

Recap: the Ising model

Let $\sigma = \pm 1$ be binary variables

the classical Ising model is denoted by:

$$H = J_{12} \sigma_1 \sigma_2 + J_{13} \sigma_1 \sigma_3 + \dots = \sum_{i < j} J_{ij} \sigma_i \sigma_j$$

Hamiltonian of a spin system coupling N binary

variables $\sigma_0 \dots \sigma_{N-1}$

J_{ij} denotes the interaction coefficients

by conv. $H = - \sum_{i < j} J_{ij} \sigma_i \sigma_j$

the quantum Ising model Hamiltonian

$$\hat{H} = \sum_{i < j} J_{ij} \hat{z}_i \otimes \hat{z}_j$$

$\sigma_i \rightarrow \hat{\sigma}_i$ (operator)

$|\uparrow\rangle \leftrightarrow |\downarrow\rangle \rightarrow \sigma = -1$, negative z direction

Both states are eigenvectors of the magnetic momentum operator

denoted by \hat{z}

if $|\underline{z}\rangle = |z_1, \dots, z_N\rangle$, the eigenstates are config. of spins

$$\hat{z}_j |z_j\rangle = (-1)^{z_j} |z_j\rangle = \sigma_j |z_j\rangle$$

the energy of the ground state

$$\langle z | \hat{H} | z \rangle = \sum_{i < j} J_{ij} \sigma_i \sigma_j$$

is the minimum energy of the classical model

the Quantum Ising Model (transverse-field)

$$\hat{H}_{QI} = - \sum_{i < j} J_{ij} \hat{z}_i \otimes \hat{z}_j - \sum_i h_i \hat{x}_i$$

here, \hat{z}_α & \hat{x}_α are the Pauli operators on qubit at index α .

* $\hat{H}_{QI} =$ terms in z -direction + terms in x -direction

h_i : excites superposition of qubits

$$\hat{H}_{QI} = \sum_{i < j} J_{ij} \hat{z}_i \hat{z}_j - \sum_i h_i \hat{x}_i$$

One qubit Quantum Ising model

$$\hat{H}_{QI} = -J\hat{Z} - h\hat{X}$$

in matrix form

$$\hat{H}_{QI} = \begin{pmatrix} -J & -h \\ -h & J \end{pmatrix}$$

eigenstates

$$\hat{H}_{QI} |E\rangle = E |E\rangle$$

eigenvalues

$$E_{\pm} = \pm \sqrt{J^2 + h^2}$$

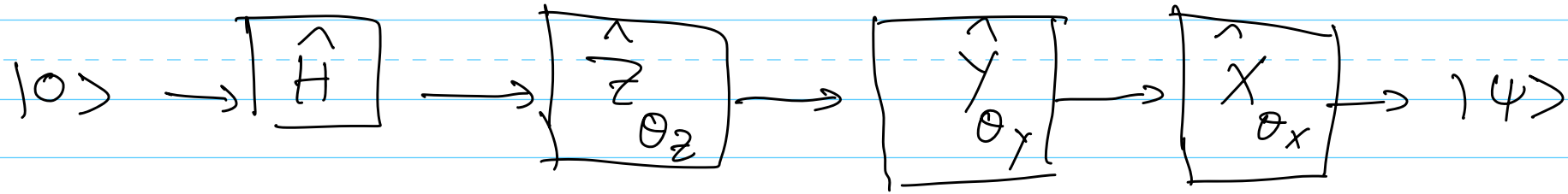
eigen vectors (normalized)

$$|E_{\pm}\rangle = \frac{1}{\sqrt{h^2 + (\pm j \pm \sqrt{h^2 + j^2})^2}} \begin{pmatrix} \pm j \pm \sqrt{h^2 + j^2} \\ h \end{pmatrix}$$

Quantum ML with Quantum Ising Model

recall: Quantum feature maps the classical data
to quantum data

target: define a variational quantum model to
minimize the quantum Ising Hamiltonian
with single qubit



$$|\psi\rangle = e^{j\theta_x \hat{x}} e^{j\theta_y \hat{y}} e^{j\theta_z \hat{z}} \hat{H} |0\rangle$$

Variational ansatz is determined by \hat{U}

$$\hat{U} = e^{j\theta_x \hat{x}} e^{j\theta_y \hat{y}} e^{j\theta_z \hat{z}} \hat{H}$$

θ_x, θ_y & $\theta_z \rightarrow$ parameters of the variational model

$$\underline{\theta} = (\theta_x, \theta_y, \theta_z)$$

\hat{U} is a quantum feature map that maps the classical $\underline{\theta}$ into a one qubit Hilbert space

Implementation with Tensorflow

A simple N.N. Layer is implemented computing the expected value of the Hamiltonian $\langle \hat{H} \rangle$ at the initial state

the model is optimized to compute $\hat{\Theta}^{\text{opt}}$ that minimize the output of the Hamilt.

Layer

the model is trained by minimizing the mean value of the Hamiltonian - a loss function

* Tests

* the model is trained where the final state is compared to

$$|E_{-}\rangle = \frac{1}{\sqrt{h^2 + (j + \sqrt{h^2 + j^2})^2}} \begin{pmatrix} j + \sqrt{h^2 + j^2} \\ h \end{pmatrix}$$

for diff. values of j & h

$$\underline{h=0, \quad j=1} \rightarrow |E_{-}\rangle = |0\rangle = |\downarrow\rangle \left\{ \begin{array}{l} \text{eigenvalue of } \hat{z} \\ \text{with } E_{-} = -j \\ \quad = -1 \end{array} \right.$$

$$\underline{h=0, \quad j=1.0} \rightarrow \langle \hat{H} \rangle = -1$$

$$h=0.5, \quad j=0 \rightarrow \langle \hat{H} \rangle = -0.5 \quad \text{i.e.} \left\{ \begin{array}{l} E_{-} = -h \\ |E_{-}\rangle = |10\rangle \end{array} \right.$$

$$h = 1.7, \quad \beta = 1 \quad \rightarrow \quad \langle \hat{H} \rangle = -1.97 \text{ (as expected)}$$

