MAGO2.0 Meeting 29.05.24

MAGO Input/Readout Theory

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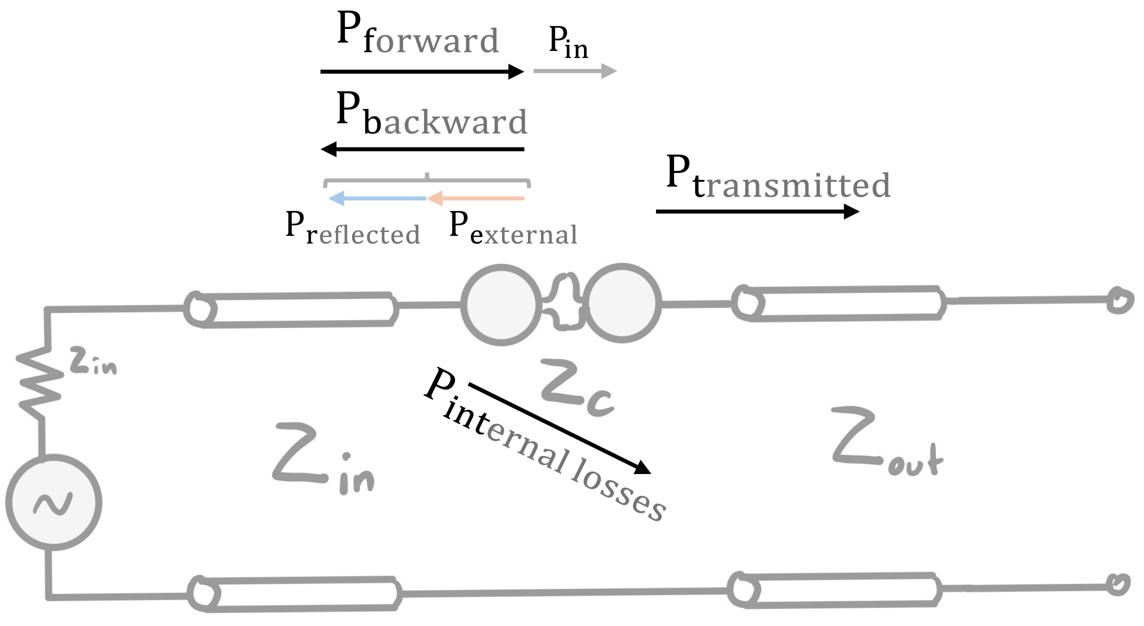
MAGO 2-Port Circuit



On resonance:

$$\frac{Z_{in}}{Z_c} = \frac{Q_{int}}{Q_e} = \frac{P_e}{P_{int}} \equiv \beta_{in}$$

$$\frac{Z_{out}}{Z_c} = \frac{Q_{int}}{Q_{out}} = \frac{P_t}{P_{int}} \equiv \beta_{out}$$

In steady state: $P_{in} = P_e + P_{int} + P_t$

$$\frac{P_b}{P_f} = \left(\frac{1+\beta_{out}-\beta_{in}}{1+\beta_{in}+\beta_{out}}\right)^2, \quad \frac{P_t}{P_f} = \frac{4\,\beta_{in}\,\beta_{out}}{(1+\beta_{in}+\beta_{out})^2}, \quad \frac{P_{int}}{P_f} = \frac{4\,\beta_{in}}{(1+\beta_{in}+\beta_{out})^2},$$ Stored energy $U_0 = \frac{4\,\beta_{in}}{(1+\beta_{in}+\beta_{out})^2}\frac{Q_{int}}{\omega_0}P_f$

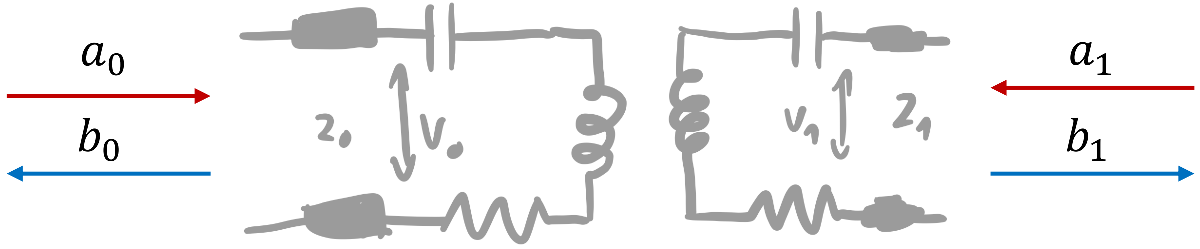
No backwards power, maximal stored energy, maximal transmission for $\boldsymbol{\beta_{in} = \beta_{out} + 1}$ (if β_{out} is fixed)

'critical coupling' $\frac{P_b}{P_f} = 0$, $\frac{P_t}{P_f} = \frac{\beta_{out}}{1+\beta_{out}}$, $\frac{P_{int}}{P_f} = \frac{1}{1+\beta_{out}}$, Stored energy $U_0 = \frac{1}{1+\beta_{out}}\frac{Q_{int}}{\omega_0}P_f$, $Q_L = \frac{1}{1+\beta_{out}}\frac{Q_{int}}{2}$

$\beta_{in} = 1$ is not really critical coupling anymore!

Taking cavity coupling into account

Consider forward- (a) and backward- (b) travelling power waves



Impedance of single cavity cell: Z_c
Coupling impedance $Z_{cpl} = i\omega L_{cpl}$

The power relationships can be obtained from the S matrix

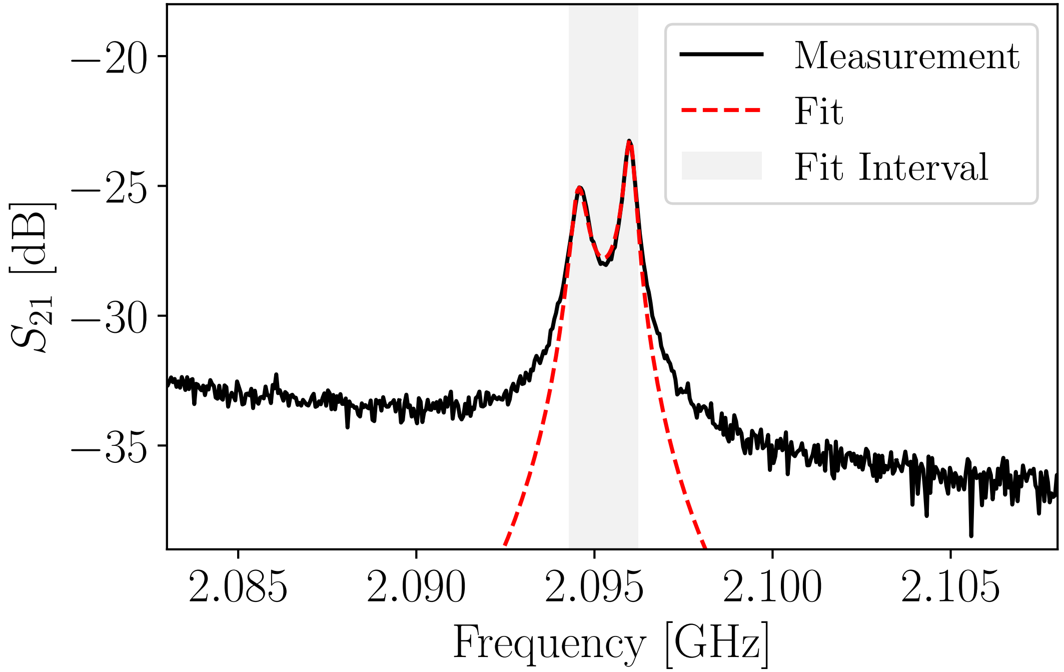
$$\begin{pmatrix} \boldsymbol{b_0} \\ \boldsymbol{b_1} \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} \boldsymbol{a_0} \\ \boldsymbol{a_1} \end{pmatrix}$$

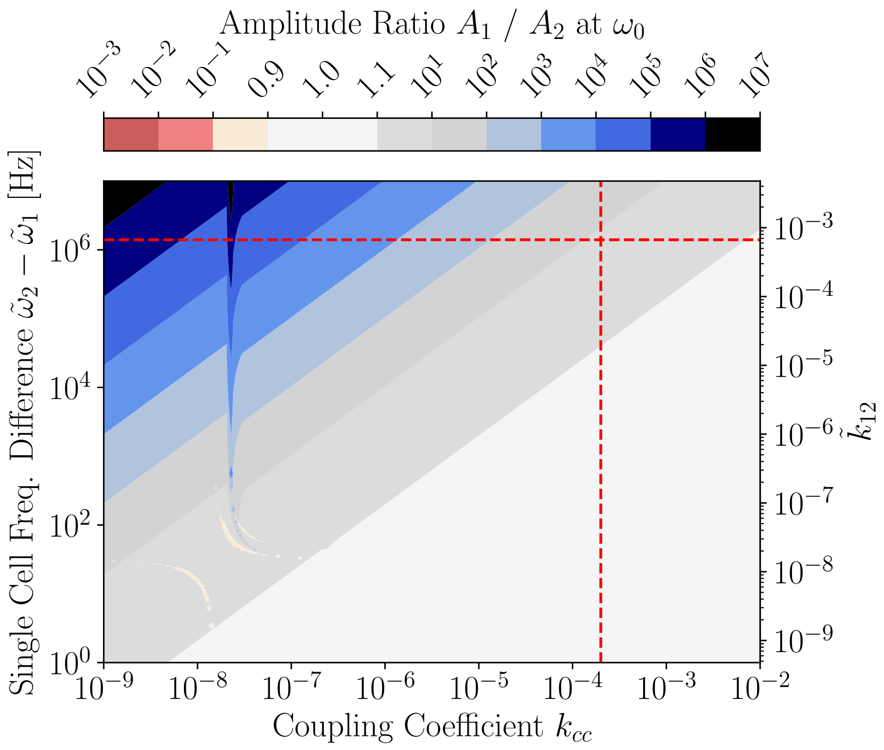
In this case: $\boldsymbol{S} = \frac{1}{(Z_c+Z_1)(Z_c+Z_0)-Z_{cpl}^2} \begin{pmatrix} (Z_c+Z_1)(Z_c-Z_0)-Z_{cpl}^2 & 2\sqrt{Z_0 Z_1}\, Z_{cpl} \\ 2\sqrt{Z_0 Z_1}\, Z_{cpl} & (Z_c-Z_1)(Z_c+Z_0)-Z_{cpl}^2 \end{pmatrix}$

If $Z_0 = Z_1$ is chosen, the most transferred power is the value which maximizes S_{21}: $Z_{0,1} = \sqrt{Z_c^2 - Z_{cpl}^2} \approx R_c$ on resonance

<=> $\beta_{0,1} = 1$ as expected

Fit to real data





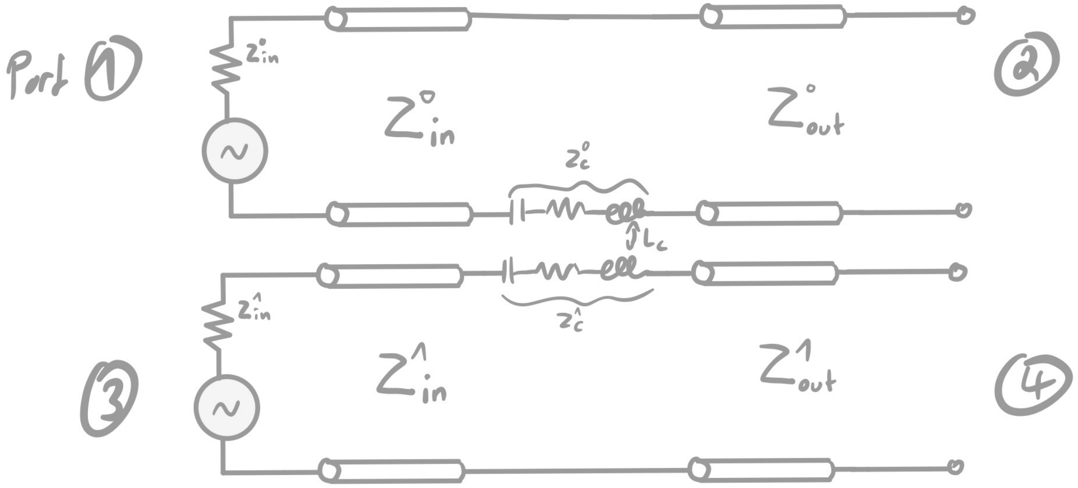
The S_{21} parameter from last slide fit to our RF measurement.

The parameters for the fit curve are $\omega_1 = 2.09457$ GHz, $\omega_1 = 2.09597$ GHz, $k_{cc} = 2 \cdot 10^{-4}$, $Q_1 = 5 \cdot 10^3$ $Q_2 = 8 \cdot 10^3$

These values are robust against the precise choice of the remaining resistances.

E.g. for $R = R_1 = R_2$ any resistances of kΩ to GΩ with $\frac{\sqrt{Z_{in} Z_{out}}}{R} = 0.013$ reproduce the above fit.

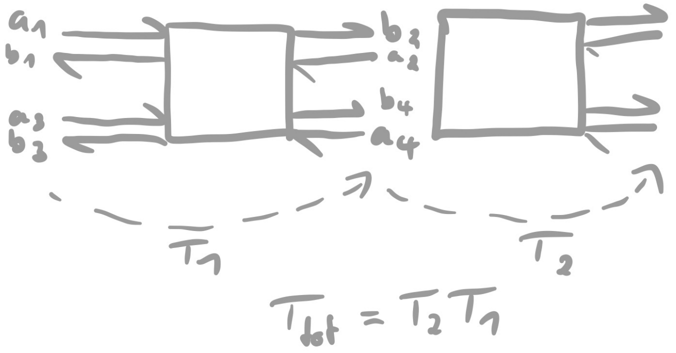
Four port system



$$S_{\text{cavity}} = \frac{1}{Z_{\text{tot}}^2 - Z_{\text{cpl}}^2} \cdot \tag{5.1}$$
$$\left[\begin{pmatrix} Z_{\text{tot}}(Z_{\text{c}} - Z_{\text{in}} + Z_{\text{out}}) & 2Z_{\text{tot}}\sqrt{Z_{\text{in}}Z_{\text{out}}} & 2Z_{\text{cpl}}Z_{\text{in}} & -2Z_{\text{cpl}}\sqrt{Z_{\text{in}}Z_{\text{out}}} \\ 2Z_{\text{tot}}\sqrt{Z_{\text{in}}Z_{\text{out}}} & Z_{\text{tot}}(Z_{\text{c}} + Z_{\text{in}} - Z_{\text{out}}) & -2Z_{\text{cpl}}\sqrt{Z_{\text{in}}Z_{\text{out}}} & 2Z_{\text{cpl}}Z_{\text{out}} \\ 2Z_{\text{cpl}}Z_{\text{in}} & -2Z_{\text{cpl}}\sqrt{Z_{\text{in}}Z_{\text{out}}} & Z_{\text{tot}}(Z_{\text{c}} - Z_{\text{in}} + Z_{\text{out}}) & 2Z_{\text{tot}}\sqrt{Z_{\text{in}}Z_{\text{out}}} \\ -2Z_{\text{cpl}}\sqrt{Z_{\text{in}}Z_{\text{out}}} & 2Z_{\text{cpl}}Z_{\text{out}} & 2Z_{\text{tot}}\sqrt{Z_{\text{in}}Z_{\text{out}}} & Z_{\text{tot}}(Z_{\text{c}} + Z_{\text{in}} - Z_{\text{out}}) \end{pmatrix} - Z_{\text{cpl}}^2 \mathbb{1}\right]$$

Also want to take magic Tees, and reflections at unused ends into account. To find total S matrix, need to consider transfer matrices T.

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \quad \text{vs} \quad \begin{pmatrix} b_2 \\ a_2 \\ b_4 \\ a_4 \end{pmatrix} = \boldsymbol{T} \begin{pmatrix} a_1 \\ b_1 \\ a_3 \\ b_3 \end{pmatrix}$$



Finding the S matrix of cascaded system:
- First, find T matrix of all components
- multiply them
- Convert the total T matrix into a S matrix

Cascaded Network

Basic approach

S Matrix

$$\begin{pmatrix} \frac{Zc+Zcpl-Zin+Zout}{Zc+Zcpl+Zin+Zout} & \frac{2\sqrt{Zin\ Zout}}{Zc+Zcpl+Zin+Zout} & 0 & 0 \\ \frac{2\sqrt{Zin\ Zout}}{Zc+Zcpl+Zin+Zout} & \frac{Zc+Zcpl+Zin-Zout}{Zc+Zcpl+Zin+Zout} & 0 & 0 \\ 0 & 0 & \frac{Zc-Zcpl-Zin+Zout}{Zc-Zcpl+Zin+Zout} & \frac{2\sqrt{Zin\ Zout}}{Zc-Zcpl+Zin+Zout} \\ 0 & 0 & \frac{2\sqrt{Zin\ Zout}}{Zc-Zcpl+Zin+Zout} & \frac{Zc-Zcpl+Zin-Zout}{Zc-Zcpl+Zin+Zout} \end{pmatrix}$$

$$U_0 = \frac{4\,\beta_{in}}{(1+\beta_{in}+\beta_{out})^2}\frac{Q_{int}}{\omega_0}P_f \leq \frac{1}{1+\beta_{out}}\frac{Q_{int}}{\omega_0}P_f$$

for $\beta_{in} = 1 + \beta_{out}$

‘Sensitivity enhancement (MAGO)’

$$\begin{pmatrix} \frac{Zc+Zcpl-Zin}{Zc+Zcpl+Zin} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{-Zc+Zcpl+Zout}{Zc-Zcpl+Zout} \end{pmatrix}$$

→ Can couple to both modes **separately**!

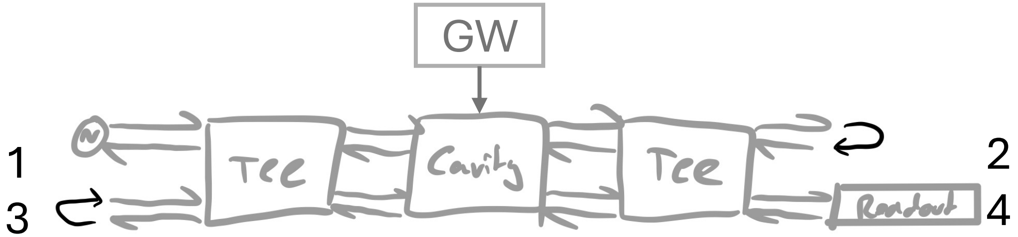
→ Pump mode can be driven at ideal efficiency

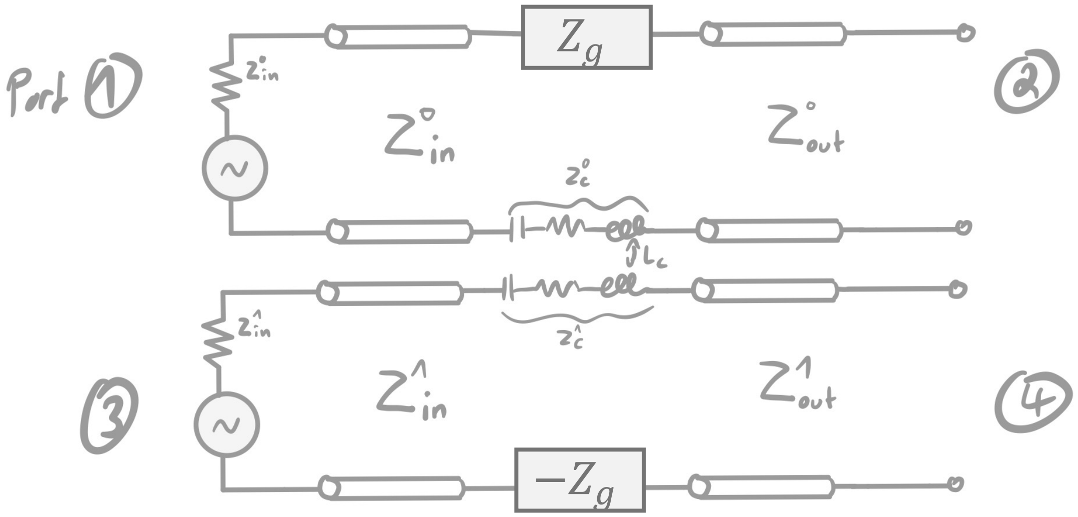
(assuming negligible power is lost in the reflection and antennas)

$$U_0 = \frac{4\,\beta_{in}}{(1+\beta_{in})^2}\frac{Q_{int}}{\omega_0}P_f \leq \frac{Q_{int}}{\omega_0}P_f$$

for $\beta_{in} = 1$ and $\beta_{out} \in \mathbb{R}$

Consider GW Signal





Add small anti-symmetric excitation through positive and negative impedance in both circuits

$$(Z_{in} + Z_{out} + Z_c)I_1 = -Z_g I_1 - Z_{cpl} I_2 + U_0$$

$$(Z_{in} + Z_{out} + Z_c)I_2 = +Z_g I_2 - Z_{cpl} I_1 + U_0$$

→ Signal strength is proportional to pump energy

→ But at the same frequency as pump mode ($\omega_{GW} = 0$)

Basic approach $$\left(\frac{P_{sig}}{P_f}\right)^{1/2} = \frac{2Z_g\sqrt{Z_{in}Z_{out}}}{(Z_c+Z_{cpl}+Z_{in}+Z_{out})(Z_c-Z_{cpl}+Z_{in}+Z_{out})}$$

‘Sensitivity enhancement (MAGO)’ $$\left(\frac{P_{sig}}{P_f}\right)^{1/2} = \frac{2Z_g\sqrt{Z_{in}Z_{out}}}{(Z_c+Z_{cpl}+Z_{in})(Z_c-Z_{cpl}+Z_{out})}$$

Ratio: $\left(\frac{P_{sig}^{SE}}{P_{sig}^{Basic}}\right)^{1/2} = \frac{(Z_c+Z_{cpl}+Z_{in}+Z_{out})(Z_c-Z_{cpl}+Z_{in}+Z_{out})}{(Z_c+Z_{cpl}+Z_{in})(Z_c-Z_{cpl}+Z_{out})} > 1$ and $\left(\frac{P_{sig}^{SE}}{P_{sig}^{Basic}}\right)^{1/2} \approx \frac{1+\beta_{out}}{1+\beta_{in}}$ for $\beta_{out} \gg \beta_{in}$

Summary

- The 'sensitivity enhancement' circuit allows coupling to the pump and signal mode separately (i.e. overcoupling the *readout* does not affect the loaded Q of the *pump mode*)

⇒ The pump mode can be loaded with the same energy using the same forward power irrespective of the output coupling

→This depends on the quality of the reflection/antennas/magic Tee

- The 'sensitivity enhancement' circuit improves the GW signal power, but not by a lot

→Only significantly if $\beta_{out} \gg \beta_{in}$ when more energy can be stored in the pump mode