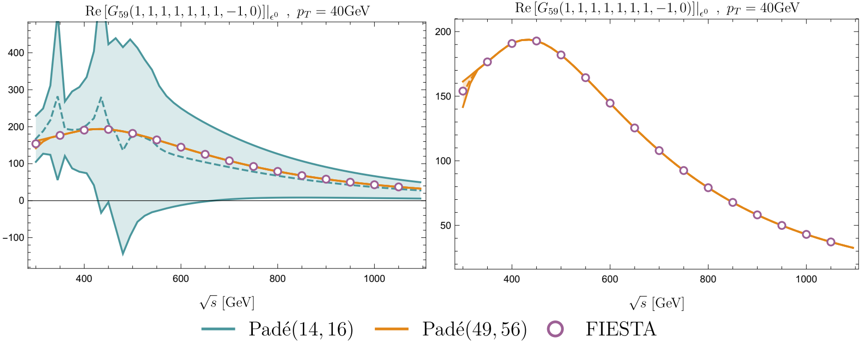
High-energy expansion $\oplus$ PA

- expansion up to $m_t^{112} = (m_t^2)^{56}$
- construct PAs with input for $(N_{\min}, N_{\max})$

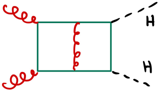
  (for each phase-space point)



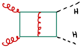
$\boxed{\text{PA is a precision tool}}$



$t \to 0$ expansion

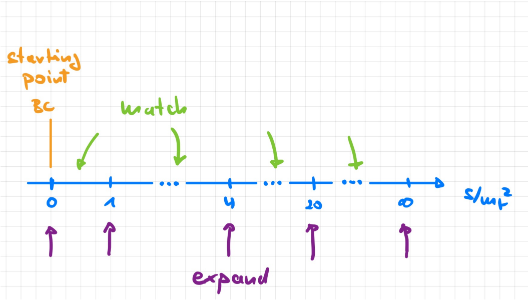


[Bonciani,Degrassi,Giardino,Gröber'18]
[Bellafronte,Degrassi,Giardino,Gröber,Vitti'22; . . .]
[Davies,Mishima,Schönwald,Steinhauser'23]

- forward scattering kinematics
- Taylor expansion in t ➪  $= f(s/m_t^2)$
- compute $f(s/m_t^2)$ with "expand and match" [Fael,Lange,Schönwald,Steinhauser'21'22]

"Expand and match" [Fael,Lange,Schönwald,Steinhauser'21'22]

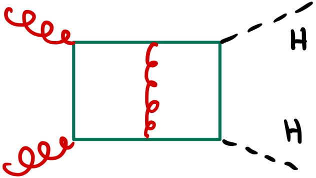
Consruct overlapping expansions using differential equations for the master integrals



Expansion of (unknown) function $f(s/m_t^2)$ around properly chosen s/m_t^2 values with precise numerical coefficients

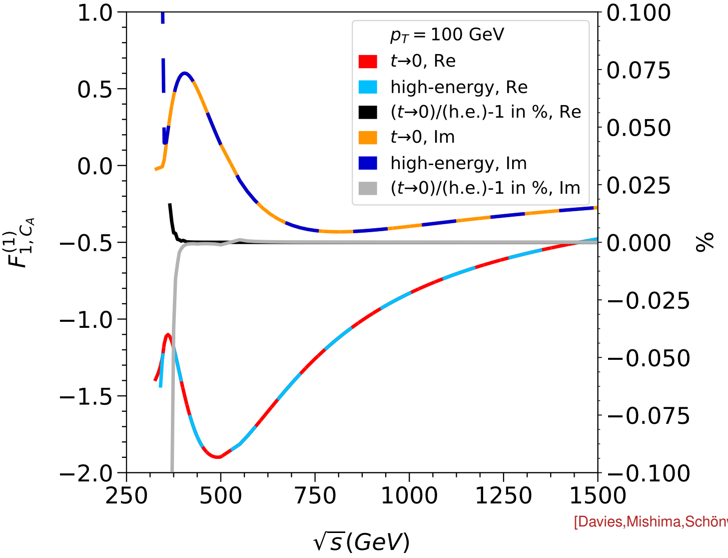
Similar approaches: [Blümlein,Czakon,Hidding,Laporta,Lee,Liu,Smirnov,. . .]

Combine $t \to 0$ and high-energy expansion



$$\begin{cases} \text{high-energy expansion} \\ t \to 0 \text{ expansion} \end{cases}$$

Combine: $t \to 0$ and h.e. at 2 loops



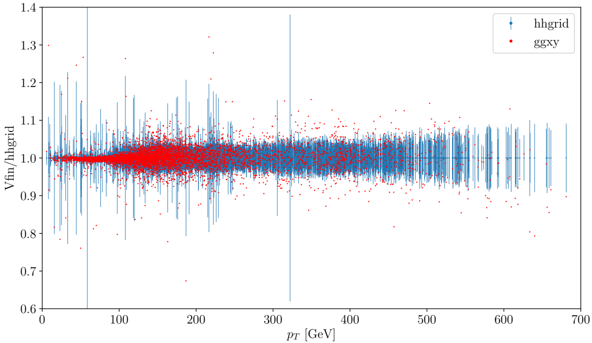
[Davies,Mishima,Schönwald,Steinhauser'23]

$\mathcal{V}_{\mathrm{fin}}$: virtual NLO QCD corrections

Normalize to “pySecDec”; 6320 data points

analytic expansions: negligible uncertainties

pySecDec err. intervals	1σ	2σ	3σ
small-t	0.57	0.85	0.92
energy	0.65	0.94	0.99



https://github.com/mppmu/hhgrid

[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke’16; Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann’19]

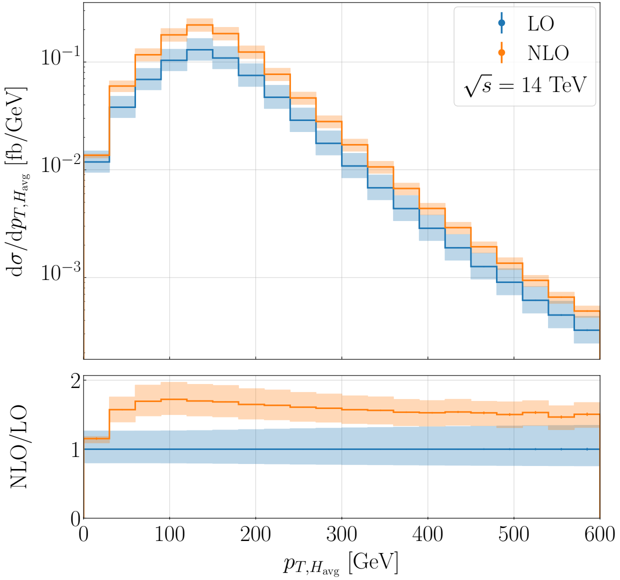
ggxy: [Davies,Schönwald,Steinhauser,Stremmer’25]

ggxy [Davies,Schönwald,Steinhauser,Stremmer'25]

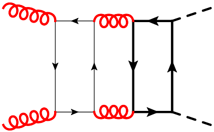
- C++ library
- Virtual corrections: t and high-energy expansion [Davies,Schönwald,Steinhauser'23]
- Real corrections: `Recola` [Actis,Denner,(Lang,)Hofer,Scharf,Uccirati'12'16], `Collier` [Denner,Dittmaier,Hofer'16], `CuTtools` [Ossola,Papadopoulos,Pittau'07], `OneLOop` [van Hameren'10]
- Additional external libraries: `avhlib` [van Hameren'07'10], `boost`, `eigen`, `LHAPDF` [Buckley et al.'14], `lievaluate` [Frellesvig,Tommasini,Wever'16], `CRunDec` [Herren,Steinhauser'16]
- Features: total and differential cross sections, parameter variation $m_t, m_H, \lambda, \ldots$, OS and $\overline{\mathrm{MS}}$ scheme for m_t, access to low-level functions (form factors)
- Typical runtime: $\approx$ 30 minutes (single core) to obtain a statistical uncertainty of 0.2% (including 7-point scale variation)
- Extendable to other gluon-induced processes

`https://gitlab.com/ggxy/ggxy-release`

ggxy: p_T distribution

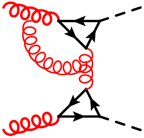


Can we go to 3 loops?

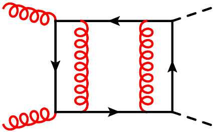


[Davies,Schönwald,Steinhauser'23]

$t = 0, m_H = 0$



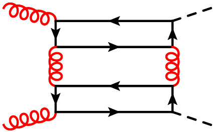
[Davies,Schönwald, Steinhauser,Vitti'24]



[Davies,Schönwald,Steinhauser'25]

$t = 0, m_H = 0$

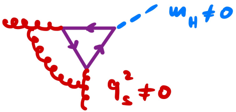
large-N_c



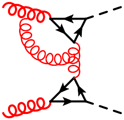
in progress

3-loop 1PR

off-shell $gg^{\star}H$ vertex at 2 loops



➪



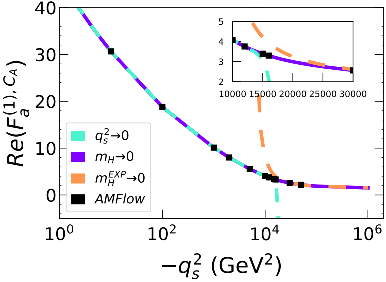
2 asymptotic expansions

1. $m_H^2 \ll q_s^2, m_t^2$
2. $q_s^2 \ll m_H^2, m_t^2$

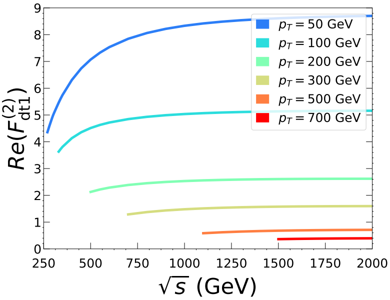
➪ cover whole PS for

$$\{s, t; m_H, m_t\}$$

$gg^{\star}H$ form factor

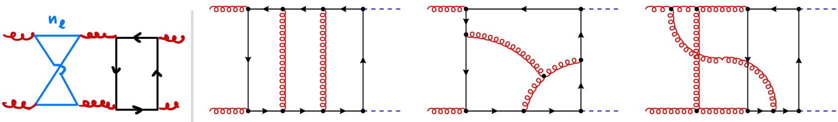


$gg \to HH$ form factor



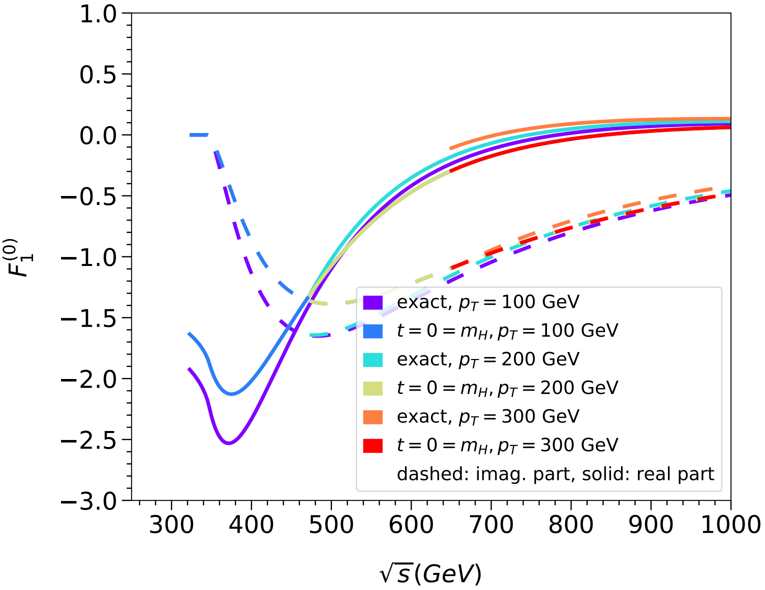
[Davies,Schönwald,Steinhauser,Vitti'24]

3-loop n_h^1, $t \to 0$



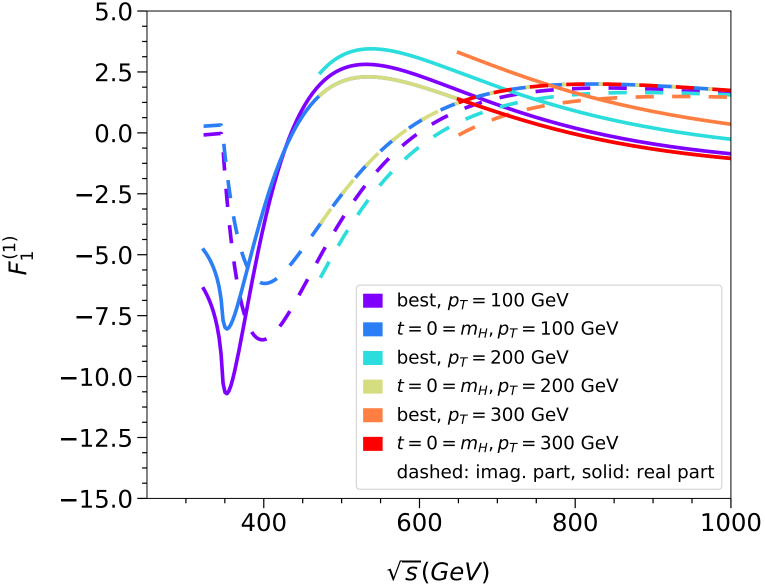
- currently: full n_l and large N_c
- also non-planar diagrams
- 203 families (full colour)
- reduction of amplitudes with `Kira` [Klappert,Lange,Maierhöfer,Usovitsch'20]
  $>$ 2 TB RAM needed ➪ 33.000 master integrals (full colour)
- minimization with `FIRE` [Smirnov,Zeng'23]
- master integrals: 1561 $\to$ 783 for large N_C
- `AMFlow` [Liu,Ma'22] $\oplus$ "expand and match" [Fael,Lange,Schönwald,Steinhauser'21'22]
- useful software: `tapir` [Gerlach,Herren,Lang'23] `Feynson`, [Magerya]
- $t = 0$, $m_H = 0$

$t \to 0$: 1 loop



1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
$F_1^{(0)}$
300
400
500
600
700
800
900
1000
$\sqrt{s}(GeV)$
exact, $p_T = 100$ GeV
$t = 0 = m_H, p_T = 100$ GeV
exact, $p_T = 200$ GeV
$t = 0 = m_H, p_T = 200$ GeV
exact, $p_T = 300$ GeV
$t = 0 = m_H, p_T = 300$ GeV
dashed: imag. part, solid: real part

$t \to 0$: 2 loops



5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−12.5
−15.0
400
600
800
1000
$F_1^{(1)}$
$\sqrt{s}(GeV)$
best, $p_T = 100$ GeV
$t = 0 = m_H, p_T = 100$ GeV
best, $p_T = 200$ GeV
$t = 0 = m_H, p_T = 200$ GeV
best, $p_T = 300$ GeV
$t = 0 = m_H, p_T = 300$ GeV
dashed: imag. part, solid: real part

IR subtraction, μ_s and μ_t

- UV renormalization: $\alpha_s^{(5)}(\mu_s)$, top quark pole mass M_t
- $M_t \longleftrightarrow \overline{m}_t(\mu_t)$
- IR subtraction [Catani'98]

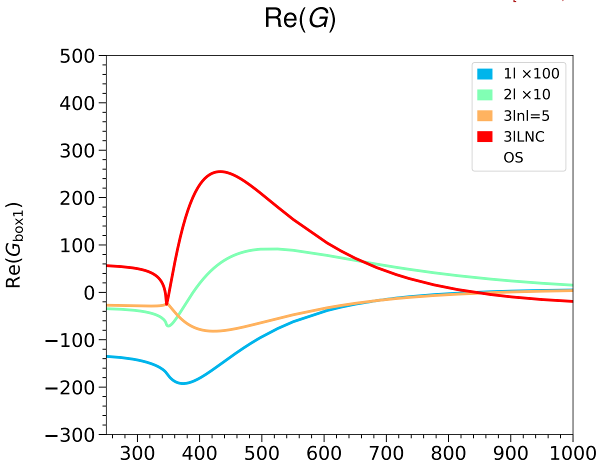
$$
\begin{aligned}
F^{(1),\mathrm{fin}} &= F^{(1)} - \frac{1}{2} I_g^{(1)} F^{(0)} \\
F^{(2),\mathrm{fin}} &= F^{(2)} - \frac{1}{2} I_g^{(1)} F^{(1)} - \frac{1}{4} I_g^{(2)} F^{(0)}
\end{aligned}
$$

$I_g^{(1)}$ and $I_g^{(2)}$ such that the renormalization scale dependence of $\alpha_s F^{\mathrm{fin}}$ is governed by $\alpha_s(\mu_s)$ if the top quark mass is renormalized OS

$$
G(\mu_s, \mu_t) = \frac{\alpha_s(\mu_s)}{\alpha_s(M_t)} F^{\mathrm{fin}}(\mu_s, \mu_t)
$$

3 loops: $G(s)$ for $t = 0, m_H = 0$

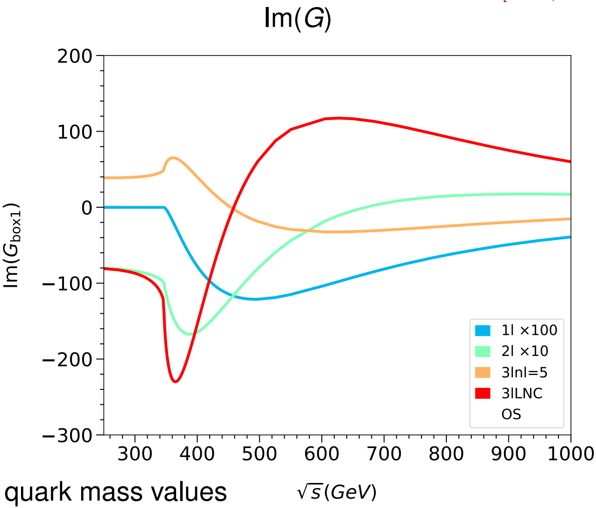
[Davies,Schönwald,Steinhauser'25]



- OS top quark mass values
- stable results for all $\sqrt{s}$

3 loops: $G(s)$ for $t = 0, m_H = 0$

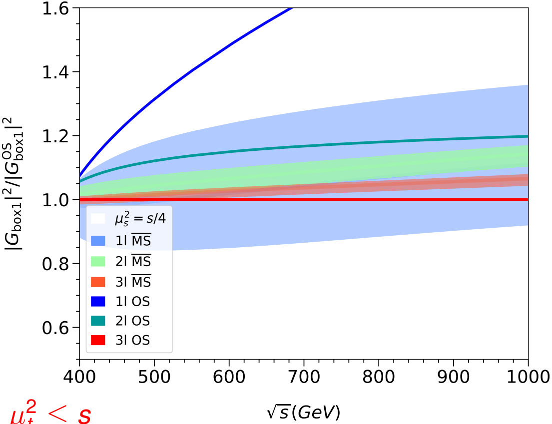
[Davies,Schönwald,Steinhauser'25]



- OS top quark mass values
- stable results for all $\sqrt{s}$

3 loops: μ_t dependence

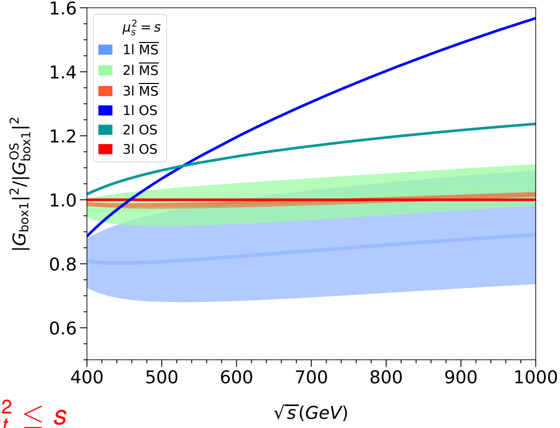
normalize to NNLO OS $\mu_s^2 = s/4$ [Davies,Schönwald,Steinhauser'25]



- $s/16 \leq \mu_t^2 \leq s$
- LO $\rightarrow$ NLO $\rightarrow$ NNLO: good overlapp of bands, widths decrease
- NNLO OS and $\overline{\mathrm{MS}}$ results are close together

3 loops: μ_t dependence

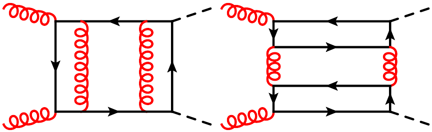
normalize to NNLO OS $\mu_s^2 = s$ [Davies,Schönwald,Steinhauser'25]



- $s/16 \leq \mu_t^2 \leq s$
- LO $\rightarrow$ NLO $\rightarrow$ NNLO: good overlapp of bands, widths decrease
- NNLO OS and $\overline{\text{MS}}$ results are close together

Challenges beyond large N_c and $t = 0$

- IBP reduction ($t = 0$) ✓
  - hardest job:
    41 days, $>$ 2 TB RAM
    took several attempts
  - 33.000 MIs accross all families



- Minimization of MIs ✓
  - cannot be done with `Kira 2`
  - Apply `FIRE`'s `FindRule` to all 2.600.000 input integrals
    + additional test reductions with `FIRE`
    ➪ 1561 MIs
- Nasty differential equation system: sheer size of the system; $1/\epsilon$ poles on diagonal; higher ϵ powers for individual MIs
- BCs: large-m_t limit ✓
- t^1 and m_H^2 terms: significantly more involved IBP reduction
- cut in t-channel

Conclusions

- Combine analytic expansions

  Here: forward limit $\otimes$ high-energy limit

  Superior to pruely numerical or analytical approaches

- ggxy

  currently: $gg \to HH$ @ NLO, fast and flexible

- NNLO for $gg \to HH$: 3-loop FF
  - large-m_t
  - $t = 0$: full n_l and large N_c
  - reducible contribution (full phase-space)
  - Next steps: beyond large-N_c,
    expansion in t and m_H^2,
    high-energy expansion at *NNLO* (?),
    real radiation