Data analysis exercises

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please copy the octave routines from
/nvme/scratch/lap24/data_analysis
please start octave

Octave basics

Octave is an interpreter:

```
octave:1> a=1
a =  1
```

Don't print results:

```
octave:1> a=1;
```

Print an expression:

```
octave:1> a
a =  1
```

Octave arrays

Octave can handle arrays well:

```
octave:1> a=[1,2,3,4,5]
a =
   1   2   3   4   5
```

Simpler:

```
octave:1> a=1:5
a =
   1   2   3   4   5
```

Also with step sizes:

```
octave:1> a=5:-0.5:3
a =
    5.0000    4.5000    4.0000    3.5000    3.0000
```

Octave treats objects as matrices:

```
octave:1> a=[1,1;0,2]
a =
   1   1
   0   2
```

Arithmetic operations are natively matrix:

```
octave:1> a^2
ans =
   1   3
   0   4
```

If you want it element-wise:

```
octave:1> a.^2
ans =
   1   1
   0   4
```

Octave vectors

Octave distinguishes between row vectors

```
octave:1> a=[1,2]
a =
   1   2
```

and column vectors

```
octave:1> b=[1;2]
b =
   1
   2
```

Transposition is simple:

```
octave:1> b'
ans =
   1   2
```

Octave matrix multiply

If we define a row vector

```
octave:1> a=[1,2]
a =
   1   2
```

there is of course a difference between

```
octave:1> a*a'
ans =  5
```

and

```
octave:1> a'*a
ans =
   1   2
   2   4
```

Octave eigenvalues

Let us define a symmetric matrix:

```
octave:1> m=[2,1;1,4]
m =
   2   1
   1   4
```

We can find the eigenvalues:

```
octave:1> eig(m)
ans =
   1.5858
   4.4142
```

Octave example

To find the eigenvectors as well, use:

```
octave:1> [vec,val]=eig(m)
vec =
   -0.92388    0.38268
    0.38268    0.92388
val =
Diagonal Matrix
    1.5858         0
         0    4.4142
```

So we can reconstruct the original matrix:

```
octave:1> vec*val*vec'
ans =
    2.00000    1.00000
    1.00000    4.00000
```

Octave matrix functions

Initializing matrix (zeros, rand, diag) example:

```
octave:1> m=rand(2)
m =
   0.089507   0.047387
   0.718905   0.878561
```

Sum (and prod) reduces one dimension:

```
octave:1> sum(m)
ans =
   0.80841   0.92595
octave:2> sum(m,2)
ans =
   0.13689
   1.59747
```

Octave cell arrays

One last detail: cell arrays

```
octave:1> a={1,"string"}
a =
{
  [1,1] =  1
  [1,2] = string
}
```

They may contain different objects and are accessed as:

```
octave:1> a{1}
ans =  1
octave:2> a{2}
ans = string
```

Octave cell arrays

Cell arrays can easily be enlarged:

```
octave:1> a=[a [1,2]]
a =
{
  [1,1] =  1
  [1,2] = string
  [1,3] =
     1   2
}
octave:2>a{3}
ans =
   1   2
```

Let's start!

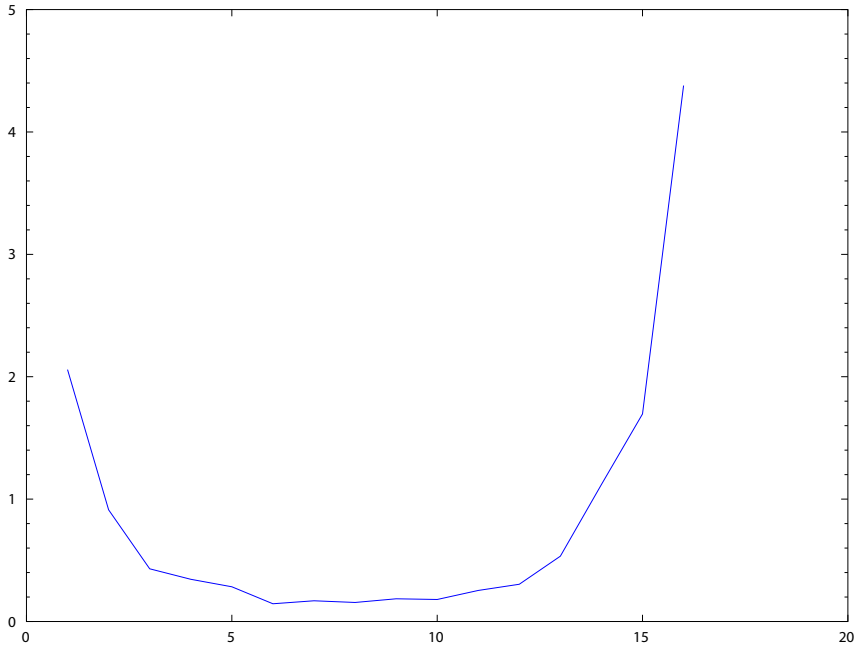
Please open the file "readme.m"

Produce one fake "pion" propagator:

```
octave:1> myprop1=piprop(16)
```

and plot it:

```
octave:1> plot(myprop1{1})
```




```
/nvme/scratch/lap24/data_analysis
```

Displaying data

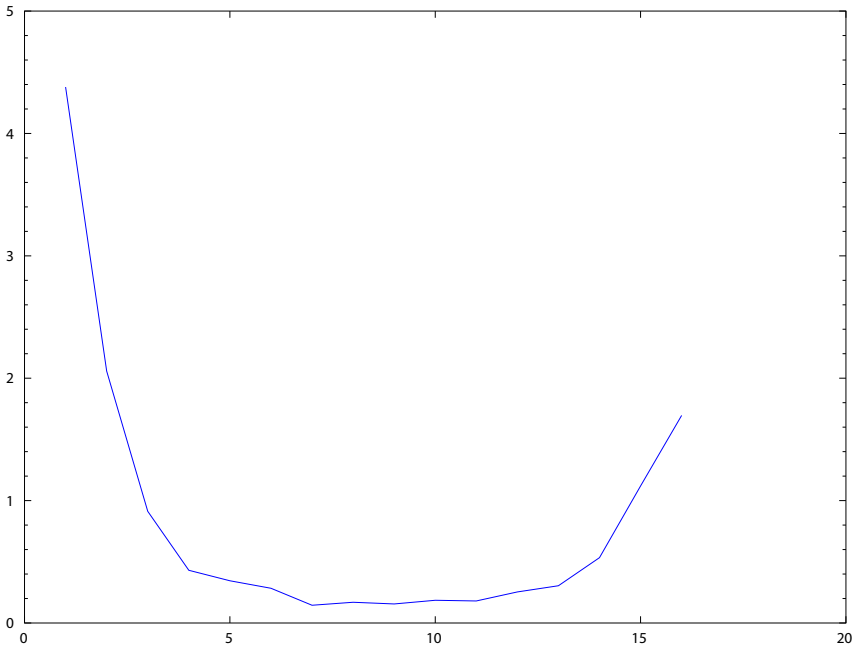
Please don't look at "piprop.m" or any "[..]prop.m" file yet!

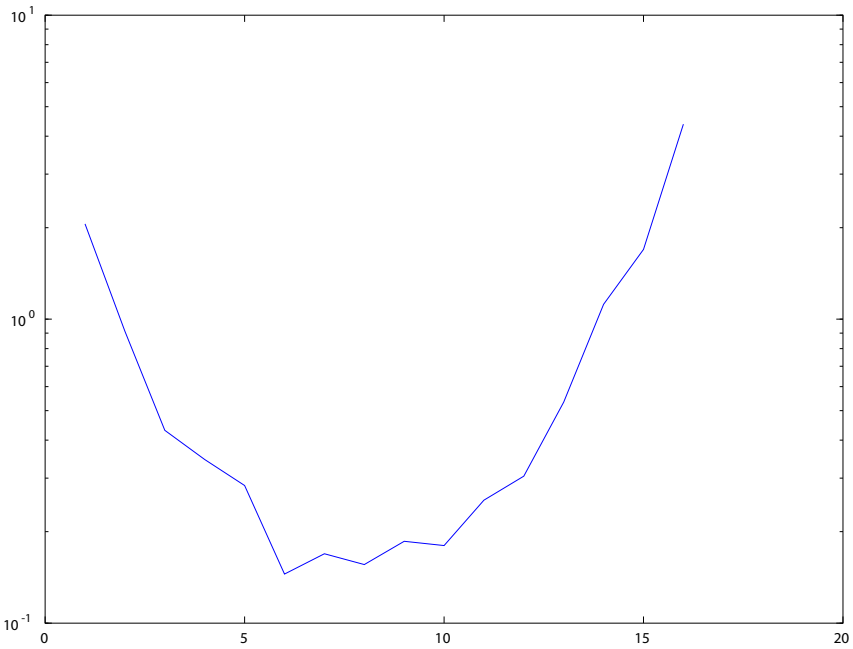
If you like the origin at 0 instead of 16:

```
octave:1> plot(myprop1{1}([end 1:end-1]))
```

For a logarithmic y-scale plot:

```
octave:1> semilogy(myprop1{1})
```





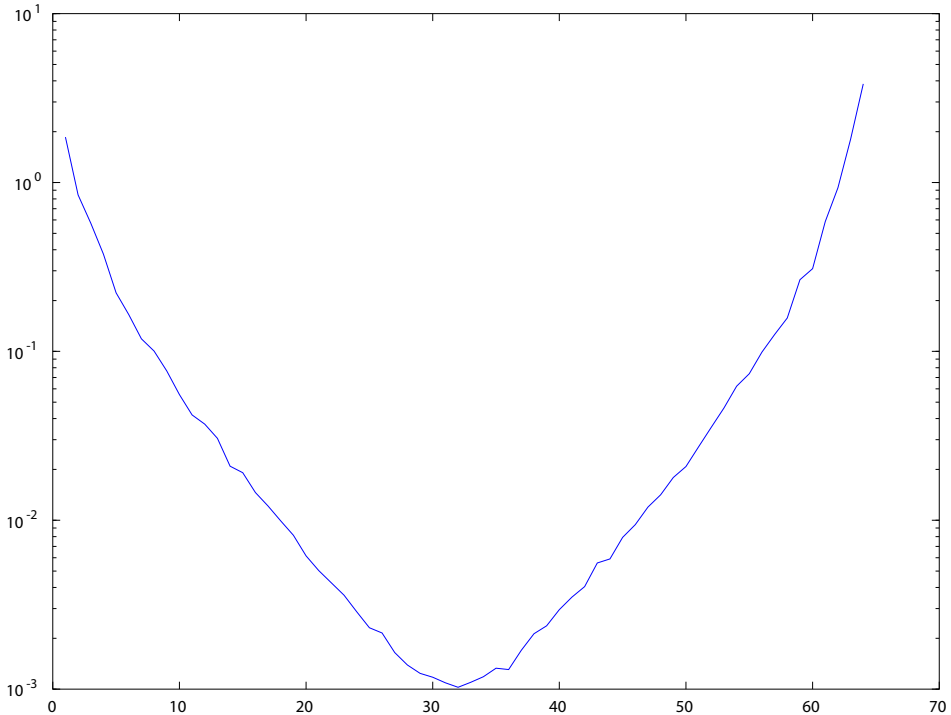
The first ensemble

Now 100 configs with $N_T = 64$:

```
octave:1> prop=piprop(64,100);
```

For a logarithmic y-scale plot:

```
octave:1> semilogy(prop{1})
```



Bootstrapping

Produce 200 bootstrap samples:

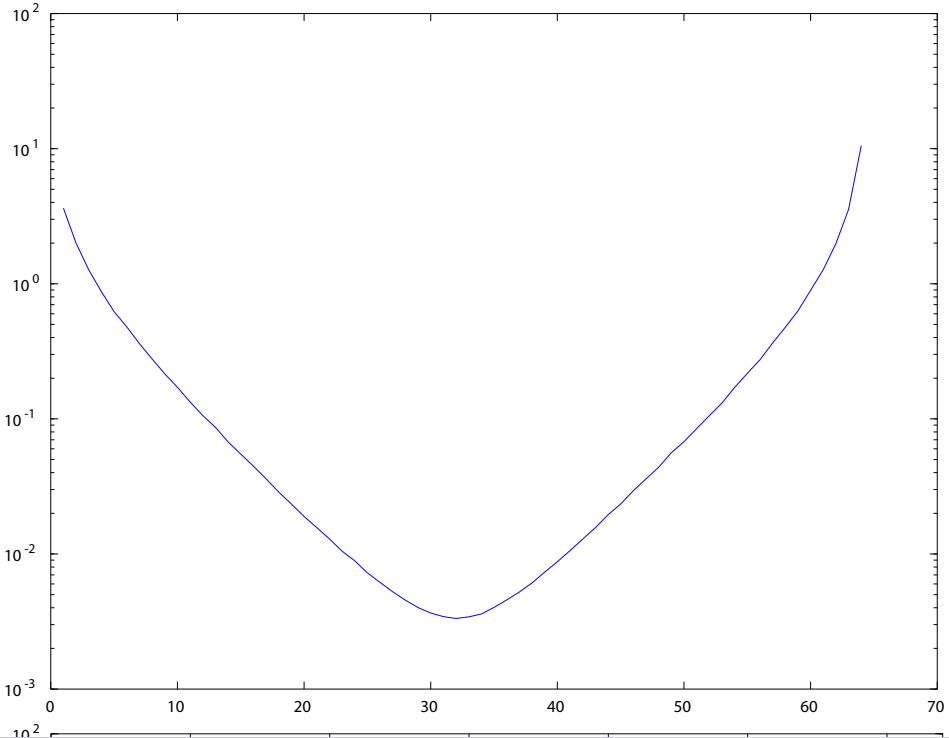
```
octave:1> bp=boot(prop,200);
```

Plot the average (column 201):

```
octave:1> semilogy(bp(:,end))
```

Routine for plotting with errors:

```
octave:1> plotprop(bp)
```



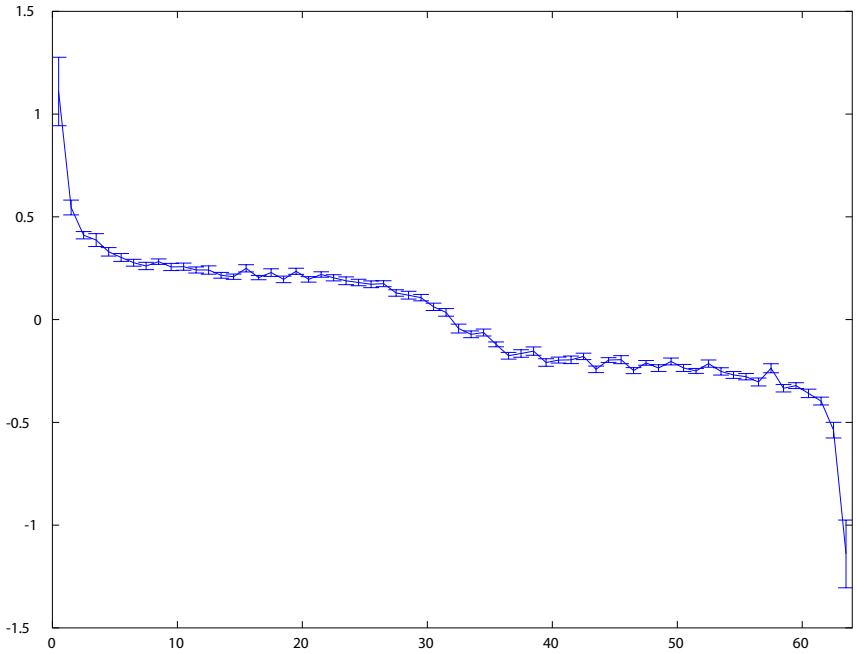
Effective mass

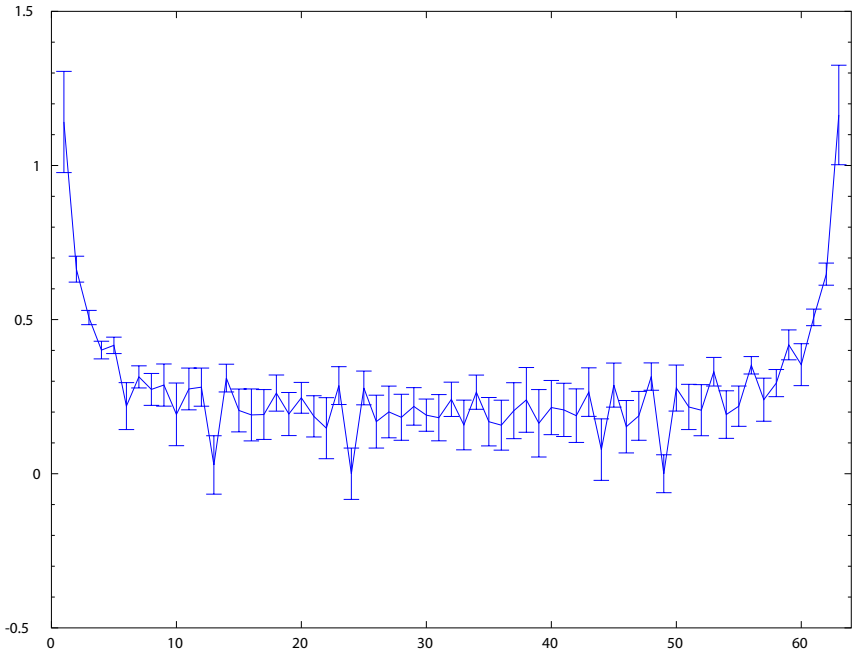
Effective mass with simple exp:

```
octave:1> plotplat(bp)
```

Effective mass with cosh:

```
octave:1> plotsymplat(bp)
```



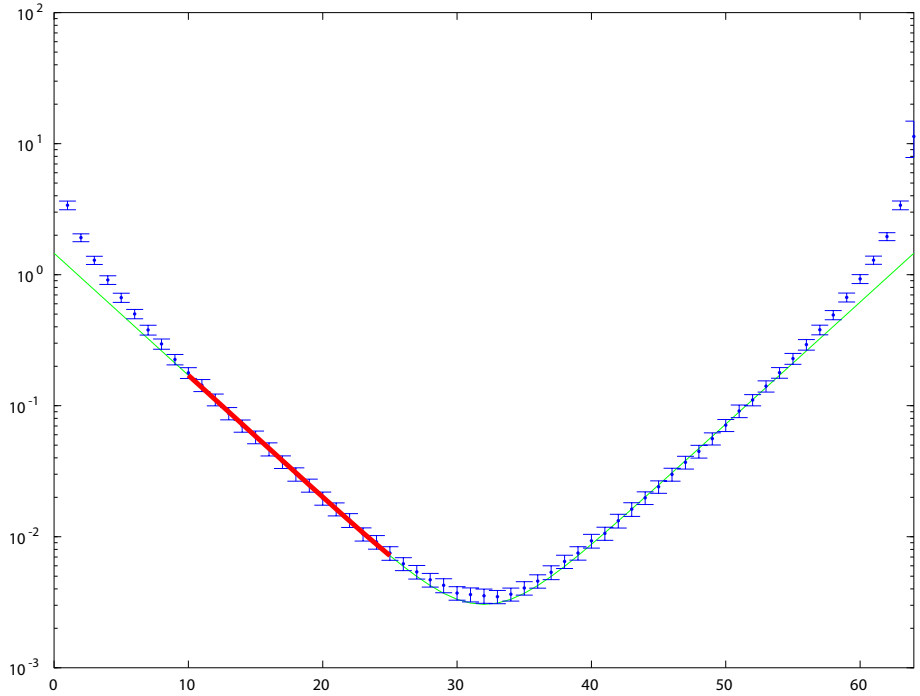


Mass fit

```
uncorrelated_fit=0
fully_correlated_fit=-1

central_value_only=1
full_bootstrap_fit=0

do_plot=1
dont_plot=0
```




```
massfit(bp,uncorrelated_fit,10,25,
                central_value_only,do_plot)
```

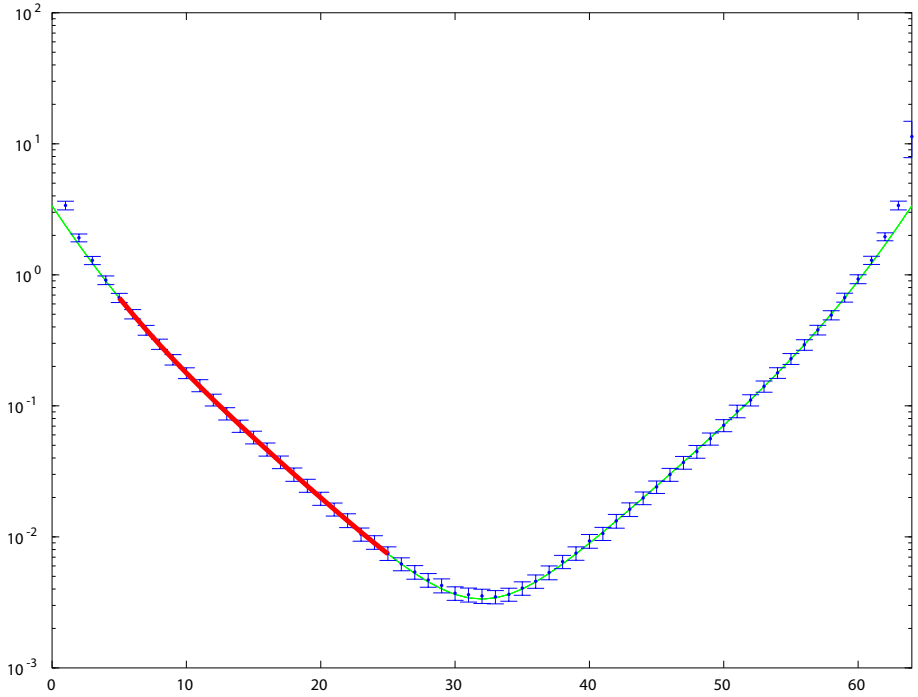
Multiple output variables

```
octave:1> [m,em,c,ec,fitq]=massfit(bp,0,10,25)
m =  0.21448
em =  0.0024118
c =  1.4575
ec =  0.13362
fitq =  0.99984
```

Excited state fit

You might want to try a fit with 2 coshs:

```
octave:1> [m,em,c,ec,fitq,m2,em2,c2,ec2]=
                    exsfit(bp,0,5,25,0,1)
m =  0.20403
em =  0.0028877
c =  1.1466
ec =  0.15216
fitq =  1.00000
m2 =  0.43349
em2 =  0.026877
c2 =  2.2185
ec2 =  0.28166
```



Correlation matrix eigenvalues

```
octave:1> correlation_matrix_solution(bp(10:25,:))
ans =
   1.1263e-03
   1.5686e-03
   1.8096e-03
   2.8204e-03
   3.8986e-03
   7.4055e-03
   8.9888e-03
   1.2718e-02
   1.6033e-02
   2.2189e-02
   2.8986e-02
   3.4987e-02
   5.9748e-02
   1.1857e-01
   2.5247e-01
   1.5427e+01
```

Your routine

Eigenvalues of correlation matrix:

```
function ev=correlation_matrix_template(bp)
# this is a template for a routine that should
# compute the normalized correation matrix and
# its eigenvalues from a bootstrap array
   N=size(bp)(1);     # number of points
   NB=size(bp)(2)-1; # number of bootstrap samples

# insert code here

   ev=eig(cor);
endfunction
```

Correlation matrix solution

```
# take out normalization
   bav=sum(bp(:,1:NB),2)/NB;
   ac=sqrt((sum(bp(:,1:NB).^2,2)/NB-bav.^2)*
                  (1+(1/(NB-1))));

# form the covariance matrix
   cor=zeros(N); # initialize cor as N*N matrix
   for x=1:N
       for y=1:N
           cor(y,x)=(bp(y,1:NB)-bav(y))*
                          (bp(x,1:NB)-bav(x))'/
                         (NB-1)/(ac(x)*ac(y));
       end
   end
```

Find the mass!

- Explore the fitting routine
- Try varying nconf, nboot, NT, fit range, correlation
- Find the “pion” mass and its error
- Give your results to me (with error!)
- If you are done, you can try “rhoprop” instead of “piprop”

have fun!

Let me know your results (central value and total error) so i can plot them

Going to the physical point

Please open the file “day2.m”

generate one well behaved ensemble at “bare quark mass” mq:

```
NT=64
NBOOT=100
NCONF=100
mq=-0.01

octave:1> prp=easy_piprop(NT,mq,NCONF);
```

Bootstrap it and look at the pion mass:

```
octave:1> bpr=boot(prp,NBOOT);
octave:2> m=massfit(bpr,0,15,30,1,1)
Only central value computed!
m =  0.57282
```

The physical point

We are at $1/a = 1.5$ GeV, so the target "physical" $m_\pi a = 0.09$

Produce ensembles that allow you to go to the physical point

- You can either interpolate or extrapolate
- Near the "physical point" propagators are more noisy

Generating ensembles

Template ensemble generation:

```
# a random table of quark masses
masstab=[0,-0.04,-0.06,-0.08]

for i=1:length(masstab)
    prp=easy_piprop(NT,masstab(i),NCONF);
    bp{i}=boot(prp,NBOOT);
# mb and cb are bootstrap results for m and c
    [m,em,c,ec,q,mb{i},cb{i}]=massfit(bp{i},0,15,30);
# extract f from c=f^2*m^2
    fb{i}=sqrt(abs(2*cb{i}./mb{i}));
end
```

Going to the physical point

Please open the file “chiralfit_template.m”

This is a long fit routine with a very simple core:

```
        stp=[d0,k0];
# do the constrained fit
        [par,ch2c,info]=sqp(stp',@constrainedfit,
                                    [],[],[],[],1000);
```

A χ^2 function is minimized, x-errors are yet ignored

Function to minimize

```
function chisq=constrainedfit(par)
# chi^2 function for constrained fpi-mpi^2 fit
   global _f _m _er _n


   chisq=0;
   for i=1:_n
       mm=_m(i);
       ff=fps(mm,par);
       ex=[_m(i)-mm,_f(i)-ff];
       chisq+=ex*_er{i}*ex';
   end
   chisq;
endfunction
```

Fit function

x-values (_m), y-values (_f) and the inverse covariance matrix (_er) are global:

```
global _f _m _er

octave:1> global _f
octave:2> _f
_f =
   0.068717   0.072484   0.075524   0.073040
```

The actual linear fit function:

```
 function ff=fps(m,c)
# the actual fit function
    ff=c(1)+c(2)*m;
endfunction
```

Full fit exercise

Then all masses are added as parameters:

```
stp=par';
for i=1:N
    stp=[stp _m(i)];
end
```

And the full fit is performed:

```
[par,ch2,info]=sqp(stp',@fullfit,[],[],[],[],1000);
```

Complete the routine fulfit!

The solution is in “chiralfit_solution.m”

Completing full fit

```
function chisq=fullfit(par)
# chi^2 function for full fpi-mpi^2 fit
   global _f _m _er _n

##################
# YOUR CODE GOES HERE
# remember: par(3:) are
# the fitted masses
# (x-axis)
##################
endfunction
```

The solution is in “chiralfit_solution.m”

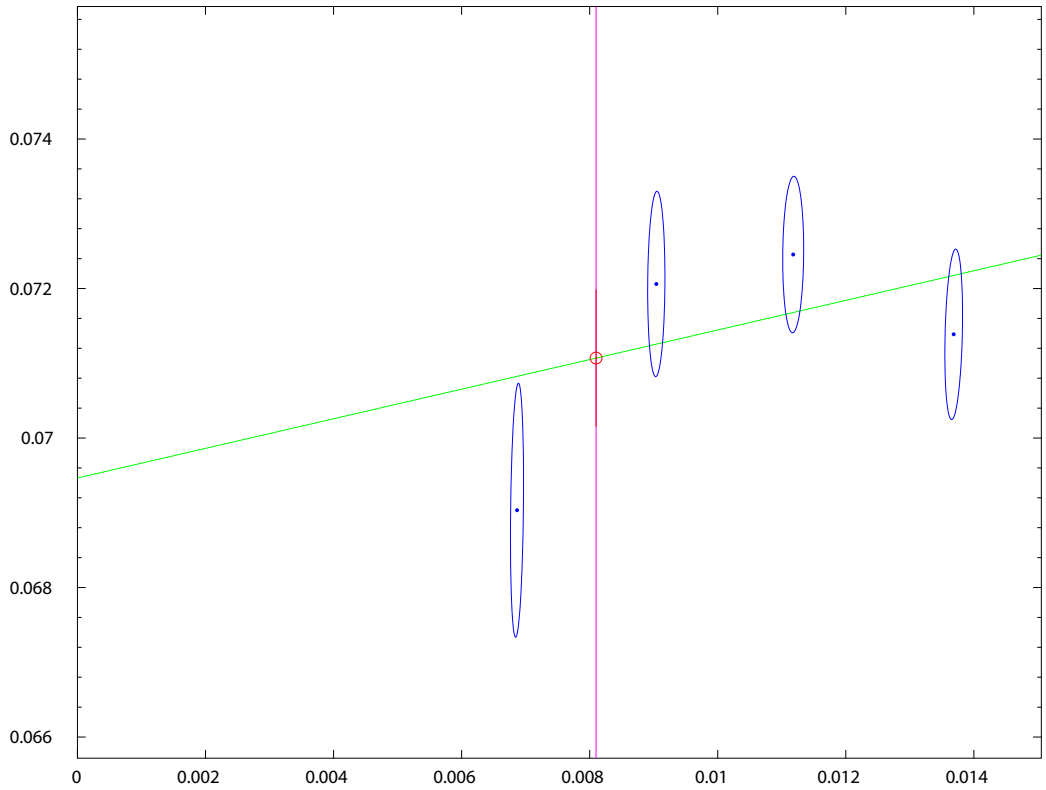
```
function chisq=fullfit(par)
# chi^2 function for full fpi-mpi^2 fit
   global _f _m _er _n

   npar=2;
   chisq=0;
   for i=1:_n
       mm=par(npar+i);
       ff=fps(mm,par(1:npar));
       ex=[_m(i)-mm,_f(i)-ff];
       chisq+=ex*_er{i}*ex';
   end
   chisq;
endfunction
```

Doing the fit

Using the chiral fit:

```
> [fpi,efpi,q,qc]=chiralfit_solution(fb,mb,-1,0,1)
fpi =  0.071069
efpi =  9.1972e-04
q =  0.51122
qc =  0.49685
```



Different fit forms

Setting the x-axis as m^2 (line 33):

```
# use mpi^2 as x-axis
      mb{i}=mb{i}.^2;
```

Vary the 2-parameter fit form (end of file):

```
function ff=fps(m,c)
# the actual fit function
    ff=c(1)+c(2)*m;
endfunction
```

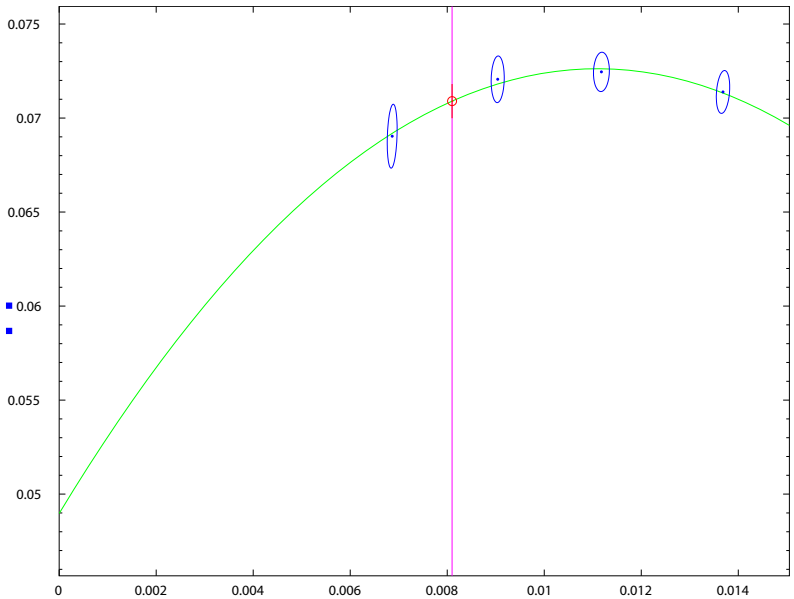
Do a 3-parameter fit:

```
[fpi,efpi,q,qc]=chiralfit_quadratic(fb,mb,-1,1,1)
```

Quadratic fit

Using the chiral fit:

```
> [fpi,efpi,q,qc]=chiralfit_quadratic(fb,mb,-1,0,1)
fpi =  0.070900
efpi =  9.0830e-04
q =  0.97827
qc =  0.97816
```



Vary the 3-parameter fit form (end of file):

```
function ff=fps(m,c)
# the actual fit function
    ff=c(1)+c(2)*m+c(3)*m.^2;
endfunction
```

Find f at the physical point!

- Explore the fitting routines - change fit forms
- Try varying ensembles and fit parameters
- Find the “decay constant” f and its error
- If you are done, try estimating the systematics
- Give your results to me (including full error estimate!)
- If you want the solution, look at “cheat.m” and “physpoint.m”

have fun!

if you have not looked at cheats, give me your results (central values and total errors) so I can plot them.