# Neutrino Physics: Part 1

## The (non)Standard Model Particle

DESY Summer School 2024 - Dr. S. Blot

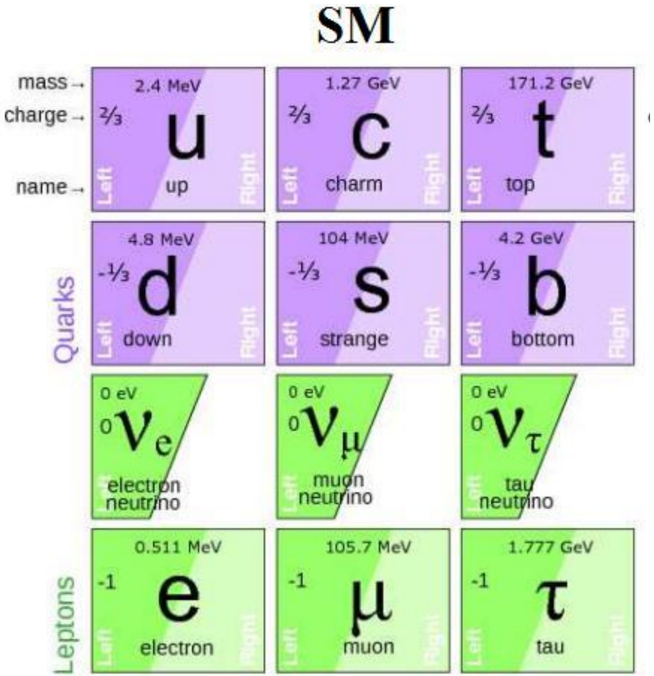
# Overview

Part 1:

- Introduction to neutrinos
- Neutrino cross sections
- Sources of neutrinos
- Massive neutrinos and oscillations

Part 2:

- Overview of neutrino detection techniques
- Review current landscape and key measurements
- Open questions and future prospects



# Let’s go back in time…. 1920s

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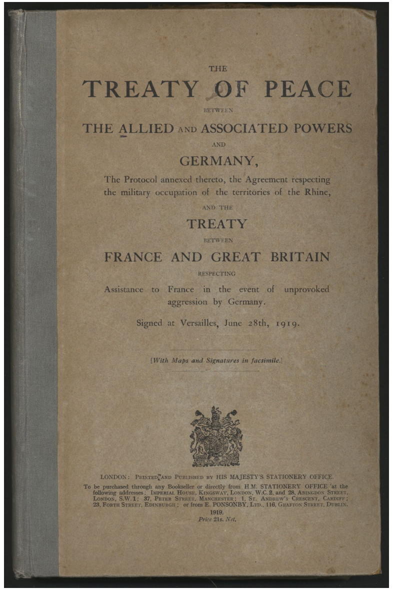
THE
TREATY OF PEACE
BETWEEN
THE ALLIED AND ASSOCIATED POWERS
AND
GERMANY,
The Protocol annexed thereto, the Agreement respecting the military occupation of the territories of the Rhine,
AND THE
TREATY
BETWEEN
FRANCE AND GREAT BRITAIN
RESPECTING
Assistance to France in the event of unprovoked aggression by Germany.
Signed at Versailles, June 28th, 1919.

[*With Maps and Signatures in facsimile.*]

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1919.
*Price* 21s. *Net.*

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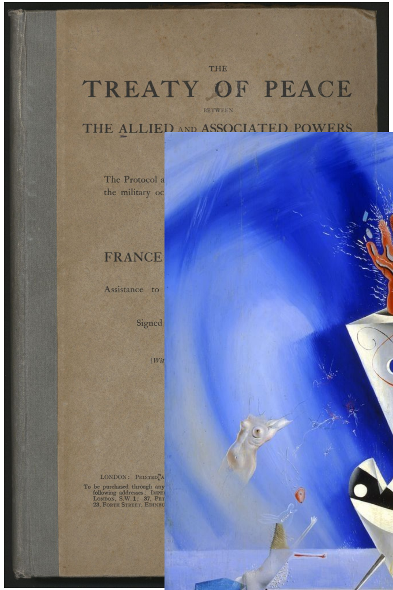


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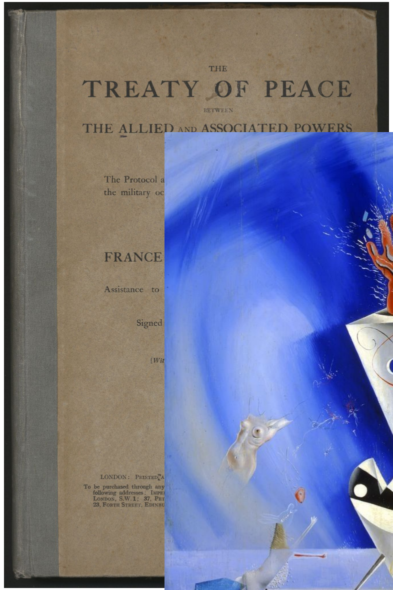


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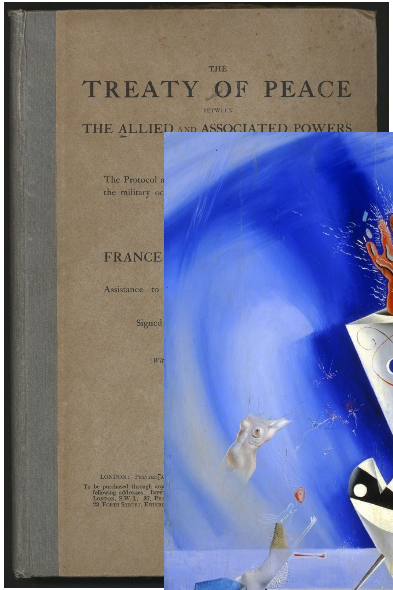


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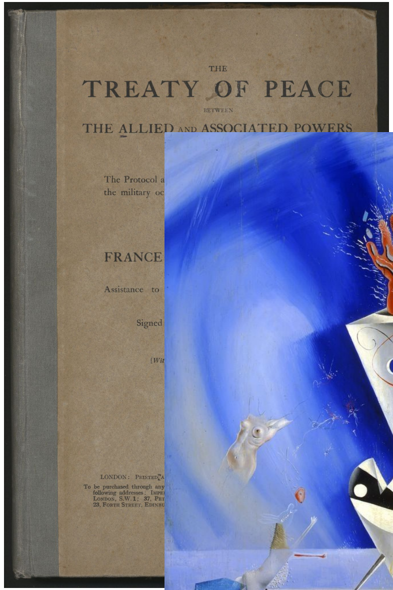


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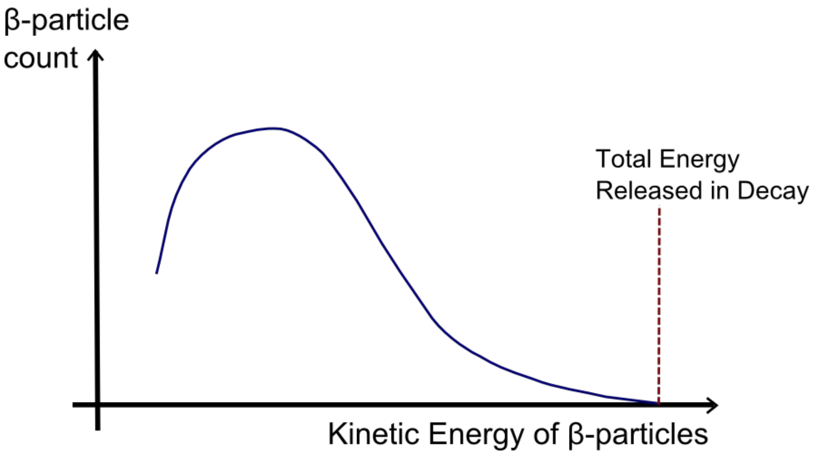

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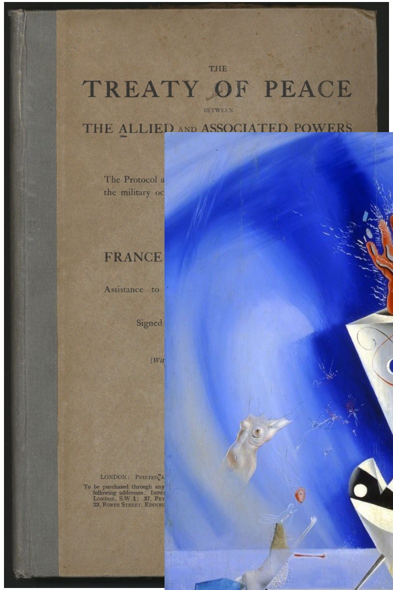


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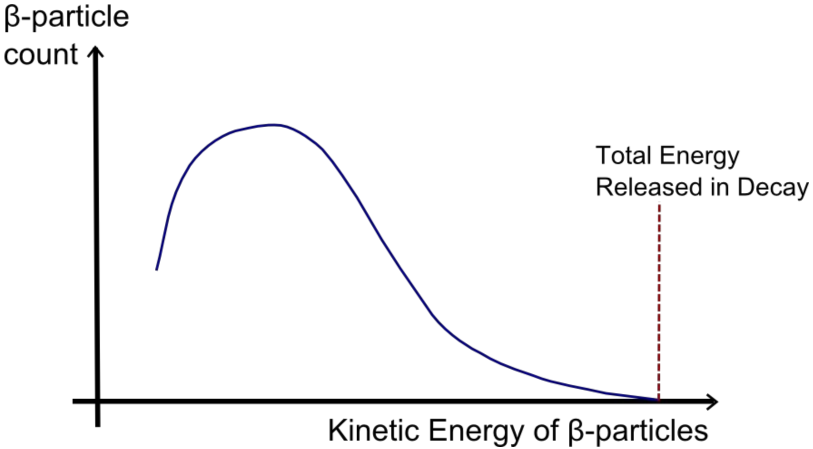


Physicists….

# A letter from Wolfgang Pauli - 4 December, 1930

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Des. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit den Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

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  - Electrically neutral
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  - Electrically neutral
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  - Very hard to detect
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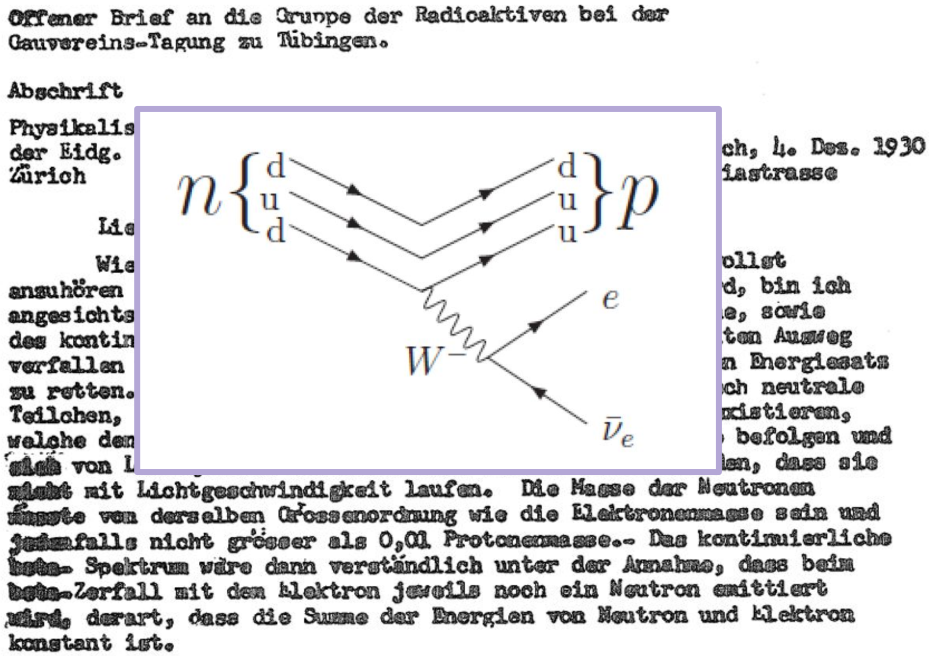
- Incorporated into theory of weak interactions by Enrico Fermi and renamed - ***neutrino***

$$n \rightarrow p + e^{-} + \overline{\nu}_e$$



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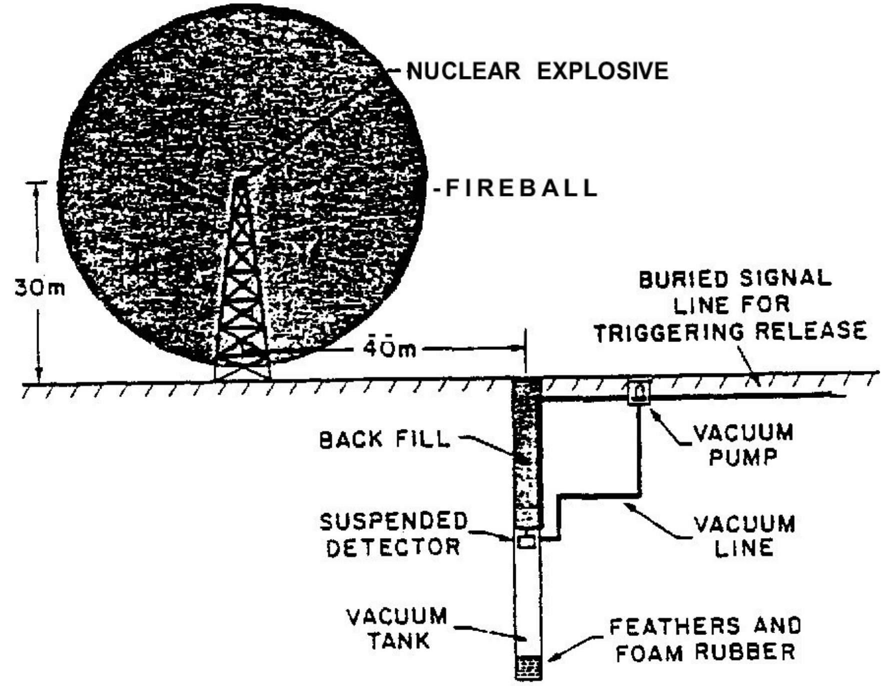
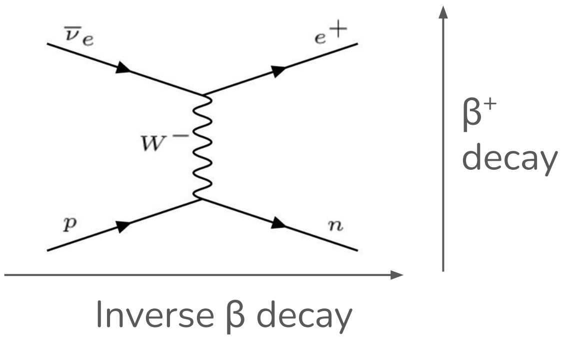
# Crazy ideas inspire more crazy ideas…

**Los Alamos proposal, 1940s**

- Use nuclear bomb as an intense source of neutrinos
- Suspend a neutrino detector in a deep hole with vacuum
- Release detector when bomb goes off
- Hope it lands softly…



# Discovery of the neutrino - 1956



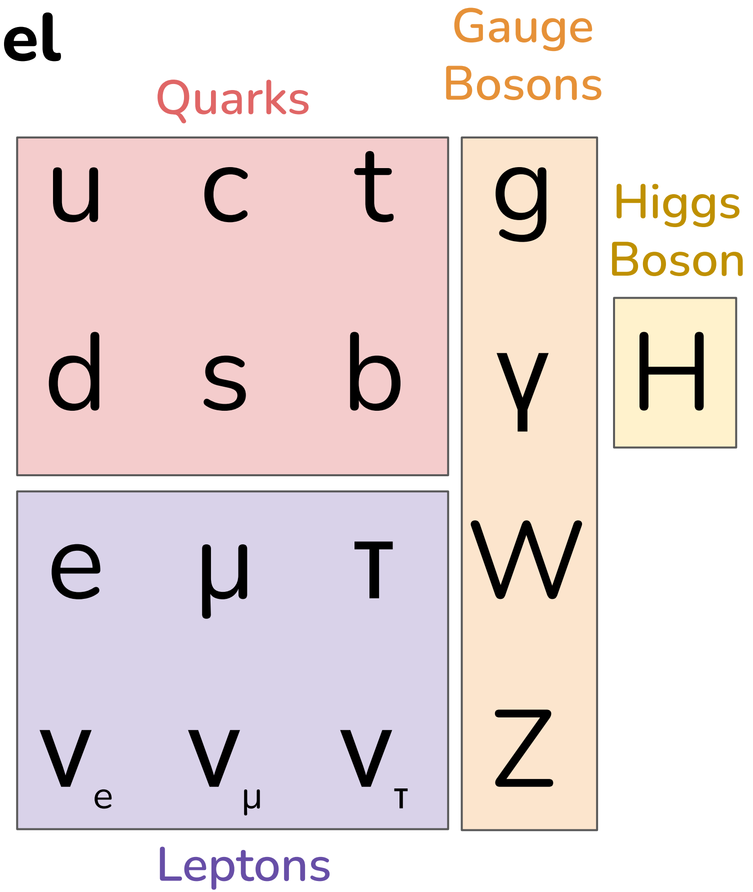

## Reines and Cowan - Project Poltergeist

- Use nuclear reactor as a neutrino source
- Capture neutrinos through inverse beta decay

$$\overline{\nu}_e + p \rightarrow e^+ + n$$

- Tank filled with water + cadmium chloride, monitored by light sensors
  - Prompt signal: $\mathbf{e^+ + e^- \rightarrow \gamma + \gamma}$
  - Delayed signal: $\mathbf{^{113}Cd + n \rightarrow {}^{114}Cd + \gamma}$

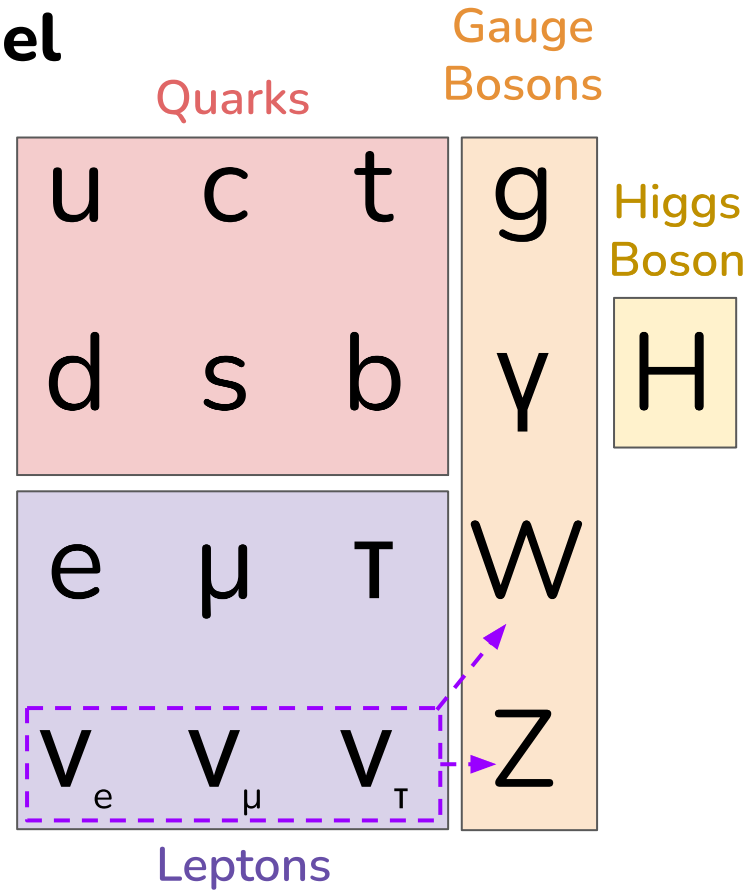
# Neutrinos in the Standard Model



# Neutrinos in the Standard Model

Key properties:

- 3 known types
- Leptons with spin ½
- No electric charge
- Only experience weak interactions



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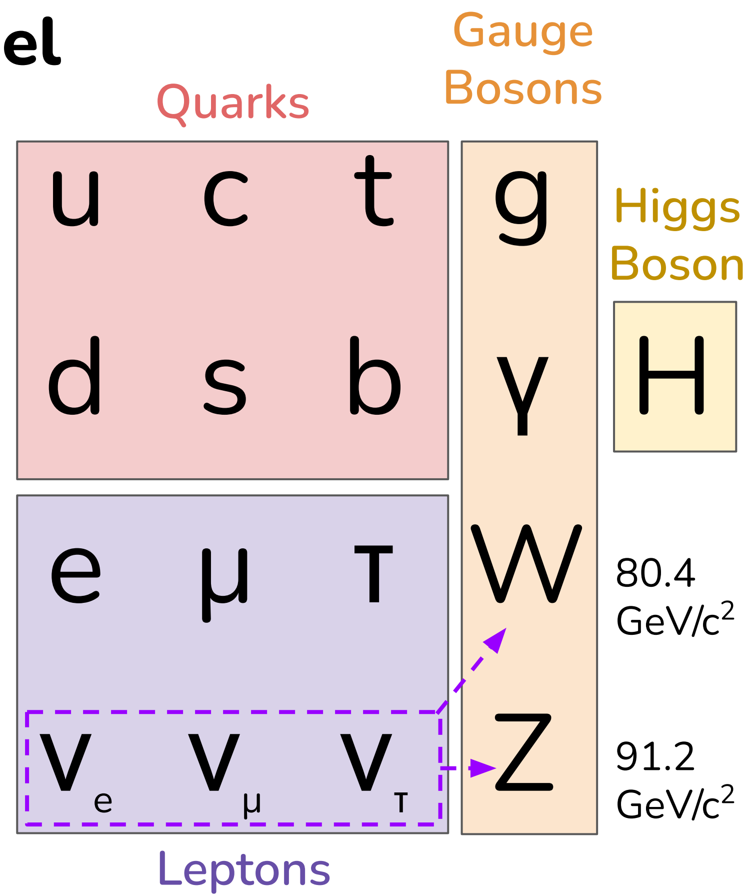
  - → W, Z bosons are heavy
    - Short-range ~$10^{-18}$m
    - Suppresses interactions



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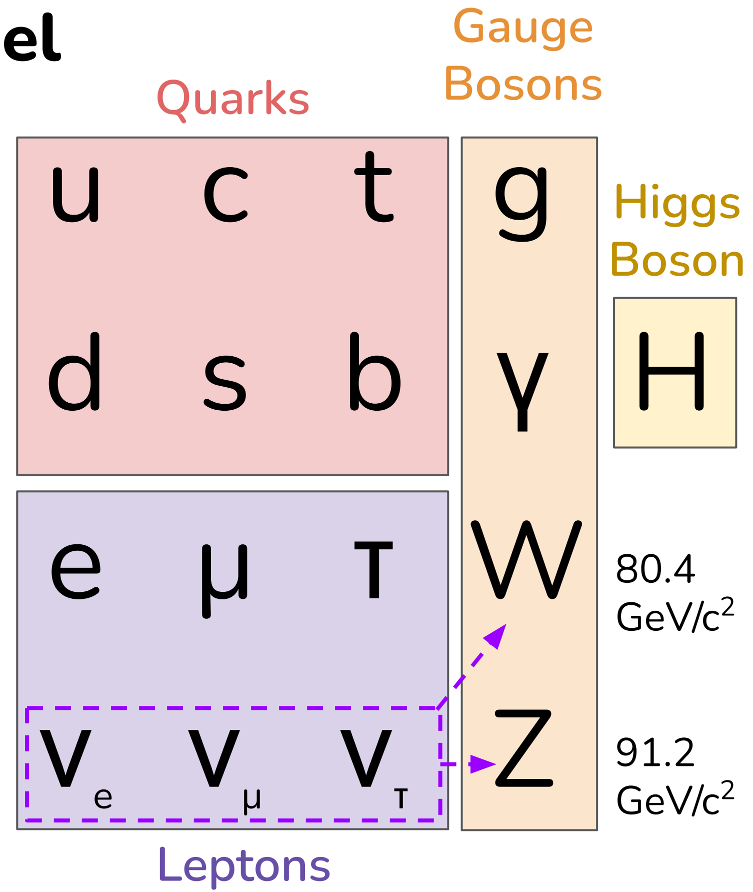

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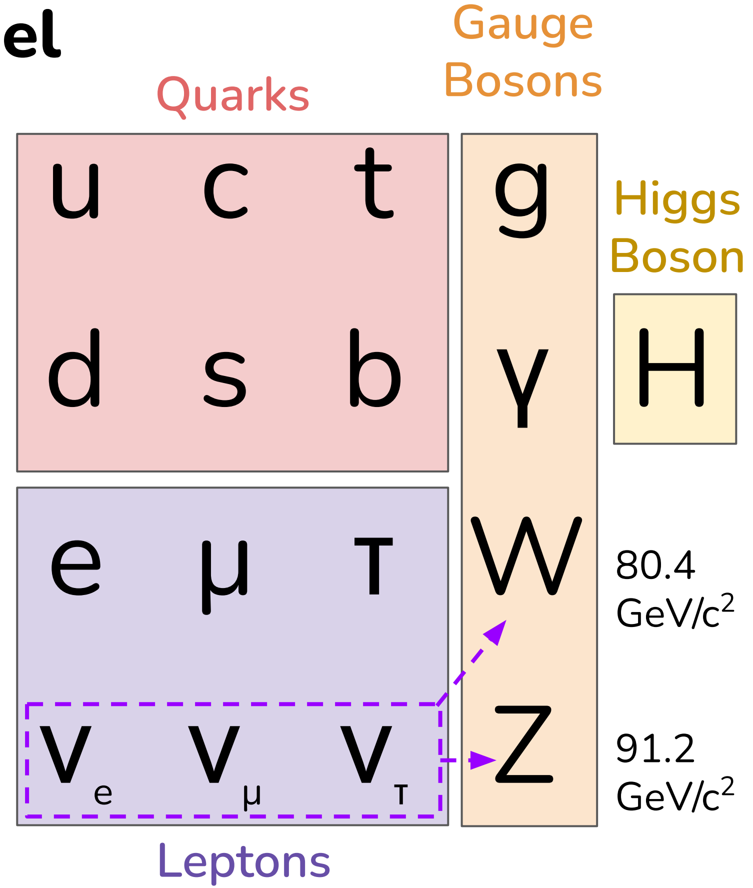

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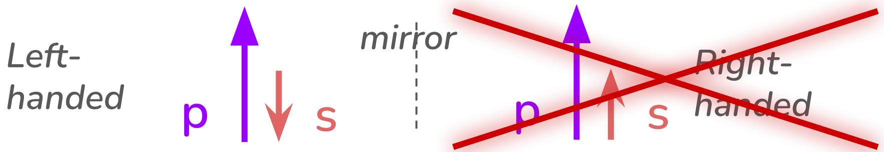
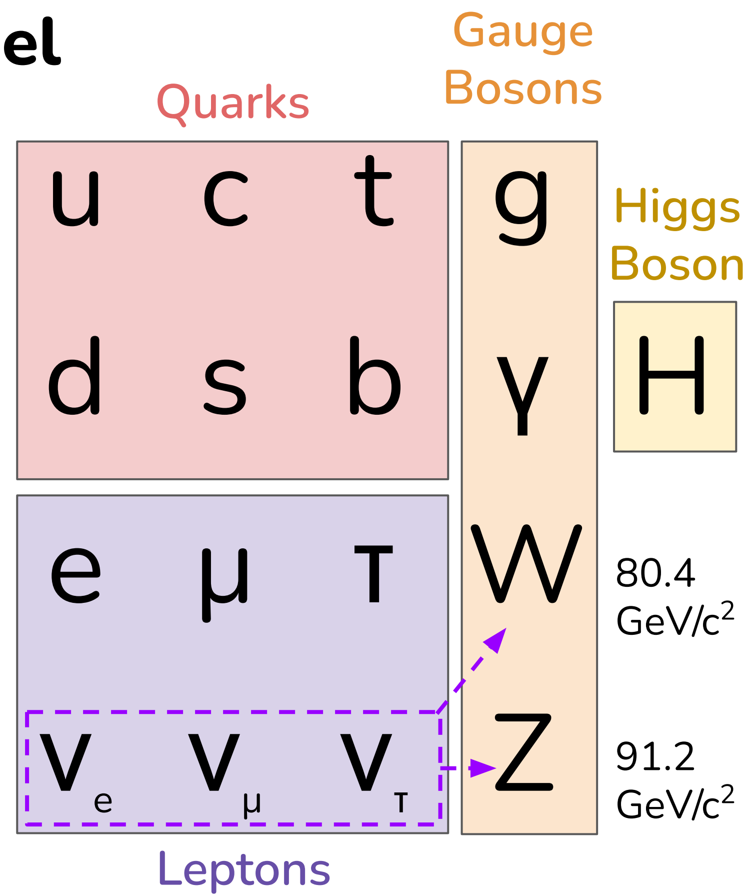
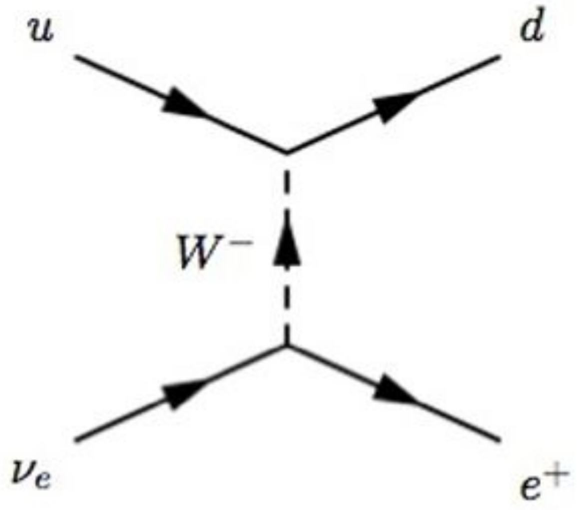


# Neutrinos interactions

Charged Current (CC)

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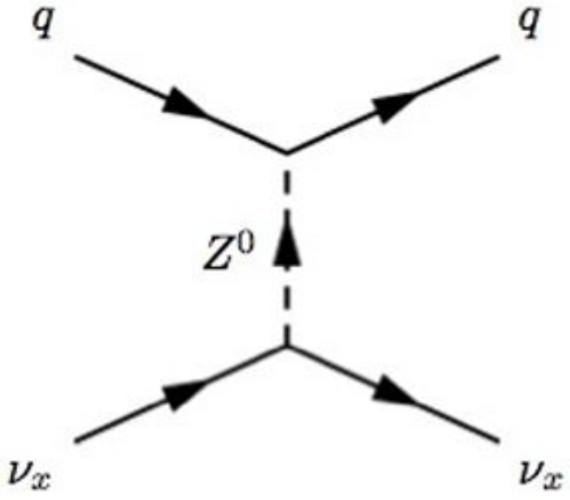

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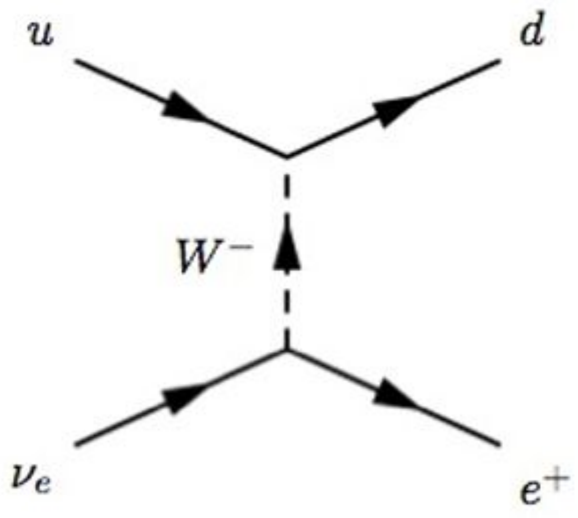
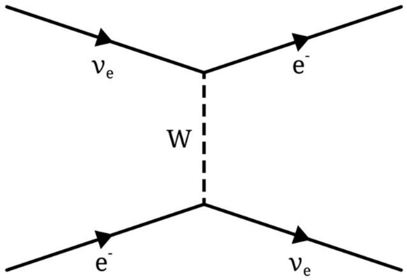
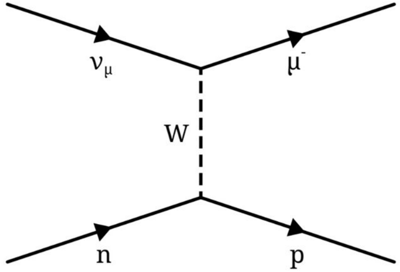



*NC interactions also categorized like this



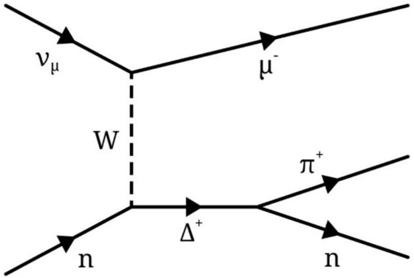

"Elastic"

Same particles in and out




"Quasi-elastic"

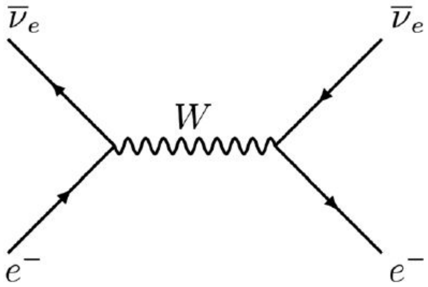
Different particles in final state, but same number in/out




"Inelastic"

Energy transferred to create new particles

# Cross-section for CC elastic scattering of anti-$\nu_e$+e




**Recall, depends on:**

- Particle energy and momentum
- Type of scattering process
- Phase space available for final state

$$d\sigma = \frac{1}{4|\mathbf{p}_{\text{initial}}|\sqrt{s}}\left(\frac{1}{(2\pi)^2}\right)|\mathcal{M}|^2 d\Phi_2$$

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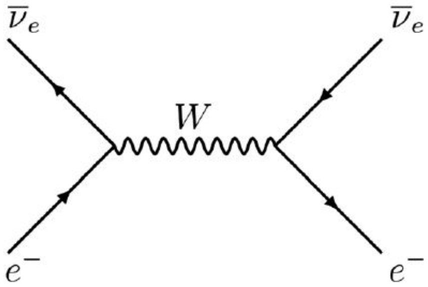


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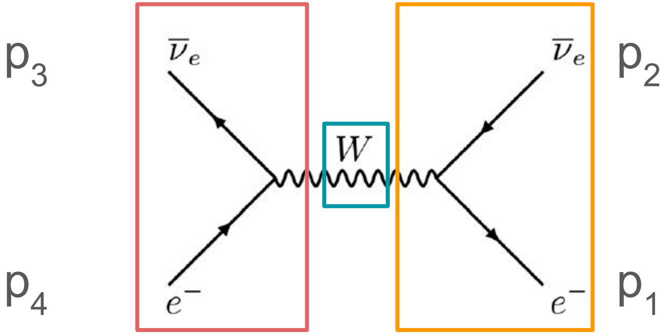


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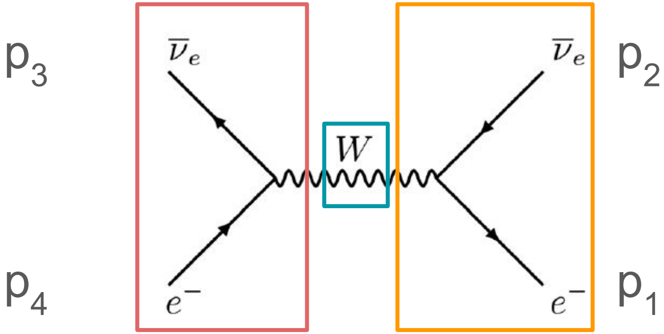


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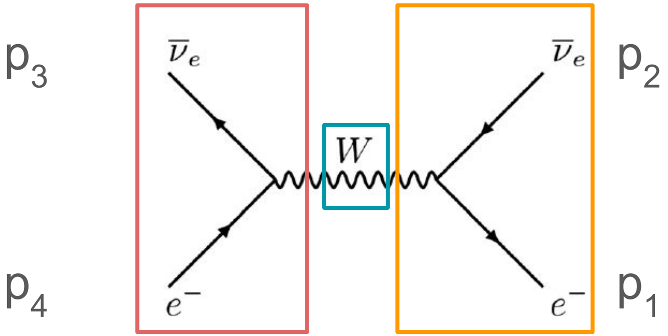


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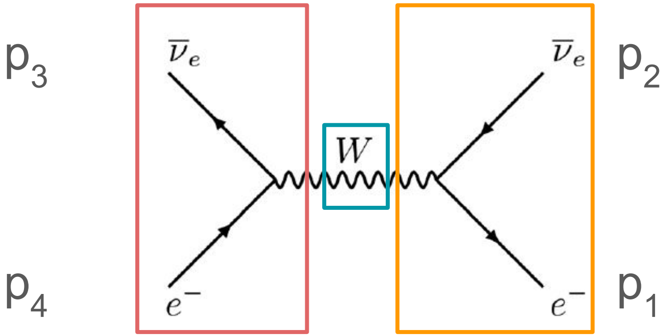
# Cross-section for CC elastic scattering of anti- $v_e$+e

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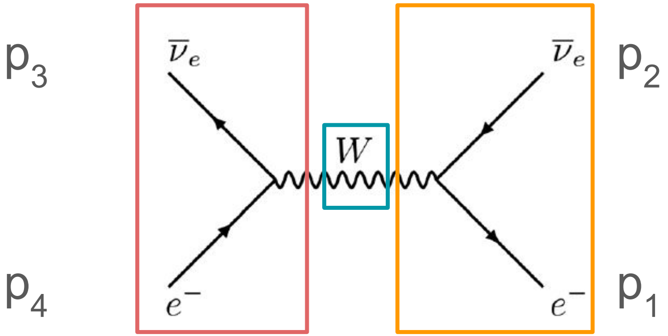


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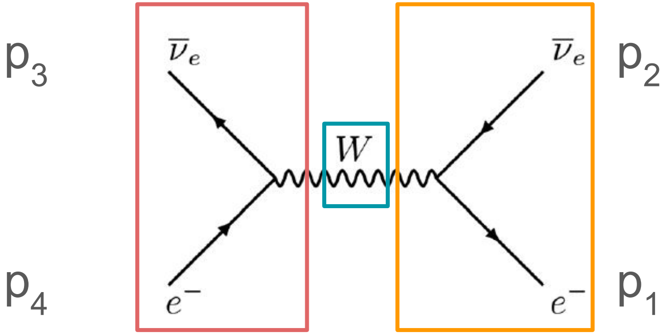


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Such that, $\mathcal{M} \propto -\frac{G_F}{\sqrt{2}}$ since…

$$g^2 = 4\sqrt{2}G_F M_W^2$$

$|\mathcal{M}|^2$ ~$10^{-10}$ $\text{GeV}^{-4}$

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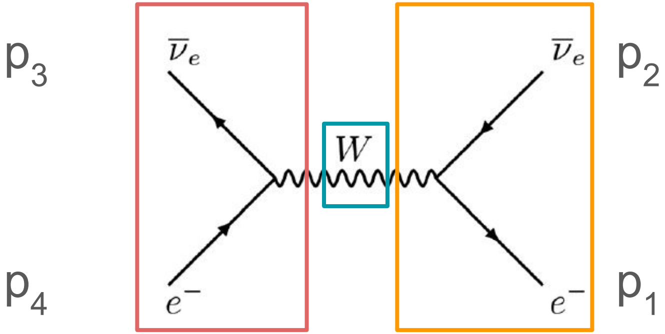


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- Phase space available for final state

$$d\sigma = \frac{1}{4|\mathbf{p}_{\text{initial}}|\sqrt{s}}\left(\frac{1}{(2\pi)^2}\right)|\mathcal{M}|^2 d\Phi_2$$

**Using Feynman rules for scattering amplitude:**

$$\mathcal{M} = \left(-\frac{g}{2\sqrt{2}}\right)\bar{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_4) \cdot \frac{-ig_{\mu\nu}}{q^2 - M_W^2} \cdot \left(-\frac{g}{2\sqrt{2}}\right)\bar{u}(p_1)\gamma^\nu(1-\gamma^5)u(p_2)$$

***Important! picks out left-handed fields***

When $q^2 << M_W{}^2$ , then $\mathcal{M} = -\frac{g^2}{8M_W^2} \cdot$ [matrix stuff]

Such that, $\mathcal{M} \propto -\frac{G_F}{\sqrt{2}}$ since…

$$g^2 = 4\sqrt{2}G_F M_W^2$$

$|\mathcal{M}|^2$ ~$10^{-10}$ GeV$^{-4}$

# Cross-section for CC elastic scattering of anti- $\nu_e$+e

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  → Excellent review with details can be found here: https://arxiv.org/abs/1305.7513
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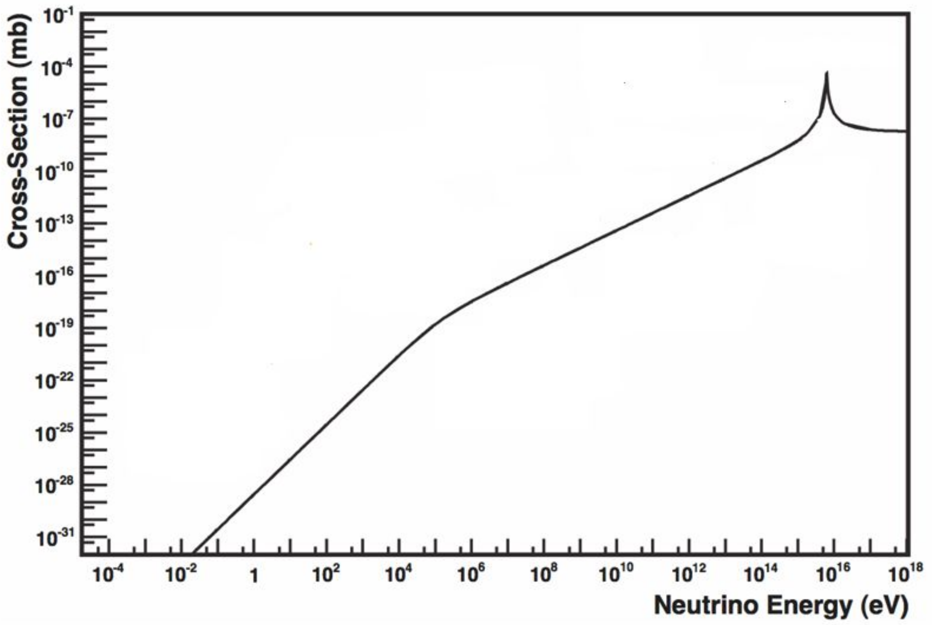
*Figure adapted from: Formaggio, Zeller*




For reference, σ(γ) ~ 1-$10^6$ barn

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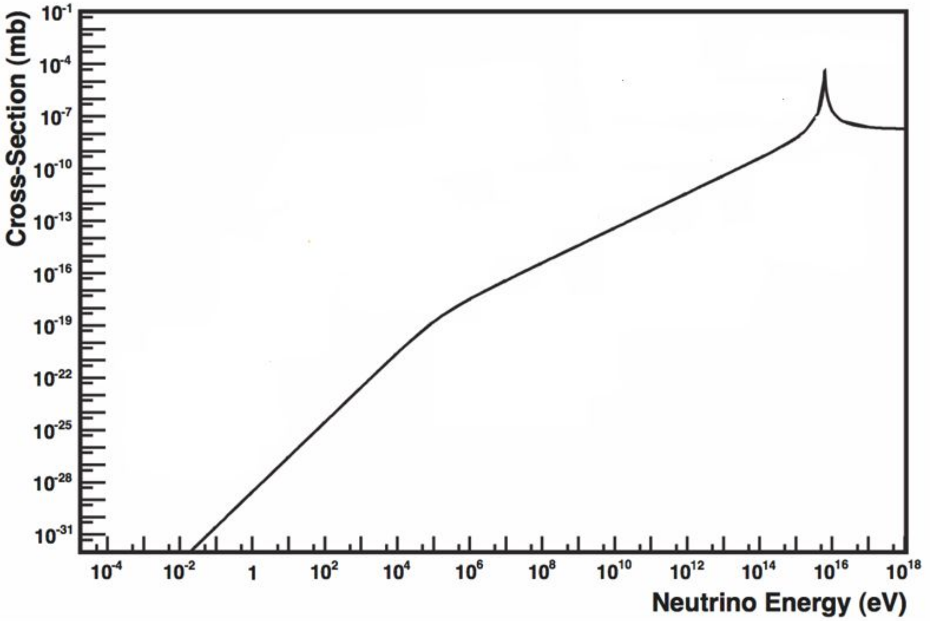


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$$\sigma_{\nu N}^{NC} = 2.31 \times 10^{-36}\ \mathrm{cm}^2 \left(\frac{E_\nu}{1\ \mathrm{GeV}}\right)^\alpha, \quad \alpha \simeq 0.363.$$

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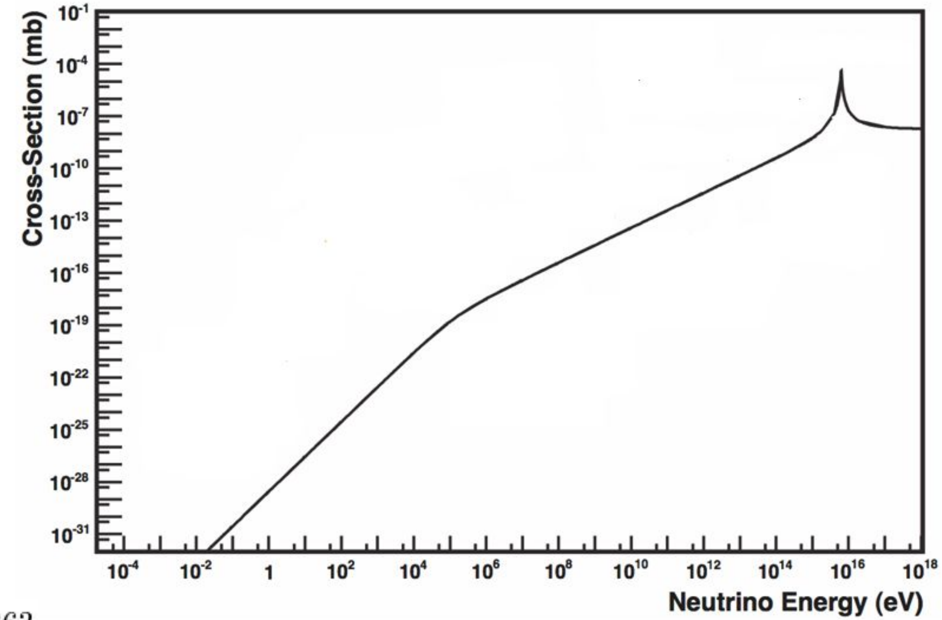


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  - → Glashow resonance ~ 6 PeV

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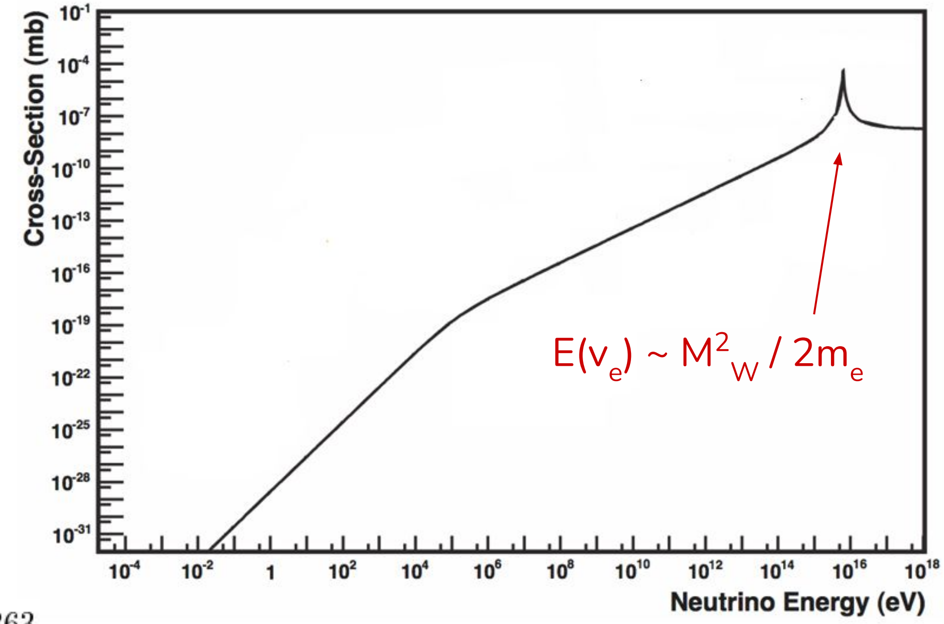


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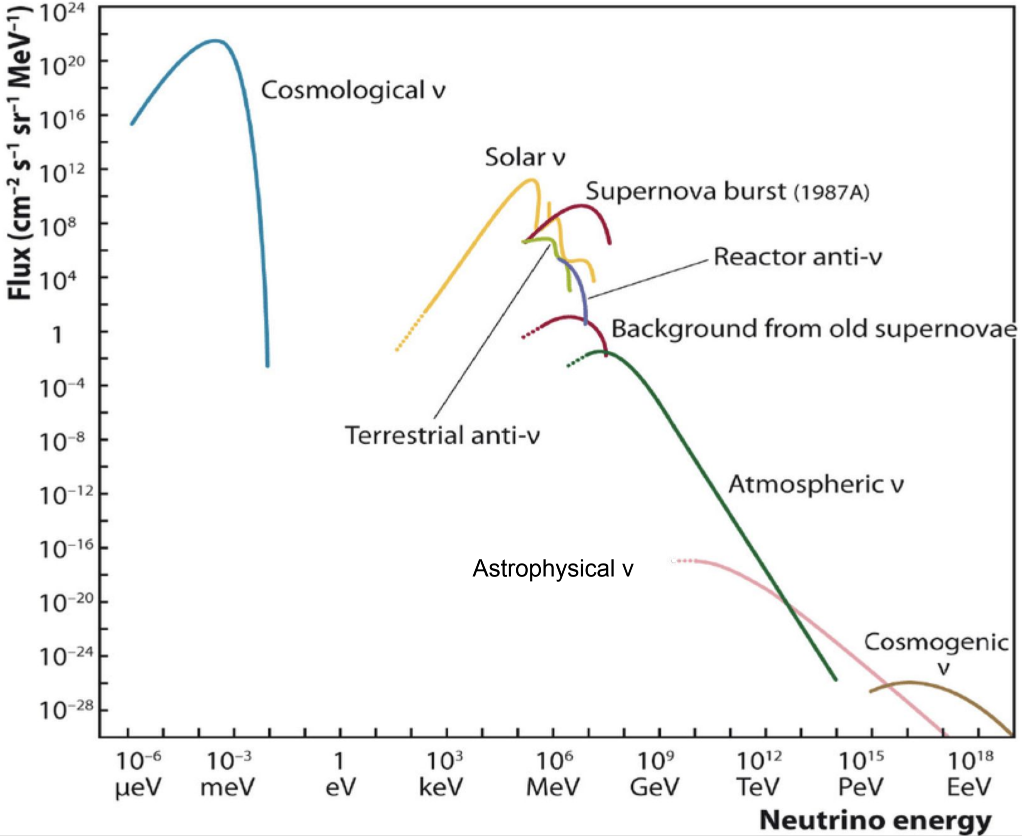
# Flux predictions from natural sources



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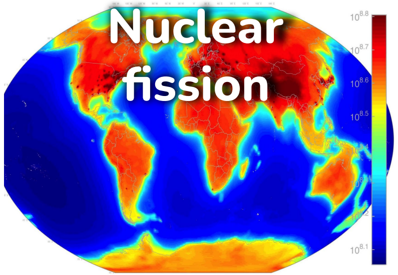


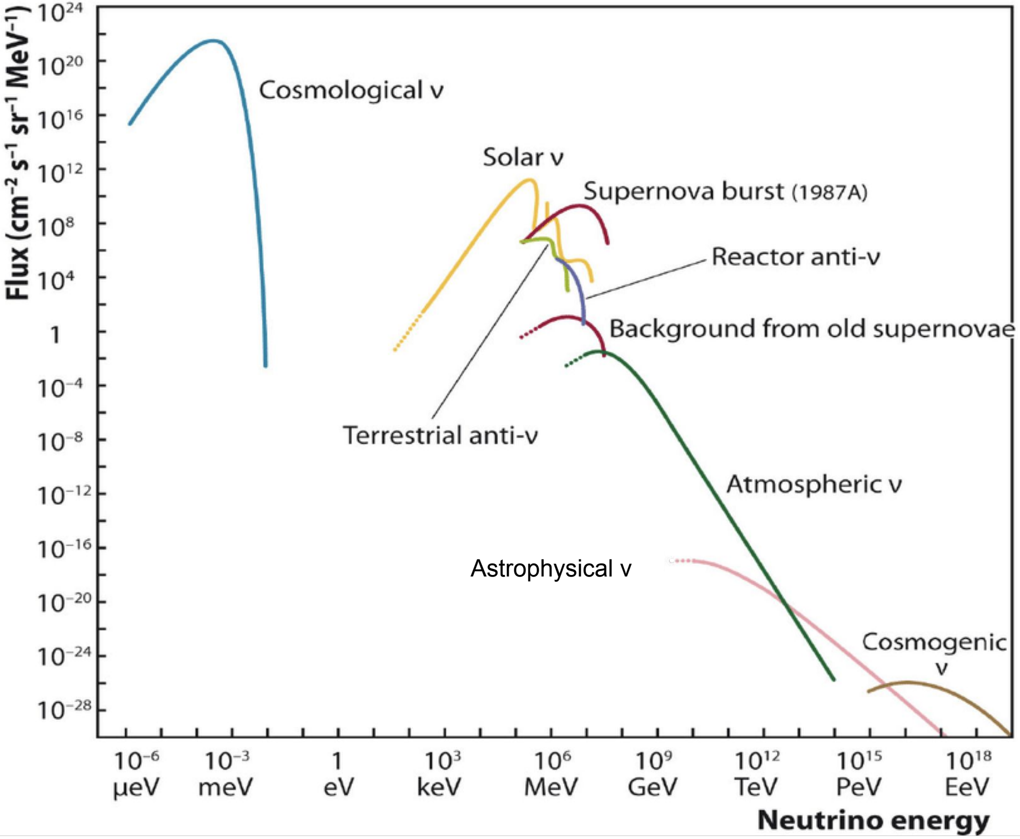

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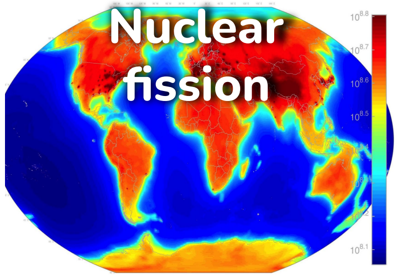


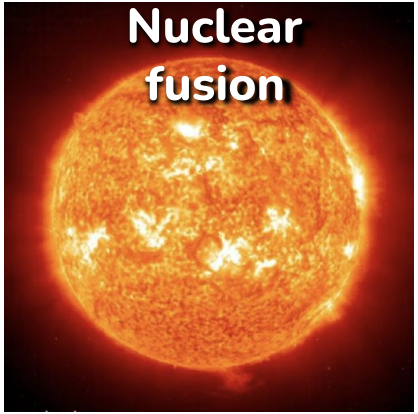


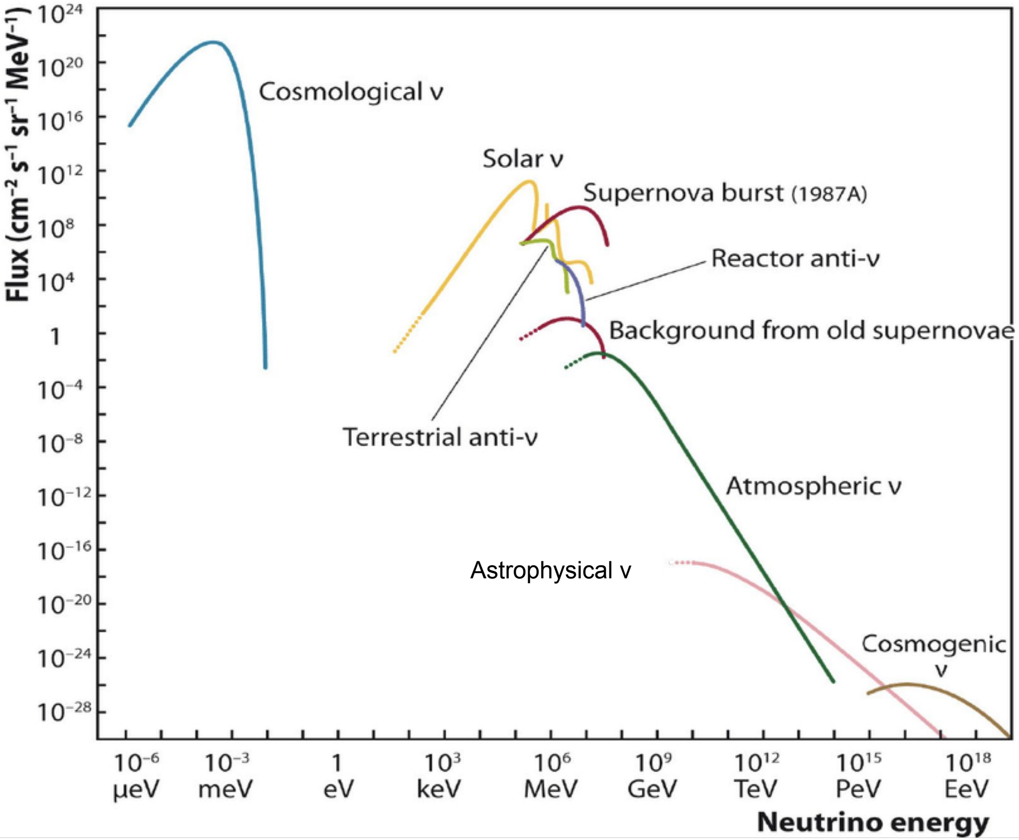

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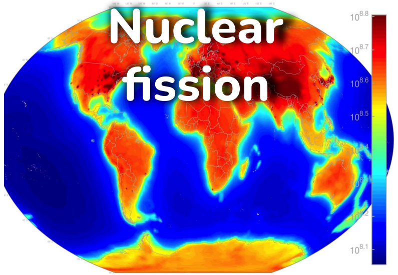


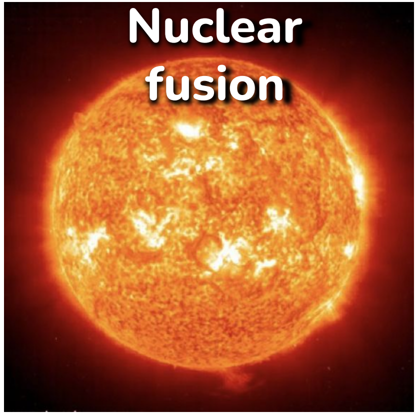


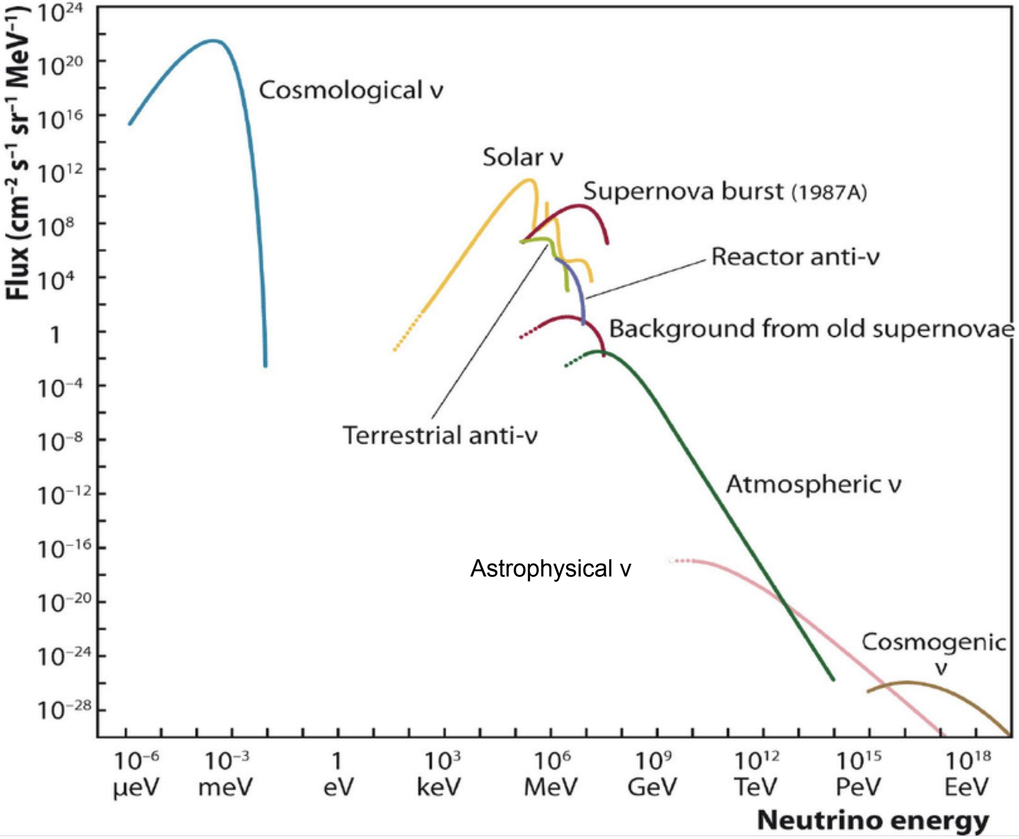


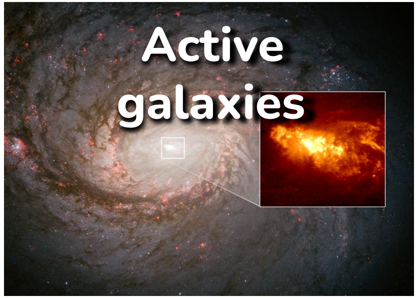

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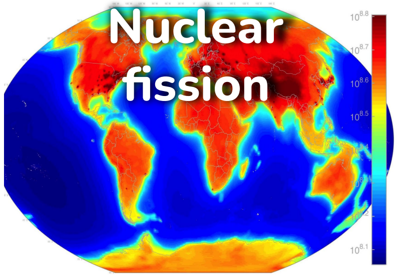


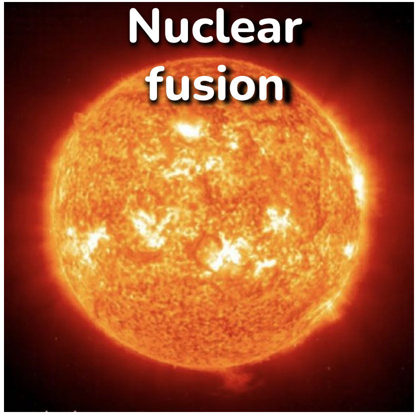


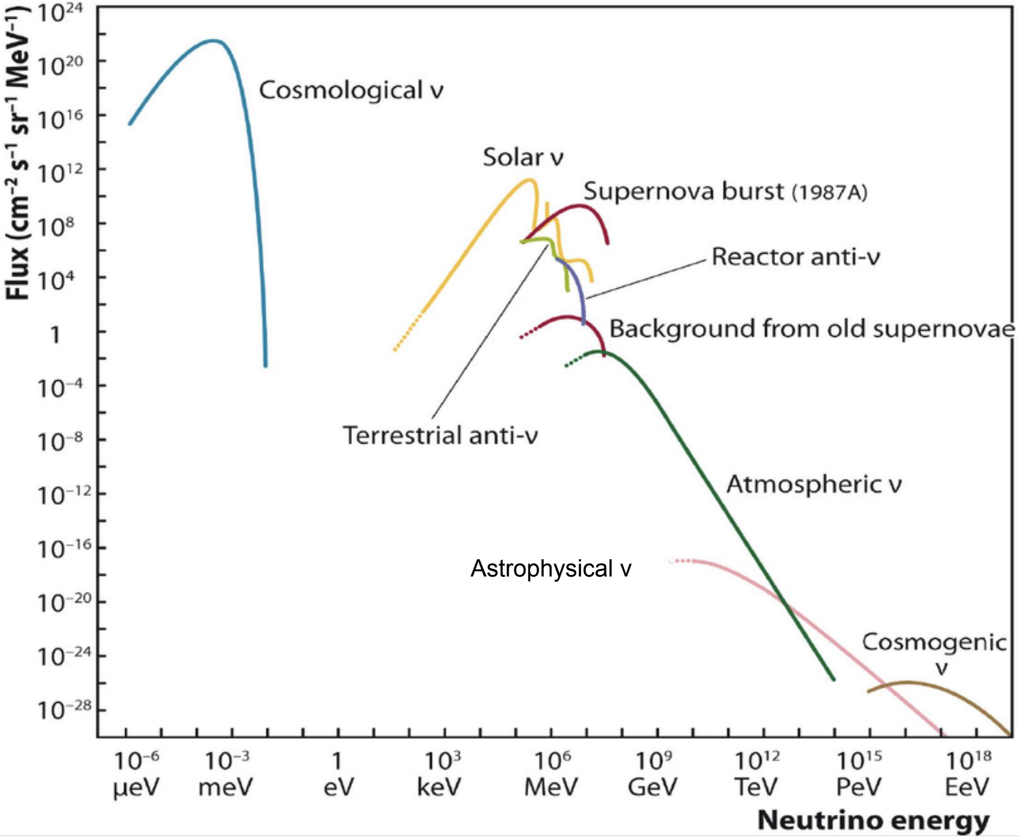


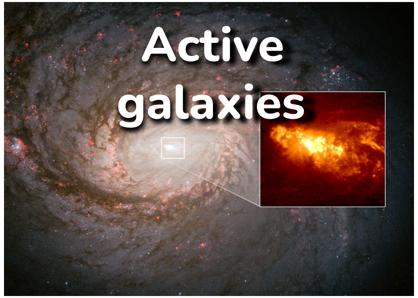


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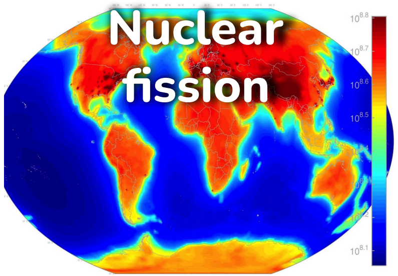
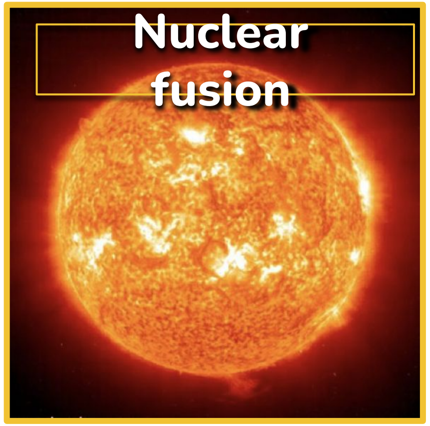
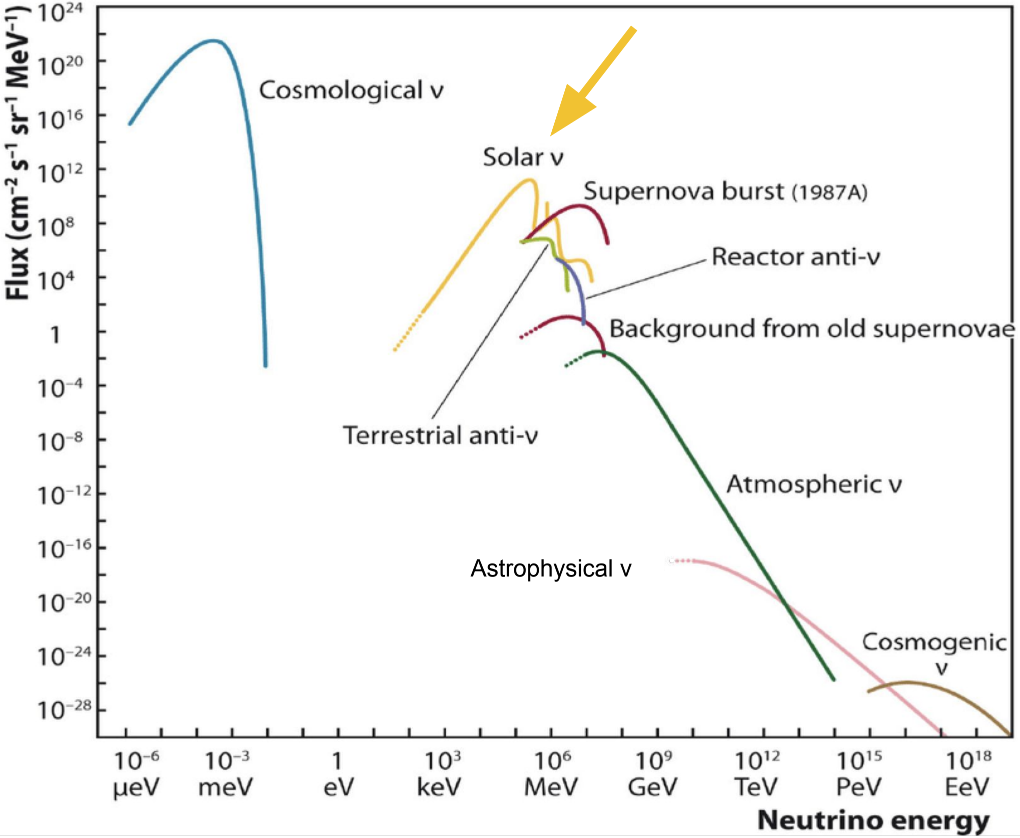
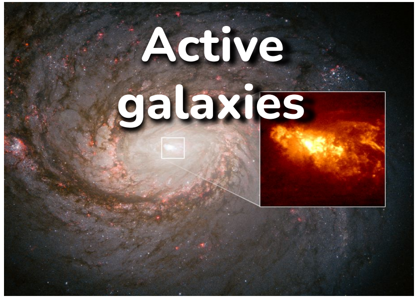


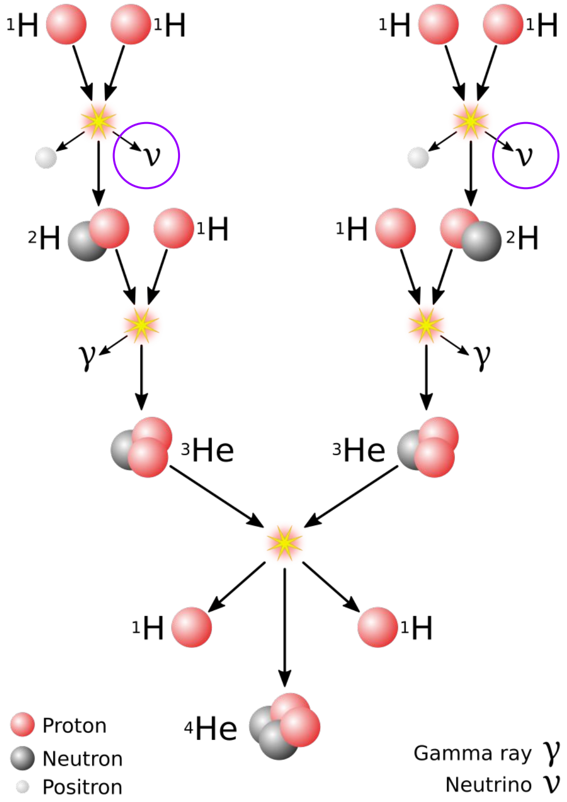




# Solar Neutrinos

- Fusion reactions in the sun generate anti-$\nu_e$ through several mechanisms
- Dominant reaction is through ***proton-proton (pp) chain*** (99%)

Image credit: Sarang



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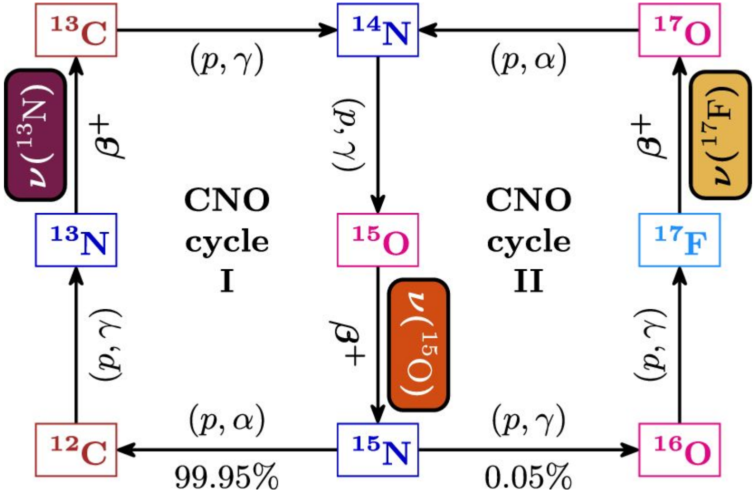

Review of Solar Neutrino Physics

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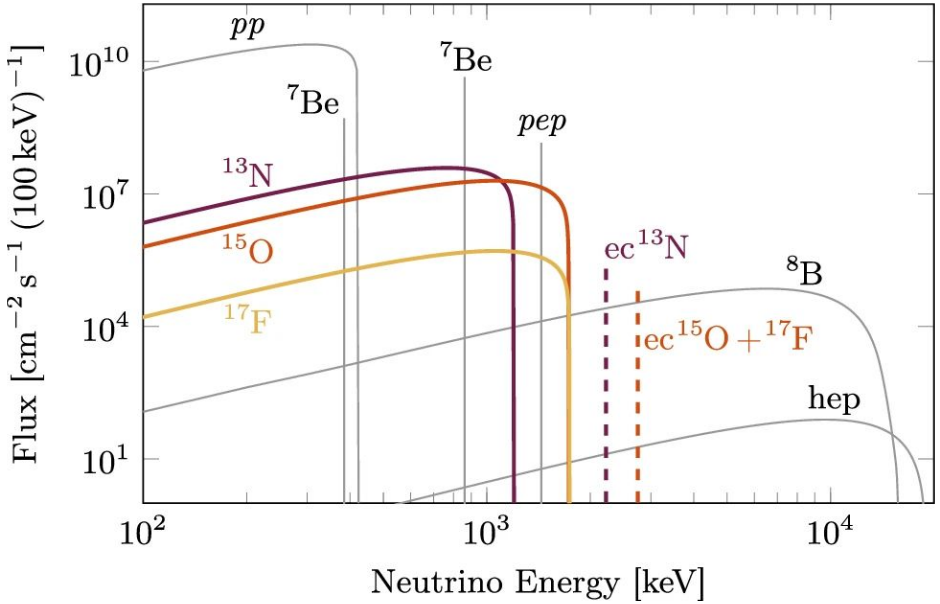
- Standard Solar Model provides estimates of neutrino fluxes



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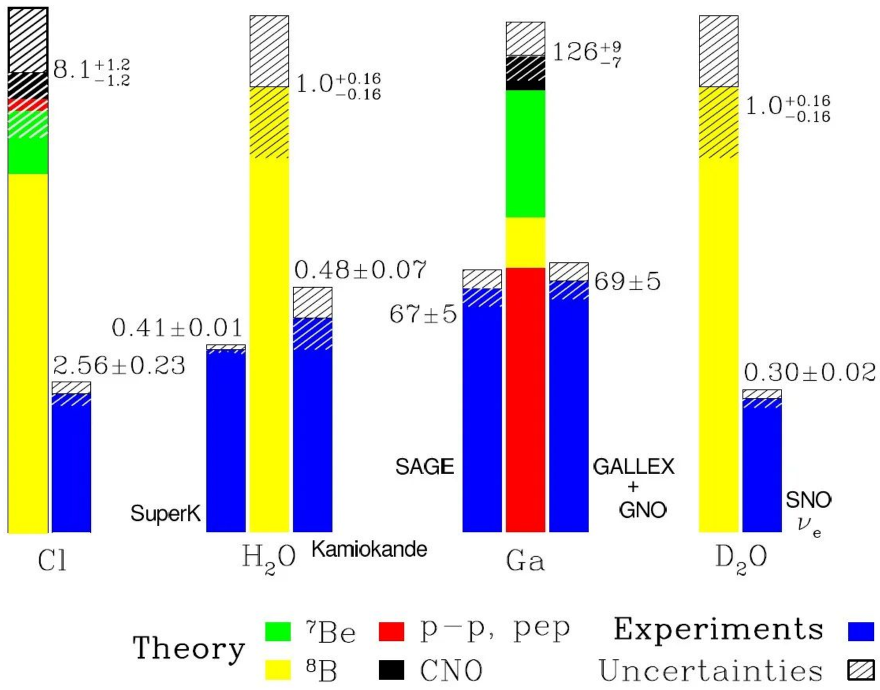

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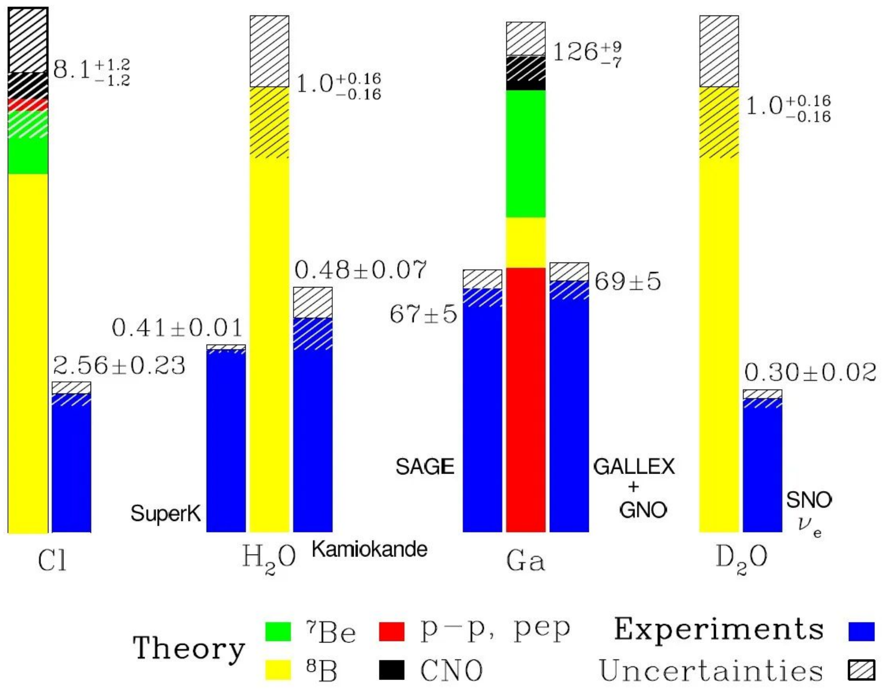

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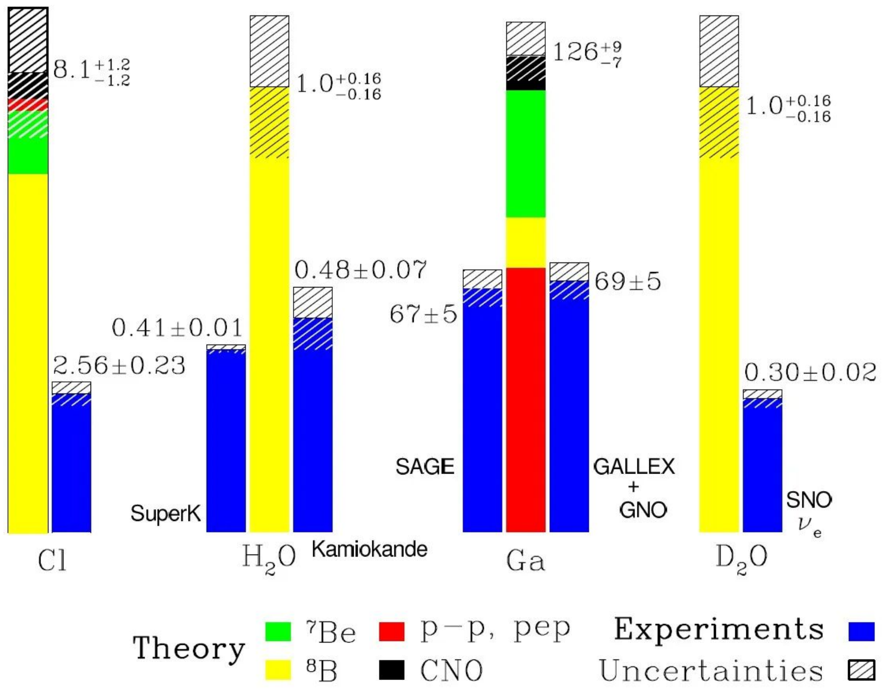
Once again, neutrinos are causing trouble…



# The solution: massive neutrinos!

- Neutrinos flavour states are a superposition of mass states

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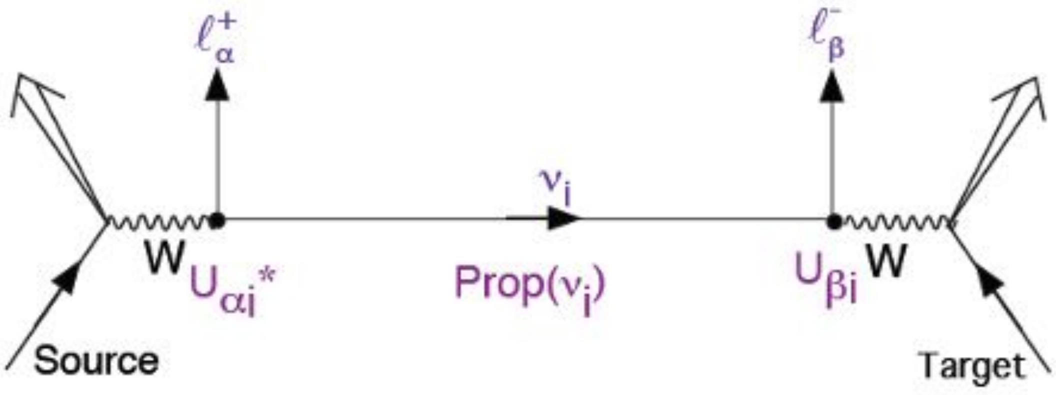

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- Unitarity constraints reduce dimensionality:

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$U^\dagger U = \mathbf{1} \rightarrow U_i \cdot U_j = \boldsymbol{\delta}_{ij}$ and $U^T_i \cdot U^T_j = \boldsymbol{\delta}_{ij}$

i.e. $|U_{e1}|^2 + |U_{\mu 1}|^2 + |U_{\tau 1}|^2 = 1$

# The Pontecorvo-Maki-Nakagawa-Sakata matrix

- PMNS matrix is most widely used parameterization

Where cij = cosθij and sij = cosθij

Free parameters: $\theta_{12}$,$\theta_{13}$,$\theta_{23}$, $\delta_{CP}$ (maybe $\alpha_1$ & $\alpha_2$)

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"atmospheric" "reactor" "solar" "Majorana"

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# Neutrino oscillations in vacuum

- To understand how this helps answer the Solar neutrino problem, we apply Schrödinger's equation and get a plane wave solution

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Transition probability for $\alpha \to \beta$

$$P_{\alpha\to\beta} = \left|\,\langle\,\nu_\beta\,|\,\nu_\alpha(L)\,\rangle\,\right|^2$$

$$= \left|\sum_j U_{\alpha j}^*\,U_{\beta j}\,e^{-i\frac{m_j^2\,L}{2E}}\right|^2$$

# Vacuum oscillation probability

$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4 \sum_{j>k} \mathcal{R}_e \left\{ U^*_{\alpha j}\, U_{\beta j}\, U_{\alpha k}\, U^*_{\beta k} \right\} \sin^2\left( \frac{\Delta_{jk} m^2\, L}{4E} \right)$$

$$+ 2 \sum_{j>k} \mathcal{I}_m \left\{ U^*_{\alpha j}\, U_{\beta j}\, U_{\alpha k}\, U^*_{\beta k} \right\} \sin\left( \frac{\Delta_{jk} m^2\, L}{2E} \right) ,$$

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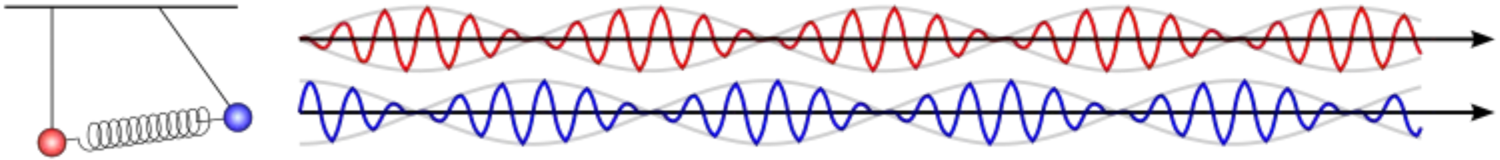
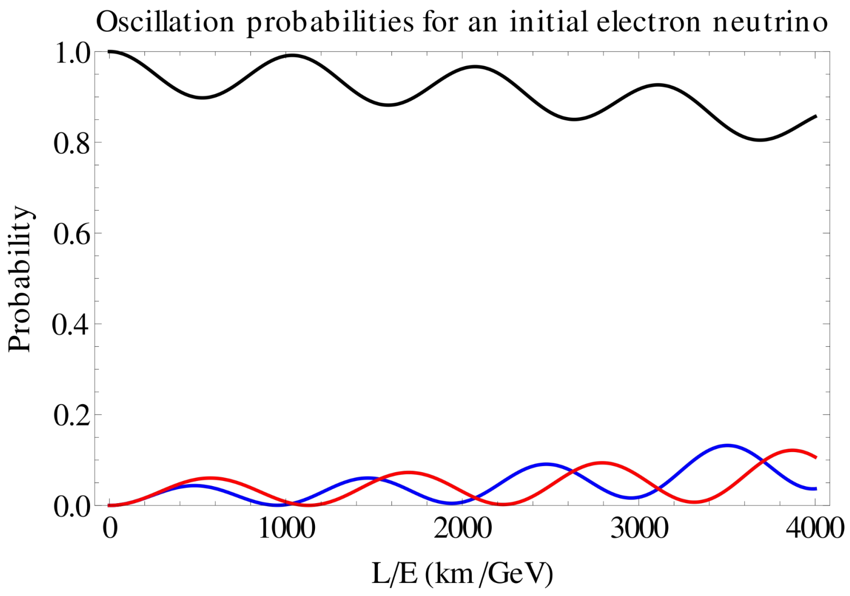
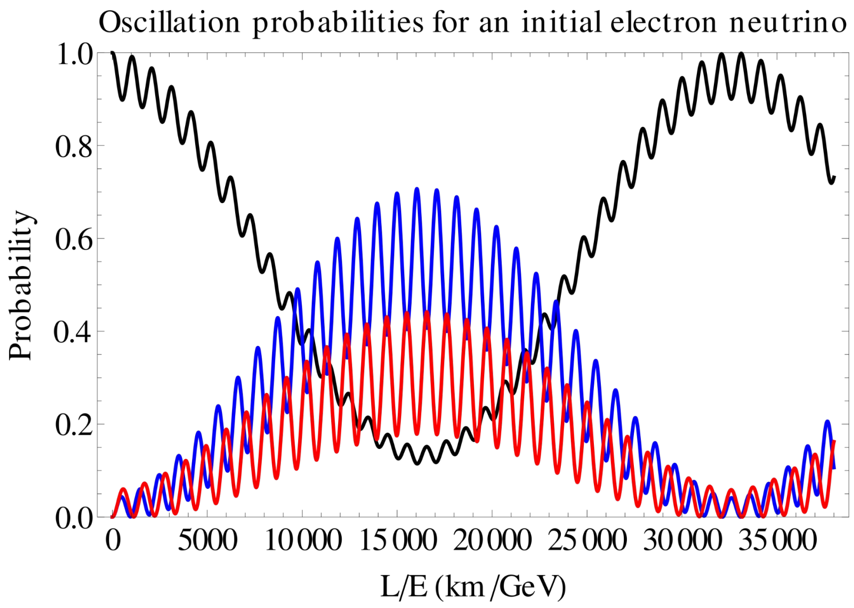


# Vacuum oscillation probability - electron neutrino

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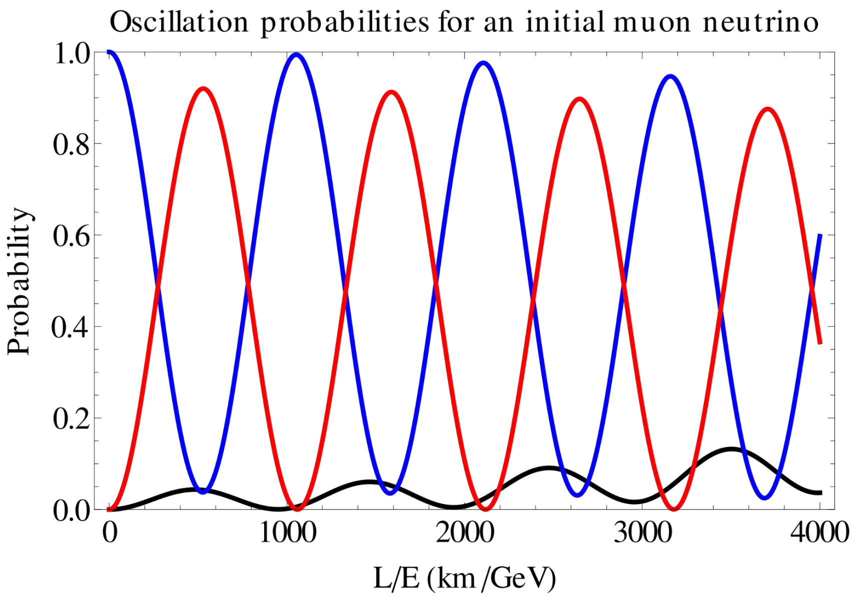
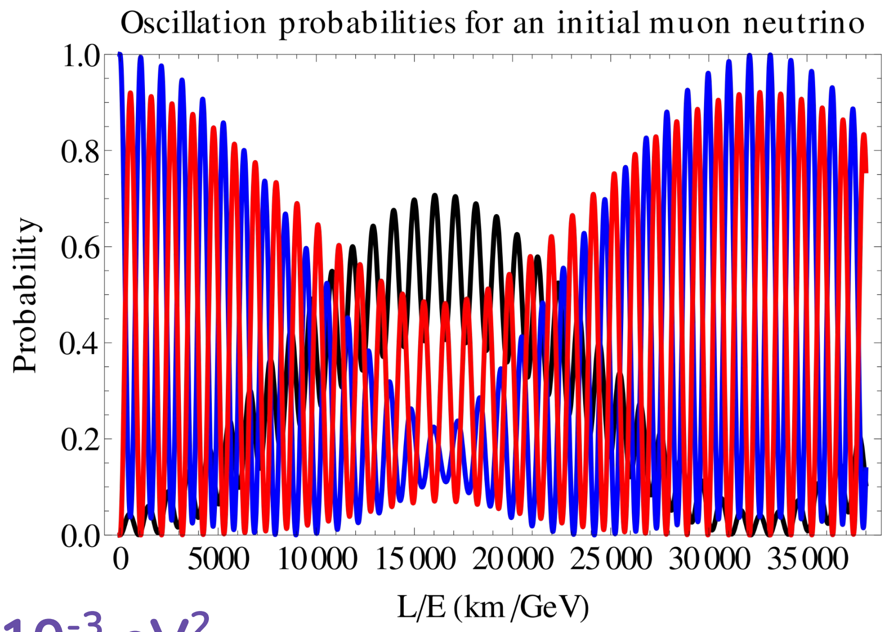
# Vacuum oscillation probability - muon neutrino



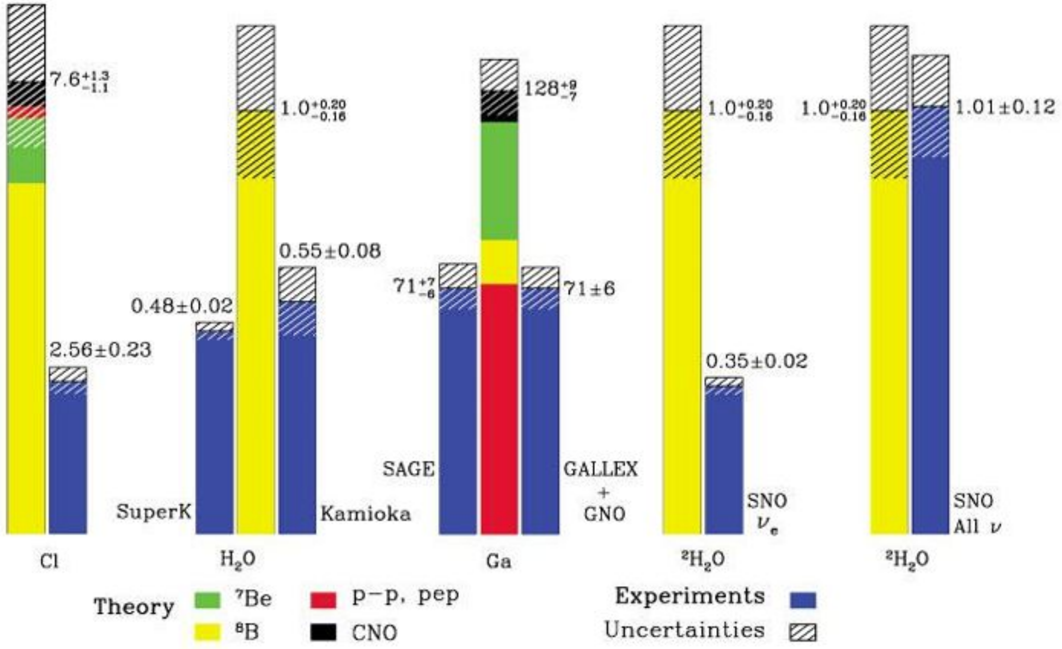




$\Delta m^2_{32} \sim 10^{-3}\ eV^2$

# Solving the solar neutrino problem

**2015**

- Solar neutrino detectors were only sensitive to anti-ve
- Low anti-ve survival probability when reaching Earth
- The SNO experiment was designed to measure both anti-ve and NC (all)
  - → **NC rates match expectation!**



# Neutrino mixing vs Quark mixing

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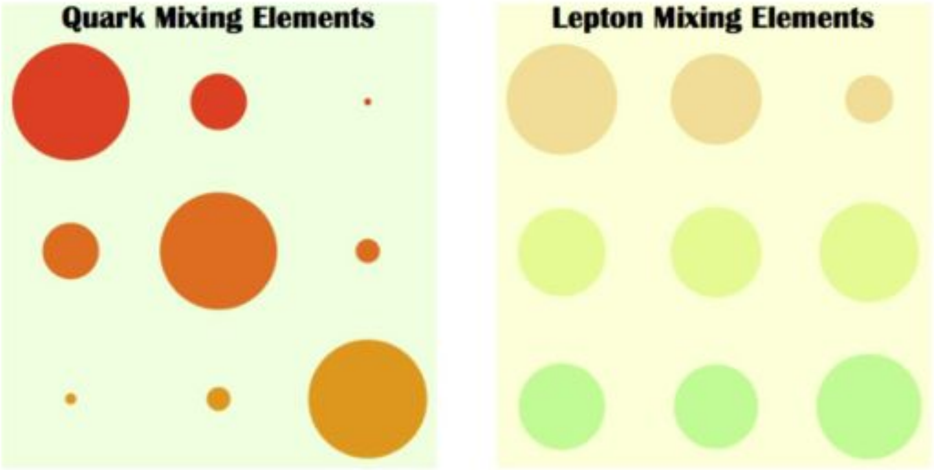
Neutrino oscillations are a ***propagation*** *effect*

Mixing in lepton sector is ***large*** *with* very different structure

Is there some **new symmetry** at play?



# Key takeaways from today

- Neutrinos are Standard Model odd-balls
- Extremely low interaction rates
- Naturally produced in abundance by many sources
- Unexpectedly massive