DEFLECTION OF HIGH-ENERGY CHARGED PARTICLES BY MEANS OF BENT CRYSTALS

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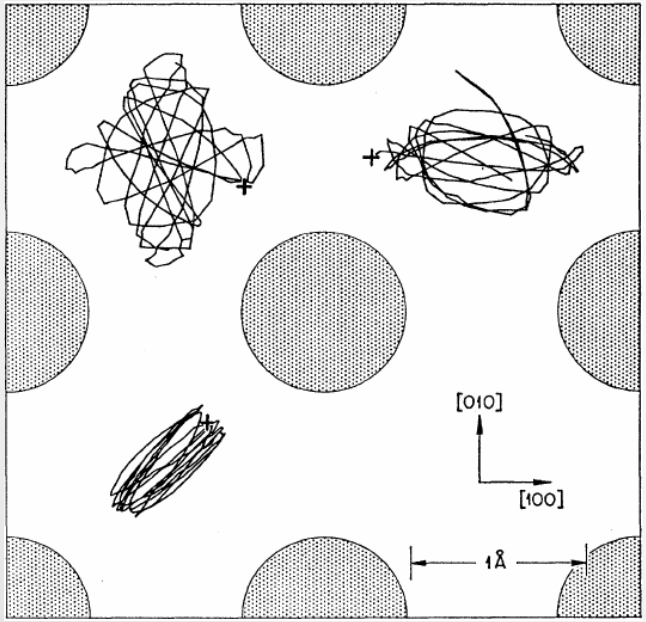
AITP NSC KIPT

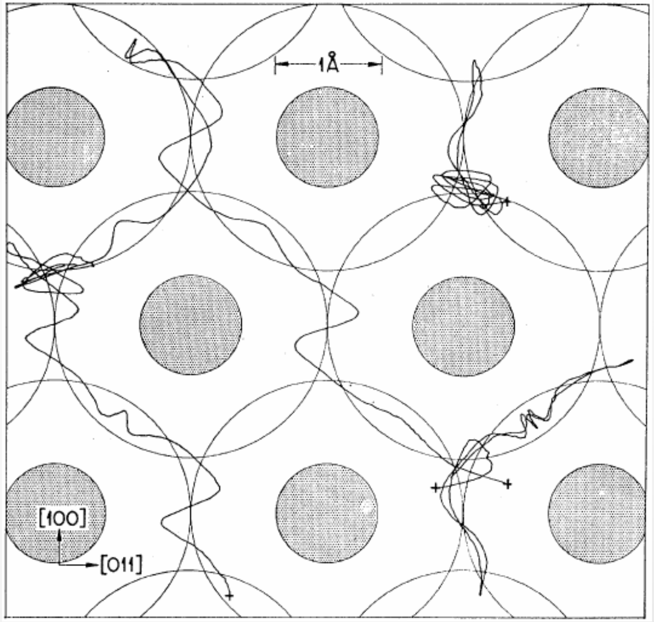
DESY-KIPT SEMINAR IN FRAME OF THE DFG PROJECT “PILOT STUDY OF A CRYSTAL-BASED EXTRACTION FOR 6 GEV ELECTRONS AT DESY”

June 20, 2024

Scattering of charged particles in a crystal

Robinson M. T., Oen O. S., Holmes D. K. Computer studies of anomalous penetration of Cu recoil atoms in Cu crystal. Proc. of Conference «Bombardment Ionique». CNRS. Paris. 1962. P. 105.
Robinson M. T., Oen O. S. Computer studies of the slowing down of energetic atoms in crystals. Phys. Rev. 1963. Vol. 132, No. 6. P. 2385.





Cu^+, E=1-10 keV

Lindhard J. Influence of crystal lattice on motion of energetic charged particles. Mat. Fys. Medd. Dan. Vid. Selsk. 1965. Vol. 34, No. 14. P. 1–64.

Approximation of continuous potential

$$\frac{d}{dt}\frac{m\mathbf{v}}{\sqrt{1-v^2/c^2}} = -q\nabla\Phi_c(\mathbf{r})$$

$$\Phi_c(\mathbf{r}) = \sum_n \Phi_a(\mathbf{r}-\mathbf{r}_n)$$

$$\Phi(\boldsymbol{\rho}) = \frac{1}{L}\int_{-\infty}^{\infty} dz\,\Phi_c(\boldsymbol{\rho}, z)$$

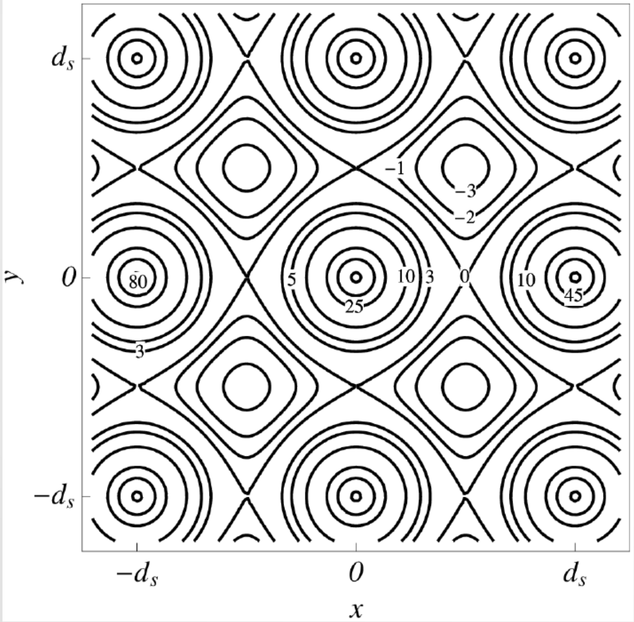
$$\ddot{\boldsymbol{\rho}} = -\frac{c^2 q}{E_\parallel}\frac{\partial}{\partial\boldsymbol{\rho}}\Phi(\boldsymbol{\rho})$$

$$E_\parallel = c\sqrt{p_\parallel^2 + (mc)^2}$$

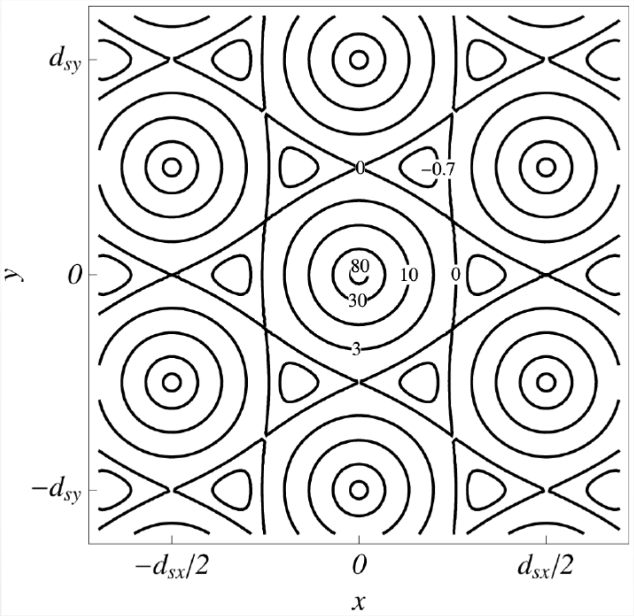
Lindhard J. Influence of crystal lattice on motion of energetic charged particles. Mat. Fys. Medd. Dan. Vid. Selsk. 1965. Vol. 34, No. 14. P. 1–64.

Potential of crystal atomic strings

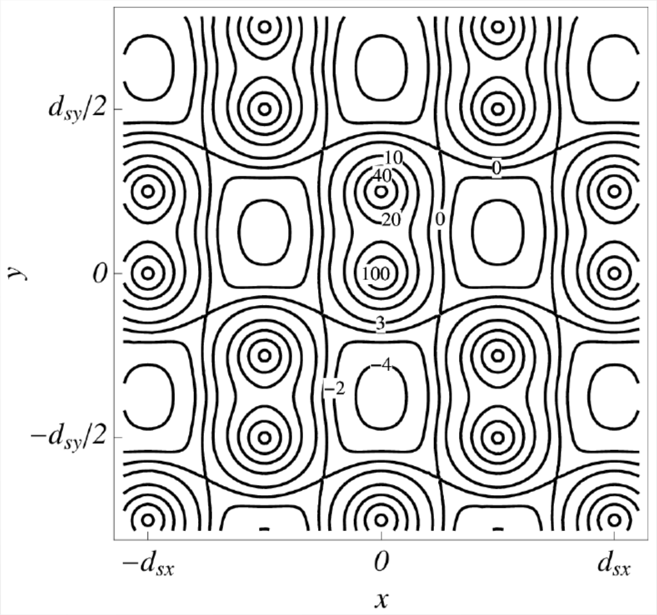
Si <100>



Si <111>

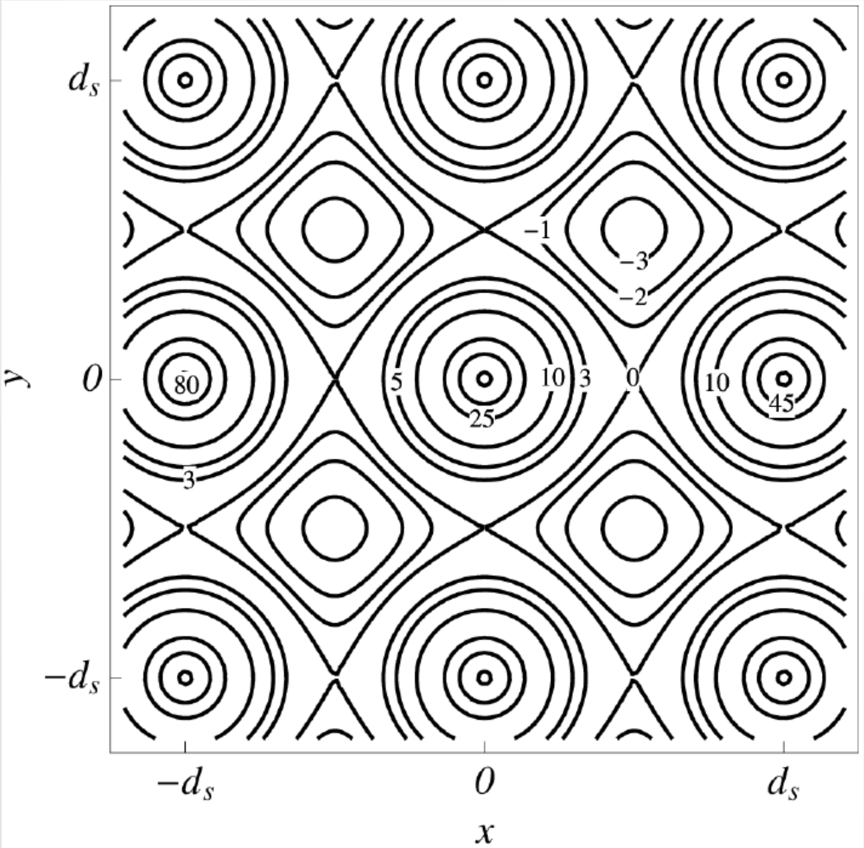


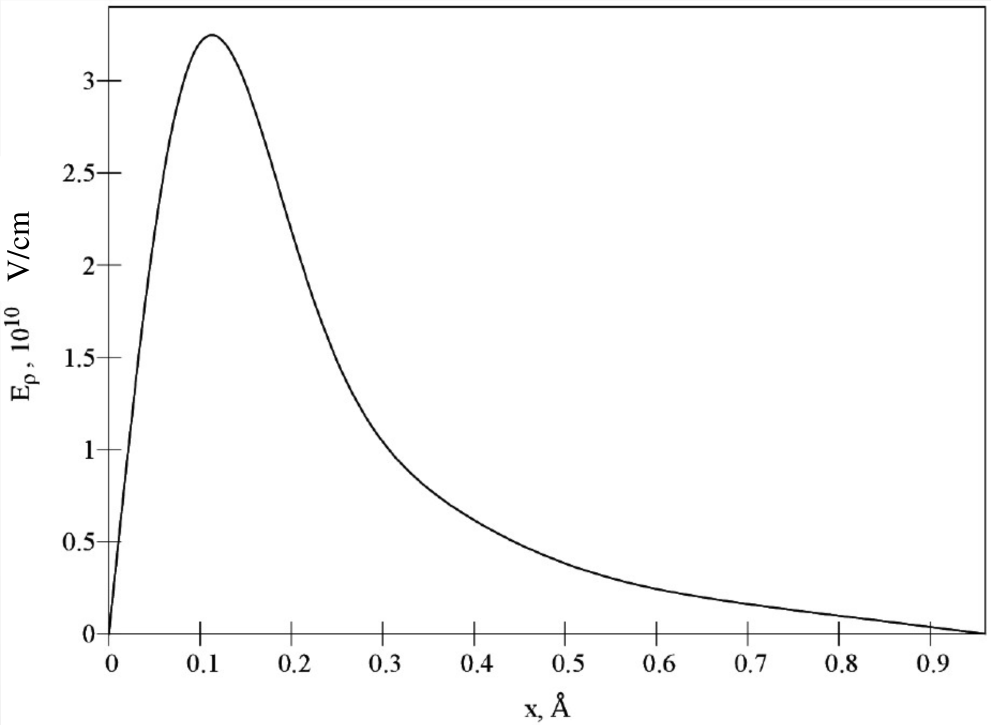
Si <110>



Potential of crystal atomic strings

Si <100>

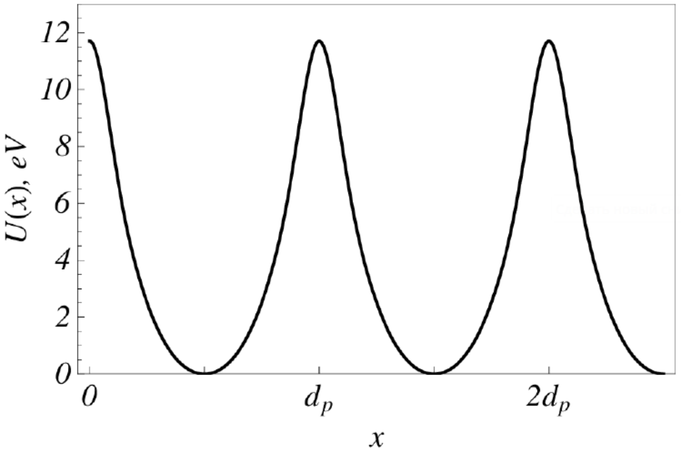


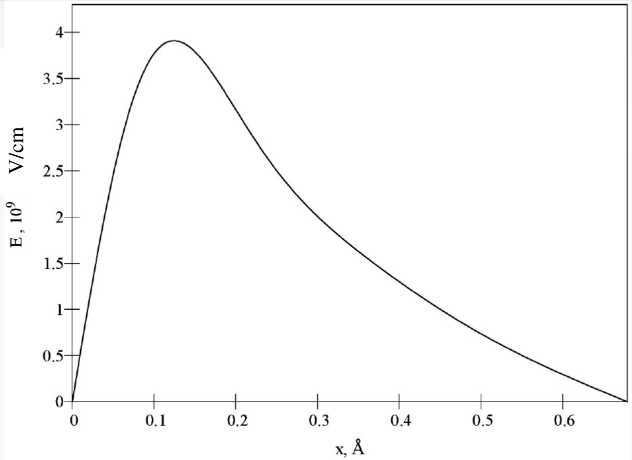


$E_{str} \sim 10^{10}$ V/cm

Potential of crystal atomic planes

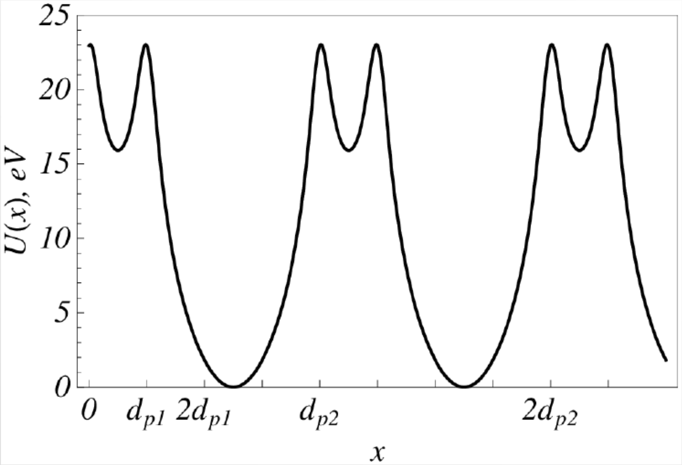
Si (100)

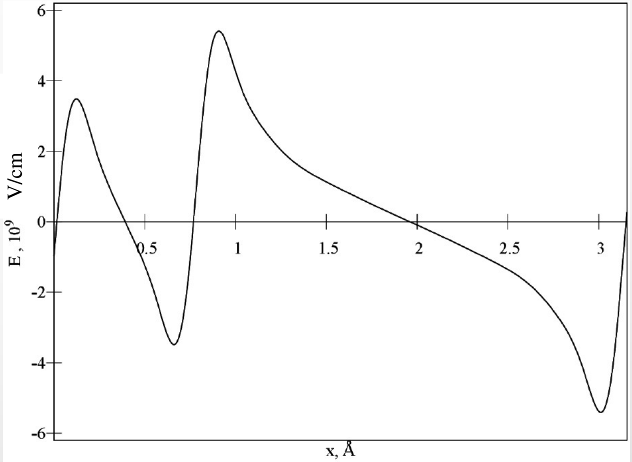




E_{str} ~ 10^{10} V/cm
E_{pl} ~ 10^{9} V/cm

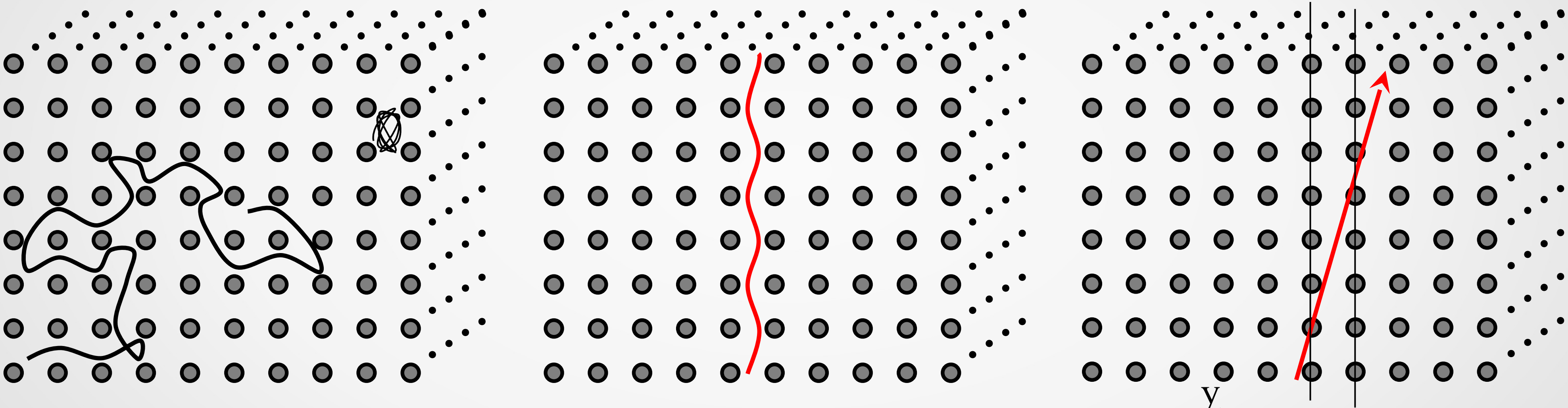
Si (110)



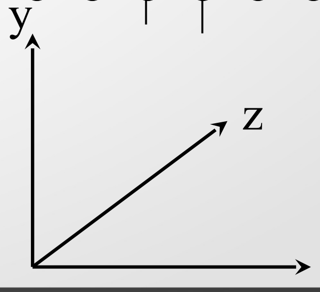


Regimes of motion in a crystal

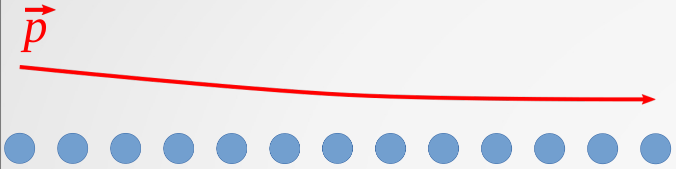
$\psi_x \approx \psi_y < \psi_c$ $\qquad$ $\psi_x < \theta_c\,,\ \psi_y \gg \psi_c$ $\qquad$ $\psi_x > \theta_c\,,\ \psi_y \gg \psi_c$



$$v_z \approx c\,, \qquad \psi_x = \frac{v_x}{c}\,, \qquad \psi_y = \frac{v_y}{c}\,, \qquad \psi_c \approx 2\,\theta_c \sim 10^{-5}$$

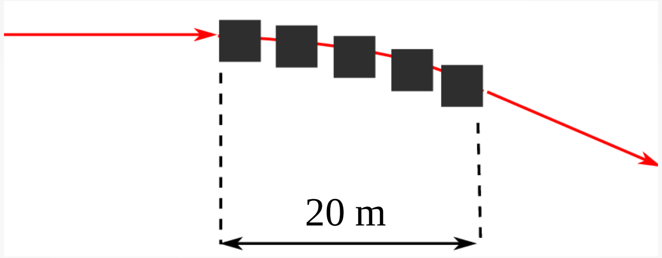


Bent crystals and magnetic deflection systems

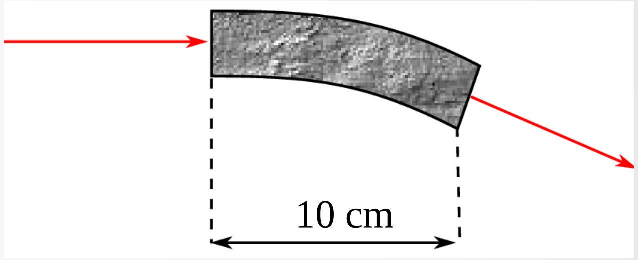


E_{str} ~ 10^{10} V/cm
E_{pl} ~ 10^{9} V/cm

Magnets

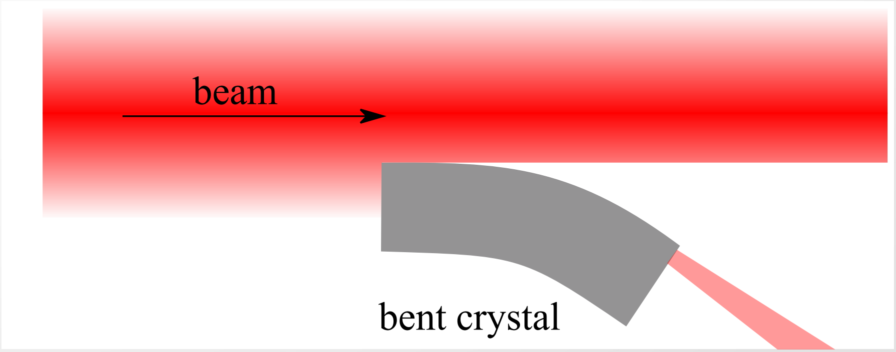


Bent crystal



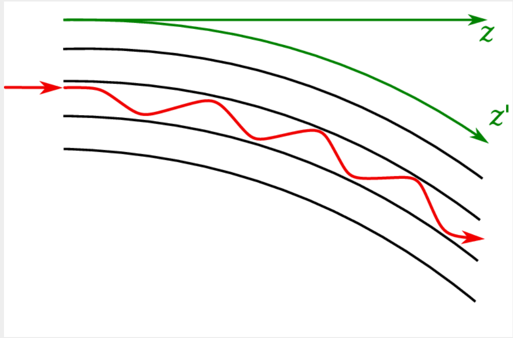
Advantages of bent crystals in comparison with magnetic deflection systems:

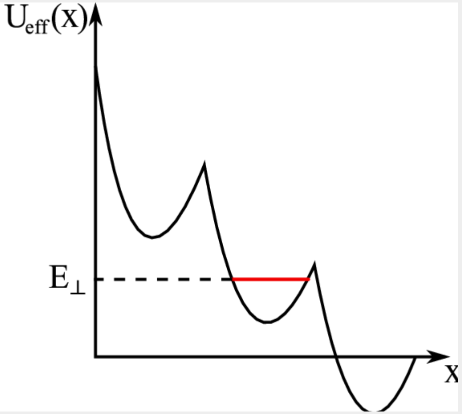
- Small size
- do not need electricity consumption
- do not need cooling



Mechanisms of deflection

Planar channeling

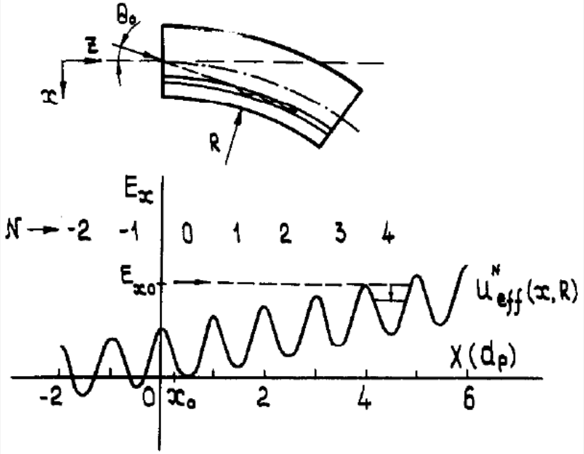


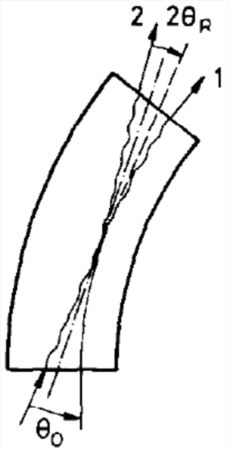


Tsyganov E. N. Fermilab TM-682, TM-684. 1976.

Volume reflection

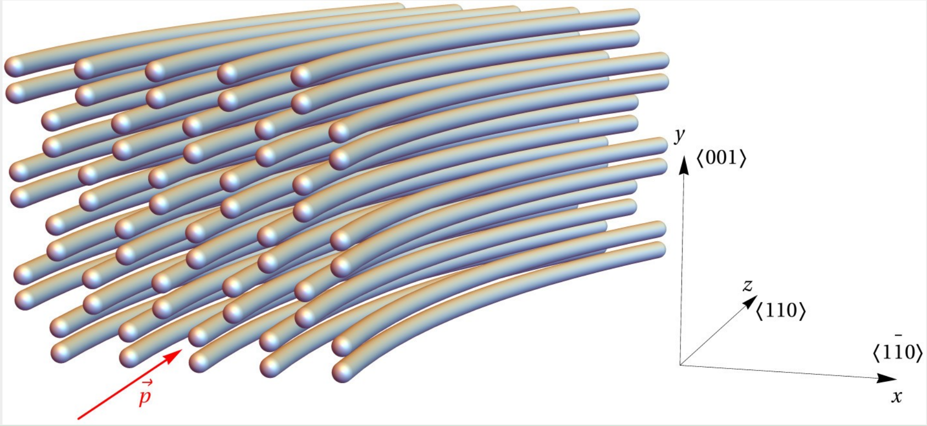
Taratin A. M., Vorobiev S. A. Phys. Lett. A. 1986. Vol. 115, No. 8. P. 398–400.





Stochastic deflection

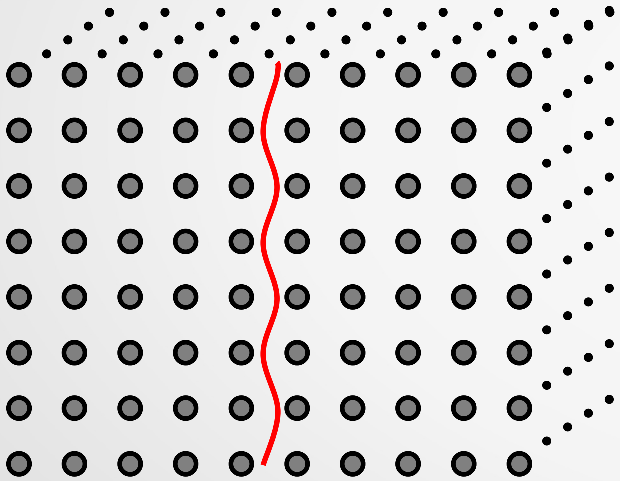
Grinenko A. A., Shul’ga N. F. J. Exp. Theor. Phys. Lett. 1991. Vol. 54. P. 524–528.

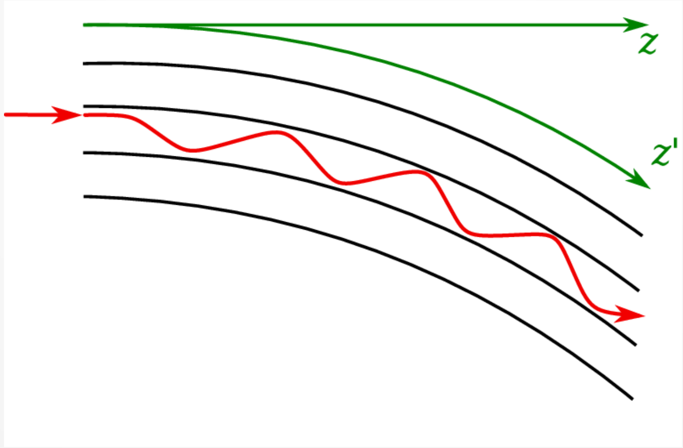


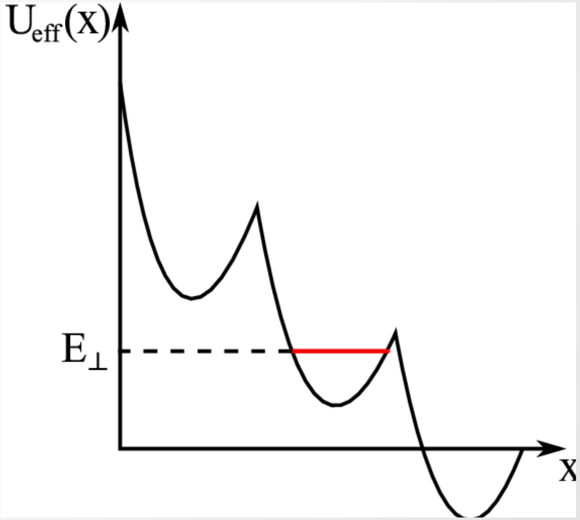
Planar channeling

Tsyganov E. N. Fermilab TM-682, TM-684. 1976.

$$\psi_x < \theta_c, \psi_y \gg \psi_c$$

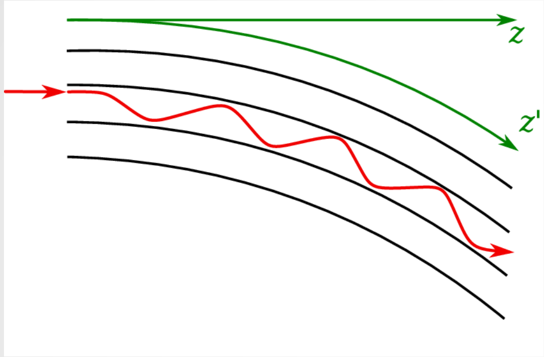


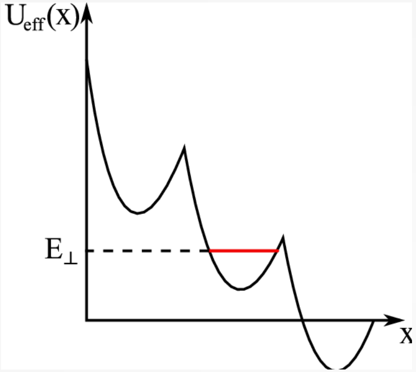




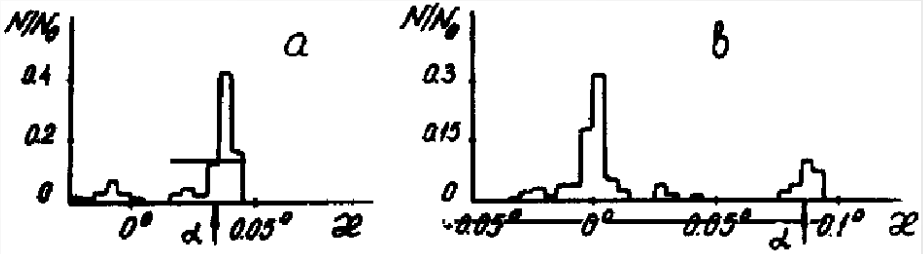
Planar channeling

Tsyganov E. N. Fermilab TM-682, TM-684. 1976.

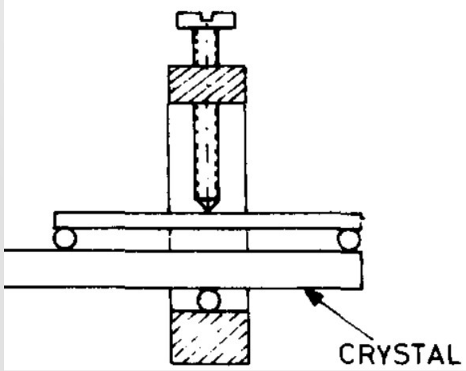


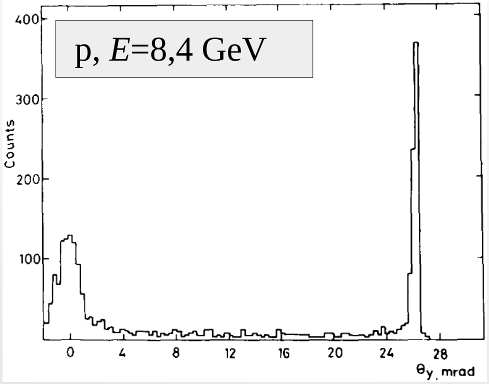


Таратин А. М., Цыганов Э. Н., Воробьев С. А. Поворот пучков заряженных частиц изогнутым монокристаллом. Численный эксперимент. Письма в ЖТФ. 1978. Т. 4. С. 947–950.
Tarantin A. M., Tsyganov E. N., Vorobiev S. A. Computer simulation of deflection effects for relativistic charged particles in a curved crystal. Phys.Lett. A. 1979. Vol. 72, No. 2. P. 145–146.



p, E=1 GeV, a) R=0,29 cm, b) R=0,112 cm





Elishev A. F., Filatova N. A., Golovatyuk V. M. et al. (I.A. Grishaev, G.D. Kovalenko, B.I. Shramenko) Steering of charged particle trajectories by a bent crystal. Phys. Lett. B. 1979. Vol. 88, No. 3-4. P. 387–391.

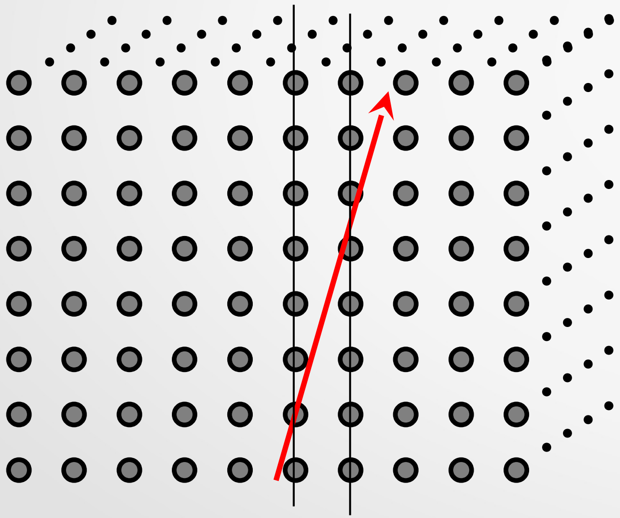
Volume reflection

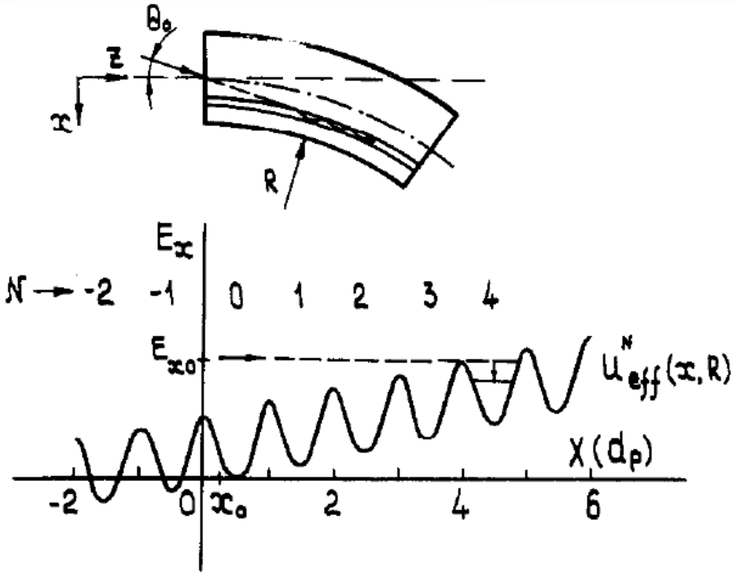
Taratin A. M., Vorobiev S. A. Phys. Lett. A. 1986. Vol. 115, No. 8. P. 398–400.
Taratin A. M., Vorobiev S. A. Nucl. Instrum. Meth. B. 1987. Vol. 26, No. 4. P. 512–521.
Taratin A. M., Vorobiev S. A. Phys. Lett. A. 1987. Vol. 119, No. 8. P. 425–428.

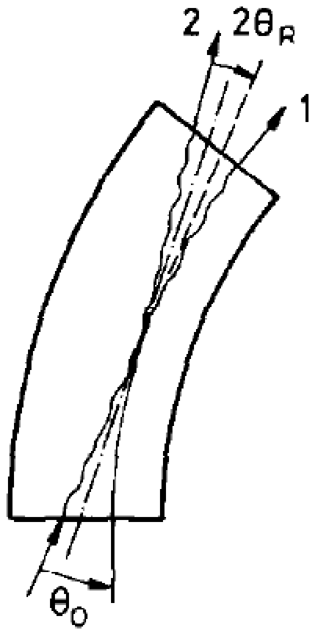
$$E_x = \frac{pv\theta_x^2}{2} + U_{eff}(x,R)$$

$$U_{eff}(x,R) = U(x) + pv\frac{x}{R}$$

$$\psi_x > \theta_c\,,\ \psi_y \gg \psi_c$$

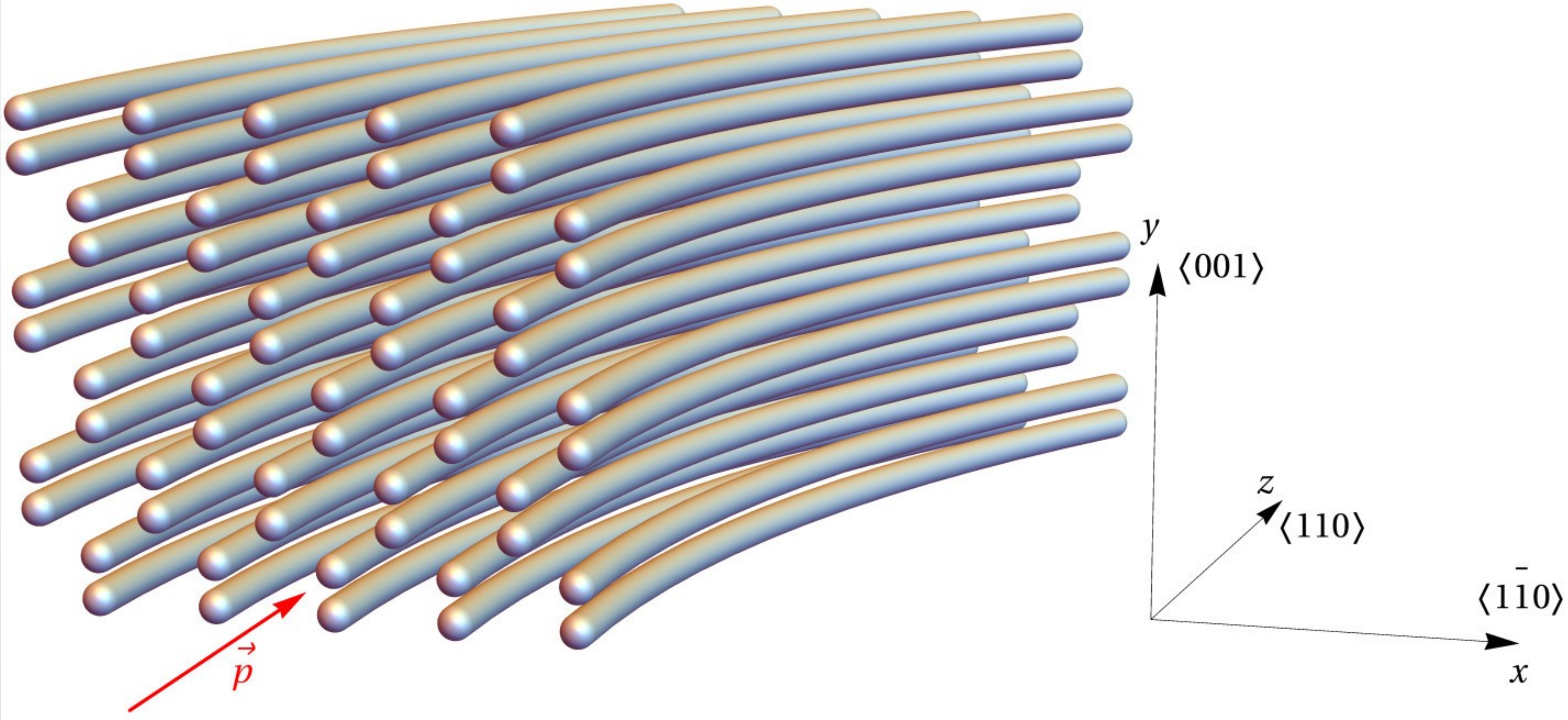






Stochastic deflection

Grinenko A. A., Shul'ga N. F. J. Exp. Theor. Phys. Lett. 1991. Vol. 54. P. 524–528.
Greenenko A. A., Shul'ga N. F. Nucl. Instrum. Meth. B. 1994. Vol. 90, No. 1-4. P. 179–182.
Shul'ga N. F., Greenenko A. A. Phys. Lett. B. 1995. Vol. 353, No. 2. P. 373–377.



Stochastic deflection

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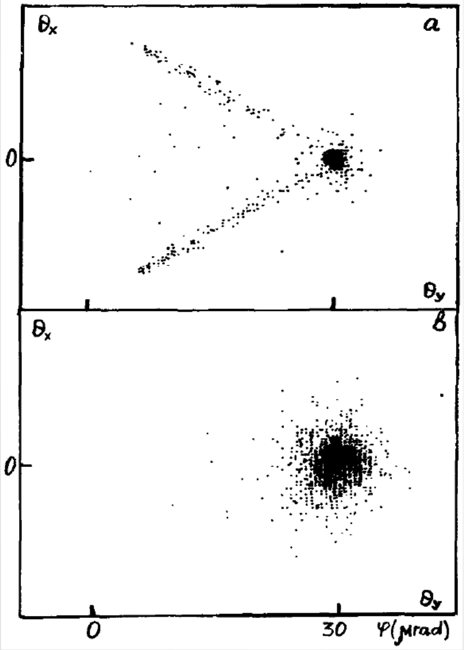
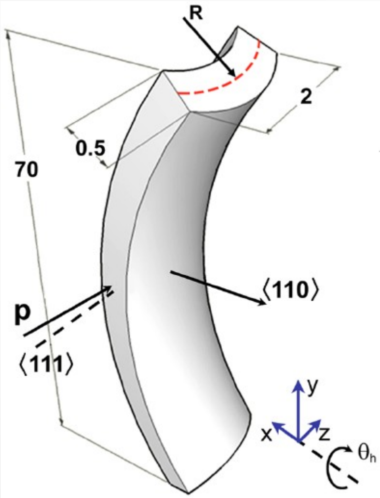
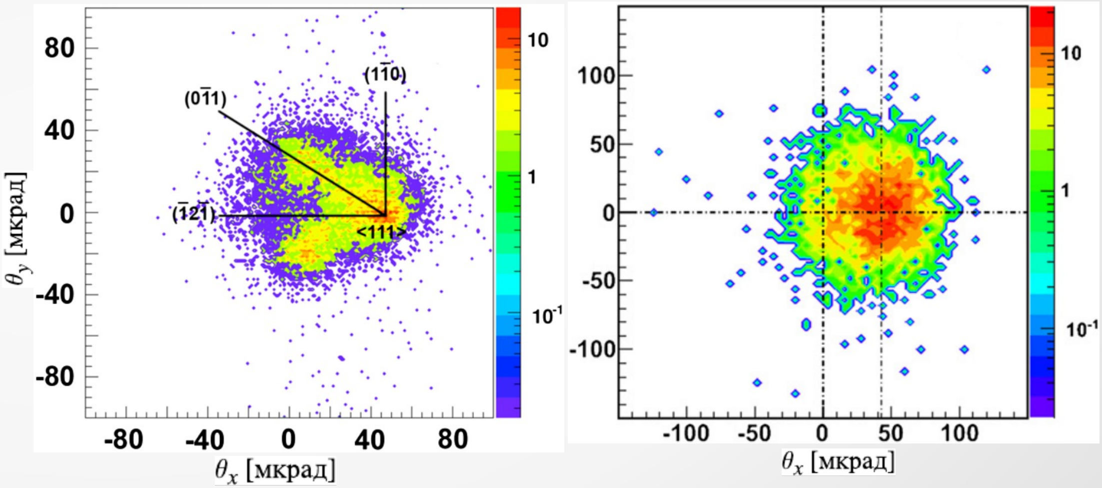
Greenenko A. A., Shul'ga N. F. Nucl. Instrum. Meth. B. 1994. Vol. 90, No. 1-4. P. 179–182.
Shul'ga N. F., Greenenko A. A. Phys. Lett. B. 1995. Vol. 353, No. 2. P. 373–377.



E=10 TeV

$$\langle \psi^2 \rangle = \frac{lL}{R^2} \leq \psi_c^2$$

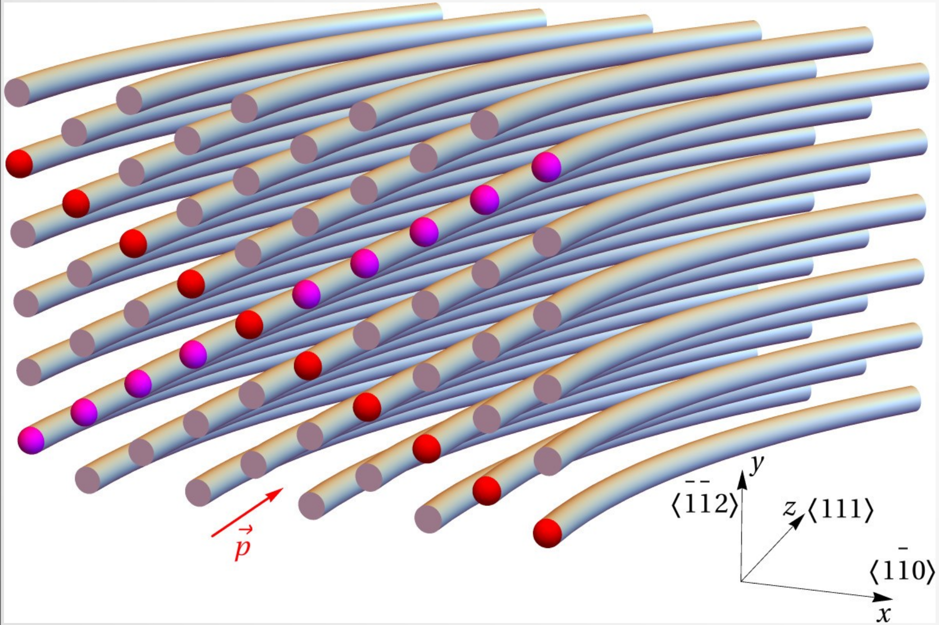




Scandale W., Vomiero A., Baricordi S. et al. High-efficiency deflection of high-energy protons through axial channeling in a bent crystal. Phys. Rev. Lett. 2008. Vol. 101, No. 16. P. 164801.
Scandale W., Vomiero A., Bagli E. et al. High-efficiency deflection of high-energy negative particles through axial channeling in a bent crystal. Phys. Lett. B. 2009. Vol. 680, No. 4. P. 301–304.

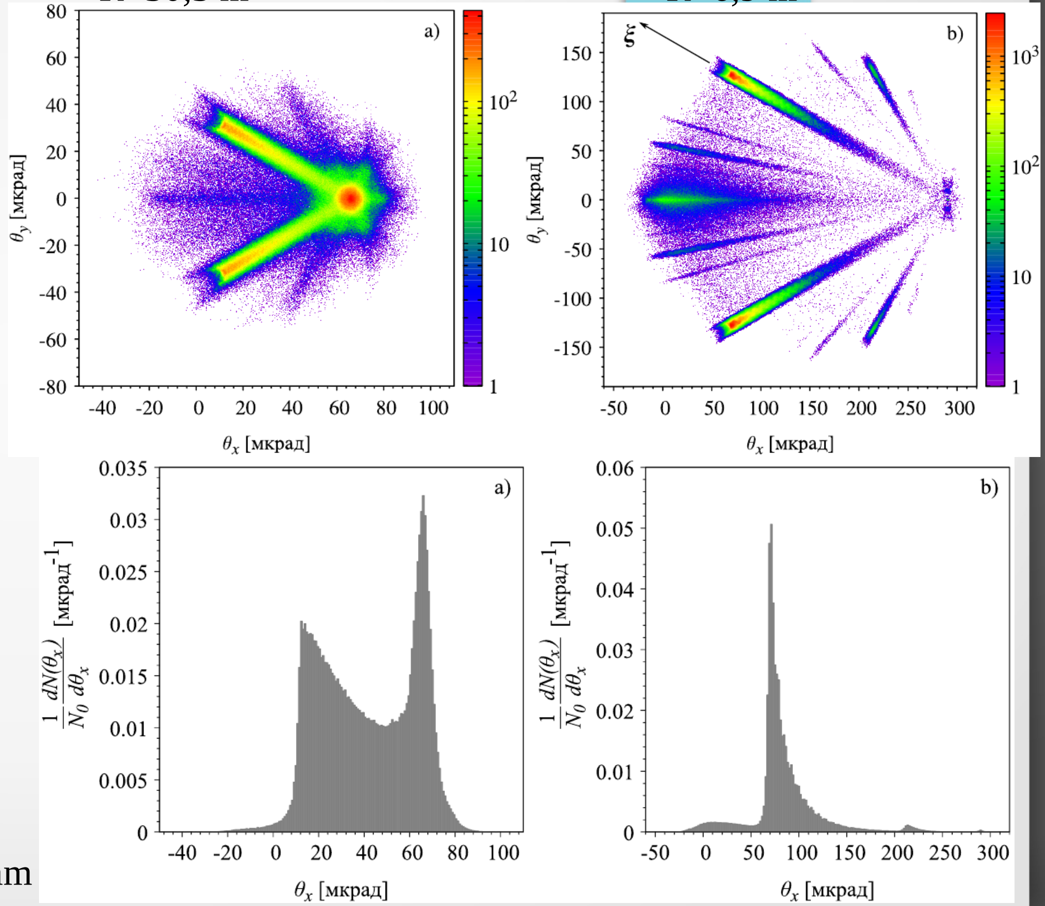
Changing the shape of the beam

$$\langle \psi^2 \rangle = \frac{lL}{R^2} \le \psi_c^2$$



R=30,3 m

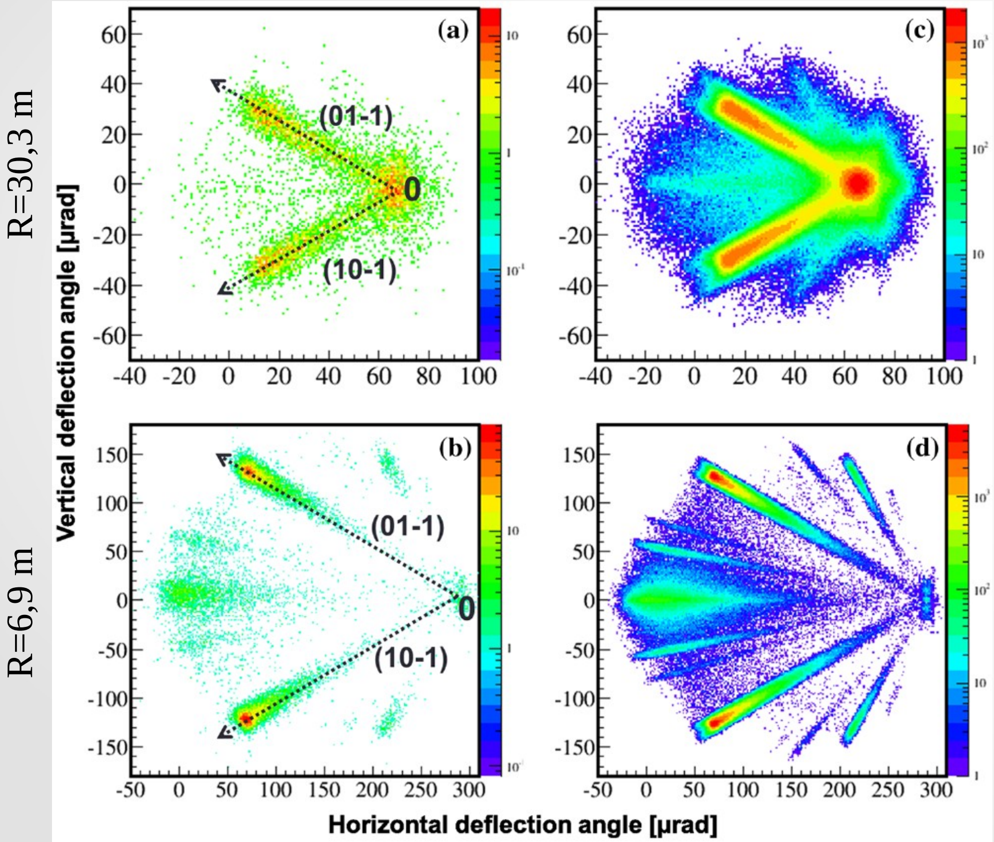
R=6,9 m



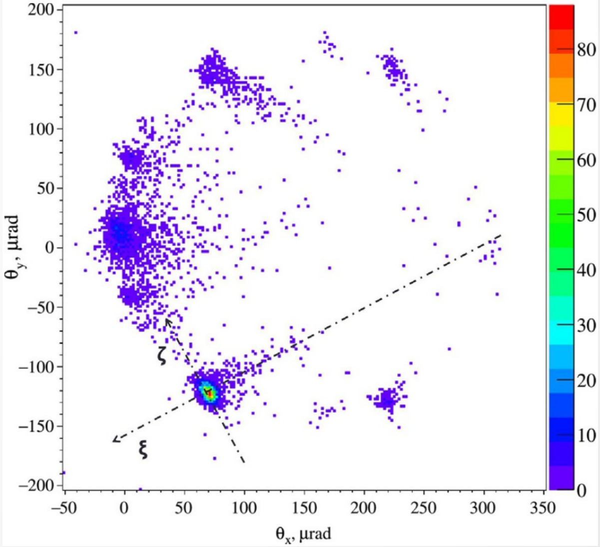
p, *E*=400 GeV, Si <111>, *L*=2 mm

Changing the shape of the beam

p, *E*=400 GeV, Si <111>, *L*=2 mm



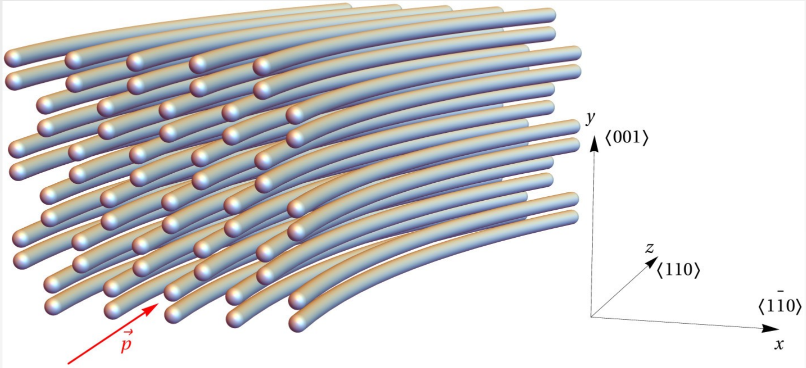
p, *E*=400 GeV, Si <111>, *L*=2 mm, R=6,9 m, $\theta_{x,in}$=-8 μrad, $\theta_{y,in}$=-4 μrad



Bandiera L., Mazzolari A., Bagli E. et al. (Kirillin I. V.). Relaxation of axially confined 400 GeV/c protons to planar channeling in a bent crystal. Eur. Phys. J. C. 2016. Vol. 76. P. 80 (1–6).
Bandiera L., Kirillin I. V., Bagli E. et al. Splitting of a high-energy positively-charged particle beam with a bent crystal. Nucl. Instr. Meth. Phys. Res. B. 2017. Vol. 402. P. 296–299.

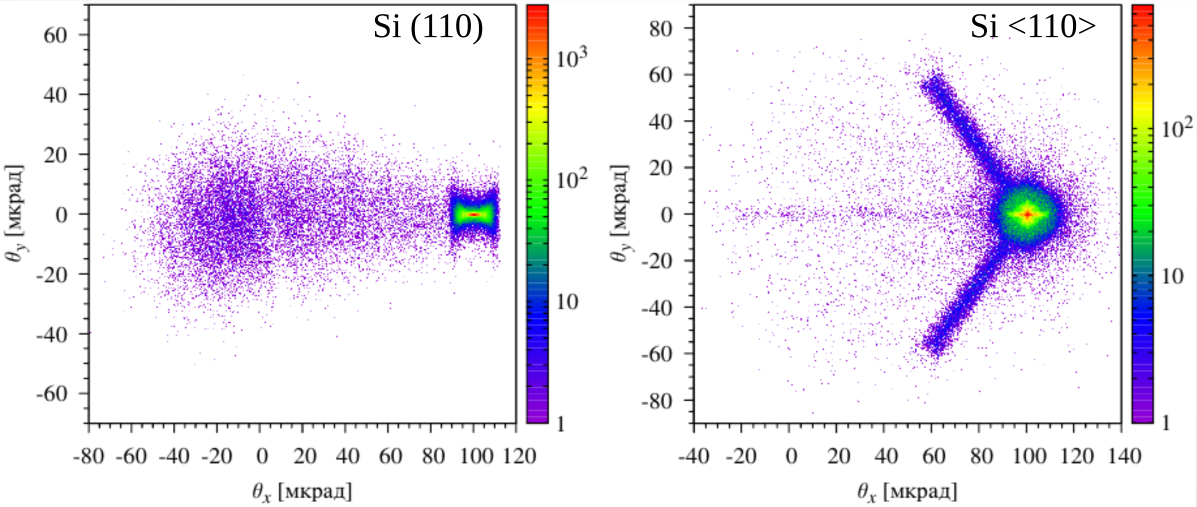
Probability of close collisions

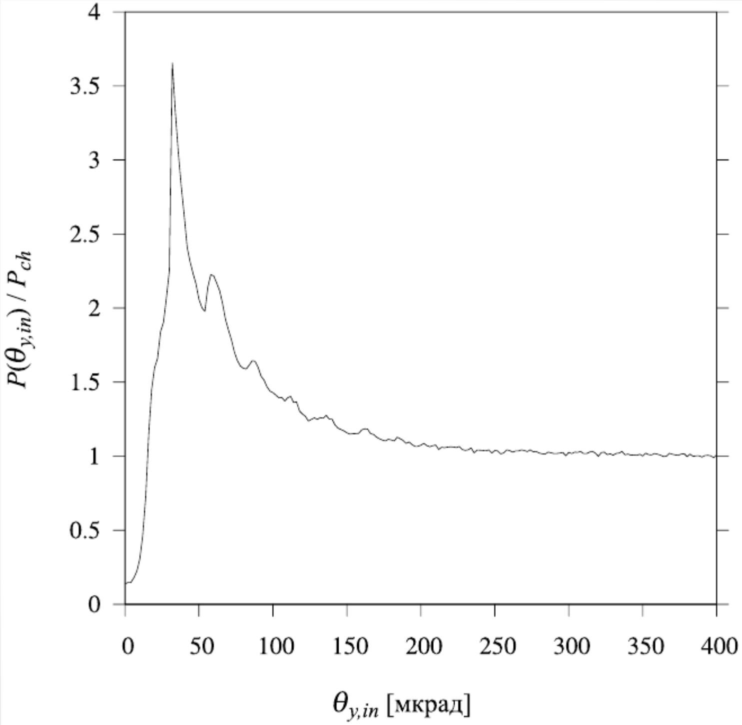
p, E=270 GeV,
Si <110>, (110),
L = 5 mm,
R = 5 m



$$w_a = \frac{4\pi r_T^2}{a_x a_y} = 4\sqrt{2}\pi r_T^2/a^2 \approx 3.39 * 10^{-3}$$

$$w_p = \frac{4 r_T}{a_x} = 4\sqrt{2} r_T/a \approx 78.12 * 10^{-3}$$

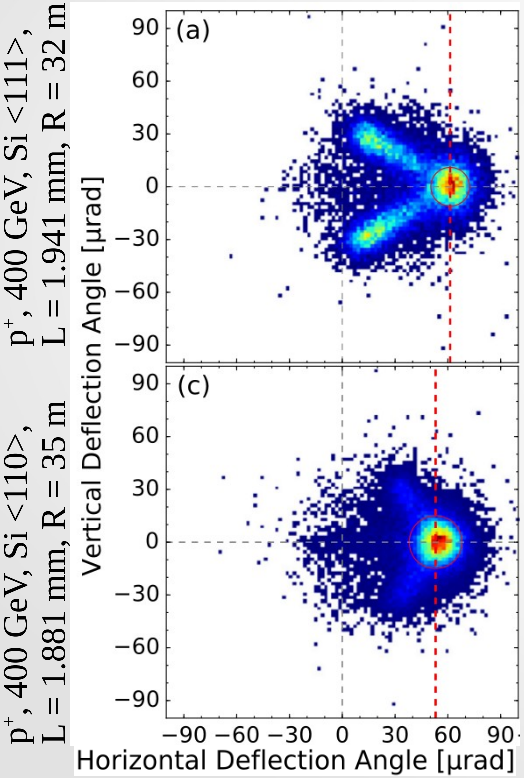




Chesnokov Yu.A., Kirillin I.V., Scandale W. et al. Phys. Lett. B. 2014. Vol. 731. P. 118–121.

Probability of close collisions

Scandale W., Arduini G., Butcher M. et al. Phys. Lett. B. 2016. Vol. 760. P. 826–831.
Scandale W., Andrisani F., Arduini G. et al. Eur. Phys. J. C. 2018. Vol. 78, No. 6. P. 505.



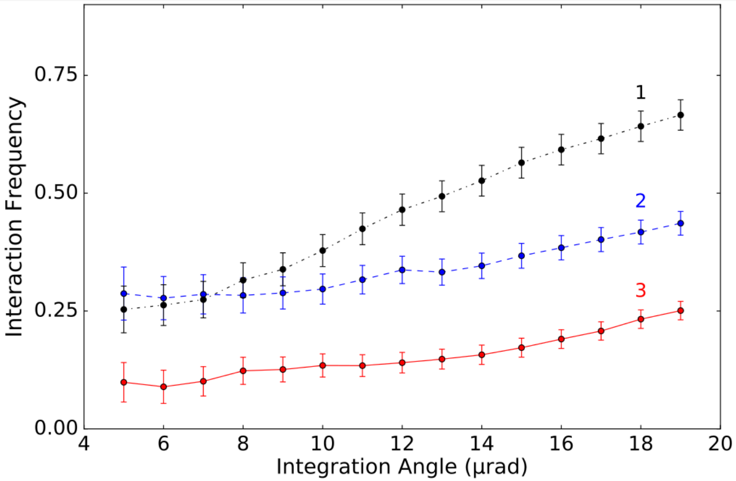
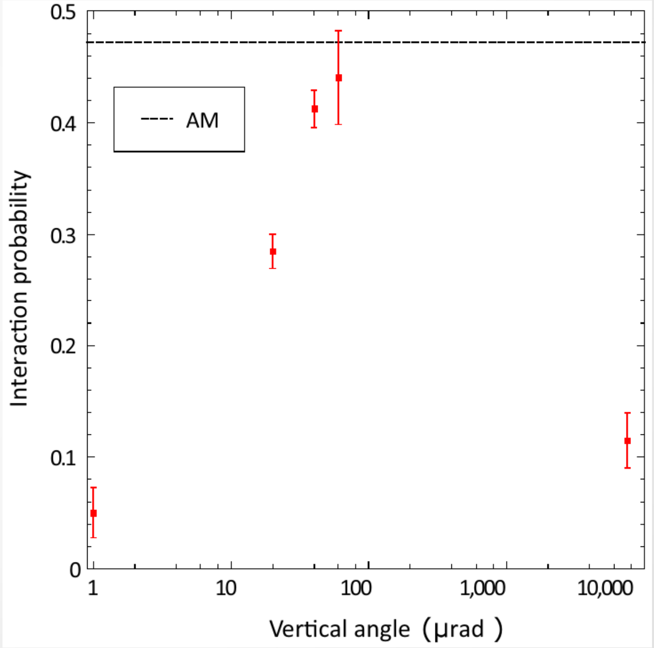


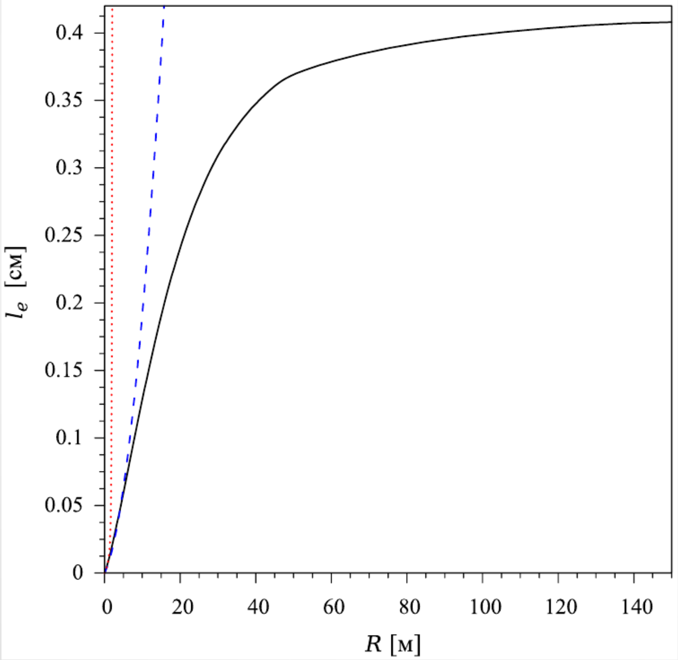
Fig. 5. Measured inelastic nuclear interaction (INI) frequency of 400 GeV/c protons interacting with the ⟨111⟩ and ⟨110⟩ crystals as a function of the angular region around the (110) planar channeling (black dash-dotted line, 1), the ⟨111⟩ axial channeling (blue dashed line, 2) and ⟨110⟩ (red continuous line, 3) orientations. The values are normalized to the INI frequencies for the amorphous crystal orientation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

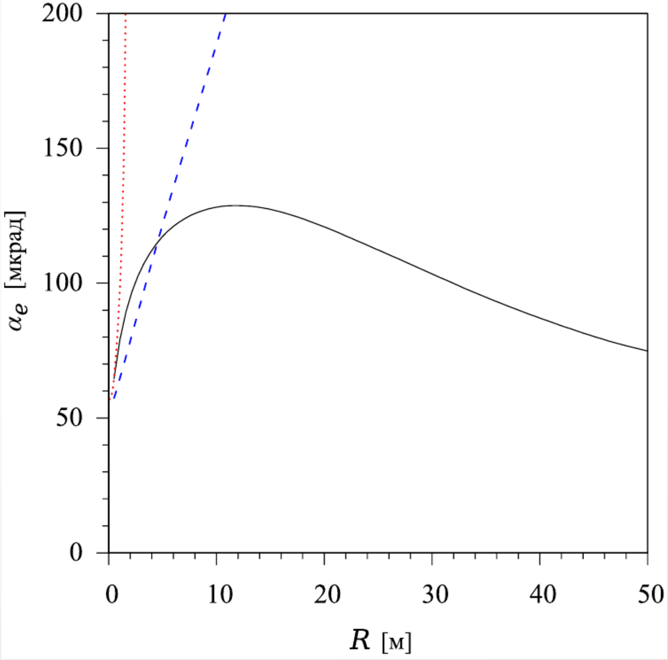


Optimal radius of curvature (stochastic deflection)

$$\langle \psi^2 \rangle = \frac{lL}{R^2} \leq \psi_c^2 \qquad \overline{\Psi_{inc}}^2 = \xi L \qquad L_{st} = \frac{\psi_m^2}{l/R^2 + \xi} \qquad \alpha_{st} = \frac{L_{st}}{R} = \frac{\psi_m^2}{l/R + \xi R}$$

π^-, E=150 GeV, Si <110>





Kirillin I.V., Shul'ga N.F., Bandiera L. et al. Eur. Phys. J. C. 2017. Vol. 77. P. 117 (1–7).

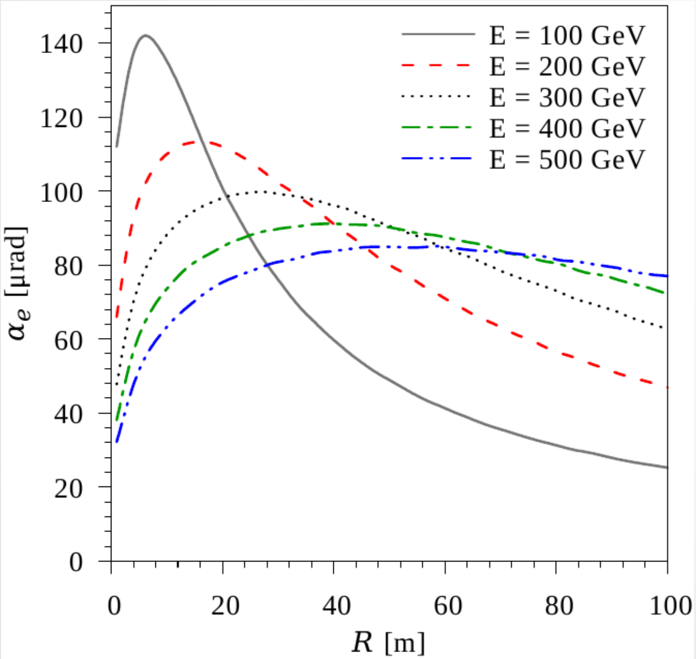
Optimal radius of curvature (stochastic deflection)

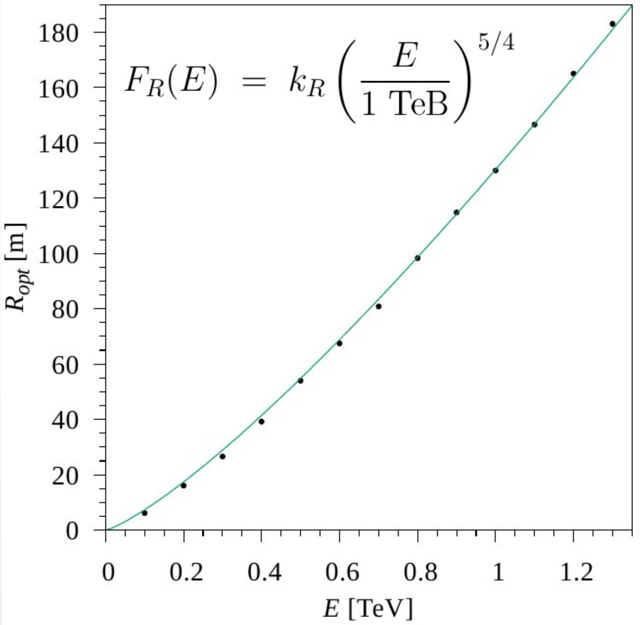
$$\overline{\psi_{inc}^2} = \zeta L/E^2 \;\Rightarrow\; \alpha_{st} = \frac{\psi_m^2}{l/R + \zeta R/E^2} \;\Rightarrow\; R_{opt} = E\sqrt{l/\zeta}$$

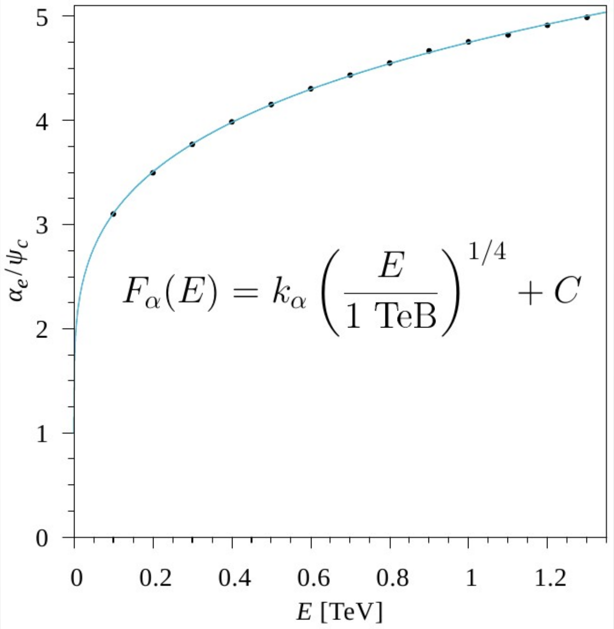
$$l \approx \frac{1}{4nda}\sqrt{\frac{E}{U_0}} \;\Rightarrow\; R_{opt} \propto E^{5/4}$$

$$\psi_m \approx 1,5\psi_c \propto E^{-1/2} \;\Rightarrow\; \max(\alpha_{st}) = \frac{\psi_m^2}{l/R_{opt} + \zeta R_{opt}/E^2} \propto E^{-1/4}$$

π^-, Si <110>





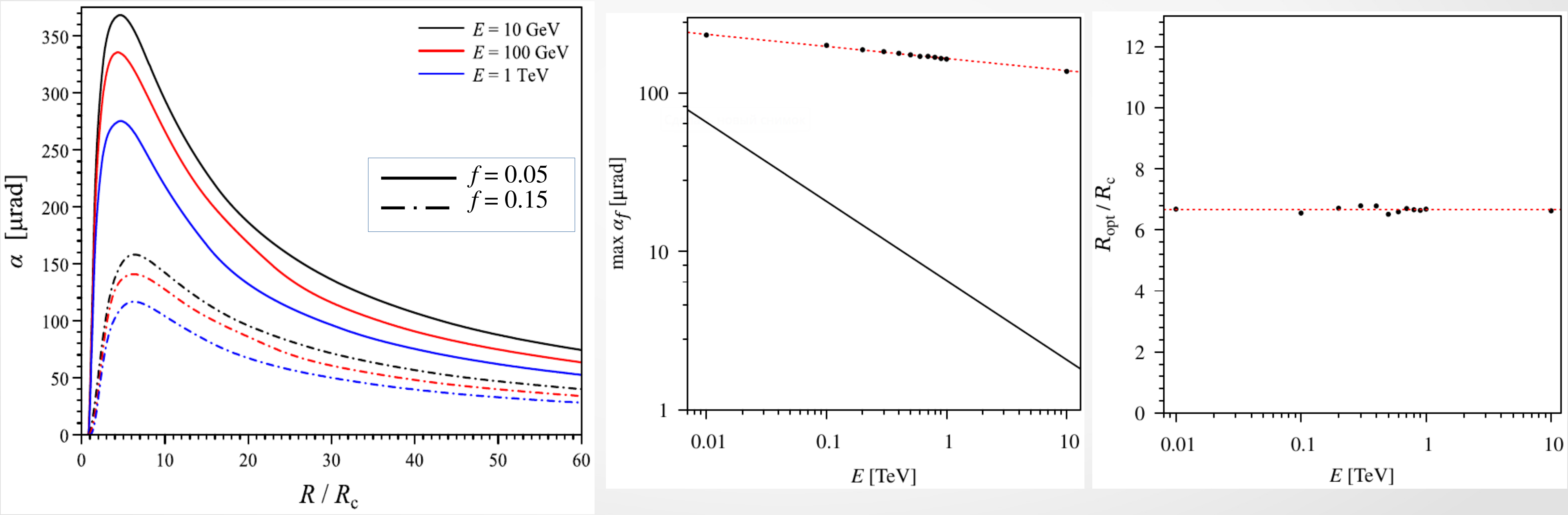


$k_R \approx 130$ м

$k_\alpha \approx 3,75$ і $C \approx 1$

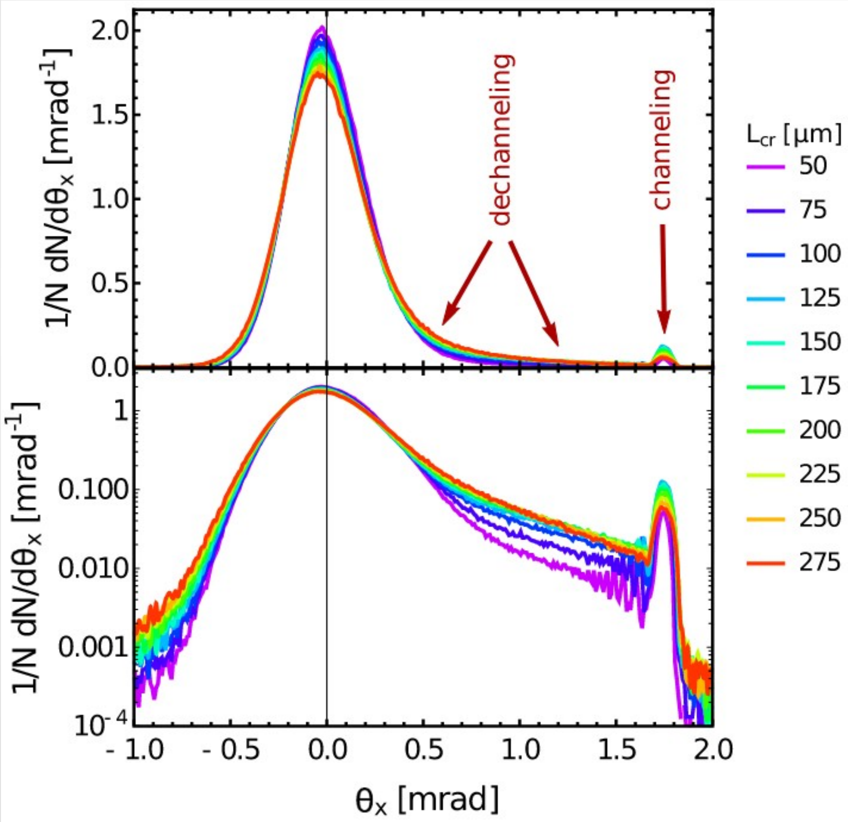
Optimal radius of curvature (planar channeling)

$$U_{\mathrm{eff}}(x) = U_{\mathrm{p}}(x) + Ex/R \qquad \left.\frac{dU_{\mathrm{eff}}(x)}{dx}\right|_{x=x_0} = 0 \qquad R_{\mathrm{c}} = \frac{E}{|U_{\mathrm{p}}'(x_0)|}$$

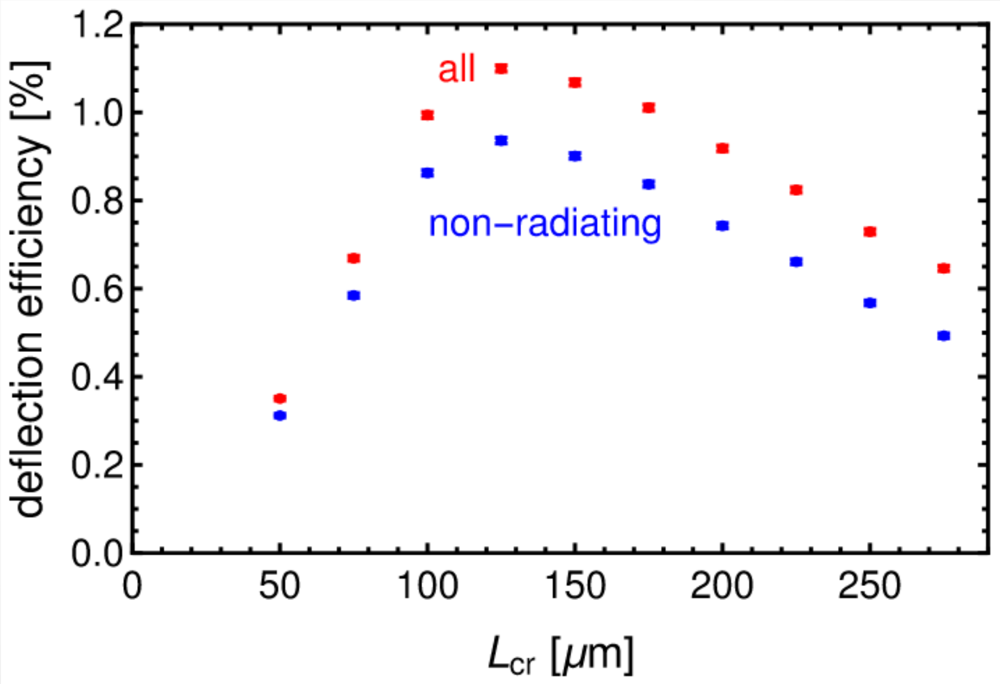


A. Sytov, G.Kube, L. Bandiera et al. First design of a crystal-based extraction of 6 GeV electrons for the DESY II Booster Synchrotron

Eur. Phys. J. C (2022) 82:197



single-turn



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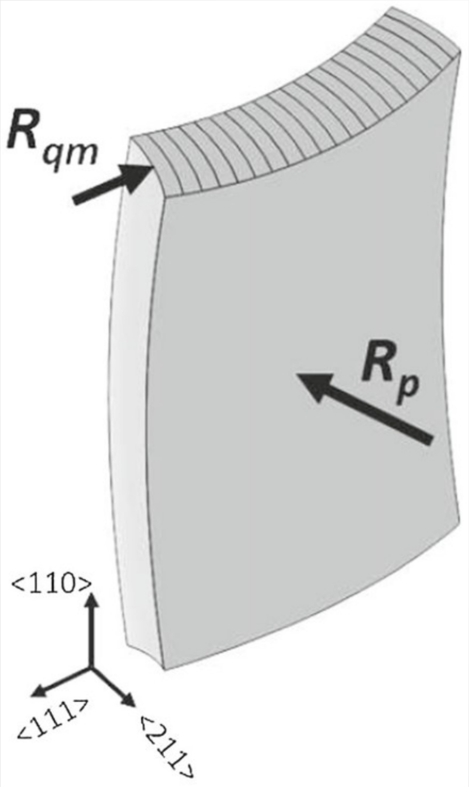
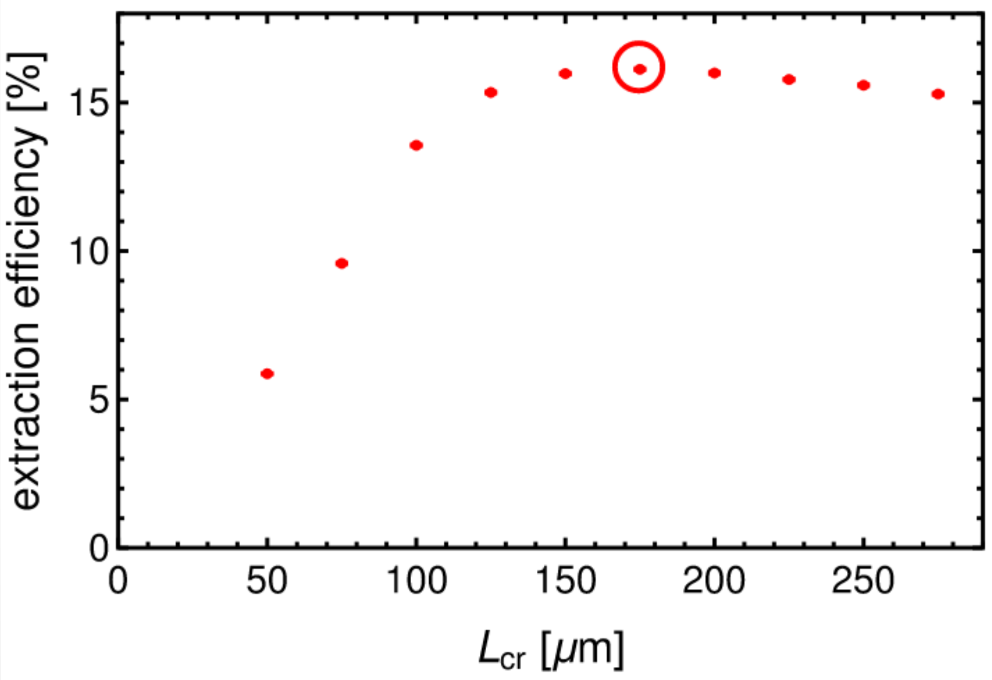


multi-turn



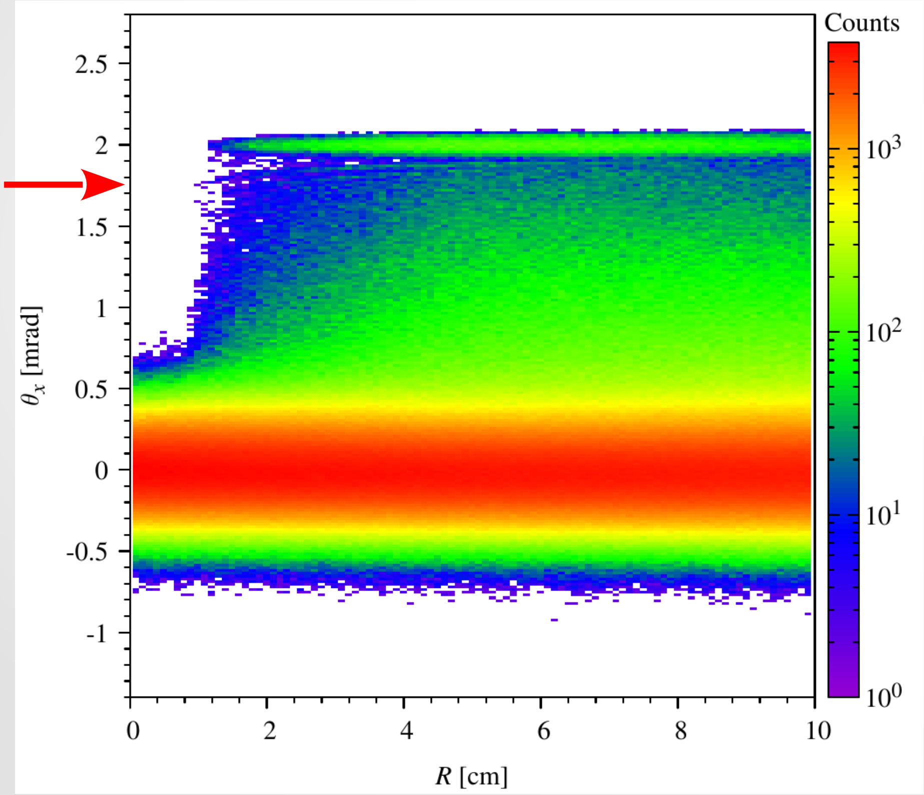
L = 175 μm,

α = 1.75 mrad

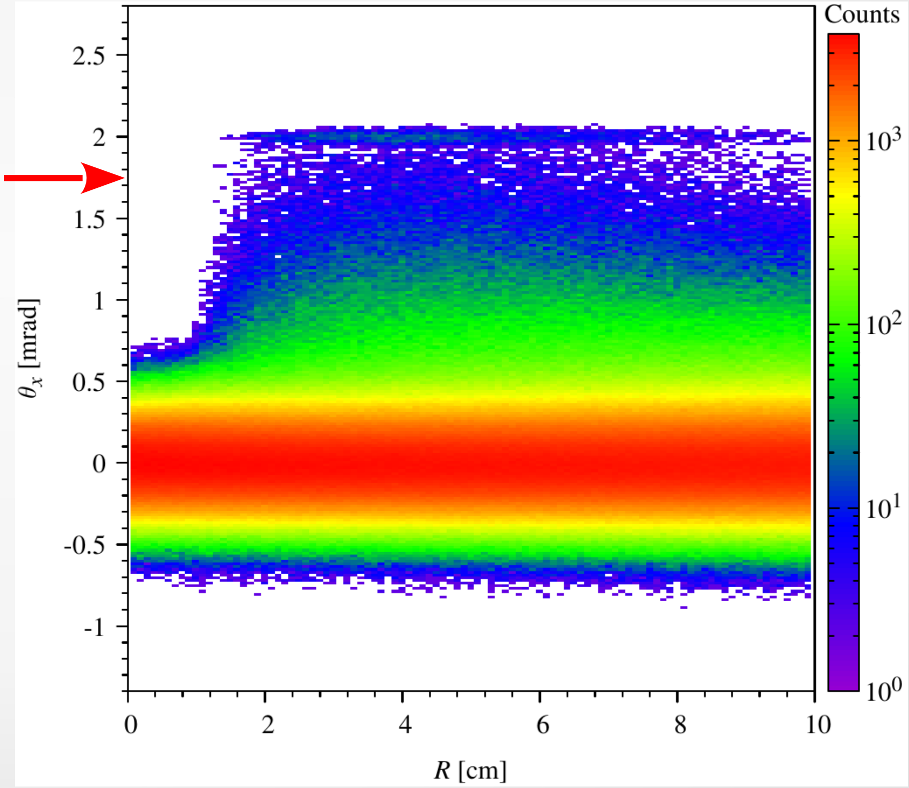
R = L / α = 10 cm

Dependence of deflection angle on R (α = 2 mrad)

(111) plane

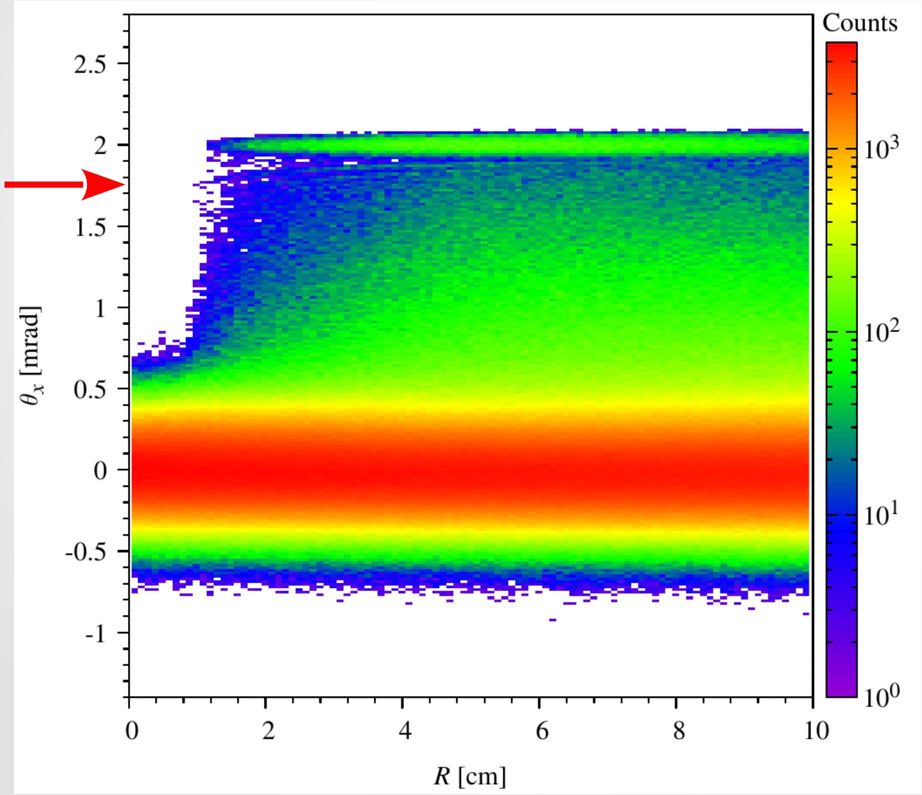


(110) plane

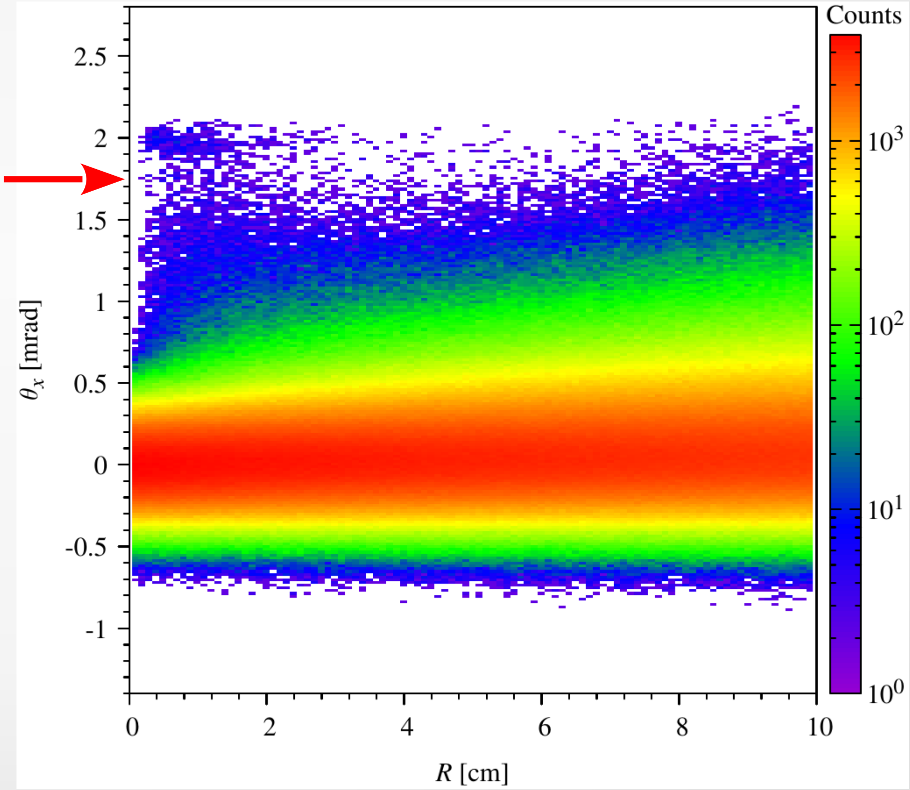


Dependence of deflection angle on R (α = 2 mrad)

(111) plane

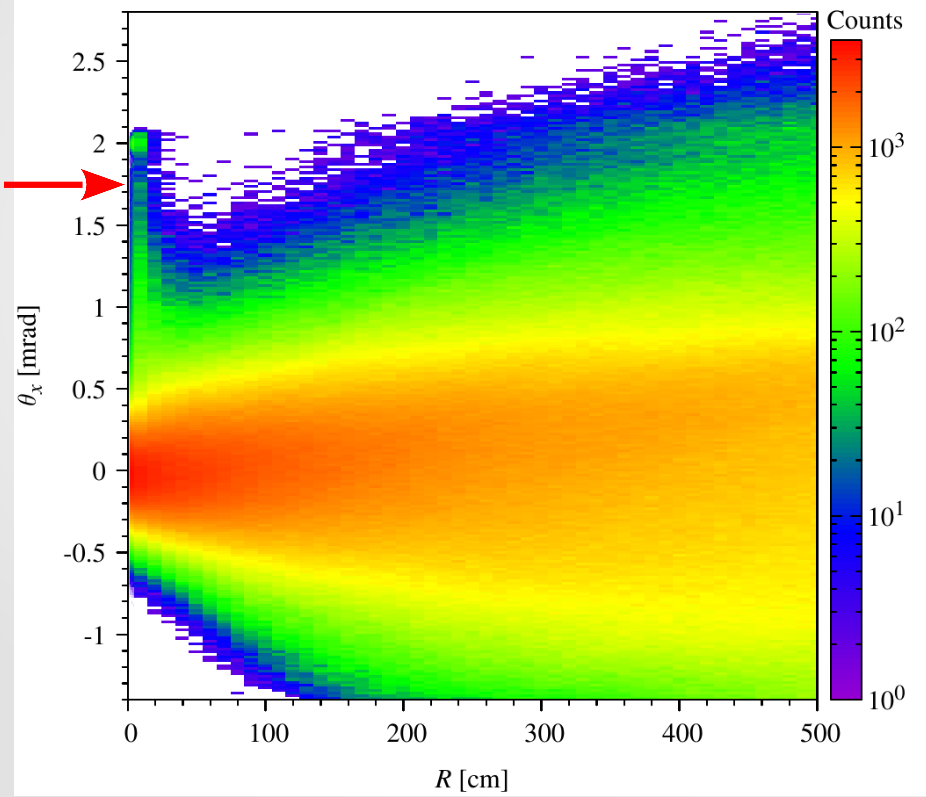


(110) axis

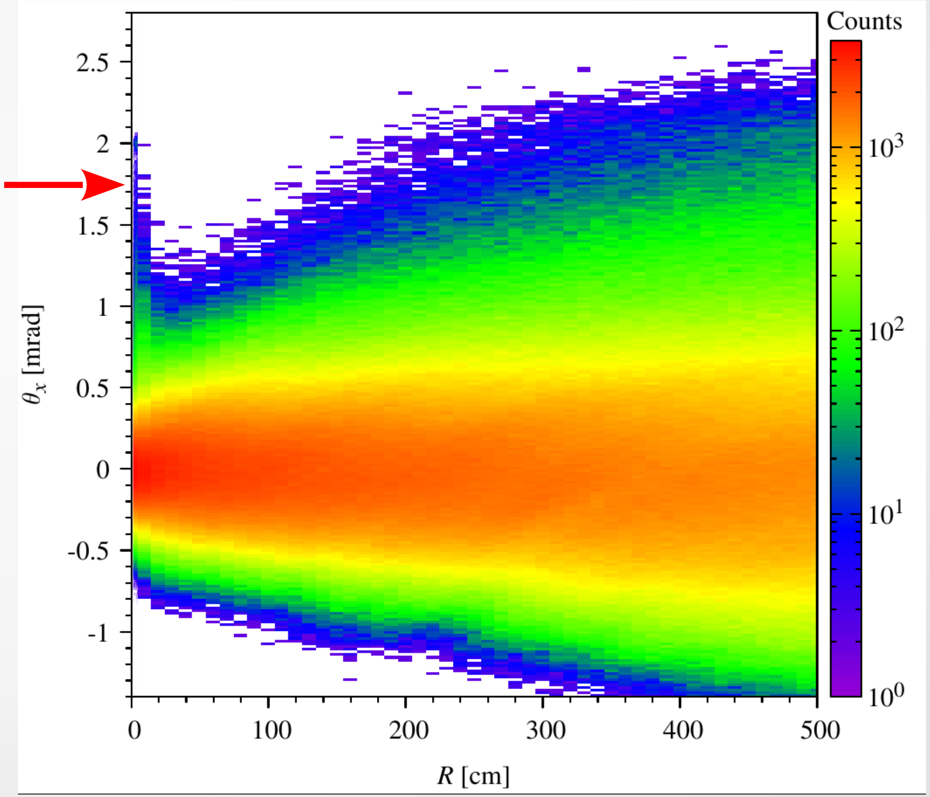


Dependence of deflection angle on R (α = 2 mrad)

(111) plane

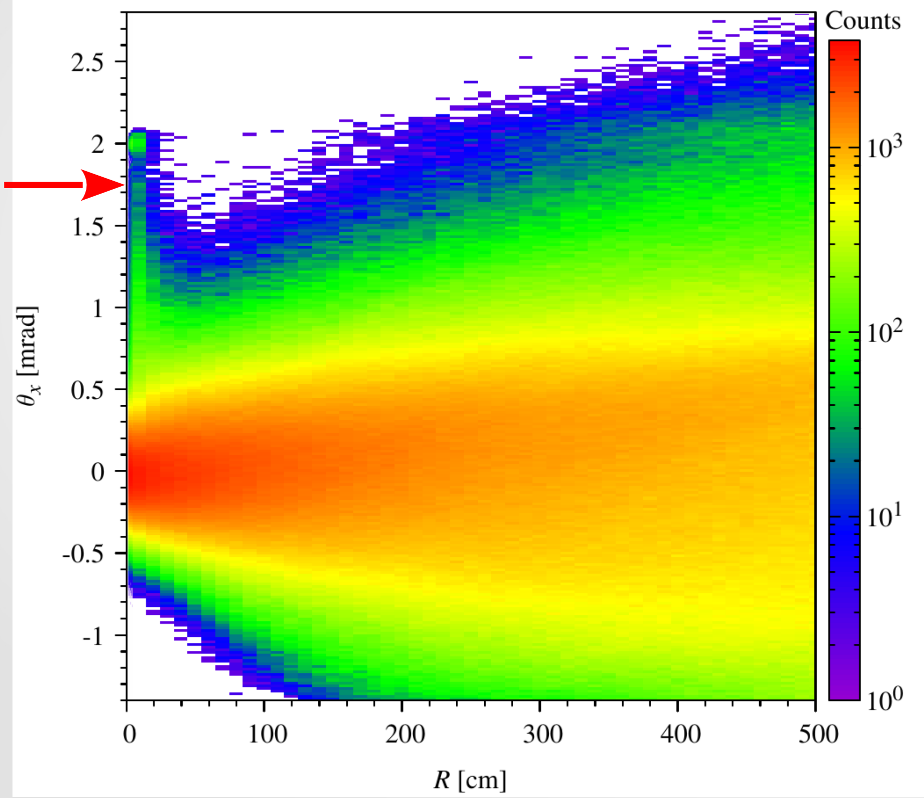


(110) plane

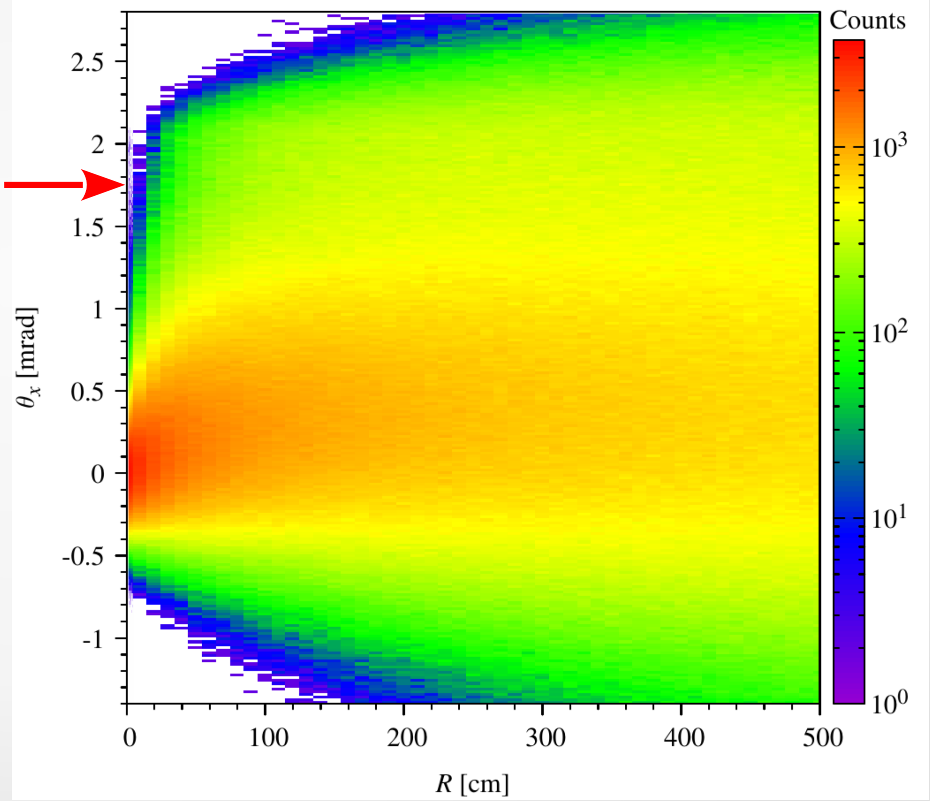


Dependence of deflection angle on R (α = 2 mrad)

(111) plane

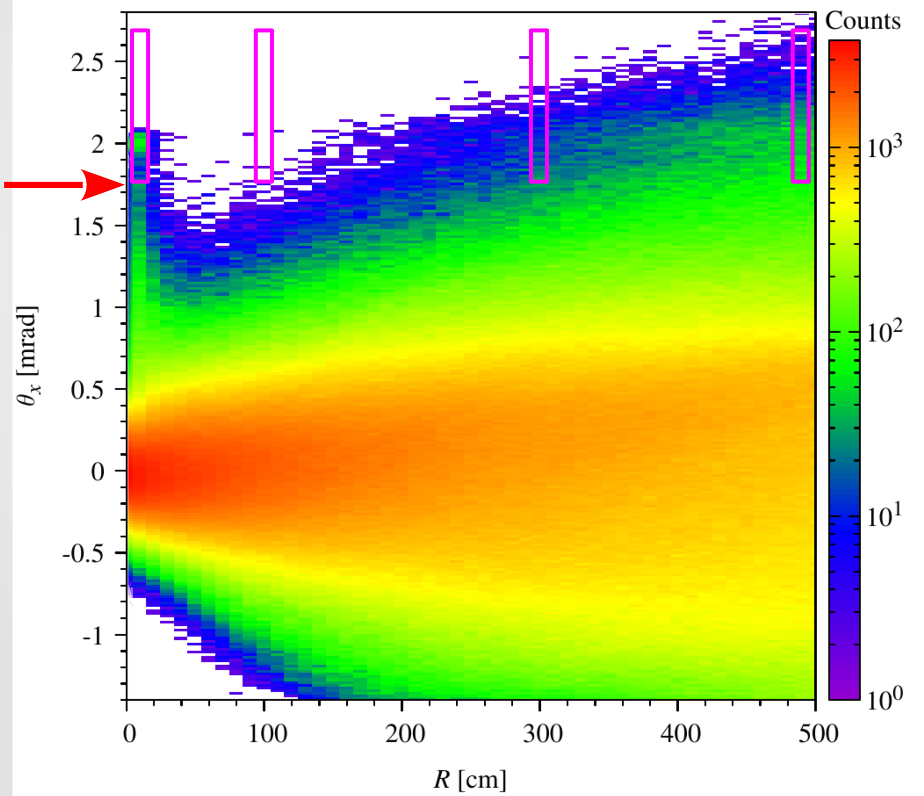


(110) axis

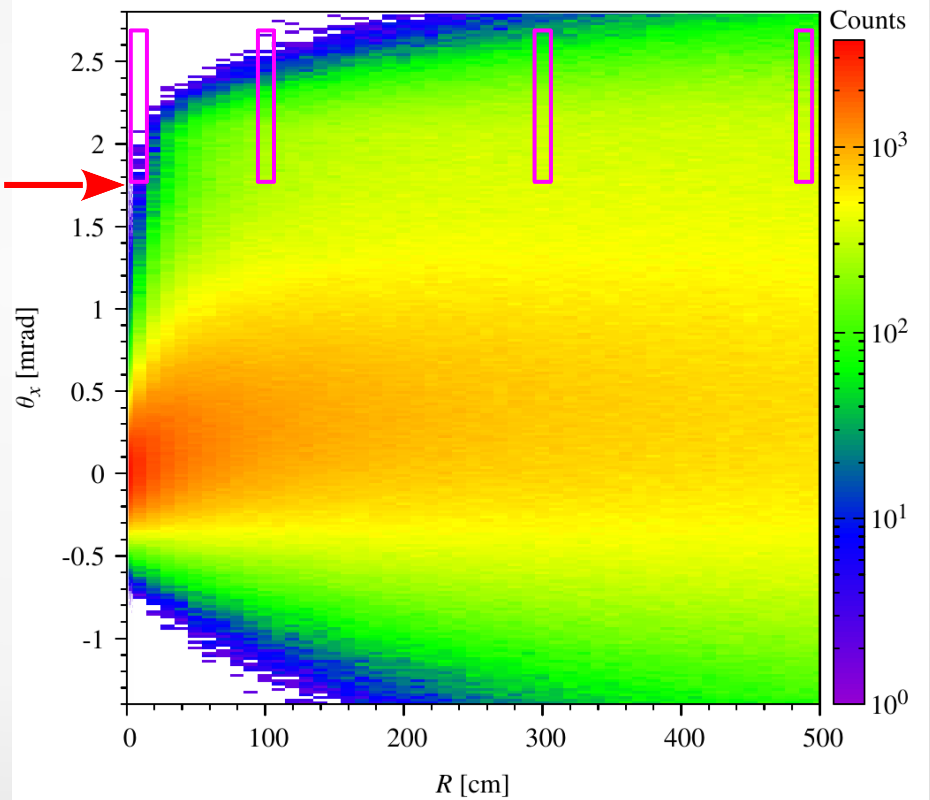


Dependence of deflection angle on R (α = 2 mrad)

(111) plane

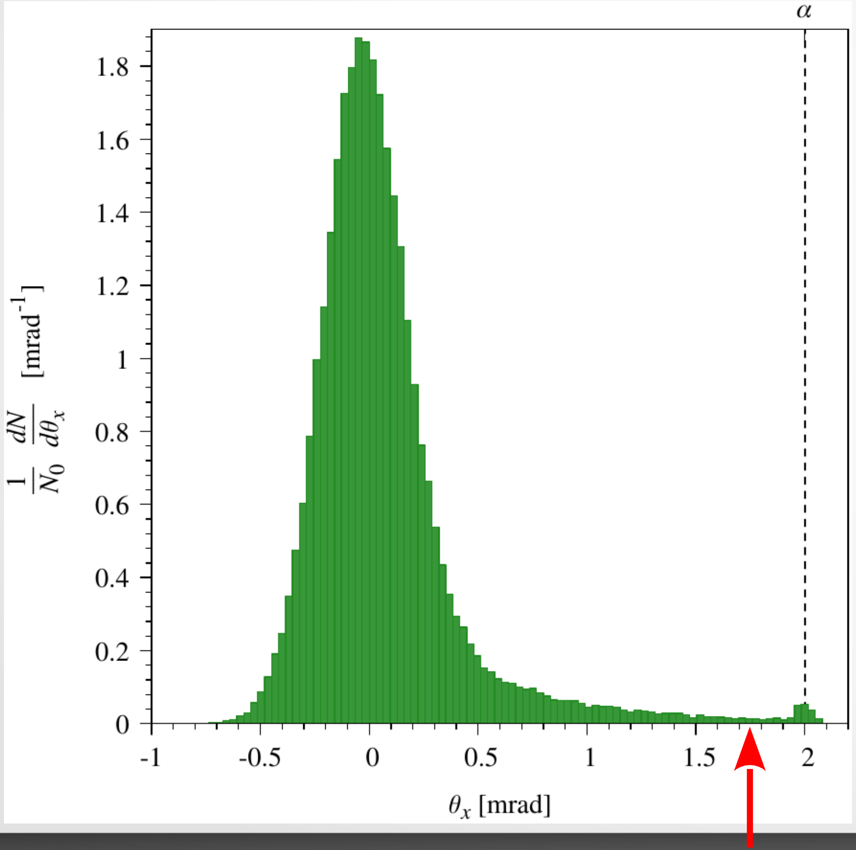


(110) axis

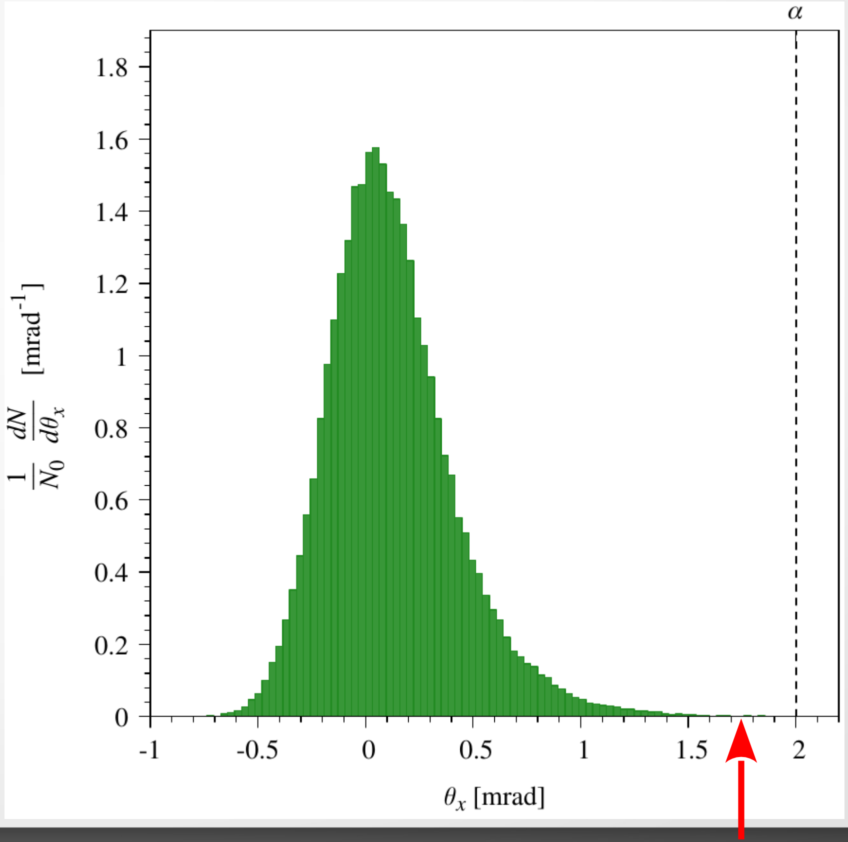


R = 8 cm (α = 2 mrad, L = 160 μm)

(111) plane

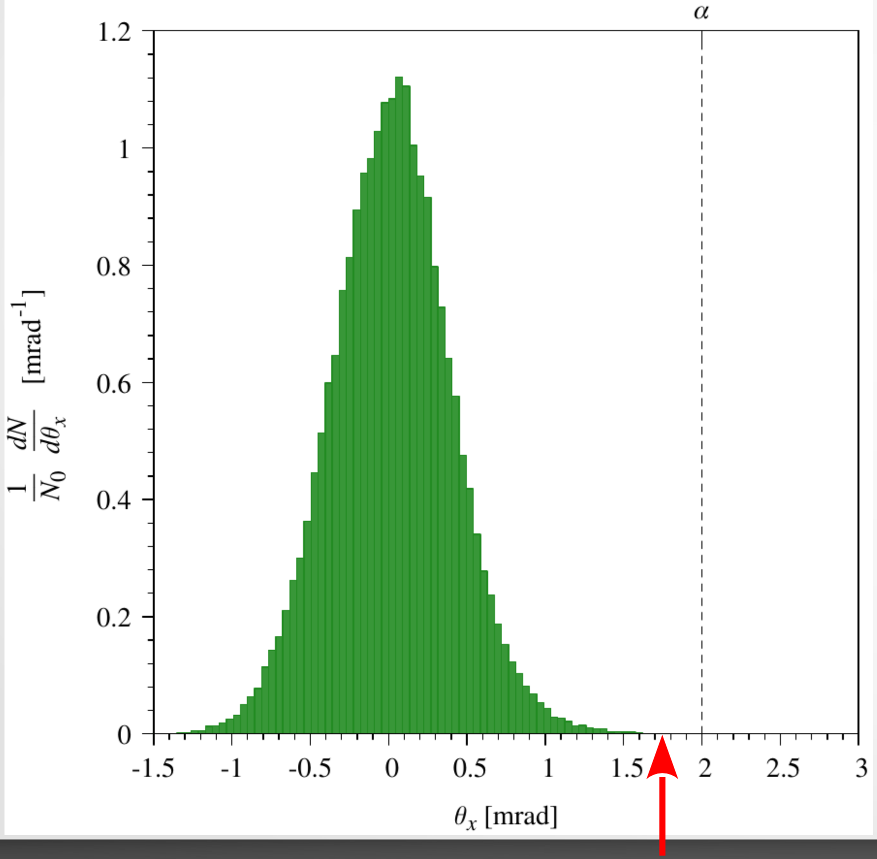


(110) axis

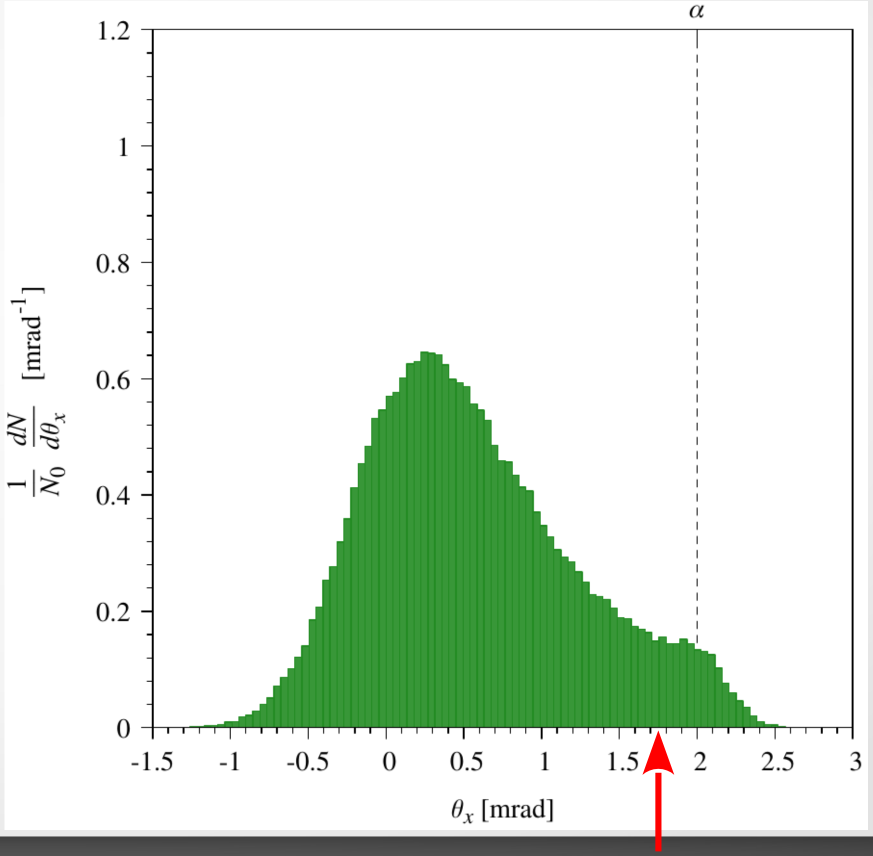


R = 1 m (α = 2 mrad, L = 2 mm)

(111) plane

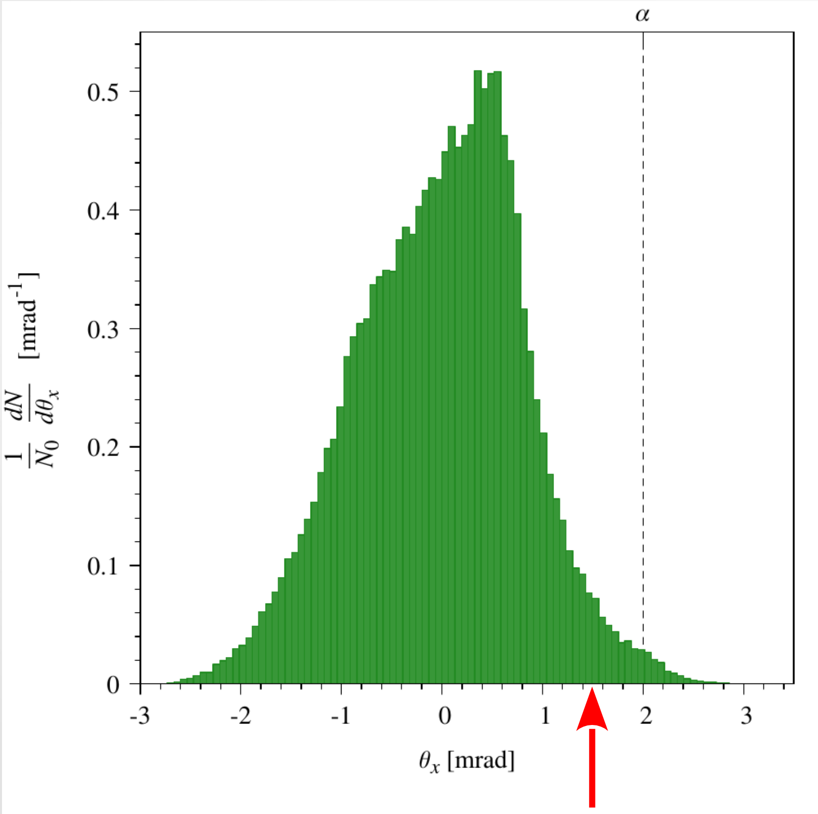


(110) axis

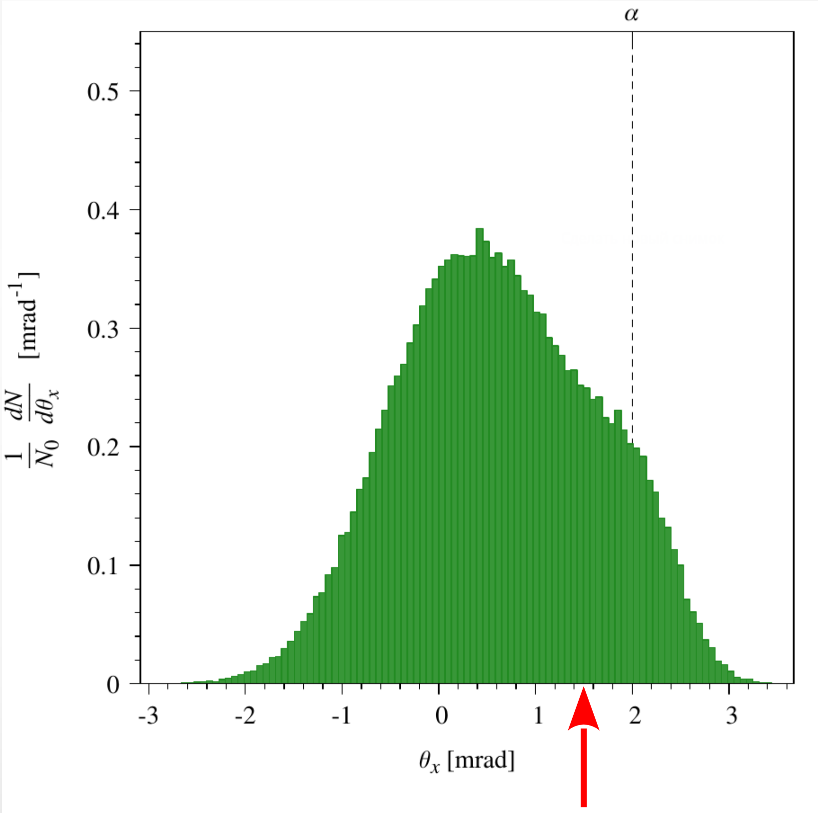


R = 5 m (α = 2 mrad, 1 cm)

(111) plane



(110) axis



Thank you

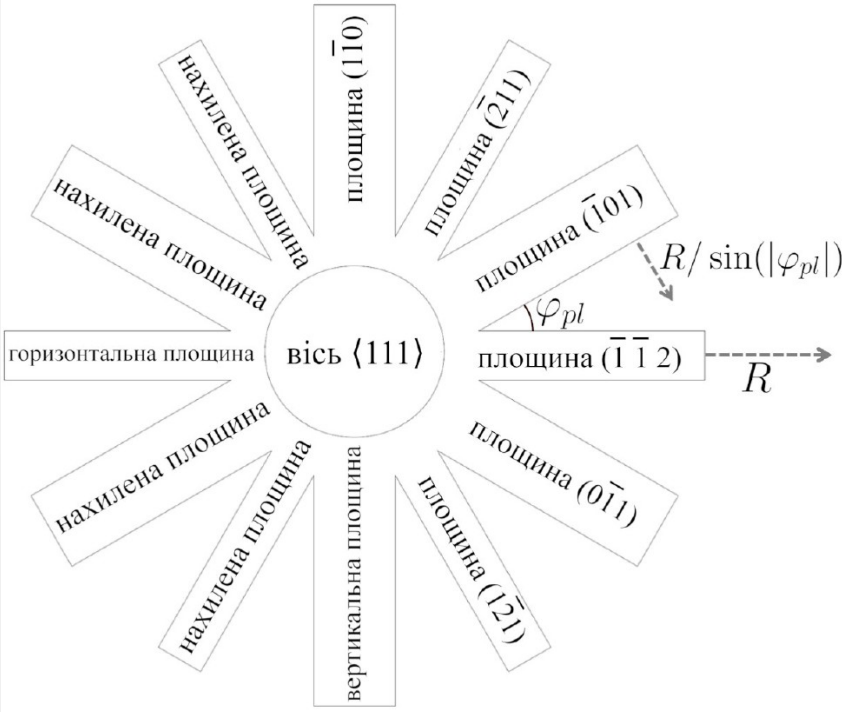
for attention

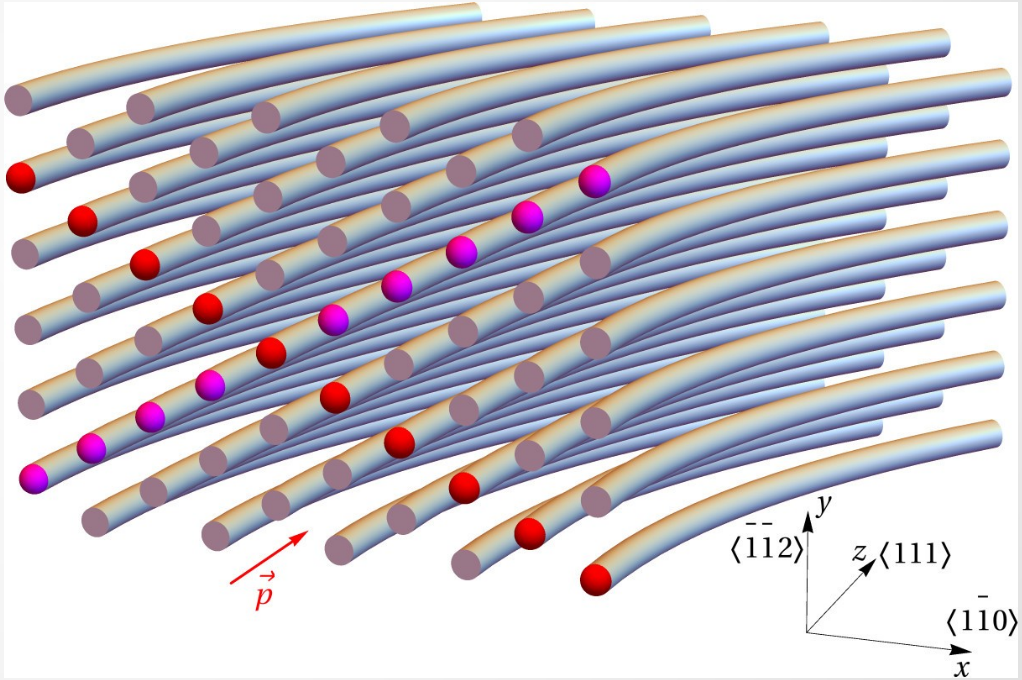
and for your help

Stochastic deflection

$$\alpha_{st} = \frac{2R\psi_c^2}{l_0}$$

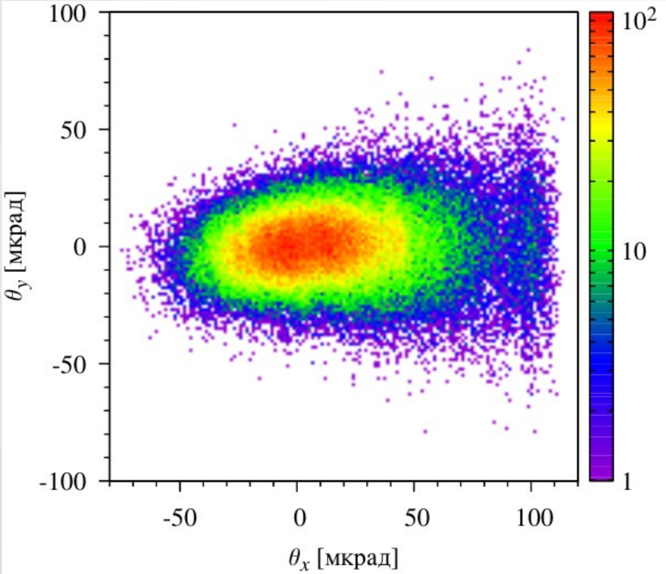
$$R_{cr} = \frac{E \sin|\varphi_{pl}|}{\max\left(\left|\frac{\partial U_{pl}(\zeta)}{\partial \zeta}\right|\right)}$$

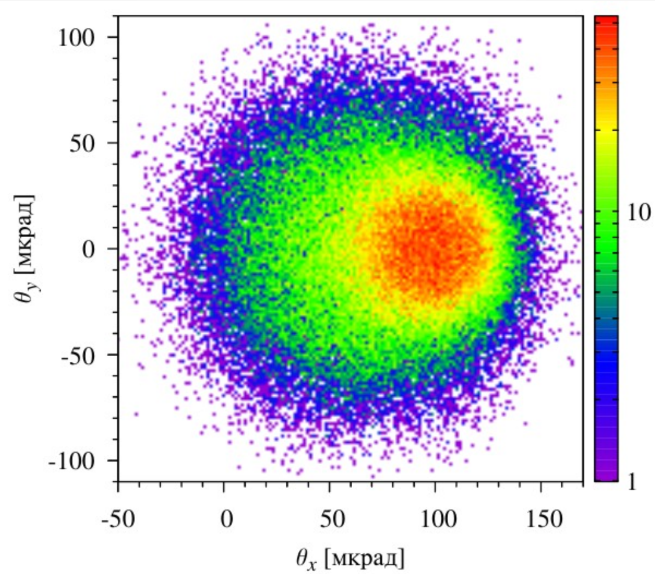


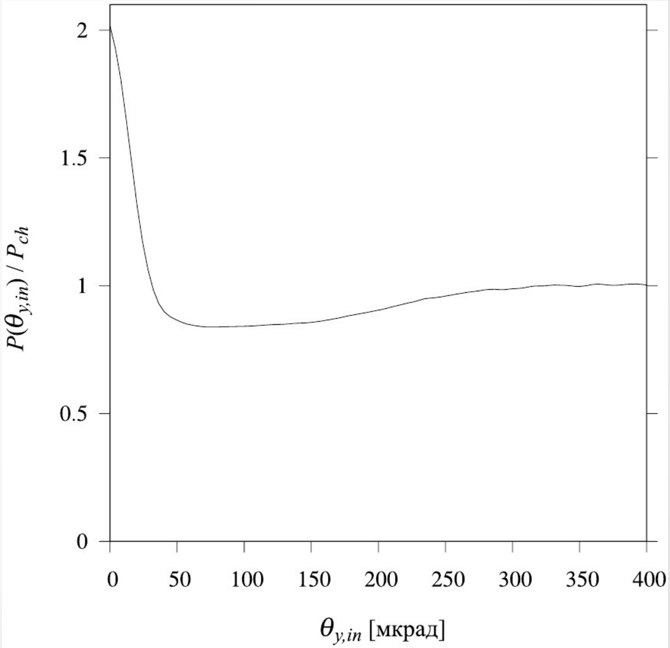


Probability of close collisions

π^-, E = 270 GeV, Si <110>, L = 5 mm, R = 5 m







Kirillin I.V., Shul'ga N.F. Nucl. Instr. Meth. Phys. Res. B. 2015. Vol. 355. P. 49–52.