Effective Lagrangians for heavy particles via functional matching

Stefan Dittmaier

universität freiburg

(based on S.D., S.Schuhmacher, M.Stahlhofen, EPJC 81 (2021) 826, arXiv:2102.12020; more in preparation)

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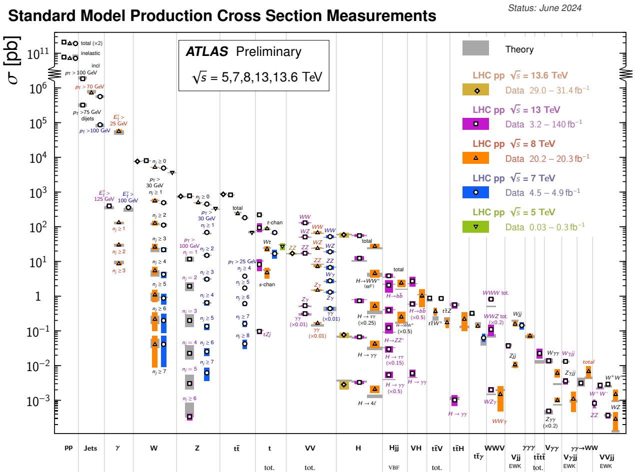
Introduction



Excellent agreement between Standard Model (SM) predictions and LHC data

⇒ Effective Field Theories (EFTs) as diagnostic tool
to identify traces of physics beyond the Standard Model (BSM)

The EFT ansatz: $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(\text{dim}-6)} + \dots, \quad \Lambda = \text{BSM scale}$

Most popular EFTs: (based on SM fields, fully SU(2)×U(1) symmetric)

- **SMEFT** Buchmüller/Wyler '85; Grzadkowski et al. 1008.4884; Brivio/Trott 1706.08945
  - ↪ Higgs field η and Goldstone fields ϕ form linear Higgs doublet

    $\mathbf{\Phi} = \frac{1}{\sqrt{2}}\left[(v+\eta)\mathbb{1} + \mathrm{i}\phi_j\tau_j\right]$ (2 × 2 matrix notation, τ_j = Pauli matrices)
  - ↪ canonical basis of 2499 dim-6 operators (59 B-conserving for 1 fermion generation)

- **Higgs EFT (HEFT)** see, e.g., Grinstein/Trott 0704.1505; Alonso et al. 1212.3305; Contino et al. 1303.3876
  - ↪ Higgs field h = singlet; gauge symmetry realized non-linearly

    $\mathbf{\Phi} = \frac{1}{\sqrt{2}}(v+h)U(\zeta), \quad U(\zeta) = \exp\left\{\mathrm{i}\zeta_j\tau_j/v\right\}$ = unitary Goldstone matrix
  - ↪ HEFT ⊃ SMEFT, more operators, no canonical basis

The EFT concept:

- **Bottom-up approach:**
  ansatz for $\mathcal{L}_{\text{eff}}$ → fit Wilson coefficients to exp. data → info on full theory

- **Top-down approach:**
  specific UV-complete theory with heavy d.o.f. (mass $M = \Lambda$)
  ↪ EFT via large-mass expansion ($M \gg v$)

Two methods for the top-down approach to construct EFTs:

$\mathcal{L}_{\rm BSM}(\varphi, H)$ $\to$ UV-complete theory

φ = light SM-like fields; H = heavy fields of mass $M \gg v$

$$\mathcal{L}_{\rm eff}(\varphi) = \mathcal{L}_{\rm SM}(\varphi) + \sum_i \frac{c_i}{M^2}\,\mathcal{O}_i^{(\rm dim-6)}(\varphi) + \dots$$

- Diagrammatic matching:

  Compare full BSM and EFT predictions for many observables X_j:

$$\left.\begin{array}{rcl} X_j^{\rm BSM} & \underset{M\to\infty}{\widetilde{}} & X_j^{\rm SM} + \frac{\Delta x_j}{M^2} \\ \stackrel{!}{=}\; X_j^{\rm EFT} & \underset{M\to\infty}{\widetilde{}} & X_j^{\rm SM} + \sum_i a_{ji}\,\frac{c_i}{M^2} \end{array}\right\} \quad \Rightarrow \quad c_i = \sum_j \left(a^{-1}\right)_{ij} \Delta x_j$$

  Note: Ansatz for generic form of $\mathcal{L}_{\rm eff}$ required (assumptions!) ☹

- Functional matching:

  Calculate functional integral over heavy fields H for $M \gg v$:

$$\int \mathcal{D}\varphi \int \mathcal{D}H \,\exp\left\{ {\rm i} \int {\rm d}^D x\, \mathcal{L}_{\rm BSM}(\varphi, H)\right\} = \int \mathcal{D}\varphi\, \exp\left\{ {\rm i} \int {\rm d}^D x\, \mathcal{L}_{\rm eff}(\varphi)\right\}$$

  Method totally agnostic w.r.t. form of EFT (no assumptions!) ☺

This talk: versatile method for functional matching

+ application to Higgs Singlet Extension of the SM

Related work on functional matching:

- first attempts: Gaillard, Chan, Cheyette '86–'88
- heavy Higgs in SM: SD/Grosse-Knetter '95,'96
- revival: Henning/Lu/Murayama '14; Boggia/Gomez-Ambrosio/Passarino '16; Fuentes-Martin/Portoles/Ruiz-Femenia '16; Buchalla et al. '17
- UOLEA: Drozd et al. '15; Ellis et al. '16–'20 Krämer/Summ/Voigt '19
- applications: Jiang et al. '18; Haisch et al. '20
- automation: Criado '17; Cohen/Lu/Zhang '20; Fuentes-Martin et al. '20
- ...

Features of our work: SD/Schuhmacher/Stahlhofen, 2102.12020; more in preparation

- fields of mass eigenstates integrated out (no problems with mixing)
- non-linear Higgs realization → flexibility (SMEFT, HEFT, (non-)decoupling)
- renormalization of BSM sector (mostly neglected elsewhere)
- validation against full NLO BSM predictions (in progress)

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Higgs Singlet Extension of the SM (HSESM)

Schabinger/Wells '05;
Patt/Wilczek '06;
Bowen/Cui/Wells '07; ...

Extended Higgs sector:

SM-like SU(2) doublet Φ & real scalar singlet σ: $v_{1,2}$ = vevs

$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}\left[(v_2+\eta_2)\mathbb{1} + \mathrm{i}\phi_j\tau_j\right] = \underbrace{\frac{1}{\sqrt{2}}(v_2+h_2)U(\zeta)}_{\text{non-linear representation as in SM}}, \qquad Y_{\mathrm{W}}(\Phi) = 1$$

$$\sigma = v_1 + \eta_1 = v_1 + h_1$$

Note: h_1, h_2 = gauge invariant (α = mixing angle)

$\hookrightarrow$ Mass basis h, H: $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R(\alpha)\begin{pmatrix} H \\ h \end{pmatrix}, \quad R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

Higgs Lagrangian: (restriction to real, $\mathbb{Z}_2$-symmetric case)

$$\mathcal{L}_{\mathrm{Higgs}} = \frac{1}{2}tr\left[(D_\mu\mathbf{\Phi})^\dagger(D^\mu\mathbf{\Phi})\right] + \frac{1}{2}(\partial\sigma)^2 - V$$

$$V = -\frac{\mu_2^2}{2}\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right] - \mu_1^2\sigma^2 + \frac{\lambda_2}{16}\left(\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right]\right)^2 + \lambda_1\sigma^4 + \frac{\lambda_{12}}{2}\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right]\sigma^2$$
$$= -\frac{\mu_2^2}{2}(v_2+h_2)^2 - \mu_1^2(v_1+h_1)^2 + \frac{\lambda_2}{16}(v_2+h_2)^4 + \lambda_1(v_1+h_1)^4$$

- rest identical to the SM
- fermion sector neglected in first step

Input parameters:

original set: $\{\lambda_1, \lambda_2, \lambda_{12}, \mu_1^2, \mu_2^2, g_1, g_2\}$

$\downarrow$

mass basis: $\{ \underbrace{M_\mathrm{H}, M_\mathrm{h}, M_\mathrm{W}, M_\mathrm{Z}, e}_{\text{renormalized on-shell}}, \underbrace{\lambda_{12}}_{\overline{\mathrm{MS}}}, \alpha \}$

Renormalization:

Bojarski et al. '15
Kanemura et al. '15,'17
Denner et al. '17,'18
Altenkamp et al. '18

Renormalization of α:

$\overline{\mathrm{MS}}$ (with various tadpole schemes), on-shell (OS), or symmetry inspired.

Large-mass limit: expansion for $\zeta \to \infty$

$M_\mathrm{H} \sim \zeta^1$, $\alpha \sim \zeta^{-1}$, all other parameters fixed ($M_\mathrm{h} = 125\,\mathrm{GeV}$)

$$\Leftrightarrow \lambda_1, \lambda_2, \lambda_{12} \sim \zeta^0, \quad \underbrace{v_2 \sim \zeta^0,}_{M_\mathrm{W} \sim \text{const.}} \quad \mu_1, \mu_2, v_1 \sim \zeta^1$$

Important:

- $\alpha \to 0$ for decoupling in lowest order of $\zeta \to \infty$
  (observed Higgs signal strengths close to 1)
- specification of scaling of all independent parameters,
  otherwise no unique large-mass limit
- $M_\mathrm{W} \sim$ const. for $\zeta \to \infty$, otherwise conflict with data on ζ trajectory

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$\hookrightarrow$ Impossible in 25min! ☹

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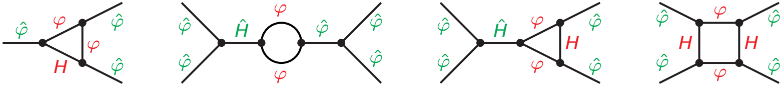

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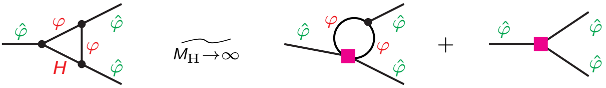

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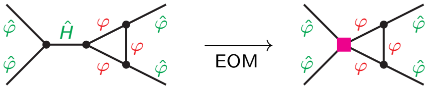

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   $\hookrightarrow$ formal solution of (Gaussian) functional integral $\int \mathcal{D}H$

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   $\hookrightarrow$ complicated result (many operators) via Neumann series

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6. Final result for the effective Lagrangian

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First result for $\delta\mathcal{L}_{\mathrm{eff}}^{\mathrm{1loop}}$ very cumbersome!

$\hookrightarrow$ Simplify result via

- integration by parts
- redefinition of light fields (leaves S-matrix invariant)

$$\varphi \to \varphi + \underbrace{F(\hat{\varphi}, \partial\hat{\varphi})}_{\text{loop suppressed}}.$$

In detail:

ansatz for $F(\hat{\varphi}, \partial\hat{\varphi})$ respecting Lorentz and $\mathrm{U(1)_{em}}$ symmetries

$\hookrightarrow$ $F(\hat{\varphi}, \partial\hat{\varphi})$ fixed by

- demanding absence of operators with dim > 6 and
- minimizing set of non-SMEFT operators

Final result: (after inverting Stueckelberg transformation, omitting hats)

$$\mathcal{L}_{\rm eff}^{\rm tree} = -\frac{s_\alpha^2}{2v_2^2}\,\mathcal{O}_{\Phi\Box}^{\rm SMEFT}$$

$$\mathcal{L}_{\rm eff}^{\rm 1loop} = \sum_i C_i^{\rm SMEFT}\,\mathcal{O}_i^{\rm SMEFT} \quad + \quad \sum_j C_j^{\rm HEFT}\,\mathcal{O}_j^{\rm HEFT}$$

non-SMEFT part!

$$\mathcal{O}_{\Phi}^{\rm SMEFT} = (\Phi^\dagger\Phi)^3$$

$$\mathcal{O}_{\Phi\Box}^{\rm SMEFT} = (\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi)$$

$$\mathcal{O}_{\Phi D}^{\rm SMEFT} = (\Phi^\dagger D^\mu\Phi)^*(\Phi^\dagger D_\mu\Phi)$$

$$\mathcal{O}_{\Phi B}^{\rm SMEFT} = (\Phi^\dagger\Phi)B_{\mu\nu}B^{\mu\nu}$$

$$\mathcal{O}_{\Phi W}^{\rm SMEFT} = (\Phi^\dagger\Phi)W^a_{\mu\nu}W^{a\mu\nu}$$

$$\mathcal{O}_{\Phi WB}^{\rm SMEFT} = -(\Phi^\dagger\tau^a\Phi)W^a_{\mu\nu}B^{\mu\nu}$$

$$\mathcal{O}_1^{\rm HEFT} = -\frac{2}{g_2^2}(v_2+h)^2\,{\rm tr}\left\{\left(D_B^\mu U^\dagger(D_\mu U)\right)\left(D_B^\nu U^\dagger(D_\nu U)\right)\right\}$$

$$\mathcal{O}_2^{\rm HEFT} = -\frac{2}{g_2^2}(v_2+h)^2\,{\rm tr}\left\{\left(D_B^\mu U^\dagger(D^\nu U)\right)\left(D_{B,\mu} U^\dagger(D_\nu U)\right)\right\}$$

$$\mathcal{O}_3^{\rm HEFT} = -\frac{2}{g_2^2}(v_2+h)^2\,{\rm tr}\left\{\left(D_B^\mu U^\dagger(D_\nu U)\right)\left(D_B^\nu U^\dagger(D_\mu U)\right)\right\}$$

$$\mathcal{O}_4^{\rm HEFT} = -\frac{2{\rm i}}{g_2^2}(v_2+h)^2\,B^{\mu\nu}{\rm tr}\left\{\tau^3(D_\mu U)^\dagger(D_\nu U)\right\}$$

$$\mathcal{O}_5^{\rm HEFT} = \frac{2}{g_2^2}(v_2+h)^2\,\partial^\mu\partial^\nu\,{\rm tr}\left\{(D_\mu U)^\dagger(D_\nu U)\right\}$$

$$\mathcal{O}_6^{\rm HEFT} = \frac{2}{g_2^2}(v_2+h)^2\,\Box\,{\rm tr}\left\{(D^\mu U)^\dagger(D_\mu U)\right\}$$

$$\mathcal{O}_7^{\rm HEFT} = \frac{2}{g_2^2}(\partial^\mu h)(\partial^\nu h)\,{\rm tr}\left\{(D_\mu U)^\dagger(D_\nu U)\right\}$$

$$\mathcal{O}_8^{\rm HEFT} = \frac{4}{g_2^4}(v_2+h)^2\,{\rm tr}\left\{\tau^a(D^\mu U)^\dagger(D^\nu U)\right\} \times {\rm tr}\left\{\tau^a(D_\mu U)^\dagger(D_\nu U)\right\}$$

$$D^\mu_{B,ab} \equiv \delta_{ab}\partial^\mu + g_1\epsilon_{ab3}B^\mu$$

Final result: (after inverting Stueckelberg transformation, omitting hats)

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- $\{\mathcal{O}_i^{\text{SMEFT}}\} \oplus$ chosen $\{\mathcal{O}_j^{\text{HEFT}}\}$ do not form a basis of HEFT

  (but an independent subset of operators)

- Decomposition into SMEFT $\oplus$ non-SMEFT operators not unique

  $\hookrightarrow$ C_i^{SMEFT} change even if only $\{\mathcal{O}_j^{\text{HEFT}}\}$ are changed

- $$C_i^{\text{SMEFT}} \propto \underbrace{\left\{\frac{1}{M_{\text{H}}^2}, \frac{s_\alpha^2}{v_2^2}\right\} f_i(\epsilon)\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{M_{\text{H}}^2}\right)}_{\text{contains contribution from } \delta s_\alpha}, \qquad \underbrace{C_j^{\text{HEFT}} \propto \frac{s_\alpha^2}{v_2^2}\, g_j(\epsilon)\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{M_{\text{H}}^2}\right)}_{\to\ 0 \text{ for } s_\alpha \to 0 \text{ (no mixing)}}$$

  But: alignment $s_\alpha \to 0$ not protected by symmetry

  $\hookrightarrow$ disturbed by corrections, depends on renormalization scheme for s_α!

Final result: (after inverting Stueckelberg transformation, omitting hats)

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  But: alignment $s_\alpha \to 0$ not protected by symmetry

  $\hookrightarrow$ disturbed by corrections, depends on renormalization scheme for s_α!

- Why non-SMEFT part at all?

  Linearly realized doublet $\mathbf{\Phi}$ destroyed by integrating out its H part:

  $$\mathbf{\Phi} = \frac{1}{\sqrt{2}}\left[(v_2 + \underbrace{\eta_2}_{= c_\alpha h + s_\alpha H})\mathbb{1} + \mathrm{i}\phi_j\tau_j\right]$$

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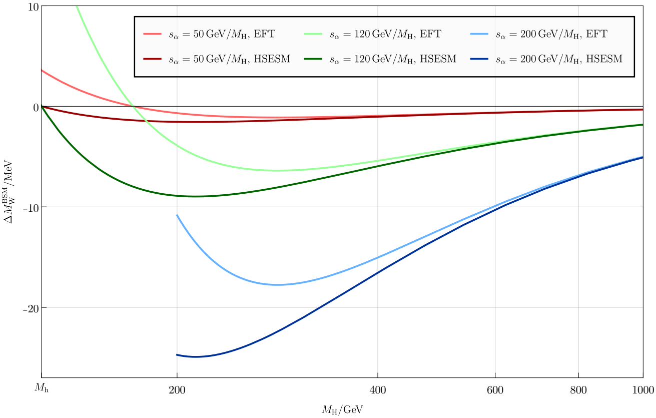
Validation and applications of the EFT

Example 1: W-boson mass from muon decay

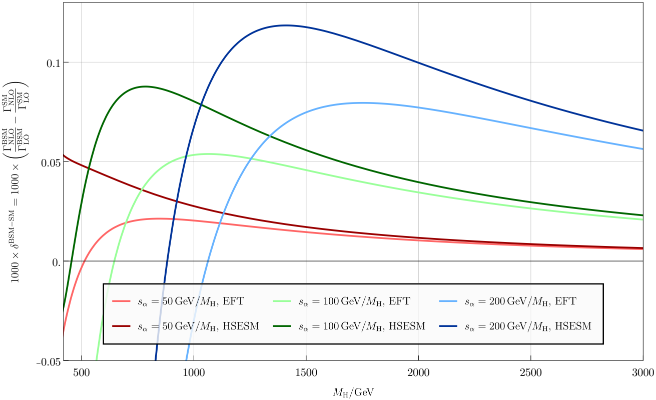


$$M_\mathrm{W} = M_\mathrm{W}^\mathrm{SM}\Big|_\mathrm{best} + \Delta M_\mathrm{W}^\mathrm{BSM}\Big|_\mathrm{NLO}$$

- good convergence HSESM $\underset{\zeta\to\infty}{\sim}$ EFT
- current exp. accuracy: $\Delta M_\mathrm{W} \sim 10\,\mathrm{MeV}$

Example 2: $\mathrm{h} \to \mathrm{WW}^* \to \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu$ (implemented in Prophecy4f)

Bredenstein et al. '06;
Altenkamp et al. '18;
Denner et al. '18



- good convergence HSESM $\underset{\zeta\to\infty}{\widetilde{}}$ EFT
- extremely small corrections in HSESM in G_μ input parameter scheme

Is SMEFT enough? – No.

Concern: Is there a field redefinition transforming away non-SMEFT operators?

$\hookrightarrow$ Try to absorb hard-mode contributions of EFT predictions into SMEFT form!
(hard contribution is unique, only source of $\ln M_\mathrm{H}$ terms)

$$X_j^\mathrm{EFT} = X_j^\mathrm{SM} + \sum_{i\,\in\mathrm{EFT}} a_{ji}^\mathrm{hard} \frac{c_i}{M_\mathrm{H}^2} + \ldots \text{(soft)}$$

$$\stackrel{?}{=} X_j^\mathrm{SM} + \sum_{i\,\in\mathrm{SMEFT}} a_{ji}^\mathrm{hard} \frac{c_i^\mathrm{new}}{M_\mathrm{H}^2} + \ldots \text{(soft)} \qquad (1)$$

Chosen observables X_j:

- M_W from μ-decay, $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$, $\Gamma_{\mathrm{Z}\to\nu\bar\nu}$
- $\Gamma_{\mathrm{H}\to\gamma\gamma}$: no hard EFT contribution, but generic SMEFT contribution
- $\mathrm{d}\Gamma_{\mathrm{H}\to\mathrm{WW}\to 4f}$: 2 conditions from differential distibutions
  (2 formfactors of $\Gamma^{HWW}_{\mu\nu}(k_1,k_2,k_3) = F_1 g_{\mu\nu} + F_2 k_{2,\nu} k_{3,\mu} + \ldots$)
- redundant: $\Gamma_{\mathrm{W}\to f\bar f'}$, $\Gamma_{\mathrm{Z}\to\ell\bar\ell,q\bar q}$, $\Gamma_{\mathrm{H}\to\mathrm{Z}\gamma}$, $\mathrm{d}\Gamma_{\mathrm{H}\to\mathrm{ZZ}\to 4f}$

Result: Eq.(1) has no solution for c_i^new with $i\in\mathrm{SMEFT}$.
$\Rightarrow$ EFT prediction cannot be reproduced by SMEFT !

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Conclusions

Functional matching for constructing EFTs for heavy fields of UV-complete models

- functional matching (EFT agnostic) superior to diagrammatic matching (needs EFT ansatz)
- functional matching very popular in recent decade:
  + several proposals/variants, generic results, steps towards automation
  – frequent shortcomings:
    fields not integrated out in mass basis, no BSM renormalization,
    no validation for observables

Presented method and results:

- approach based on background-field method and strategy of regions
  (similar to some other approaches)
- fields integrated out in mass basis (no issues with mixing)
- BSM renormalization included
- non-linear Higgs realization used (linear possible as well)
  ↪ elegant treatment of gauge d.o.f.s, flexibility if non-decoupling effects exist
- EFT for Higgs Singlet Extension of the SM ($M_{\mathrm{H}} \to \infty$, $\alpha \sim M_{\mathrm{H}}^{-1}$):
  - EFT validated against several BSM NLO calculations for observables
  - EFT Lagrangian of HEFT type (not SMEFT!)

Backup slides

Functional matching

1. Background-field method — DeWitt '67,'80; 't Hooft '75; Boulware '81; Abbott '81; Denner/SD/Weiglein '94

$\varphi \;\to\; \hat{\varphi} + \varphi,\qquad H \;\to\; \hat{H} + H,$

special gauge fixing of quantum fields

$\Downarrow$

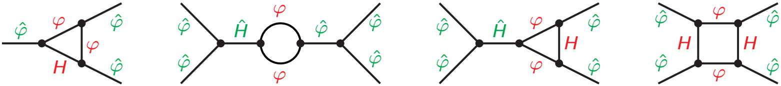
$\hat{\Gamma}[\hat{\varphi}, \hat{H}]$ gauge invariant

background fields

- effective action $\hat{\Gamma}[\hat{\varphi}, \hat{H}]$
- tree lines in diagrams

quantum fields

- $\int \mathcal{D}\varphi \int \mathcal{D}H \; \mathrm{e}^{\mathrm{i}\int \mathrm{d}^D x\, \mathcal{L}}$
- loop lines in diagrams



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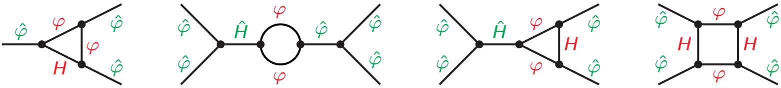


Further details/implications:

- non-linear Higgs realization: $\quad U(\zeta) \to U(\hat{\zeta})\, U(\zeta)$ SD/Grosse-Knetter '95,'96

  $\hookrightarrow$ take unitary gauge $U(\hat{\zeta}) \to \mathbb{1}$ and revert at the end (Stueckelberg trafo)

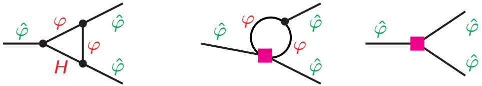
- 1-loop approximation: only terms bilinear in φ, H relevant

$$\mathcal{L}^{\rm 1loop} = \underbrace{-\tfrac{1}{2} H \Delta_H H + \varphi \mathcal{X}_{\varphi H} H - \tfrac{1}{2} \varphi \mathcal{A} \varphi}_{\Delta_H,\ \mathcal{X}_{\varphi H},\ \mathcal{A} \text{ contain } \partial,\ \hat{\varphi},\ \hat{H}} + \ldots \qquad \varphi = (h, \zeta_j, W_a^\mu, B^\mu)$$

2. Method of regions

Beneke/Smirnov '97

Concept for integrals: $$\int \mathrm{d}^D p\, f(p) \;=\; \underbrace{\int\limits_{p\sim m_\varphi} \mathrm{d}^D p\, f(p) \;+\; \int\limits_{p\sim M_\mathrm{H}} \mathrm{d}^D p\, f(p)}_{\text{Taylor expansions of } f(p) \text{ for } M_\mathrm{H} \gg m_\varphi}$$



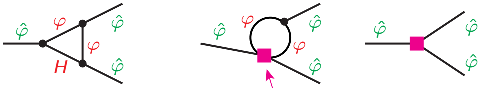
$\hookrightarrow$ decompositions of fields $\varphi(x)$, $H(x)$ into light and heavy modes:

$$\varphi(x) = \varphi_l(x) + \varphi_h(x), \qquad \int \mathcal{D}\varphi = \int \mathcal{D}\varphi_l \int \mathcal{D}\varphi_h$$

$$H(x) = H_l(x) + H_h(x), \qquad \int \mathcal{D}H = \int \mathcal{D}H_l \int \mathcal{D}H_h$$

2. Method of regions Beneke/Smirnov '97

Concept for integrals: $$\int \mathrm{d}^D p\, f(p) \;=\; \underbrace{\int\limits_{p\sim m_\varphi} \mathrm{d}^D p\, f(p) \;+\; \int\limits_{p\sim M_\mathrm{H}} \mathrm{d}^D p\, f(p)}_{\text{Taylor expansions of } f(p) \text{ for } M_\mathrm{H} \gg m_\varphi}$$



$\hookrightarrow$ decompositions of fields $\varphi(x)$, $H(x)$ into light and heavy modes:

$$\varphi(x) = \varphi_l(x) + \varphi_h(x), \qquad \int \mathcal{D}\varphi = \int \mathcal{D}\varphi_l \int \mathcal{D}\varphi_h$$

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$\downarrow$

$\delta\mathcal{L}_\mathrm{eff}^\mathrm{tree}(\hat\varphi + \varphi)$
(essentially from EOM for H_l)

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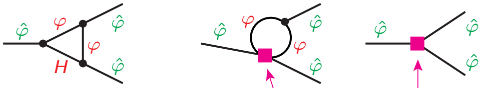

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$\longrightarrow \delta\mathcal{L}_\mathrm{eff}^\mathrm{1loop}(\hat\varphi)$

$\downarrow$

$\delta\mathcal{L}_\mathrm{eff}^\mathrm{tree}(\hat\varphi+\varphi)$

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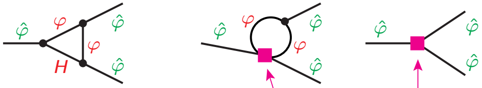

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$\boxed{\int \mathcal{D}\varphi_h \int \mathcal{D}H_h} \longrightarrow \delta\mathcal{L}_\mathrm{eff}^\mathrm{1loop}(\hat\varphi)$

$\boxed{\int \mathcal{D}H_l} \downarrow$ $\delta\mathcal{L}_\mathrm{eff}^\mathrm{tree}(\hat\varphi+\varphi)$
(essentially from EOM for H_l)

Note: Background fields do not have heavy modes, since $p_\mathrm{ext} \ll M_\mathrm{H}$ per def.

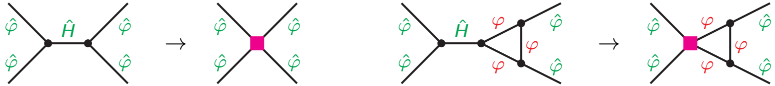
3. Elimination of heavy background fields via EOM(s)

EOM for heavy fields $\hat{H}$: (from field shift $H \to H + \Delta\hat{H}$)

$$\frac{\delta\mathcal{L}}{\delta H} = 0 \quad \to \text{ can be solved iteratively for } \hat{H}_l, \text{ since } |\partial\hat{H}_l|, |\partial H_l| \ll M_\mathrm{H}\hat{H}_l, M_\mathrm{H} H_l$$

$$\Rightarrow \hat{H}_l = -\frac{s_\alpha}{2v_2}\left(\hat{h}_l + h_l + h_h\right)^2 + \mathcal{O}(\zeta^{-3})$$

$$\mathcal{L}^{\mathrm{tree}}(\hat{\varphi}_l + \varphi_l, \hat{H}_l + H_l) \xrightarrow[\text{EOM}]{} \mathcal{L}^{\mathrm{tree}}_{\mathrm{eff}}(\hat{\varphi}_l + \varphi_l)$$



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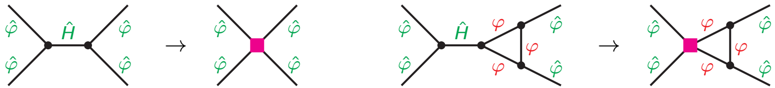

Important: Include BSM renormalization in this step!
(otherwise information might get lost)

Renormalization of the HSESM:

Renormalization trafo for parameters and fields at NLO:

$$c_{i,0} = c_i + \delta c_i, \qquad \hat\varphi_0 = (1+\tfrac{1}{2}\delta Z_{\hat\varphi})\hat\varphi,$$

$$\begin{pmatrix}\hat h_0\\ \hat H_0\end{pmatrix} = \begin{pmatrix}1+\frac{1}{2}\delta Z_{\hat h\hat h} & \frac{1}{2}\delta Z_{\hat h\hat H}\\ \frac{1}{2}\delta Z_{\hat H\hat h} & 1+\frac{1}{2}\delta Z_{\hat H\hat H}\end{pmatrix}\begin{pmatrix}\hat h\\ \hat H\end{pmatrix}$$

$\hookrightarrow$ General issue:

Include scaling of renormalization constants with $\zeta \sim M_\mathrm{H}$ in power-counting!

BSM parameters: M_H, s_α, λ_{12} (or λ_1); BSM field: H

- SM parameters and fields renormalized "as usual" (e.g. OS scheme)
- unproblematic: $M_\mathrm{H} \to$ OS scheme; $\lambda_{12} \to \overline{\mathrm{MS}}$ scheme
- delicate: $s_\alpha \to$ various schemes known, Bojarski et al. '15; Kanemura et al. '15,'17 Denner et al. '17,'18; Altenkamp et al. '18

  but: $\delta s_\alpha^{\overline{\mathrm{MS}}}$ depends on tadpole scheme (PRTS, FJTS, GIVS → backup)

  $$\delta s_\alpha^{\overline{\mathrm{MS}}/\mathrm{PRTS}} \sim \mathcal{O}(\zeta^{-1}), \qquad \delta s_\alpha^{\overline{\mathrm{MS}}/\mathrm{FJTS}} \sim \mathcal{O}(\zeta^{1})$$

$\Rightarrow$ $\overline{\mathrm{MS}}/\mathrm{FJTS}$ scheme destroys EFT decoupling for $\zeta \to \infty$: SD/Schuhmacher /Stahlhofen '21

$$\mathcal{L}_\mathrm{eff} = \begin{cases} \mathcal{O}(\zeta^{-2}) & \text{for } \overline{\mathrm{MS}}/\mathrm{PRTS} \text{ and OS schemes } \checkmark \\ \mathcal{O}(\zeta^{0}) & \text{for } \overline{\mathrm{MS}}/\mathrm{FJTS} \text{ scheme } ✗ \end{cases}$$

4. Calculation of $\delta\mathcal{L}_{\mathrm{eff}}^{\mathrm{1loop}}$

$$\delta\mathcal{L}^{\mathrm{1loop}} = -\tfrac{1}{2} H\Delta_H H + \varphi\mathcal{X}_{\varphi H}H - \tfrac{1}{2}\varphi\mathcal{A}\varphi, \qquad \varphi \to \varphi + \mathcal{A}^{-1}\mathcal{X}_{\varphi H}H$$

$$\to -\tfrac{1}{2} H\tilde{\Delta}_H H - \tfrac{1}{2}\varphi\mathcal{A}\varphi$$

$$\tilde{\Delta}_H = \Delta_H - \mathcal{X}_{\varphi H}^{\dagger}\mathcal{A}^{-1}\mathcal{X}_{\varphi H}$$

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$$\tilde{\Delta}_H = \Delta_H - \mathcal{X}_{\varphi H}^{\dagger}\mathcal{A}^{-1}\mathcal{X}_{\varphi H}$$

$\delta\mathcal{L}_{\text{eff}}^{\text{1loop}}$ from functional integral over hard modes:

$$\exp\left\{\mathrm{i}\int \mathrm{d}^D x\, \delta\mathcal{L}_{\text{eff}}^{\text{1loop}}\right\} = \int \mathcal{D}H_h \int \mathcal{D}\varphi_h \exp\left\{\mathrm{i}\int \mathrm{d}^D x\, \delta\mathcal{L}_{\text{hard}}^{\text{1loop}}\right\}$$
$$= \int \mathcal{D}H_h \exp\left\{-\tfrac{\mathrm{i}}{2}\int \mathrm{d}^D x\, H_h\tilde{\Delta}_H H_h\right\} \int \mathcal{D}\varphi_h \exp\left\{-\tfrac{\mathrm{i}}{2}\int \mathrm{d}^D x\, \varphi_h\mathcal{A}\varphi_h\right\}$$

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&\to -\tfrac{1}{2} H\tilde{\Delta}_H H - \tfrac{1}{2}\varphi\mathcal{A}\varphi \\
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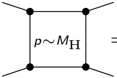
&= \int \mathcal{D}H_h \exp\left\{-\tfrac{\mathrm{i}}{2}\int \mathrm{d}^D x\, H_h\tilde{\Delta}_H H_h\right\} \cancel{\int \mathcal{D}\varphi_h \exp\left\{-\tfrac{\mathrm{i}}{2}\int \mathrm{d}^D x\, \varphi_h\mathcal{A}\varphi_h\right\}}
\end{aligned}$$



$= 0$ (scaleless integrals)

4. Calculation of $\delta\mathcal{L}_{\rm eff}^{\rm 1loop}$

$$\delta\mathcal{L}^{\rm 1loop} = -\tfrac{1}{2} H\Delta_H H + \varphi\mathcal{X}_{\varphi H} H - \tfrac{1}{2}\varphi\mathcal{A}\varphi, \qquad \varphi \to \varphi + \mathcal{A}^{-1}\mathcal{X}_{\varphi H} H$$

$$\to -\tfrac{1}{2} H\tilde{\Delta}_H H - \tfrac{1}{2}\varphi\mathcal{A}\varphi$$

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$$= \int \mathcal{D}H_h \exp\left\{ -\tfrac{\mathrm{i}}{2}\int \mathrm{d}^D x\, H_h \tilde{\Delta}_H H_h \right\}$$

$$\propto \left\{ \mathcal{D}et_h\left[\delta(x-y)\tilde{\Delta}_H(x,\partial_x) \right] \right\}^{-\frac{1}{2}}$$

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&= \int\mathcal{D}H_h\,\exp\left\{-\tfrac{\rm i}{2}\int {\rm d}^D x\,H_h\tilde{\Delta}_H H_h\right\} \\
&\propto \left\{\mathcal{D}et_h\left[\delta(x-y)\tilde{\Delta}_H(x,\partial_x)\right]\right\}^{-\frac{1}{2}} \\
&= \exp\left\{-\frac{1}{2}\mathcal{T}r_h\left[\ln\left(\delta(x-y)\tilde{\Delta}_H(x,\partial_x)\right)\right]\right\}
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$$= \exp\left\{-\frac{1}{2}\mathcal{T}r_h\Big[\ln\Big(\delta(x-y)\tilde{\Delta}_H(x,\partial_x)\Big)\Big]\right\}$$

$$\Rightarrow\quad \delta\mathcal{L}_{\rm eff}^{\rm 1loop} = \frac{\mathrm{i}}{2}\,\mu^{4-D}\int_{p\sim M_{\rm H}} \frac{\mathrm{d}^D p}{(2\pi)^D}\,\ln\Big(\tilde{\Delta}_H(x,\partial_x+\mathrm{i}p)\Big)$$

5. Large-mass expansion

$$\tilde{\Delta}_H = \Delta_H - \mathcal{X}^\dagger_{\varphi H}\mathcal{A}^{-1}\mathcal{X}_{\varphi H}, \qquad (\Delta_H = \partial^2 + M_\mathrm{H}^2 + \dots)$$

Expansion of the inverse of $\mathcal{A} = \mathcal{D} - \mathcal{X}$ via Neumann series: $(\mathcal{D} = \partial^2 + \dots)$

$$\mathcal{A}^{-1} = (\mathcal{D} - \mathcal{X})^{-1} = \mathcal{D}^{-1} + \mathcal{D}^{-1}\mathcal{X}\mathcal{D}^{-1} + \mathcal{D}^{-1}\mathcal{X}\mathcal{D}^{-1}\mathcal{X}\mathcal{D}^{-1} + \dots$$

$\mathcal{A}$, $\mathcal{D}$, $\mathcal{X}$ = "matrix-valued in field space"

Large-mass expansion in momentum space:

$$\tilde{\Delta}_H(x, \partial_x + \mathrm{i}p) = -(p^2 - M_\mathrm{H}^2) + \Pi\Big(\hat{\varphi}(x), p, \partial_x\Big)$$

$$\ln\Big(\tilde{\Delta}_H(x, \partial_x + \mathrm{i}p)\Big) = \ln(-p^2 + M_\mathrm{H}^2) - \sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{\Pi}{p^2 - M_\mathrm{H}^2}\right)^n$$

$$\Rightarrow \delta\mathcal{L}_\mathrm{eff}^\mathrm{1loop} = -\frac{\mathrm{i}}{2}\sum_{n=1}^{4}\frac{1}{n}\,\mu^{4-D}\int\frac{\mathrm{d}^D p}{(2\pi)^D}\,\frac{\Pi\big(\hat{\varphi}, p, \partial\big)^n}{(p^2 - M_\mathrm{H}^2)^n} + \mathcal{O}(\zeta^{-4})$$

- simple 1-loop vacuum integrals

  $\hookrightarrow$ terms $f(\epsilon)\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{M_\mathrm{H}^2}\right)$ $\qquad (D = 4 - 2\epsilon)$
- quite lengthy algebraic manipulations

Warsaw basis of SMEFT

Buchmüller/Wyler '85; Grzadkowski et al. 1008.4884

Bosonic operators:

Φ^6 *and* $\Phi^4 D^2$	X^3
$\mathcal{O}_\Phi = (\Phi^\dagger\Phi)^3$	$\mathcal{O}_G = -f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{\Phi\Box} = (\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi)$	$\mathcal{O}_{\widetilde{G}} = -f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{\Phi D} = (\Phi^\dagger D^\mu\Phi)^*(\Phi^\dagger D_\mu\Phi)$	$\mathcal{O}_W = -\epsilon^{abc} W_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu}$
	$\mathcal{O}_{\widetilde{W}} = -\epsilon^{abc} \widetilde{W}_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu}$
$X^2\Phi^2$	
$\mathcal{O}_{\Phi G} = (\Phi^\dagger\Phi) G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\Phi B} = (\Phi^\dagger\Phi) B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\Phi\widetilde{G}} = (\Phi^\dagger\Phi) \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\Phi\widetilde{B}} = (\Phi^\dagger\Phi) \widetilde{B}_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\Phi W} = (\Phi^\dagger\Phi) W_{\mu\nu}^a W^{a\mu\nu}$	$\mathcal{O}_{\Phi WB} = -(\Phi^\dagger\tau^a\Phi) W_{\mu\nu}^a B^{\mu\nu}$
$\mathcal{O}_{\Phi\widetilde{W}} = (\Phi^\dagger\Phi) \widetilde{W}_{\mu\nu}^a W^{a\mu\nu}$	$\mathcal{O}_{\Phi\widetilde{W}B} = -(\Phi^\dagger\tau^a\Phi) \widetilde{W}_{\mu\nu}^a B^{\mu\nu}$

Non-linear Higgs representation for the SM

Lee/Zinn-Justin '72; ...
Grosse-Knetter/Kögerler '93
SD/Grosse-Knetter '95

Higgs and Goldstone fields

Relation between fields of the linear and non-linear representations:

$$\eta = \cos\left(\frac{\zeta}{v}\right)(v+h) - v = h - \frac{\zeta^2}{2v}\left(1+\frac{h}{v}\right) + \mathcal{O}(\zeta^4), \quad \zeta \equiv |\vec{\zeta}|$$

$$\vec{\phi} = \frac{1}{\zeta}\sin\left(\frac{\zeta}{v}\right)(v+h)\vec{\zeta} = \left(1+\frac{h}{v}\right)\vec{\zeta} + \mathcal{O}(\zeta^3)$$

Gauge transformations: (θ_a = group parameters, g_n = gauge couplings)

$$\mathbf{\Phi} \to \underbrace{S(\boldsymbol{\theta})}_{\exp(\frac{\mathrm{i}}{2}g_2\theta_j\sigma_j)}\,\mathbf{\Phi}\,\underbrace{S_Y(\theta_Y)}_{\exp(\frac{\mathrm{i}}{2}g_1\theta_Y\sigma_3)}, \qquad h \to h, \qquad U(\boldsymbol{\zeta}) \to S(\boldsymbol{\theta})\,U(\boldsymbol{\zeta})\,S_Y(\theta_Y)$$

Higgs potential:

$$V = -\frac{\mu_2^2}{2}\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right] + \frac{\lambda_2}{16}\left(\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right]\right)^2 = -\frac{\mu_2^2}{2}(v_2+h)^2 + \frac{\lambda_2}{16}(v_2+h)^4$$

= independent of Goldstone fields

Kinetic Higgs Lagrangian: $\vec{C}^\mu = \left(W_1^\mu, W_2^\mu, Z^\mu/c_\mathrm{W}\right)^\mathrm{T}$

$$\mathcal{L}_{\mathrm{H,kin}} = \frac{1}{2}(\partial h)^2 + \frac{(v_2+h)^2}{2v^2}\Big\{(\partial_\mu\vec{\zeta})\cdot(\partial^\mu\vec{\zeta}) + \frac{g_2^2v^2}{4}\vec{C}_\mu\cdot\vec{C}^\mu$$
$$+ g_1g_2B_\mu\left[-W_3^\mu\zeta^2 + (\vec{W}^\mu\cdot\vec{\zeta})\zeta_3\right] - g_2^2 v\,\vec{C}_\mu\cdot(\vec{W}^\mu\times\vec{\zeta})$$
$$- g_2 v\,\vec{C}_\mu\cdot\partial^\mu\vec{\zeta} - g_2(\vec{C}_\mu - 2\vec{W}_\mu)\cdot(\vec{\zeta}\times\partial^\mu\vec{\zeta})\Big\} + \mathcal{O}(\zeta^3)$$

= non-polynomial with arbitrarily high powers in ζ_j

Basics of the Background-Field Method (BFM)

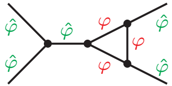
DeWitt '67,'80; 't Hooft '75; Boulware '81; Abbott '81   SM: Denner/SD/Weiglein '94

Fields φ split into background and quantum parts: $\varphi \to \hat{\varphi} + \varphi$

- Background fields $\hat{\varphi}$:
  - sources of the BFM effective action $\hat{\Gamma}[\hat{\varphi}]$
  - external and tree lines in Feynman diagrams
  - gauge of background gauge fields $\hat{A}^a_\mu$ not fixed in $\hat{\Gamma}[\hat{\varphi}]$
- Quantum fields φ:
  - integration variables of the functional integral $\int \mathcal{D}\varphi \exp\{\mathrm{i}\int \mathrm{d}^D x\, \mathcal{L}\}$
  - loop lines of Feynman diagrams
  - R_ξ-type gauge-fixing to support gauge invariance of $\hat{\Gamma}[\hat{\varphi}]$

BFM gauge invariance and gauge fixing of $\hat{\varphi}$:

- $\hat{\Gamma}[\hat{\varphi}]$ fully invariant under "ordinary" gauge transformations of $\hat{\varphi}$
  $\hookrightarrow$ "ghost-free" QED-like Ward identities for BFM vertex fcts. $\hat{\Gamma}^{\hat{\varphi}\cdots}$
- Reducible Green fcts. $\hat{G}^{\hat{\varphi}\cdots}$ and S-matrix elements:
  - gauge-fixing of $\hat{\varphi}$ required for bkg. propagators $\hat{G}^{\hat{\varphi}\hat{\varphi}}$
  - formed from trees with vertex fcts. $\hat{\Gamma}^{\hat{\varphi}\cdots}$



Mixing-angle renormalization

On the role of Higgs mixing angles α, ...

- generic feature of extended Higgs sectors
- typically rescale SM couplings by factors $\cos\alpha$, $\sin\alpha$, ...
  ↪ phenomenologically well accessible
  ↪ appropriate input quantities requiring renormalization

Desirable properties for the renormalized mixing angles:
Freitas/Stöckinger '02; Denner/SD/Lang '18

- gauge independence
  ↪ S-matrix = gauge-independent function of input parameters
- symmetry w.r.t. mixing degrees of freedom
- process independence
- perturbative stability
  ↪ higher-order corrections should not get artificially large
- smoothness for degenerate masses or extreme mixing angles
  ↪ no singularities like $1/(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})$ or $1/\sin\alpha$, $1/\cos\alpha$, etc.

+ addendum: decoupling behaviour in large-mass limits
  should be respected in transition LO → NLO → ...
  see, e.g., SD/Schuhmacher/Stahlhofen '21

Types of renormalization schemes for mixing angles

1. $\overline{\mathrm{MS}}$ scheme

   + process independence, simplicity
   + scale dependence as diagnostic tool to check perturbative stability
   - • dependence on tadpole scheme:

     FJTS: gauge independence, potentially large corrections
     PRTS: gauge dependence, corrections better behaved
     GIVS: gauge independence, corrections similar to PRTS
   - − prone to problems for extreme parameter scenarios (especially FJTS)

2. Momentum-subtraction schemes

   - • procedures based on $\Sigma^{ij}(p^2)$ at some momentum transfer p^2
   + process independence
   - − potential gauge dependence (often removed ad hoc)
   - − physical meaning and generalizability of ad hoc procedures unclear

3. Process-specific on-shell (OS) conditions: e.g. $\Gamma^{\mathrm{h}\to XY} \overset{!}{=} \Gamma^{\mathrm{h}\to XY}_{\mathrm{LO}}$

   + gauge independence
   - − process dependence
   - − "contamination" of $\delta\alpha$ by all types of different corrections
     $\hookrightarrow$ perturbative instabilities & "dead corners" in parameter space

Types of renormalization schemes for mixing angles (continued)

4. OS conditions on amplitude/formfactor ratios: e.g. $\frac{\mathcal{M}^{\mathrm{h}\to XY}}{\mathcal{M}^{\mathrm{H}\to XY}} \stackrel{!}{=} \frac{\mathcal{M}_{\mathrm{LO}}^{\mathrm{h}\to XY}}{\mathcal{M}_{\mathrm{LO}}^{\mathrm{H}\to XY}}$
   Denner/SD/Lang '18
   - \+ no dependence on gauge or tadpole scheme
   - • process independence in specific cases (e.g. HSESM)
   - \+ great perturbative stability, no "dead corners" in parameter space
5. Symmetry-inspired schemes Kanemura et al. '03; Krause et al. '16; Denner/SD/Lang '18
   - • exploits symmetry relations of UV divergences ("rigid" invariance, background-field gauge invariance)
   - \+ process independence
   - − gauge dependence
   - \+ "dead corners" in parameter space avoidable

OS renormalization schemes for the HSESM (and THDM) Denner/SD/Lang '18

Idea: Renormalization condition on ratio of S-matrix elements

$$\frac{\mathcal{M}^{H_1\to XY}}{\mathcal{M}^{H_2\to XY}} \stackrel{!}{=} \frac{\mathcal{M}_0^{H_1\to XY}}{\mathcal{M}_0^{H_2\to XY}} = \rho(\alpha) = \text{function of } \alpha \text{ only}$$

- $X, Y \stackrel{!}{=}$ neutral, otherwise problem with IR divergences
- vertex corrections $\delta^{H_i XY}$ to $\delta\alpha$ might be avoidable for clever choice of X, Y

HSESM: extension by fermionic singlet ψ with Yukawa coupling $y_\psi \to 0$

$$\mathcal{L}_\psi = \mathrm{i}\bar{\psi}\partial\!\!\!/\psi - y_\psi \bar{\psi}\psi \underbrace{(v_1 + H_1 c_\alpha - H_2 s_\alpha)}_{=\sigma} = \text{ gauge invariant}$$

- ratio $\rho(\alpha) = -c_\alpha / s_\alpha$ for $XY = \bar{\psi}\psi$
- vertex corrections $\delta^{H_i\bar{\psi}\psi} \to 0$ for $y_\psi \to 0$
- $\delta\alpha = \frac{1}{2}(\delta Z^H_{11} - \delta Z^H_{22}) c_\alpha s_\alpha + \frac{1}{2}(\delta Z^H_{12} c^2_\alpha - \delta Z^H_{21} s^2_\alpha)$

  $=$ gauge independent, symmetric in $\mathrm{H}_1/\mathrm{H}_2$, perturbatively stable

Tadpole renormalization

Note: Tadpole renormalization has no effect on observables in OS schemes.

Starting point: bare Lagrangian contains $t_0\eta$ term with

$$t_0 = \frac{1}{4} v_0 \left(4\mu_{2,0}^2 - \lambda_{2,0} v_0^2\right)$$

$\hookrightarrow$ Set $t_0 = 0$ in leading order to get the usual free propagators

Question: How to set t_0 in higher orders? $\rightarrow$ part of "tadpole scheme"

1. Parameter-Renormalized Tadpole Scheme (PRTS): e.g. Böhm/Hollik/Spiesberger '86 Denner '93

- interpret $t_0\eta$ as tadpole counterterm in Lagrangian: $\delta t = t_0$
- technically equivalent:
  δt term generated in ren. transformation of bare parameters $\mu_{2,0}^2$, $\lambda_{2,0}$:

$$\mu_{2,0}^2 \rightarrow \mu_{2,0}^2 + \frac{3\delta t^{\mathrm{PRTS}}}{2v}, \qquad \lambda_{2,0} \rightarrow \lambda_{2,0} + \frac{2\delta t^{\mathrm{PRTS}}}{v^3}$$

- expansion of η field about corrected minimum of effective potential
- drawback: δt can enter relations between bare input parameters
  $\hookrightarrow$ gauge-dependent terms $\propto \delta t$ enter relations
  between renormalized parameters and predicted observables

2. Fleischer–Jegerlehner Tadpole Scheme (FJTS):

Fleischer/Jegerlehner '80
Actis et al. '06

- set $t_0 = 0$ to all orders
- δt generated via field shift $\eta \;\to\; \eta + \Delta v$ with $\Delta v = -\delta t^{\mathrm{FJTS}}/M_{\mathrm{H}}^2$
- field shift has no physical effect (only redistributes tadpole terms)
  $\hookrightarrow$ FJTS equivalent to just including all tadpole loops
- expansion of η field about minimum of lowest-order Higgs potential
- advantage: no δt terms in relations between bare parameters
  $\hookrightarrow$ gauge-independent counterterms and relations between renormalized parameters and observables
- drawback: potentially large corrections in $\overline{\mathrm{MS}}$ schemes

3. Gauge-Invariant Vacuum expectation value Scheme (GIVS):

SD/Rzehak '22

- hybrid scheme of PRTS and FJTS: $\delta t = \delta t_1 + \delta t_2$
  - PRTS part δt_1: gauge independent, from non-linear Higgs realization
  - FJTS part δt_2: gauge dependent, needed for full tadpole compensation
- expansion of η field about minimum of lowest-order Higgs potential
- perturbative stability (like PRTS) and gauge invariance (like FJTS)