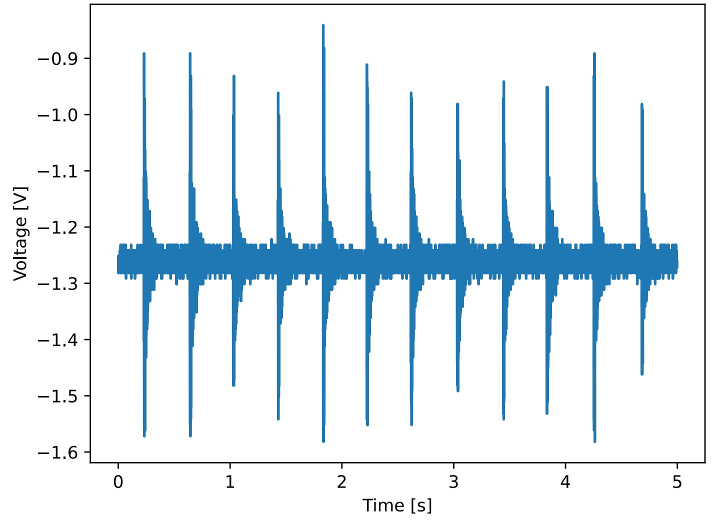
MAGO2.0 Meeting 15.05.24

Update Accelerometers, S Matrix Formalism

Tom Krokotsch

Vibration Measurement With New Accelerometers

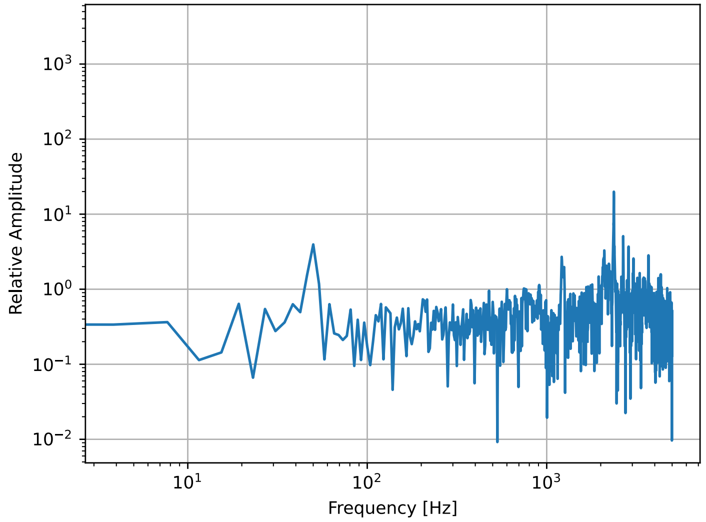
- Accelerometer mounted on cavity
- Cavity struck with hammer
- Nominal sensitivity: 1 - 10 mV / ms^{-2}



Problem:

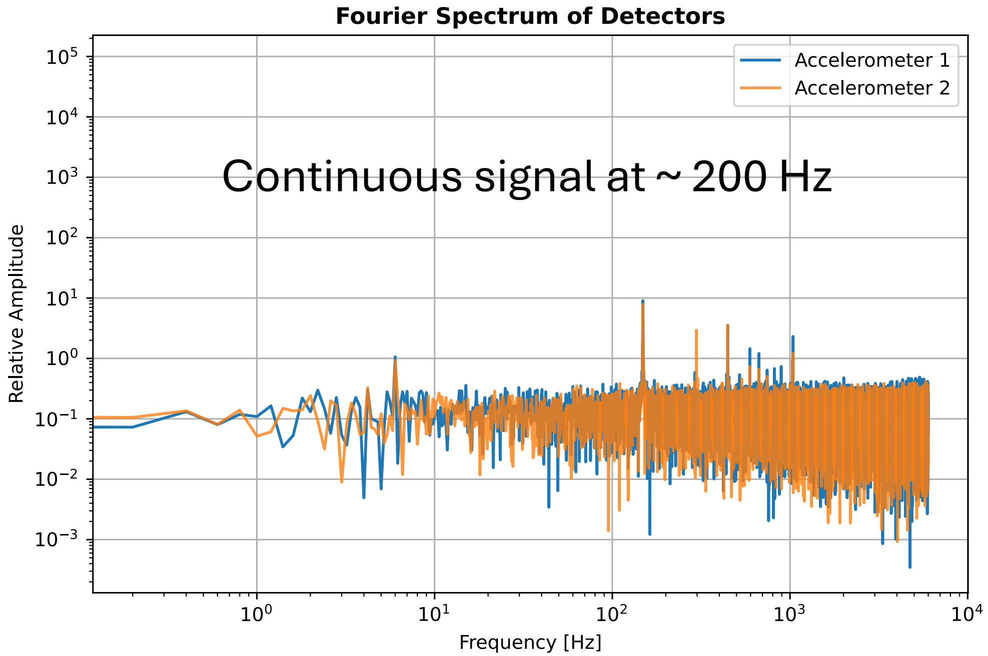
Noise floor too high. We want to resolve vibrational noise background, not just hammer blows

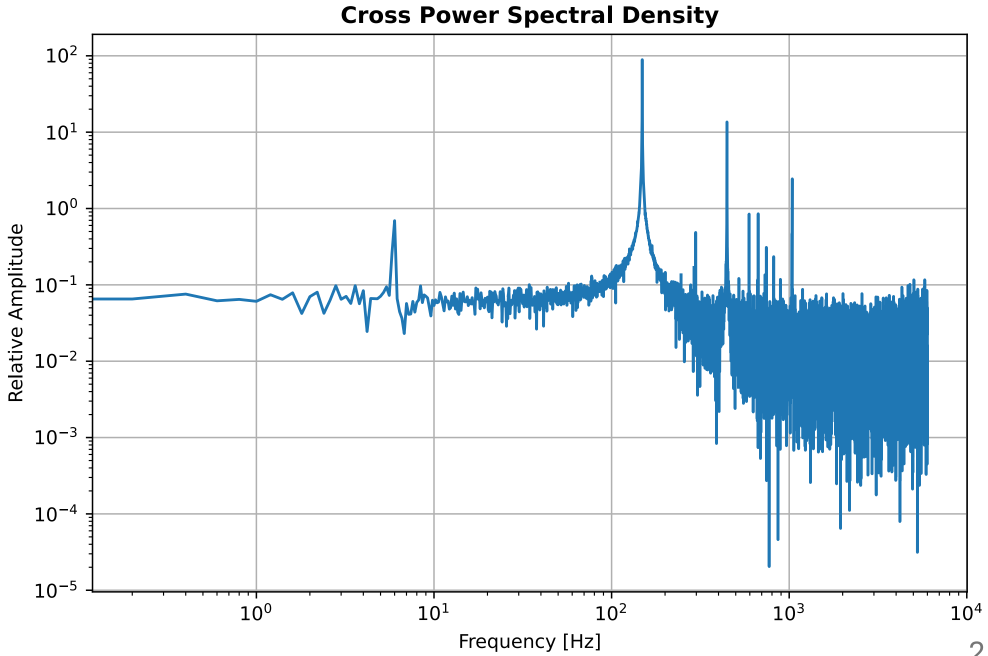
→Better power source, amplifier or DAQ may help



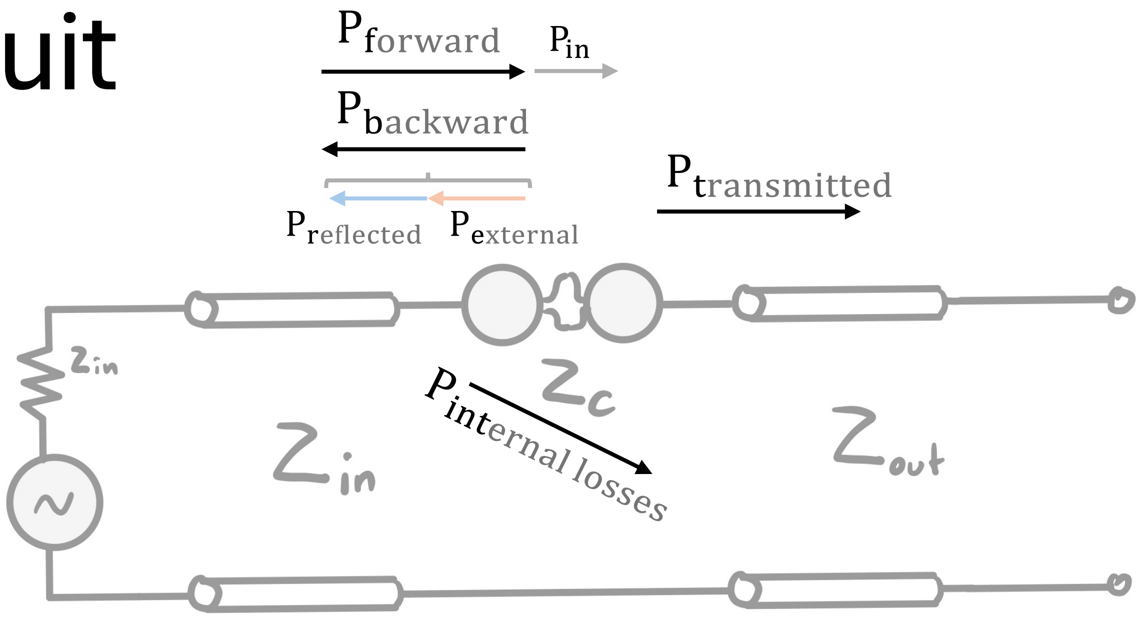
Improved Sensitivity

- Use internal amplifier in DAQ for better resolution
- Cross correlate two detectors to suppress electronic noise





MAGO Equivalent Circuit



On resonance:

$$\frac{Z_{in}}{Z_c} = \frac{Q_{int}}{Q_e} = \frac{P_e}{P_{int}} \equiv \beta_{in}$$

$$\frac{Z_{out}}{Z_c} = \frac{Q_{int}}{Q_{out}} = \frac{P_t}{P_{int}} \equiv \beta_{out}$$

In steady state: $P_{in} = P_e + P_{int} + P_t$

$$\frac{P_b}{P_f} = \left(\frac{1+\beta_{out}-\beta_{in}}{1+\beta_{in}+\beta_{out}}\right)^2, \quad \frac{P_t}{P_f} = \frac{4\,\beta_{in}\,\beta_{out}}{(1+\beta_{in}+\beta_{out})^2}, \quad \frac{P_{int}}{P_f} = \frac{4\,\beta_{in}}{(1+\beta_{in}+\beta_{out})^2},$$ Stored energy $U_0 = \frac{4\,\beta_{in}}{(1+\beta_{in}+\beta_{out})^2}\frac{Q_{int}}{\omega_0}P_f$

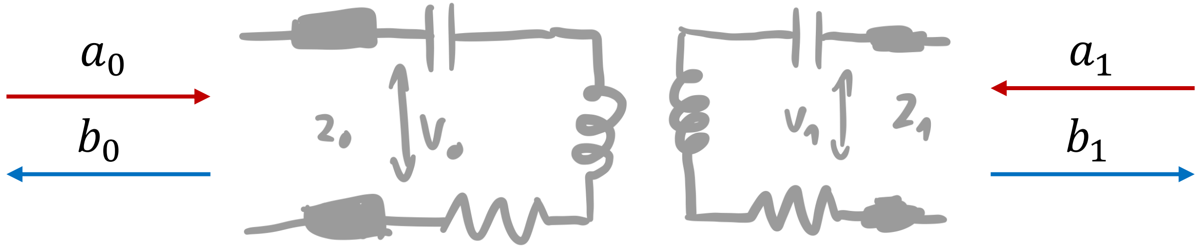
No backwards power, maximal stored energy, maximal transmission for $\boldsymbol{\beta_{in} = \beta_{out} + 1}$ (if β_{out} is fixed)

'critical coupling' $\frac{P_b}{P_f} = 0$, $\frac{P_t}{P_f} = \frac{\beta_{out}}{1+\beta_{out}}$, $\frac{P_{int}}{P_f} = \frac{1}{1+\beta_{out}}$, Stored energy $U_0 = \frac{1}{1+\beta_{out}}\frac{Q_{int}}{\omega_0}P_f$, $Q_L = \frac{1}{1+\beta_{out}}\frac{Q_{int}}{2}$

$\beta_{in} = 1$ is not really critical coupling anymore!

Taking cavity coupling into account

Consider forward- (a) and backward- (b) travelling power waves



Impedance of single cavity cell: Z_c
Coupling impedance $Z_{cpl} = i\omega L_{cpl}$

The power relationships can be obtained from the S matrix

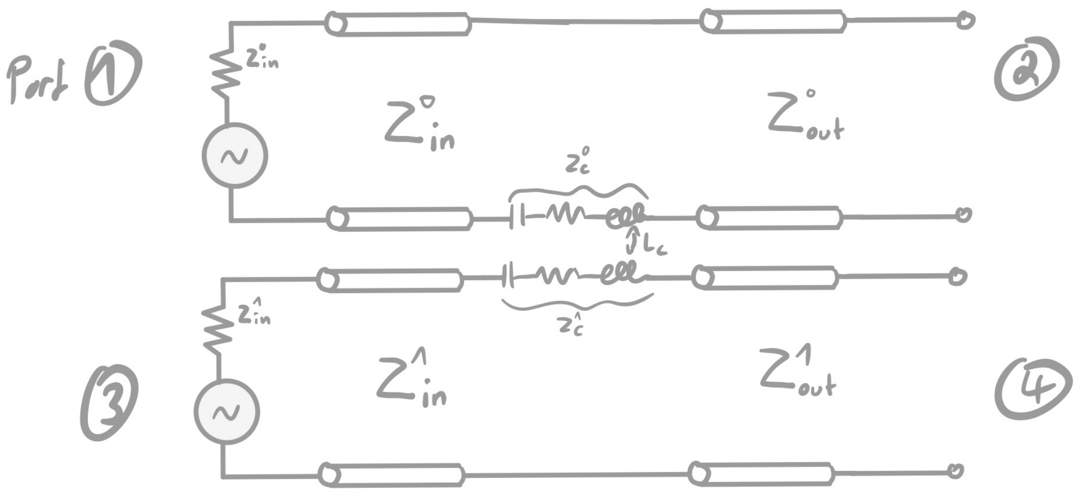
$$\begin{pmatrix} \boldsymbol{b_0} \\ \boldsymbol{b_1} \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} \boldsymbol{a_0} \\ \boldsymbol{a_1} \end{pmatrix}$$

In this case: $\boldsymbol{S} = \frac{1}{(Z_c+Z_1)(Z_c+Z_0)-Z_{cpl}^2} \begin{pmatrix} (Z_c+Z_1)(Z_c-Z_0)-Z_{cpl}^2 & 2\sqrt{Z_0 Z_1}\, Z_{cpl} \\ 2\sqrt{Z_0 Z_1}\, Z_{cpl} & (Z_c-Z_1)(Z_c+Z_0)-Z_{cpl}^2 \end{pmatrix}$

If $Z_0 = Z_1$ is chosen, the most transferred power is the value which maximizes S_{21}: $Z_{0,1} = \sqrt{Z_c^2 - Z_{cpl}^2} \approx R_c$ on resonance

<=> $\beta_{0,1} = 1$ as expected

Four port system

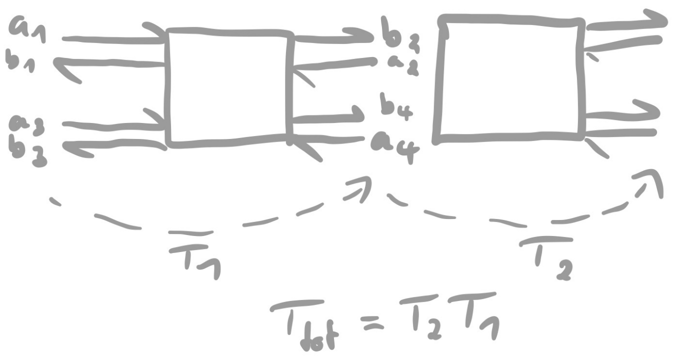


$$S = \frac{1}{Z_{tot}^2 - Z_{cpl}^2}\begin{pmatrix} (Z_{out}+Z_c-Z_{in})Z_{tot} - Z_{cpl}^2 & -2\sqrt{Z_{in}Z_{out}}\,Z_{tot} & -2Z_{in}Z_{cpl} & 2\sqrt{Z_{in}Z_{out}}\,Z_{cpl} \\ -2\sqrt{Z_{in}Z_{out}}\,Z_{tot} & (-Z_{out}+Z_c+Z_{in})Z_{tot} - Z_{cpl}^2 & 2\sqrt{Z_{in}Z_{out}}\,Z_{cpl} & -2Z_{out}Z_{cpl} \\ -2Z_{in}Z_{cpl} & 2\sqrt{Z_{in}Z_{out}}\,Z_{cpl} & Z_{tot}(Z_c - Z_{in} + Z_{out}) - Z_{cpl}^2 & -2\sqrt{Z_{in}Z_{out}}\,Z_{tot} \\ 2\sqrt{Z_{in}Z_{out}}\,Z_{cpl} & -2Z_{out}Z_{cpl} & -2\sqrt{Z_{in}Z_{out}}\,Z_{tot} & Z_{tot}(Z_c + Z_{in} - Z_{out}) - Z_{cpl}^2 \end{pmatrix}$$

for $Z_c^0 = Z_c^1$, $Z_{in}^0 = Z_{in}^1$, $Z_{out}^0 = Z_{out}^1$ and $Z_{tot} := Z_c + Z_{in} + Z_{out}$

Also want to take magic Tees, and reflections at unused ends into account. To find total S matrix, need to consider transfer matrices T.

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = S \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \quad \text{vs} \quad \begin{pmatrix} b_2 \\ a_2 \\ b_4 \\ a_4 \end{pmatrix} = T \begin{pmatrix} a_1 \\ b_1 \\ a_3 \\ b_3 \end{pmatrix}$$



Finding the S matrix of cascaded system:

- First, find T matrix of all components
- multiply them
- Convert the total T matrix into a S matrix

Cascaded Network

S Matrix

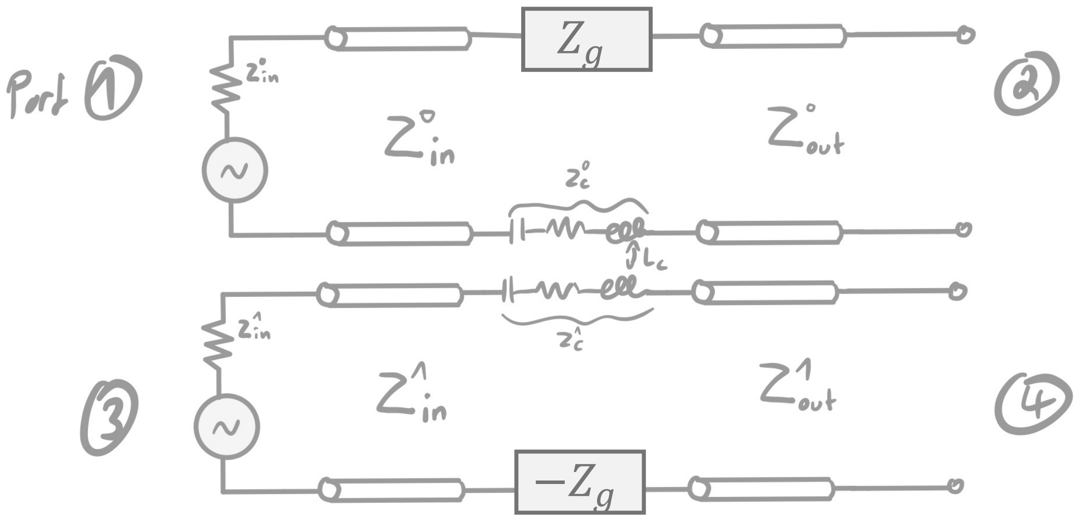
Basic approach

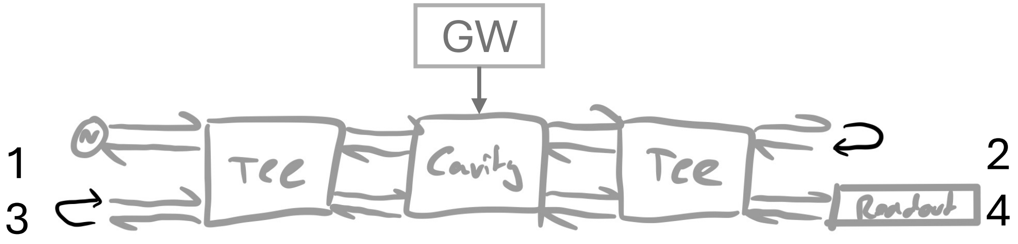
$$\begin{pmatrix} 1-\frac{2\,\text{Zin}}{\text{Zc}-\text{Zcpl}+\text{Zin}+\text{Zout}} & -\frac{2\sqrt{\text{Zin}\,\text{Zout}}}{\text{Zc}+\text{Zcpl}+\text{Zin}+\text{Zout}} & 0 & 0 \\ -\frac{2\sqrt{\text{Zin}\,\text{Zout}}}{\text{Zc}+\text{Zcpl}+\text{Zin}+\text{Zout}} & 1-\frac{2\,\text{Zout}}{\text{Zc}-\text{Zcpl}+\text{Zin}+\text{Zout}} & 0 & 0 \\ 0 & 0 & 1-\frac{2\,\text{Zin}}{\text{Zc}+\text{Zcpl}+\text{Zin}+\text{Zout}} & -\frac{2\sqrt{\text{Zin}\,\text{Zout}}}{\text{Zc}-\text{Zcpl}+\text{Zin}+\text{Zout}} \\ 0 & 0 & -\frac{2\sqrt{\text{Zin}\,\text{Zout}}}{\text{Zc}-\text{Zcpl}+\text{Zin}+\text{Zout}} & 1-\frac{2\,\text{Zout}}{\text{Zc}+\text{Zcpl}+\text{Zin}+\text{Zout}} \end{pmatrix}$$

‘Sensitivity enhancement (MAGO)’

$$\begin{pmatrix} 1-\frac{8\,\text{Zcpl}\,\text{Zin}\,(\text{Zc}+\text{Zin}+\text{Zout})}{(\text{Zc}-\text{Zcpl}+\text{Zin}+\text{Zout})\,(\text{Zc}+\text{Zcpl}+\text{Zin}+\text{Zout})^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1+\frac{8\,\text{Zcpl}\,\text{Zout}\,(\text{Zc}+\text{Zin}+\text{Zout})}{(\text{Zc}-\text{Zcpl}+\text{Zin}+\text{Zout})^2\,(\text{Zc}+\text{Zcpl}+\text{Zin}+\text{Zout})} \end{pmatrix}$$

Consider GW Signal





Add small anti-symmetric excitation through positive and negative impedance in both circuits

$$(Z_{in} + Z_{out} + Z_c)I_1 = -Z_g I_1 - Z_{cpl} I_2 + U_0$$

$$(Z_{in} + Z_{out} + Z_c)I_2 = +Z_g I_2 - Z_{cpl} I_1 + U_0$$

→ Signal strength is proportional to pump energy

→ But at the same frequency as pump mode ($\omega_{GW} = 0$)

Basic approach $\frac{P_{sig}}{P_f} = 4\, Z_g^2 \frac{Z_{in}\, Z_{out}}{Z_{cpl}^2 - Z_{tot}^2}$

'Sensitivity enhancement (MAGO)' $\frac{P_{sig}}{P_f} = 64\, Z_g^2 \frac{Z_{in}\, Z_{out}\, Z_{cpl}^4}{(Z_{cpl}^2 - Z_{tot}^2)\hat{}4}$