Towards the transverse SSA for $ep^{\uparrow} \to hX$ at NLO and its connection to $ep^{\uparrow} \to \gamma X$

Daniel Rein, Marc Schlegel, Patrick Tollkühn, Werner Vogelsang

Table of Contents

Introduction

- $pp^{\uparrow} \to hX$: Large transverse single-spin asymmetries (TSSA) $A_{UT} \equiv \frac{\sigma(\vec{S}_T)-\sigma(-\vec{S}_T)}{\sigma(\vec{S}_T)+\sigma(-\vec{S}_T)}$ of up to ~50%

Klem, R. D., et al., 1976, Phys. Rev. Lett. 36, 929
Dragoset, W. H., et al., 1978, Phys. Rev. D 18, 3939
Antille, J., et al., 1980, Phys. Lett. 94B, 523
Apokin, V. D., et al., 1990, Phys. Lett. B 243, 461
Saroff, S., et al., 1990, Phys. Rev. Lett. 64, 995

- proper treatment ↔ collinear twist-3 formalism
  -however: very challenging
  -NLO so far not on the horizon

J.-w. Qiu, G. Sterman, Phys. Rev. Lett. 67, 2264 (1991)
Y. Kanazawa, Y. Koike, Phys. Lett. B478, 121 (2000)
C. Kouvaris, J.-W. Qiu, W. Vogelsang, F. Yuan, Phys. Rev. D74, 114013 (2006)
F. Yuan, J. Zhou, Phys. Rev. Lett. 103, 052001 (2009).
Kanazawa, Y. Koike, A. Metz, D. Pitonyak, Phys. Rev. D89, 111501 (2014)

Introduction

Simpler similar processes?

$lp^{\uparrow} \to lX$ (DIS)

TSSA vanishes for one-photon exchange
→ time-reversal symmetry
→ non-zero TSSA for two-photon exchange

N. Christ, T. D. Lee, Phys. Rev. 143, 1310 (1966)

$lp^{\uparrow} \to hX$

Accessible at the EIC

- Computation at NLO
- Jet production $lp^{\uparrow} \to jet\ X$

$lp^{\uparrow} \to \gamma X$

No twist-3 fragmentation, but interference

$lp^{\uparrow} \to l\gamma X$ (γSIDIS)

W.S. Albaltan, A. Prokudin, M. Schlegel, Phys. Lett. B 804, 135367 (2020)

Collinear twist-3 factorization approach

Intrinsic

higher twist terms of familiar quark-quark correlator

Kinematical

Effects due to transverse motion of the partons, non-zero k_T

Dynamical

Quark-gluon-quark or tri-gluon correlations inside the hadron

The notation in the following slides follows the conventions from K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak, M. Schlegel, [Phys. Rev. D 93, 054024 (2016)]

Kinematical twist-3 distribution functions

Correlator with non-zero k_T:

$$\Phi_{ij}^{q}(x, k_T) = \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} \int \frac{d^{d-2} z_T}{(2\pi)^{d-2}} e^{i\lambda x + i k_T \cdot z_T} \langle P, S | \bar{q}_j(0) q_i(\lambda n + z_T) | P, S \rangle$$

Example:

$$\frac{E_h d\sigma}{d^{d-1} P_h} \propto \int dx d^{d-2} k_T \int d(1/z)\, \Phi_{ij}^{q}(x, k_T) H_{ij}(x, k_T, z) D_1^q(z)$$

$$\xrightarrow{\mathcal{O}(k_T)} \int dx \int d(1/z) \left(\int d^{d-2} k_T\, k_T^{\rho} \Phi_{ij}^{q}(x, k_T) \right) \left(\frac{d}{dk_T^{\rho}} H_{ij}(x, k_T, z) \bigg|_{k_T = 0} \right) D_1^q(z)$$

Kinematical twist-3 distribution functions

Kinematical twist-3 correlator:

$$\Phi^{q,\rho}_{\partial,ij}(x) = \int d^2k_T\, k_T^\rho \Phi^q_{ij}(x,k_T)$$
$$= \frac{1}{2} M \epsilon^{Pn\rho S} \not{P}_{ij} f_{1T}^{\perp(1),q}(x) + \ldots$$

First k_T-moment of the Sivers function appears:

$$f_{1T}^{\perp(1),q}(x) = \int d^2k_T\, \frac{\vec{k}_T^2}{2M^2} f_{1T}^{\perp q}(x, k_T^2)$$

Dynamical twist-3 distribution functions

Multi-parton correlators:

$$\Phi^{q,\rho}_{F,ij}(x,x') = \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} \int_{-\infty}^{+\infty} \frac{d\mu}{2\pi} e^{ix'\lambda + i(x-x')\mu} \langle P,S | \bar{q}_j(0) igF^{n\rho}(\mu n) q_i(\lambda n) | P,S \rangle$$

$$= \frac{M}{2}\left(\epsilon^{Pn\rho S} \not{p}\, i F^q_{FT}(x,x') - S_T^\rho \not{P} \gamma_5 G^q_{FT}(x,x') + \ldots\right)$$

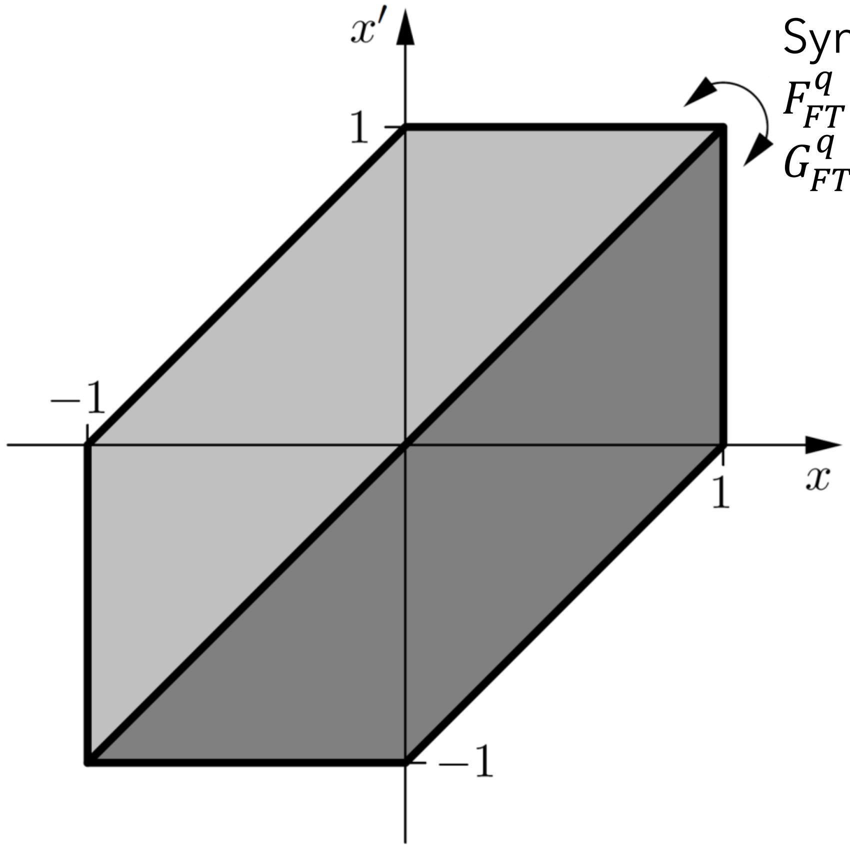
Connection with Sivers function

$$\pi F^q_{FT}(x,x) = f_{1T}^{\perp(1),q}(x)$$

D. Boer, P.J. Mulders, F. Pijlman, Nucl. Phys. B 667 (2003)

Dynamical twist-3 distribution functions

Support:
$|x| \leq 1, |x'| \leq 1$ and
$|x - x'| \leq 1$



Symmetry:
$F_{FT}^{q}(x, x') = F_{FT}^{q}(x', x)$
$G_{FT}^{q}(x, x') = -G_{FT}^{q}(x', x)$

Key features of the calculation

LO result for $e(l)p^{\uparrow}(P) \to h(P_h)X$:

Twist-3 effects on the fragmentation side, which we do not consider in this talk

$$E_h \frac{d\sigma_{LO}}{d^{d-1}P_h}(S) = \sigma_0(S) \int_{v_0}^{v_1} dv \int_{x_0}^{1} \frac{dw}{w} \hat{\sigma}_{LO}(v,w) \sum_q e_q^2 \left[\left(1 - x\frac{d}{dx}\right) F_{FT}^q(x,x) D_1^q(z)\right]\Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}} + h_1^q \otimes \Im[\hat{H}_{FU}^q]$$

$$s = (l+P)^2 \cong 2l\cdot P, \qquad t = (P-P_h)^2 \cong -2P\cdot P_h, \qquad u = (l-P_h)^2 \cong -2l\cdot P_h$$

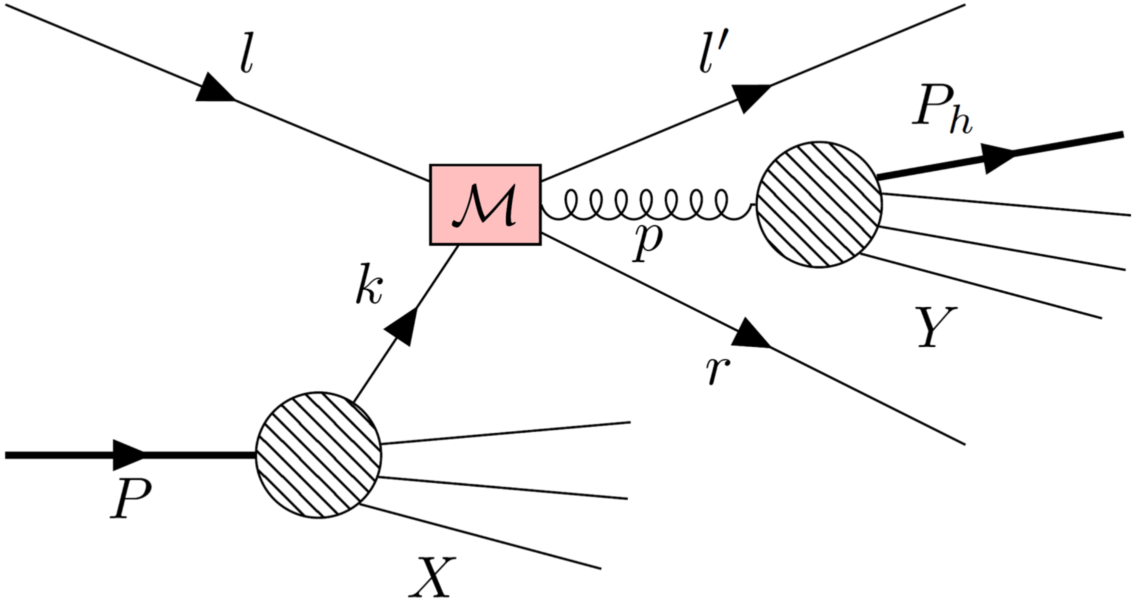
$$x_0(v) = \frac{1-v}{v}\frac{u}{t}, \qquad v_0 = \frac{u}{t+u}, \qquad v_1 = \frac{s+t}{s}$$

Key features of the calculation

NLO Calculation

- Virtual corrections…
  …not part of this talk!
- Real corrections:
  - Distribution side:
    $qg \to g$
    $qg \to q$
    $qq \to q$
    $qq \to q'$
    $gg \to q'$
  - Fragmentation side:
    Not considered here!

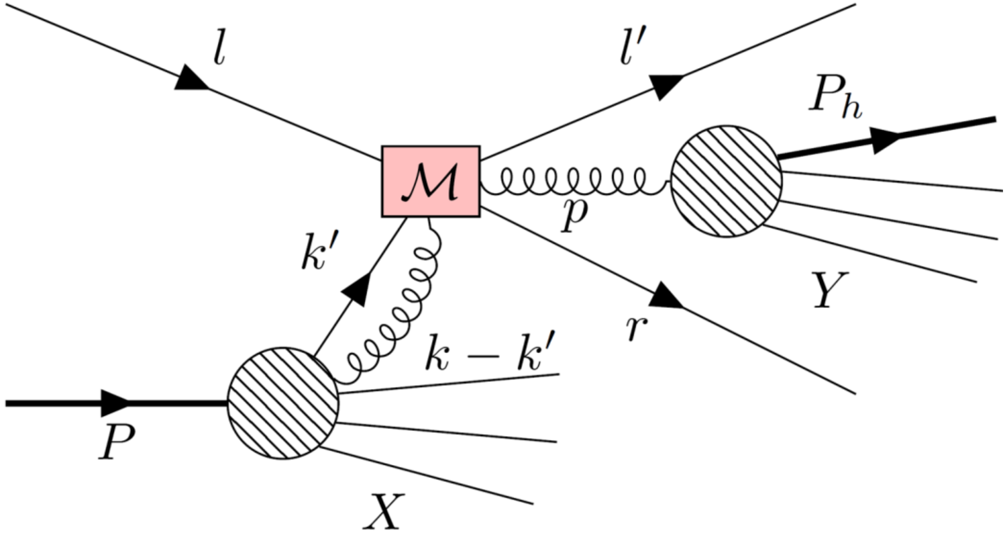
Key features of the kinematical part



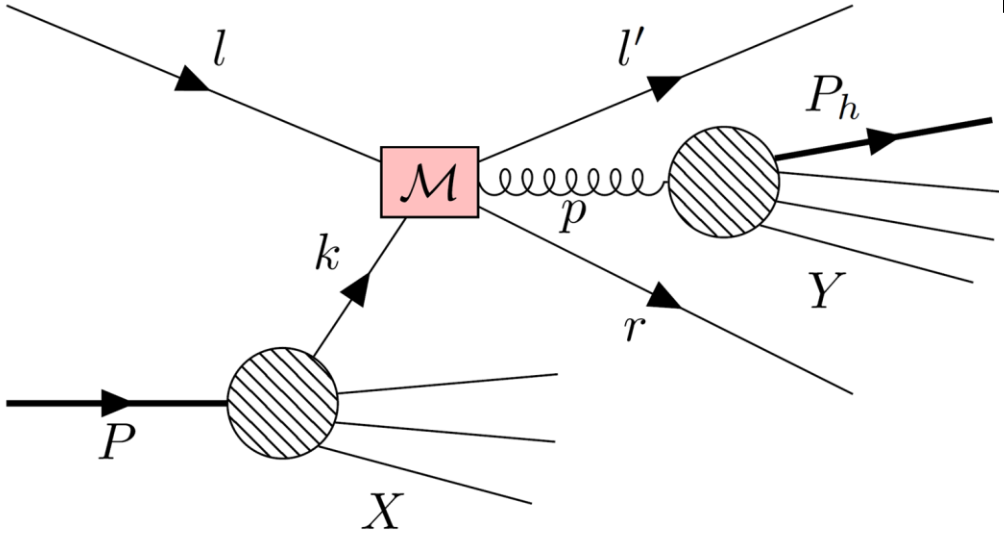
$$\frac{E_h d\sigma_{kin}}{d^{d-1}P_h} = \int dx \Phi^{q,\rho}_{\partial,ij}(x) \int d(1/z)\, D_1^q(z)$$

$$\times \frac{d}{dk_T^\rho} H_{ij}\left(k = xP + k_T, p = \frac{P_h}{z}\right)\Big|_{k_T=0}$$

Key features of the dynamical part



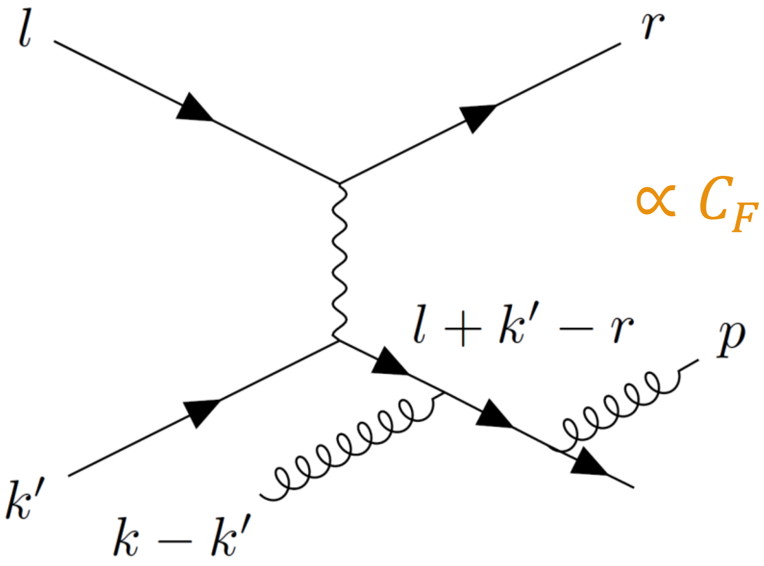
×



† $+c.c.$

$$\frac{E_h d\sigma_{dyn}}{d^{d-1}P_h} = \int dx \int dx' \int d(1/z) z^{2\varepsilon} D_1^q(z)\, H_{ij}\left(k = xP, k' = x'P, p = \frac{P_h}{z}\right) \frac{i\Phi_{F,ij}^{q,\rho}(x,x')}{x' - x} + c.c.$$

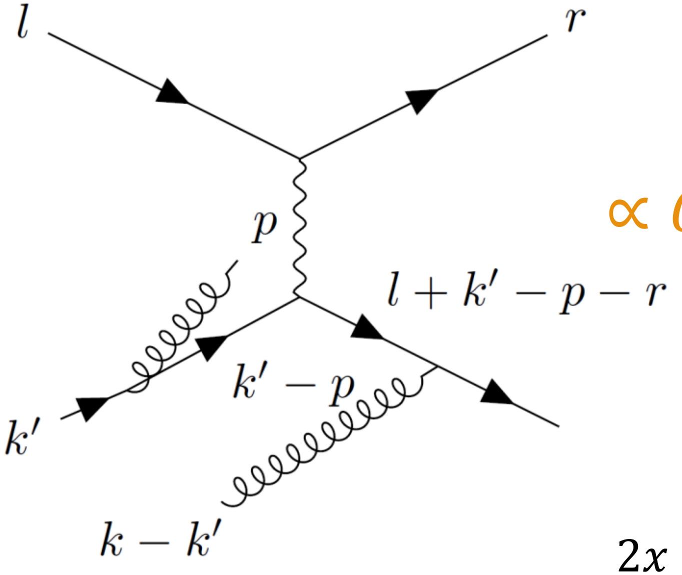
Key features of the dynamical part



$\propto C_F$

$$\frac{1}{(l+k'-p-r)^2 + i\,\eta}$$

$$\propto \left(\mathcal{P}\frac{1}{x'-x} - i\pi\delta(x'-x) \right)$$

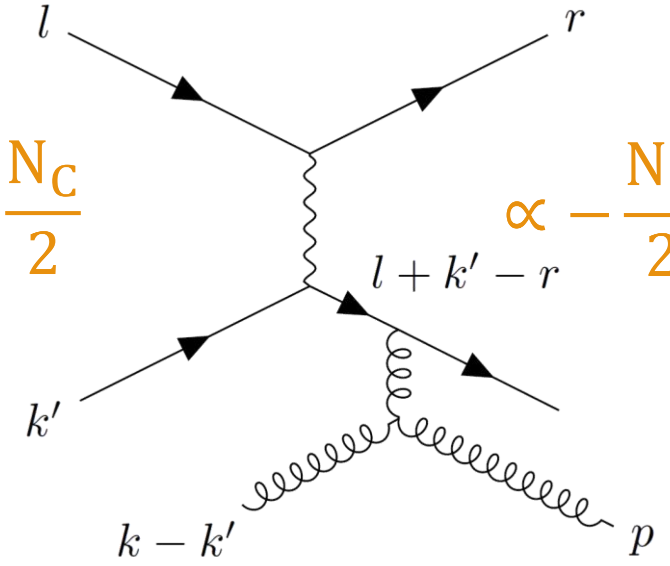


$\propto C_F - \frac{N_C}{2}$

$$\tilde{x} \equiv \frac{2x\, l\cdot r}{\hat{s} - 2k\cdot r}$$

$$\frac{1}{(l+k'-r)^2 + i\,\eta}$$

$$\propto \left(\mathcal{P}\frac{1}{x'-\tilde{x}} - i\pi\delta(x'-\tilde{x}) \right)$$



$\propto -\frac{N_C}{2}$

$$\frac{1}{(k'-p)^2 + i\,\eta}$$

$$\propto \left(\mathcal{P}\frac{1}{x'} + i\pi\delta(x') \right)$$

Key features of the dynamical part

$\frac{\delta(x'-x)}{x'-x}$ ⇨ $F_{FT}^q(x,x), (G_{FT}^q(x,x)=0),$ $\frac{d}{dx}F_{FT}^q(x,x), \frac{\partial}{\partial x'}G_{FT}^q(x,x')\Big|_{x'=x}$ Soft Gluon Pole (SGP)

$\delta(x')$ ⇨ $F_{FT}^q(x,0), G_{FT}^q(x,0)$ Soft Fermion Pole (SGP)

$\delta(x'-\tilde{x})$ ⇨ $F_{FT}^q(x,\tilde{x}), G_{FT}^q(x,\tilde{x})$ Hard Pole (HP)

Key features of the dynamical part

But Careful! $\tilde{x} = \frac{2x\, l\cdot r}{\hat{s}-2k\cdot r}$ so $F_{FT}^q(x,\tilde{x}), G_{FT}^q(x,\tilde{x}) \propto r$

⇩

Keep $i\eta$ in the propagator, perform the phase space integral first, then take the imaginary part

⇩

Extract imaginary part via

$$\log[x \pm i\eta] \xrightarrow{\eta\to 0^+} \log[|x|] \pm \Theta(-x) i\pi$$

Key features of the dynamical part

Def. $\zeta = \frac{x'}{x}$; discontinuity at $\zeta = w$ and poles at $\zeta = 0, \zeta = 1$

→ Need to regularize these poles!

Finite

$$\frac{E_h d\sigma_{dyn}}{d^{d-1}P_h} \propto \int_{v_0}^{v_1} dv \int_{x_0}^{1} \frac{dw}{w} \int_0^1 d\zeta \left[\left(\hat{\sigma}(v,w,\zeta) \frac{F_{FT}^q(x,x\zeta)}{\zeta(1-\zeta)^2} + \hat{\sigma}_5(v,w,\zeta) \frac{G_{FT}^q(x,x\zeta)}{\zeta(1-\zeta)^2} \right) z^{2\varepsilon} D_1^q(z) \right] \Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}}$$

Poles

Key features of the dynamical part

Idea: add and subtract suitable terms!

$$\frac{F_{FT}^q(x,x\zeta)}{\zeta(1-\zeta)^2} \to -\frac{(1-\zeta)^2 F_{FT}^q(x,0) + \zeta(2-\zeta)F_{FT}^q(x,x) - \frac{x}{2}\zeta(1-\zeta)\frac{d}{dx}F_{FT}^q(x,x) - F_{FT}^q(x,x\zeta)}{\zeta(1-\zeta)^2}$$

Extra SFP

Extra SGP

$$\frac{G_{FT}^q(x,x\zeta)}{\zeta(1-\zeta)^2} \to -\frac{(1-\zeta)^2 G_{FT}^q(x,0) - x\,\zeta(1-\zeta)\frac{\partial}{\partial x'}G_{FT}^q(x,x')|_{x'=x} - G_{FT}^q(x,x\zeta)}{\zeta(1-\zeta)^2}$$

No ζ-dependence in F_{FT}^q, G_{FT}^q → go back and apply $\delta(x' - \tilde{x})$

Final result

$$E_h \frac{d\sigma}{d^3P_h}(S) = E_h \frac{d\sigma^{SGP}}{d^3P_h}(S) + E_h \frac{d\sigma^{SFP}}{d^3P_h}(S) + E_h \frac{d\sigma^{int}}{d^3P_h}(S)$$

$$\sigma_1(S) \equiv \frac{8\pi\alpha_{\text{em}}^2}{s^2}\frac{\alpha_s}{2\pi}\frac{M\epsilon^{lPP_hS}}{u^2}, \qquad (\partial_{x'}G_{FT}^q)(x,x) \equiv \frac{\partial}{\partial x'}G_{FT}^q(x,x')\Big|_{x'=x}$$

$$E_h \frac{d\sigma^{int}}{d^3P_h} = \sigma_1(S)\sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^{1}\frac{dw}{w}\int_0^1 d\zeta \Bigg(\left[C_F\hat{\sigma}_{CF}^{int}(v,w,\zeta) + \frac{N_C}{2}\hat{\sigma}_{NC}^{int}(v,w,\zeta)\right]\left[\frac{(1-\zeta)^2F_{FT}^q(x,0) + \zeta(2-\zeta)F_{FT}^q(x,x) - \frac{x}{2}\zeta(1-\zeta)\frac{d}{dx}F_{FT}^q(x,x) - F_{FT}^q(x,x\zeta)}{\zeta(1-\zeta)^2}D_1^g(z)\right]$$
$$+\left[C_F\hat{\sigma}_{5,CF}^{int}(v,w,\zeta) + \frac{N_C}{2}\hat{\sigma}_{5,NC}^{int}(v,w,\zeta)\right]\left[\frac{(1-\zeta)^2G_{FT}^q(x,0) - x\,\zeta(1-\zeta)\big(\partial_{x'}G_{FT}^q\big)(x,x) - G_{FT}^q(x,x\zeta)}{\zeta(1-\zeta)^2}D_1^g(z)\right]\Bigg)\Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}}$$

$$E_h \frac{d\sigma^{SFP}}{d^3P_h} = \sigma_1(S)\sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^{1}\frac{dw}{w}\left(\left[C_F\hat{\sigma}_{CF}^{SFP}\left(v,w,\frac{su}{tm^2}\right) + \frac{N_C}{2}\hat{\sigma}_{NC}^{SFP}\left(v,w,\frac{su}{tm^2}\right)\right]\left[F_{FT}^q(x,0)D_1^g(z)\right] + \left[C_F\hat{\sigma}_{5,CF}^{SFP}\left(v,w,\frac{su}{tm^2}\right) + \frac{N_C}{2}\hat{\sigma}_{5,NC}^{SFP}\left(v,w,\frac{su}{tm^2}\right)\right]\left[G_{FT}^q(x,0)D_1^g(z)\right]\right)\Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}}$$

$$E_h \frac{d\sigma^{SGP}}{d^3P_h} = \sigma_1(S)\sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^{1}\frac{dw}{w}\left(\left[C_F\hat{\sigma}_{CF}^{SGP}\left(v,w,\frac{su}{t\mu^2}\right) + \frac{N_C}{2}\hat{\sigma}_{NC}^{SGP}\left(v,w,\frac{su}{tm^2}\right)\right]\left[F_{FT}^q(x,x)D_1^g(z)\right] + \left[C_F\hat{\sigma}_{5,CF}^{SGP}(v,w) + \frac{N_C}{2}\hat{\sigma}_{5,NC}^{SGP}(v,w)\right]\left[x\big(\partial_{x'}G_{FT}^q\big)(x,x)D_1^g(z)\right]\right)\Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}}$$

Connection to $ep^{\uparrow} \to \gamma X$ (w/o interference)

The result for the related process $ep^{\uparrow} \to \gamma X$ is readily obtained by the replacements:

N_C	$\Rightarrow$	0
C_F	$\Rightarrow$	e_q^2
α_s	$\Rightarrow$	α_{em}
$D_1^g(z)$	$\Rightarrow$	$\delta(1-z)$

Connection to $ep^{\uparrow} \to \gamma X$ – model ansatz

Ansatz for the twist-3 correlation functions

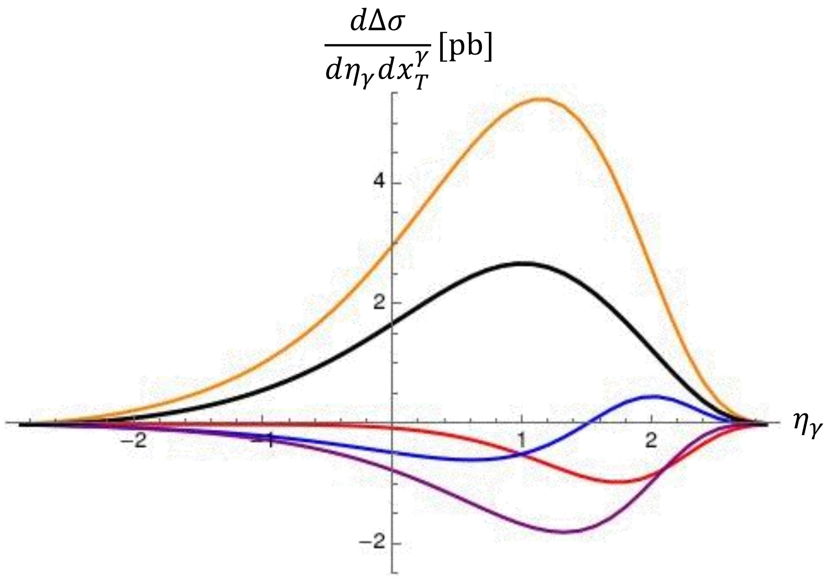
$$F_{FT}^{q}(r,\varphi) = \left(\frac{1}{2\pi}\left(f_{1T}^{\perp(1),q} + f_{1T}^{\perp(1),\bar{q}}\right)\left(\frac{r}{\sqrt{2}}\right) + \frac{1}{2\pi}\left(f_{1T}^{\perp(1),q} - f_{1T}^{\perp(1),\bar{q}}\right)\left(\frac{r}{\sqrt{2}}\right)\cos\varphi \right.$$
$$\left. + \sum_{n=1}^{\infty} a_{2n}^{q}(r)(\cos(2n\varphi) - 1) + \sum_{n=1}^{\infty} a_{2n+1}^{q}(r)\Big(\cos\big((2n+1)\varphi\big) - \cos\varphi\Big) \right)$$
$$\times \left(\frac{(1-x^2)(1-(x')^2)}{(1-xx')^2}\right)^{\delta} (1-(x-x')^2)^{\epsilon}\Theta(1-|x|)\Theta(1-|x'|)\Theta(1-|x-x'|)\Bigg|_{x=r\cos\left(\varphi+\frac{\pi}{4}\right)}^{x'=r\sin\left(\varphi+\frac{\pi}{4}\right)}$$

Similar for $G_{FT}^{q}(r,\varphi)$

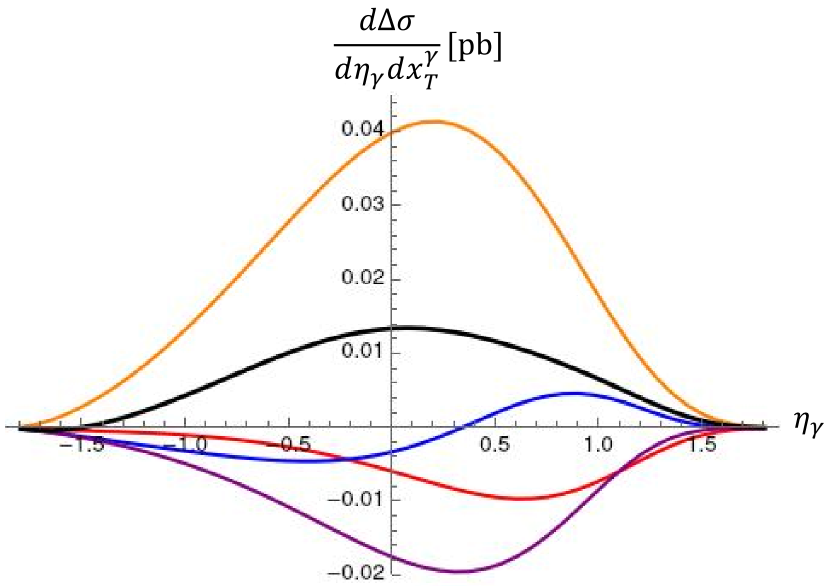
Rapidity plots for $ep^{\uparrow} \to \gamma X$

$$x_T^\gamma = \frac{2p_T^\gamma}{\sqrt{s}}$$

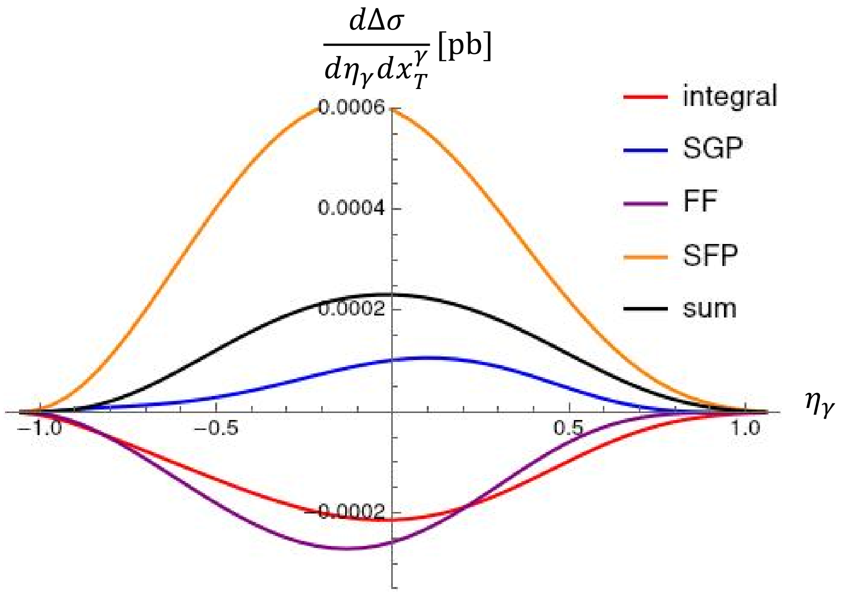
$\sqrt{s} = 100$ GeV, $x_T^\gamma = 0.1, \mu = p_T^\gamma = 5$ GeV



$\sqrt{s} = 100$ GeV, $x_T^\gamma = 0.3, \mu = p_T^\gamma = 15$ GeV



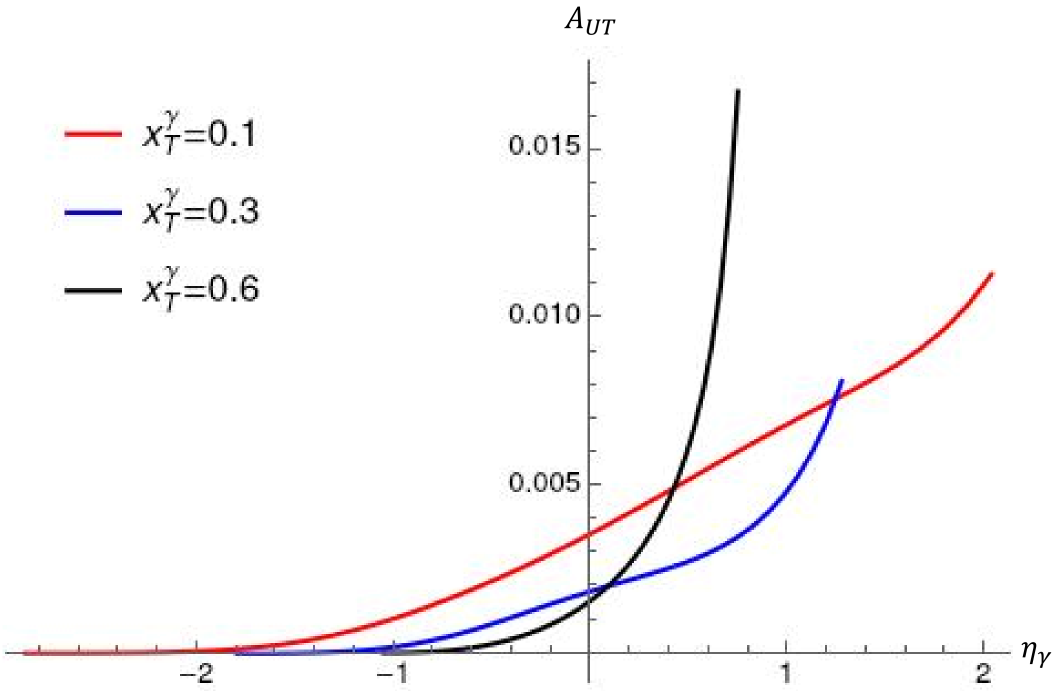
$\sqrt{s} = 100$ GeV, $x_T^\gamma = 0.6, \mu = p_T^\gamma = 30$ GeV



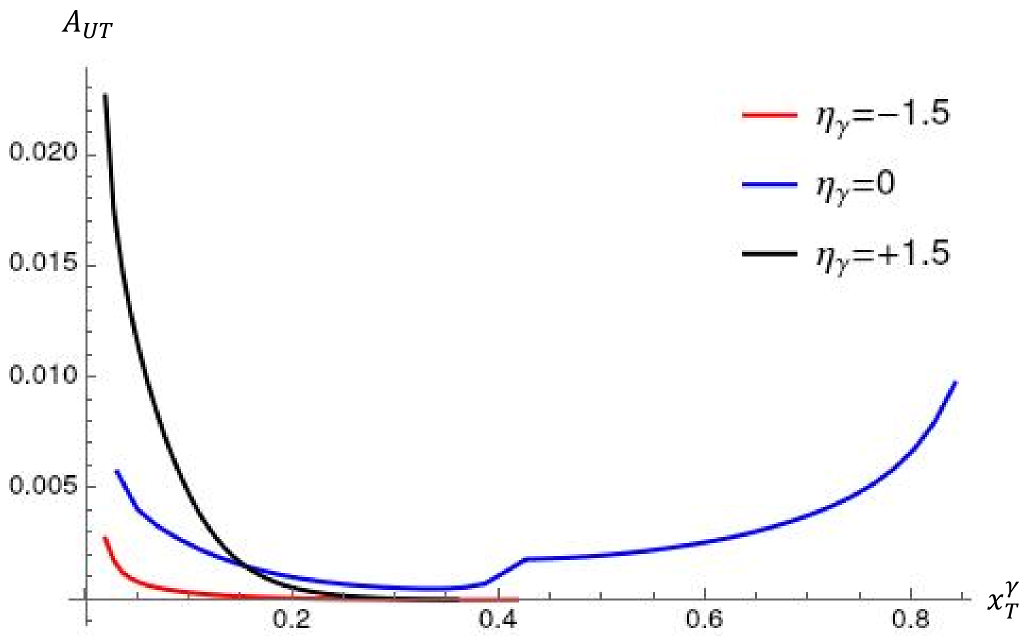
Asymmetries for $ep^\uparrow \to \gamma X$

$$x_T^\gamma = \frac{2p_T^\gamma}{\sqrt{s}}$$

$\sqrt{s} = 100$ GeV, $\mu = p_T^\gamma$



$\sqrt{s} = 100$ GeV, $\mu = p_T^\gamma$



Summary/Outlook

- Twist-3 formalism - plethora of different contributions + distribution functions
- Combine using EoM-relations and/or lorentz invariance relations
  → express result for the $qg \to g$ channel entirely in terms of F_{FT}^{q}, G_{FT}^{q} and twist-2 FFs
- We also obtain (part of) the result for $ep^{\uparrow} \to \gamma X$
  → model predictions show potential for large A_{UT} in certain kinematic regions
- Finish the calculation for remaining real correction channel $gg \to q'$
- Derive corresponding results for $ep^{\uparrow} \to jet\ X$
- Include twist-3 fragmentation effects

Backup

Higher twist terms from the quark-quark correlator familiar from unpolarized and longitudinally polarized twist-2 calculations:

$$\Phi_{ij}^{q}(x) = \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | \bar{q}_j(0) q_i(\lambda n) | P, S \rangle$$

Wilson lines, like $\mathcal{W}[0;\lambda n]$ in this case, are omitted for simplicity!

$$\Phi^{q}(x) = \frac{1}{2} \not{P} f_1^{q}(x) + \frac{1}{2} M\, e^{q}(x) - \frac{1}{2} M (S \cdot n) \not{P} \gamma_5 g_1^{q}(x) + \frac{1}{4} M^2 (S \cdot n) [\not{P}, \not{n}] \gamma_5 h_L^{q}(x)$$
$$- \frac{1}{4} [\not{P}, \not{S}] \gamma_5 h_1^{q}(x) - \frac{1}{2} M (\not{S} - (S \cdot n) \not{P}) \gamma_5 g_T^{q}(\mathrm{x})$$

Backup

The phase space integral can be solved analytically, choosing a frame where $\vec{l} + \vec{k} - \vec{p} = \vec{0}$ one finds

$$J \equiv \frac{1}{(2\pi)^{d-1}} \int d^d r\, \delta^+(r^2)\delta^+\big((l + xP + k_T - P_h/z - r)^2\big) f(r)$$

$$\to \frac{1}{4(2\pi)^3} S_\varepsilon \left(\frac{-u(1-w) + 2wk_T \cdot l}{w\mu^2}\right)^{-\varepsilon} \frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \int_0^\pi \sin^{1-2\varepsilon}(\theta) d\theta \int_0^\pi \sin^{-2\varepsilon}(\phi) d\phi f(\theta,\phi)$$

$\downarrow$ $\frac{\partial}{\partial k_T}$; $k_T \to 0$

$$\varepsilon(1-w)^{-1-\varepsilon}$$

$\downarrow$

$$(1-w)^{-1-a\varepsilon} = -\frac{1}{a\varepsilon}\delta(1-w) + \frac{1}{(1-w)_+} + \mathcal{O}(\varepsilon)$$

For further evaluation of the angular integrals we followed:
V.E. Lyubovitskij, F. Wunder, A.S. Zhevlakov,
J. High Energ. Phys. 2021, 66 (2021)

Backup

Alternative approach (this was used for $lp^{\uparrow} \to l\gamma X$ (γSIDIS) in W.S. Albaltan, A. Prokudin, M. Schlegel, Phys. Lett. B 804, 135367 (2020)):
pull the k_T-derivative inside the phase space integral:

$$\frac{\partial}{\partial k_T^\rho}\delta\big((l + xP + k_T - P_h/z - r)^2\big)\,\hat{\sigma}_{ij}(x, k_T, z)\Big|_{k_T=0}$$
$$= \delta\big((l + k - p - r)^2\big)\left[\frac{\partial\hat{\sigma}_{ij}}{\partial k_T^\rho}\Big|_{k_T=0} - \frac{2x(l-r)_\rho}{\hat{s} + \hat{t} - 2k\cdot r}\left(\frac{\partial\hat{\sigma}_{ij}}{\partial x}\Big|_{k_T=0} + \hat{\sigma}_{ij}\Big|_{k_T=0}\frac{\partial}{\partial x}\right)\right]$$

Leading to:

$k = xP$

$$\frac{E_h d\sigma_{kin}}{d^{d-1}P_h} = \int dx \int d(1/z)\, D_1^q(z) \int d^d r\, \delta^+(r^2)\delta^+\big((l + k - p - r)^2\big)$$
$$\times\left[\Phi_{\partial,ij}^{q,\rho}(x)\left(\frac{\partial\hat{\sigma}_{ij}}{\partial k_T^\rho}\Big|_{k_T=0} - \frac{2x(l-r)_\rho}{\hat{s}+\hat{t}-2k\cdot r}\frac{\partial\hat{\sigma}_{ij}}{\partial x}\Big|_{k_T=0}\right) - \frac{\partial\Phi_{\partial,ij}^{q,\rho}(x)}{\partial x}\frac{2x(l-r)_\rho}{\hat{s}+\hat{t}-2k\cdot r}\hat{\sigma}_{ij}\Big|_{k_T=0}\right]$$

Backup

Another possible regularization: $\frac{\hat{\sigma}'(v,w,\zeta)F_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)}, \frac{\hat{\sigma}_5{}'(v,w,\zeta)G_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)}$ with

$$\frac{F_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)} \to -\frac{\frac{(1-\zeta)(w-\zeta)}{w}F_{FT}^q(x,0) - \frac{\zeta(w-\zeta)}{1-w}F_{FT}^q(x,x) + \frac{\zeta(1-\zeta)}{w(1-w)}F_{FT}^q(x,xw) - F_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)}$$

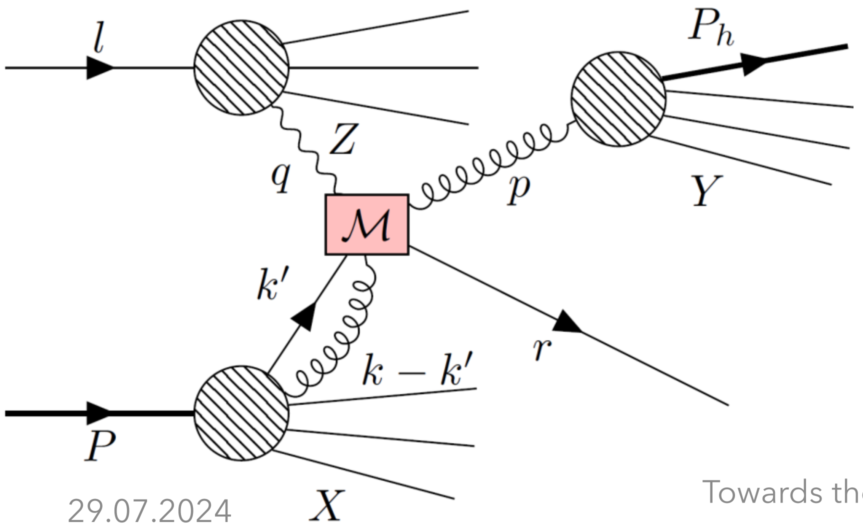
„genuine" hard pole

$$\frac{G_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)} \to -\frac{\frac{(1-\zeta)(w-\zeta)}{w}G_{FT}^q(x,0) + \frac{\zeta(1-\zeta)}{w(1-w)}G_{FT}^q(x,xw) - G_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)}$$

Backup

There are two types of collinear singularities left at this point

Singularities from neglecting the electron mass
→removed by Weizsäcker-Williams contribution



Singularities from collinear emission of the gluon from the quark
→removed by fragmentation function renormalization

$$\hat{\sigma}_{LO} \otimes \frac{\alpha_s}{2\pi} \frac{S_\varepsilon}{\varepsilon} \int_z^1 \frac{dw}{w} P_{gq}(w) D_1^q\left(\frac{z}{w}\right)$$

Quark-gluon splitting function

Backup

Extraction of Sivers function at scale $\mu = 1.55\ \mathrm{GeV}$ (Anselmino et al., Eur. Phys. J. A 39, 89–100 (2009))

$$f_{1T}^{\perp(1),q}(x) = -\frac{1}{2}\mathcal{N}^q(x) f_1^q(x)\sqrt{2e}\,\frac{M_1^3\langle k_T^2\rangle}{M_p(M_1^2+\langle k_T^2\rangle)^2}$$

$$\mathcal{N}^q(x) = N^q x^{\alpha_q}(1-x)^{\beta_q}\frac{(\alpha_q+\beta_q)^{\alpha_q+\beta_q}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

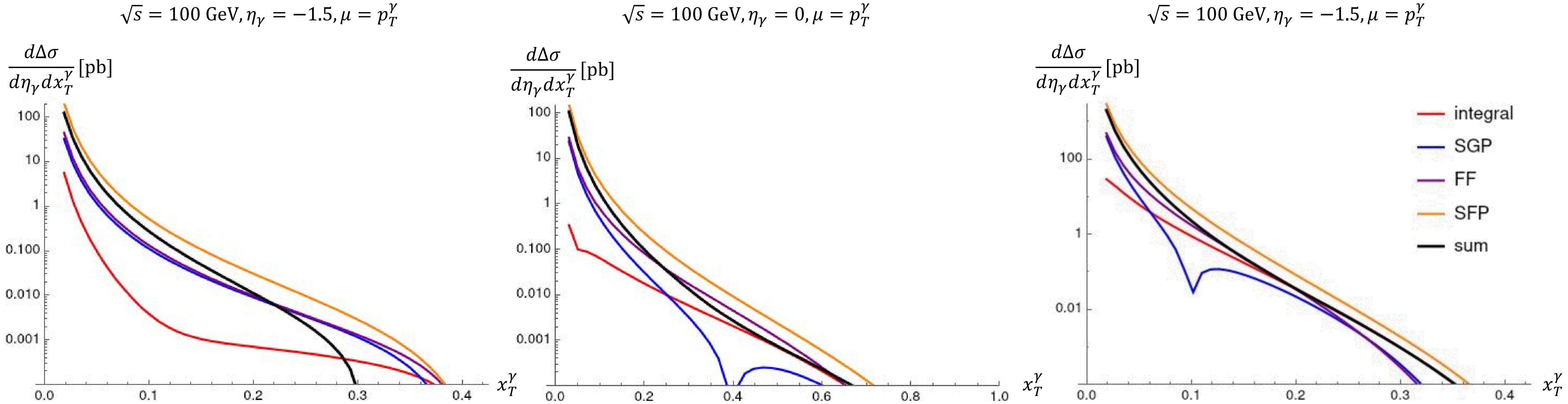
$M_p = 0.93827\ \mathrm{GeV}; \langle k_T^2\rangle = 0.25\ \mathrm{GeV}; M_1 = 0.583\ \mathrm{GeV}; \beta = \beta_q = 3.46; \alpha_u = 0.73; \alpha_d = 1.08; \alpha_{sea} = 0.79;$
$N^u = 0.35; N^d = -0.9; N^s = -0.24; N^{\bar{u}} = 0.04; N^{\bar{d}} = -0.4; N^{\bar{s}} = 1$

For the plots:
NLO unpolarized parton distributions MSTW2008 A. D. Martin, W. J. Stirling, R. S. Thorne, G. Watt, Eur. Phys. J. C 63, 189 (2009)
Parton-to-photon fragmentation function GRV NLO M. Gluck, E. Reya, A. Vogt, Phys. Rev. D 48, 116 (1993) [Erratum: Phys.Rev.D 51, 1427 (1995)]

$$\epsilon = \delta = 0.01; a_{2-7}^u = \left(-\frac{1}{2}, 0, 1, 0, \frac{1}{3}, 0\right); a_{2-7}^d = a_{2-7}^s = 0; b_{1-6}^u = \left(0, -\frac{1}{2}, 0, 1, 0, \frac{1}{3}\right); b_{1-6}^d = b_{1-6}^s = 0$$

Backup

$$x_T^\gamma = \frac{2p_T^\gamma}{\sqrt{s}}$$



Backup

$$x_T^\gamma = \frac{2p_T^\gamma}{\sqrt{s}}$$