Spinning Gluon and Long Range Correlations at the LHC and EIC

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Guo, Liu, Yuan, Zhu, arXiv: 2406.05880;

to appear

EIC Science: from quark/gluon to cosmo

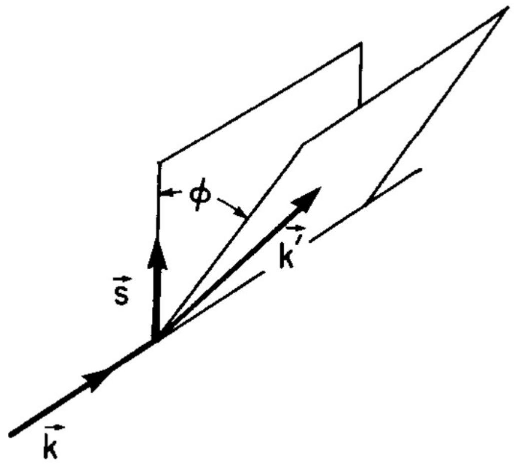
- How do the nucleonic properties such as mass and spin emerge from partons and their underlying interactions?
- How are partons inside the nucleon distributed in both momentum and position space?
- What happens to the gluon density in nucleons and nuclei at small *x*? Does it saturate at high energy, giving rise to gluonic matter with universal properties in all nuclei (and perhaps even in nucleons)?
- How do color-charged quarks and gluons, and jets, interact with a nuclear medium? How do confined hadronic states emerge from these quarks and gluons? How do the quark-gluon interactions generate nuclear binding?
- Do signals from beyond-the-standard-model physics manifest in electron-proton/ion collisions? If so, what can we learn about the nature of these new particles and forces?

EIC Whitepaper for LRP
QCD Whitepaper, 2303.02579, NPA 2024

Spinning gluon: nontrivial part of nucleon tomography

- Gluon is always spinning in high energy/small-x
  - Spinning gluon in inclusive DIS
  - Spinning gluon (helicity-flip) in GPD
  - Spinning gluon (linearly polarized) in TMD
- Spinning gluon in Nucleon EEC
  - Two particle correlations in DIS
  - Long range correlation at the LHC

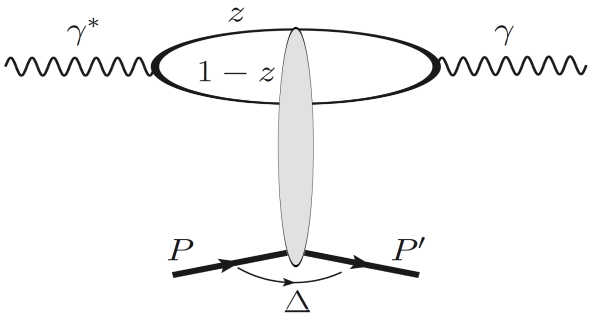
In the context of inclusive DIS: nuclear gluonometry for J>1



Jaffe, Manohar, 1989

- Structure function difference between polarization along x and y directions, i.e., $\cos(2\phi_S)$ asymmetry
- For nucleons, the asymmetry vanishes in QCD
- Nontrivial asymmetry for nucleus with J>1, e.g., deuteron, only receives contributions from the spinning gluons

Spinning gluon in exclusive processes: GPD framework



Gluon Tomography and Wigner distribution:

$$x\mathcal{W}_g^T(x,|\vec{q}_\perp|,|\vec{b}_\perp|)+2\cos(2\phi)x\mathcal{W}_g^\epsilon(x,|\vec{q}_\perp|,|\vec{b}_\perp|)$$

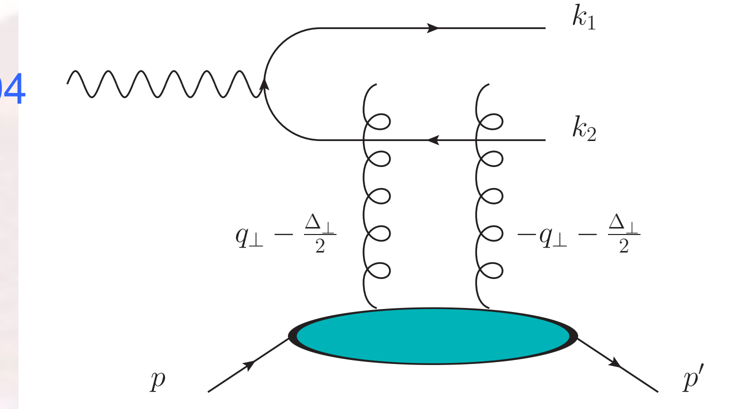
$$\frac{1}{P^+}\int\frac{d\zeta^-}{2\pi}e^{ixP^+\zeta^-}\langle p'|F^{+i}(-\zeta/2)F^{+j}(\zeta/2)|p\rangle$$
$$=\frac{\delta^{ij}}{2}xH_g(x,\Delta_\perp)+\frac{xE_{Tg}(x,\Delta_\perp)}{2M^2}\left(\Delta_\perp^i\Delta_\perp^j-\frac{\delta^{ij}\Delta_\perp^2}{2}\right)+$$

Hoodbhoy-Ji 98
Diehl 01

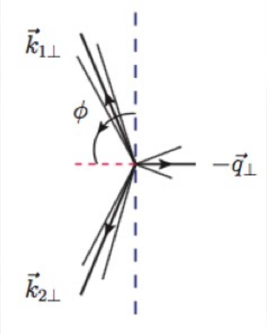
- A nontrivial tomography distribution of gluon inside the nucleon
- It contributes to a $\cos(2\phi)$ in photon production (DVCS)

Semi-exclusive process: Diffractive Dijet to Probe the Gluon Tomoraphy

Earlier studies:
Nikolaev, Zakharov 1994
Bartles, Ewerz, Lotter, Wusthoff 1996
Diehl 1996
Braun, Ivanov 2005
…



Hatta-Xiao-Yuan,1601.01585



cos(2ϕ) anisotropy

- The diffractive dijet cross section is proportional to the square of the Wigner distribution → nucleon/nucleus tomography

$$xW_g^T(x, |\vec{q}_\perp|, |\vec{b}_\perp|) + 2\cos(2\phi)xW_g^\epsilon(x, |\vec{q}_\perp|, |\vec{b}_\perp|)$$

More correlations to study OAM, Spin-Orbital Correlations, …

Boussarie-Hatta-Bhattacharya, 2022,2024

Spinning gluon in semi-inclusive process: TMD framework

- Linearly polarized gluon distribution

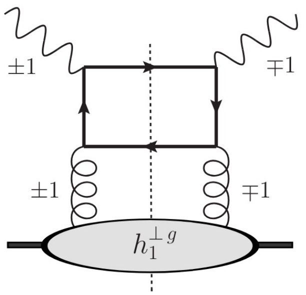
Mulders, Rodrigues, 20021

$$\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P,S|F^{+i}(0)\,F^{+j}(\xi)|P,S\rangle$$

$$\left[-g_T^{ij}G(x,\boldsymbol{k}_T^2)+\left(\frac{k_T^i k_T^j}{M^2}+g_T^{ij}\frac{\boldsymbol{k}_T^2}{2M^2}\right)H^{\perp}(x,\boldsymbol{k}_T^2)\right]$$

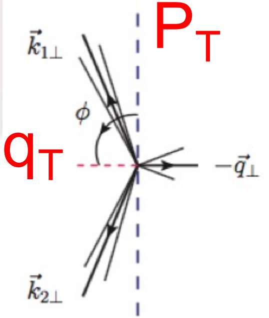
- Can be measured through TMD processes, such as heavy quark pair production in DIS

Boer, Brodsky, Mulders, Pisano, 2011

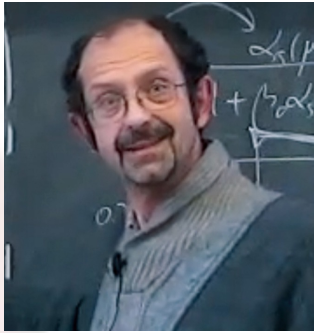


$$\left[A+\frac{\boldsymbol{q}_T^2}{M^2}\,B\,\cos 2(\phi_T-\phi_\perp)\right]$$

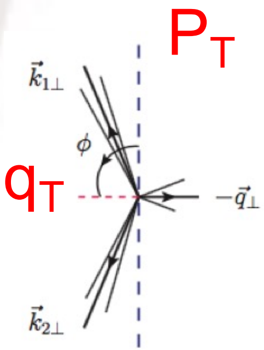
Cos(2ϕ) between the total and difference of the two leading transverse momenta



Soft gluon radiations can generate and mixes the azimuthal asymmetry



- Azimuthal angular asymmetries arise from soft gluon radiations
  - ϕ is defined as angle between total and different transverse momenta of the two final state particles
- Infrared safe but divergent
  - $\langle\cos(\phi)\rangle$, $\langle\cos(2\phi)\rangle$, … divergent, $\sim 1/q_T^2$
  - Examples discussed include Vj, top quark pair production



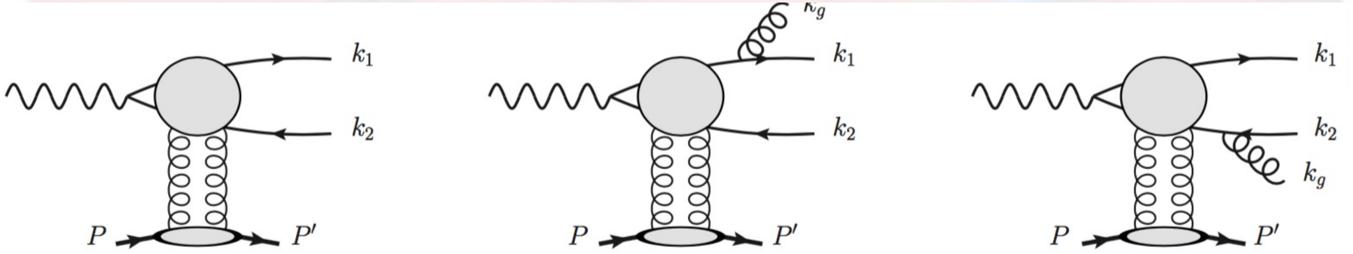
Catani-Grazzini-Sargsyan 2017

Diffractive dijet production

- Gluon radiation tends to be aligned with the jet direction

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2}\int dy_g \left(\frac{k_1 \cdot k_2}{k_1 \cdot k_g k_2 \cdot k_g}\right)_{\vec{q}_\perp = -\vec{k}_{g\perp}}$$

$$S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|) + \cdots$$



Hatta-Xiao-Yuan-Zhou, 2010.10774, 2106.05307

Leading power contributions, explicit result at α_s

$$S_J(q_\perp) = S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|)$$

$$S_{J0}(q_\perp) = \delta(q_\perp) + \frac{\alpha_0}{\pi}\frac{1}{q_\perp^2}\,, \quad S_{J2}(q_\perp) = \frac{\alpha_2}{\pi}\frac{1}{q_\perp^2}\,,$$

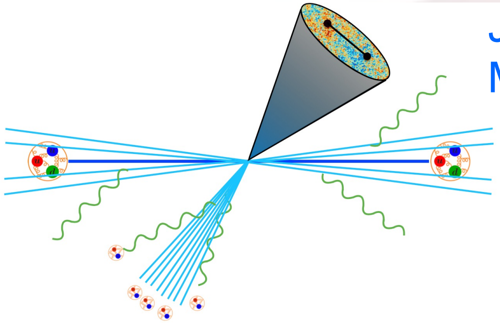
where

$$\alpha_0 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_0}{R^2}\,, \quad \alpha_2 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_2}{R^2}\,.$$

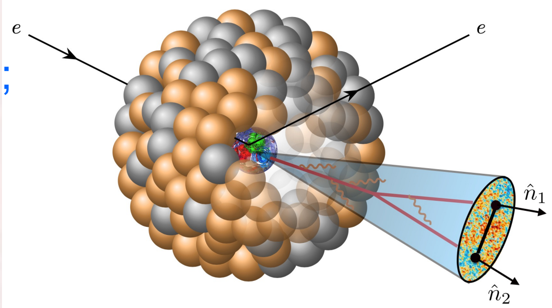
a_0,a_2 are order 1 constants, so,

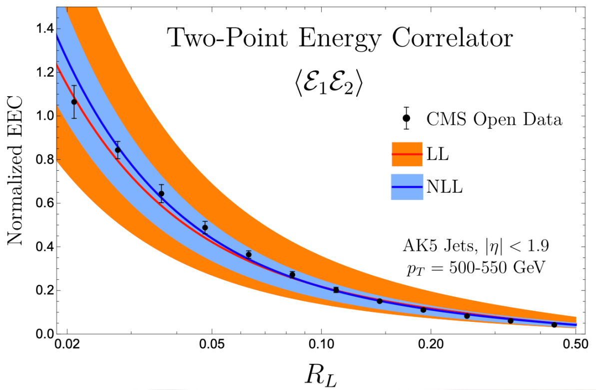
in the small-R limit, <cos(2ϕ)> goes to 1

Energy-Energy Correlators at Colliders

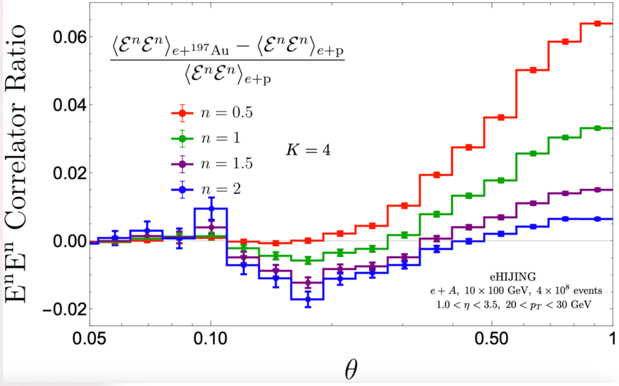


Major focus of recent studies:
Jet substructure
Moult, Zhu, Lee, et al, 20-24;





Lee, Mecaj, Moult, 2205.03414



Devereaux, Fan, Ke, Lee, Moult, 2303.08143

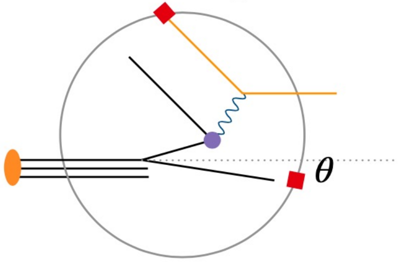
New Avenue: Nucleon EEC and DIS

Liu, Zhu, 2209.02080
Cao, Liu, Zhu, 2303.01530

$$f_{q,EEC}(x,\theta) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi E_P} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle p | \bar{\psi}(0) \mathcal{E}(\theta) \mathcal{L} \psi(y^-) | p \rangle$$

$$= \sum_X \sum_{i \in X} \frac{E_i}{E_P} \delta(\theta_i^2 - \theta^2) \delta((1-x)p^+ - p_X^+) \frac{\gamma^+}{2} \langle p | \bar{\psi}(0) | X \rangle \langle X | \mathcal{L} \psi(0) | p \rangle$$

$$x_B = \frac{Q^2}{2P \cdot q}$$



$$\Lambda_{QCD} \ll Q\theta \ll Q$$

$$\Sigma(x_B, Q^2, \theta) = \int \frac{dz}{z} \hat{\sigma}\left(\frac{x_B}{z}, Q^2, \mu\right) f_{\rm EEC}(z, \theta, \mu)$$

$$\propto \int \frac{dz}{z} \hat{\sigma}\left(\frac{x_B}{z}\right) \frac{1}{\theta^2} \int \frac{d\xi}{\xi} (1 - \frac{z}{\xi}) P(\frac{z}{\xi}) [\xi f(\xi)]$$

→ Perturbative scaling

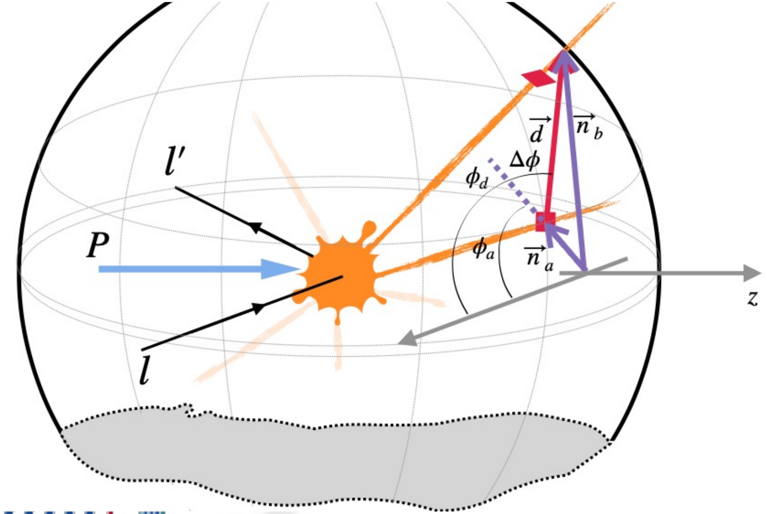
- θ-distribution solely determined by $f_{\rm EEC}$
- In the collinear factorization:
  - $d\Sigma / d\ln\mu = P \otimes \Sigma$, solely determined by the vacuum splitting function
  - $\Sigma \sim \theta^{-2}$ at LO, $\Sigma \sim \theta^{-2+\gamma[\alpha_s]}$ to all orders

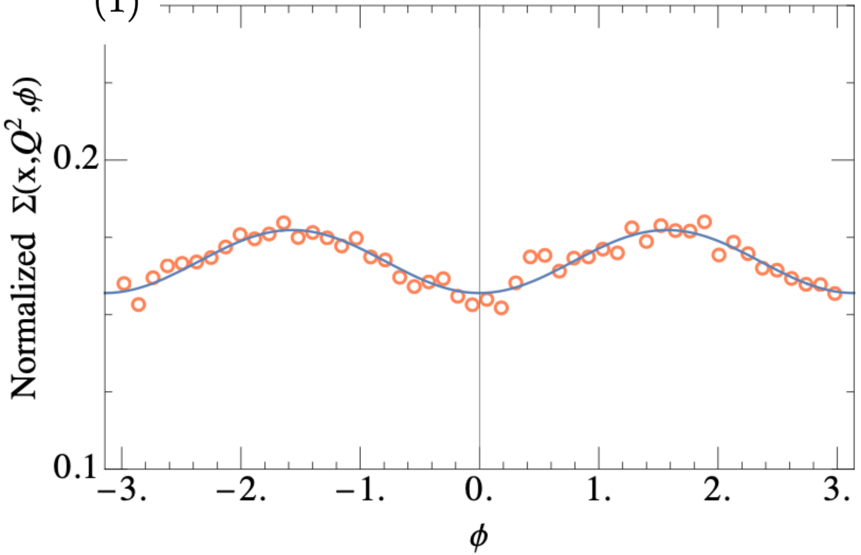
→ All order resummation

Spinning gluon in EEC and two particle cos(2ϕ) correlation in DIS

$$f_{g,\mathrm{EEC}}^{\alpha\beta}(x,\vec{n}_a) = \int \frac{dy^-}{2\pi x P^+} e^{-ixP^+\frac{y^-}{2}} \times \langle P|\mathcal{F}^{+\alpha}(y^-)\,\mathcal{L}^\dagger[\infty,y^-]\hat{\mathcal{E}}(\vec{n}_a)\mathcal{L}[\infty,0]\mathcal{F}^{+\beta}(0)|P\rangle = \left(-g_T^{\alpha\beta}/2\right) f_{g,\mathrm{EEC}} + h_T^{\alpha\beta} d_{g,\mathrm{EEC}}\,, \quad (1)$$

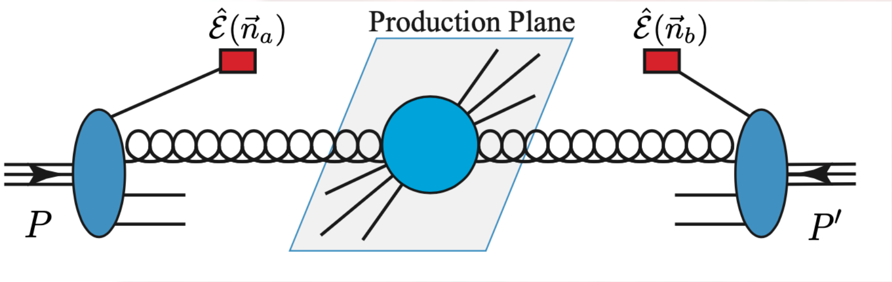
$$h_T^{\alpha\beta} = n_{a,T}^\alpha n_{a,T}^\beta / |n_{a,T}^2| + g_T^{\alpha\beta}/2$$



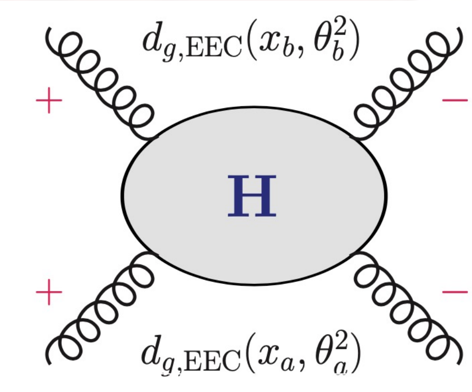


Spinning gluon and long range azimuthal correlation at the LHC

Guo, Liu, Yuan, Zhu, arXiv: 2406.05880



$$\Sigma(Q^2;\theta_{a,b},\phi)$$
$$= \int d\Omega \left\{ x_a f_{g,\mathrm{EEC}}\left(x_a,\theta_a^2\right) x_b f_{g,\mathrm{EEC}}\left(x_b,\theta_b^2\right) \hat{\sigma}_0 \right. \qquad (3)$$
$$\left. + x_a d_{g,\mathrm{EEC}}\left(x_a,\theta_a^2\right) x_b d_{g,\mathrm{EEC}}\left(x_b,\theta_b^2\right) \hat{\sigma}_2(Q^2)\cos(2\phi) \right\} ,$$



Two examples: Higgs, Top quark pair

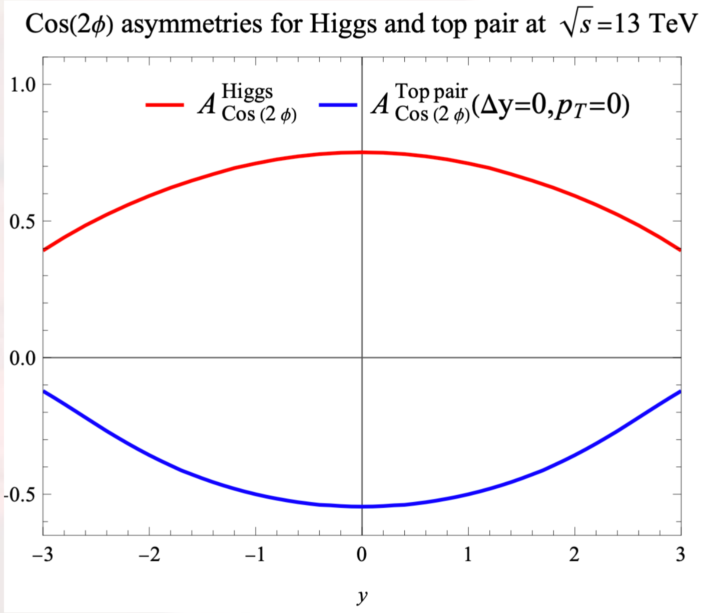
Higgs couples to the spinning gluon directly

$$\hat{\sigma}_2 = \hat{\sigma}_0 = \pi g_\phi^2/64$$

Top quark pair is different

$$\hat{\sigma}_0 = \frac{\alpha_s^2\pi}{\hat{s}^2}\left[\frac{1}{6}\frac{1}{\hat{t}_1\hat{u}_1} - \frac{3}{8}\frac{1}{\hat{s}^2}\right]\left[\hat{t}_1^2 + \hat{u}_1^2 + 4m_t^2\hat{s} - \frac{4m_t^4\hat{s}^2}{\hat{t}_1\hat{u}_1}\right]$$

$$\hat{\sigma}_2 = \frac{\alpha_s^2\pi}{\hat{s}^2}\left[\frac{3}{8}\frac{1}{\hat{s}^2} - \frac{1}{6}\frac{1}{\hat{t}_1\hat{u}_1}\right]\frac{2m_t^4\hat{s}^2}{\hat{t}_1\hat{u}_1}, \quad (7)$$



Quantum Entanglement and Bell Inequality

Define Observable:

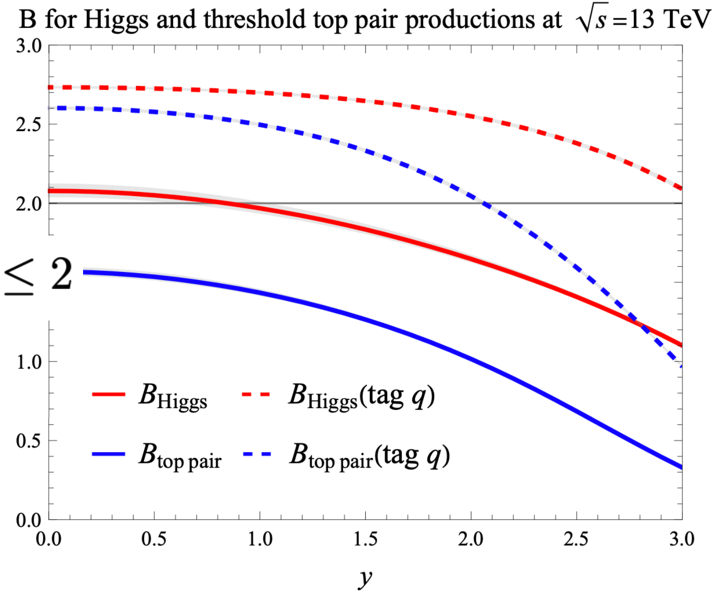
$$S(\phi_a,\phi_b) \equiv \frac{\Sigma(\phi_a,\phi_b)+\Sigma(\phi'_a,\phi'_b)-\Sigma(\phi'_a,\phi_b)-\Sigma(\phi_a,\phi'_b)}{\Sigma(\phi_a,\phi_b)+\Sigma(\phi'_a,\phi'_b)+\Sigma(\phi'_a,\phi_b)+\Sigma(\phi_a,\phi'_b)}$$

Construct the Clauser-Horne-Shimony-Holt (CHSH) Inequality:

$$B \equiv |S(\phi_a,\phi_b) - S(\phi_a,\tilde{\phi}_b) + S(\tilde{\phi}_a,\phi_b) + S(\tilde{\phi}_a,\tilde{\phi}_b)| \leq 2$$

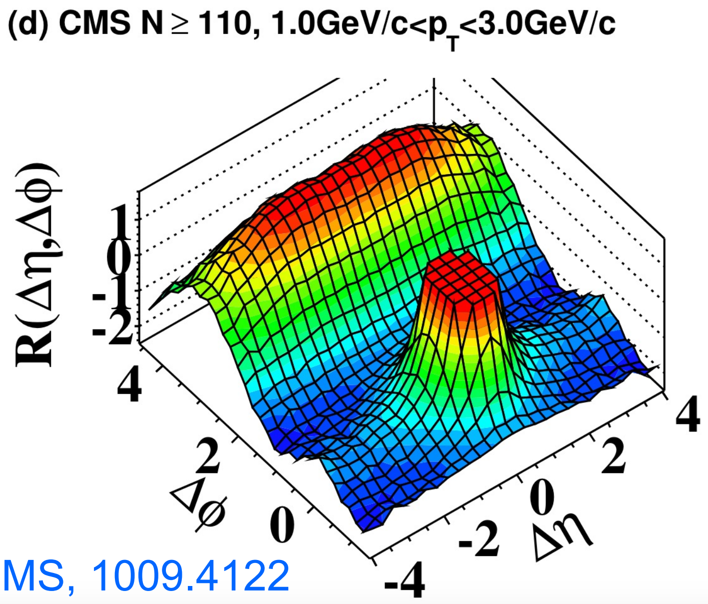
$$\phi' = \phi + \frac{\pi}{2},$$

$$\phi_a = 0,\ \phi_b = \frac{\pi}{8}, \widetilde{\phi_a} = \frac{\pi}{4},\ \widetilde{\phi_b} = \frac{3\pi}{8}$$



Recent review on testing Bell Inequality: Barr et al., 2402.07972

Extension to multi-jet production: ridge phenomena in pp collisions at the LHC



CMS, 1009.4122

First step, understand EEC:

$$\Sigma^{jet}(Q^2;\theta_{a,b},\phi) = \sum_{ij}\int d\sigma^{jet}(Q^2)\frac{E_i}{E_P}\frac{E_j}{E_P}\mathcal{F}(\phi;\vec{n}_{a,b}) \times \delta(\vec{n}_a-\vec{n}_i)\delta(\vec{n}_b-\vec{n}_j)\,, \quad (\text{:}$$

$$= \int d\Omega \left\{x_a f_{g,\mathrm{EEC}}\left(x_a,\theta_a^2\right) x_b f_{g,\mathrm{EEC}}\left(x_b,\theta_b^2\right)\hat{\sigma}_0 + x_a d_{g,\mathrm{EEC}}\left(x_a,\theta_a^2\right) x_b d_{g,\mathrm{EEC}}\left(x_b,\theta_b^2\right)\hat{\sigma}_2(Q^2)\cos(2\phi)\right\}$$

Cos(2ϕ) and helicity amplitudes in QCD

$$\hat{\sigma}_2 \propto \sum_{\lambda_3 \lambda_4} \mathcal{A}(\pm, \pm, \lambda_3, \lambda_4) \mathcal{A}^*(\mp, \mp, \lambda_3, \lambda_4)$$

- Cos(2ϕ) comes from interference between double helicity-flip with the same helicity for the incoming gluons
- QCD amplitude vanishes for same helicity for all external partons or only one has different helicity

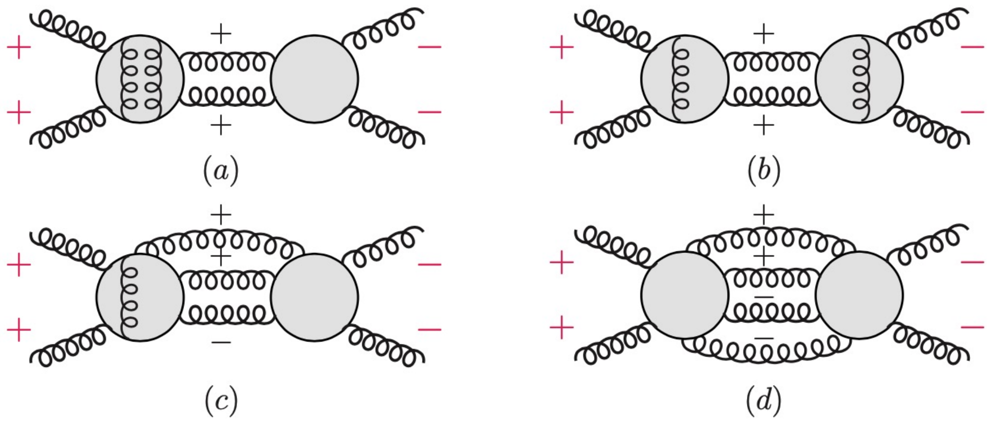
Parke, Taylor, 1986
Berends, Giele, 1988

- Any combinations of λ_3 and λ_4 the above vanishes, and one-loop amplitude contribution also vanishes

Bern, Kosower, 1992; …

- Nonvanishing contribution only comes from two-loop amplitudes

A power counting rule



- Similar conclusion holds for three jet final state, vanishing at the leading order, but survives at NLO
- Four jet final state, cos(2ϕ) is leading order

Number of Jets	2	3	≥ 4
$\langle\cos(2\phi)\rangle$ asymmetry	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(1)$

Dijet $\cos(2\phi)$ at NNLO

- Applying the two-loop amplitudes

Ahmed, Henn, Mistlberger, 1910.06684; Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi, 2112.11097

$$\hat{\sigma}_2^{(2)} = A^{(1)}(+++-)A^{(1)*}(--+-) + A^{(1)}(++++)A^{(1)*}(--++) + A^{(2)}(++++)A^{(0)*}(--++) + h.c.$$

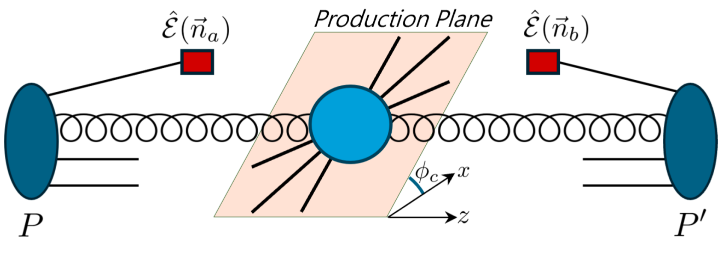
- There are IR divergences, which will be cancelled by soft real gluon radiation (with a jet veto q_0)

$$\hat{\sigma}_2^{(2)} = \hat{\sigma}_{2,0}^{(2)}(x)\ln\left(\frac{\hat{s}}{q_0^2}\right) + \hat{\sigma}_{2,2}^{(2)}(x) - \hat{\sigma}_{2,1}^{(2)}(x)\left[\mathcal{P}_{gg}^d \otimes d_g(x_a) + \mathcal{P}_{gg}^d \otimes d_g(x_b)\right]$$

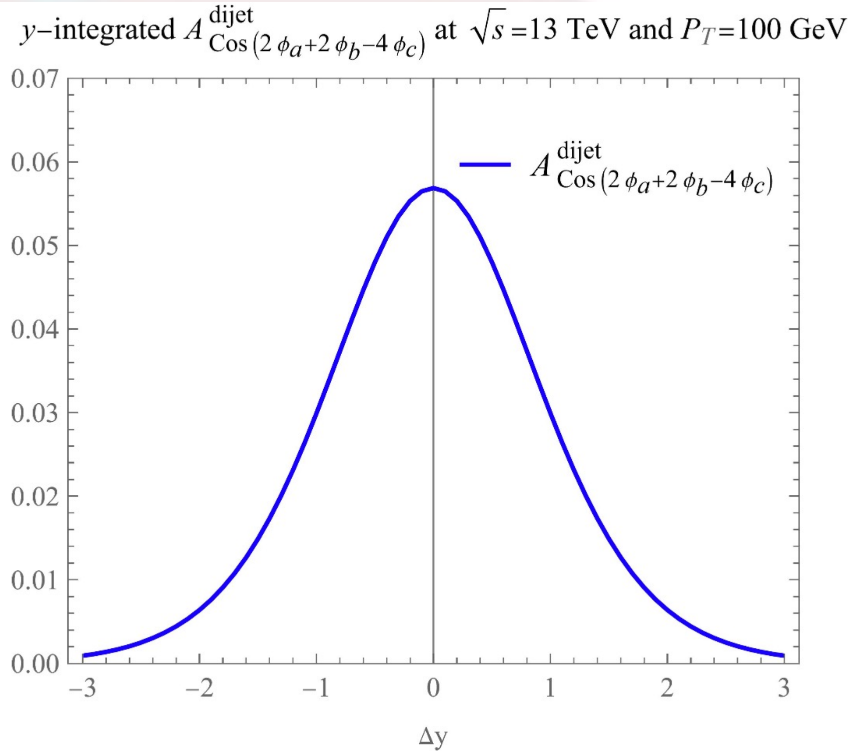
Comments on phenomenological results

- Suppressed by $(\alpha_s/4\pi)^2$, order 10^{-4}- 10^{-3}
  - Roughly same order as that found in the long range ridge for lower end of multiplicity events
- There are cancellations between different terms
- Strongly depends on the jet veto q_0
  - we may see sign changes depending on q_0

Cross check with semi-long range correlation



$$\langle \cos(2\phi_a + 2\phi_b - 4\phi_c) \rangle_{\text{dijet}} = \frac{\int d\Omega d_{g,\text{EEC}}(x_a) d_{g,\text{EEC}}(x_b) \frac{9}{2} \frac{\hat{s}^2+\hat{t}^2+\hat{u}^2}{2\hat{s}^2}}{\int d\Omega f_{g,\text{EEC}}(x_a) f_{g,\text{EEC}}(x_b) \frac{9}{2} \frac{(\hat{s}^2+\hat{t}^2+\hat{u}^2)^3}{8\hat{s}^2\hat{t}^2\hat{u}^2}}$$



Next step

- Compute three-jet, four-jet asymmetry
  - one-loop amplitudes for three-jet final state
  - tree amplitudes for four-jet final state
- Build simulation for the ridge measurements
  - Keep interference and spin information in the parton shower simulations
  - And/or include high number of jets in the final states

Back-up

Universal IR Structure in QCD Amplitudes

Catani 1998; Sterman-Tejeda-Yeomans 2003

$$\boldsymbol{I}^{(1)} \equiv \left[-\sum_i \left(\frac{\gamma_K^{[i](1)}}{2\epsilon^2} + \frac{\mathcal{G}_0^{[i](1)}}{\epsilon} \right) \mathbf{1} + \frac{\mathbf{\Gamma}^{(1)}}{\epsilon} \right] \left(-\frac{\mu^2}{s} \right)^{\epsilon} \qquad \mathbf{\Gamma}^{(1)} = \frac{1}{2} \sum_i \sum_{j \neq i} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \ln \left(\frac{-\mu^2}{s_{ij}} \right)$$

- Applying in our case

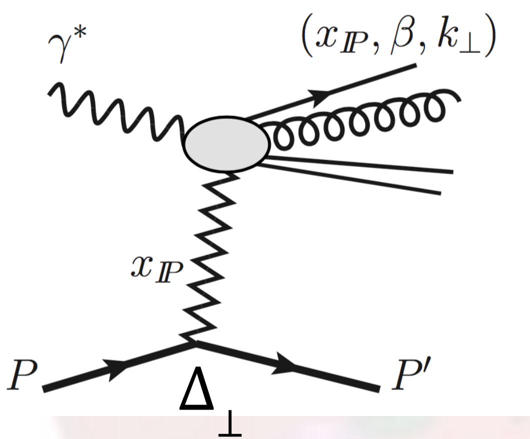
$$\hat{\sigma}_2^{(2)v} = \frac{1}{\epsilon} \left(\hat{\sigma}_{2,J}^{(2)v} + \hat{\sigma}_{2,S}^{(2)v} \right)$$

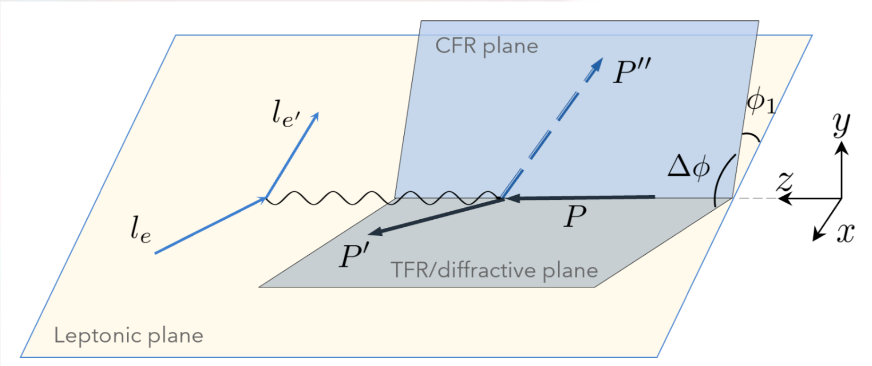
$$\hat{\sigma}_{2,S}^{(2)v} = \frac{2}{\mathcal{V}} \left\langle A^{(1)}(++++) | 2\mathrm{Re} \left[\mathbf{\Gamma}^{(1)} \right] | A^{(0)}(--++) \right\rangle$$

$$\hat{\sigma}_{2,J}^{(2)v} = -\sum_i \gamma_K^{[i](1)} \hat{\sigma}_2^{(1)\epsilon} = -8 C_A \hat{\sigma}_2^{(1)\epsilon}$$

Helicity amplitudes can be found in Refs. for two loop results:
Ahmed, Henn, Mistlberger, 1910.06684;
Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi, 2112.11097

More: semi-inclusive diffractive DIS

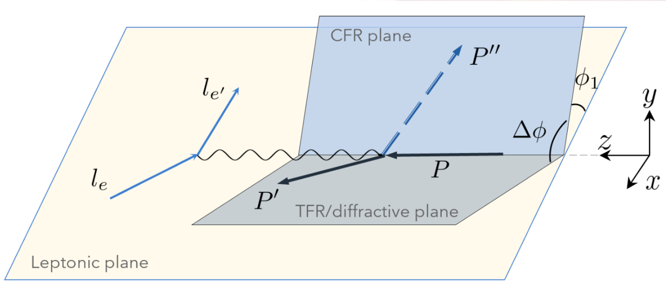


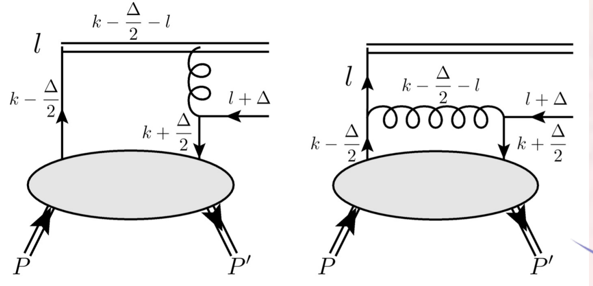


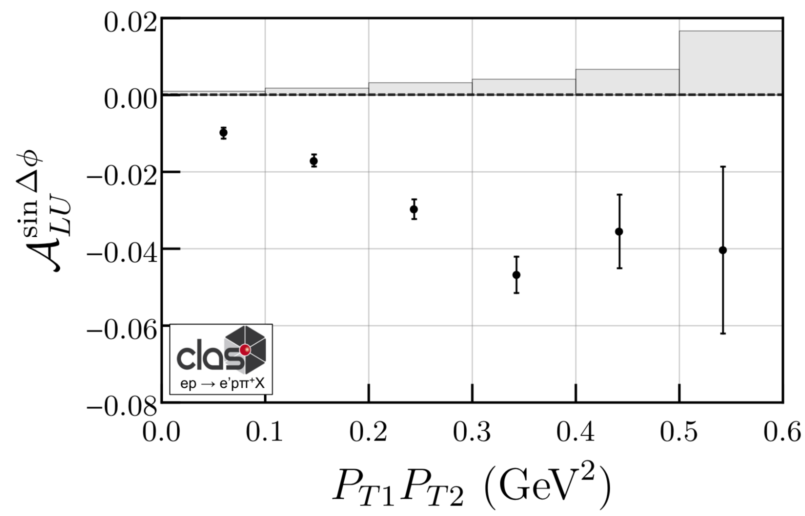
Iancu-Mueller-Triantafyllopoulos, 2112.06353;
Hatta-Xiao-Yuan, 2205.08060, Hatta-Yuan, 2403.19609;
Fucilla, Grabovsky, Li, Szymanowski, Wallon, 2310.11066 (NLO);
Guo, Yuan, 2312.01008

- Flavor dependence in the diffractive PDFs
- TMD dependence can be measured and so as the correlation between $k_\perp$ and $\Delta_\perp$, long range correlation in small-x diffractive processes

Compute the Diffractive PDFs at moderate-x and the spin asymmetries in semi-inclusive diffractive DIS







CLAS Coll., 2208.05508

Guo, Yuan, 2312.01008;
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