μTRISTAN:

μ^+e^- and $\mu^+\mu^+$ collider

Yu Hamada (DESY, Theory)

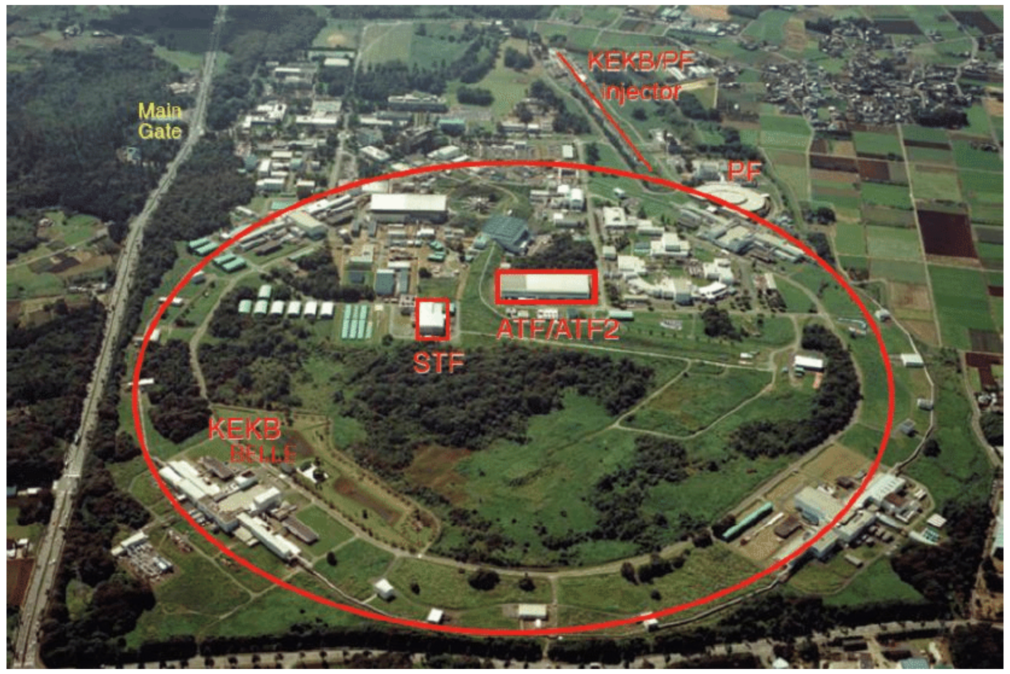
Based on:

- arXiv:2201.06664, **YH**, R. Kitano, R. Matsudo, H. Takaura, and M. Yoshida
- arXiv:2210.11083, **YH**, R. Kitano, R. Matsudo, and H. Takaura
- arXiv:2406.04500, L. Chen, **YH**, and S. Iguro
- arXiv:2407.XXXXX, **YH**, R. Kitano, R. Matsudo, S. Okawa, R. Takai, H. Takaura, and L. Treuer

21st Jun 2024, Future Colliders @ DESY Meeting

About myself

- Name: Yu Hamada
- got my Ph.D @ Kyoto U., Japan (2021)
- 1st postdoc @ KEK Theory Group, Tsukuba City, Japan 2021-2023
- 2nd postdoc @ DESY Theory Group (Cosmology) since October 2023



Plan of talk

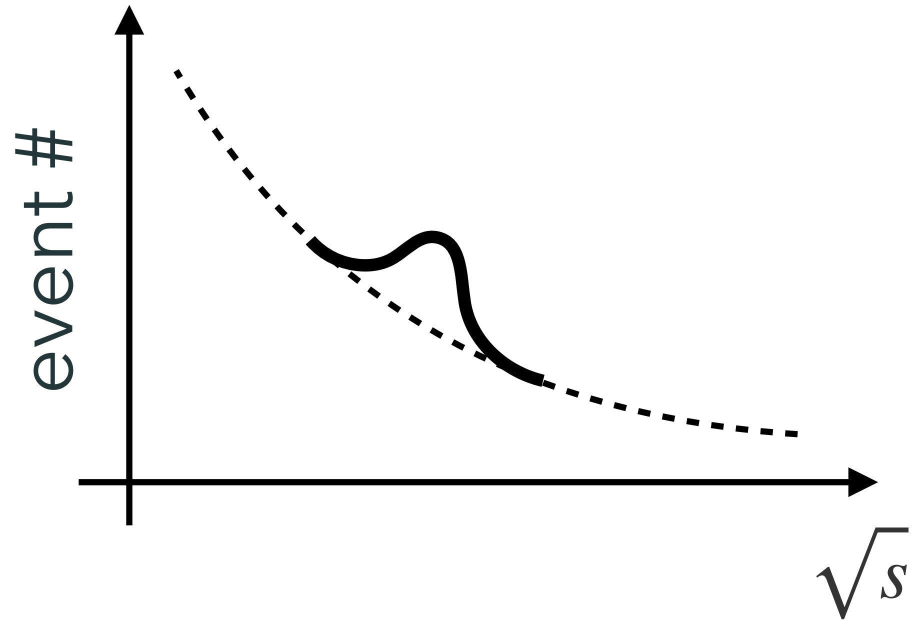
- Introduction: What is μTRISTAN?
- Higgs physics at μTRISTAN
- New physics search at μTRISTAN
- Summary

Introduction: What is μTRISTAN?

Task of future collider

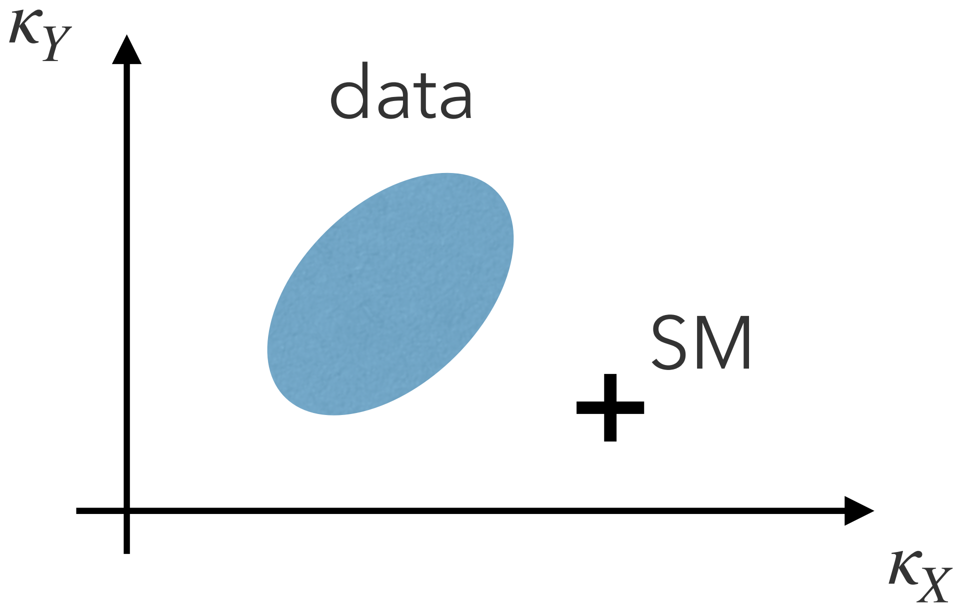
1. **Go to higher energy**

   production of new particle
   → direct search



2. **Precision measurement**

   probe deviation from SM
   → indirect search



- **Muon colliders are very nice for both!**

  precision measurements ← lepton collider

  high energy e.g. O(10) TeV ← less synchrotron rad. than $e^\pm$

Basics for collider

The formula of the number of the collision events:

$$N_{\mathrm{events}} = \sigma \times \mathscr{L} \times t_{\mathrm{run}}$$

t_{run}: running time

σ: cross section

# of particles

How frequently collisions occur

luminosity: $$\mathscr{L} = \frac{N_{\mathrm{beam1}}\, N_{\mathrm{beam2}}}{4\pi\, \sigma_x\, \sigma_y} f_{\mathrm{rep}}$$

beam size

The statistical error of the cross section is given as:

$$\frac{\Delta_{\mathrm{stat.}}\sigma}{\sigma} \propto \frac{1}{\sqrt{N_{\mathrm{events}}}} \propto \sqrt{\frac{\sigma_x \sigma_y}{N_{\mathrm{beam1}} N_{\mathrm{beam2}}}}$$

# of particles and narrow beam size are important

Current difficulties of $\mu^+\mu^-$ collider

- **MAP** (Muon Accelerator Program)

  $\mu^\pm$ produced from $\pi^\pm$ decay

  → **too hot, randomly distributed**

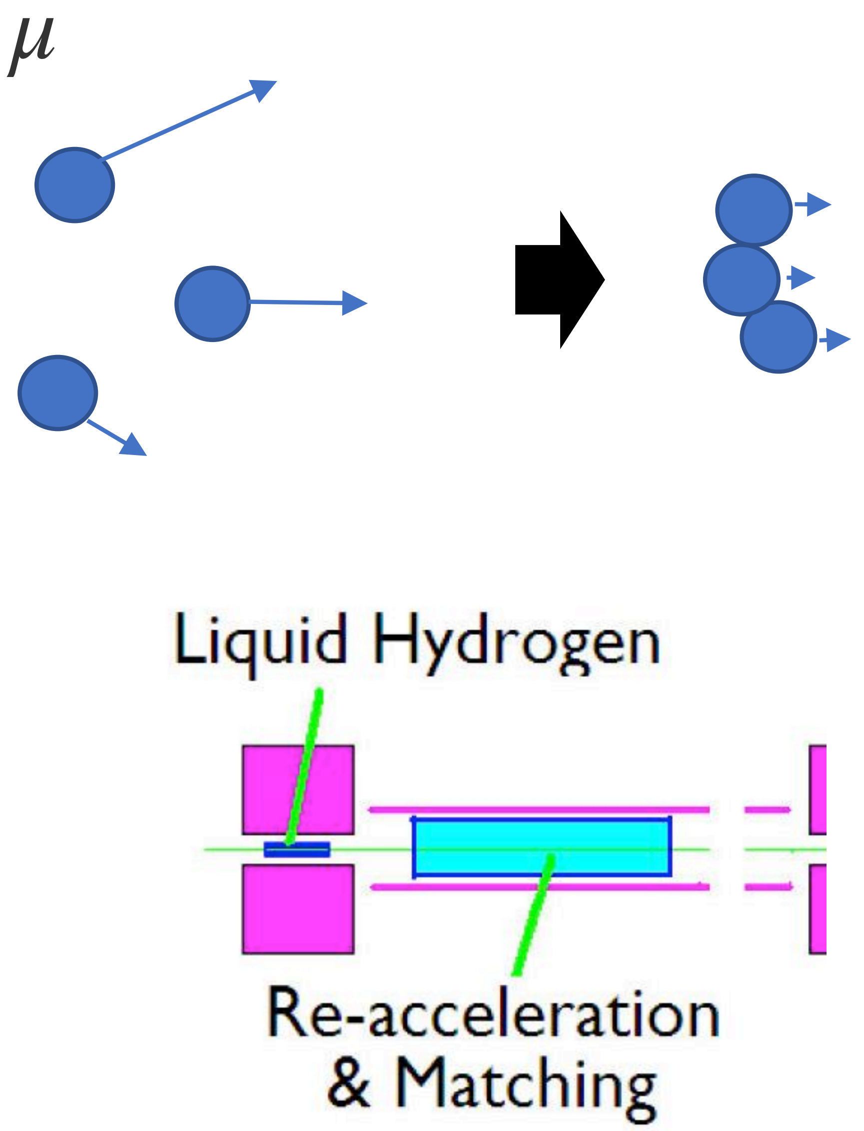
  → cooling with ionized material

  principle works, but not yet established

- **LEMMA** (Low Emittance Muon Accelerator)

  $e^+e^- \to \mu^+\mu^-$ at threshold

  → almost at rest i.e. **already cool, but less amount**



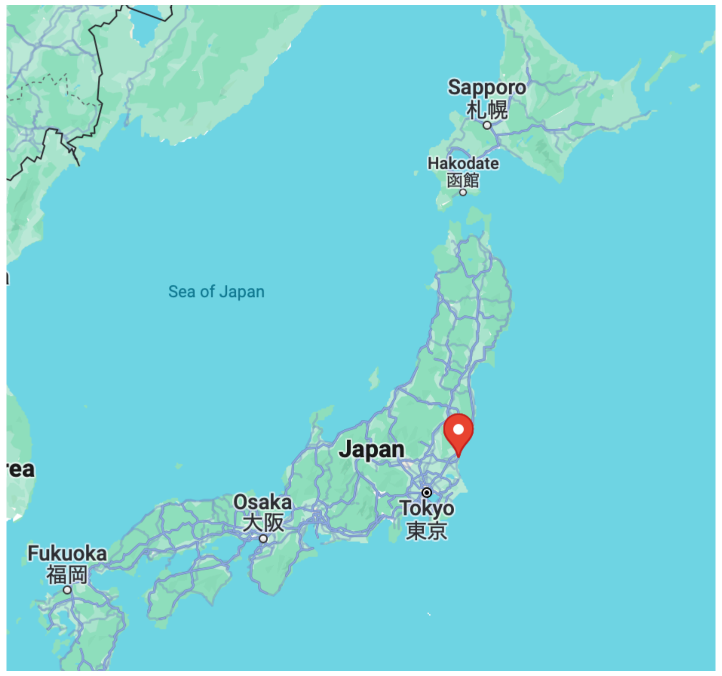
[from slide by Daniel Schulte]

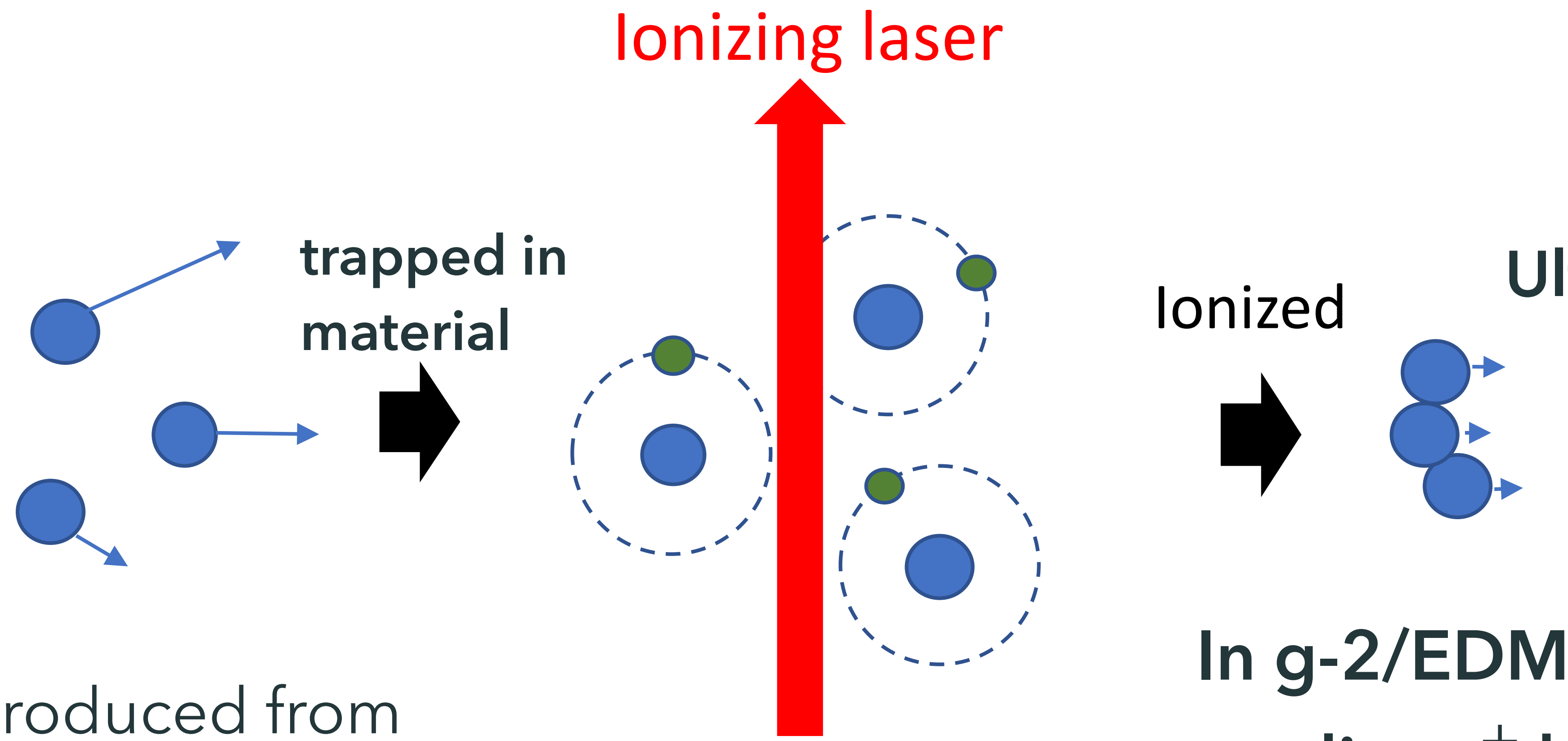
→ μ cooling w/ large amount is very difficult!

Technology for μ^+ cooling exists!

- J-PARC is planning μ g-2/EDM experiment.

The key technology is cooling of μ^+, which is available today!





μ^+ produced from π^+ decay

μ^+e^- bound state (muonium)

In g-2/EDM exp., used as a high-quality μ^+ beam by accelerating to 300 MeV.

fig from slide by Takaura

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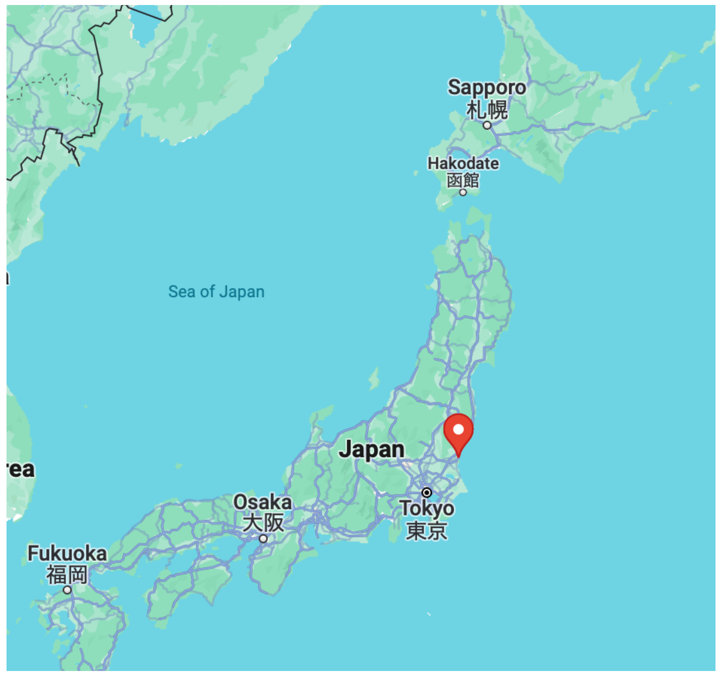

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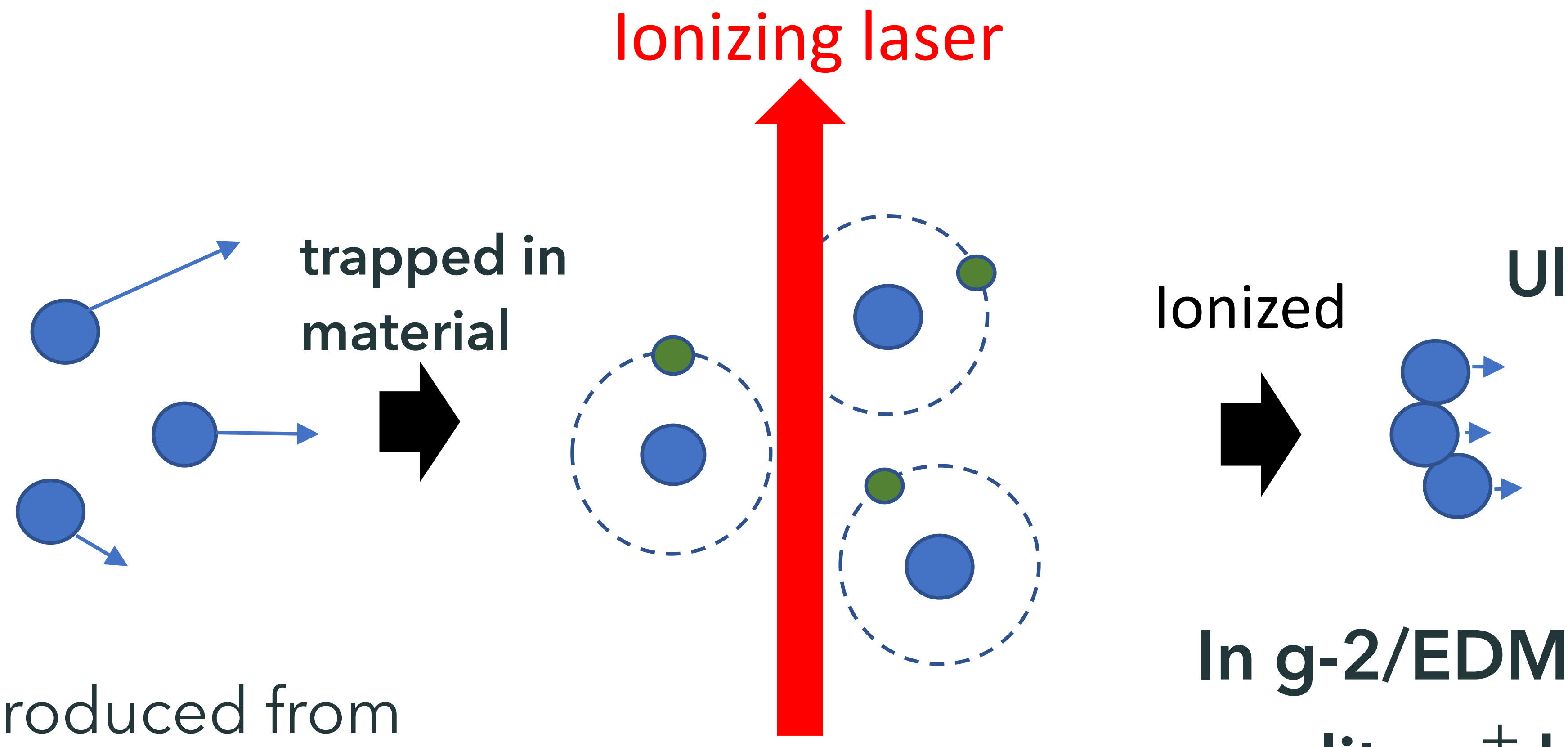


μ^+ produced from π^+ decay

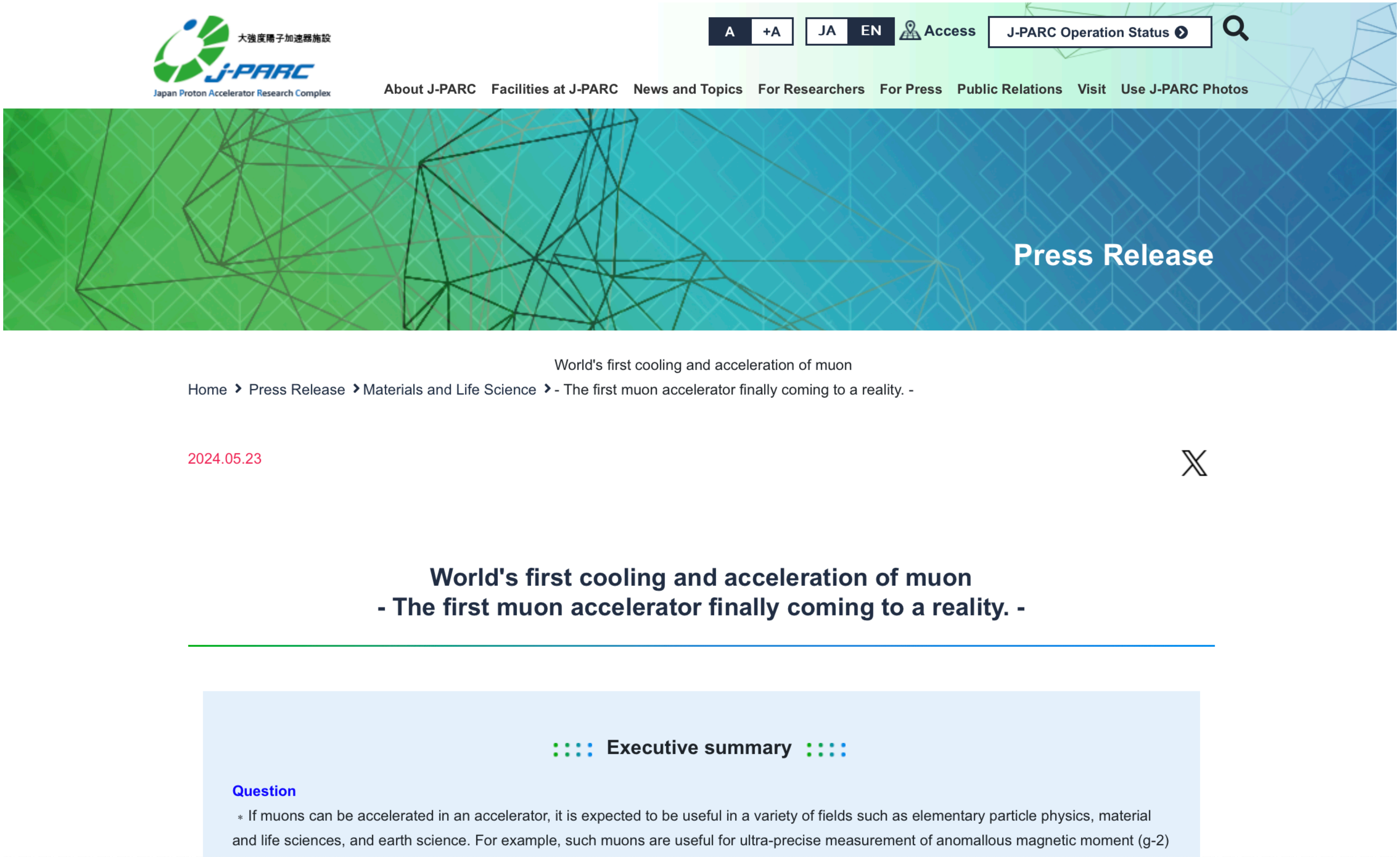
μ^+e^- bound state (muonium)

In g-2/EDM exp., used as a high-quality μ^+ beam by accelerating to 300 MeV.

Our proposal: accelerate it to O(1)TeV!

fig from slide by Takaura

Technology for μ^+ cooling exists!



World's first cooling and acceleration of muon

Home › Press Release › Materials and Life Science › - The first muon accelerator finally coming to a reality. -

2024.05.23

World's first cooling and acceleration of muon
- The first muon accelerator finally coming to a reality. -

Executive summary

Question

＊If muons can be accelerated in an accelerator, it is expected to be useful in a variety of fields such as elementary particle physics, material and life sciences, and earth science. For example, such muons are useful for ultra-precise measurement of anomallous magnetic moment (g-2)

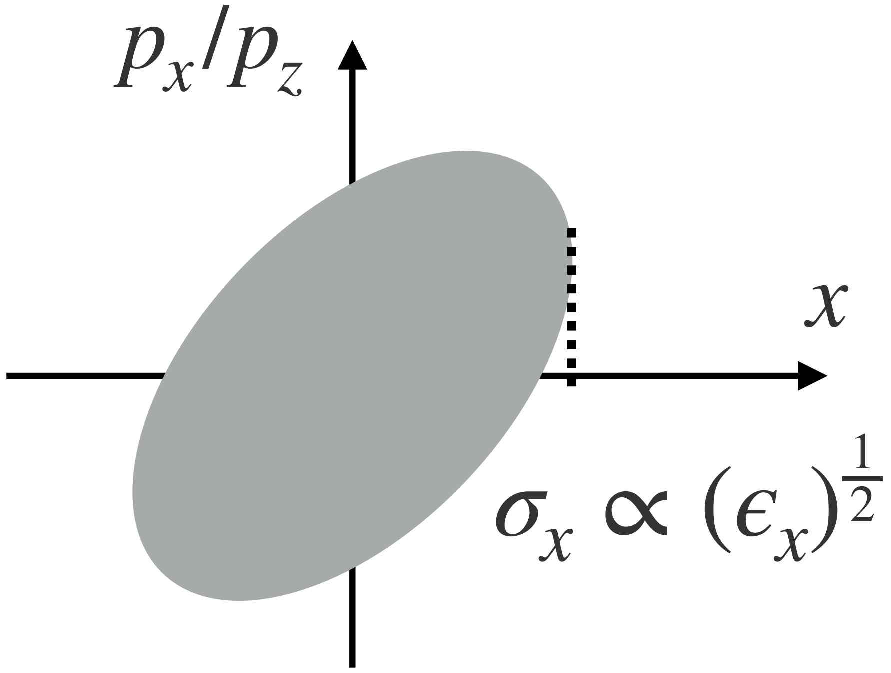
https://www.j-parc.jp/c/en/press-release/2024/05/23001341.html

ready to be accelerated

Beam emittance

- Beam size is determined by emittance $\epsilon_{x,y}$

  emittance = area of distribution in phase space
  → **reflects quality of beam**



Beam emittance

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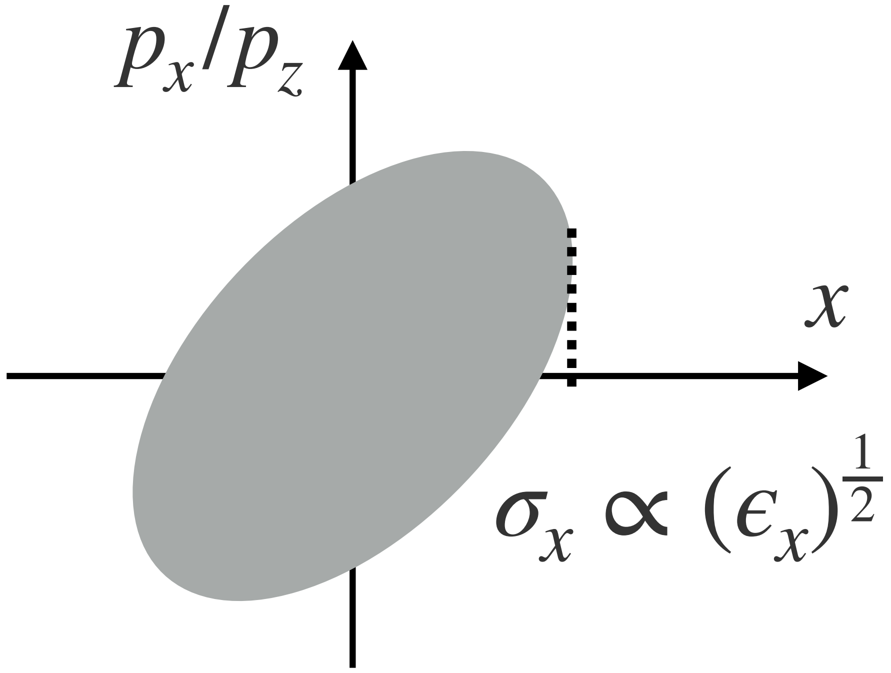
[J-PARC EDM/g-2 ,1901.03047]



For ultra-cold μ^+: $\epsilon_x, \epsilon_y = \dfrac{4\,\mu\mathrm{m}}{\beta\gamma}$

Beam emittance

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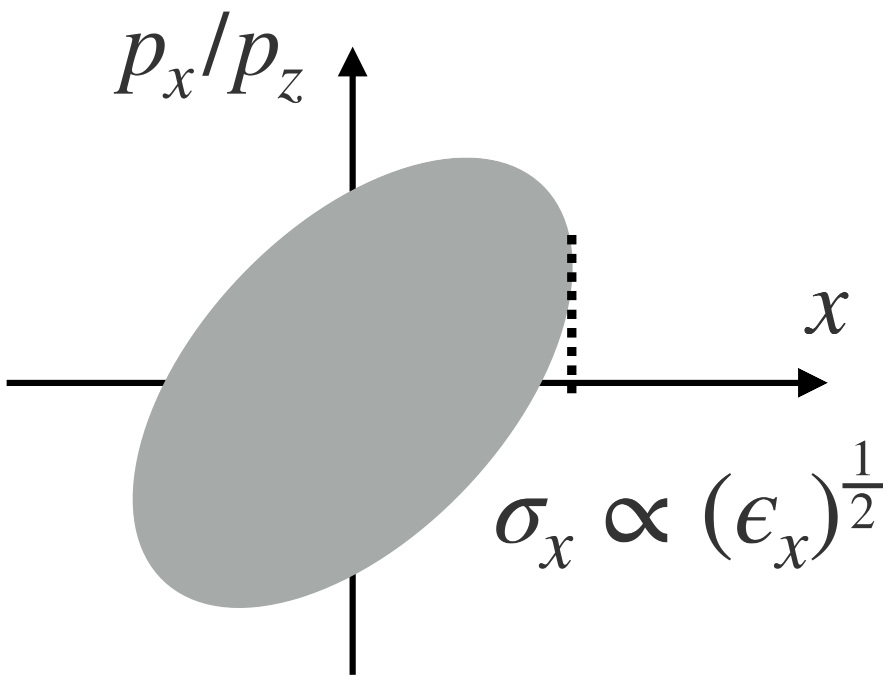

[J-PARC EDM/g-2 ,1901.03047]

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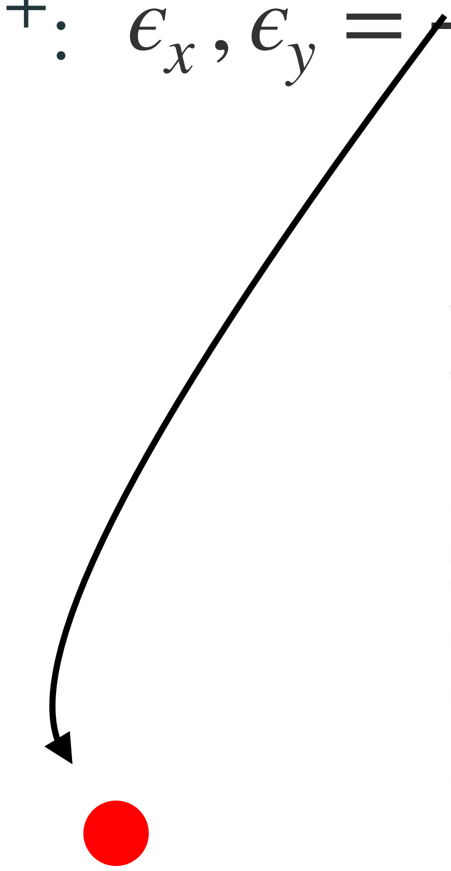
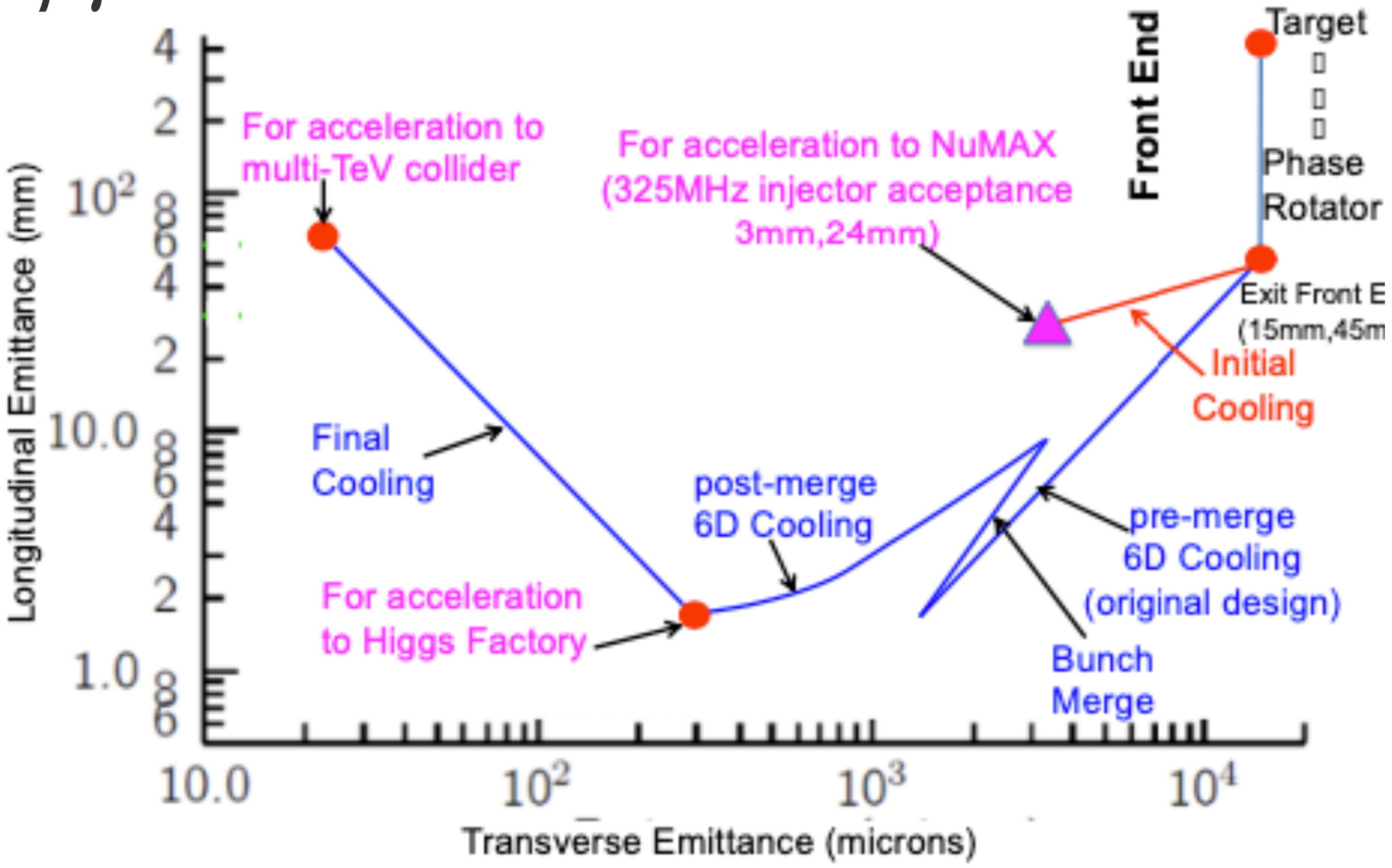


ultra-cold μ^+ is here!



planned emittance in MAP

Proposal of new experiment: μTRISTAN!

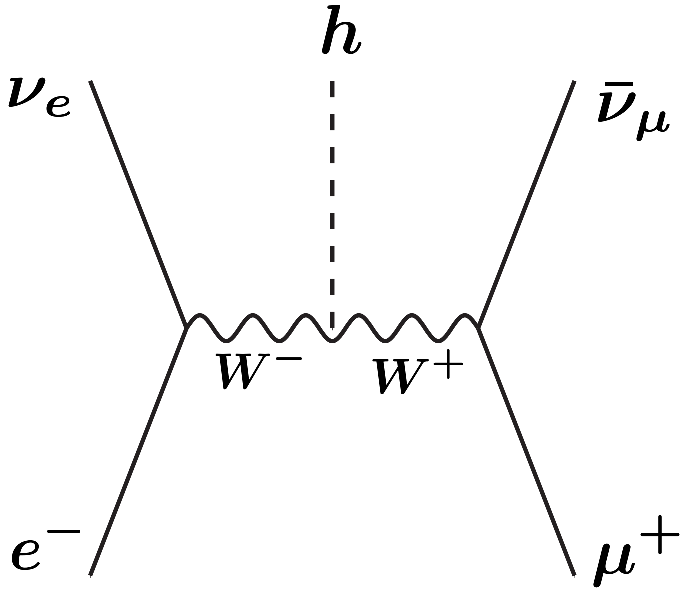
high-quality μ^+ beam accelerated to O(1) TeV

- $\mu^+ e^-$ collider

$$E_{\mu^+} = 1\,\mathrm{TeV}, \quad E_{e^-} = 30\,\mathrm{GeV}$$ (TRISTAN energy)

$$\to \sqrt{s} = 346\,\mathrm{GeV} \qquad \mathscr{L}_{\mu^+e^-} = 4.6 \times 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Higgs factory/EW precision



- $\mu^+\mu^+$ collider (instead of $\mu^+\mu^-$)

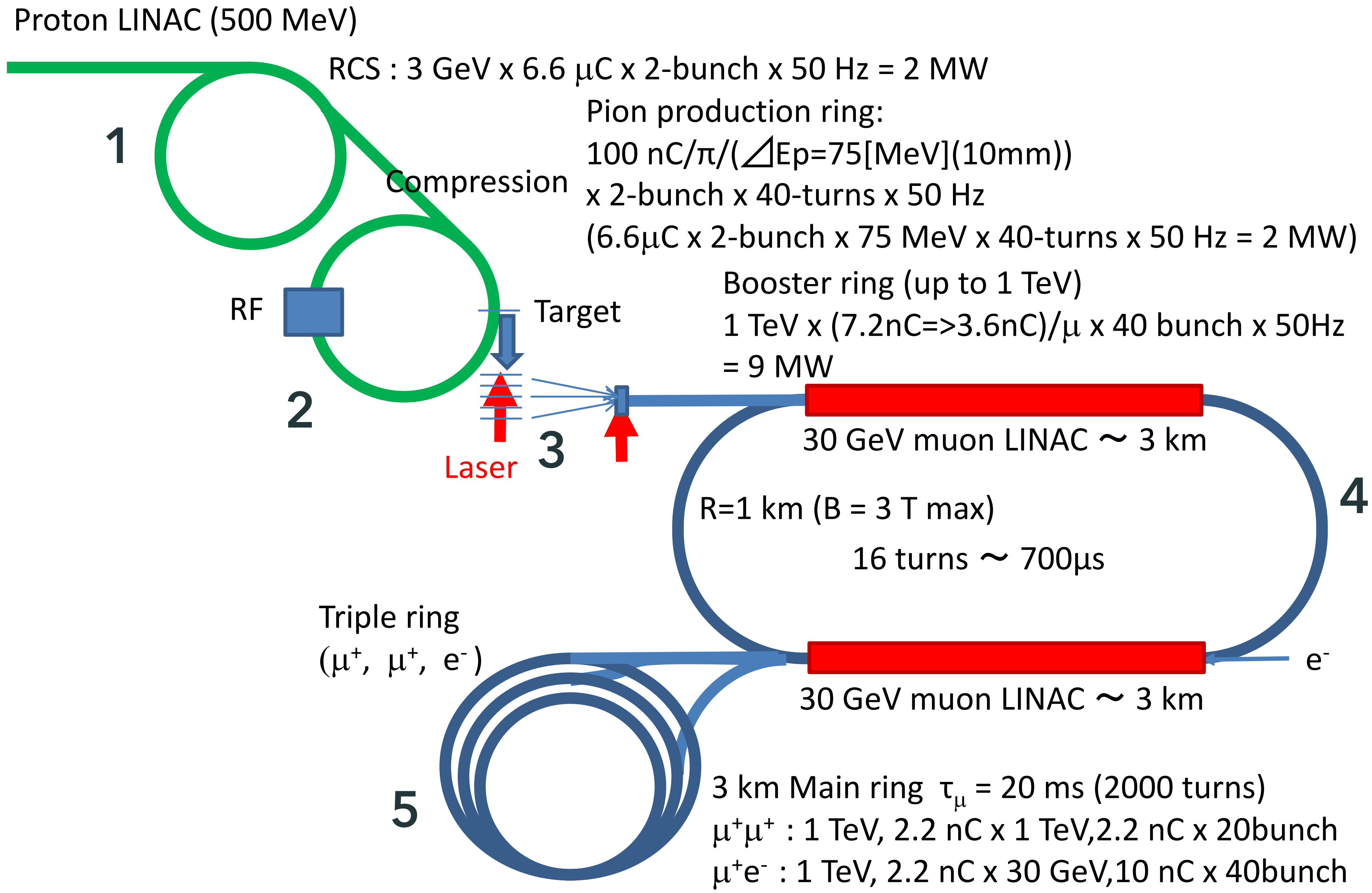
$$E_{\mu^+} = 1\,\mathrm{TeV}, \quad E_{\mu^+} = 1\,\mathrm{TeV}$$

$$\to \sqrt{s} = 2\,\mathrm{TeV} \qquad \mathscr{L}_{\mu^+\mu^+} = 5.7 \times 10^{32}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

New physics search/Higgs factory(!?)

Design of μTRISTAN

[**YH**, Kitano, Matsudo, Takaura, Yoshida, 2201.06664]



Proton LINAC (500 MeV)

RCS : 3 GeV x 6.6 μC x 2-bunch x 50 Hz = 2 MW

Pion production ring:
100 nC/π/(⊿Ep=75[MeV](10mm))
x 2-bunch x 40-turns x 50 Hz
(6.6μC x 2-bunch x 75 MeV x 40-turns x 50 Hz = 2 MW)

Booster ring (up to 1 TeV)
1 TeV x (7.2nC=>3.6nC)/μ x 40 bunch x 50Hz
= 9 MW

30 GeV muon LINAC ∼ 3 km

R=1 km (B = 3 T max)

16 turns ∼ 700μs

30 GeV muon LINAC ∼ 3 km

Triple ring
(μ^+, μ^+, e^-)

3 km Main ring τ_μ = 20 ms (2000 turns)
$\mu^+\mu^+$: 1 TeV, 2.2 nC x 1 TeV,2.2 nC x 20bunch
μ^+e^- : 1 TeV, 2.2 nC x 30 GeV,10 nC x 40bunch

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Higgs physics at μTRISTAN

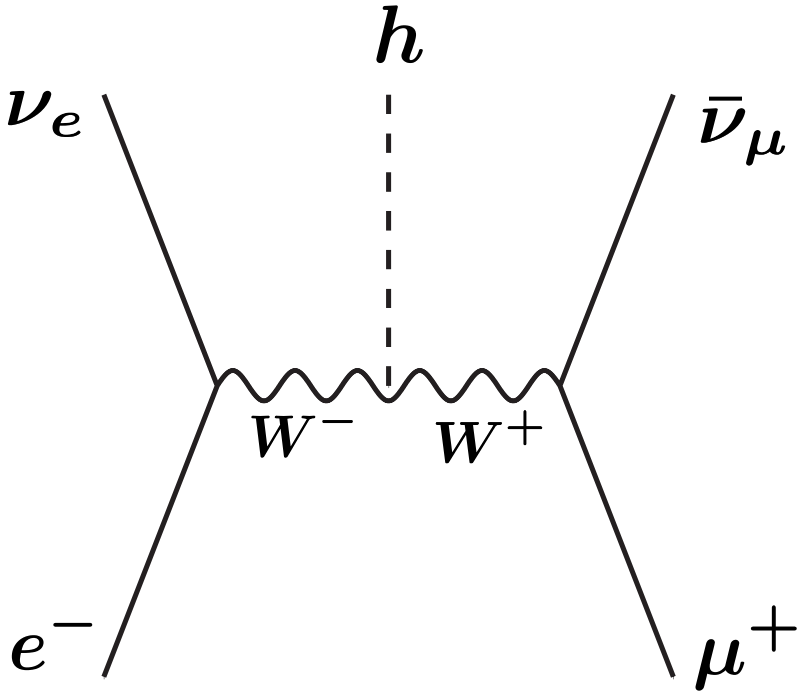
Higgs production

[**YH**, Kitano, Matsudo, Takaura, Yoshida, 2201.06664]

Due to the large luminosity, the μ^+e^- collider is more suitable.

$$(E_{\mu^+}, E_{e^-}) = (1\,\mathrm{TeV}, 30\,\mathrm{GeV}) \qquad \sqrt{s} = 346\,\mathrm{GeV}$$

$$(P_{\mu^+}, P_{e^-}) = (0.8, -0.7)$$



Main process: W-boson fusion $\sigma_{\mathrm{WBF}} \simeq 91\,\mathrm{fb}$

- **luminosity** $\mathscr{L}_{\mu^+e^-} = 4.6 \times 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$
- $\int dt\, \mathscr{L}_{\mu^+e^-} = 1.0\,\mathrm{ab}^{-1}$ w/ ten-year running

$$\longrightarrow \quad N(\mathrm{Higgs}) = 9.5 \times 10^4 \times \frac{(\text{integrated luminosity})}{1.0\,\mathrm{ab}^{-1}}$$

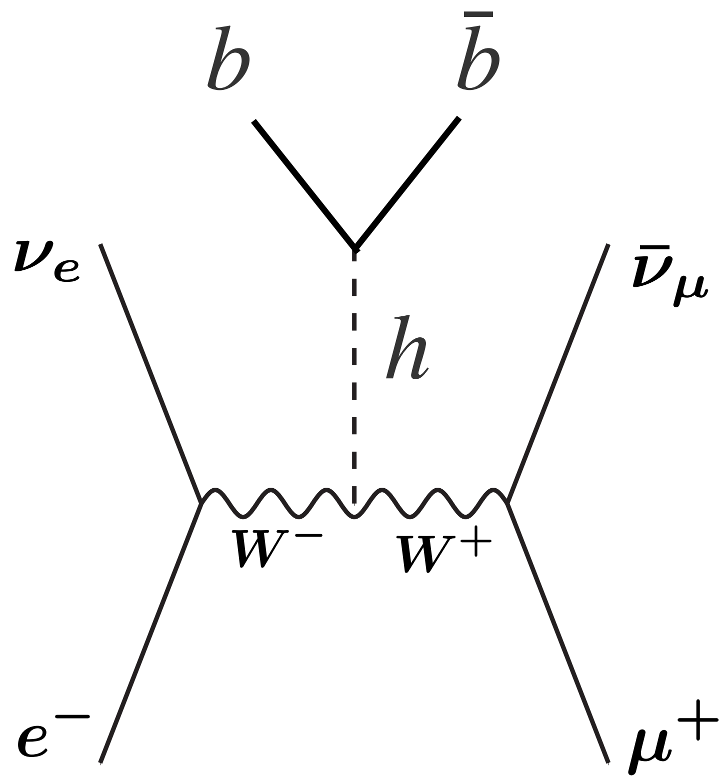
Higgs precision measurement is possible!

Coupling measurement

[**YH**, Kitano, Matsudo, Takaura, Yoshida, 2201.06664]

Higgs mainly decays into $b\bar{b}$ (Br. = 58.2 % in SM)

$$\sigma_{\rm SM} = \sigma^{\rm SM}_{\rm WBF} \underbrace{\boxed{\frac{\Gamma^{\rm SM}_{H\to b\bar{b}}}{\Gamma^{\rm SM}_{H,tot}}}}_{{\rm Br}(H\to b\bar{b})}$$



All couplings are parameterized by κ's (κ-scheme)

$$g_{hWW} = \kappa_W \, g^{\rm SM}_{hWW}$$

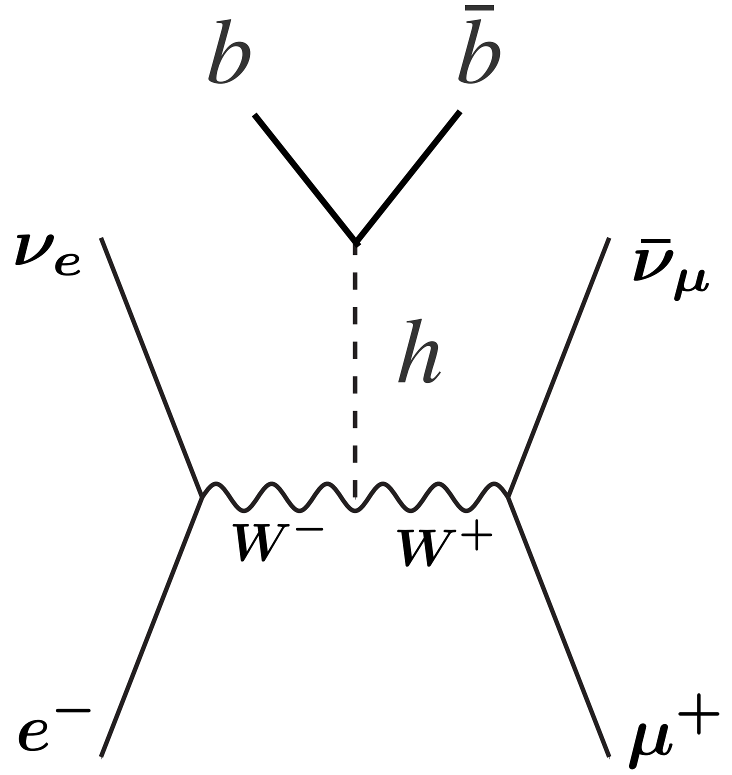
$$g_{hb\bar{b}} = \kappa_b \, g^{\rm SM}_{hb\bar{b}} \qquad \longrightarrow \qquad \sigma = \frac{\kappa_W^2 \kappa_b^2}{\kappa_H^2} \sigma_{\rm SM}$$

$$\Gamma_{H,tot} = \kappa_H^2 \, \Gamma^{\rm SM}_{H,tot}$$

Coupling measurement

[**YH**, Kitano, Matsudo, Takaura, Yoshida, 2201.06664]

$$\sigma = \frac{\kappa_W^2 \kappa_b^2}{\kappa_H^2} \sigma_{\rm SM}$$



$\kappa_\bullet = 1 + \Delta\kappa_\bullet$ (SM corresponds to $\Delta\kappa_\bullet = 0$)

κ's can be measured with the statistical uncertainty:

$$\left|\Delta\kappa_W + \Delta\kappa_b - \Delta\kappa_H\right| \lesssim \frac{1}{2}\frac{\Delta_{\rm stat.}\sigma}{\sigma} \simeq \frac{1}{2}\frac{1}{\sqrt{N({\rm Higgs}) \times {\rm Br} \times {\rm efficiency}}}$$

$$\simeq 3.1 \times 10^{-3} \times \left(\frac{\text{integrated luminosity}}{1.0\,{\rm ab}^{-1}}\right)^{-1/2} \left(\frac{\text{efficiency}}{0.5}\right)^{-1/2}$$

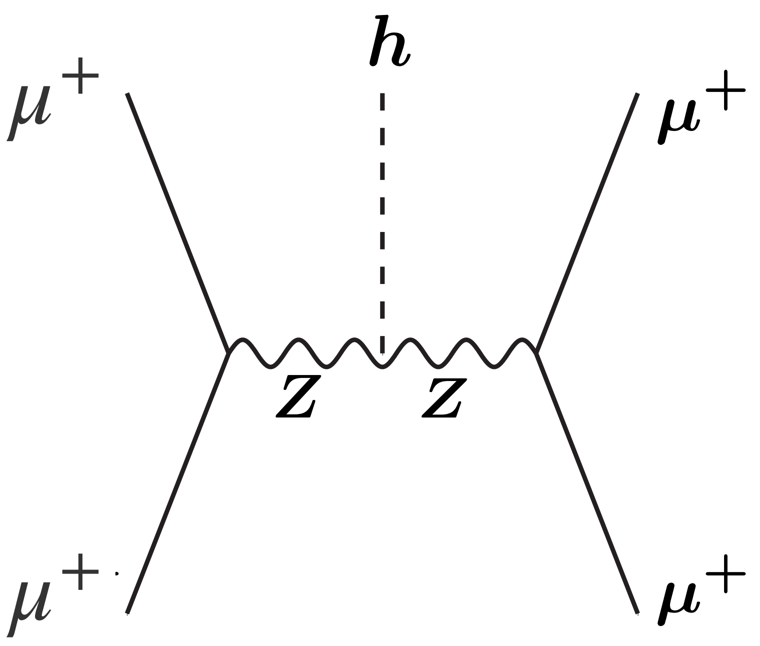
O(0.1)% measurement!

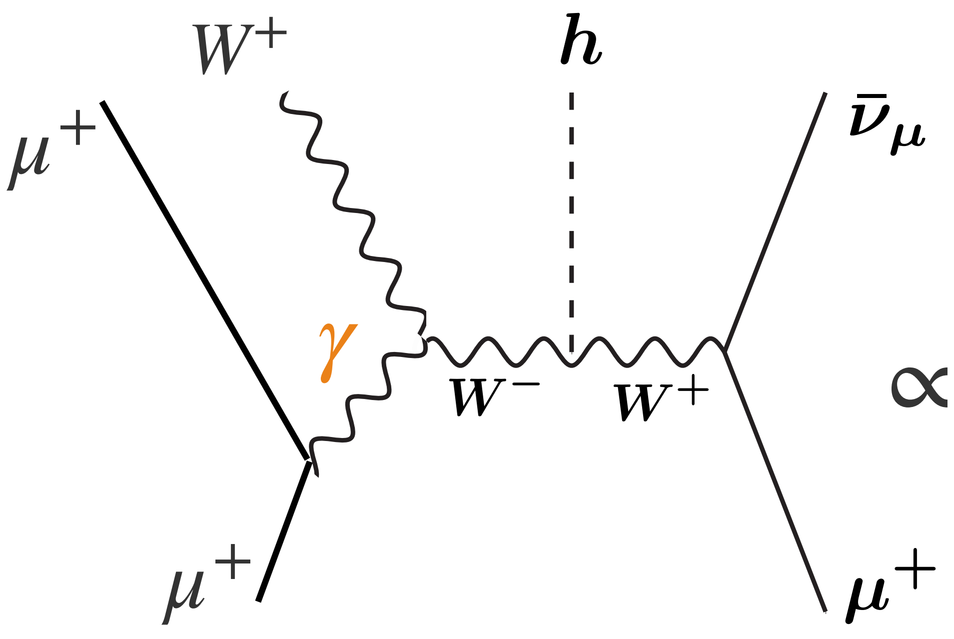
Higgs production @ $\mu^+\mu^+$?

[2407.XXXXX, **YH**, Kitano, Matsudo, Okawa, Takai, Takaura, and Treuer]

- **Naively, only ZBF is possible at $\mu^+\mu^+$ collider → suppressed?**

- **However, γ-emitted WBF can be significant at high energy!!**

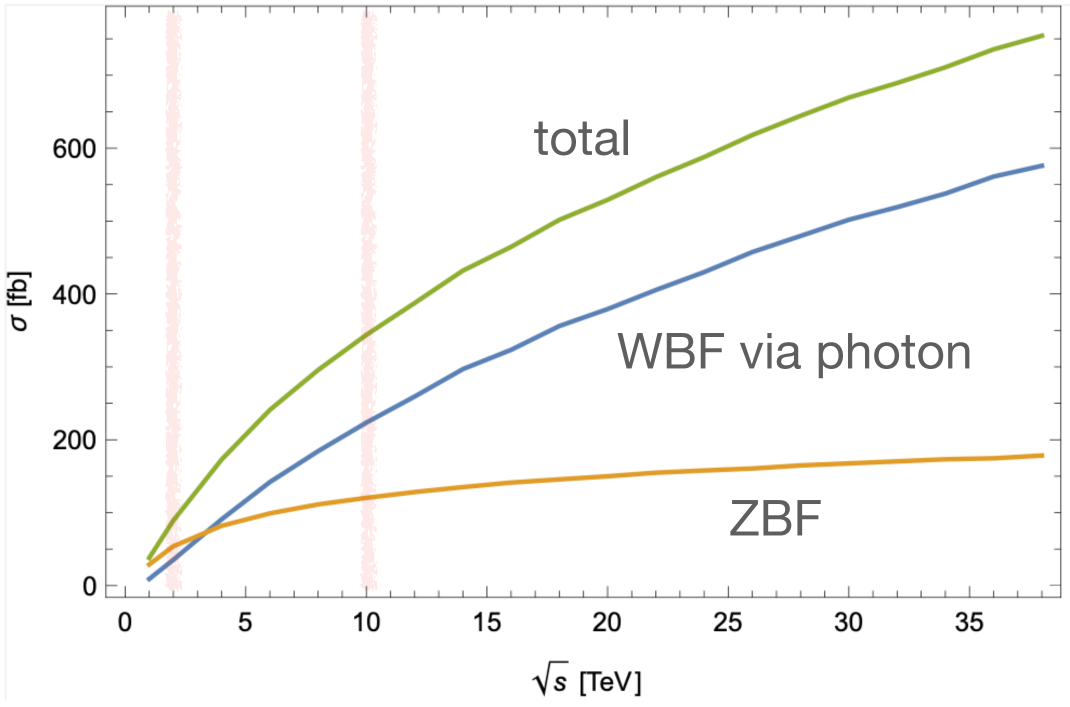




$$\propto \alpha \log(E/m_\mu)$$

$\sqrt{s}$ [TeV]	ZBF [fb]	Photon emission [fb]
2	54	35
10	121	224
20	150	376

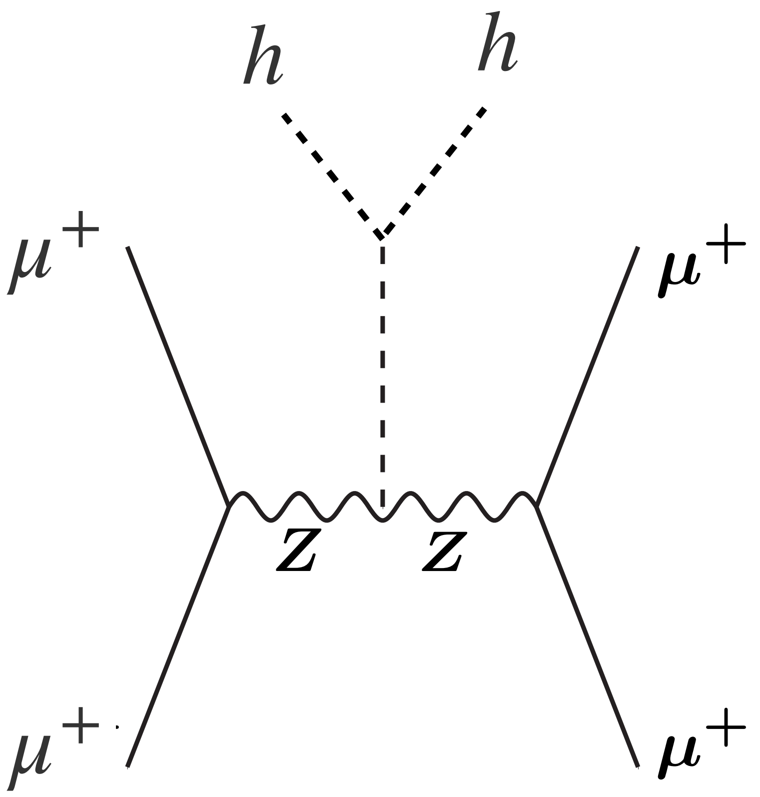
about a factor of two smaller than μ⁺μ⁻
(not too bad?)

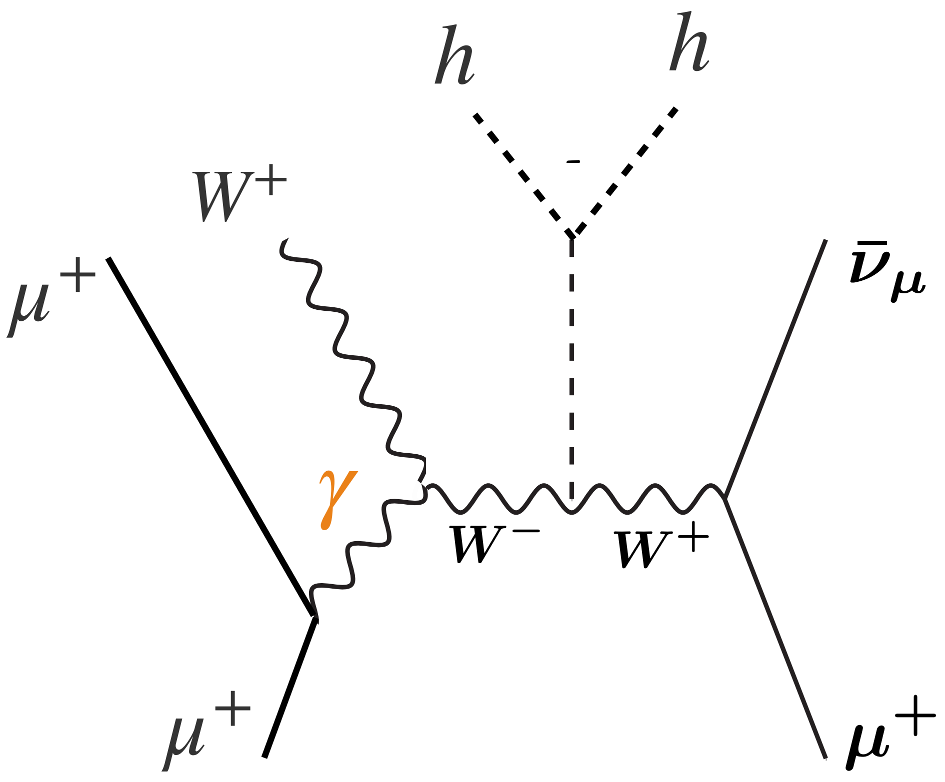


Higgs production @ $\mu^+\mu^+$?

[2407.XXXXX, **YH**, Kitano, Matsudo, Okawa, Takai, Takaura, and Treuer]

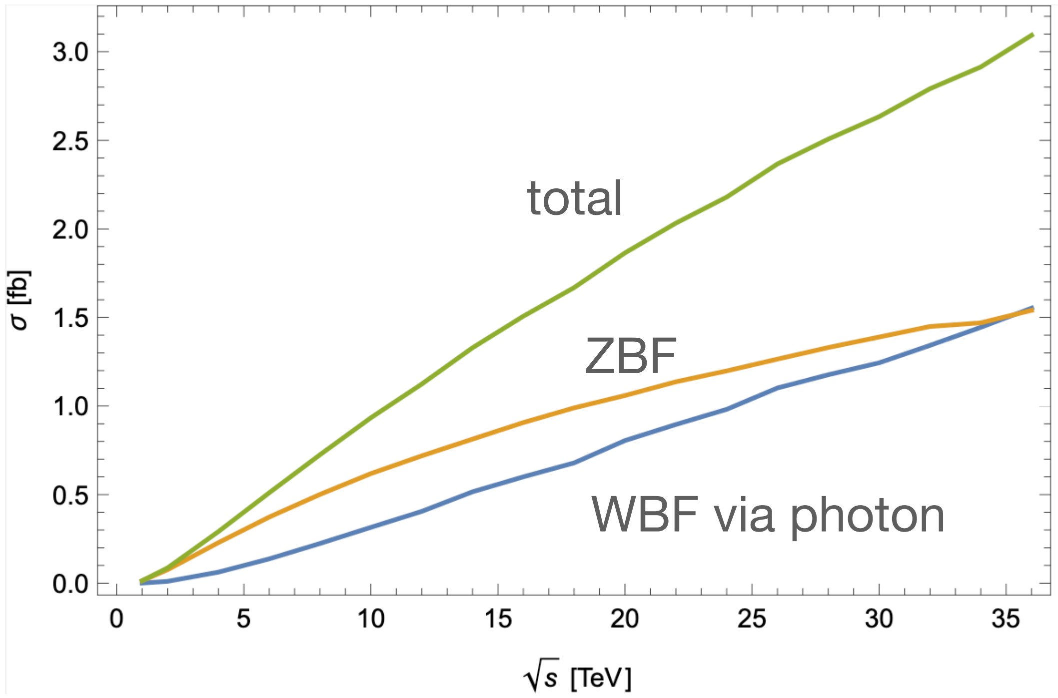
- **di-Higgs production at $\mu^+\mu^+$ collider**





$\sqrt{s}$ [TeV]	ZBF [fb]	Photon emission [fb]
2	0.075	0.010
10	0.62	0.30
20	1.1	0.75

hhh coupling at 5-10% level? (@10TeV, 10ab^{-1})



Plan of talk

- Introduction: What is TRISTAN?
- Higgs physics at μTRISTAN
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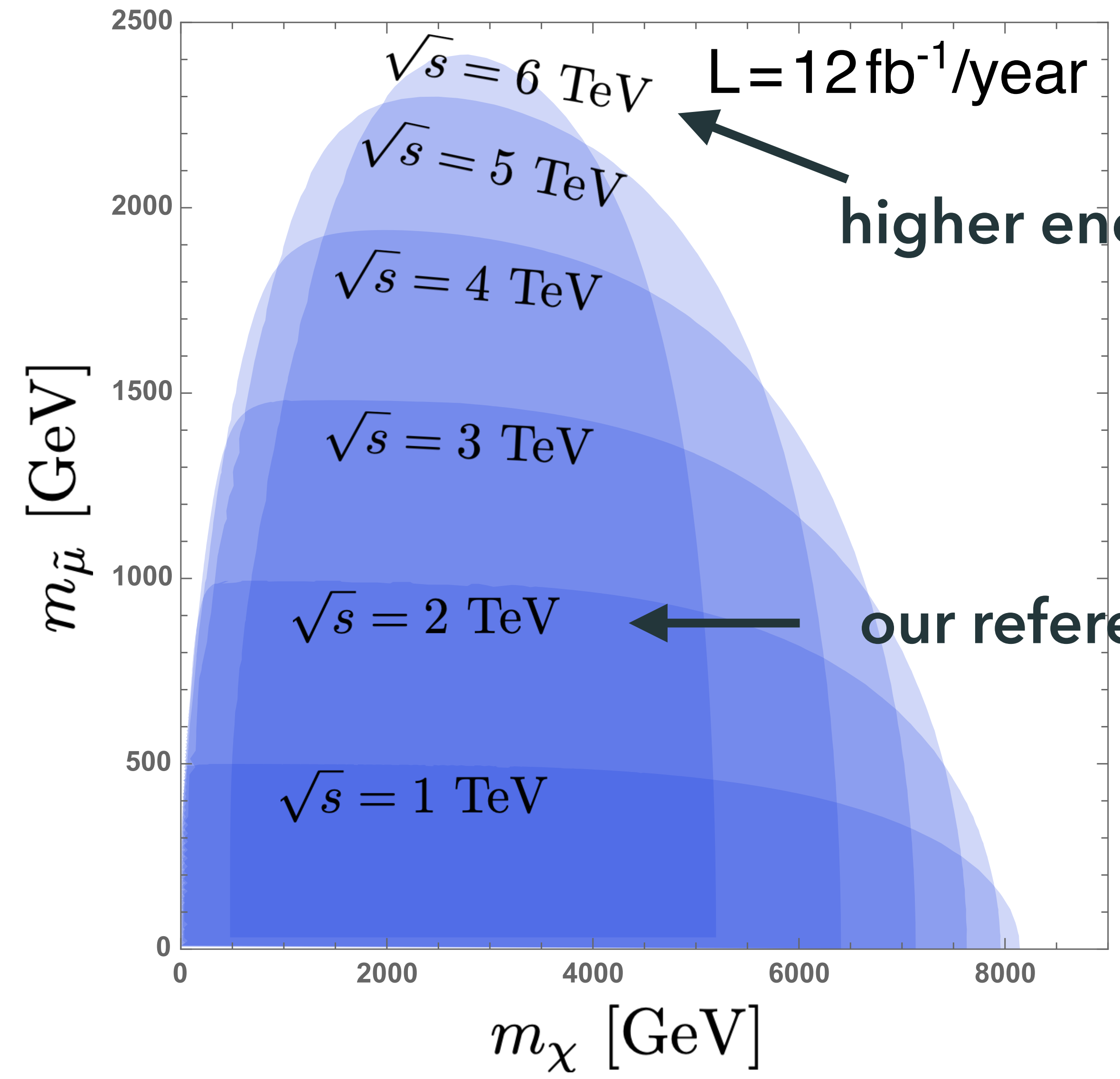
New physics search at μTRISTAN

Slepton production at μTRISTAN

[**YH**, Kitano, Matsudo, Takaura, Yoshida, 2201.06664]

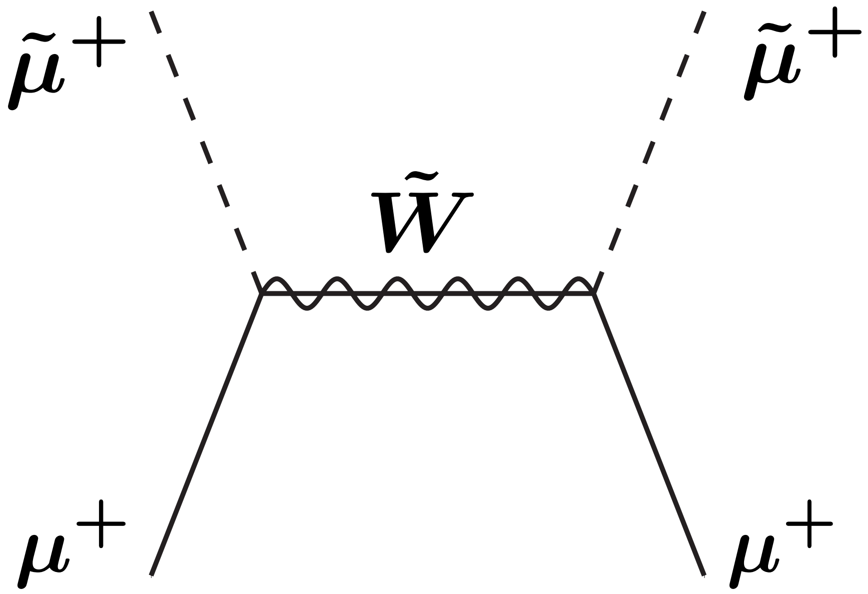
- For simplicity, only Wino $\tilde{W}$ exchange (no other neutralinos)
- Mass parameter region where # of events exceeds 100

(We do not consider decay of sleptons)



higher energy option

our reference energy



$\mu^+\mu^+$ collider

$m_{\tilde{\mu}} \lesssim 1$ TeV can be explored

Indirect new physics search at μTRISTAN

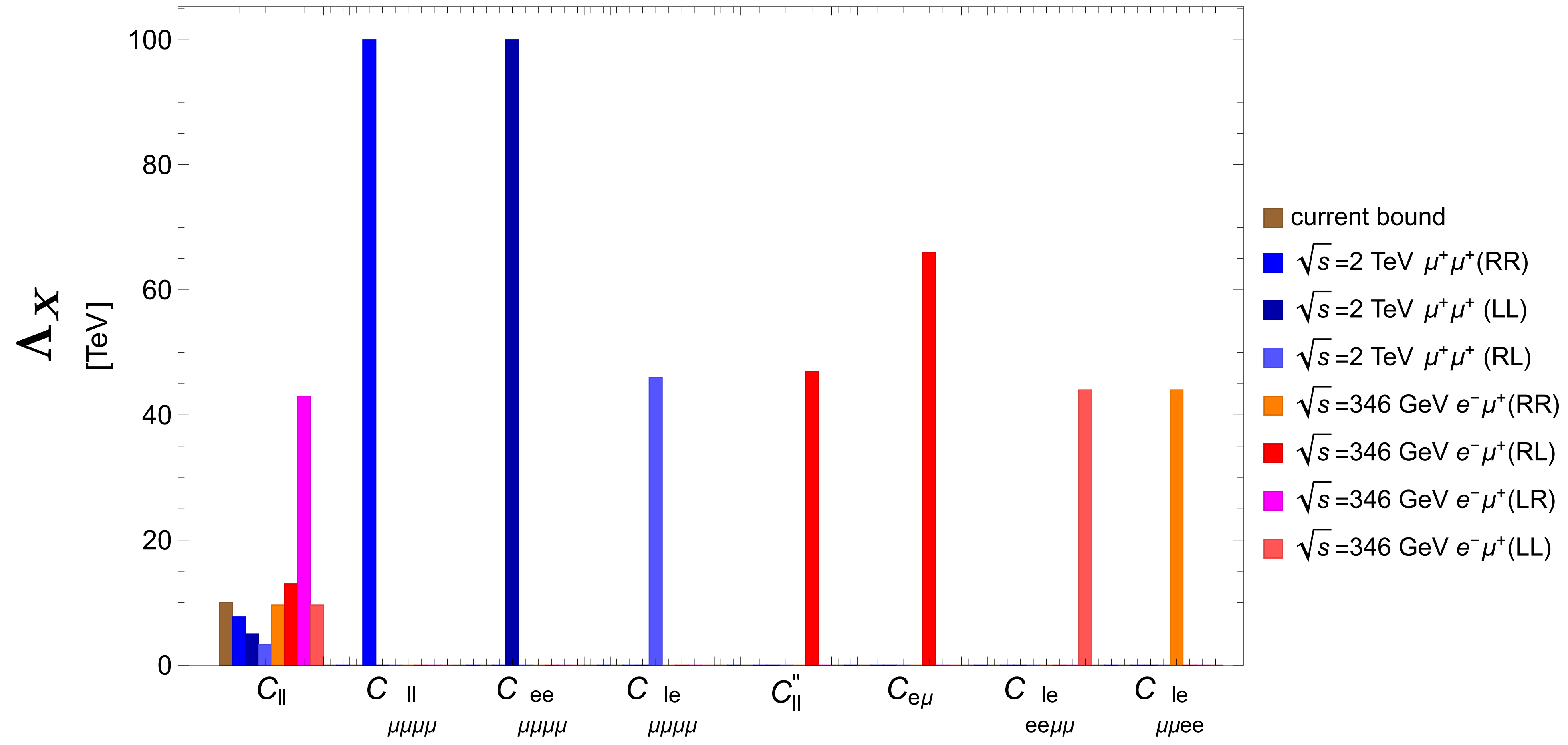
[**YH**, Kitano, Matsudo, Takaura, 2210.11083]

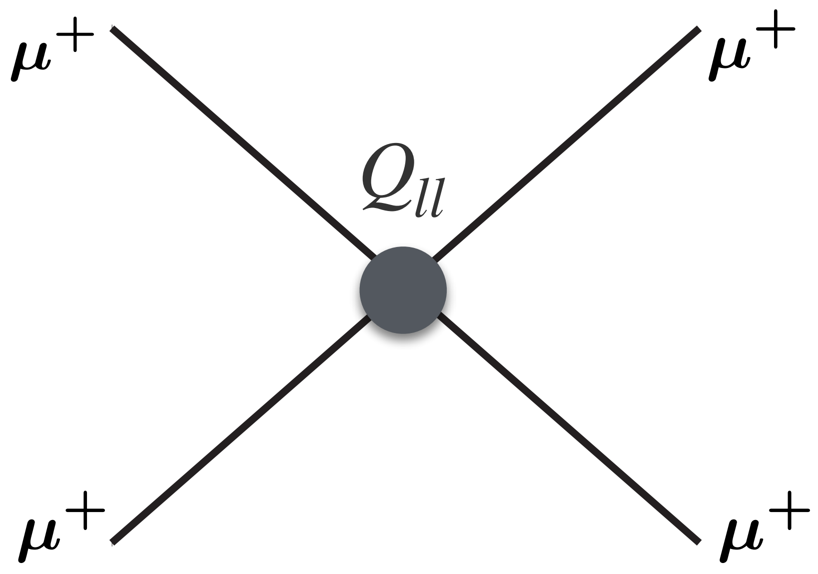
Constrain SMEFT dim-6 operators via Møller scattering

$$Q_{ll} = \frac{1}{\Lambda^2}(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L) \qquad Q_{HD} = \frac{1}{\Lambda^2}(H^\dagger D_\mu H)^* (H^\dagger D^\mu H) \qquad \text{etc.}$$

$$C_X = \frac{1}{\Lambda_X^2}$$

Current bound from 2204.05260 by Bagnaschi et al.





with 95% C.L.

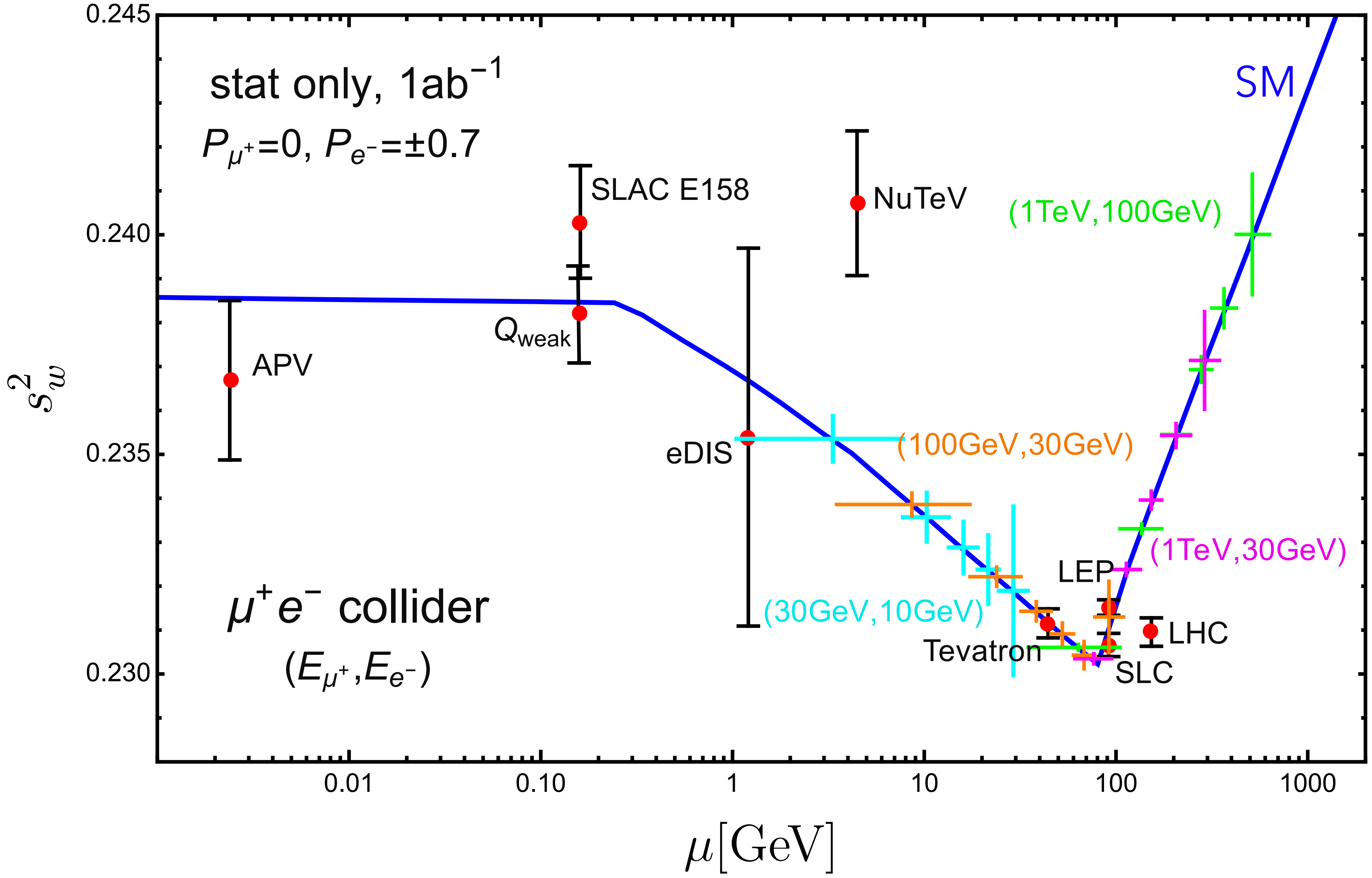
We can detect $\Lambda \lesssim 100\,\mathrm{TeV}$ at most!

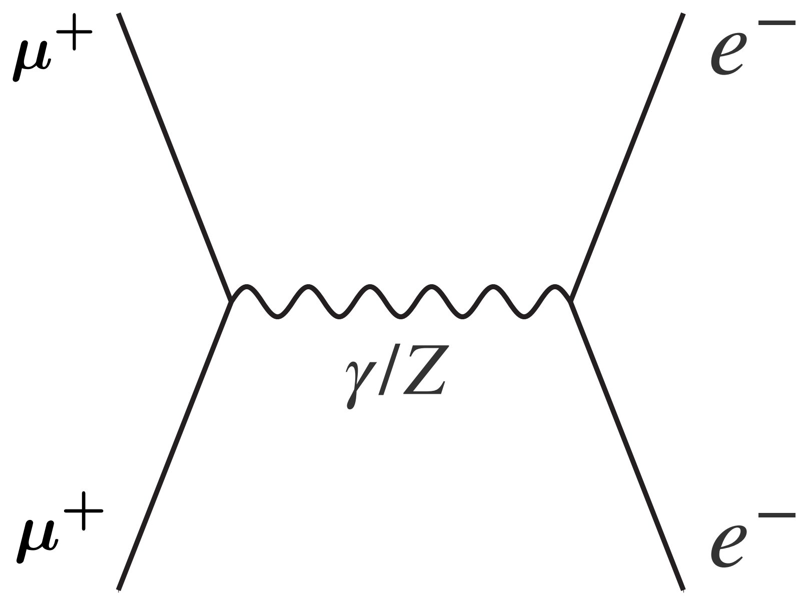
Weak mixing angle at μTRISTAN

[Chen, **YH**, Iguro, 2406.04500]

Weak mixing angle: $\sin^2\theta_W = \dfrac{g_Y^2}{g_W^2 + g_Y^2}$ **(@ tree)**

important quantity of EW sector and sensitive to NP through RGE





scan over wide range of μ with nice precision!

Summary

- **We proposed a new collider μTRISTAN using ultra-cold μ^+, which is already available!**

μ^+e^- **collider** $\mathscr{L}_{\mu^+e^-} = 4.6 \times 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ $\sqrt{s} = 346\,\mathrm{GeV}$

  - Higgs factory → coupling measurement w/ O(0.1) % precision
  - Weak mixing angle measurement

$\mu^+\mu^+$ **collider** $\mathscr{L}_{\mu^+\mu^+} = 5.7 \times 10^{32}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ $\sqrt{s} = 2\,\mathrm{TeV}$

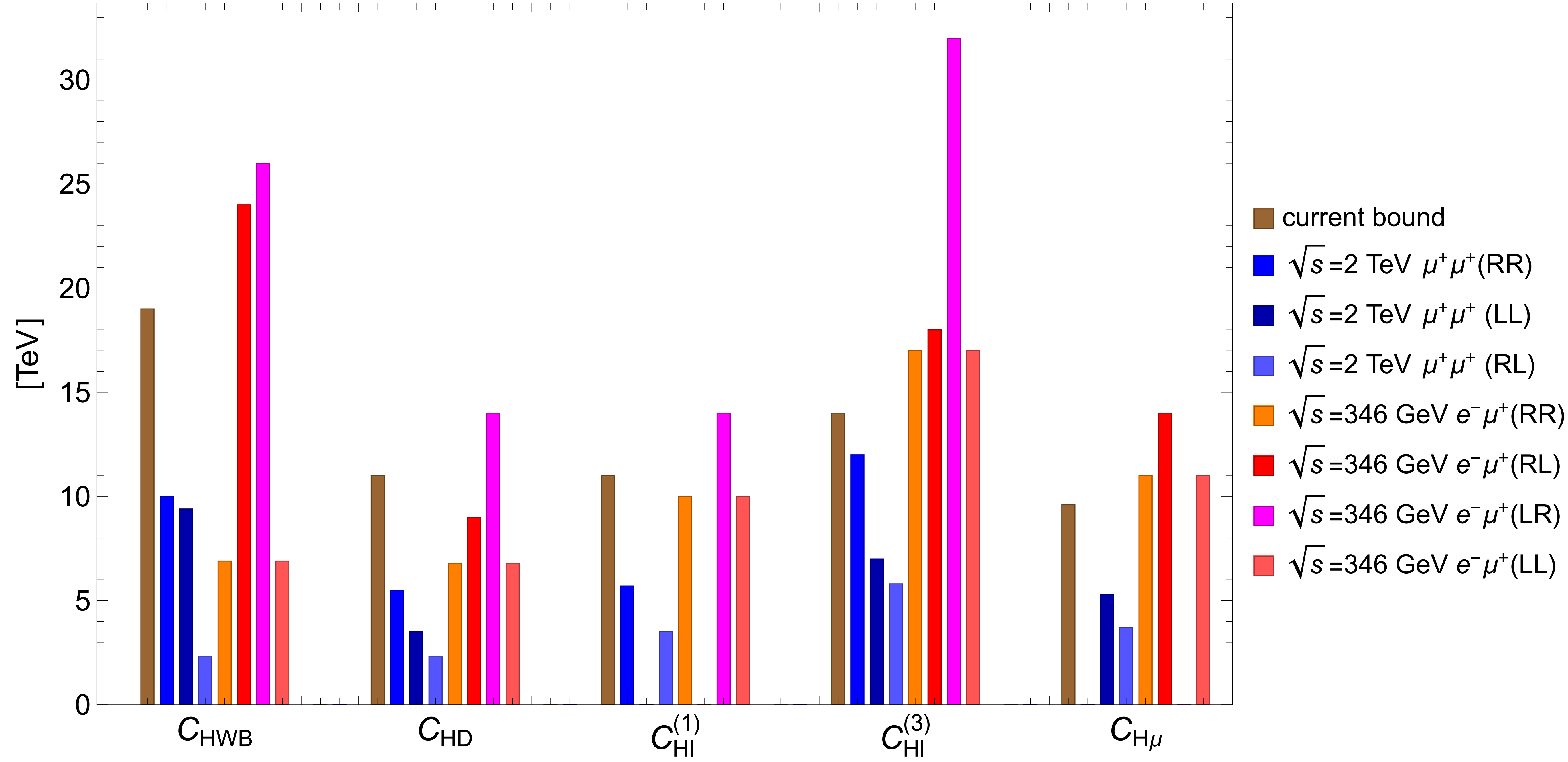
  - direct NP search like slepton
  - indirect NP search based on SMEFT → $\Lambda \simeq 100\,\mathrm{TeV}$ at most
  - Higgs factory at higher energy (γ-emit WBF)

Backup

Indirect new physics search at μTRISTAN

$$C_X = \frac{1}{\Lambda_X^2}$$

Current bound from 2204.05260 by Bagnaschi et al.



with 95% C.L.

Indirect new physics search at μTRISTAN

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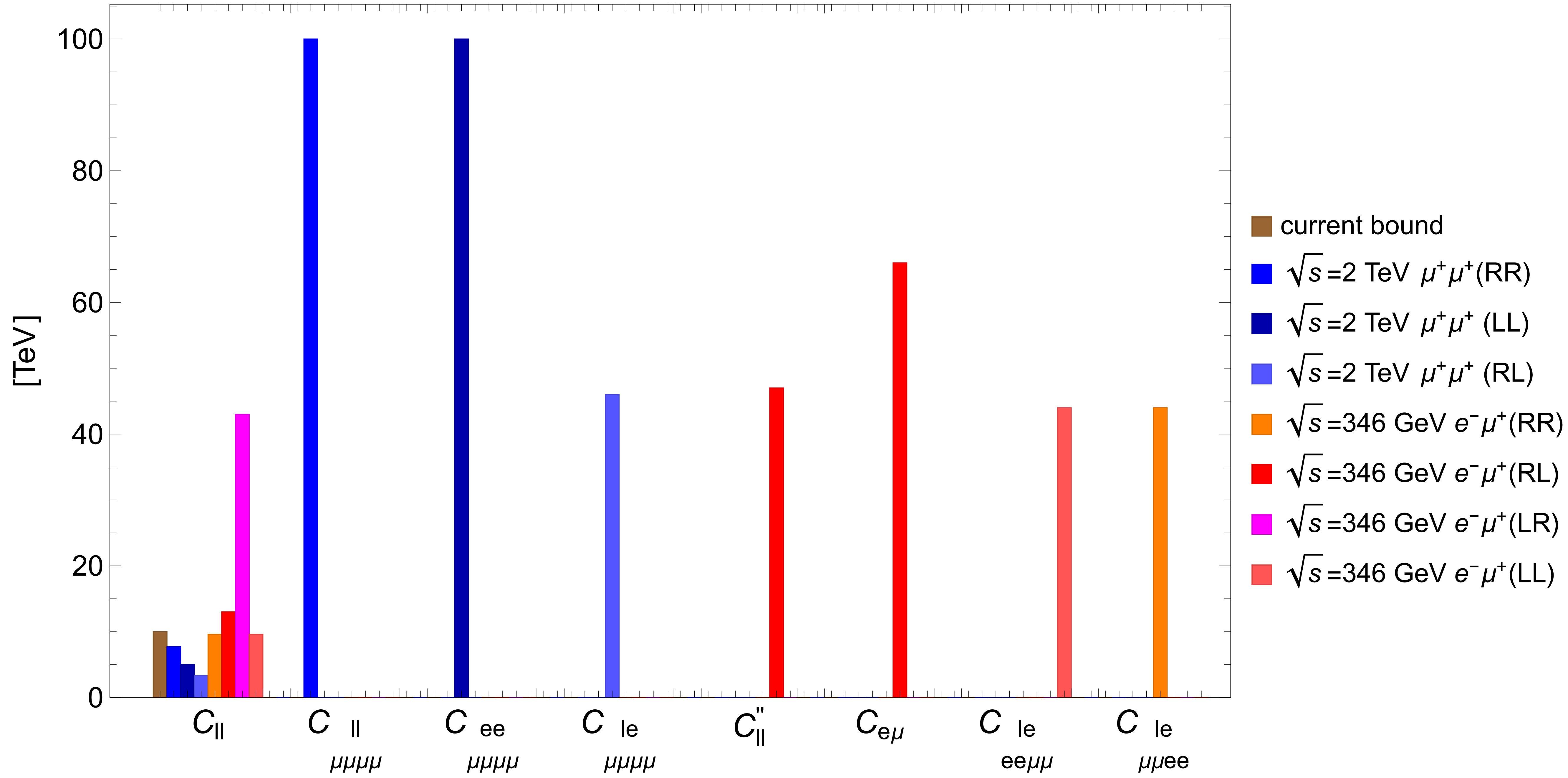
Current bound from 2204.05260 by Bagnaschi et al.



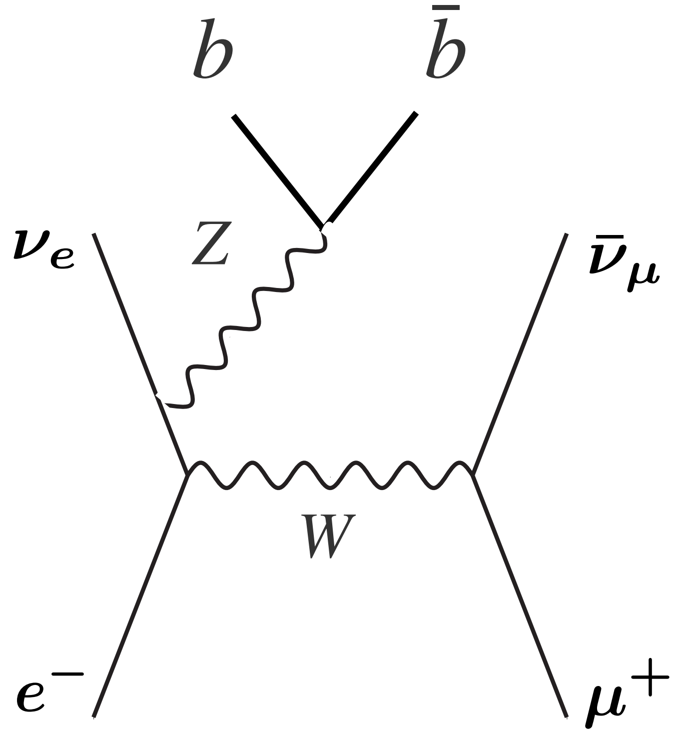
with 95% C.L.

We can detect $\Lambda \lesssim 100\,\mathrm{TeV}$ at most!

Efficiency

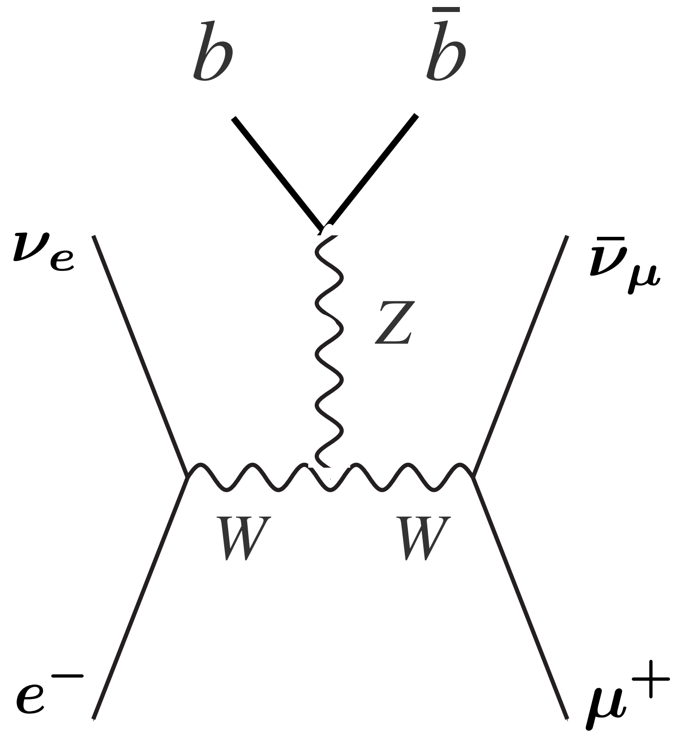
in progress with experimentalists

The efficiency to detect the events is important and under studied.



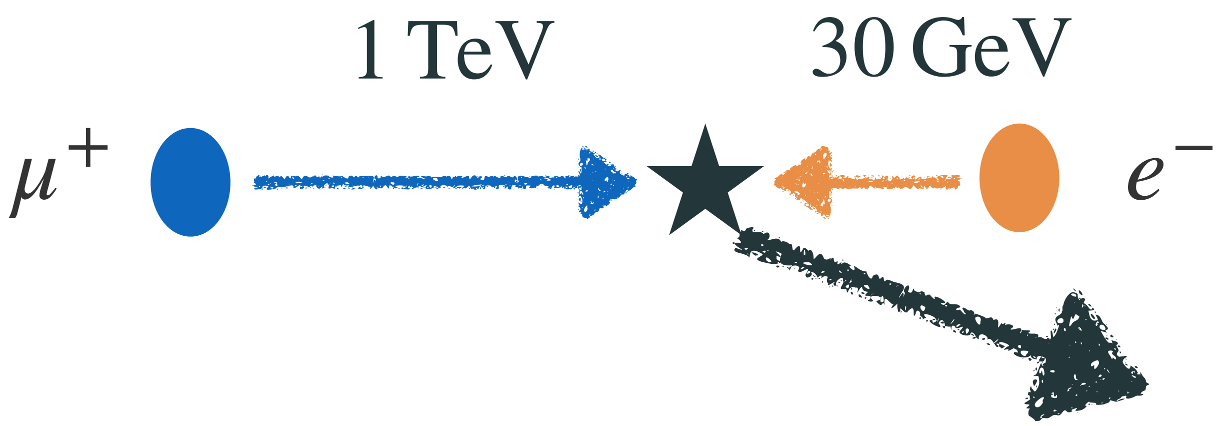
- **Background events**

  By reconstructing the invariant mass from b-jets, we would suppress the BG events.



- **Coverage of detector in a small angle region**

  The produced particle is strongly boosted.



Efficiency

in progress with experimentalists

from slide by Toshiaki Kaji

Benchmark Detector : HL-LHC

Coverage of HL-LHC @ Delphes

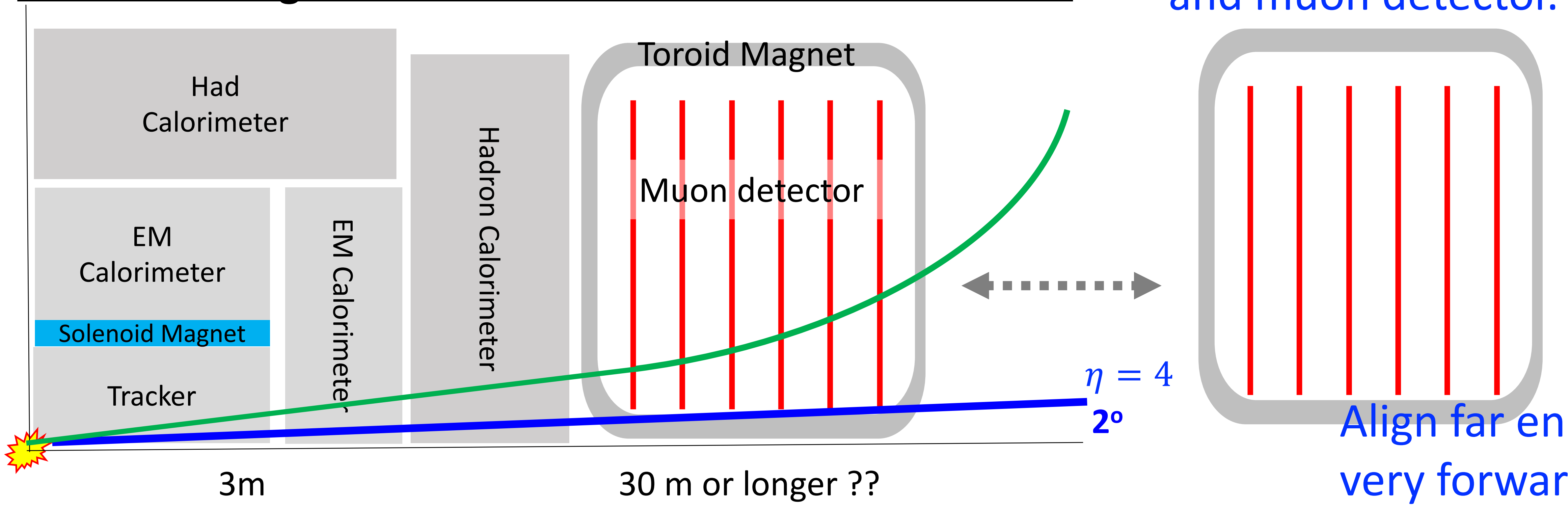
- Muon |η| < 4.0
- Electron |η| < 4.0
- B-jet |η| < 4.0, pT > 25 GeV

Delphes card for HL-LHC is used for this study.

https://github.com/delphes/delphes/blob/3.5.0/cards/delphes_card_HLLHC.tcl

Forward muons should be detected by combination of forward magnet and muon detector.

Tentative rough schematic for muon collider detector



Align far enough to detect very forward muons

Efficiency

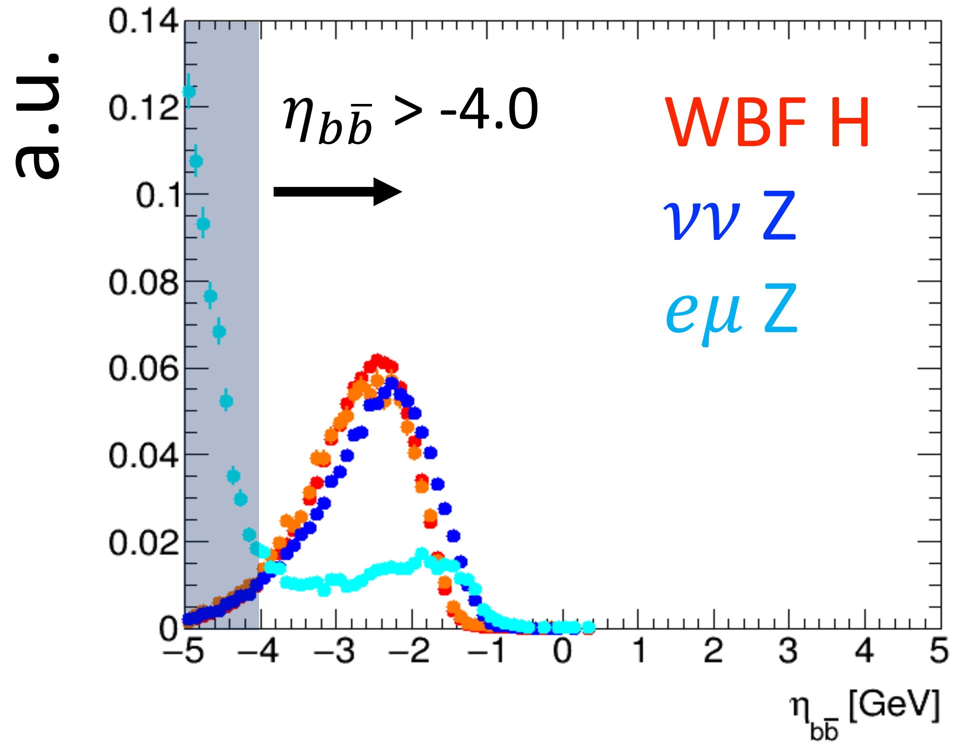
in progress with experimentalists

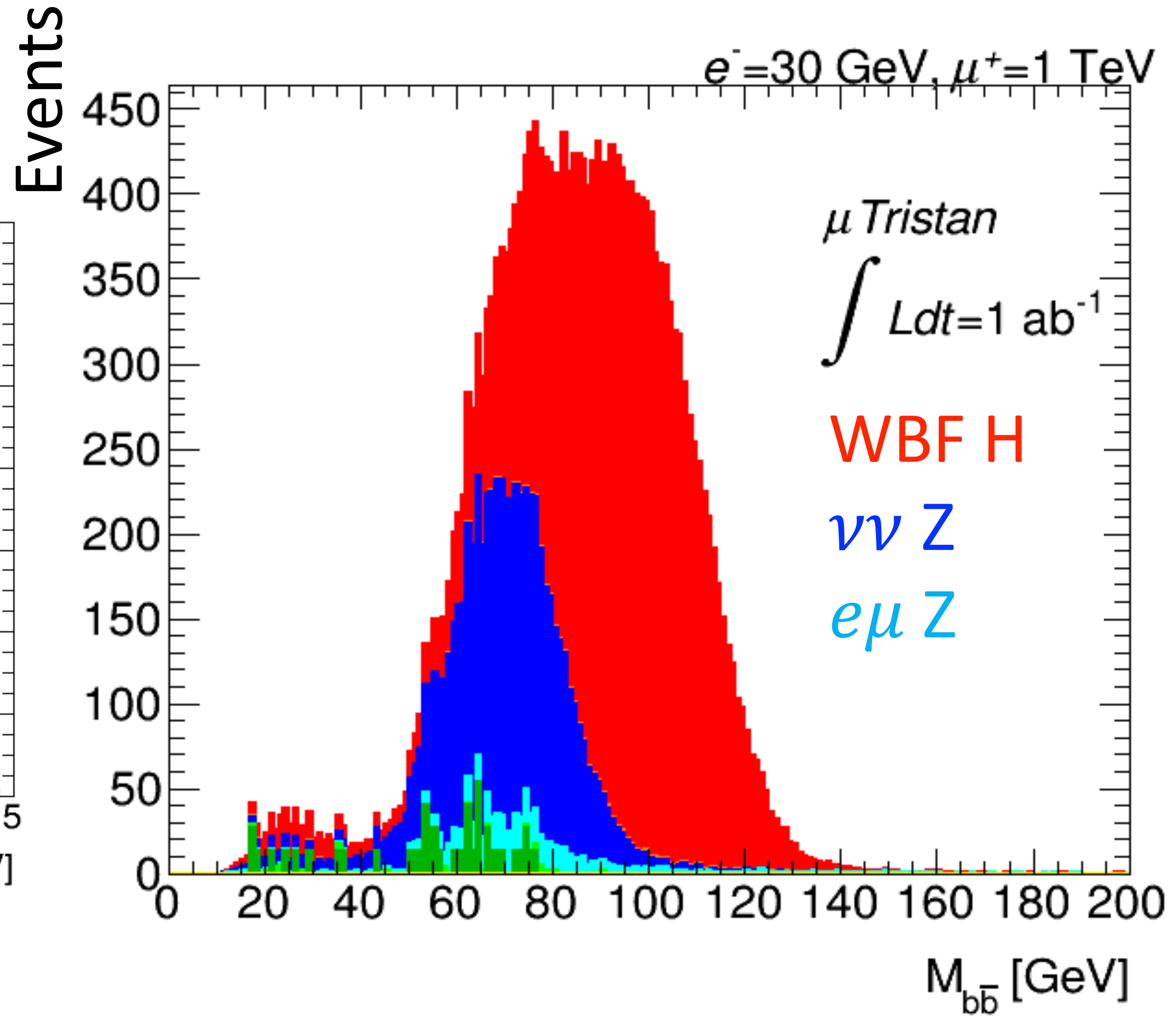
from slide by Toshiaki Kaji

WBF Higgs Measurements

Requirements

- No muon
- No electron
- Exact 2 b-jets
- $\eta_{b\bar{b}}$ > -4.0





Radiative Z background
can be removed by lepton veto and $\eta_{b\bar{b}}$ cut.

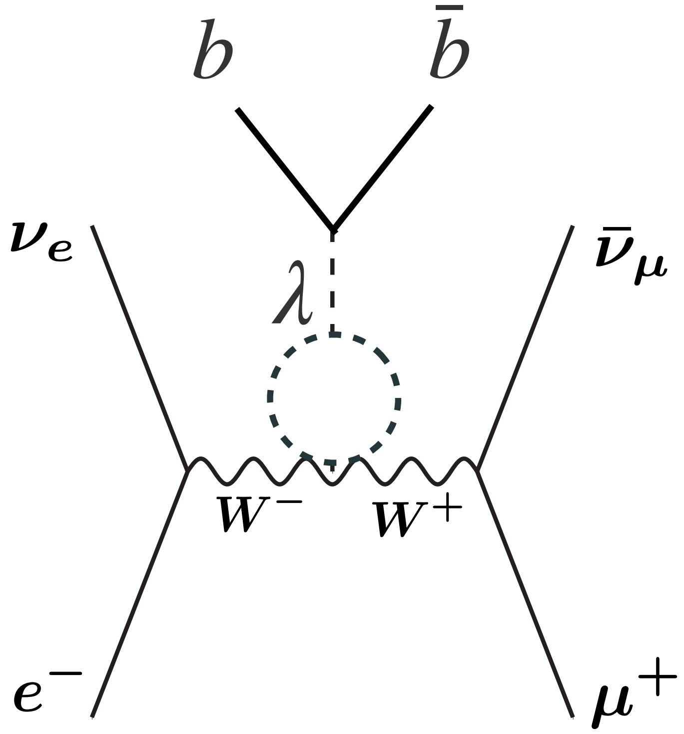
Efficiency : ~ 23%

→ 12k events @ 1 ab^{-1}
→ $\Delta(\kappa_W + \kappa_b - \kappa_H)_{\rm stat} =$ 0.5% @ 1 ab^{-1}

Trilinear coupling in higher energy case

$$E_{\mu^+} = 3\,\mathrm{TeV}, \quad E_{e^-} = 50\,\mathrm{GeV}$$

$$\sqrt{s} = 775\,\mathrm{GeV} \qquad (P_{\mu^+}, P_{e^-}) = (0.8, -0.7)$$



- WBF process $\sigma_{\mathrm{WBF}} \simeq 472\,\mathrm{fb}$

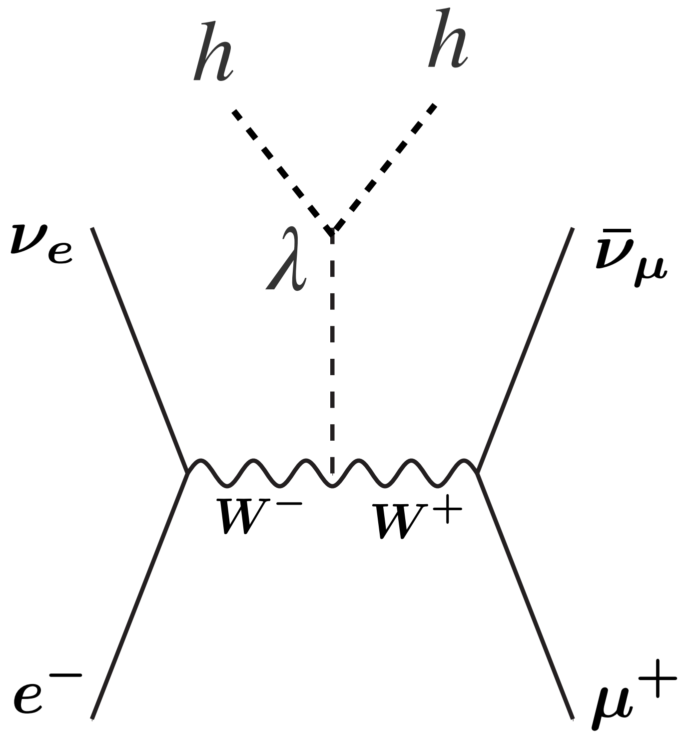
can probe Higgs trilinear coupling via 1-loop

[Di Vita+, 1711.03978]

$$\left|\Delta\kappa_W + \Delta\kappa_b - \Delta\kappa_H + 0.006\Delta\kappa_\lambda\right|$$

$$\lesssim 1.3\times 10^{-3} \times \left(\frac{\text{integrated luminosity}}{1.0\,\mathrm{ab}^{-1}}\right)^{-1/2} \left(\frac{\text{efficiency}}{0.5}\right)^{-1/2}$$

$$\longrightarrow \quad \underline{\left|\Delta\kappa_\lambda\right| \lesssim 20\,\%} \qquad \text{(if other } \Delta\kappa\text{'s are zero)}$$



- also probed by Higgs pair production

$$N(\text{di Higgs}) \simeq 89 \times \left(\frac{\text{integrated luminosity}}{1.0\,\mathrm{ab}^{-1}}\right)^{-1/2} \left(\frac{\text{efficiency}}{0.5}\right)^{-1/2} \longrightarrow \left|\Delta\kappa_\lambda\right| \lesssim 100\,\%$$

Proposal of new experiment: μTRISTAN!

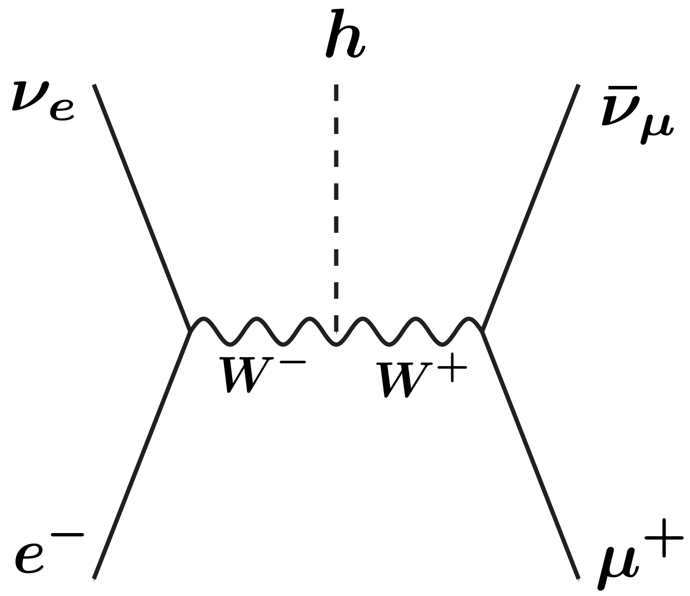
If a larger 6 (or 9) km ring is available (Tevatron size), we can explore higher energy:

- $\mu^+ e^-$ collider

$$E_{\mu^+} = 3\,\mathrm{TeV}, \quad E_{e^-} = 50\,\mathrm{GeV}$$

$$\rightarrow \sqrt{s} = 775\,\mathrm{GeV}$$

Higgs factory



- $\mu^+ \mu^+$ collider (instead of $\mu^+\mu^-$)

$$E_{\mu^+} = 3\,\mathrm{TeV}, \quad E_{\mu^+} = 3\,\mathrm{TeV}$$

$$\rightarrow \sqrt{s} = 6\,\mathrm{TeV}$$

New physics search

Comment on polarization

Polarization is important to enhance cross sections

- **Electron beam polarization:** $P_{e^-} = \pm 0.7$ same polarization as superKEKB
- **Muon beam polarization:** $P_{\mu^+} = \pm 0.8$

eg., $P = 0.4$ means 40 % is RH

while 60% is unpolarized.

muon production sequence:

$\pi^+ \rightarrow \mu^+ + \nu_\mu$ ⟶ Muonium $(\mu^+ e^-)$ formation in target silica aerogel ⟶ Ultra-cold muon

ionized by laser

$P_{\mu^+} = +1.0$

initial muonium states: $|+\rangle_{\mu^+} |+\rangle_{e^-}$ & $|+\rangle_{\mu^+} |-\rangle_{e^-}$

By applying magnetic field of 0.3 T,

the polarization can be maintained.

oscillate $|-\rangle_{\mu^+} |+\rangle_{e^-}$

[CDR for muon g-2/EDM exp. at J-PARC]

Thus $P_{\mu^+} = \pm 0.8$ would be a reasonable estimation.

SMEFT operator

$$Q_{HWB} = H^\dagger \tau^I H W^I_{\mu\nu} B^{\mu\nu} \qquad \longleftarrow \quad \text{S parameter}$$

$$Q_{HD} = (H^\dagger D_\mu H)^* (H^\dagger D_\mu H) \qquad \longleftarrow \quad \text{T parameter}$$

$$Q^{(1)}_{H\ell} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{L} \gamma^\mu L)$$

$$Q^{(3)}_{H\ell} = (H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{L} \tau^I \gamma^\mu L)$$

$$Q_{H\mu} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\mu} \gamma^\mu P_+ \mu)$$

$$Q_{\substack{ll \\ prst}} = (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$$

$$Q_{\substack{le \\ prst}} = (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$$

$$Q_{\substack{ee \\ prst}} = (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$$

$$C_{\substack{\ell\ell \\ \mu\mu\mu\mu}}, \quad C''_{\ell\ell} \equiv \frac{1}{2}\left(C_{\substack{\ell\ell \\ ee\mu\mu}} + C_{\substack{\ell\ell \\ \mu\mu ee}}\right), \quad C_{\substack{\ell e \\ \mu\mu\mu\mu}}, \quad C_{\substack{\ell e \\ ee\mu\mu}}, \quad C_{\substack{\ell e \\ \mu\mu ee}}, \quad C_{\substack{ee \\ \mu\mu\mu\mu}},$$

$$C_{e\mu} \equiv \frac{1}{4}\left(C_{\substack{ee \\ \mu\mu ee}} + C_{\substack{ee \\ ee\mu\mu}} + C_{\substack{ee \\ \mu ee\mu}} + C_{\substack{ee \\ e\mu\mu e}}\right).$$

$$C_{\substack{\ell\ell \\ e\mu\mu e}} = C_{\substack{\ell\ell \\ \mu ee\mu}} \equiv C_{\ell\ell}.$$

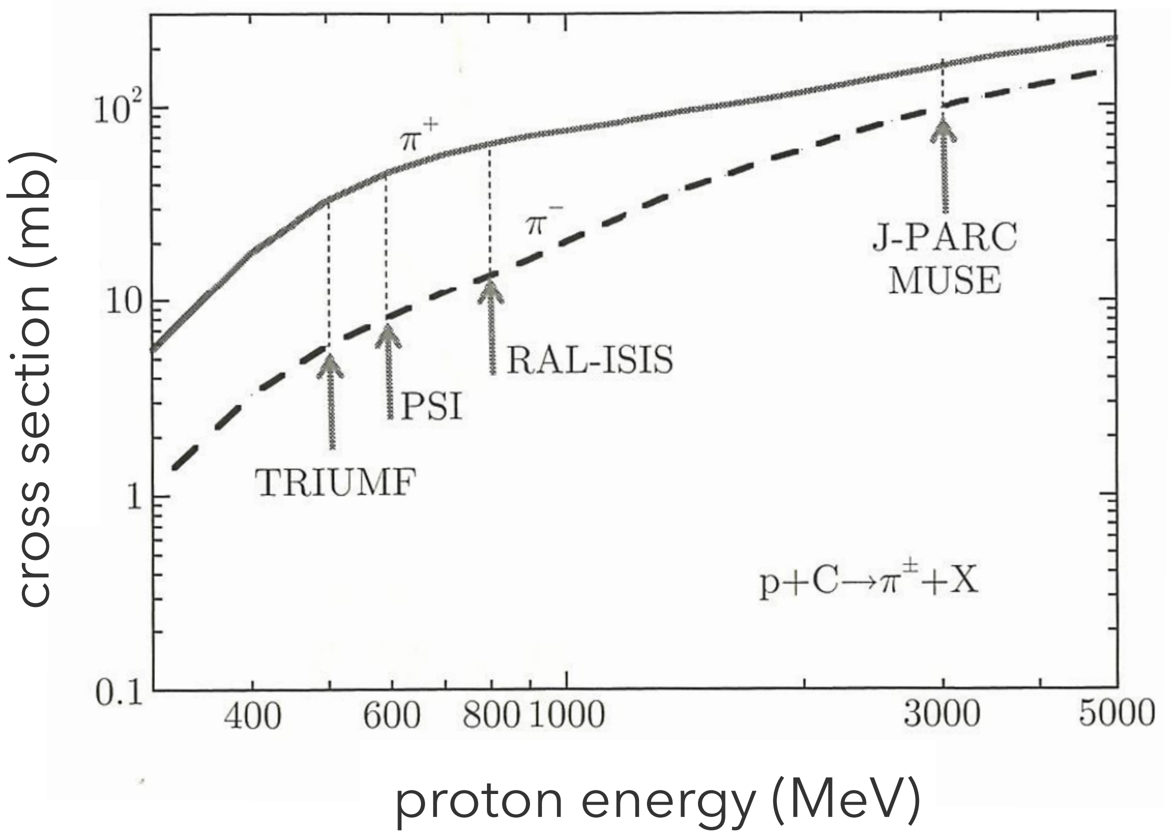
Current difficulties of $\mu^+\mu^-$ collider

The μ^- beam is difficult!

- Production of large amount of μ^-

  μ^- is produced by $\pi^- \to \mu^- + \bar{\nu}_\mu$

  But π^- production is less than π^+



- **Cooling of μ^-**

  To make narrow beam, cooling is necessary. But cooling technology of μ^- hasn't been established yet!

→ large beam size → **low luminosity!**

Luminosity

$$\mathscr{L} = \frac{N_{\text{beam1}}\, N_{\text{beam2}}}{4\pi\, \sigma_x\, \sigma_y} f_{\text{rep}}$$

- Collision frequency (per detector)

$$f_{\text{rep}}^{(\mu^+ e^-)} = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}/(3\,\text{km}) \times 40 = 4\,\text{MHz}$$

speed of beam

circumference of ring

# of bunches

$$f_{\text{rep}}^{(\mu^+ \mu^+)} = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}/(3\,\text{km}) \times 20 = 2\,\text{MHz}$$

Luminosity

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$$\beta\gamma \sim 10^4$$

- Beam size is determined by emittance

emittance: $\epsilon_x \, , \epsilon_y = \dfrac{4 \, \text{mm mrad}}{\beta\gamma}$

$$\sigma_i = \sqrt{\epsilon_i \beta_i}$$

beta function

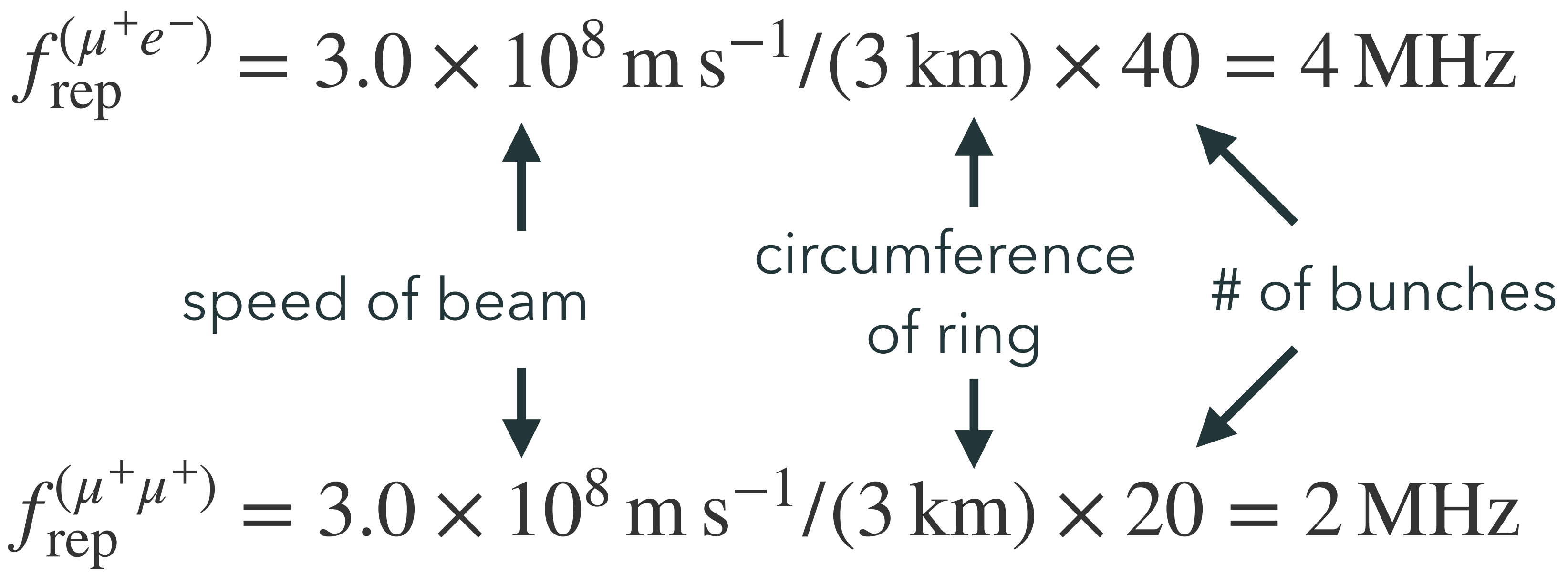
$\beta_x = 30 \, \text{mm}$ $\sigma_x = 3.6 \, \mu\text{m}$

$\beta_y = 7 \, \text{mm}$ $\sigma_y = 1.7 \, \mu\text{m}$

Luminosity

- Collision frequency (per detector)

$$\mathscr{L} = \frac{N_{\rm beam1}\,N_{\rm beam2}}{4\pi\,\sigma_x\,\sigma_y} f_{\rm rep}$$



$$f_{\rm rep}^{(\mu^+e^-)} = 3.0\times10^{8}\,{\rm m\,s^{-1}}/(3\,{\rm km})\times 40 = 4\,{\rm MHz}$$

speed of beam

circumference of ring

# of bunches

$$f_{\rm rep}^{(\mu^+\mu^+)} = 3.0\times10^{8}\,{\rm m\,s^{-1}}/(3\,{\rm km})\times 20 = 2\,{\rm MHz}$$

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Thanks to ultra-cold muon, low emittance is realized!

$$\sigma_i = \sqrt{\epsilon_i\beta_i}$$

beta function

$$\beta_x = 30\,{\rm mm} \qquad \sigma_x = 3.6\,\mu{\rm m}$$

$$\beta_y = 7\,{\rm mm} \qquad \sigma_y = 1.7\,\mu{\rm m}$$

Luminosity

$$\mathscr{L} = \frac{N_{\text{beam1}}\, N_{\text{beam2}}}{4\pi\, \sigma_x\, \sigma_y} f_{\text{rep}}$$

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speed of beam

circumference of ring

# of bunches

$$f_{\text{rep}}^{(\mu^+\mu^+)} = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}/(3\,\text{km}) \times 20 = 2\,\text{MHz}$$

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$\beta_y = 7\,\text{mm}$ $\sigma_y = 1.7\,\mu\text{m}$

Thanks to ultra-cold muon, low emittance is realized!

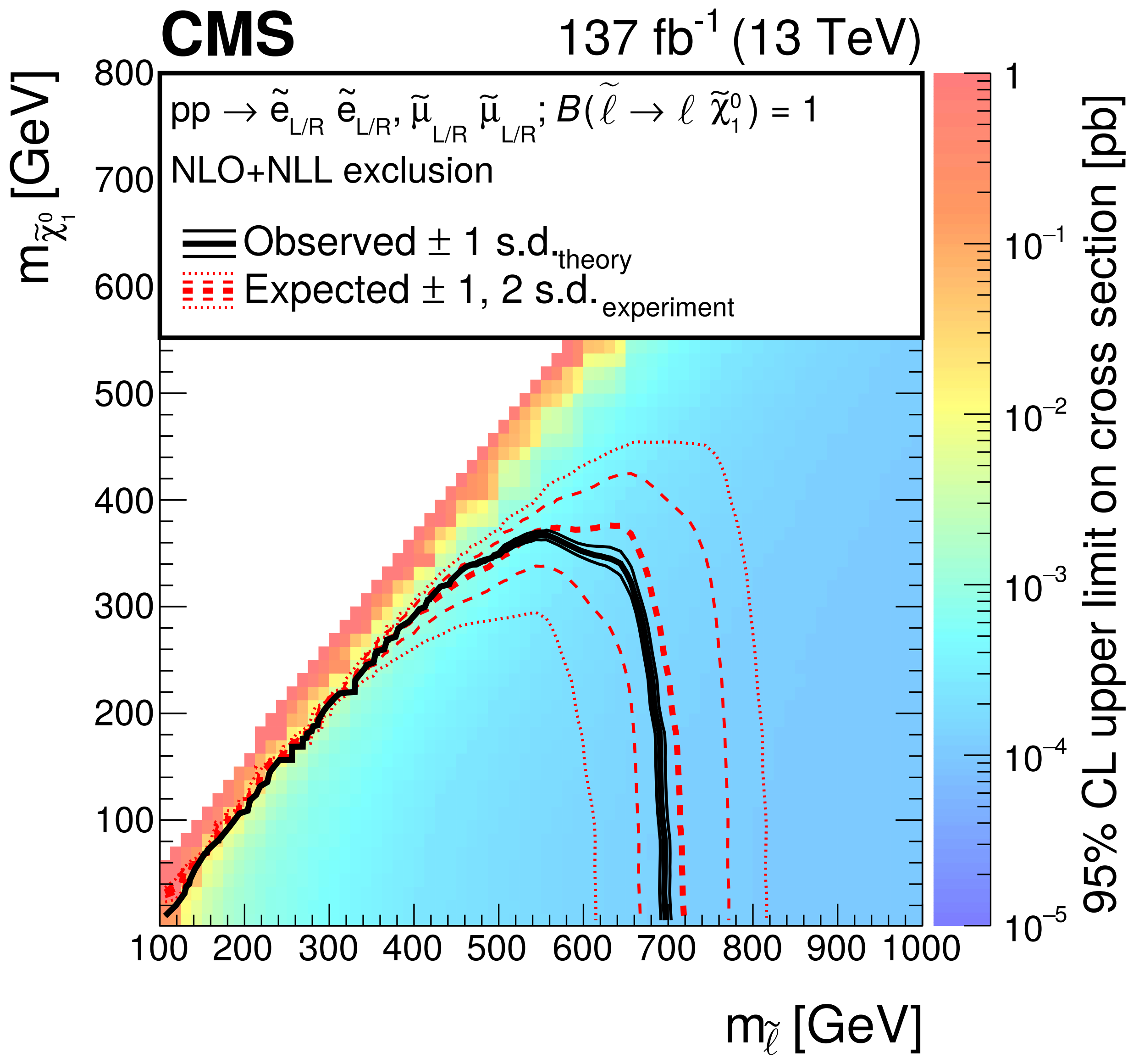
- # of beam particles

$N_{e^-} = 10\,\text{nC}$ per bunch

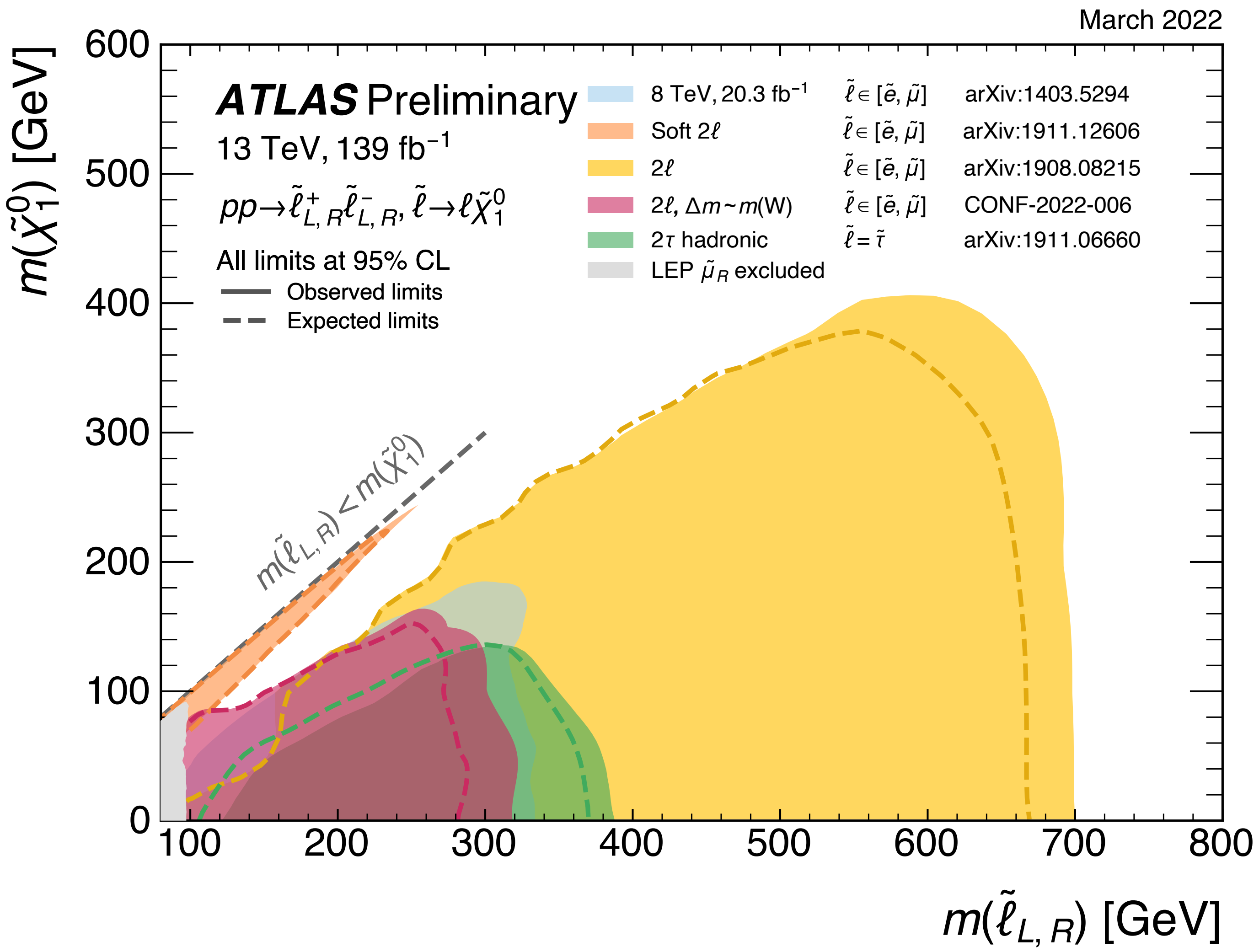
$N_{\mu^+} = 3.6\,\text{nC} \rightarrow 1.3\,\text{nC}$ per bunch due to decay

Current status of SUSY search

[2012.08600]



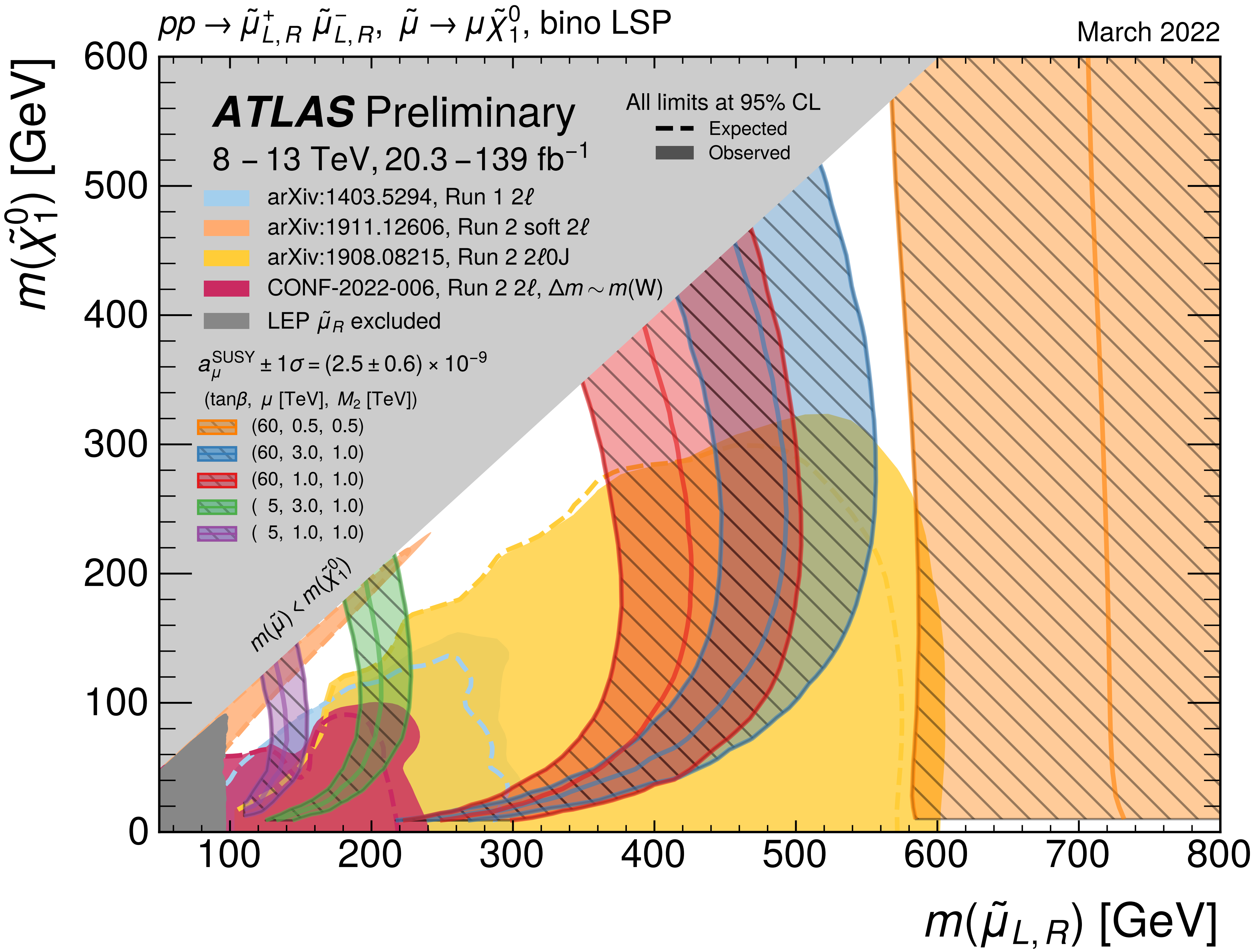
[ATL-PHYS-PUB-2022-013]



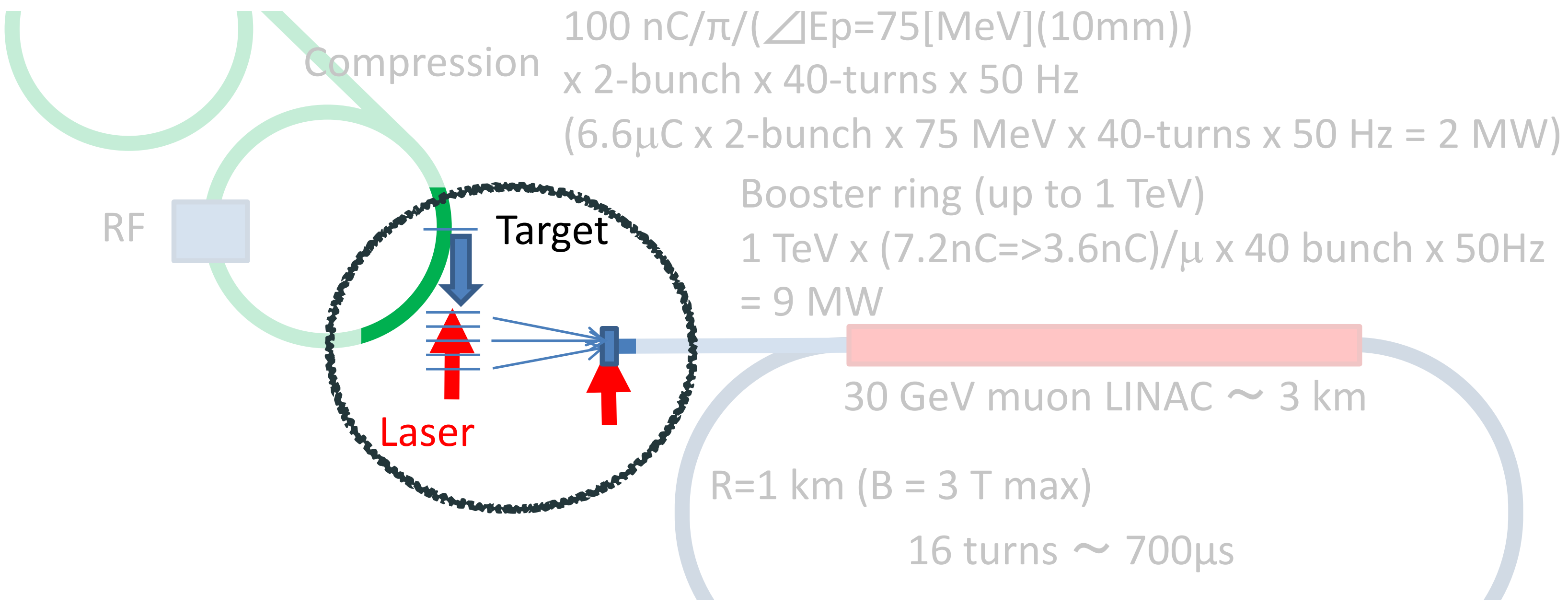
Typically, slepton w/ mass $m_{\tilde{l}} \lesssim 700\,\mathrm{GeV}$ is excluded.

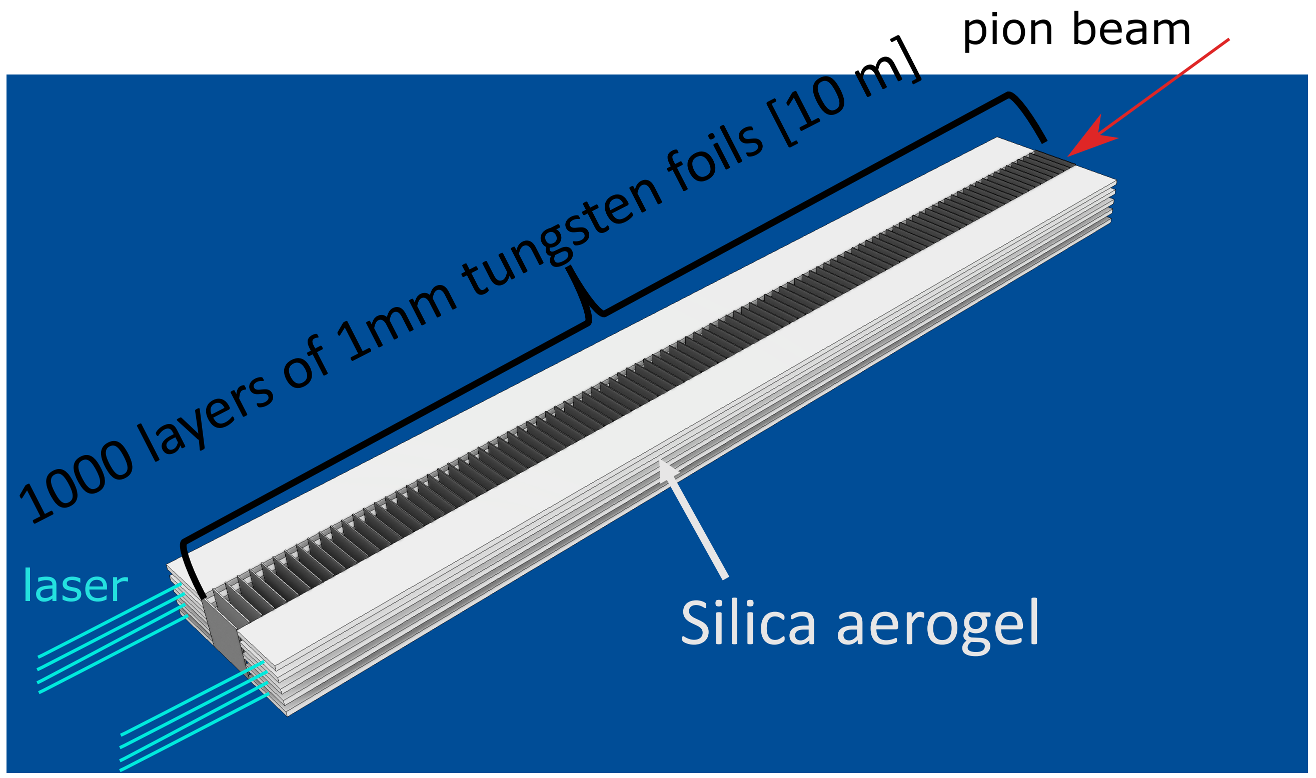
SUSY search and muon g-2

[ATL-PHYS-PUB-2022-013]



Ultra-cold muons





1. Pions are stopped at tungsten foils and decay into muons.

pion transportation to the target: 50%

2. Muons are transported into the aerogel target and form muoniums.

muonium formation: 52 %

3. Neutral muoniums become thermalized w/ $E_K \sim 25\,\mathrm{meV}$ and thermally diffused from the target. → ionized by laser

muonium emission: 60 % & decay loss: 60 %

laser ionization: 73%

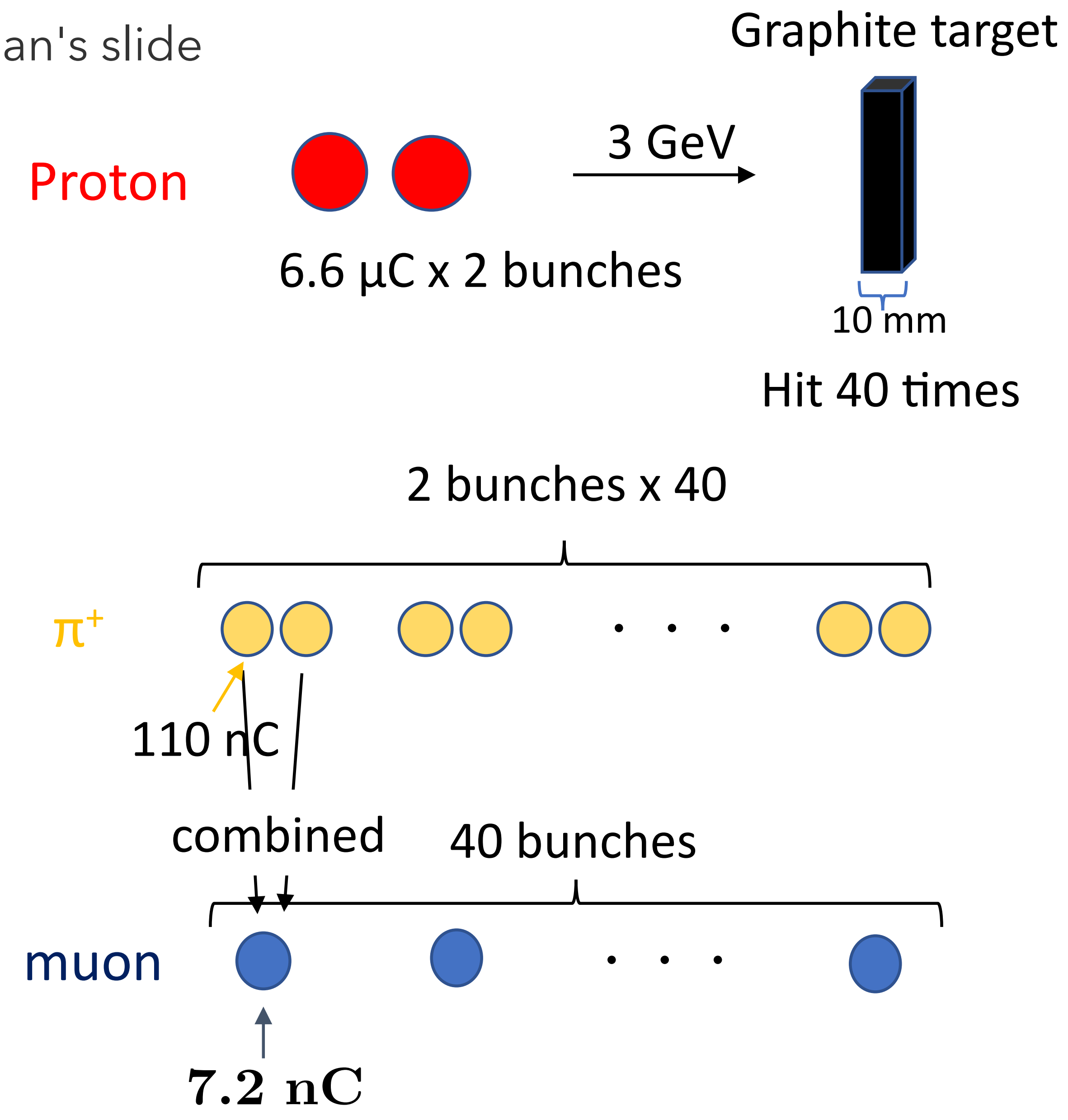
repeat step 2 and 3 twice: **1st target size ~ 10m, 2nd target ~ O(1) cm**

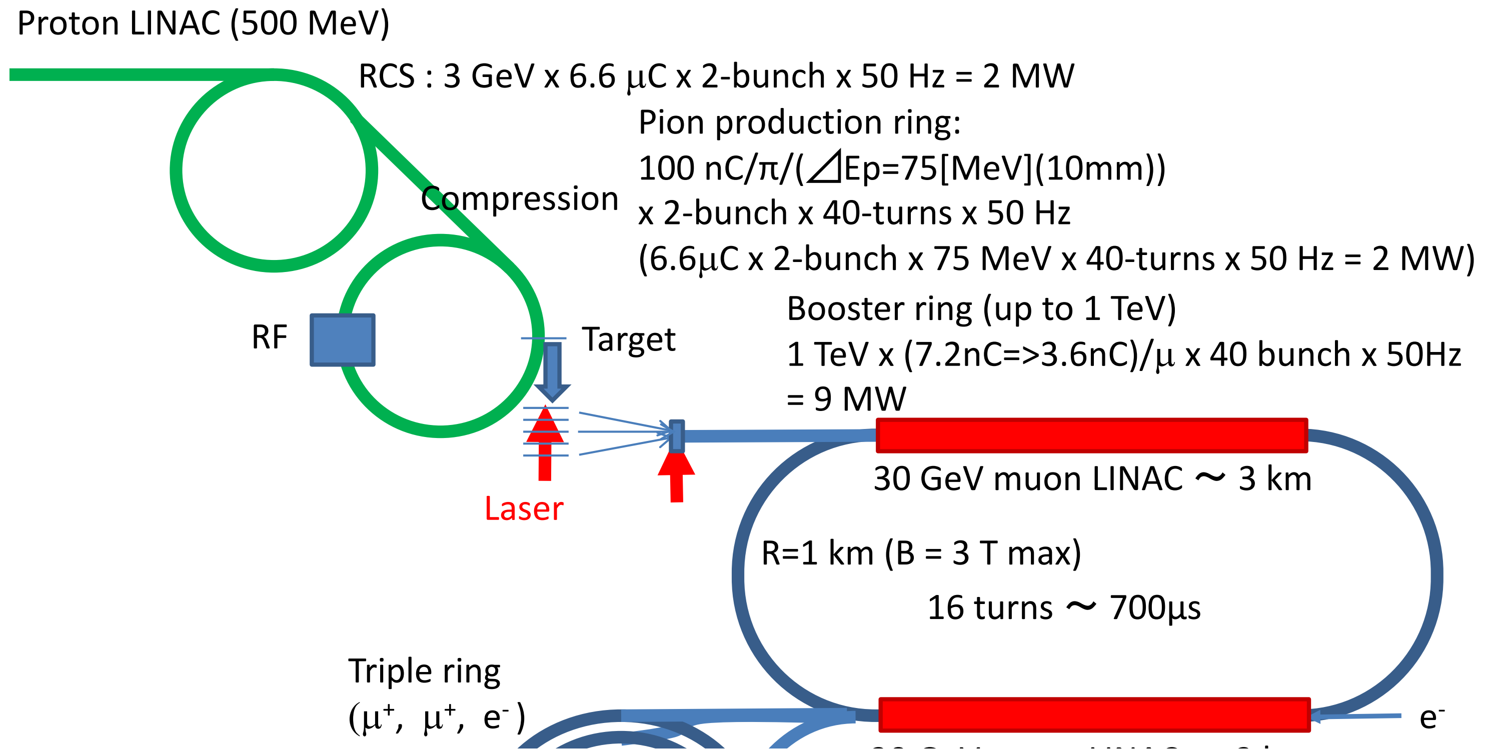
50% at second time because of a thin target

$$\therefore N_{\mu^+}/N_{\pi^+} \simeq 0.5 \times 0.52 \times 0.73 \times 0.6 \times 0.6 \times 0.5 \simeq 3.4\,\%$$

Estimation of N_{μ^+}

Fig taken from
Takaura-san's slide





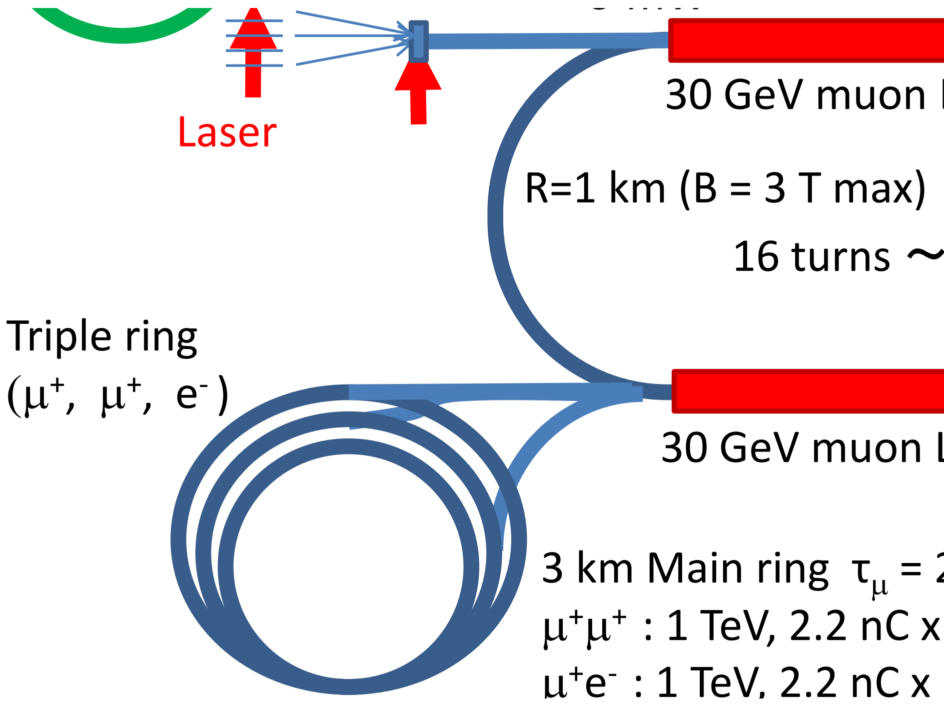
These operations are repeated at every 20 ms.

Initial # of ultra-cold muons $= 7.2\,\mathrm{nC}\times 40\,\mathrm{bunch}/(20\,\mathrm{ms}) = 9.0\times 10^{13}/\mathrm{s}$

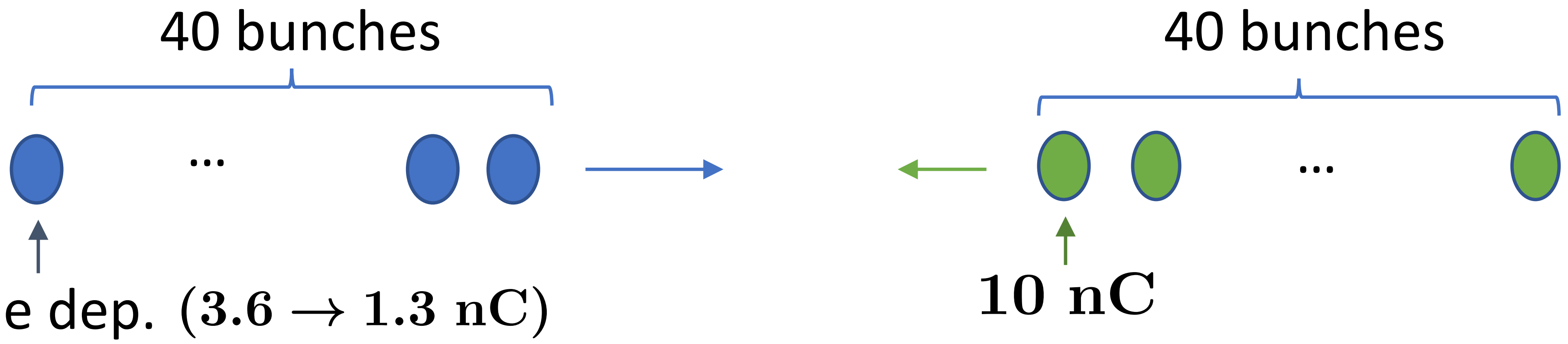
acceleration
→ # of ultra-cold muons $= 3.6\,\mathrm{nC}\times 40\,\mathrm{bunch}/(20\,\mathrm{ms}) = 4.5\times 10^{13}/\mathrm{s}$

Beam in main ring

Fig taken from
Takaura-san's slide



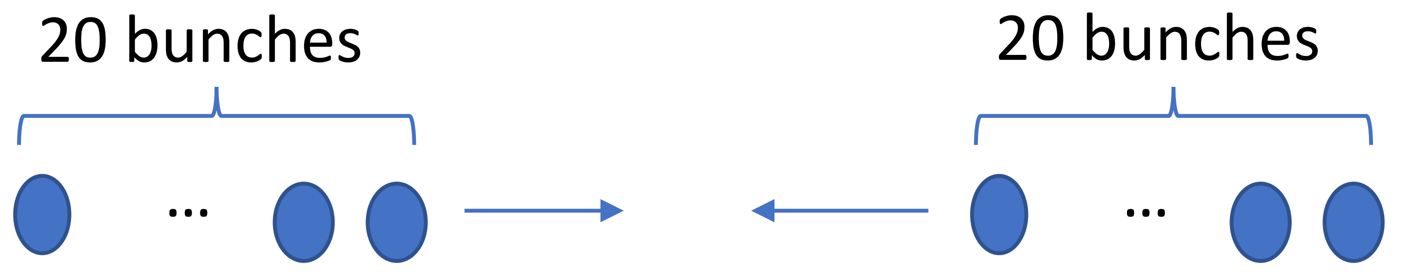
- μ^+e^- collider



Time dep. $(\mathbf{3.6 \rightarrow 1.3\ nC})$ $\mathbf{10\ nC}$

muons (1 TeV) electrons (30 GeV)

- $\mu^+\mu^+$ collider



muons (1 TeV) muons (1 TeV)

Luminosity

$$\mathscr{L} = \frac{N_{\text{beam1}} N_{\text{beam2}}}{4\pi \sigma_x \sigma_y} f_{\text{rep}}$$

- Collision frequency (per detector)

$$f_{\text{rep}}^{(\mu^+ e^-)} = 3.0 \times 10^8 \, \text{m s}^{-1} / (3 \, \text{km}) \times 40 = 4 \, \text{MHz}$$

speed of beam

circumference of ring

# of bunches

$$f_{\text{rep}}^{(\mu^+ \mu^+)} = 3.0 \times 10^8 \, \text{m s}^{-1} / (3 \, \text{km}) \times 20 = 2 \, \text{MHz}$$

$$\beta\gamma \sim 10^4$$

- Beam size

emittance: $\epsilon_x, \epsilon_y = \frac{4 \, \text{mm mrad}}{\beta\gamma}$

$$\sigma_i = \sqrt{\epsilon_i \beta_i}$$

beta function

$\beta_x = 30 \, \text{mm}$ $\sigma_x = 3.6 \, \mu\text{m}$

$\beta_y = 7 \, \text{mm}$ $\sigma_y = 1.7 \, \mu\text{m}$

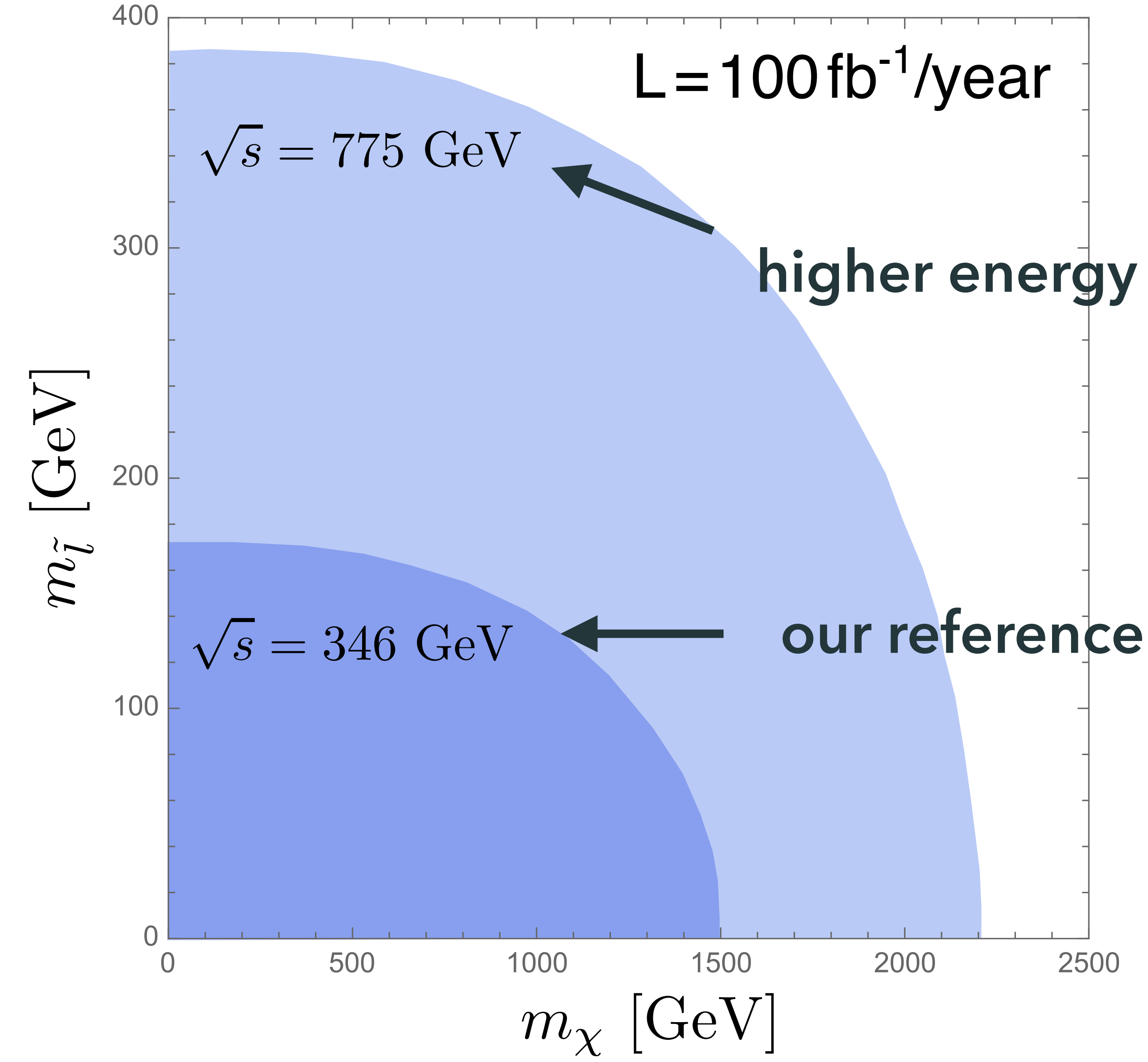
Thanks to ultra-cold muon, low emittance is realized!

Without cooling, the normalized emittance is $\sim 10^3 \pi \, \text{mm mrad}$

Slepton production at μTRISTAN

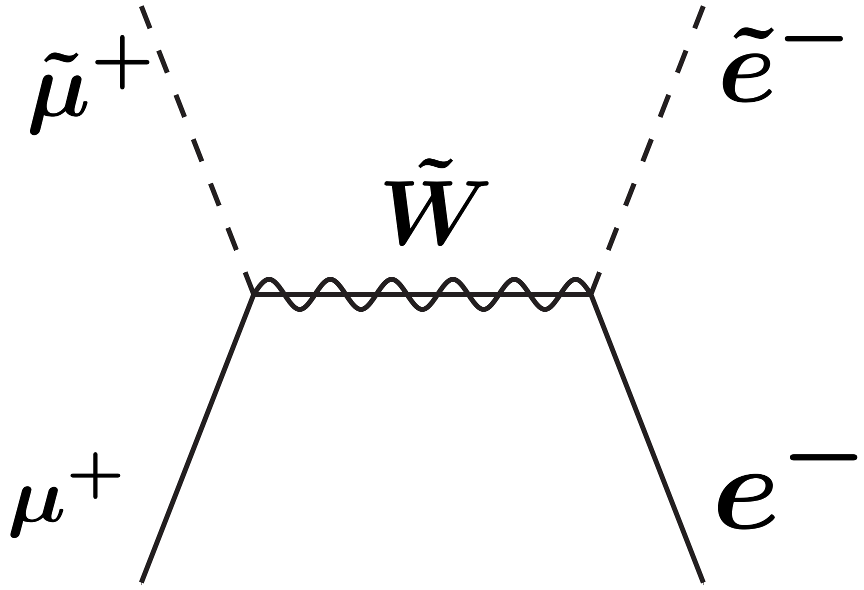
- $\mu^+ e^-$ collider

Mass parameter region where # of events exceeds 100.



higher energy option (large ring)

our reference energy



$m_{\tilde{\mu}} = m_{\tilde{e}}$ is assumed

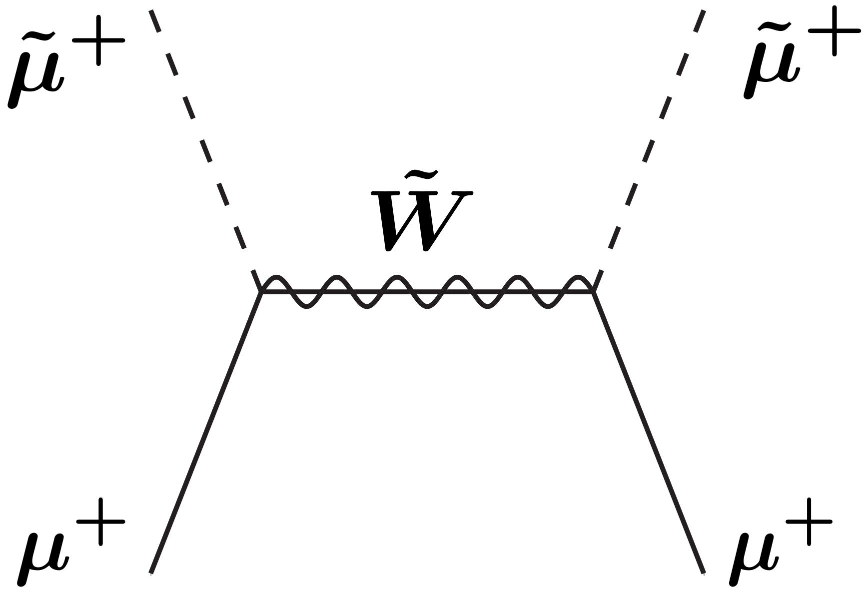
The mass reach is not very high, but we can cover different region from LHC.

Cross section

$$d\sigma = \frac{d\cos\theta}{32\pi}\frac{\beta}{s}|M_{\mathrm{RR}}|^2\frac{(1+P_{\mu 1})(1+P_{\mu 2})}{4}, \quad 0 \le \cos\theta \le 1,$$

$$M_{\mathrm{RR}} = -\frac{g_2^2}{2}\cdot\frac{4\sqrt{x}_A(1+2x_A-2x_3)}{(1+2x_A-2x_3)^2-\beta^2\cos^2\theta},$$

$$x_A = \frac{m_\chi^2}{s}, \quad x_3 = \frac{m_{\tilde{\mu}}^2}{s}, \quad \beta = \sqrt{1-4x_3}.$$



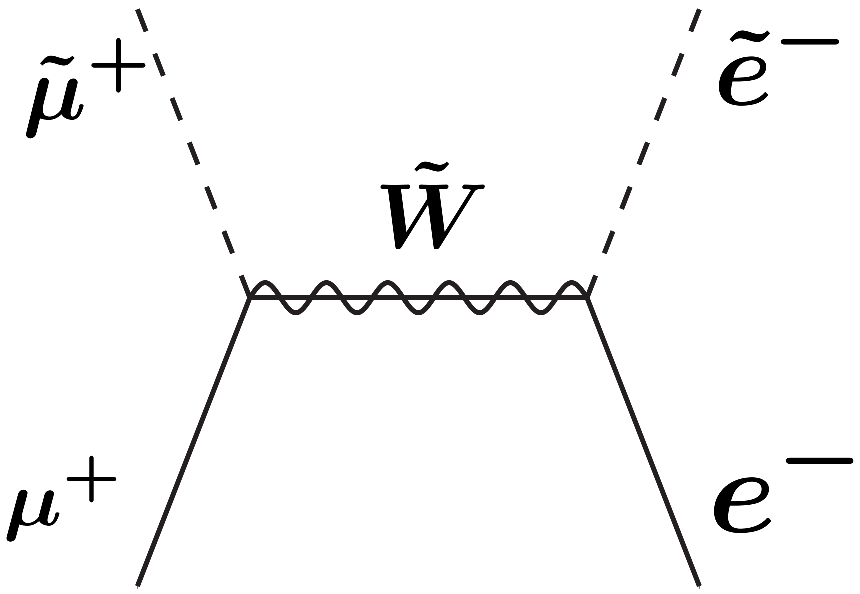
$$\sigma = \frac{g_2^4}{64\pi}\frac{1}{s}\left[\frac{\beta x_A}{x_A+(x_A-x_3)^2} + \frac{2x_A}{1+2x_A-2x_3}\log\frac{1+2x_A-2x_3+\beta}{1+2x_A-2x_3-\beta}\right] \times \frac{(1+P_{\mu 1})(1+P_{\mu 2})}{4}.$$

Cross section

$$d\sigma = \frac{d\cos\theta}{32\pi}\frac{(1+x_3-x_4)\beta}{s}|M_{\rm LR}|^2\frac{(1-P_{e^-})(1+P_{\mu^+})}{4}, \quad -1 \le \cos\theta \le 1,$$

$$M_{\rm LR} = -\frac{g_2^2}{2}\cdot\frac{(1+x_3-x_4)\beta\sin\theta}{1+2x_A-x_3-x_4-(1+x_3-x_4)\beta\cos\theta},$$

$$x_A = \frac{m_\chi^2}{s}, \quad x_3 = \frac{m_{\tilde{e}}^2}{s}, \quad x_4 = \frac{m_{\tilde{\mu}}^2}{s}, \quad \beta = \frac{\sqrt{1-2x_3-2x_4+(x_3-x_4)^2}}{1+x_3-x_4}.$$



Comparison with ILC

- **μTRISTAN: μ^+e^- collider**

$$\sqrt{s} = 346\,\text{GeV} \qquad \mathscr{L}_{\mu^+e^-} = 4.6 \times 10^{33}\,\text{cm}^{-2}\text{s}^{-1}$$

main ring: 3km circumference

booster ring: 2km LINAC x2 + R=1km arc

construction cost: 5000 billion yen? (with large uncertainty)

(1 dollar ~120 yen)

- **ILC: e^+e^- collider**

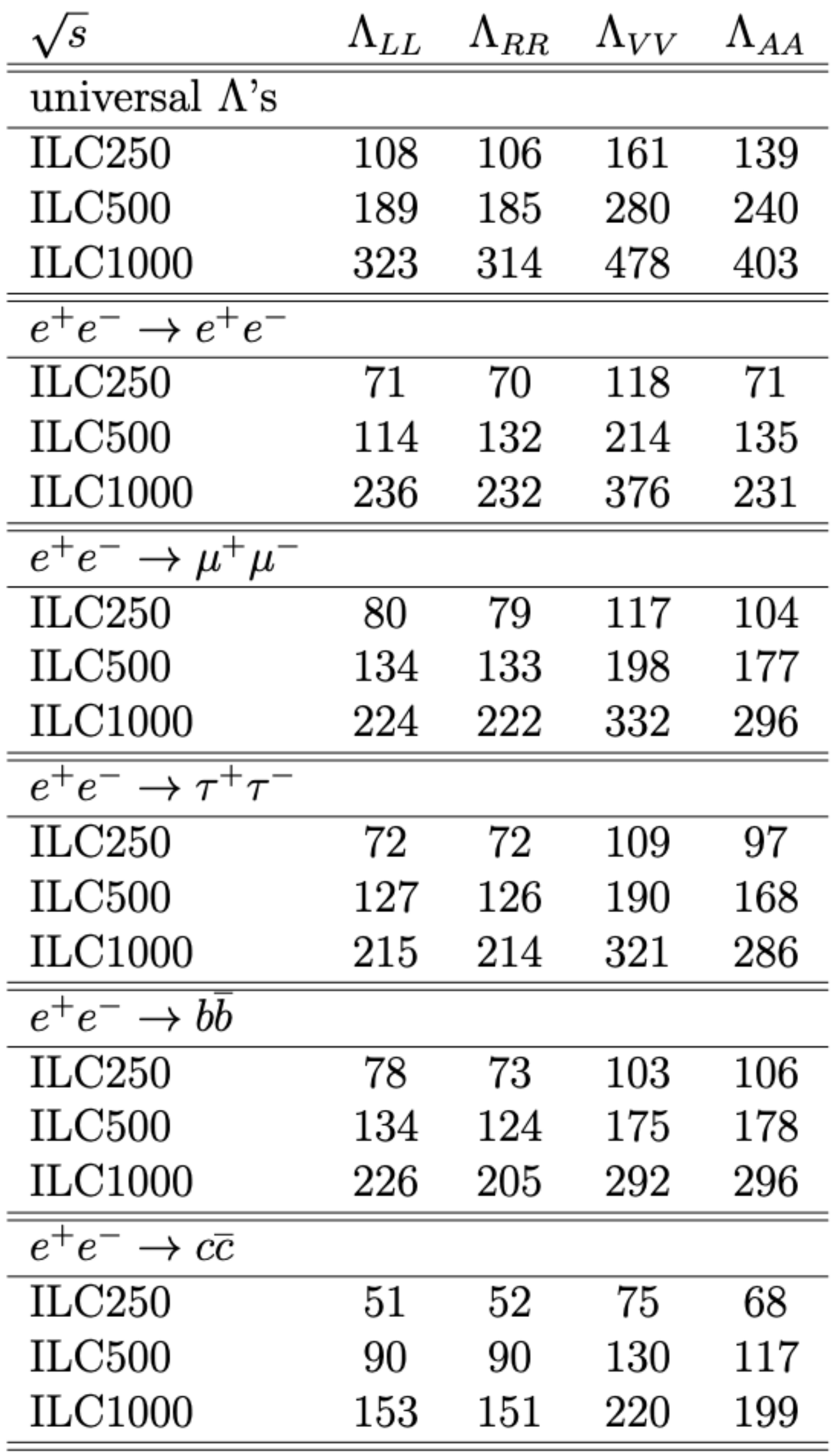
$$\sqrt{s} = 250\,\text{GeV} \qquad \mathscr{L} = 1.35 \times 10^{34}\,\text{cm}^{-2}\text{s}^{-1}$$

length: 20km

construction cost: 7300-8000 billion yen

Four fermi measurement at ILC

[1908.11299]



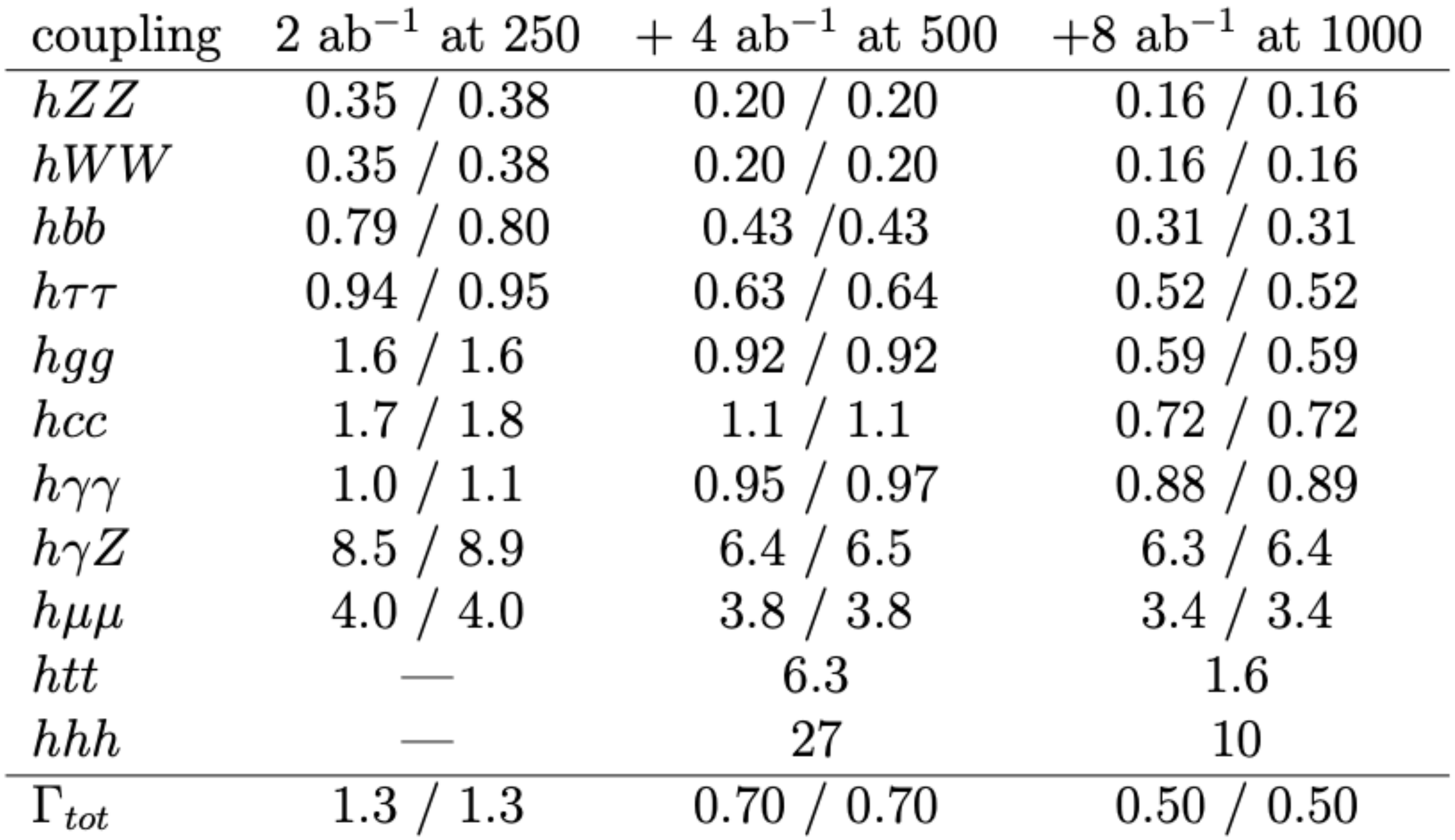
$\sqrt{s}$	Λ_{LL}	Λ_{RR}	Λ_{VV}	Λ_{AA}
universal Λ's				
ILC250	108	106	161	139
ILC500	189	185	280	240
ILC1000	323	314	478	403
$e^+e^- \to e^+e^-$				
ILC250	71	70	118	71
ILC500	114	132	214	135
ILC1000	236	232	376	231
$e^+e^- \to \mu^+\mu^-$				
ILC250	80	79	117	104
ILC500	134	133	198	177
ILC1000	224	222	332	296
$e^+e^- \to \tau^+\tau^-$				
ILC250	72	72	109	97
ILC500	127	126	190	168
ILC1000	215	214	321	286
$e^+e^- \to b\bar{b}$				
ILC250	78	73	103	106
ILC500	134	124	175	178
ILC1000	226	205	292	296
$e^+e^- \to c\bar{c}$				
ILC250	51	52	75	68
ILC500	90	90	130	117
ILC1000	153	151	220	199

[TeV]

Note: They use a different convention: $\frac{4\pi}{\Lambda^2}(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$

Higgs coupling measurement at ILC

[1908.11299]



coupling	2 ab^{-1} at 250	+ 4 ab^{-1} at 500	+8 ab^{-1} at 1000
hZZ	0.35 / 0.38	0.20 / 0.20	0.16 / 0.16
hWW	0.35 / 0.38	0.20 / 0.20	0.16 / 0.16
hbb	0.79 / 0.80	0.43 /0.43	0.31 / 0.31
$h\tau\tau$	0.94 / 0.95	0.63 / 0.64	0.52 / 0.52
hgg	1.6 / 1.6	0.92 / 0.92	0.59 / 0.59
hcc	1.7 / 1.8	1.1 / 1.1	0.72 / 0.72
$h\gamma\gamma$	1.0 / 1.1	0.95 / 0.97	0.88 / 0.89
$h\gamma Z$	8.5 / 8.9	6.4 / 6.5	6.3 / 6.4
$h\mu\mu$	4.0 / 4.0	3.8 / 3.8	3.4 / 3.4
htt	—	6.3	1.6
hhh	—	27	10
Γ_{tot}	1.3 / 1.3	0.70 / 0.70	0.50 / 0.50

[%]

Magnet

Dipole magnet with the magnetic field of 10T

→ Main ring 3km for $(E_{\mu^+},\ E_{e^-}) = (1\,\mathrm{TeV},\ 30\,\mathrm{GeV})$

9km for $(E_{\mu^+},\ E_{e^-}) = (3\,\mathrm{TeV},\ 50\,\mathrm{GeV})$

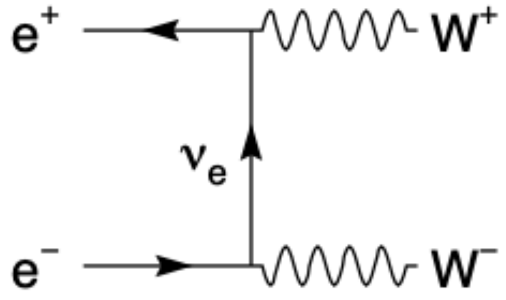
cf. High-luminosity LHC: 11T

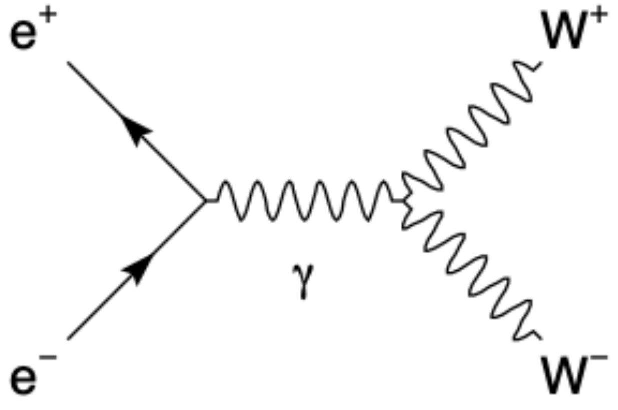
If dipole magnet with the magnetic field of 16T is possible,

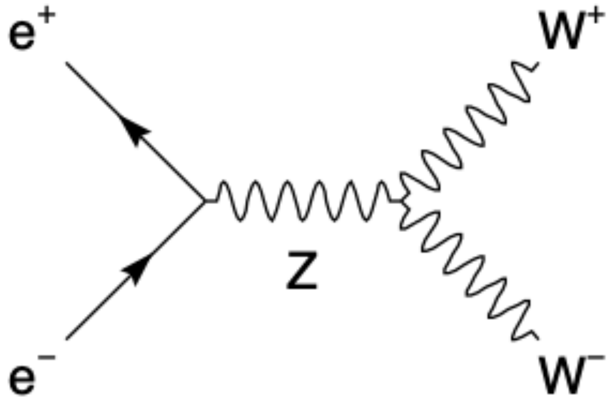
→ Main ring 6km for $(E_{\mu^+},\ E_{e^-}) = (3\,\mathrm{TeV},\ 50\,\mathrm{GeV})$

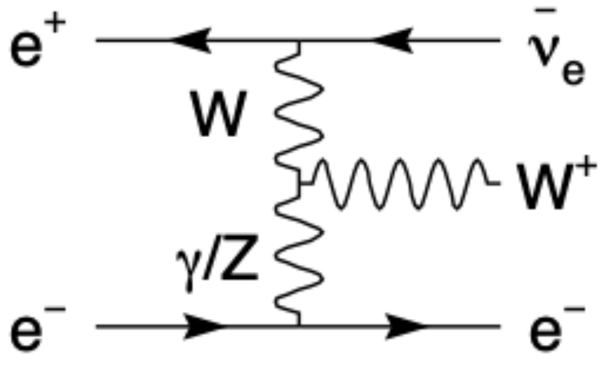
W boson mass

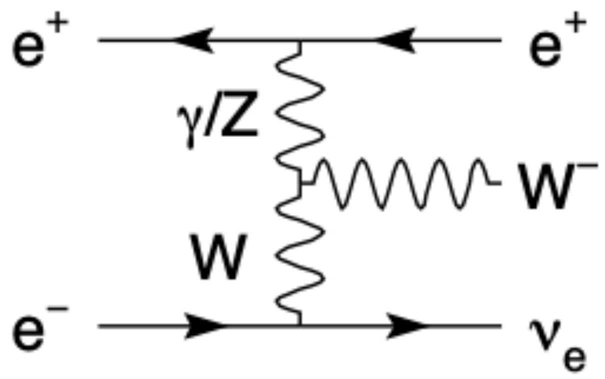
- ILC (and LEP):





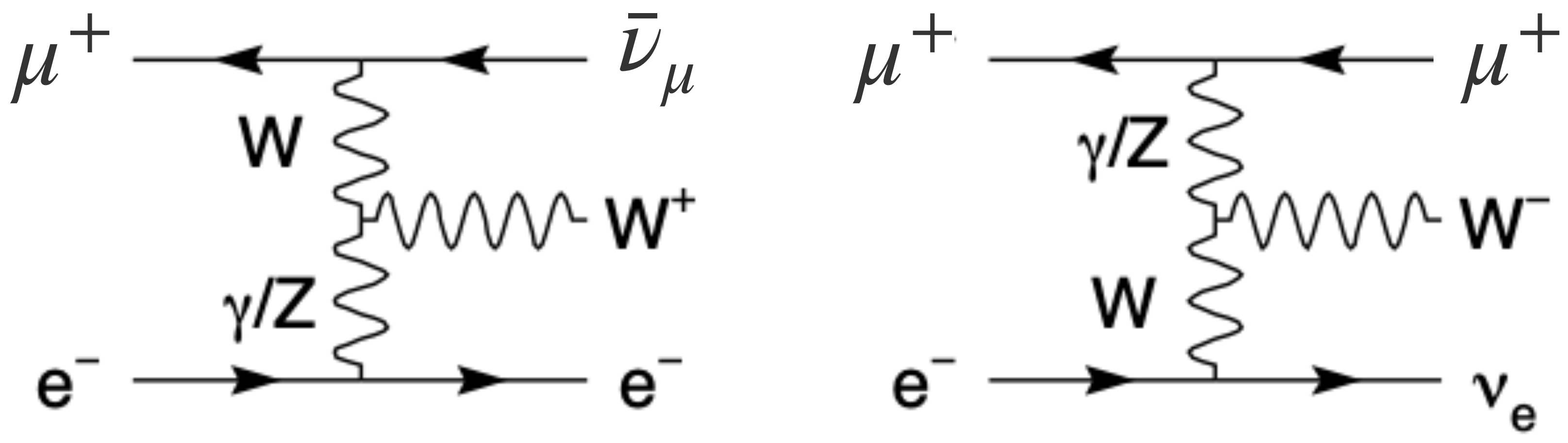






W boson mass

- μTRISTAN



& hadronic decay $W \to q\bar{q}$

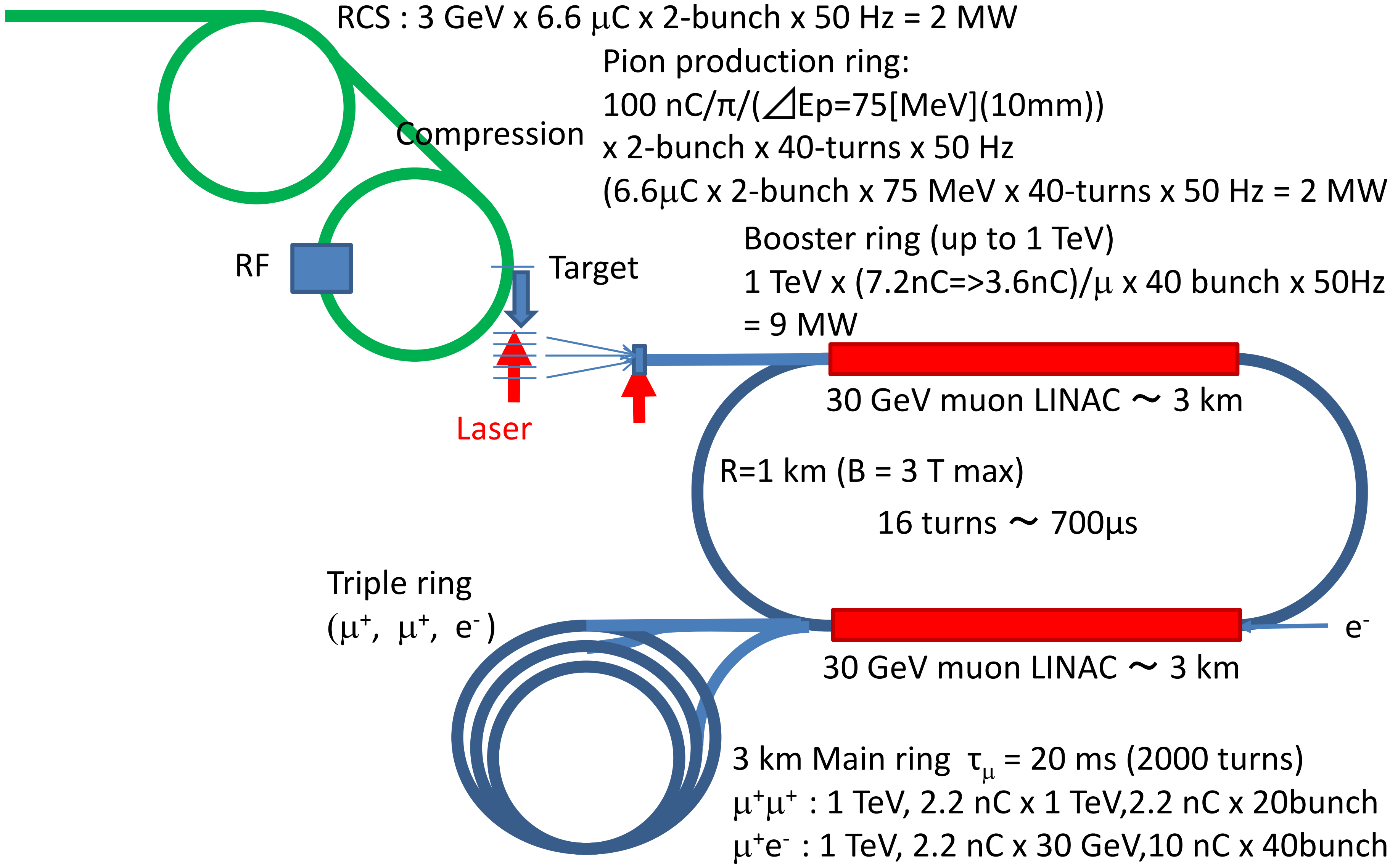
In ILC 250 study, $\Delta M_W \simeq 3.7\,\mathrm{MeV}$, which is dominated by systematic uncertainty (particularly hodronization). [1310.6708]

We expect a similar precision at μTRISTAN.

Cf.) CDF II result: $M_W = 80{,}433.5 \pm 9.4\,\mathrm{MeV}$

Collider design

Design of μTRISTAN



Proton LINAC (500 MeV)

RCS : 3 GeV x 6.6 μC x 2-bunch x 50 Hz = 2 MW

Pion production ring:
100 nC/π/(⊿Ep=75[MeV](10mm))
x 2-bunch x 40-turns x 50 Hz
(6.6μC x 2-bunch x 75 MeV x 40-turns x 50 Hz = 2 MW)

Compression

RF

Target

Laser

Booster ring (up to 1 TeV)
1 TeV x (7.2nC=>3.6nC)/μ x 40 bunch x 50Hz
= 9 MW

30 GeV muon LINAC ∼ 3 km

R=1 km (B = 3 T max)

16 turns ∼ 700μs

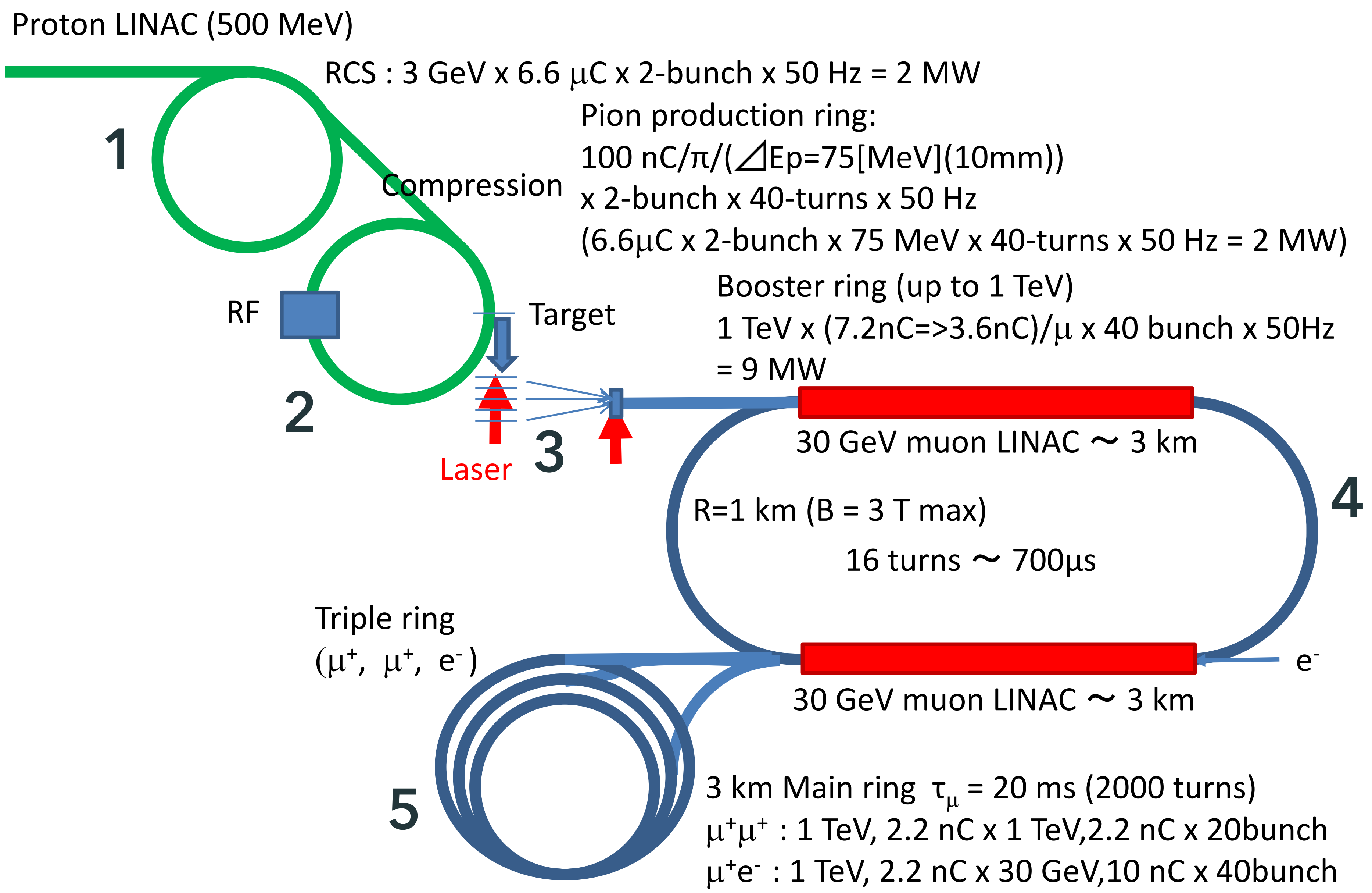
Triple ring
(μ^+, μ^+, e^-)

e^-

30 GeV muon LINAC ∼ 3 km

3 km Main ring τ_μ = 20 ms (2000 turns)
$\mu^+\mu^+$: 1 TeV, 2.2 nC x 1 TeV,2.2 nC x 20bunch
μ^+e^- : 1 TeV, 2.2 nC x 30 GeV,10 nC x 40bunch

Design of μTRISTAN



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1

Compression

RF

2

Target

Laser

3

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Triple ring
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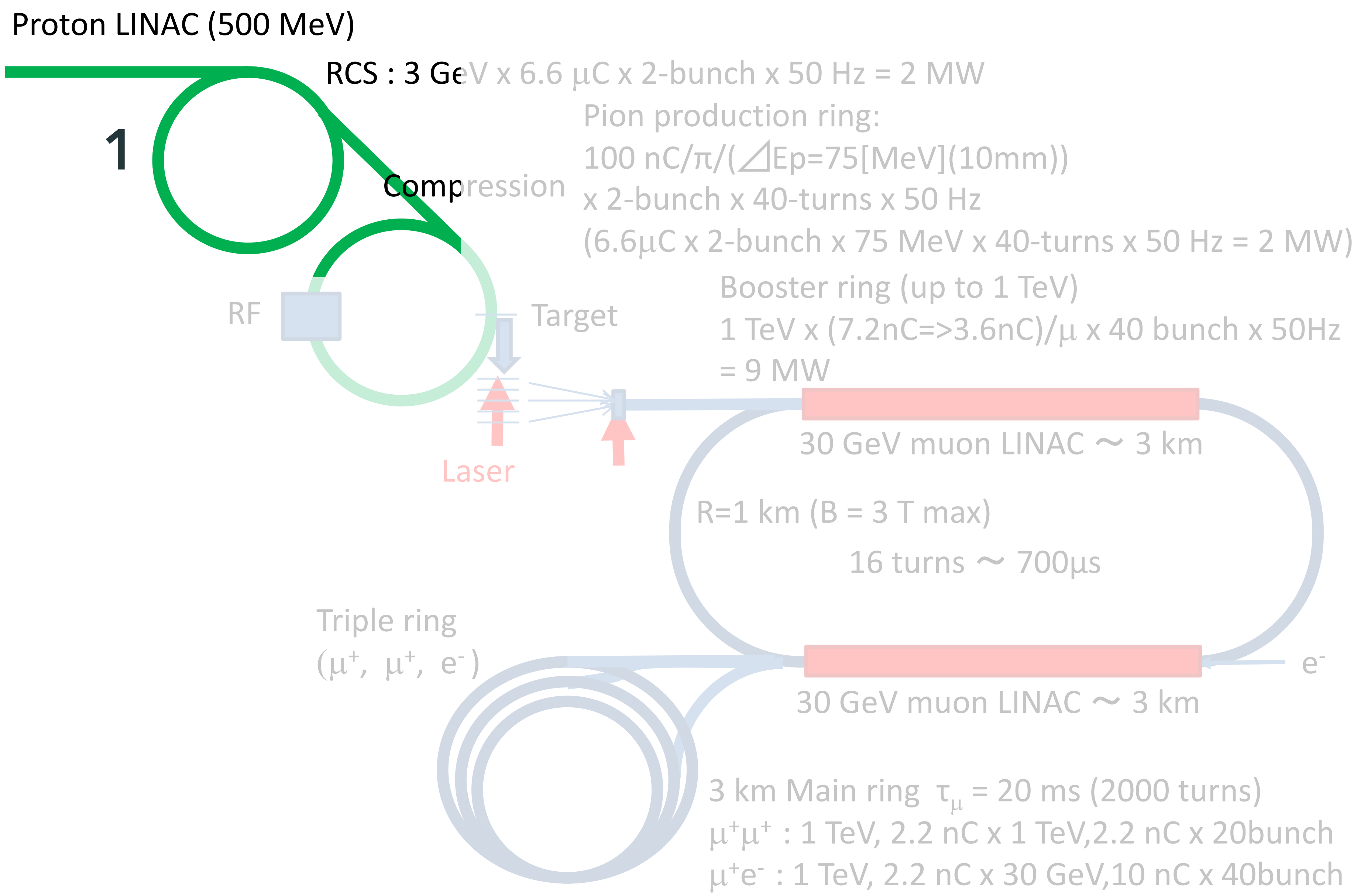
Design of μTRISTAN

Proton acceleration (proton LINAC & RCS)

$p(3\,\mathrm{GeV})$



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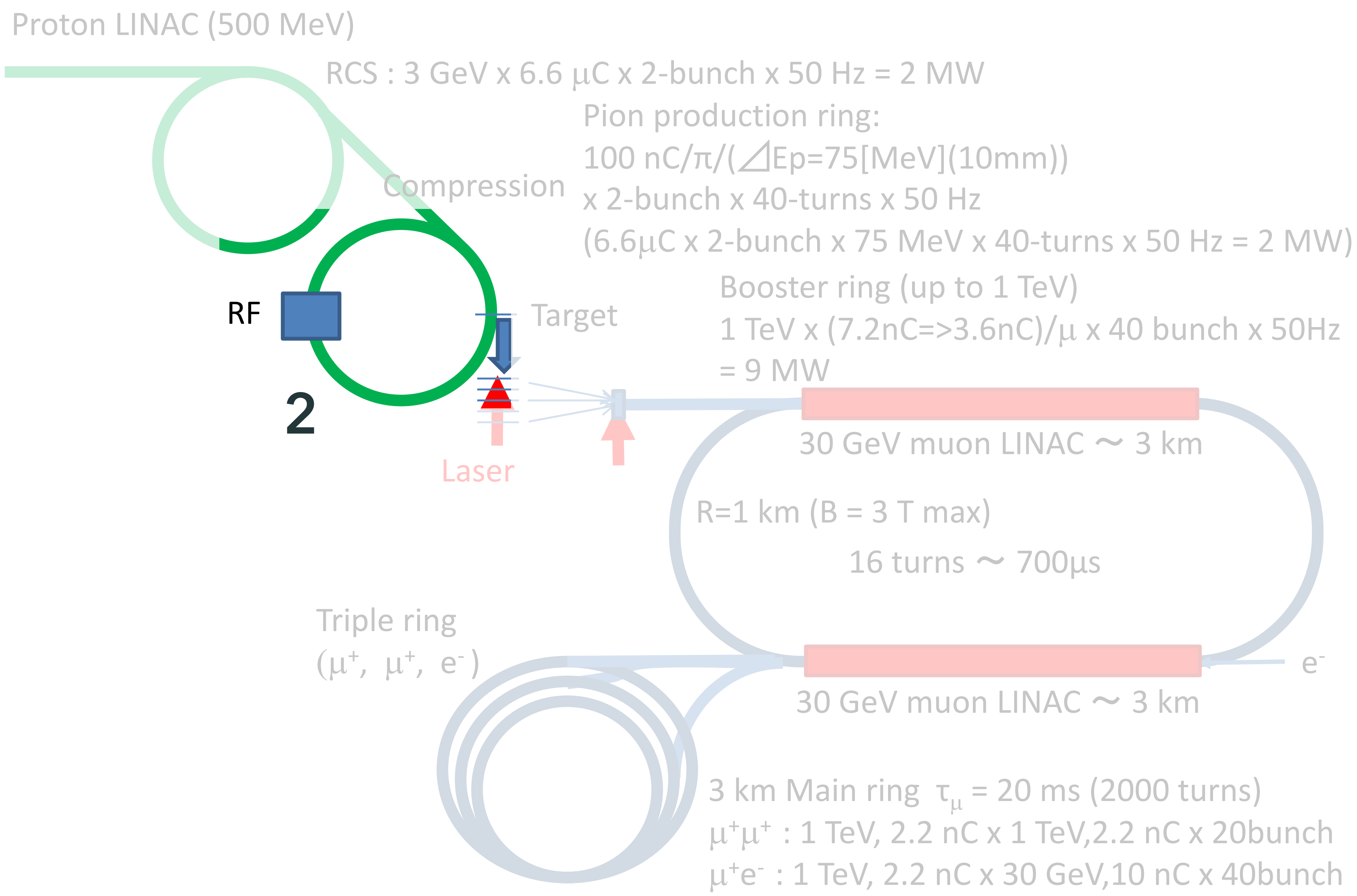
Design of μTRISTAN

Proton acceleration (proton LINAC & RCS) ⟶ Pion production (pion production ring)

$$p(3\,\mathrm{GeV})$$

$$p(3\,\mathrm{GeV}) + C \to \pi^+ + X$$



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Design of μTRISTAN

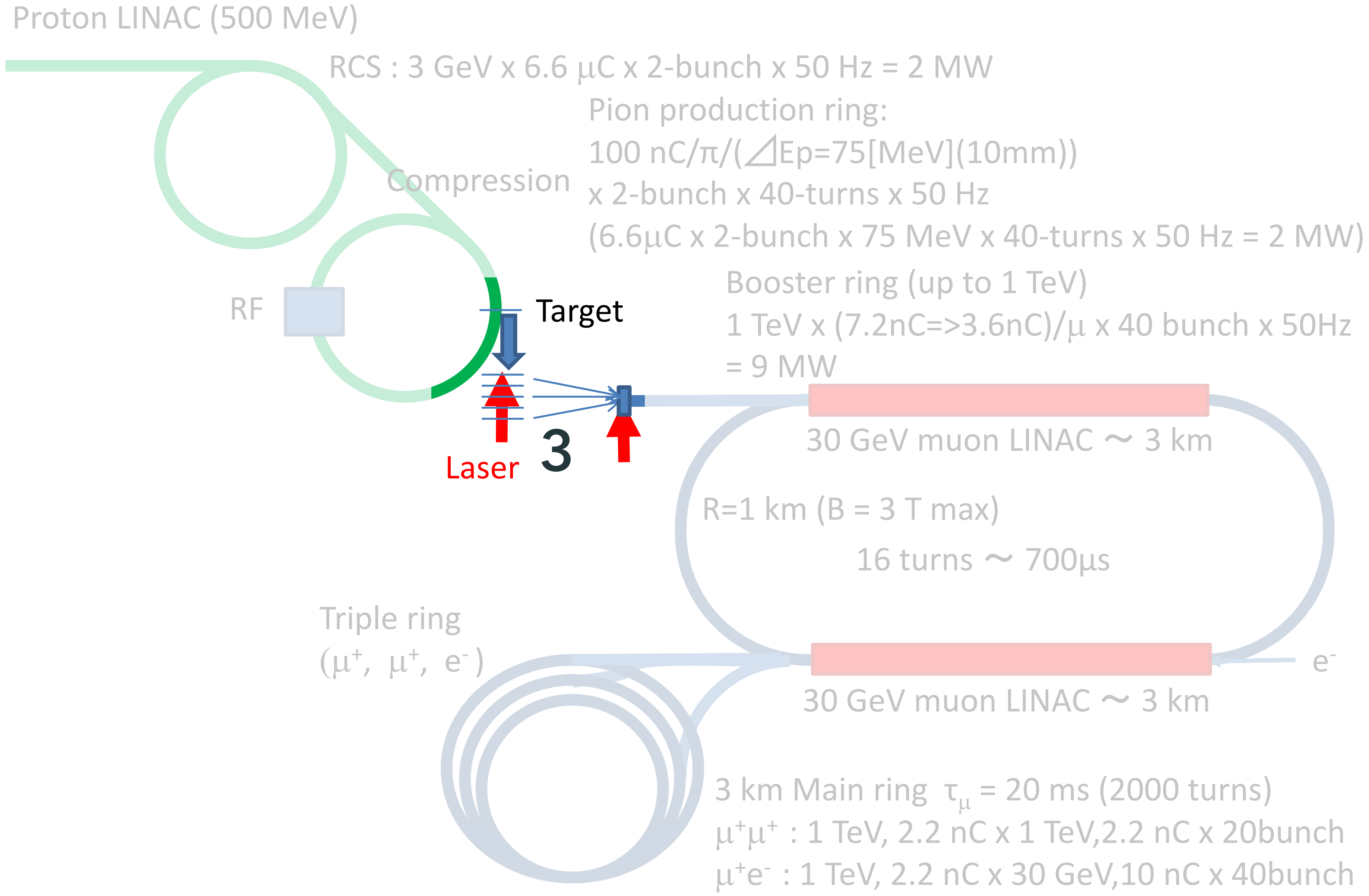
Proton acceleration (proton LINAC & RCS) ⟶ Pion production (pion production ring)

$p(3\,\mathrm{GeV})$ $p(3\,\mathrm{GeV}) + C \to \pi^+ + X$

⟶ Ultra-cold muon production (explained later)

$\pi^+ \to \mu^+ + \nu_\mu$ ⟶ Muonium ($\mu^+ e^-$) formation in target silica aerogel ⟶ Ultra-cold muon

ionized by laser



Design of μTRISTAN

Proton acceleration (proton LINAC & RCS) → Pion production (pion production ring)

$p(3\,\mathrm{GeV})$ $\qquad$ $p(3\,\mathrm{GeV}) + C \to \pi^+ + X$

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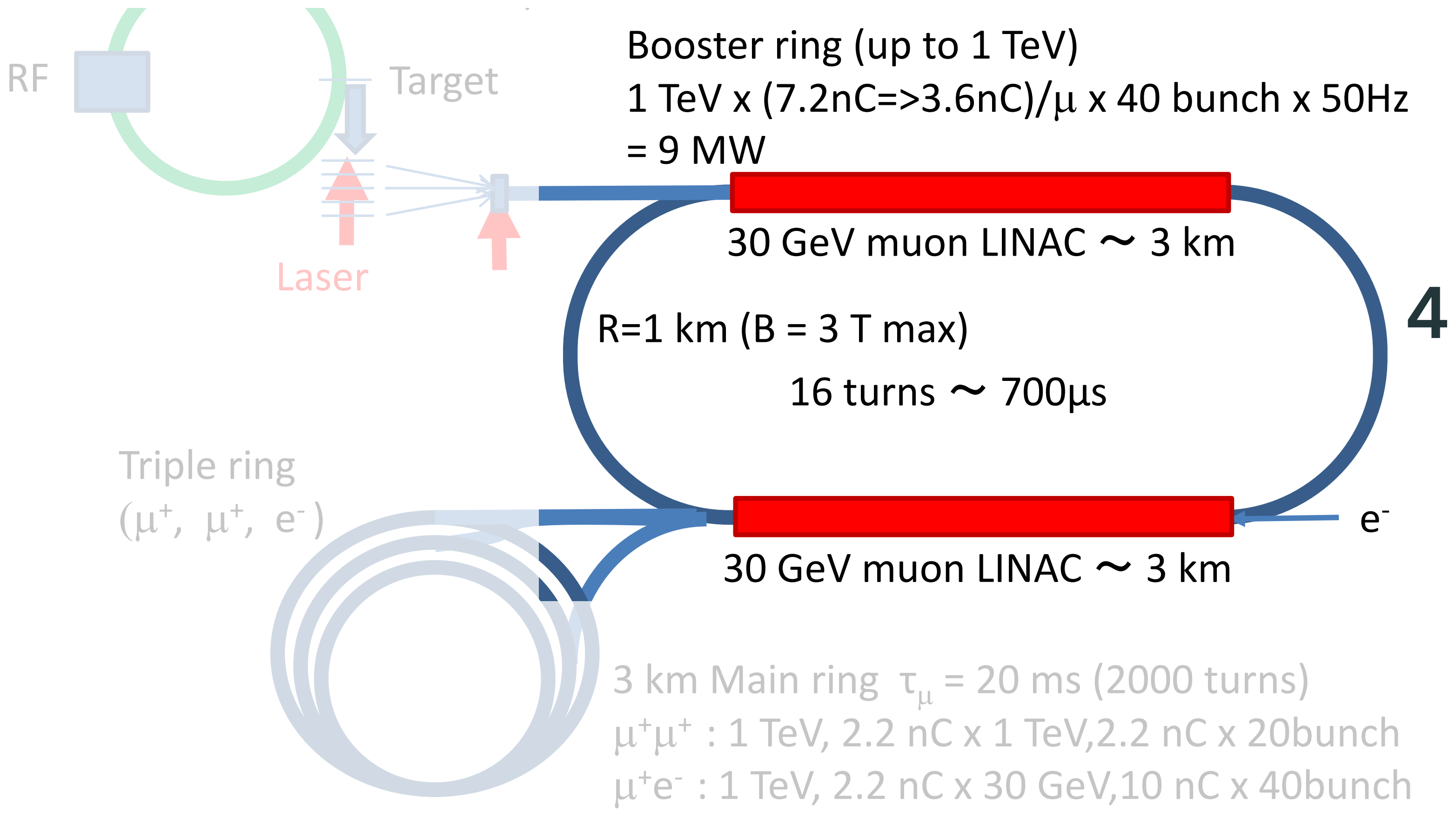
ionized by laser

→ Muon acceleration (booster ring) to 1TeV & electron acceleration (LINAC part)



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Design of μTRISTAN

Proton acceleration (proton LINAC & RCS) ⟶ Pion production (pion production ring)

$p(3\,\mathrm{GeV})$ $p(3\,\mathrm{GeV}) + C \rightarrow \pi^+ + X$

⟶ Ultra-cold muon production (explained later)

$\pi^+ \rightarrow \mu^+ + \nu_\mu$ ⟶ Muonium ($\mu^+ e^-$) formation in target silica aerogel ⟶ Ultra-cold muon **ionized by laser**

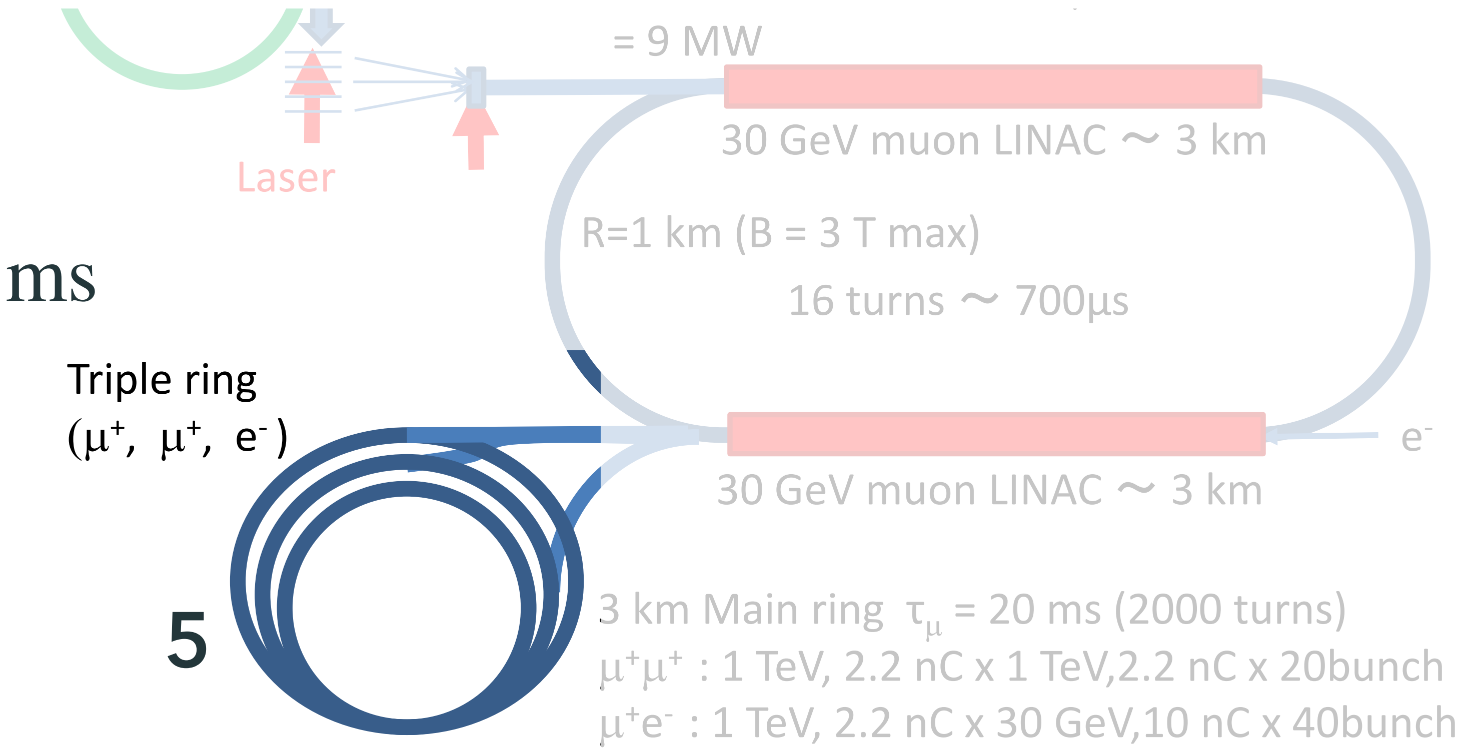
⟶ Muon acceleration (booster ring) to 1TeV & electron acceleration (LINAC part)

⟶ Collide (main ring) : $[\mu^+\,(1\,\mathrm{TeV}),\ e^-\,(30\,\mathrm{GeV})]$ or $[\mu^+\,(1\,\mathrm{TeV}),\ \mu^+\,(1\,\mathrm{TeV})]$

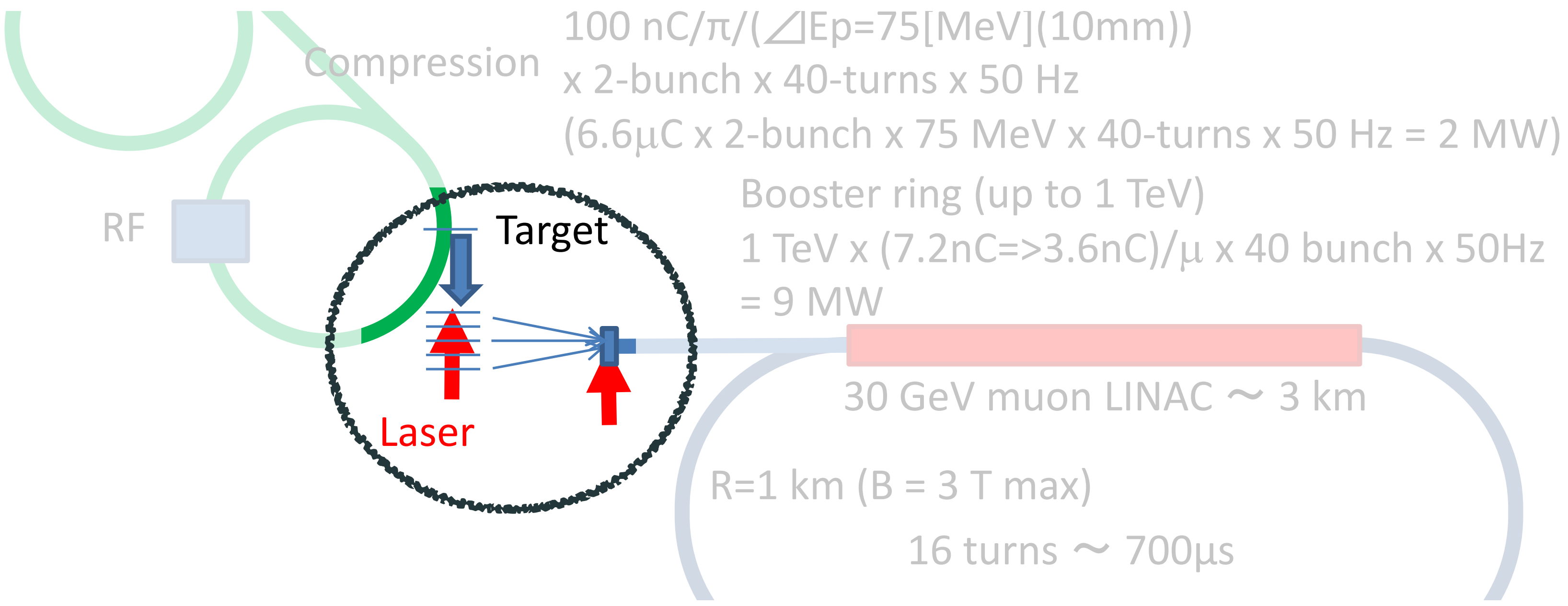
$\tau_{\mu(1\mathrm{TeV})} = 20\,\mathrm{ms}$

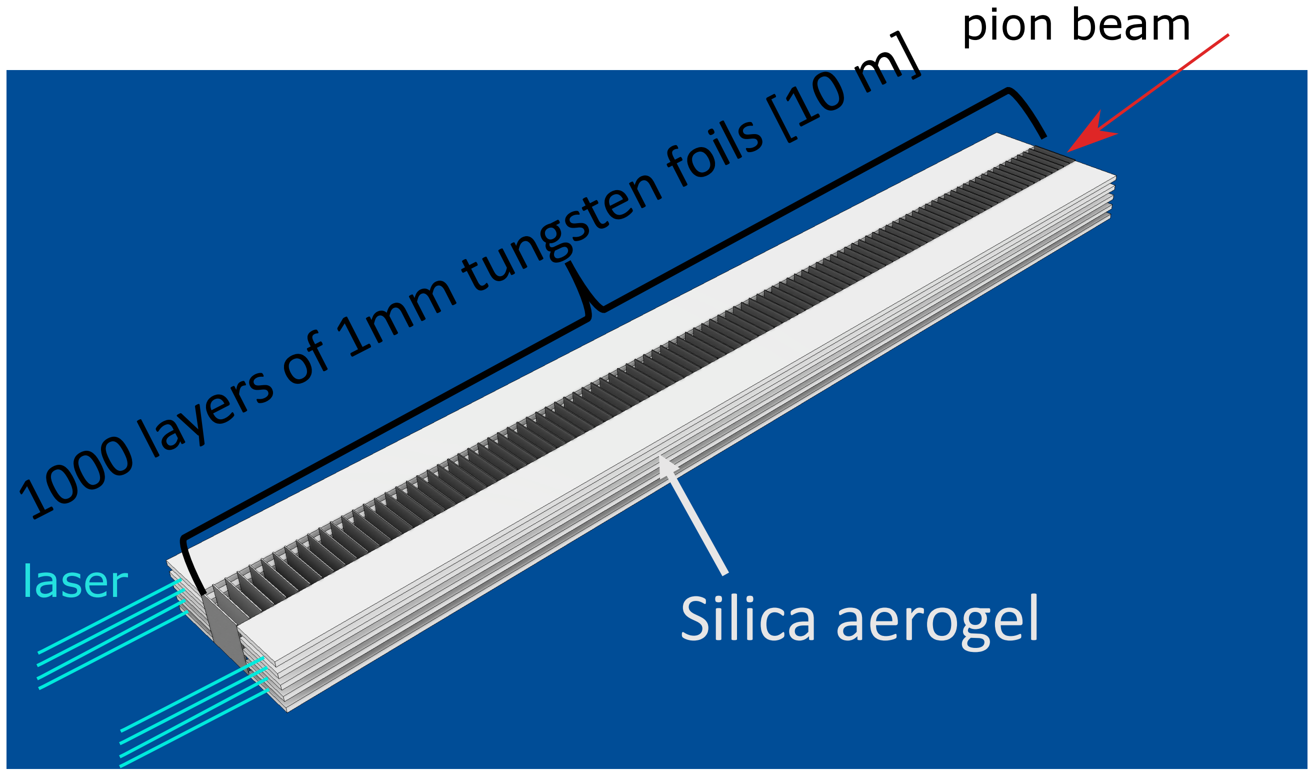
New beam injected every $20\,\mathrm{ms}$

Ring circumference 3 km



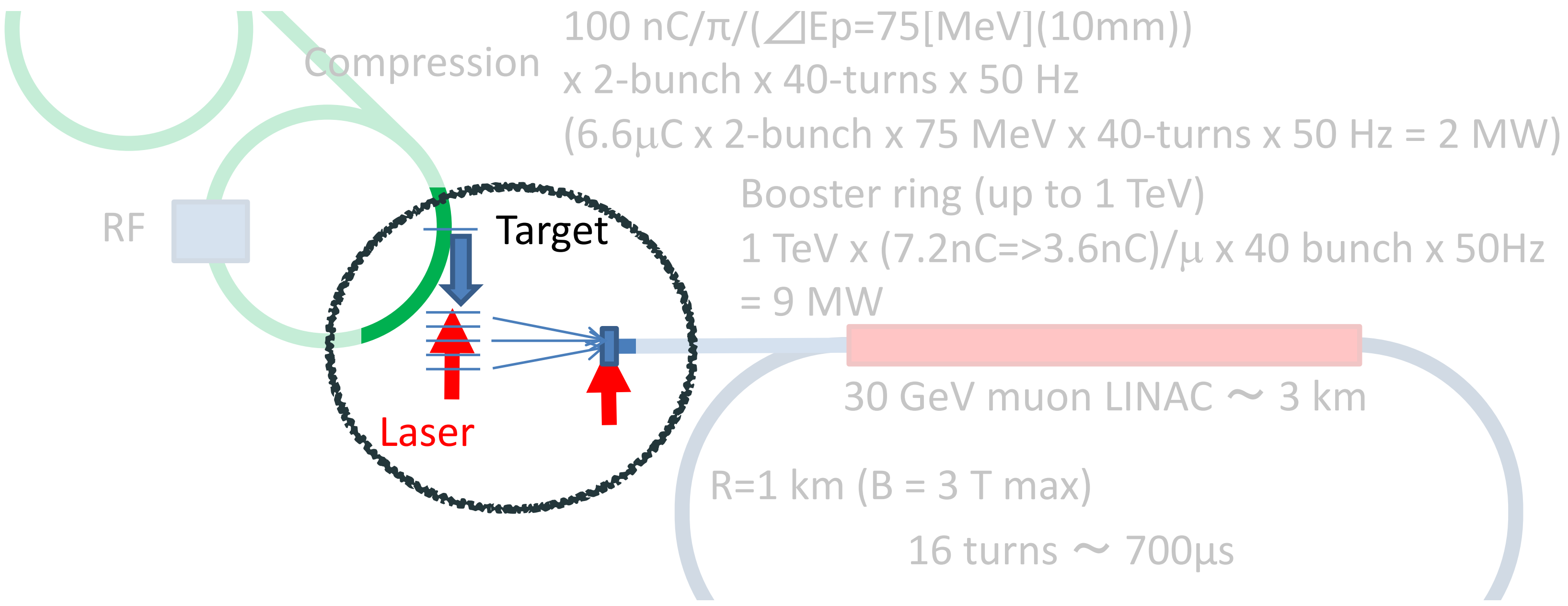
Ultra-cold muons

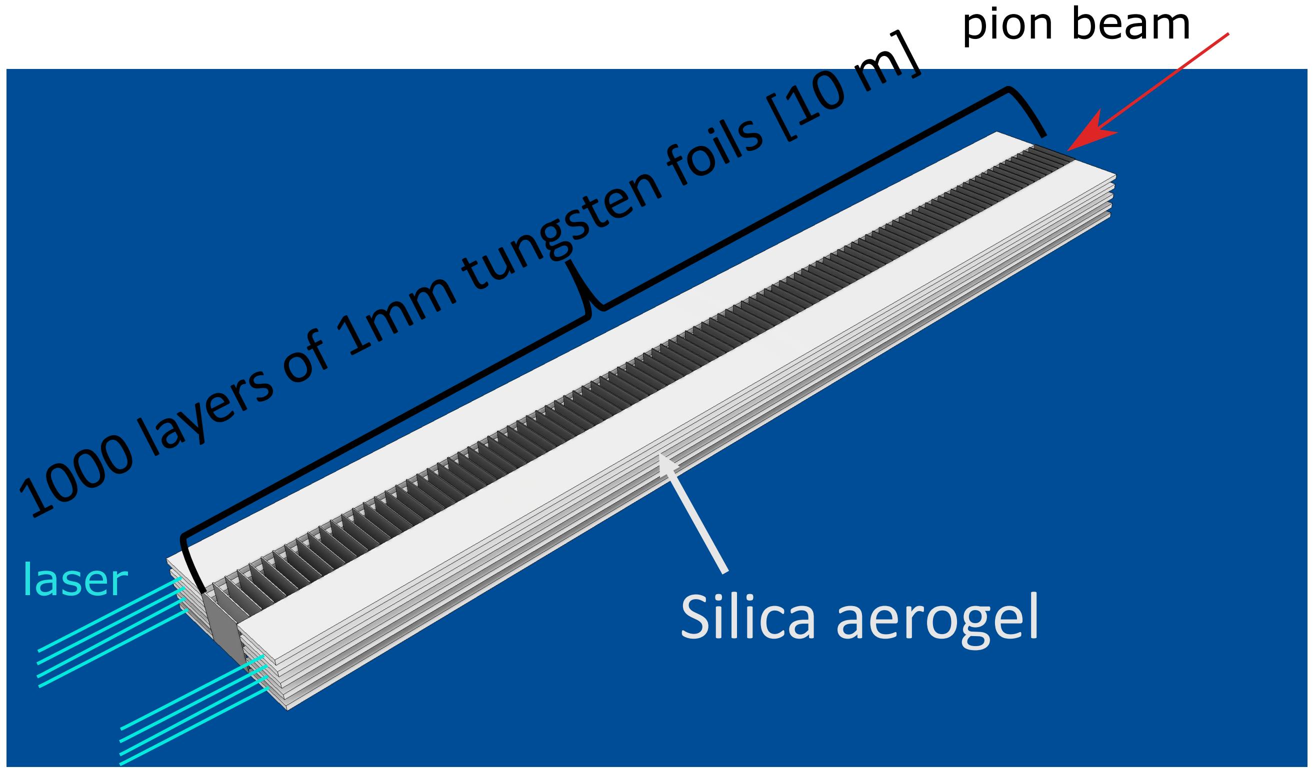




1. Pions are stopped at tungsten foils and decay into muons.
2. Muons are transported into the aerogel target and form muoniums.
3. Neutral muoniums become thermalized w/ $E_K \sim 25\,\mathrm{meV}$ and thermally diffused from the target. → ionized by laser (Lyman-α)

Ultra-cold muons





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repeat step 2 and 3 twice: **1st target size ~ 10m, 2nd target ~ O(1) cm**

→ **obtain ultra-cold muons, whose # is** $9.0 \times 10^{13}/\mathrm{sec}$ **(or** $14\,\mu\mathrm{C}/\mathrm{sec}$ **)**

Luminosity

$$\mathscr{L} = \frac{N_{\text{beam1}} N_{\text{beam2}}}{4\pi \sigma_x \sigma_y} f_{\text{rep}}$$

$$f_{\text{rep}}^{(\mu^+ e^-)} = 3.0 \times 10^8 \,\text{m}\,\text{s}^{-1}/(3\,\text{km}) \times 40 = 4\,\text{MHz}$$

$$f_{\text{rep}}^{(\mu^+ \mu^+)} = 3.0 \times 10^8 \,\text{m}\,\text{s}^{-1}/(3\,\text{km}) \times 20 = 2\,\text{MHz}$$

$$\sigma_x = 3.6\,\mu\text{m} \qquad \sigma_y = 1.7\,\mu\text{m}$$

$N_{e^-} = 10\,\text{nC}$ per bunch $\qquad N_{\mu^+} = 3.6\,\text{nC} \to 1.3\,\text{nC}$ per bunch due to decay

Our estimate:

(10 years running w/ 70 % duty factor)

$\mathscr{L}_{\mu^+ e^-} = 4.6 \times 10^{33}\,\text{cm}^{-2}\text{s}^{-1}$ ➡ $\int dt\, \mathscr{L}_{\mu^+ e^-} \simeq 1.0\,\text{ab}^{-1}$

$\mathscr{L}_{\mu^+ \mu^+} = 5.7 \times 10^{32}\,\text{cm}^{-2}\text{s}^{-1}$ ➡ $\int dt\, \mathscr{L}_{\mu^+ \mu^+} \simeq 130\,\text{fb}^{-1}$