Dimensional regularization and non-anticommuting γ_5 — towards the EWSM

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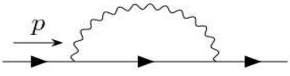
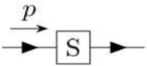
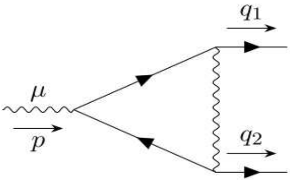
Collaborators: Bélusca-Maïto, Ilakovac; Ebert, **Kühler**, Mađor-Božinović, **Weisswange**, **v. Manteuffel**

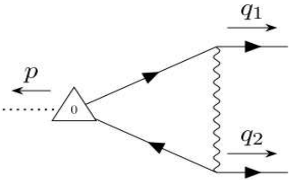
1-Loop: [2004.], 2-loop: [2109.], 3-loop [2312.], 4-loop [2506.] review: [2303.], 1-loop "couplings" [2411.] 2-loop YM [2504.]

Preview: Procedure in a nutshell









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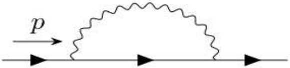

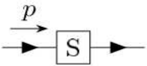

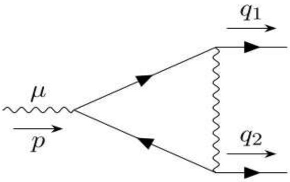

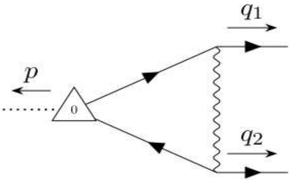

Ward identity
violated

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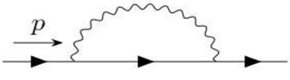

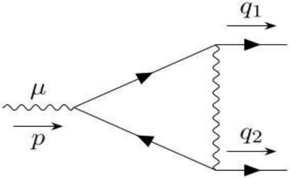

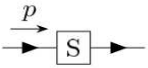

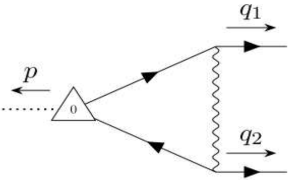

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compensated by
special c.t.

(our main task)

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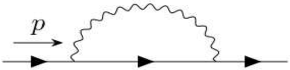

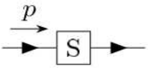

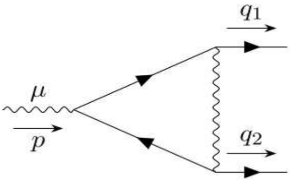

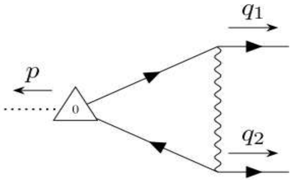

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$$S^1_{\text{fct}} = \frac{e^2}{16\pi^2} \int d^4 x \left\{ \ldots + \left(\frac{5+\xi}{6}\right) (\mathcal{Y}^j_R)^2 \left(\bar{\psi}_j i \bar{\not{\partial}}\, P_{\text{R}}\, \psi_j \right) \right\}.$$

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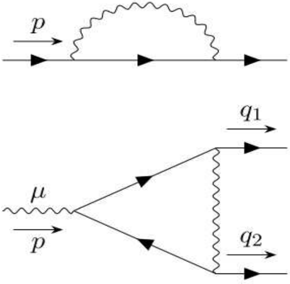

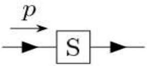

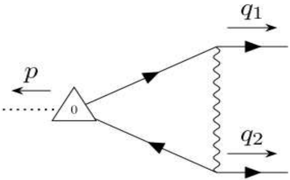

Ward identity violated

compensated by special c.t.

(our main task)

Alternative: breaking via $\frac{\epsilon}{\epsilon}$ term

(tool)

Status

2021 $\longrightarrow$ 2025

Abelian model	1L	2L	3L	4L
Non-Abelian	1L	2L		
SM-like, with scalars	1L			

EWSM: in progress

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Outline

The problem: γ_5 and DReg

Three properties in 4-dimensions:

$$\{\gamma_5, \gamma^\mu\} = 0, \tag{1}$$
$$\mathrm{Tr}(\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \tag{2}$$
$$\mathrm{Tr}(\Gamma_1 \Gamma_2) = \mathrm{Tr}(\Gamma_2 \Gamma_1)\,. \tag{3}$$

Inconsistent in $D \neq 4$ (can prove that trace=0).
Give up at least one $\Rightarrow$ many proposals!

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Give up at least one $\Rightarrow$ many proposals!

- BMHV (non-anticommuting, breaks gauge inv. But unitary, consistent)
- "*D*-dim space" split into pure 4-dim space $\oplus$ (-2ϵ)-dim space

$$X^\mu = \bar{X}^\mu + \hat{X}^\mu \qquad \{\gamma_5, \bar{\gamma}^\mu\} = 0$$
$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 \qquad [\gamma_5, \hat{\gamma}^\mu] = 0$$

Details on symmetries

gauge invariance $\rightarrow$ BRST invariance $\rightarrow$ Slavnov-Taylor identity is requirement on renormalized theory

To be fulfilled after adding counterterms:

$$S(\Gamma_{\text{ren}}) = 0$$

combines well-known STI/WIs between 1PI Green functions e.g.

- photon self energy = transverse
- Ward identity between fermion self energy and vertex function, etc

Details on dimensional regularization

Regularized diagrams $\leftrightarrow$ regularized Lagrangian!

E.g. QED or QCD:

$$D\text{-dim propagator} \leftrightarrow \quad \mathcal{L}_{\text{kin}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi$$

$$\text{fully } D\text{-dim gauge interaction} \leftrightarrow \quad \mathcal{L}_{\text{int}} = \bar{\psi} \gamma^\mu A_\mu \psi$$

$\Rightarrow$ in total: *D*-dim gauge invariance

Details on dimensional regularization

Regularized diagrams $\leftrightarrow$ regularized Lagrangian!
However, e.g. neutrino and B^μ (hypercharge):

$$D\text{-dim propagator} \leftrightarrow \qquad \mathcal{L}_{\text{kin}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi$$

$$\text{purely "L" gauge interaction} \leftrightarrow \qquad \mathcal{L}_{\text{int}} = \bar{\psi} P_R \gamma^\mu B_\mu P_L \psi$$

$\Rightarrow$ in total: not gauge invariant!

Details on dimensional regularization

Regularized diagrams $\leftrightarrow$ regularized Lagrangian! $\leftrightarrow$ path integral

Simple path integral relations “quantum action principle” [Breitenlohner, Maison '77]

$$“\quad \delta \int \mathcal{D}\phi \; e^{i\int \mathcal{L}} = \int \mathcal{D}\phi \; (i \textstyle\int \delta\mathcal{L}) e^{i\int \mathcal{L}} \quad ”$$

are literally valid if path integral is defined by DReg diagrams **and** *D*-dim Lagrangian

Technical tool therefore:

$$\mathcal{S}(\textstyle\int \mathcal{L}) = \Delta \qquad \Rightarrow \qquad \mathcal{S}(\Gamma_{\text{reg}}) = \Delta \cdot \Gamma \qquad \text{in } D \text{ dimensions}$$

Compute this! Then determine counterterms such that

$$\mathcal{S}(\Gamma_{\text{ren}}) = 0 \qquad \text{in } D \to 4 \text{ limit}$$

Abelian case: Lagrangian and BRST symmetry breaking

$$\mathcal{L}_{\text{kin+int}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \bar{\psi} P_L \gamma^\mu P_R A_\mu \psi + \ldots$$

- Mismatch breaks gauge invariance of $\mathcal{L}_D$

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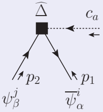
$$\mathcal{L}_{\text{kin+int}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \bar{\psi} P_L \gamma^\mu P_R A_\mu \psi + \ldots$$

- Mismatch breaks gauge invariance of $\mathcal{L}_D$
  $\Rightarrow$ and leads to breaking of tree-level Slavnov-Taylor identity

$$\mathcal{S}_d(S_0) = \widehat{\Delta} \equiv \int \mathrm{d}^d x \, (e\mathcal{Y}_{Ri}) c \left\{ \overline{\psi}_i \left(\overleftarrow{\widehat{\partial\!\!\!/}} P_\text{R} + \overrightarrow{\widehat{\partial\!\!\!/}} P_\text{L} \right) \psi_i \right\} .$$

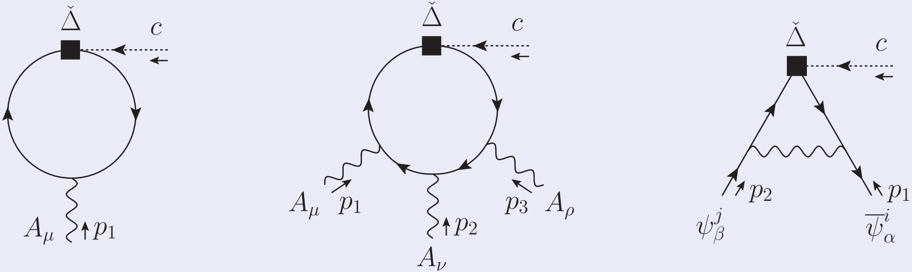


$$= (e\mathcal{Y}_{Ri}) \left(\widehat{p\!\!\!/}_1 P_\text{R} + \widehat{p\!\!\!/}_2 P_\text{L} \right)_{\alpha\beta}$$

Insertions $\widehat{\Delta} \cdot \Gamma$ determine STI breaking
Only $\frac{1}{\epsilon^n}$ can contribute!

Full 1-loop calculation of $\widehat{\Delta} \cdot \Gamma$

The complete set of power-counting divergent 1-loop diagrams with insertion of $\widehat{\Delta}$:



Results mean: breaking of three concrete WI/STIs.

They have the form $\frac{\epsilon/\text{evanescent}}{\epsilon} \times$ (local)

$\rightsquigarrow$ local counterterms can repair the symmetry!

(There is an additional diagram corresponding to the fermion triangle loop and the true anomaly (assumed absent))

Details: Results for counterterms

$$0 \stackrel{!}{=} \underbrace{\mathcal{S}(\mathrm{LIM}_{D\to 4}\Gamma^{(1)}_{\mathrm{reg}}}_{\substack{\text{from}\\ \hat{\Delta}\cdot\Gamma^{1}_{\mathrm{reg}}}} + \underbrace{S^1_{\mathrm{sct}}}_{\text{finiteness}} + \underbrace{S^1_{\mathrm{fct}})}_{\text{sym.restoration}}$$

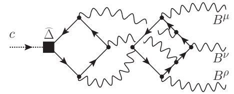
Full 1-loop symmetry-restoring counterterms:

$$S^1_{\mathrm{fct}} = \frac{e^2}{16\pi^2}\int \mathrm{d}^4 x \left\{ \frac{-\operatorname{Tr}(\mathcal{Y}_R^2)}{6}\bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + \frac{e^2 \operatorname{Tr}(\mathcal{Y}_R^4)}{12}(\bar{A}^2)^2 + \left(\frac{5+\xi}{6}\right)(\mathcal{Y}_R^j)^2 \left(\bar{\psi}_j i \bar{\partial\!\!\!/} P_{\mathrm{R}}\, \psi_j\right) \right\}.$$

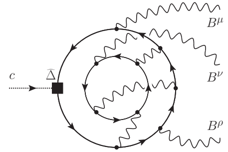
$\rightsquigarrow$ Correspondence to the three diagrams/three Ward identities

Higher orders (subrenormalization!) [Paul Kühler…Matthias Weisswange…]

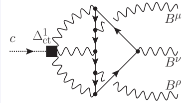
4-loop insertion of $\widehat{\Delta}$



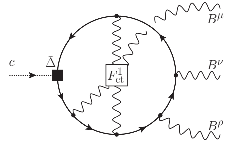
4-loop insertion of $\widehat{\Delta}$



3-loop insertion of Δ^1_{ct}



3-loop counterterm diagram



Checks: 1. The sum gives $\mathcal{S}_d(\Gamma^{(n)})|_{\text{finite}}$ =local.

2. We can solve $\mathcal{S}_d(\Gamma^{(n)})|_{\text{finite}} + \mathcal{S}_d S^n_{\text{fct}} \stackrel{!}{=} 0$ for local CTs.

General pattern at n =2-, 3-, 4-loop

Requiring the renormalized STI to hold leads to the result

$$S_{\text{fct}}^{n} = \frac{e^{2n}}{(16\pi^2)^n} \int d^4 x \left\{ \mathcal{F}_{AA}^{n,\text{break}} \frac{1}{2} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu - \frac{e^2}{8} \mathcal{F}_{AAAA}^{n,\,\text{break}} (\bar{A}^2)^2 - \mathcal{F}_{\psi\bar{\psi},\,ji}^{n,\,\text{break}} \left(\bar{\psi}_j i \bar{\not\partial}\, P_{\text{R}}\, \psi_j \right) \right\}$$

$$\mathcal{F}_{AA}^{2,\text{break}} = \tfrac{11}{48} \text{Tr}\left(\mathcal{Y}_R^4\right)$$

$$\mathcal{F}_{AA}^{3,\text{break}} = -\left(\tfrac{35242}{21600} + \tfrac{8448}{21600}\,\zeta_3\right) \text{Tr}\left(\mathcal{Y}_R^6\right) - \tfrac{1639}{21600}\, \text{Tr}\left(\mathcal{Y}_R^4\right) \text{Tr}\left(\mathcal{Y}_R^2\right)$$

$$\mathcal{F}_{AA}^{4,\text{SM}} = \frac{105574465087}{604661760} + \frac{2665684621}{6998400}\zeta(3) - \frac{499349}{174960}\zeta(4) - \frac{99009133}{209952}\zeta(5)$$

Technicalities: (A) Mathematica+FeynArts+FeynCalc up to 3-loop, (B) QGraf+FORM+Fire/Reduze2 at 4-loop

[Bélusca-Maïto, Ilakovac, **Kühler**, Mađor-Božinović, DS ’21], [DS, Weisswange ’23], [v. Manteuffel, DS, Weisswange ’25]

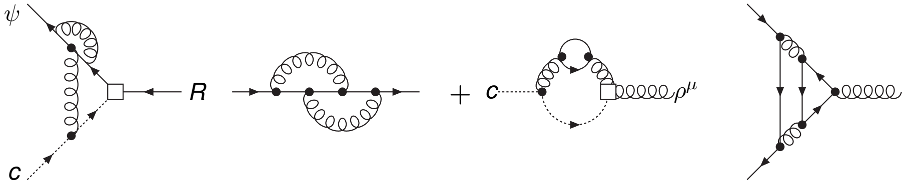
Three dimensions of difficulty

1. higher loops
2. abelian/non-abelian
3. intricacies of the EWSM

Non-abelian complications 1

- Gauge-boson and ghost interactions
- Non-linear gauge/BRST transformations receive loop corrections
- Appear in Ward/ST identities, e.g.

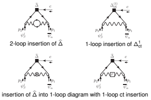
$$
\begin{array}{ccccc}
eQ & \Gamma_{\psi\bar{\psi}}(p') - \ldots + & q^{\mu} & \Gamma_{\psi\bar{\psi}A^{\mu}} & \stackrel{!}{=} 0 \\
\downarrow & & \downarrow & & \\
\Gamma_{\psi c \bar{R}} & \Gamma_{\psi\bar{\psi}}(p') - \ldots + & \Gamma_{c\rho_{\mu}} & \Gamma_{\psi\bar{\psi}A^{\mu}} & \stackrel{!}{=} 0
\end{array}
$$

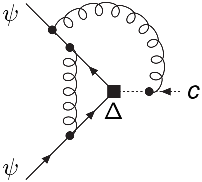


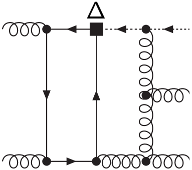
Non-abelian complications 2

2. Alternative calculation of breaking via $\Delta \cdot \Gamma_{\psi\bar{\psi}c}$:

lhs $=$ (like abelian case) $+$ (new structures, worse powercounting) elsewhere even 5-point fcts







- All implemented in FeynArts:
  (loop-corrected BRST prefactors, R =bosonic spinor, ρ^{μ} =fermionic vector with antighost number,
  Δ = ghost-number=1 vertex)
- Validation: both methods agree (different Feynman rules appear)
- 2-loop counterterm result obtained [Kühler, DS '25]

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In SM: e.g. electrons $\psi_{L,R}$ with $\mathcal{Y}_L \neq \mathcal{Y}_R \neq 0$

1. straightforward: $\psi = \psi_L + \psi_R$

$$\mathcal{L} = \underbrace{\overline{\psi} i \partial\!\!\!/ \psi}_{=\overline{\psi_L} i \overline{\partial\!\!\!/} \psi_L + \overline{\psi_R} i \hat{\partial\!\!\!/} \psi_L + \ldots} - g\mathcal{Y}_L \overline{\psi_L} \overline{B\!\!\!/} \psi_L - g\mathcal{Y}_R \overline{\psi_R} \overline{B\!\!\!/} \psi_R$$

breaks local **and** global hypercharge ⇝ CTs $(\Phi^\dagger\Phi)^2$ and $(\Phi\Phi)^2$

related: [Cornella, Feruglio, Vecchi'22][Naterop, Stoffer '23]

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2. add evanescent interaction (try to preserve QED g. inv.)

$$\mathcal{L} = \ldots - g\mathcal{Y}_{RL}\overline{\psi_R}\hat{B\!\!\!/}\psi_L$$

breaks local/global hypercharge more strongly

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breaks local/global hypercharge more strongly

3. fictitious sterile fields (like toy models): $\psi_1 = \psi_L + \psi_R^{\text{st}}$, $\psi_2 = \psi_r + \psi_L^{\text{st}}$

$$\mathcal{L} = \overline{\psi_1} i \partial\!\!\!/ \psi_1 + \overline{\psi_2} i \partial\!\!\!/ \psi_2 - g\mathcal{Y}_L \overline{\psi_L} \overline{B\!\!\!/} \psi_L - g\mathcal{Y}_R \overline{\psi_R} \overline{B\!\!\!/} \psi_R - m \overline{\psi_R} \psi_L$$

does not produce usual *D*-dim propagators

also: [C.P. Martin et al '99]

General exploration — U(1) part of SM

General framework containing all above as special cases
$\Rightarrow$ option 1 ist most promising [Ebert, Kühler, DS, Weisswange '24]

$$S^1_{\mathrm{fct}} = \frac{1}{16\pi^2}\Big[-\frac{g^2}{3}\overline{B}_\mu \Box \overline{B}^\mu + \frac{g^4}{24}\overline{B}_\mu \overline{B}^\mu \overline{B}_\nu \overline{B}^\nu$$
$$- i\frac{g}{3}\delta y_2\Big[\big(\overline{\partial}^\mu \phi_2^\dagger\big)\phi_2 - \phi_2^\dagger\big(\overline{\partial}^\mu \phi_2\big)\Big]\overline{B}_\mu - \frac{g^2}{4}\delta y_2\Big[\phi_1^\dagger\phi_1\overline{B}^\mu\overline{B}_\mu + \frac{5}{3}\phi_2^\dagger\phi_2\overline{B}^\mu\overline{B}_\mu\Big]$$
$$+ g\,\overline{\psi}_f \overline{\not{B}}\Big[\delta F^{\mathrm{opt1}}_{\mathrm{R},f}\mathbb{P}_{\mathrm{R}} + \delta F^{\mathrm{opt1}}_{\mathrm{L},f}\mathbb{P}_{\mathrm{L}}\Big]\psi_f$$
$$- \Big\{\delta Y^{\mathrm{opt1}}_{e,2}\, y_e \overline{e_L}\phi_2^\dagger e_R + \delta Y^{\mathrm{opt1}}_{u,2}\, y_u \overline{u_L}\phi_2 u_R + \delta Y^{\mathrm{opt1}}_{d,2}\, y_d \overline{d_L}\phi_2^\dagger d_R + \mathrm{h.c.}\Big\}$$
$$+ \frac{1}{6}\delta y_2\Big[\overline{\partial}_\mu\phi_2\overline{\partial}^\mu\phi_2 - \frac{3}{2}ig\overline{\partial}_\mu\big(\phi_2\phi_2\big)\overline{B}^\mu + \frac{3}{4}g^2\phi_2\phi_2\overline{B}_\mu\overline{B}^\mu + \mathrm{h.c.}\Big]$$
$$- \Big(\frac{1}{12}\delta y_4\,\phi_2\phi_2\phi_2\phi_2 + \frac{2}{3}\big(\delta y_4 - \delta y_{ud}\big)\phi_1^\dagger\phi_1\phi_2\phi_2 + \frac{2}{3}\delta y_4\,\phi_2^\dagger\phi_2\phi_2\phi_2 + \mathrm{h.c.}\Big)\Big],$$

Conclusions

Apply BMHV non-anticommuting γ_5 scheme to chiral gauge theories

- gauge/BRST invariance broken via $\Delta \cdot \Gamma$
- need symmetry-restoring and evanescent divergent counterterms

Status:

- Method established, many crosschecks
- two computations of breaking, locality, cancellability
- Feynman rules implemented in FeynArts and QGRAF setups

Results and outlook:

- 4-loop abelian model, 2-loop non-abelian
- 1-loop general fermions+scalars exploration
- work in progress: EWSM

Plan here: chiral "QED" (only $P_R\psi$) at 1-/2-loop

[Bélusca-Maïto, Ilakovac,Kühler Mađor-Božinović, DS '21]

1. Define D-dimensional Lagrangian compute symmetry breaking
2. Determine 1-loop UV divs $\rightsquigarrow \mathcal{L}_{\text{sct}}$
3. Determine 1-loop violation of Slavnov-Taylor identity
4. Determine 1-loop symmetry-restoring counterterms $\rightsquigarrow \mathcal{L}_{\text{fct}}$
5. Repeat at 2-loop new features?

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1. Define D-dimensional Lagrangian

Abelian theory like $U(1)_Y$-part of SM, only ψ_{Ri} interact

Description of symmetry: gauge invariance $\rightarrow$ BRST invariance $\rightarrow$ Slavnov-Taylor identity is required for renormalized theory: $S(\Gamma_{\text{ren}}) = 0$

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Description of symmetry: gauge invariance $\to$ BRST invariance $\to$
Slavnov-Taylor identity is required for renormalized theory: $S(\Gamma_{\text{ren}}) = 0$

$\mathcal{L}$ has D-dim kinetic but 4-dim interaction term!

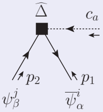
$$\mathcal{L}_{\text{fermions}} = i\overline{\psi}_i \partial\!\!\!/ \psi_i + e\mathcal{Y}_{Ri}\overline{\psi_R}_i A\!\!\!/ \psi_{Ri}\,.$$

1. Define *D*-dimensional Lagrangian

Abelian theory like $\mathrm{U}(1)_Y$-part of SM, only ψ_{Ri} interact

$\mathcal{L}$ breaks *D*-dim gauge/BRST invariance
$\Rightarrow$ and leads to breaking of tree-level Slavnov-Taylor identity

$$\mathcal{S}_d(S_0) = \widehat{\Delta} \equiv \int \mathrm{d}^d x\, (e\mathcal{Y}_{Ri}) c \left\{ \overline{\psi}_i \left(\overleftarrow{\widehat{\partial\!\!\!/}} P_\mathrm{R} + \overrightarrow{\widehat{\partial\!\!\!/}} P_\mathrm{L} \right) \psi_i \right\} .$$



$$= (e\mathcal{Y}_{Ri}) \left(\widehat{p\!\!\!/}_1 P_\mathrm{R} + \widehat{p\!\!\!/}_2 P_\mathrm{L} \right)_{\alpha\beta}$$

This is the core of the difficulties.
Can be written as a
local Feynman rule

2. Compute Green functions to determine UV divs

Many 1-loop diagrams (not shown) $\rightsquigarrow$ divergent counterterms:

First part as usual

second part is special for BMHV, sym-breaking and “evanescent”

3. Determine 1-loop violation of Slavnov-Taylor id.

Ultimate structure at 1-loop (finite ct to be determined)

$$\Gamma^{(1)}_{\text{DReg}} = \Gamma^{(1)} + S^1_{\text{sct}} + S^1_{\text{fct}}\,,$$

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Evaluate STI at 1-loop order, div-parts cancel, fin-parts t.b.d.

$$\mathcal{S}_d(\Gamma^{(1)}_{\text{DReg}}) = \underbrace{\mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}}} + \mathcal{S}_d S^1_{\text{fct}}$$

Left term means: breaking of regularized STI; must be computed.

In principle this corresponds to checking all STIs/WIs, e.g. Fermion 2-point/3-point function etc.

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Left term means: breaking of regularized STI; must be computed.

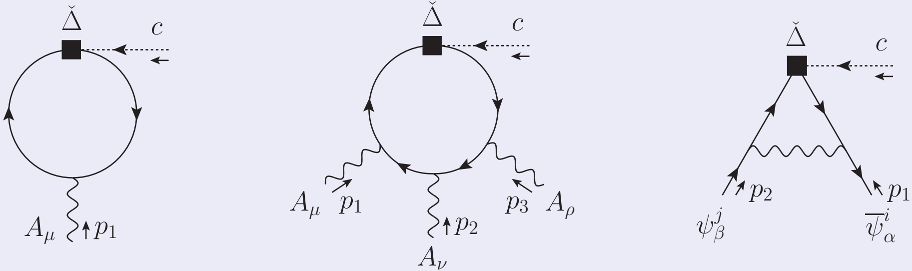
In principle this corresponds to checking all STIs/WIs, e.g. Fermion 2-point/3-point function etc.

But can be simplified by using quantum action principle (BM)

$$\mathcal{S}_d(\Gamma^{(1)}) = \widehat{\Delta} \cdot \Gamma^{(1)} \,,$$

Bonneau (1980): only power-counting divergent diagrams matter!

The complete set of power-counting divergent 1-loop diagrams with insertion of $\widehat{\Delta}$:



Results mean: breaking of three concrete WI/STIs.

They have the form $\frac{\epsilon/\text{evanescent}}{\epsilon} \times$ (local)

$\rightsquigarrow$ local counterterms can repair the symmetry!

(There is an additional diagram corresponding to the fermion triangle loop and the true anomaly (assumed absent))

4. Determine symmetry-restoring counterterms

$$\mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}} + \mathcal{S}_d S^1_{\text{fct}} \stackrel{!}{=} 0$$

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$$\mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}} + \mathcal{S}_d S^1_{\text{fct}} \stackrel{!}{=} 0$$

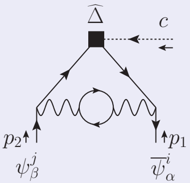
Requiring this renormalized STI to hold leads to the result

$$S^1_{\text{fct}} = \frac{e^2}{16\pi^2} \int d^4 x \left\{ \frac{-\operatorname{Tr}(\mathcal{Y}_R^2)}{6} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + \frac{e^2 \operatorname{Tr}(\mathcal{Y}_R^4)}{12} (\bar{A}^2)^2 + \left(\frac{5+\xi}{6}\right) (\mathcal{Y}_R^j)^2 \left(\bar{\psi}_j i \bar{\partial\!\!\!/} P_R \psi_j\right) \right\}.$$

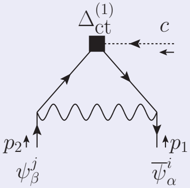
This is the full 1-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.
Finite, NON-evanescent counterterms. Not gauge invariant!
Modify both self-energies and A^4 interaction

5. Repeat at 2-loop (subrenormalization!)

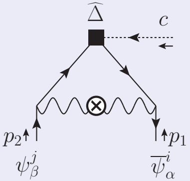
2-loop Slavnov-Taylor breaking — many diagrams of four types:

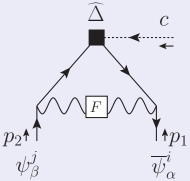


2-loop insertion of $\widehat{\Delta}$



1-loop insertion of Δ^1_{ct}





insertion of $\widehat{\Delta}$ into 1-loop diagram with 1-loop ct insertion

Sum gives $\mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}}$ =local. Can cancel by local counterterms

Determine sym-restoring counterterms at 2-loop

$$\mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d S^2_{\text{fct}} \stackrel{!}{=} 0$$

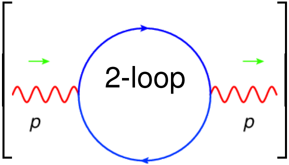
Requiring this renormalized STI to hold leads to the result

$$S^2_{\text{fct}} = \frac{e^4}{(16\pi^2)^2} \int d^4 x \left\{ \text{Tr}(\mathcal{Y}_R^4) \frac{11}{48} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + e^2 \frac{\text{Tr}(\mathcal{Y}_R^6)}{8} (\bar{A}^2)^2 \right.$$
$$\left. -(\mathcal{Y}_R^j)^2 \left(\frac{127}{36} (\mathcal{Y}_R^j)^2 - \frac{1}{27} \text{Tr}(\mathcal{Y}_R^2) \right) \left(\bar{\psi}_j i \bar{\not{\partial}} P_\text{R} \psi_j \right) \right\}$$

This is the full 2-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.
Finite, NON-evanescent counterterms. Not gauge invariant!
Same structure as at 1-loop

Application: restoration of 2-loop photon self energy

This is how the breaking looks like:



$$\overset{\text{fin-part}}{\propto} \frac{ie^4}{3\cdot 256\pi^4}\left[\left(\frac{673}{23} - 6\,\log(-\overline{p}^2) - 24\zeta(3)\right)(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu}) + \frac{11}{8}\,\overline{p}^\mu\overline{p}^\nu\right],$$

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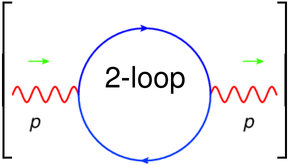


$$\stackrel{\text{fin-part}}{\propto} \frac{ie^4}{3 \cdot 256\pi^4} \Bigg[\left(\frac{673}{23} - 6\,\log(-\overline{p}^2) - 24\zeta(3) \right) (\overline{p}^\mu \overline{p}^\nu - \overline{p}^2 \overline{g}^{\mu\nu}) + \frac{11}{8} (\overline{p}^\mu \overline{p}^\nu - \overline{p}^2 \overline{g}^{\mu\nu}) \Bigg],$$

The breaking is compensated by the counterterm of the previous slide:

$$\mathcal{L}_{\text{fin-ct}} \propto -\frac{e^4}{3 \cdot 256\pi^4} \frac{11}{16} \bar{A}_\mu \overline{\partial}^2 \bar{A}^\mu$$

Generalization to YM

[Bélusca-Maïto, Ilakovac, Mađor-Božinović, DS, 2020]

symmetry-restoring counterterm for YM+fermions+scalars (1-loop)

$$S^1_{\text{fct,restore}} =$$

$$\frac{\hbar}{16\pi^2}\Big\{g^2\frac{S_2(R)}{6}\Big(5S_{GG}+S_{GGG}-\int d^4x\, G^{a\,\mu}\partial^2 G^a_\mu\Big)+\frac{Y_2(S)}{3}\overline{S_{\Phi\Phi}}$$

$$+g^2\frac{(T_R)^{abcd}}{3}\int d^4x\,\frac{g^2}{4}G^a_\mu G^{b\,\mu}G^c_\nu G^{d\,\nu}-\frac{(\mathcal{C}_R)^{ab}_{mn}}{3}\int d^4x\,\frac{g^2}{2}G^a_\mu G^{b\,\mu}\Phi^m\Phi^n$$

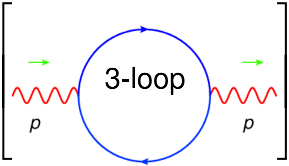
$$+g^2\Big(1+\frac{\xi-1}{6}\Big)C_2(R)S_{\overline{\psi}\psi}-\frac{\left((Y^m_R)^*T_{\overline{R}}{}^aY^m_R\right)_{ij}}{2}\int d^4x\, g\overline{\psi}_i \not{G}^a P_R\psi_j$$

$$-g^2\frac{\xi C_2(G)}{4}\big(S_{\bar{R}c\psi_R}+S_{Rc\overline{\psi_R}}\big)\Big\}\,,$$

Finite, NON-evanescent counterterms. Not gauge invariant!
Modify all self-energies and some interactions!
But rather compact, universal, can be/is implemented e.g. in FeynArts

3-loop outlook — photon self energy breaks transversality (preliminary) [Matthias Weisswange]

$$i\widetilde{\Gamma}_{AA}^{\nu\mu}(p)\Big|_{\mathrm{div}}^{3} =$$



$$\propto \frac{i\,e^6}{(16\pi^2)^3}\Bigg[\bigg(\frac{10}{81}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) - \frac{2}{27}\mathrm{Tr}(\mathcal{Y}_R^6)\bigg)\frac{1}{\epsilon^2} + \bigg(\frac{61}{1620}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) + \frac{638}{405}\mathrm{Tr}(\mathcal{Y}_R^6)\bigg)\frac{1}{\epsilon}\Bigg]\Big(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu}\Big)$$

$$+ \frac{i\,e^6}{(16\pi^2)^3}\Bigg[-\frac{1}{18}\mathrm{Tr}(\mathcal{Y}_R^6)\frac{1}{\epsilon^3} + \bigg(\frac{61}{540}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) + \frac{529}{1080}\mathrm{Tr}(\mathcal{Y}_R^6)\bigg)\frac{1}{\epsilon^2} + \bigg(\frac{4187}{32400}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) + \Big(\frac{49427}{64800} - \frac{544}{225}\zeta_3\Big)\mathrm{Tr}(\mathcal{Y}_R^6)\bigg)\frac{1}{\epsilon}\Bigg]\widehat{p}^2\overline{g}^{\mu\nu}$$

$$+ \frac{i\,e^6}{(16\pi^2)^3}\bigg(\frac{79}{1080}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) + \frac{1}{60}\mathrm{Tr}(\mathcal{Y}_R^6)\bigg)\frac{1}{\epsilon}\,\overline{p}^2\overline{g}^{\mu\nu}$$

General Summary

Background:

- γ_5 is problematic in DReg, BMHV scheme is rigorous
- γ_5 non-anticommuting, distinguish 4-dim and ϵ-dim quantities
- gauge invariance broken already in $\mathcal{L}_D$ and at loop level

Renormalization in general: $\Gamma_{\text{ren}} = \Gamma_{\text{reg}} + \Gamma_{\text{ct}}$

- Γ_{ren} should be finite
- $\mathcal{S}(\Gamma_{\text{ren}}) = 0$ should hold
- this fixes divergent and symmetry-restoring counterterms
- in addition, counterterms derived from field/parameter renormalization may be added

General Summary

Results:

- Symmetry-restoring counterterms: 1-/2-loop YM, 3/4-loop “QED”
- Method established: determine violation of Ward/Slavnov-Taylor identities from $\widehat{\Delta}$-diagrams
- Result has compact simple structure

Outlook:

- 1-, 2-loop EWSM, 3-loop YM ...
- many details not talked about relevant for SM!
- automatize, implement in FeynArts, FeynRules etc
- alternative $\mathcal{L}_D$, schemes (FDH, DRed, etc, other γ_5 schemes)
- RGEs
- Fierz problem...

2. Compute Green functions to determine UV divs

Many 1-loop diagrams (not shown) $\rightsquigarrow$ divergent counterterms:

$$S^1_{\text{sct}} = S^1_{\text{sct,inv}} + S^1_{\text{sct,break}}\,,$$

First part as usual

$$S^1_{\text{ct,inv}} = \frac{\delta Z^1_A}{2} L_A + \frac{\delta Z^1_c}{2} L_c + \frac{\delta Z^1_{\psi_R}}{2} \overline{L_{\psi_R}} + \frac{\delta e^1_A}{e_A} L_{e_A}\,,$$

second part is special for BMHV, sym-breaking and “evanescent”

$$S^1_{\text{sct,break}} = \frac{-\hbar\, e_A^2}{16\pi^2\epsilon} \frac{\text{Tr}(\mathcal{Y}_R^2)}{3} \left(2(\overline{S}_{AA} - S_{AA}) + \int \mathrm{d}^d x\, \frac{1}{2} \bar{A}^\mu \widehat{\partial}^2 \bar{A}_\mu \right)\,.$$

Divergences for evanescent operators with independent coefficients, beyond the usual field/parameter renormalization

well-known in DRed: needed for unitarity/finiteness at higher orders

2. Determine UV divs at 2-loop

Many 2-loop diagrams (not shown) $\rightsquigarrow$ divergent counterterms:

$$S^2_{\text{sct}} = S^2_{\text{sct,inv}} + S^2_{\text{sct,break}}\,,$$

First part as usual $\sim$ field and parameter renormalization
second part is special for BMHV, “sym-breaking” (partially non-evan.)

$$S^2_{\text{sct,break}} = -\frac{e^4}{256\pi^4\epsilon}\frac{\mathrm{Tr}(\mathcal{Y}_R^4)}{3}\left(2(\overline{S}_{AA} - S_{AA}) + \left(\frac{1}{2\epsilon} - \frac{17}{24}\right)\int \mathrm{d}^d x\ \frac{1}{2}\bar{A}^\mu \widehat{\partial}^2$$

$$-\frac{e^4}{256\pi^4}\frac{(\mathcal{Y}_R^j)^2}{3\epsilon}\left(\frac{5}{2}(\mathcal{Y}_R^j)^2 - \frac{2}{3}\mathrm{Tr}(\mathcal{Y}_R^2)\right)\overline{S^j_{\bar{\psi}\psi_R}}$$

3. Determine 2-loop violation of Slavnov-Taylor id.

Ultimate structure at 2-loop (fct to be determined)

$$\Gamma^{(2)}_{\text{DReg}} = \Gamma^{(2)} + S^2_{\text{sct}} + S^2_{\text{fct}} ,$$

Evaluate STI at 2-loop order, div-parts cancel, fin-parts t.b.d.

$$\mathcal{S}_d(\Gamma^{(2)}_{\text{DReg}}) = \mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d S^2_{\text{fct}}$$

Left term (breaking of regularized STI) must be computed, use q.a.p.

$$\mathcal{S}_d(\Gamma^{(2)}) = \widehat{\Delta} \cdot \Gamma^{(2)} + \Delta^1_{\text{ct}} \cdot \Gamma^{(1)}$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

How do Green functions behave?

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

Path integral:

Symmetry transformations of Green functions

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Path integral:

$$Z(J) = \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi}$$

Symmetry transformations of Green functions

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$$Z(J) = \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi}$$

(measure invariant)

$$= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+\delta\mathcal{L}+J\phi+J\delta\phi}$$

Symmetry transformations of Green functions

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(measure invariant) $$= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+\delta\mathcal{L}+J\phi+J\delta\phi}$$

(1st order in δ) $$= \int \mathcal{D}\phi \; (1 + i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}$$

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Formal “derivation” for $\delta\mathcal{L} = 0$ gives form of ST identities

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = 0$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

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result: $$0 = \int \mathcal{D}\phi\ (i\int \delta\mathcal{L} + J\delta\phi)e^{i\int \mathcal{L}+J\phi}$$

"derivation" is valid in DReg and gives breaking DREG: [Breitenlohner, Maison '77], DRED: [DS '05],review[2303.09120]

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\textstyle\int\delta\mathcal{L})\rangle$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

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result: $$0 = \int \mathcal{D}\phi\ (i\int \delta\mathcal{L} + J\delta\phi)e^{i\int \mathcal{L}+J\phi}$$

This is exactly true in DREG (where $\delta\mathcal{L}$ might be $\neq 0$)

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

Symmetry transformations of Green functions — really

Regularized quantum action principle

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

Interpret this as an identity between regularized Feynman diagrams

- becomes a property of regularization scheme, does not necessarily hold (no fundamental QFT requirement)
- if desired, must be proven for each regularization
- valid in BPHZ: [Lowenstein et al '71], DREG: [Breitenlohner, Maison '77], DRED: [DS '05]

Symmetry transformations of Green functions — really

Regularized quantum action principle

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

Interpret this as an identity between regularized Feynman diagrams

Idea of proof in DREG/DRED: look at possible Wick contractions

- $\delta\mathcal{L} = \delta\mathcal{L}_{\text{quadratic}} + \delta\mathcal{L}_{\text{int}}, \qquad \delta\mathcal{L}_{\text{quadratic}} = (\delta\phi_i)D_{ij}\phi_j$
- Use properties of DREG/DRED: D is inverse propagator even on regularized level, scaleless integrals vanish
- then, combinatorics leads to above identity