QCD and Gravity in the Regge limit

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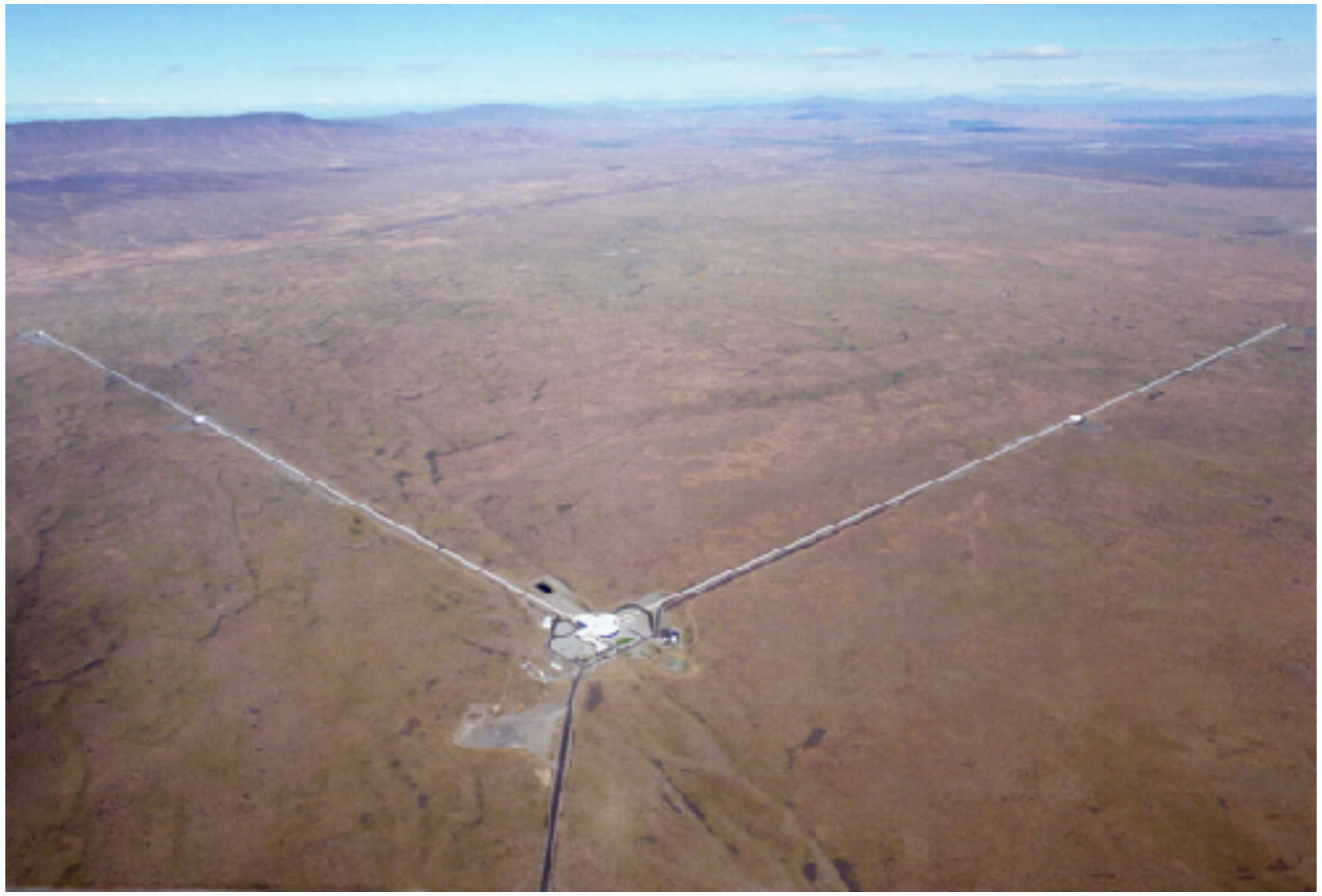
Loop Summit Cadenabbia 23 July 2025

Gravitational waves

- 1915: from GR, Einstein predicts GWs
- 2015: first GW signal, GW150914:
two black holes, each about 30 $M_\odot$, $1.5 \cdot 10^9$ ly away



LIGO

KAGRA

$f = 30$ Hz → $\lambda = 10^7$ m

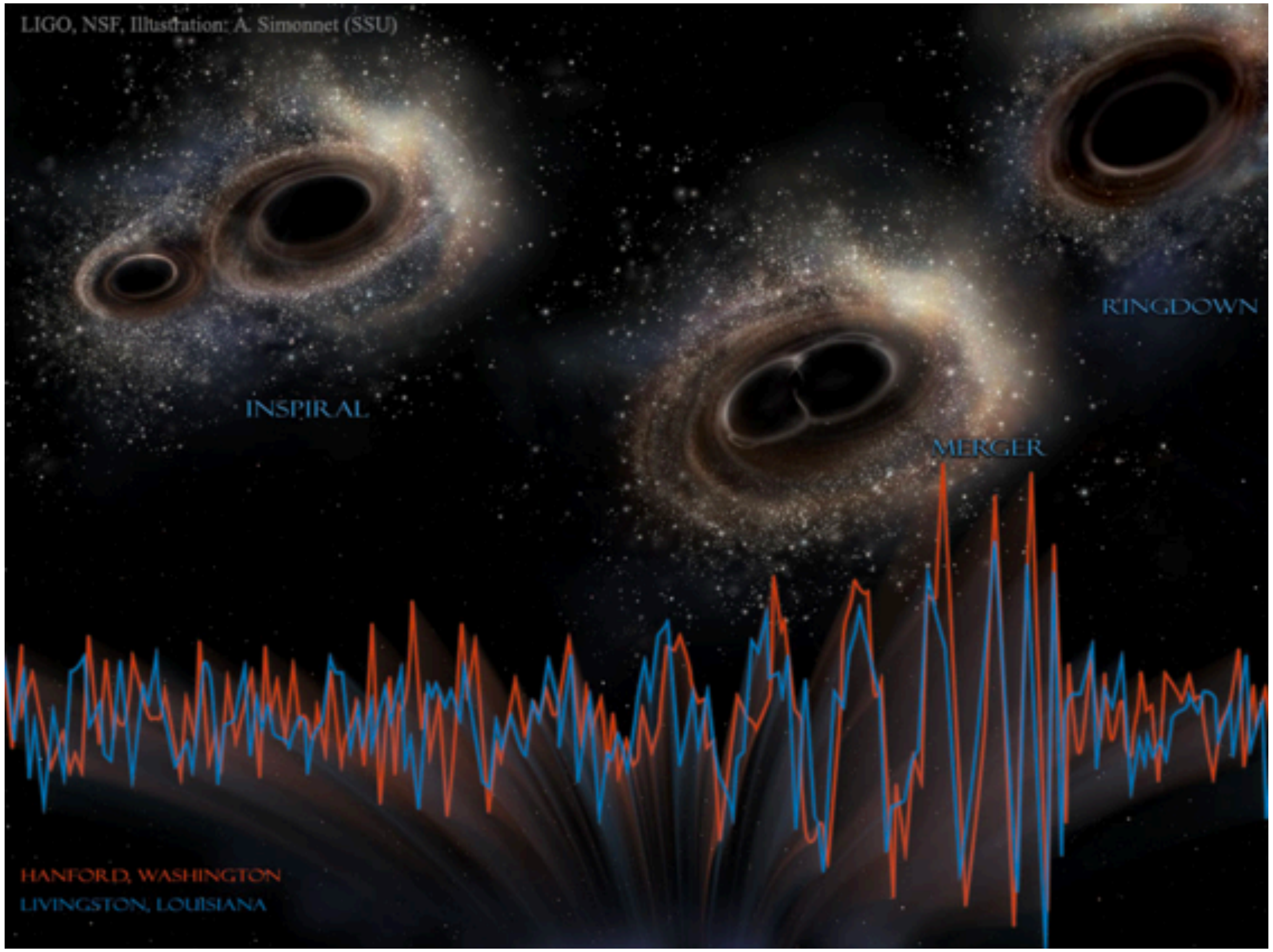
$f = 10^4$ Hz → $\lambda = 3 \cdot 10^4$ m

mass $\lesssim 200\, M_\odot$

distance $z = 0.25 \sim 3.3 \cdot 10^9$ ly

Virgo

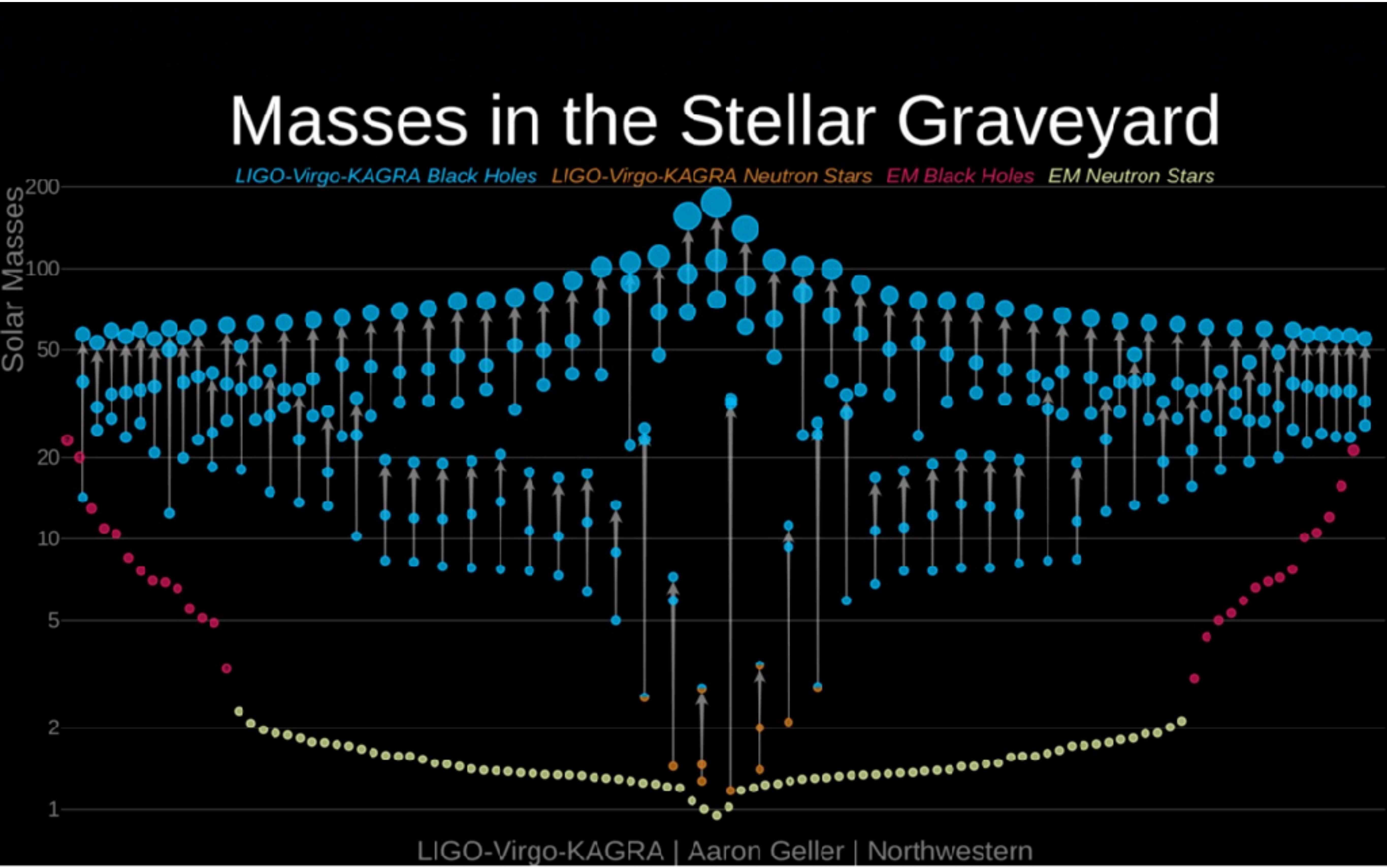
LIGO/National Science Foundation/Aurore Simmonet/Somona State University



GW150914

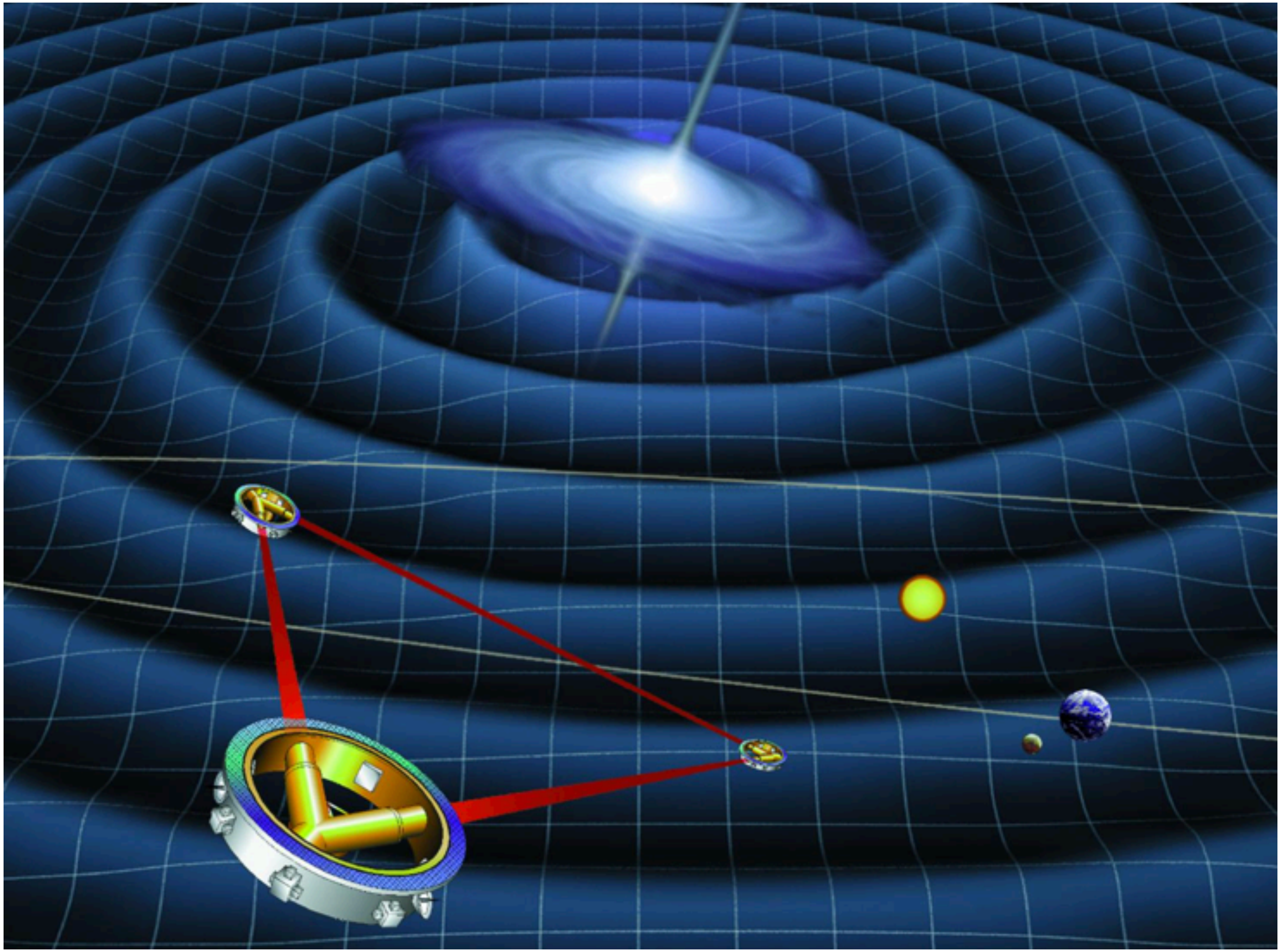
inspiral merger ringdown

PN, PM, SF expansions NR BHPT



observed for < 200 $M_\odot$ so far

taxonomy from models, not from GWs



LISA

launch ~ 2035

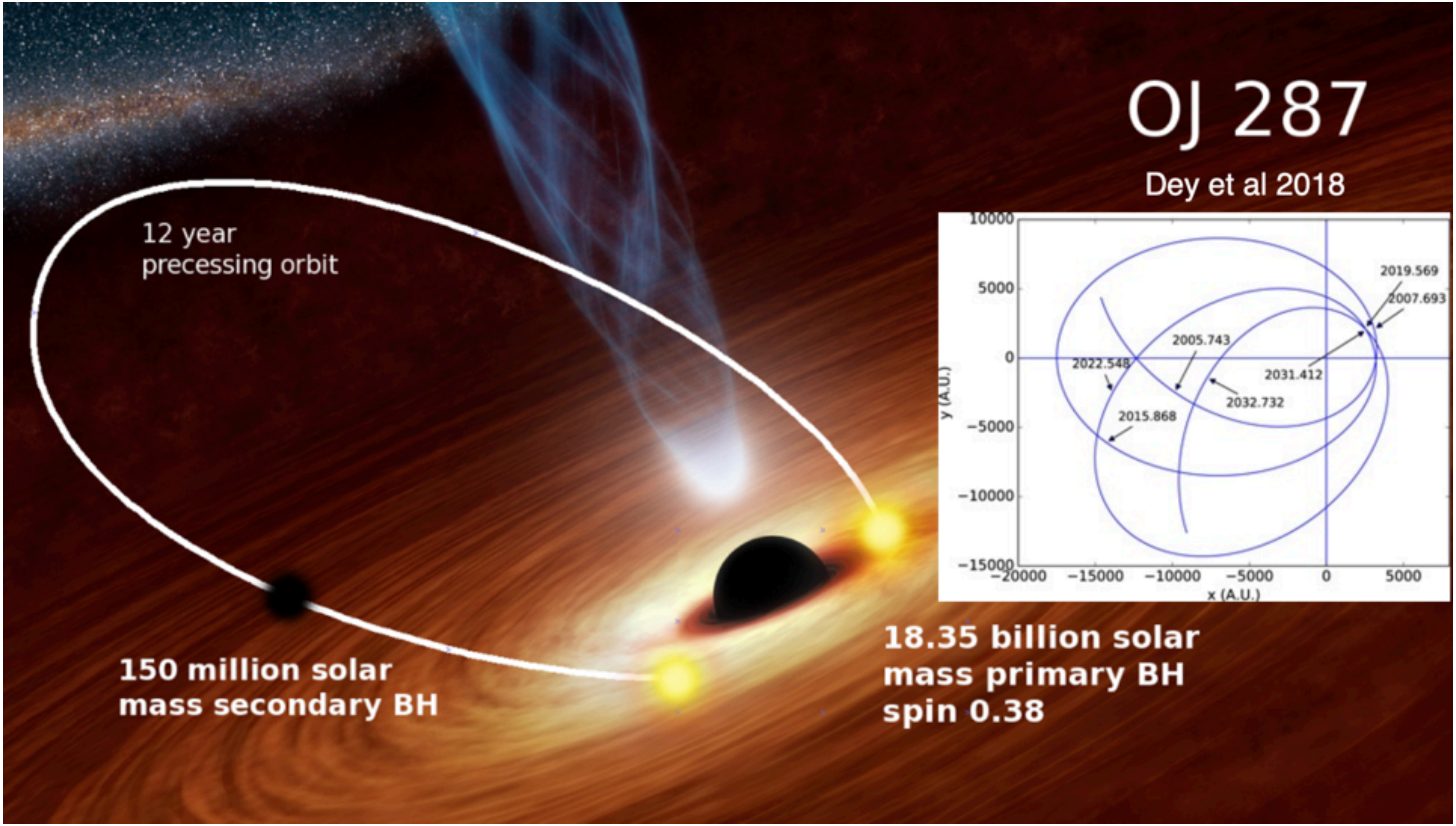
f = 0.1 Hz → $\lambda = 3 \cdot 10^9$ m

f = 10^{-4} Hz → $\lambda = 3 \cdot 10^{12}$ m

mass ~ 10^4 - 10^7 $M_\odot$ (MBH) distance z = 10 ~ $13.4 \cdot 10^9$ ly

M_1 ~ 10 $M_\odot$ M_2 ~ 10^5 $M_\odot$ (EMRI) distance z = 4 ~ $12 \cdot 10^9$ ly
inspiral with up to ~ 10^4 cycles

Binary system of SMBH

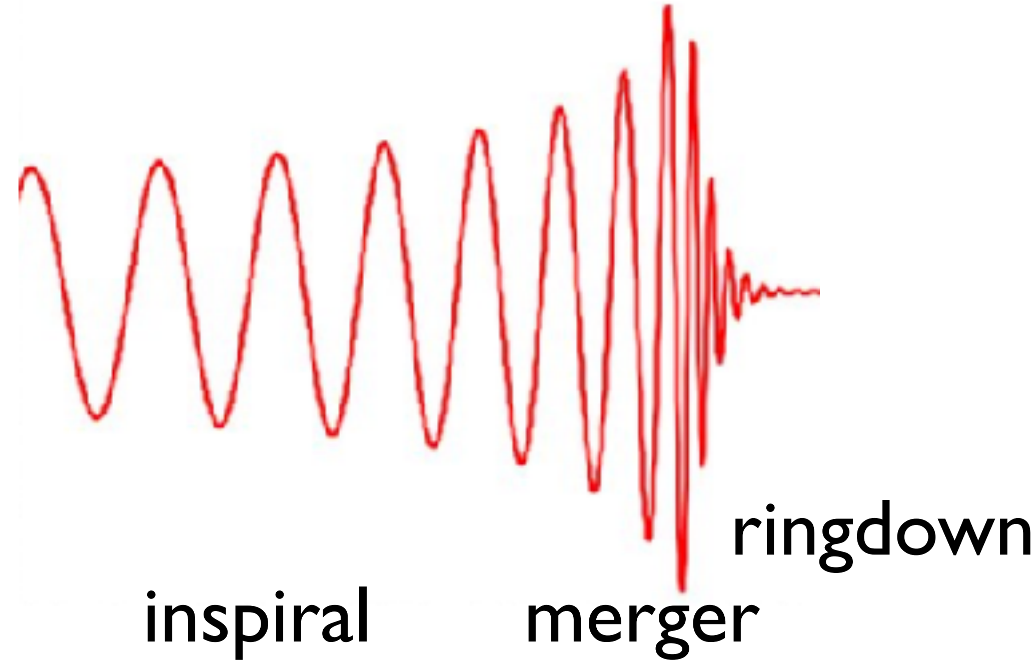


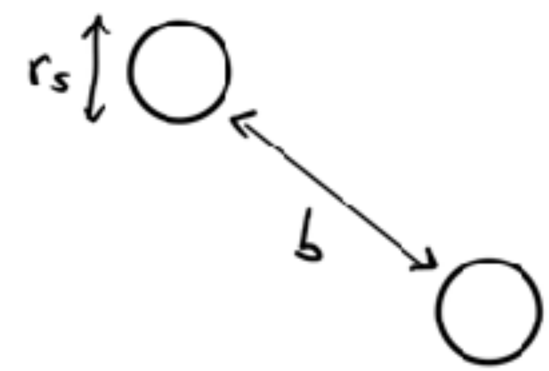
Waveform

- interested in modelling inspired phase with analytic expansions

  $r_s << b$

  GWs carry info about the potential of a BH binary system

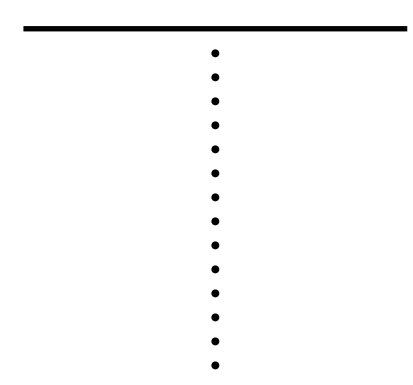




- Post Newtonian (PN) expansion: v^2/c^2

  (deals with 3-dim integrals in config. space)

1 PN: Lorentz Droste 1917; Einstein Infeld Hoffmann 1938
...
6 PN: Bini Damour Geralico 2020-2021
Blümlein Maier Marquard Schäfer 2020-2021



Potential

Caveat:
GR deals with bodies as finite-size objects
analytic expansions deal with point-like objects

- EFT of point particles coupled to gravity

  $r_s << b$ Goldberger Rothstein hep-th/0409156

  use RG methods to organise $\log(r_s/b)$

from the gravitational dynamics that governs the orbital evolution and radiation emission. Using the EFT formulation, it is easy to show that the divergences which arise at v^6 in the PN expansion can be attributed to the existence of new operators in the effective point particle description. However, these operators can be removed via a point transformation of the metric tensor and thus never contribute to physical quantities. This leads to the conclusion that there are no finite size effects at order v^6. Practically, this means that *whenever one encounters a log divergent integral at order v^6 in the potential, one may simply set it to zero. Its value cannot affect physical predictions.*

- Post Minkowskian (PM) expansion: G/r

  ideal for eccentric orbits

  matches loop expansion of amplitude:
  1 PM = O(G/r) = tree level
  2 PM = $O(G^2/r^2)$ = one loop, etc.

2 PM: Westpfahl Goller 1979 … Cheung Rothstein Solon 1808.02489

potential coefficients are obtained by matching EFT and full theory amplitudes order by order in κ (because EFT and full theory describe same IR dynamics)

How classical is a quantum loop?

- put $\hbar$ back

- $\hbar$ counting $1/\hbar$ from vertex $\exp\left(\frac{i}{\hbar}\int d^4x\mathcal{L}_{int}(\varphi)\right)$

  $\hbar$ from (massless) propagator $[\varphi(\vec{x}),\pi(\vec{y})] = i\hbar\delta^3(\vec{x}-\vec{y})$

  get $\hbar^{\text{I-V+1}} = \hbar^{\text{L}}$

$$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4}\frac{i\hbar\, e^{ik(x-y)}}{k^2 - i\varepsilon}$$

  Note that k is wavenumber, with $p=\hbar k$

- with masses   Klein-Gordon is $\left(\Box + \frac{m^2}{\hbar^2}\right)\varphi(x) = 0$

$$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4}\frac{i\hbar\, e^{ik(x-y)}}{k^2 - \frac{m^2}{\hbar^2} - i\varepsilon}$$

  so, effectively

  $e \to e/\sqrt{\hbar}$ $\kappa \to \kappa/\sqrt{\hbar}$

  massless momenta $p \to \hbar p$

Boulware Deser 1975
Gupta Radford 1980
....
Donoghue Holstein hep-th/0405239
Kosower Maybee O'Connell 1811.10950

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High-energy gravitational scattering and the general relativistic two-body problem

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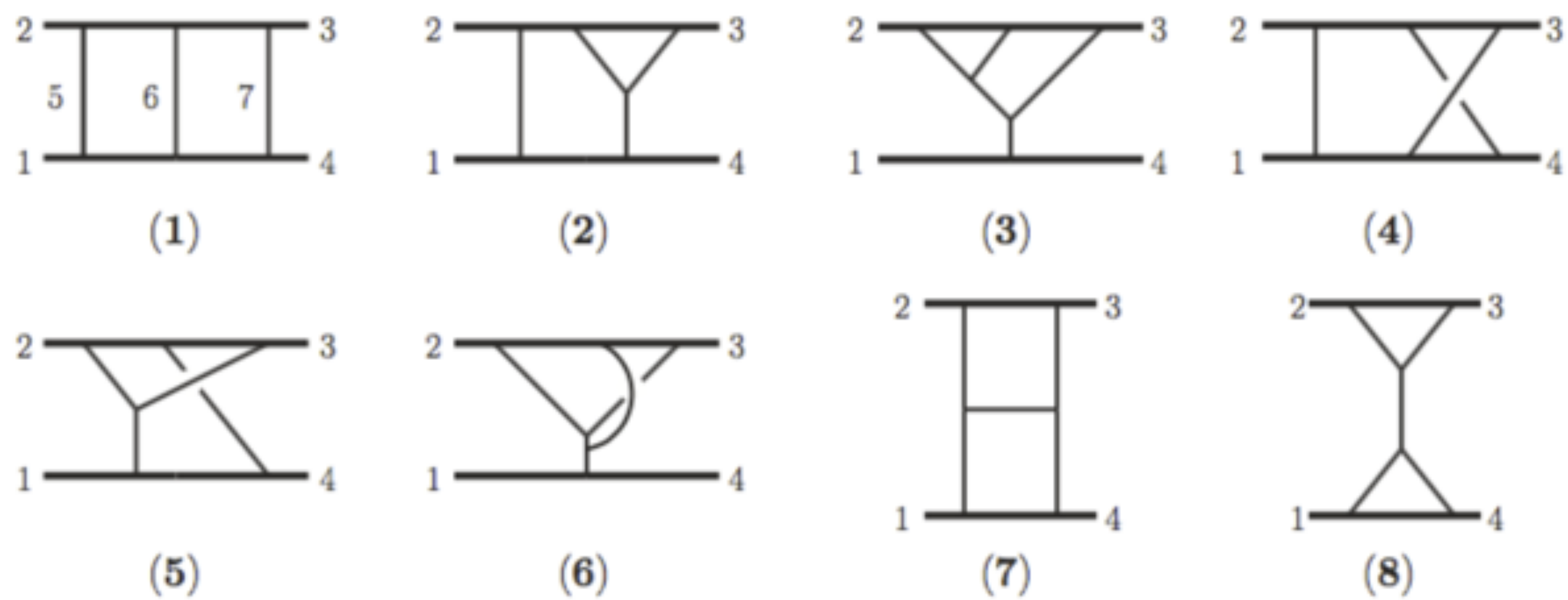
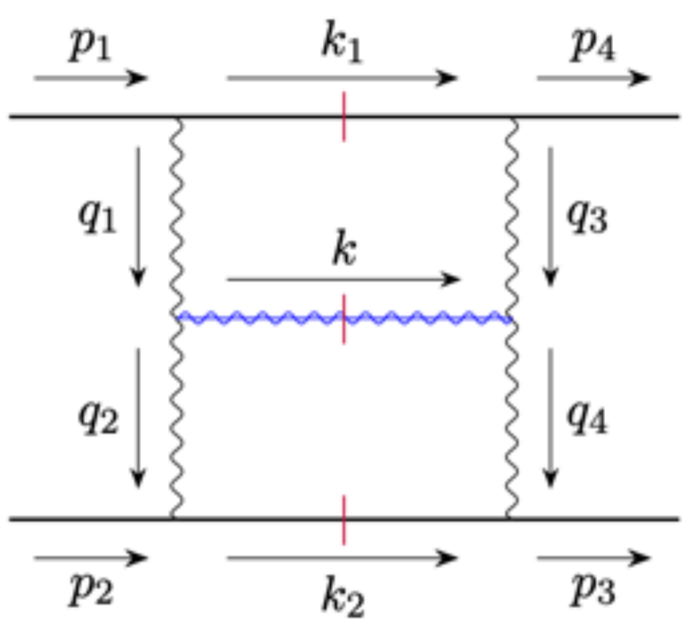
A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [Phys. Rev. D **94**, 104015 (2016)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the two-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian.

3 PM = $O(G^3/r^3)$: Bern Cheung Roiban Shen Solon Zeng 1901.04424 & 1908.01493





H diagram

Amati Ciafaloni Veneziano 1990

Regge limit $s >> |t|$

$$\mathrm{Im}A^{(2)}(s,t) \simeq G^3 s^3 \log(s/t) \cdot (\mathrm{poles\,in}\,\varepsilon)$$

Di Vecchia Heissenberg Russo Veneziano 2008.12743

$$\mathrm{Re}A^{(2)}(s,t) \simeq \frac{1}{\log(s/t)} \mathrm{Im}A^{(2)}(s,t)$$

Heavy effective field theory (HEFT)

Brandhuber Chen Travaglini Wen 2108.04216

2 PM: $A^{(1)}(s,t) \simeq G^2 m_1^2 m_2^2 \,(m_1+m_2)\, y^2(1+O(1/y^2))$

$$y = \frac{s-m_1-m_2}{2m_1 m_2}$$

(probe limit)

3 PM, 0 SF:

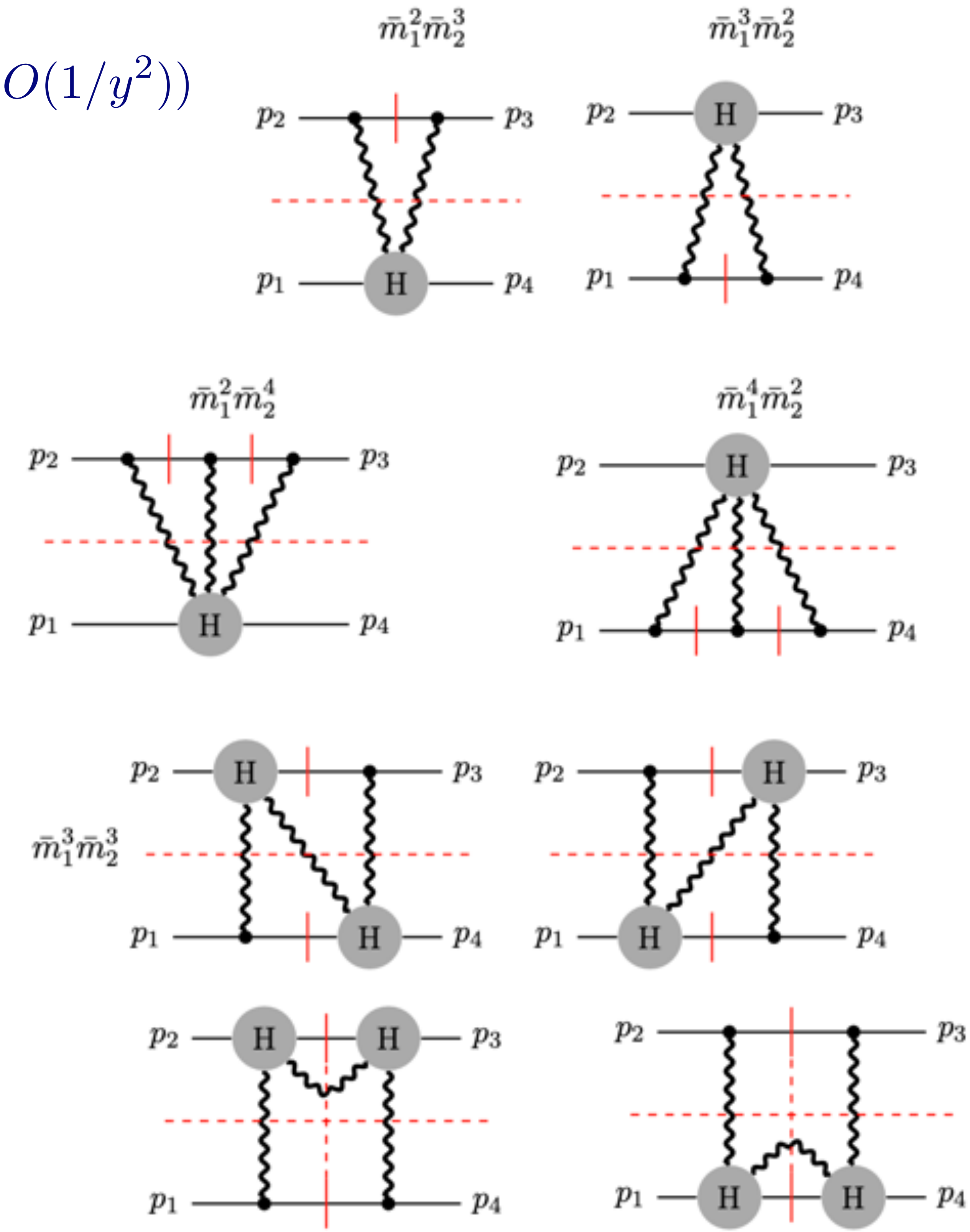
$$A^{(2)}_{0\mathrm{SF}}(s,t) \simeq G^3 m_1^2 m_2^2 \,(m_1^2+m_2^2)\, y^2(1+O(1/y^2))$$

3 PM, 1 SF: (beyond the probe)

$$\mathrm{Re}A^{(2)}_{1\mathrm{SF}}(s,t) \simeq G^3 m_1^3 m_2^3\, y^3(1+O(1/y))$$

$$\mathrm{Im}A^{(2)}_{1\mathrm{SF}}(s,t) \simeq G^3 m_1^3 m_2^3\, y^3(\log(2y)+O(1/y))$$

radiation reaction needed
to cancel additional power of log(s/|t|)

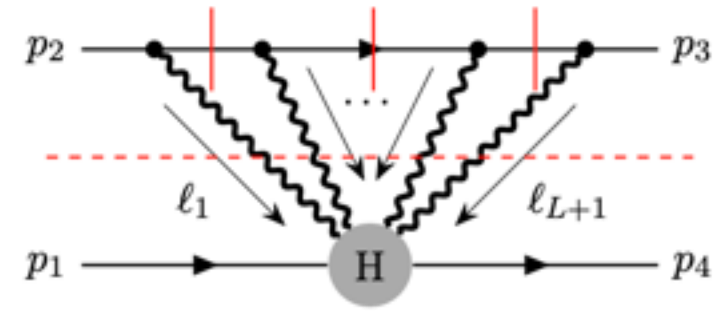


(radiation reaction)

Probe limit

(*L*+1)-PM, 0 SF: $A^{(L)}_{0\mathrm{SF}}(s,t) \simeq G^{L+1} m_1^2 m_2^L \, y^2 (1 + O(1/y^2))$

$1 + O(1/y^2) = 2(L+1)-$degree polynomial in $1/y^2$



Brandhuber Chen Travaglini Wen 2108.04216 conjectured polynomial form, with unknown coefficients

Sasank Chava (2023, MSc thesis unpublished)

obtained polynomial coefficients leveraging geometric info
from geodesic eq. for test particle in a Schwarzschild background

Cheung Shah Solon 2010.08568
Damour 1710.10599

next-to-probe

- an analytic resummation of next-to-probe, or 1 SF, terms is not yet known

Self force

- Probe limit corresponds to Schwarzschild geometry: static (infinitely heavy) black hole

- in Self Force (SF) expansion,
  one expands Einstein's equation in powers of m/M
  and solves them (numerically)

  0 SF = Probe limit = Schwarzschild
  1 SF is known for generic orbits
  2 SF is known for specific cases (e.g. quasi circular orbits)

4 PM, 0 SF: $A^{(3)}_{0\mathrm{SF}}(s,t) \simeq G^4 m_1^2 m_2^2\,(m_1^3 + m_2^3)\,y^2(1 + O(1/y^2))$

4 PM, 1 SF: $A^{(3)}_{1\mathrm{SF}}(s,t) \simeq G^4 m_1^3 m_2^3\,(m_1 + m_2)\,y^3(\log(2y) + O(1/y))$

Bern Parra-Martinez Roiban Ruf Shen Solon Zeng 2101.07254 & 2112.10750
Dlapa Kälin Liu Neef Porto 2106.08276, 2112.11296 & 2210.05541

5 PM, 0 SF: $A^{(4)}_{0\mathrm{SF}}(s,t) \simeq G^5 m_1^2 m_2^2\,(m_1^4 + m_2^4)\,y^2(1 + O(1/y^2))$

5 PM, 1 SF: $A^{(4)}_{1\mathrm{SF}}(s,t) \simeq G^5 m_1^3 m_2^3\,(m_1^2 + m_2^2)\,y^3(\log(2y) + O(1/y))$

Driesse Jakobsen Mogull Plefka Sauer Usovitsch 2403.07781 & 2411.11846
Dlapa Kälin Liu Porto 2506.20665

mostly through world-line formulation

$$S = -\sum_{i=1}^{2} \frac{m_i}{2} \int d\tau_i g_{\mu\nu}(x_i(\tau_i))\, v_i^{\mu}(\tau_i) v_i^{\nu}(\tau_i)$$

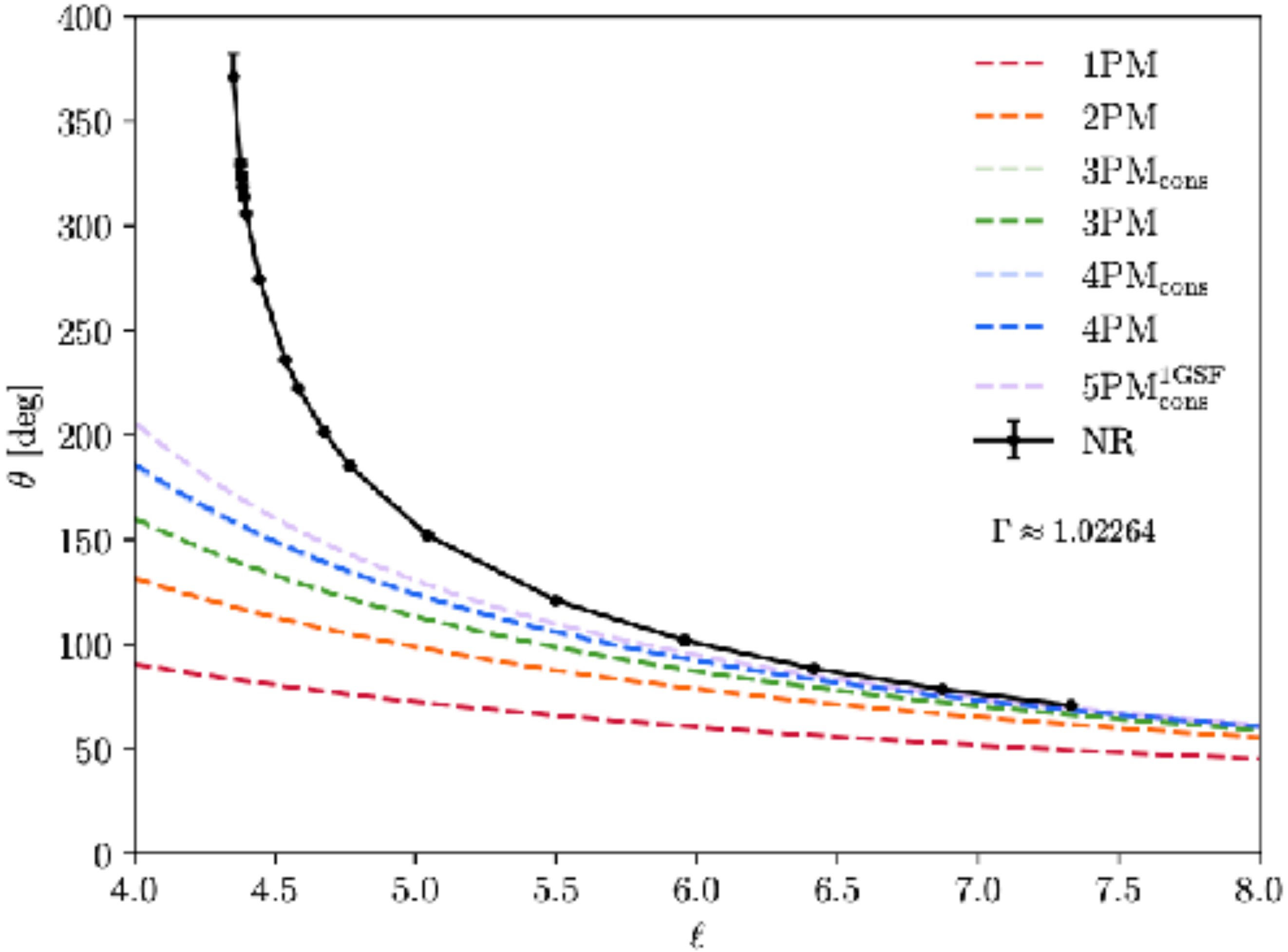
5 PM, 2 SF: $A^{(4)}_{2\mathrm{SF}}(s,t) \simeq G^5 m_1^4 m_2^4\,y^4(\log^2(2y) + O(1/y))$?

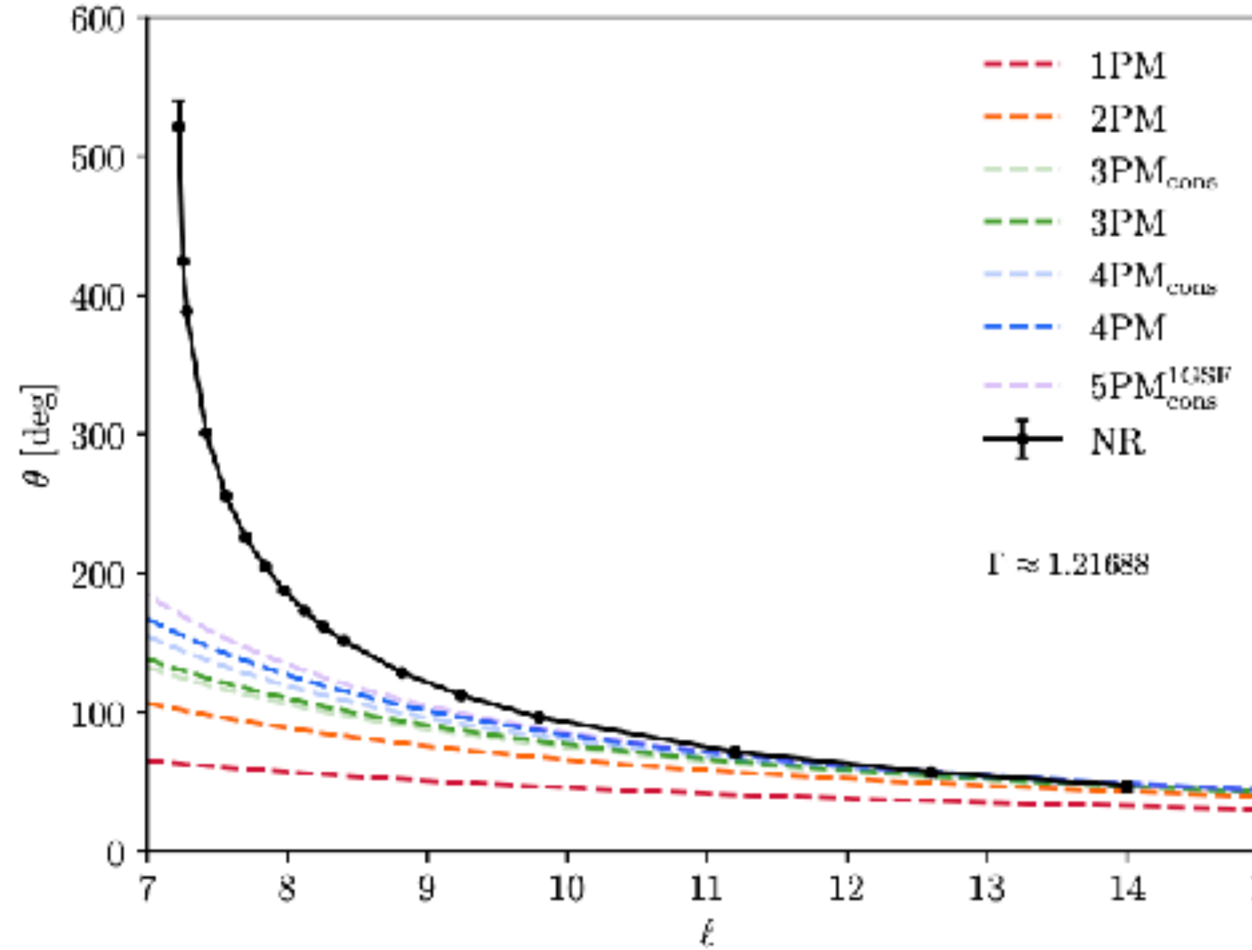
unknown yet

Comparison to Numerical Relativity

Pratten Schmidt Swain 2411.09652

scattering angle from NR data vs. PM





Γ: highest and lowest energy in the NR simulation

- comparison worsens as we approach strong-field regime

Regge limit - Foreword

- In the Regge limit, radiative corrections to 2→2 amplitudes display iterative patterns of the evolution in rapidity $y \simeq \log(s/|t|)$, either if the evolution occurs in the t channel or in the s channel

- In the Regge limit of QCD, the leading radiative corrections are associated to a gluon ladder exchanged in the t channel, the Reggeised or Glauber gluon.
  t channel two-gluon ladder and s channel terms are logarithmically suppressed

  BFKL 1976-77

$$\mathcal{A} \simeq \exp\left(i\pi \mathbf{T}_s^2 + \mathbf{T}_t^2 \log\left(\frac{s}{-t}\right)\right)$$

- In the Regge limit of gravity, the leading radiative corrections are due to the eikonal phase terms (Weinberg's soft gravitons).
  t channel one-graviton ladder is power suppressed in t/s

  Bartels Lipatov Sabio-Vera 1208.3423
  Melville Naculich Schnitzer White 1306.6019

  colour-kinematics duality

  $\mathbf{T}_s^2 \to s \qquad \mathbf{T}_t^2 \to t$

- s channel ladders win over t channel ladders

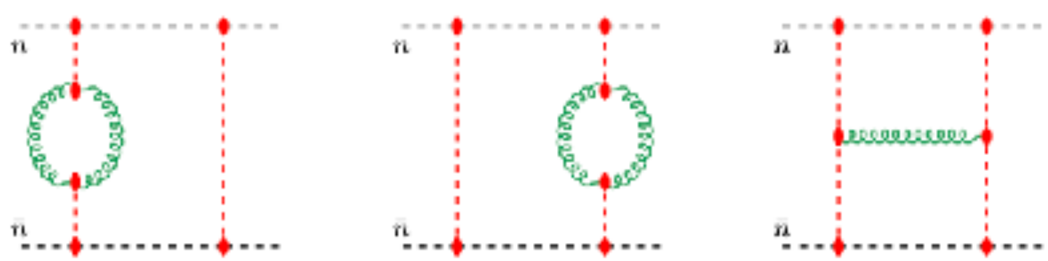
Glauber EFT of gravity

- Mimicking Glauber EFT of QCD   Rothstein Stewart 1601.04695

$$\mathcal{M}_{2\to 2} = i \sum_M J_{(M)} \otimes S_{(M)} \otimes \bar{J}_{(M)}$$

Rothstein Saavedra 2412.04428 showed that the exchange of a *t*-channel two-graviton ladder is ruled by a rapidity RGE, whose anomalous dimension is Lipatov gravity (BFKL-like) kernel with graviton trajectory and graviton central-emission vertex (CEV)   Lipatov 1982

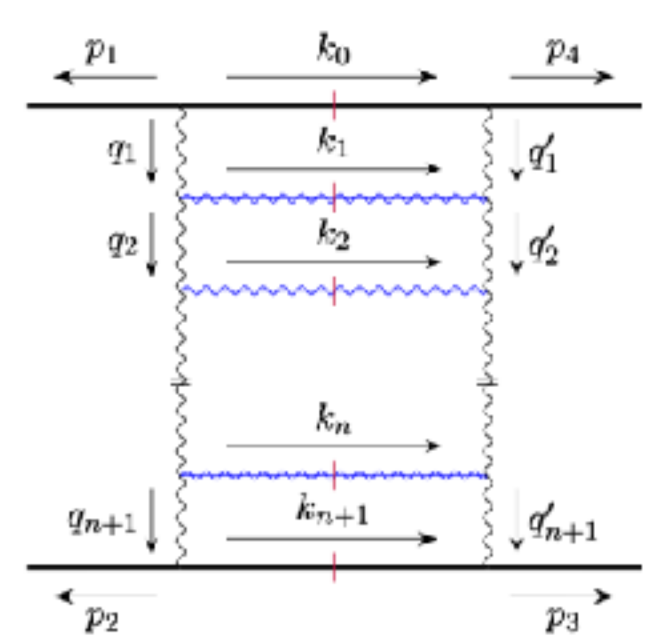
$$\nu \frac{d}{d\nu} S_{(N)} = -\gamma^{\nu}_{(N)} \otimes S_{(N)} - S_{(N)} \otimes \gamma^{\nu}_{(N)}$$



$$\gamma^{\nu}_{(M)} \sim \sum_j \omega_G(q_i) I_{\perp(M-1)} + \sum_{\mathrm{Pairs}\, i,j} \mathcal{K}^{\mathrm{GR}}(q_i, q_j; q)\, I_{\perp(M-2)}$$

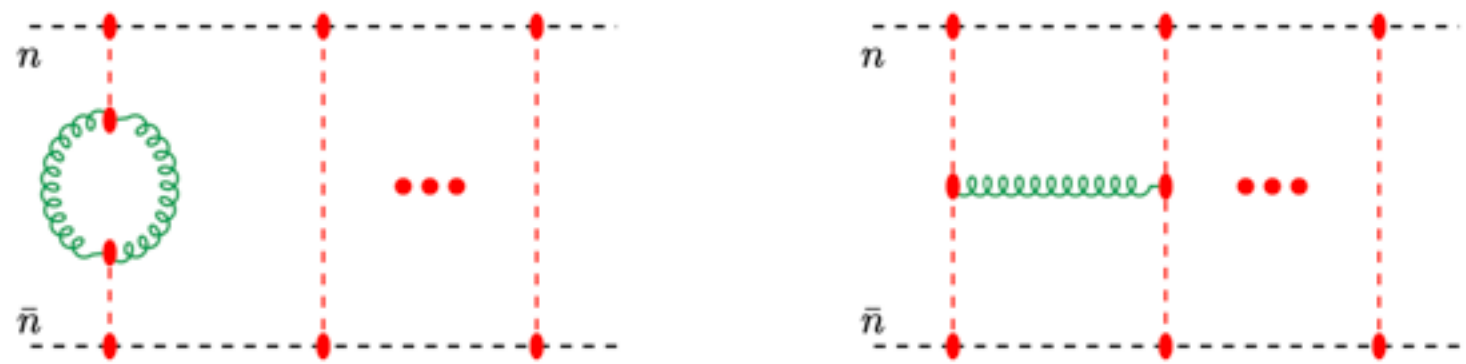
through $\hbar$ counting, easy to see that trajectory is quantum,
in fact the whole two-graviton ladder is quantum,
except for one graviton CEV emission: the *H diagram*

$$\mathrm{Im}\, \mathcal{M}^{(n+1)}_{2\to 2}(s, q^2) \sim s^3 \log\left(\frac{s}{-t}\right)^n \frac{G}{q_\perp^2}\, G\, q_\perp^n\, (G\, q_\perp)^n$$



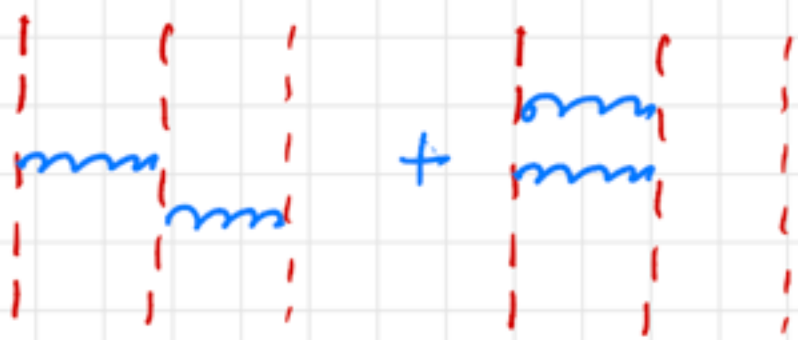
Glauber EFT of gravity

Rothstein Saavedra 2412.04428 showed that the exchange of a *t*-channel multi-graviton ladder is ruled by a rapidity RGE, whose anomalous dimension is a convolution of gravity BFKL-like kernels



Expanding the rapidity RGE to second order in G

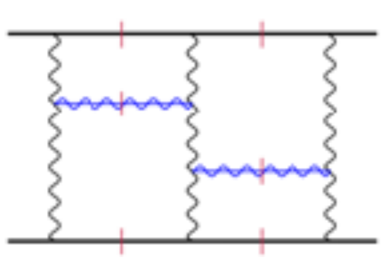
$S^{(2)}_{(3,3)} \sim$



(note these are $G^2 s \ln s$ corrections to $S^{(1)}_{(2,2)}$)

the first is classical, the second is quantum

through $\hbar$ counting, easy to see that the whole three-graviton ladder is quantum, except for the convolution of two graviton CEVs

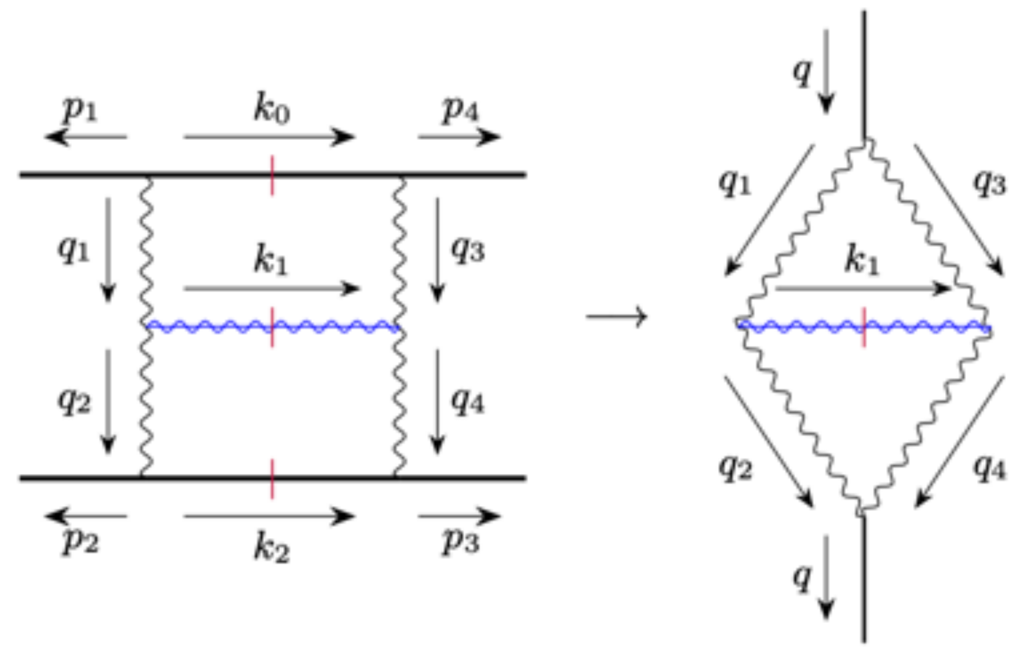


$$\mathrm{Im} A^{(4)}(s,t) \simeq G^5 s^4 \log^2(s/t) \cdot (\text{poles in } \varepsilon)$$

In fact, a *t*-channel ladder with *(n+2)* gravitons in the *s* channel features a classical term of (2n+3)-PM order, and provides a correction of $O((G^2\, s \log(s/|t|))^n)$ to the *H* diagram

s-channel discontinuities

- Revisiting the *H* diagram   Amati Ciafaloni Veneziano 1990



in MRK, the light-cone dof's decouple from the transverse dof's

$$\text{Disc}_s \mathcal{M}^{(2)}_{2\to 2}(s,q^2) \simeq \frac{\kappa^6 s^3}{8\pi} \log(s) H_1(q_\perp^2),$$

$$H_1(q_\perp^2) \equiv \int_{q_{1\perp}} \int_{q_{2\perp}} \frac{\mathcal{K}^{\text{GR}}(q_1,q_2;q)}{q_{1\perp}^2 q_{2\perp}^2 (q-q_1)_\perp^2 (q-q_2)_\perp^2}$$

gravity (BFKL-like) kernel

$$\mathcal{K}^{\text{GR}}(q_1,q_2;q_3,q_4) = \left[(q_{1\perp}+q_{3\perp})\cdot(q_{2\perp}+q_{4\perp}) - \frac{q_{2\perp}^2 q_{3\perp}^2 + q_{1\perp}^2 q_{4\perp}^2}{k_\perp^2}\right]^2 + \frac{4}{k_\perp^4}\left[q_{1\perp}^2 q_{2\perp}^2 q_{3\perp}^2 q_{4\perp}^2 - q_{3\perp}^2 q_{4\perp}^2 (q_{1\perp}\cdot q_{2\perp})^2 - q_{1\perp}^2 q_{2\perp}^2 (q_{3\perp}\cdot q_{4\perp})^2\right]$$

in momentum space $\quad \text{Disc}\, \mathcal{M}^{(2)}_{2\to 2}(s,q^2) = 8G^3 s^3 \log(s)\left(-\frac{1}{\epsilon^2} + \frac{2}{\epsilon} + \zeta_2 + \mathcal{O}(\epsilon^0)\right)$

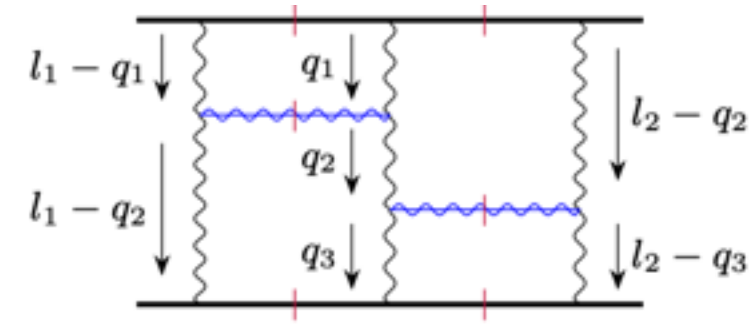
Fourier transforming in impact parameter space

$$\tilde{\mathcal{M}}_{2\to 2}(s,b) \simeq \frac{1}{2s}\int \hat{d}^d q_\perp e^{ib\cdot q_\perp} \mathcal{M}_{2\to 2}(s,-q_\perp^2) \equiv \text{F.T.}[\mathcal{M}_{2\to 2}](s,b)$$

$$\text{Disc}\, \tilde{\mathcal{M}}^{(2)}_{2\to 2}(s,b) = \frac{8G^3 s^2}{b^2} \log(s)\left(-\frac{1}{\epsilon} + 2 + \mathcal{O}(\epsilon^0)\right)$$

H^2 diagram

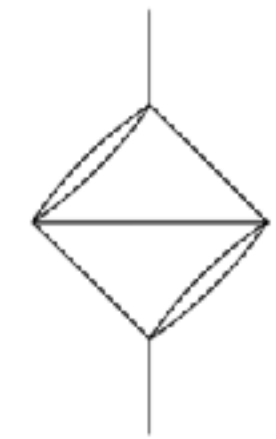
- through iterated s-channel discontinuities



$$\mathrm{Disc}_s^2\,\mathcal{M}^{(4)}_{2\to 2}(s,q^2) \simeq c\frac{\kappa^{10}s^4}{64\pi^2}\log(s)^2 H_2(q_\perp^2)$$

with H_2 the convolution of two gravity kernels

the result is reduced to bubbles and a kite integral



in momentum space

$$\mathrm{Disc}_s^2\mathcal{M}^{(4)}_{2\to 2}(s,q^2) \simeq c\frac{\kappa^{10}s^4}{32\pi^2}\log(s)^2\frac{1}{(q^2)^3}\left(\frac{q^2}{4\pi}\right)^d\left[-\frac{3}{\epsilon^3}-\frac{3}{\epsilon^2}-\frac{27}{\epsilon}+\frac{6}{\epsilon}\zeta_2+\frac{6}{\epsilon}\zeta_3+O(1)\right]$$

i.e. an $O(G^5\, s^4 \log(s/|t|)^2)$ term

Alessio VDD Gonzo Rosi Rothstein Saavedra 25xx.yyyyy

in impact parameter space

$$\mathrm{Disc}_s^2\tilde{\mathcal{M}}^{(4)}_{2\to 2}(s,b^2) \simeq c\frac{\kappa^{10}s^3}{128\pi^2}\log(s)^2\frac{(b^2)^3}{(\pi b^2)^{5d/2}}e^{9\gamma\epsilon}\left[\frac{3}{8\epsilon^2}-\frac{3}{\epsilon}+\frac{15}{2}+\frac{15}{16}\zeta_2-\frac{3}{4}\zeta_3+O(\epsilon)\right]$$

Conclusions

- "It has been shown that the non-resummed PM-expanded scattering angles demonstrate poor convergence towards NR. This motivates the exploration of *resummation* strategies… … in particular we focus on the approach to the high-energy limit for equal-mass non-spinning binaries, which proves to be challenging for all resummation schemes considered …"

  Pratten Schmidt Swain 2411.09652

- In the Regge limit, at leading logarithmic accuracy, there is an s channel sequence of classical corrections of $O((G^2\, s \log(s/|t|))^n)$ to the *H* diagram

- Those terms are $(2n+3)$-PM and $(n+1)$-SF order, i.e. maximal SF order within a fixed PM order

- Using EFT and iterated s-channel discontinuities, we computed the H^2 diagram, which is an $O(G^5\, s^4 \log(s/|t|)^2)$ term, i.e. a 5 PM, 2 SF term and represents the first correction to the *H* diagram