



Precise predictions for semi-inclusive DIS

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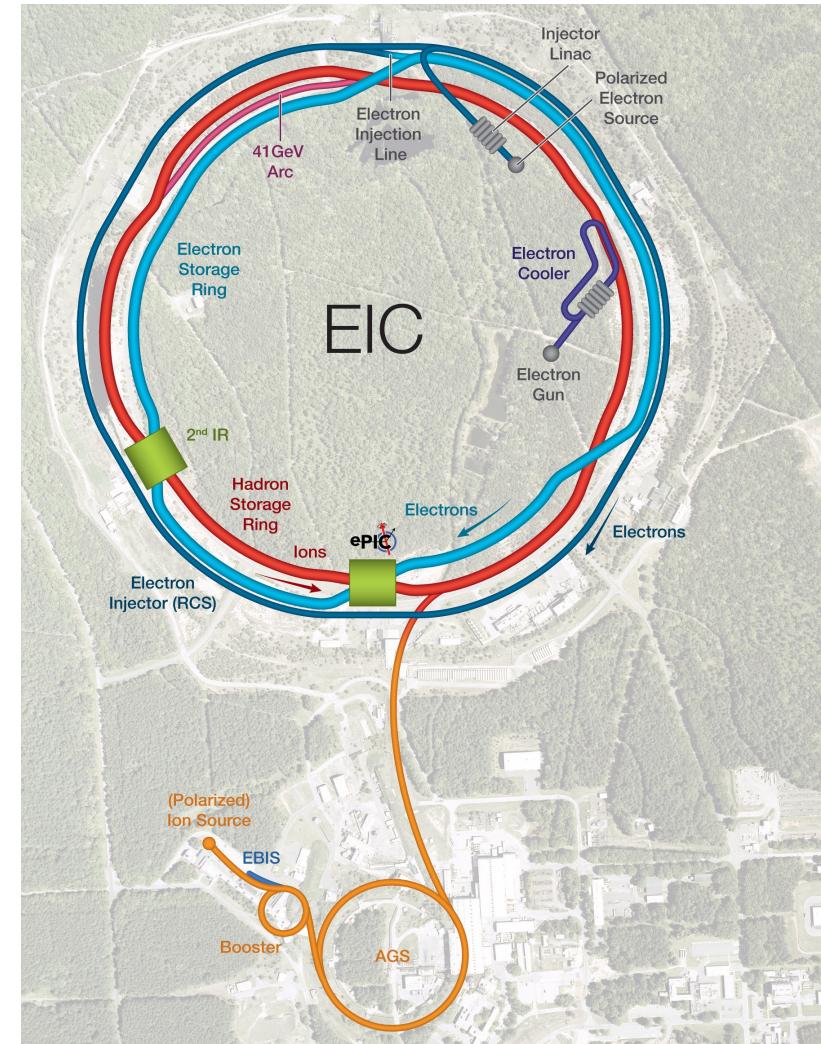
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Motivation

Precision physics at the Electron-Ion Collider (EIC)

- EIC under construction at BNL
- High-luminosity: $10^{33} \dots 10^{34} \text{ cm}^{-2}\text{s}^{-1}$
- Centre-of-mass energy range: 40..140 GeV
- Full identification of hadronic final state
- Polarized collisions

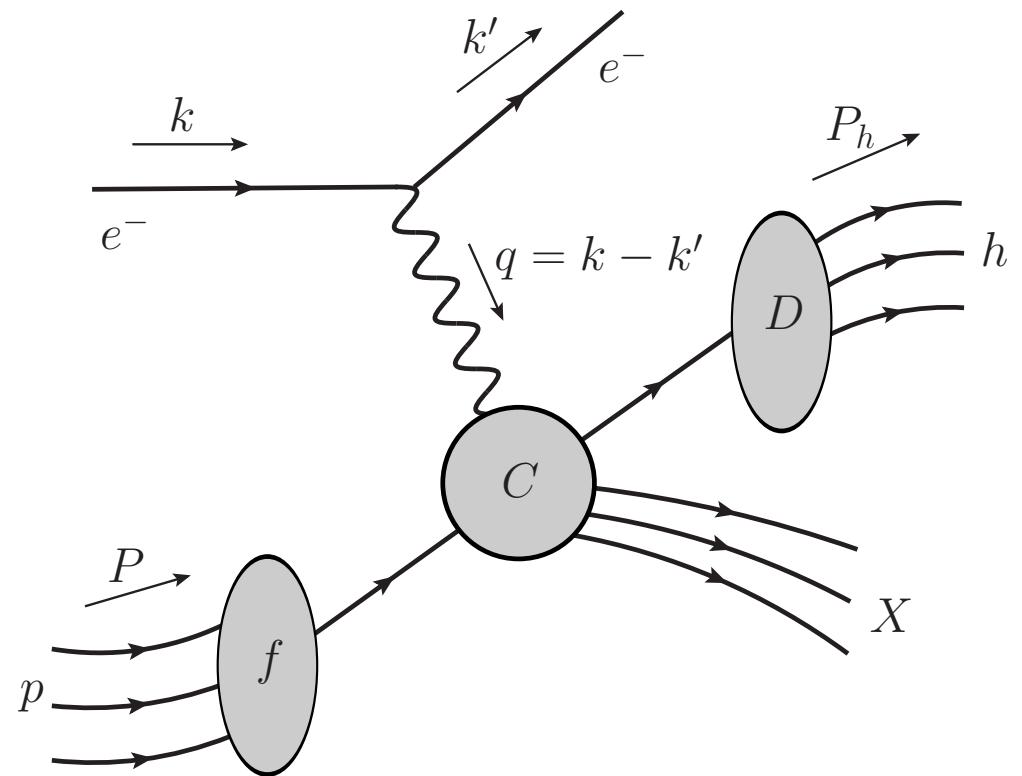
Physics interpretation of EIC data requires precision theory predictions



Semi-inclusive deep-inelastic scattering (SIDIS)

Production of identified hadrons in DIS

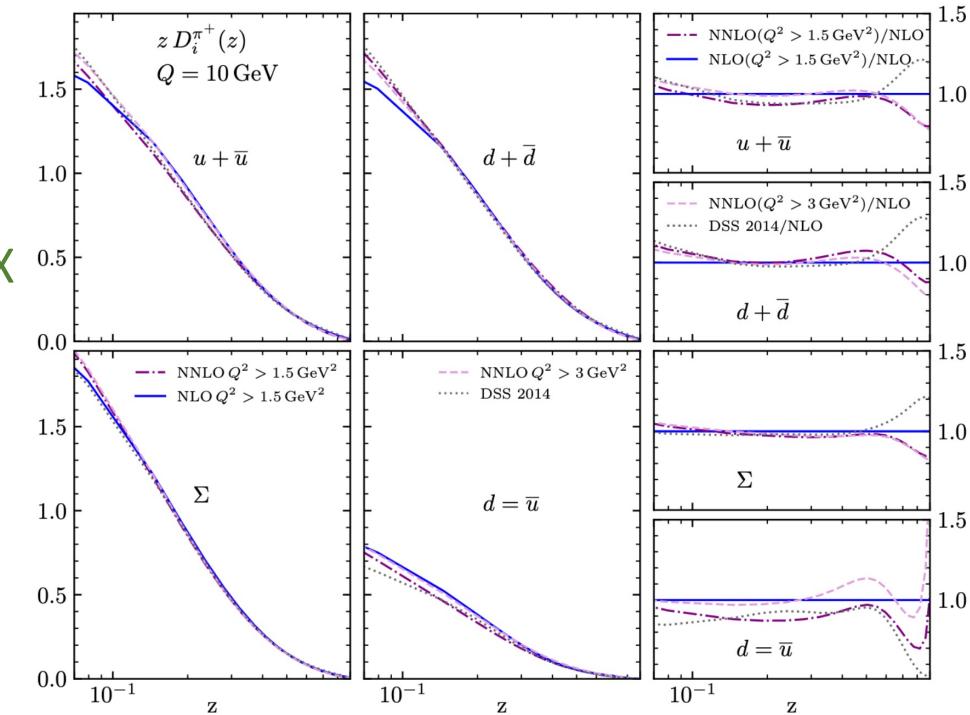
- multiple hadron species: $\pi, K, D, p, n, \Lambda, \dots$
- hadron structure:
parton distribution function (PDF) $f(x, \mu)$
- parton-to-hadron fragmentation:
fragmentation function (FF) $D(z, \mu)$
- probe flavour structure of PDFs and FFs
- flavour decomposition of sea quark PDFs
- constraints on polarized PDFs
- flavour separation of FFs for different hadrons



State-of-the-art: fragmentation functions

Global fits of fragmentation functions

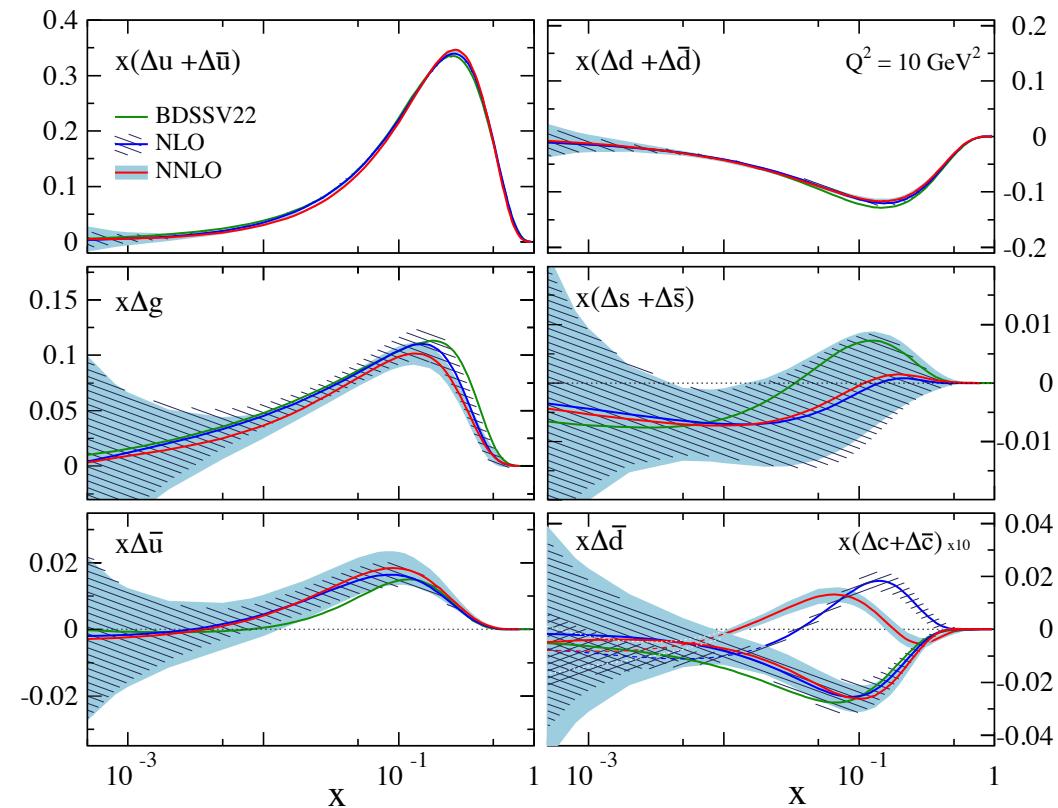
- data sets
 - Semi-inclusive annihilation (SIA) $e^+e^- \rightarrow h+X$
 - Semi-inclusive deep-inelastic scattering (SIDIS) $ep \rightarrow eh+X$
 - Single-inclusive hadro-production (SIH) $pp \rightarrow h+X$
- perturbative order
 - NNLO for time-like Altarelli-Parisi splitting functions
[A.Almasy, S.Moch, A.Vogt]
 - NNLO for SIA coefficient functions [P.Rijken, W. van Neerven]
 - NLO for SIDIS and SIH coefficient functions
[G.Altarelli, K.Ellis, G.Martinelli, S.Pi; F.Aversa, P.Chiappetta, M.Greco, J.P.Gillet]
 - approximate NNLO from threshold resummation for SIDIS [M.Abele, D.de Florian, W.Vogelsang]
- NNLO SIA and aNNLO SIA+SIDIS fits [D.Anderle, F.Ringer, M.Stratmann; V.Bertone, S.Carrazza, N.Hartland, E.Nocera, J.Rojo (NNFF); I.Borsa, R.Sassot, D.de Florian, W.Vogelsang; R.Abdul Khalek, V.Bertone, A.Khoudii, E.Nocera (MAP)]



State-of-the-art: polarized PDFs

Global fits of polarized parton distributions

- data sets
 - inclusive polarized DIS
 - polarized SIDIS
 - polarized proton-proton (RHIC)
- perturbative order
 - NNLO splitting functions
[S.Moch, J.Vermaseren, A.Vogt; J. Blümlein, C.Schneider, K.Schönnwald]
 - NNLO inclusive DIS and Drell-Yan
[E.Zijstra, W.van Neerven; R.Boughezal, H.Li, F.Petriello]
 - aNNLO SIDIS
- most recently first NNLO fits: MAP24, BDSSV24
[MAP: V. Bertone, E.Cieza, E.Nocera;
BDSSV: I.Borsa, D.de Florian, R.Sassot, M.Stratmann, W.Vogelsang]



SIDIS kinematics

$$l(k) + p(P) \rightarrow l(k') + h(P_h) + X$$

variables

$$Q^2 = -q^2 \quad x = \frac{Q^2}{2P \cdot q} \quad z = \frac{P \cdot P_h}{P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k} = \frac{Q^2}{x s_{lp}} \quad W^2 = (q + P)^2 = Q^2 \frac{1 - x}{x}$$

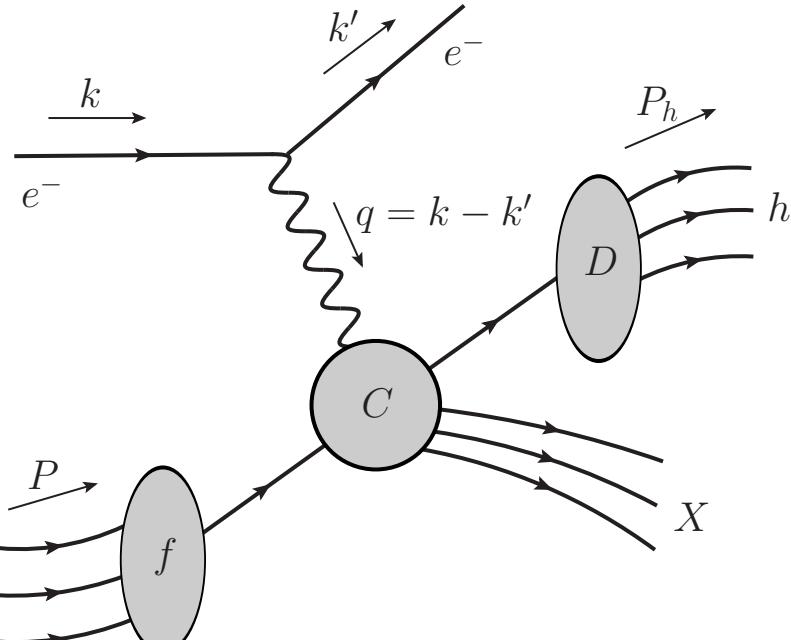
cross sections

$$\frac{d^3\sigma^h}{dxdydz} = \frac{4\pi\alpha^2}{Q^2} \left[\frac{1 + (1 - y)^2}{2y} F_T^h(x, z, Q^2) + \frac{1 - y}{y} F_L^h(x, z, Q^2) \right]$$

$$\frac{d^3\Delta\sigma^h}{dxdydz} = \frac{4\pi\alpha^2}{Q^2} (2 - y) g_1^h(x, z, Q^2)$$

multiplicity $\frac{dM_h}{dz} = \frac{d^3\sigma^h}{dxdydz} \Big/ \frac{d^2\sigma^{\text{DIS}}}{dxdy}$

spin asymmetry $A_1^h(x, z, Q^2) = \frac{g_1^h(x, z, Q^2)}{F_1^h(x, z, Q^2)}$



SIDIS coefficient functions

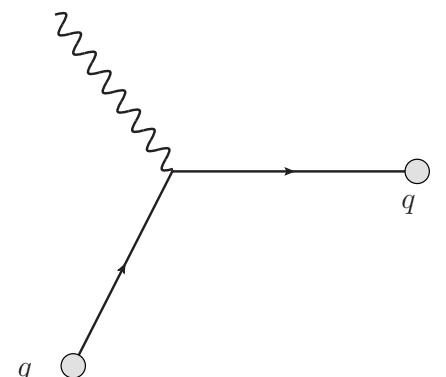
Parton model

$$F_i^h(x, z, Q^2) = \sum_{p,p'} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} f_p \left(\frac{x}{\hat{x}}, \mu_F \right) D_{p'}^h \left(\frac{z}{\hat{z}}, \mu_A \right) C_{p'p}^i \left(\hat{x}, \hat{z}, Q^2, \mu_R, \mu_F, \mu_A \right)$$

$$\begin{aligned} C_{p'p}^i \left(\hat{x}, \hat{z}, Q^2, \mu_R, \mu_F, \mu_A \right) = & C_{p'p}^{i,(0)} \left(\hat{x}, \hat{z} \right) + \frac{\alpha_s(\mu_R)}{2\pi} C_{p'p}^{i,(1)} \left(\hat{x}, \hat{z}, Q^2, \mu_R, \mu_F, \mu_A \right) \\ & + \left(\frac{\alpha_s(\mu_R)}{2\pi} \right)^2 C_{p'p}^{i,(2)} \left(\hat{x}, \hat{z}, Q^2, \mu_R, \mu_F, \mu_A \right) + \dots \end{aligned}$$

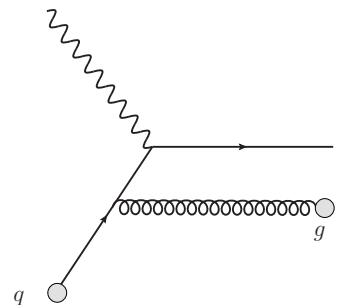
Leading order

$$C_{qq}^{T,(0)} \left(\hat{x}, \hat{z} \right) = e_q^2 \delta(1 - \hat{x}) \delta(1 - \hat{z}) \quad C_{qq}^{L,(0)} \left(\hat{x}, \hat{z} \right) = 0$$

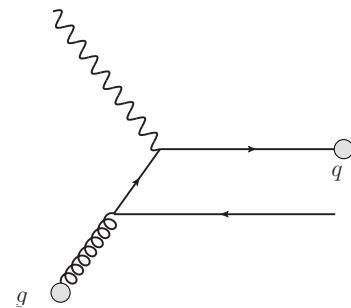


SIDIS coefficient functions

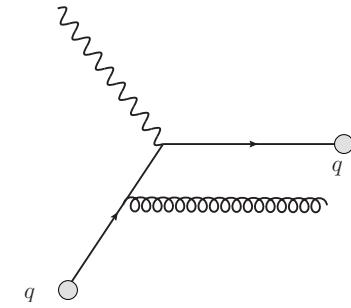
Next-to-leading order [G.Altarelli, K.Ellis, G.Martinelli, S.Pi; D.de Florian, M.Stratmann, W.Vogelsang]



$$C_{gq}^{i,(1)}$$



$$C_{qg}^{i,(1)}$$



$$C_{qq}^{i,(1)}$$

- real radiation at NLO: $q + p_i \rightarrow k_p^{\text{id}} + k_j$

$$C_{\text{R}}^{(1)} \sim \int d\phi_2(k_p, k_j; q, k_i) |\mathcal{M}|_{\text{R}}^2(s_{ip}, s_{ij}, s_{jp}) \delta \left(z - \frac{s_{ip}}{s_{ip} + s_{ij}} \right)$$

$$(1-x)^{-1-2\epsilon} = -\frac{1}{2\epsilon} \delta(1-x) + \sum_n \frac{(-2\epsilon)^n}{n!} \mathcal{D}_n(1-x)$$

$$s_{ip} = -Q^2 \frac{\hat{z}}{\hat{x}}$$

$$s_{ij} = -Q^2 \frac{1-\hat{z}}{\hat{x}}$$

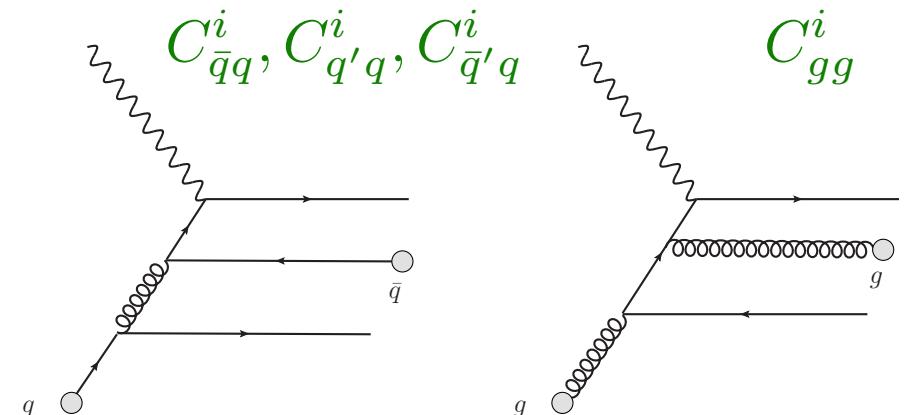
$$s_{jp} = Q^2 \frac{1-\hat{x}}{\hat{x}}$$

SIDIS coefficient functions at NNLO

Partonic channels

$$C_{qq}^{i,(2)} = C_{\bar{q}\bar{q}}^{i,(2)} = e_q^2 C_{qq}^{i,\text{NS}} + \left(\sum_j e_{q_j}^2 \right) C_{qq}^{i,\text{PS}},$$

- $C_{\bar{q}q}^{i,(2)} = C_{q\bar{q}}^{i,(2)} = e_q^2 C_{\bar{q}q}^i,$
- $C_{q'q}^{i,(2)} = C_{\bar{q}'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} + e_q e_{q'} C_{q'q}^{i,3},$
- $C_{\bar{q}'q}^{i,(2)} = C_{q'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} - e_q e_{q'} C_{q'q}^{i,3},$
- $C_{gq}^{i,(2)} = C_{g\bar{q}}^{i,(2)} = e_q^2 C_{gq}^i,$
- $C_{qg}^{i,(2)} = C_{\bar{q}g}^{i,(2)} = e_q^2 C_{qg}^i,$
- $C_{gg}^{i,(2)} = \left(\sum_j e_{q_j}^2 \right) C_{gg}^i,$



new channels at NNLO

SIDIS coefficient functions at NNLO

Partonic channels

► $C_{qq}^{i,(2)} = C_{\bar{q}\bar{q}}^{i,(2)} = e_q^2 C_{qq}^{i,\text{NS}} + \left(\sum_j e_{q_j}^2 \right) C_{qq}^{i,\text{PS}},$

$$C_{\bar{q}q}^{i,(2)} = C_{q\bar{q}}^{i,(2)} = e_q^2 C_{\bar{q}q}^i,$$

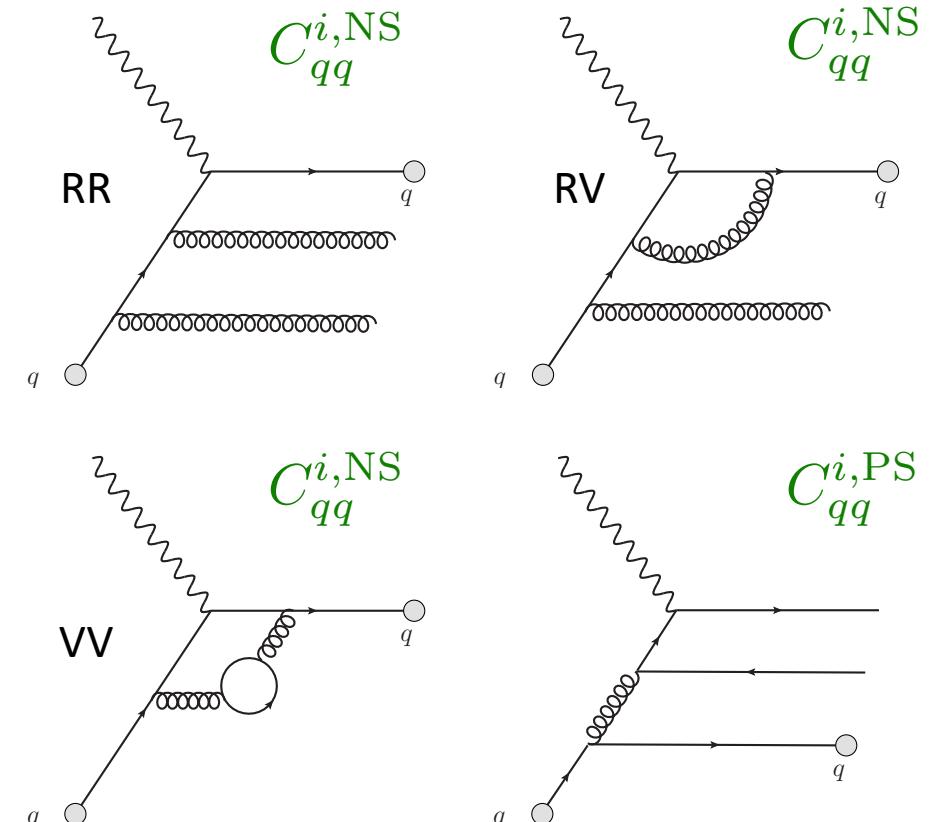
$$C_{q'q}^{i,(2)} = C_{\bar{q}'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} + e_q e_{q'} C_{q'q}^{i,3},$$

$$C_{\bar{q}'q}^{i,(2)} = C_{q'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} - e_q e_{q'} C_{q'q}^{i,3},$$

$$C_{gq}^{i,(2)} = C_{g\bar{q}}^{i,(2)} = e_q^2 C_{gq}^i,$$

$$C_{qg}^{i,(2)} = C_{\bar{q}g}^{i,(2)} = e_q^2 C_{qg}^i,$$

$$C_{gg}^{i,(2)} = \left(\sum_j e_{q_j}^2 \right) C_{gg}^i,$$



not a forward scattering amplitude:
evaluate all contributions separately

NNLO corrections

VV: known massless two-loop form factors [T.Matsuura, W.van Neerven]

RV: one-loop single-real matrix elements

$$C_{\text{RV}}^{(2)} \sim \int d\phi_2(k_p, k_j; q, k_i) |\mathcal{M}|_{\text{RV}}^2(s_{ip}, s_{ij}, s_{jp}) \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip} + s_{ij}}\right)$$

- phase space integral fully constrained, expand in distributions [R.Schürmann, TG]

RR: tree-level double-real matrix elements

$$C_{\text{RR}}^{(2)} \sim \int d\phi_3(k_p, k_j, k_k; q, k_i) |\mathcal{M}|_{\text{RR}}^2(\{s_{ab}\}) \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip} + s_{ij} + s_{ik}}\right)$$

- phase space integrals with kinematical constraint
- reduce to phase-space master integrals, computed through differential equations [L.Bonino, M.Marcoli, R.Schürmann, G.Stagnitto, TG]

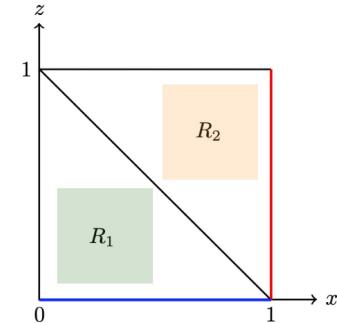
NNLO corrections: RV

RV: one-loop single-real matrix elements: massless bubble and box functions

$$C_{\text{RV}}^{(2)} \sim \int d\phi_2(k_p, k_j; q, k_i) |\mathcal{M}|_{\text{RV}}^2(s_{ip}, s_{ij}, s_{jp}) \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip} + s_{ij}}\right)$$

- requires careful analytic continuation of one-loop functions (4 segments)
- example: Box($s_{12}=s_{ij}$, $s_{23}=s_{pj}$)

$$\begin{aligned} \text{Box}(s_{ij}, s_{ik}) &= \frac{2(1-2\epsilon)}{\epsilon} A_{2,LO} \frac{1}{s_{ij}s_{ik}} \\ &\times \left[\left(\frac{s_{ij}s_{ik}}{s_{ij}-s_{ijk}} \right)^{-\epsilon} {}_2F_1 \left(-\epsilon, -\epsilon; 1-\epsilon; \frac{s_{ijk}-s_{ij}-s_{ik}}{s_{ijk}-s_{ij}} \right) \right. \\ &+ \left(\frac{s_{ij}s_{ik}}{s_{ik}-s_{ijk}} \right)^{-\epsilon} {}_2F_1 \left(-\epsilon, -\epsilon; 1-\epsilon; \frac{s_{ijk}-s_{ij}-s_{ik}}{s_{ijk}-s_{ik}} \right) \\ &- \left. \left(\frac{-s_{ijk}s_{ij}s_{ik}}{(s_{ij}-s_{ijk})(s_{ik}-s_{ijk})} \right)^{-\epsilon} {}_2F_1 \left(-\epsilon, -\epsilon; 1-\epsilon; \frac{s_{ijk}(s_{ijk}-s_{ij}-s_{ik})}{(s_{ijk}-s_{ij})(s_{ijk}-s_{ik})} \right) \right]. \end{aligned}$$



$$\begin{aligned} a_1(s_{12}, s_{23}) &= \frac{s_{123} - s_{12} - s_{23}}{s_{123} - s_{12}} = -\frac{z}{1-x-z}, \\ a_2(s_{12}, s_{23}) &= \frac{s_{123} - s_{12} - s_{23}}{s_{123} - s_{23}} = z, \\ a_3(s_{12}, s_{23}) &= \frac{s_{123}s_{13}}{(s_{13} + s_{23})(s_{12} + s_{13})} = -\frac{xz}{1-x-z} \end{aligned}$$

$$\begin{aligned} \tilde{a}_1(s_{12}, s_{23}) &= 1 - \frac{1}{a_1(s_{12}, s_{23})} = \frac{1-x}{z}, \\ \tilde{a}_3(s_{12}, s_{23}) &= 1 - \frac{1}{a_3(s_{12}, s_{23})} = \frac{(1-x)(1-z)}{xz} \end{aligned}$$

R₁

R₂

NNLO corrections: RR

RR: tree-level double-real matrix elements

$$C_{\text{RR}}^{(2)} \sim \int d\phi_3(k_p, k_j, k_k; q, k_i) |\mathcal{M}|_{\text{RR}}^2(\{s_{ab}\}) \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip} + s_{ij} + s_{ik}}\right)$$

- correspond to cut two-loop integrals

$$C_{\text{RR}}^{(2)} \sim \int d^{4-2\epsilon}k_p d^{4-2\epsilon}k_j |\mathcal{M}|_{\text{RR}}^2(\{s_{ab}\}) \delta^+(k_p^2) \delta^+(k_j^2) \delta^+((q + k_i - k_p - k_j)^2) \delta\left(\hat{z} - \frac{s_{ip}}{s_{ip} + s_{ij} + s_{ik}}\right)$$

- use Cutkosky rule to arrive at Standard integral form

$$I_{t,r,s}(p_1, \dots, p_n) = \int \frac{d^d k}{(2\pi)^d} \frac{d^d l}{(2\pi)^d} \frac{1}{D_1^{m_1} \dots D_t^{m_t}} S_1^{n_1} \dots S_q^{n_q}$$

$$\delta^+(p_i^2) = \frac{1}{2\pi i} \left(\frac{1}{p_i^2 + i0} - \frac{1}{p_i^2 - i0} \right)$$

$$S_i = \{k \cdot p_j, l \cdot p_j\}$$

$$D_i = \{(k - p_j)^2, (l - p_j)^2, (k - l - p_j)^2\}$$

NNLO corrections: RR

Reduction to master integrals: integration-by-part (IBP) equations [K. Chetyrkin, F. Tkachov]

$$\int \frac{d^d k}{(2\pi)^d} \frac{d^d l}{(2\pi)^d} \frac{\partial}{\partial a^\mu} [b^\mu f(k, l, p_i)] = 0 \quad \text{with } a^\mu = k^\mu, l^\mu; b^\mu = k^\mu, l^\mu, p_i^\mu$$

- yield large system of linear relations among integrals
- solved using lexicographic ordering (Laporta algorithm): Reduze2 [A. von Manteuffel, C. Studerus]

Computation of master integrals: differential equations [E. Remiddi, TG]

- differential equations in x and z derived at integrand level
- generic solution by direct integration
- specific solution (matching to boundary condition) by integration over z and comparison with inclusive RR integrals (DIS coefficient functions)

NNLO corrections: RR

Integrals [L.Bonino, M.Marcoli, R.Schürmann, G.Stagnitto, TG]

- 12 propagators (4 cut),
7 of them linearly independent
- 13 integral families with
total 21 master integrals
- analytical results throughout

$$T_{12}(y) = \int_0^y \frac{\arctan x}{x} dx$$

- family A,B,C previously
computed for photon
fragmentation

[R.Schürmann, TG]

$$\begin{aligned} D_1 &= (q - k_p)^2, \\ D_2 &= (p_i + q - k_p)^2, \\ D_3 &= (p_i - k_l)^2, \\ D_4 &= (q - k_l)^2, \\ D_5 &= (p_i + q - k_l)^2, \\ D_6 &= (q - k_p - k_l)^2, \\ D_7 &= (p_i - k_p - k_l)^2, \\ D_8 &= (k_p + k_l)^2, \\ D_9 &= k_p^2, \\ D_{10} &= k_l^2, \\ D_{11} &= (q + p_i - k_p - k_l)^2, \\ D_{12} &= (p_i - k_p)^2 + Q^2 \frac{z}{x}, \end{aligned}$$

family	master	deepest pole	at $x = 1$	at $z = 1$
A	$I[0]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[5]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2, 3, 5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
B	$I[7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-2, 7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-3, 7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2, 3, 7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
C	$I[5, 7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[3, 5, 7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
D	$I[1]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1, 4]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1, 3, 4]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
E	$I[1, 3, 5]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
G	$I[1, 3, 8]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
H	$I[1, 4, 5]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
I	$I[2, 4, 5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
J	$I[4, 7]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[3, 4, 7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
K	$I[3, 5, 8]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
L	$I[4, 5, 7]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
M	$I[4, 5, 8]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$

Table 1. Summary of the double real radiation master integrals.

NNLO corrections: RR

Example: $I[3,5,8]$

$$\begin{aligned}\frac{\partial I[358](Q^2, x, z)}{\partial Q^2} &= -\frac{2(1+\epsilon)}{Q^2} I[358](Q^2, x, z), \\ \frac{\partial I[358](Q^2, x, z)}{\partial x} &= \left(\frac{1+2\epsilon}{1-x} + \frac{2+2\epsilon}{x} \right) I[358](Q^2, x, z), \\ \frac{\partial I[358](Q^2, x, z)}{\partial z} &= -\frac{1+2\epsilon}{z} I[358](Q^2, x, z) - \frac{2x^3(1-2\epsilon)^2(1+z)}{(Q^2)^3(1-x)^2\epsilon z^2(1-z)^2} I[0](Q^2, x, z) \\ &\quad + \frac{2x^2\epsilon}{(Q^2)^2(1-x)z^2} I[5](Q^2, x, z).\end{aligned}$$

$$I_{\text{inc}}[358](Q^2, x) = \frac{3(1-2\epsilon)(4-6\epsilon)(2-6\epsilon)}{\epsilon^3} \frac{x^3}{(Q^2)^3(1-x)^2} I[0](Q^2, x),$$

$$I_{\text{inc}}[0](Q^2, x) = N_\Gamma(Q^2)^{1-2\epsilon} (1-x)^{1-2\epsilon} x^{-1+2\epsilon} \frac{\Gamma(2-2\epsilon)\Gamma(1-\epsilon)}{\Gamma(3-3\epsilon)}.$$

$$\begin{aligned}I[358](Q^2, x, z) &= N_\Gamma \left(\frac{1-2\epsilon}{\epsilon} \right)^2 (Q^2)^{-2-2\epsilon} (1-x)^{-1-2\epsilon} x^{2+2\epsilon} z^{-1-2\epsilon} \\ &\quad \times \left(-2(1-z)^{-2\epsilon} z^\epsilon + 2z^\epsilon {}_3F_2(\epsilon, \epsilon, 2\epsilon, 1+\epsilon, 1+\epsilon, z) \right. \\ &\quad \left. - \frac{2\epsilon\Gamma(1-2\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-\epsilon)} (\pi \cot(\pi\epsilon) + \ln(z)) \right).\end{aligned}$$

Numerical results

$$\frac{d^3\sigma^{\pi^+}}{dxdydz} = \frac{4\pi\alpha^2}{Q^2} \left[\frac{1 + (1 - y)^2}{2y} F_T^{\pi^+}(x, z, Q^2) + \frac{1 - y}{y} F_L^{\pi^+}(x, z, Q^2) \right]$$

Unpolarized SIDIS

[L.Bonino, G.Stagnitto, TG]

- COMPASS kinematics

$\sqrt{s} = 17.35$ GeV

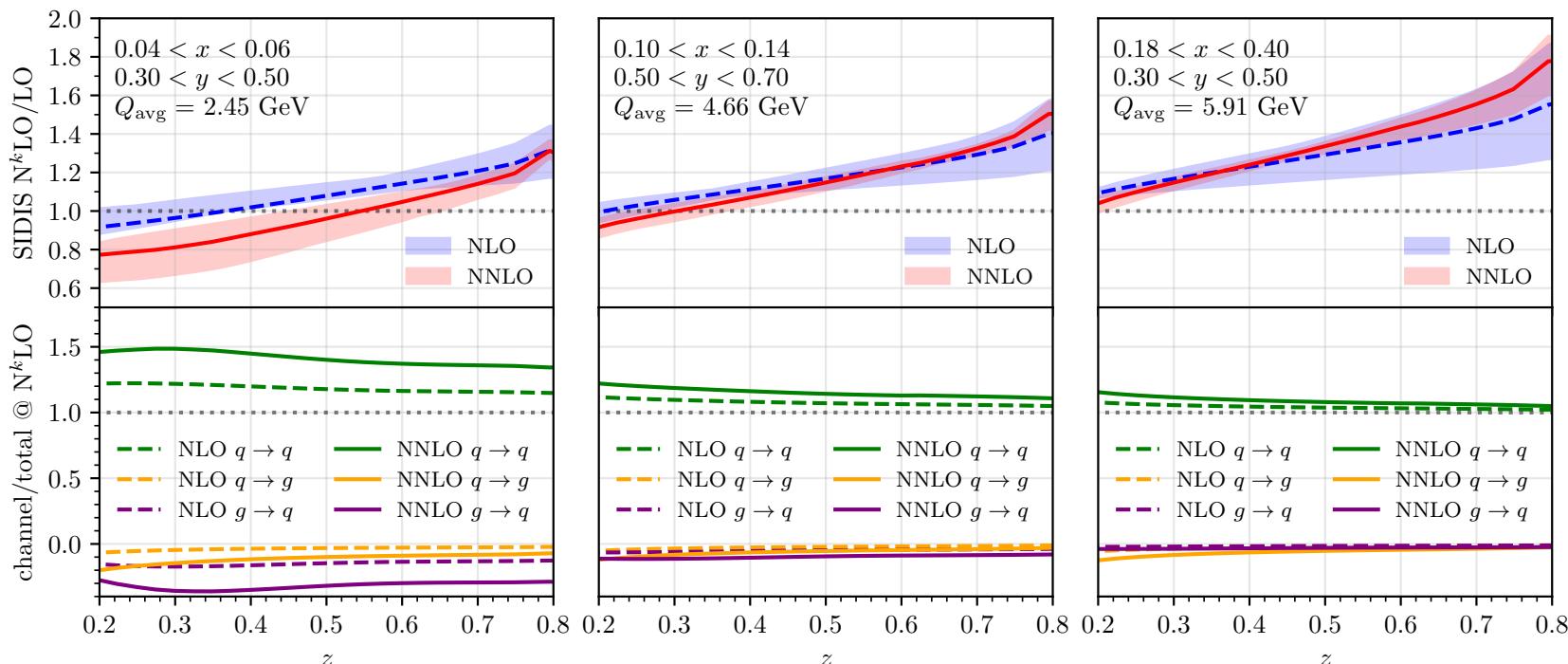
$Q^2 > 1$ GeV 2

$W > 5$ GeV

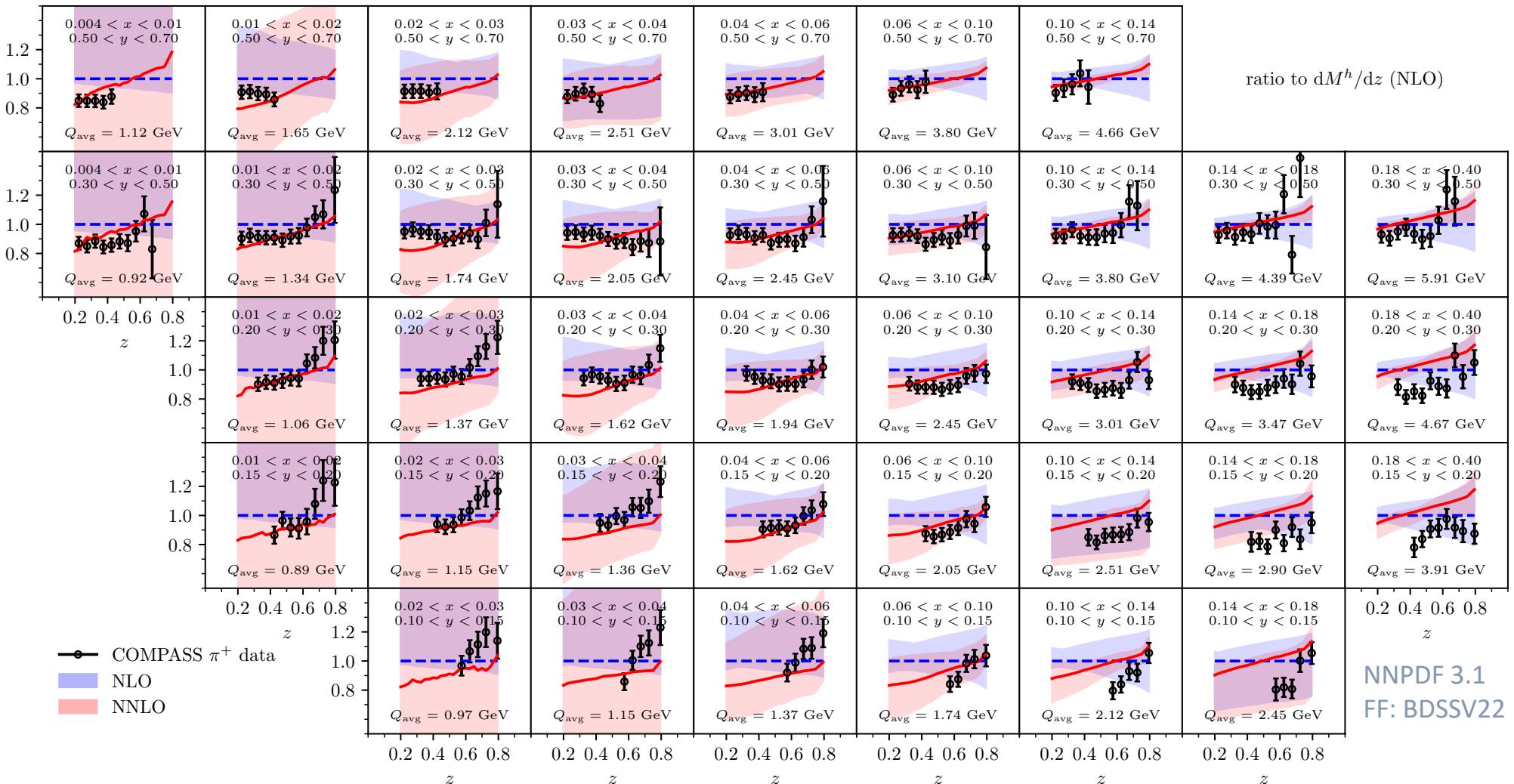
- PDF: NNPDF3.1

- FF: BDSSV22 [I.Borsa, D.de Florian, R.Sassot, M.Stratmann, W.Vogelsang]

- 7-point scale variation



Numerical results: π^+ multiplicity



Numerical results

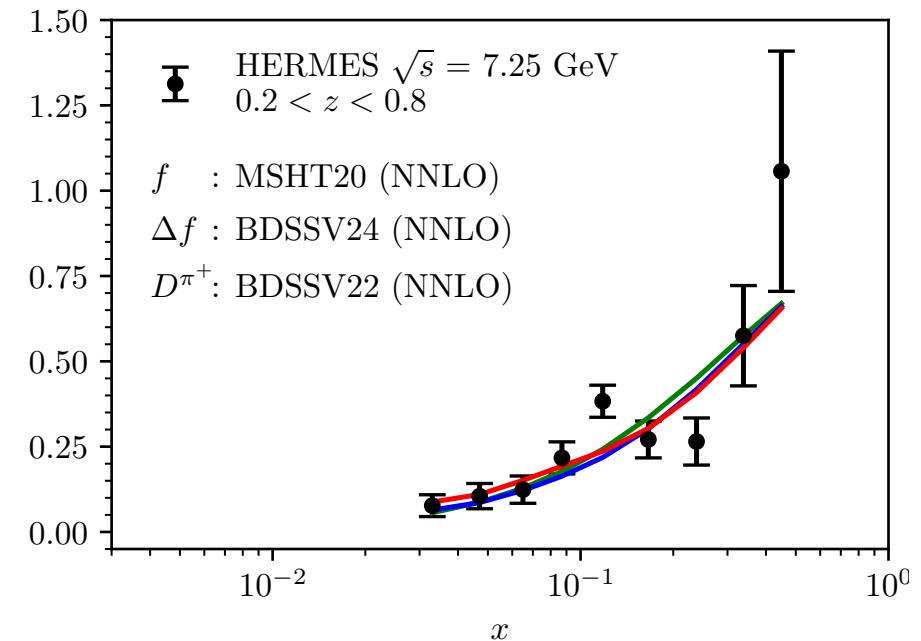
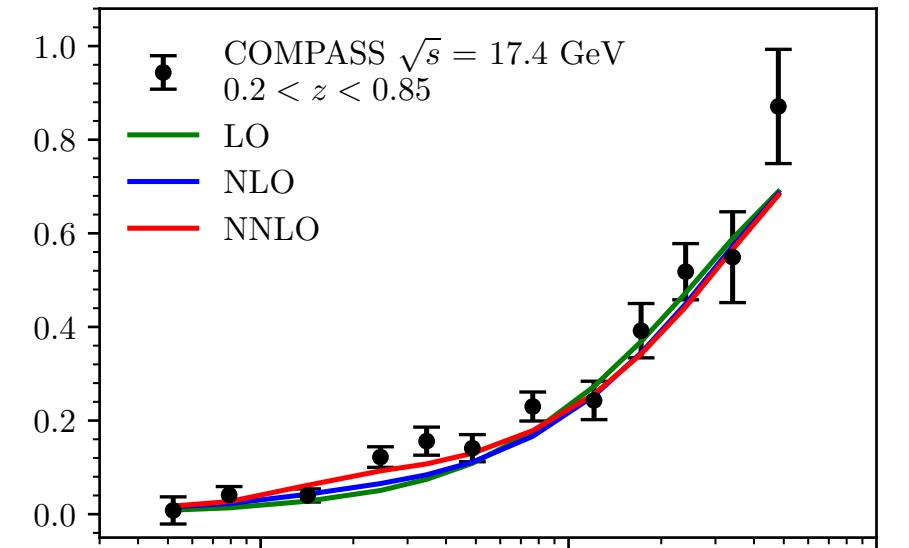
Polarized SIDIS [L.Bonino, M.Löchner, K.Schönwald, G.Stagnitto, TG]

- use Larin γ_5
- finite scheme transformation of polarized PDFs to MS scheme

$$2g_1^h(x, z, Q^2) = \sum_{p,p'} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \Delta f_p \left(\frac{x}{\hat{x}}, \mu_F \right) D_{p'}^h \left(\frac{z}{\hat{z}}, \mu_A \right) \\ \times \Delta C_{p'p} \left(\hat{x}, \hat{z}, Q^2, \mu_R, \mu_F, \mu_A \right)$$

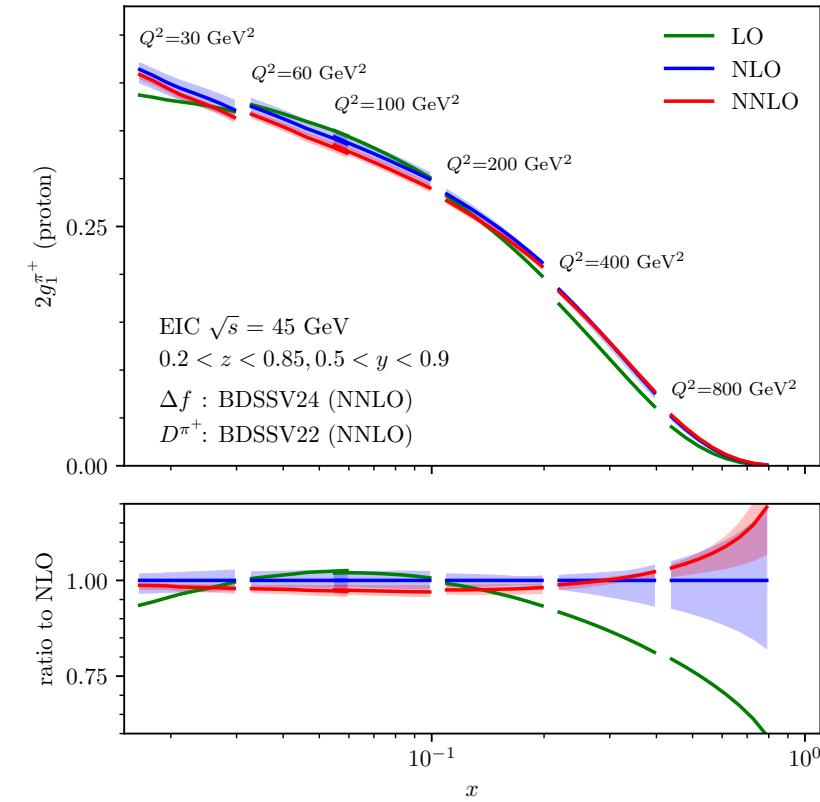
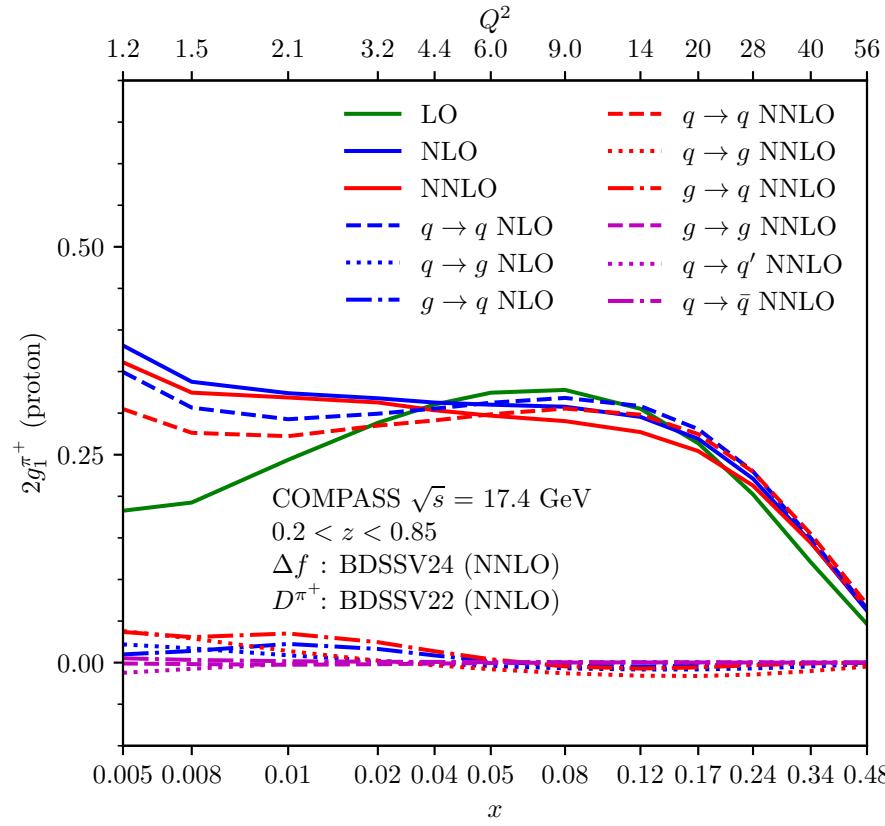
- PDF: BDSSV24 (pol) / MSHT20 (unpol)
[I.Borsa, D.de Florian, R.Sassot, M.Stratmann, W.Vogelsang;
S.Bailey, T.Cridge, L.Harland-Lang, A.Martin, R.Thorne]
- FF: BDSSV21

$$A_1^h(x, z, Q^2) = \frac{g_1^h(x, z, Q^2)}{F_1^h(x, z, Q^2)}$$



Numerical results

Polarized SIDIS [L.Bonino, M.Löchner, K.Schönwald, G.Stagnitto, TG]

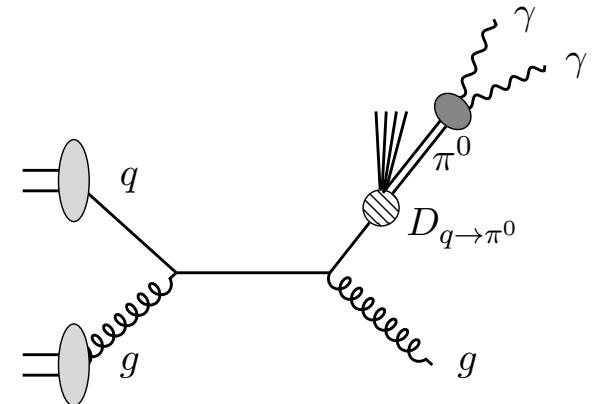


- analytical agreement with an independent calculation [S.Goyal, R.Lee, S.Moch, V.Pathak, N.Rana, V.Ravindran]

Outlook

Single-inclusive hadro-production (SIH) $pp \rightarrow h + X$

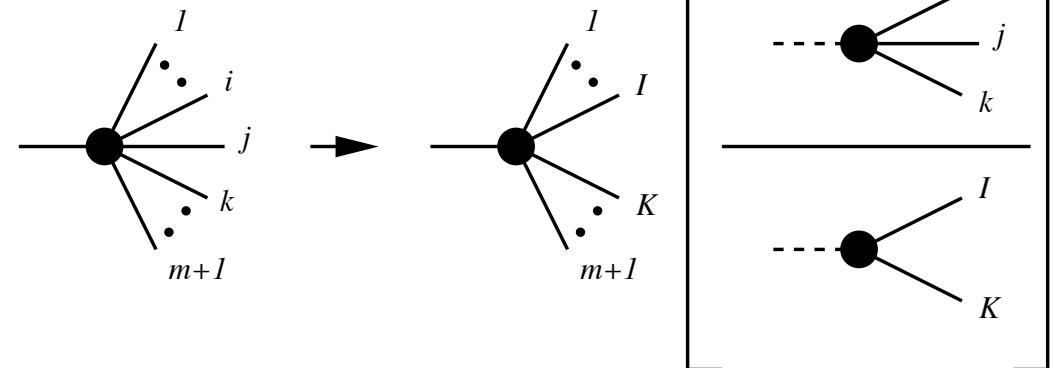
- many precise data sets available
- currently known to NLO [F.Aversa, P.Chiappetta, M.Greco, J.P.Gillet]



NNLO corrections to SIH

- NNLO parton-level event generator: separate numerical evaluation of RR, RV, VV
- requires infrared subtraction scheme at NNLO
- antenna subtraction for hadron fragmentation processes up to NNLO

[L.Bonino, M.Marcoli, R.Schürmann, G.Stagnitto, TG]



Summary

NNLO corrections to SIDIS coefficient functions now available

- fully analytical expressions
- two independent calculations by competing groups

Reduced theory uncertainty on SIDIS cross sections

- enable precision phenomenology with SIDIS data
- allow consistent NNLO fits of fragmentation functions and polarized PDFs