Analysis of DESY – Test Beam data on bremsstrahlung by electrons and positrons in Aluminum

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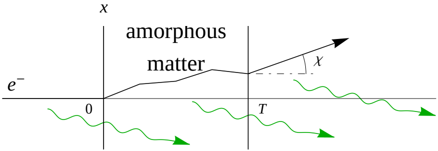
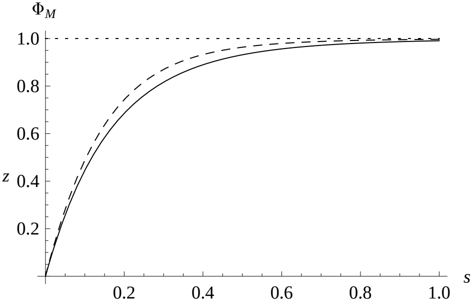
Contents

LPM effect

L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **92** (1953) 535;
A. B. Migdal, Phys. Rev. **103** (1956) 1811:
Since the electron scatters in the target during the radiation emission, the bremsstrahlung spectrum profile will differ from that for a single scattering of a sequence of incoherent scatterings and emissions.
The effect is the most pronounced at $\omega \to 0$:





Experimental verification of the LPM effect (1970-2013)

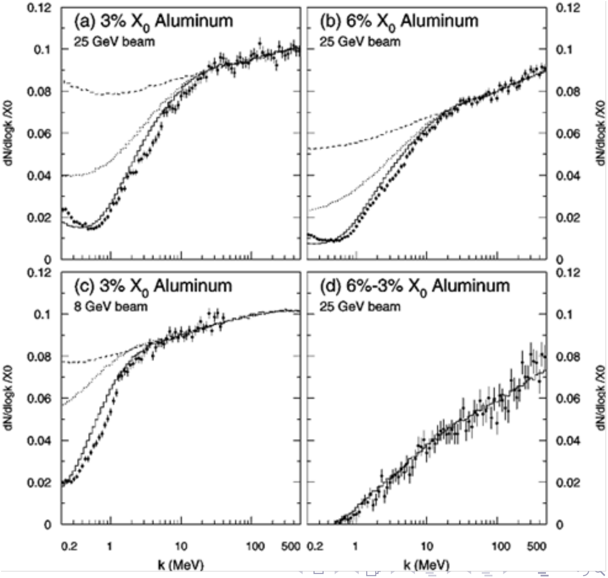


Figure: Spectra of bremsstrahlung in aluminum targets 3.12 and 5.3 mm thick, measured by a BGO calorimeter [P. L. Anthony et al., 1997]. Dashed curves, BH emission spectra without account for suppression effects (the increase being due to multiphoton effects alone). Dotted, LPM spectra without dielectric suppression. Solid, predictions taking into account both LPM and dielectric suppression. A rise in the spectra at small $k = \omega$ is attributed to edge effects.

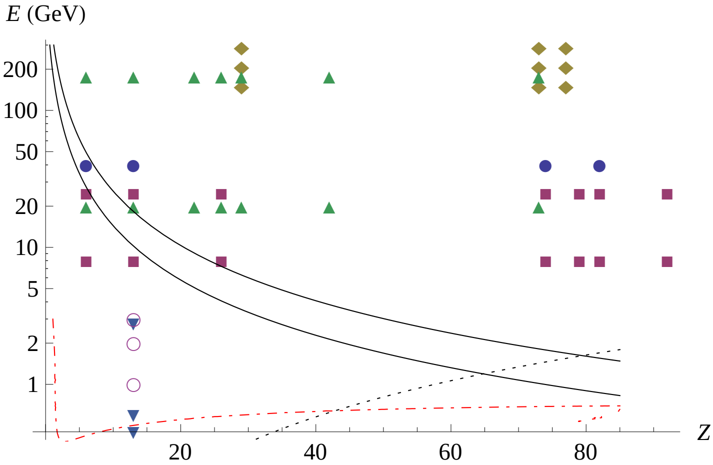


Figure: Optimal parameters for experimental observation of LPM and dielectric suppression effects. Solid curve is the lower boundary of the LPM suppression domain. Dotted curve , lower boundary of the ideal plasma dispersion law. Dot-dashed curve is the upper boundary of absorption region. Points: experiments [1, 2, 3, 4, 5, 6], our experiment (hollow circles).

List of experiments on LPM suppression

F. R. Arutyunyan *et al.*,
Sov. Phys. JETP Lett. **4**, 187 (1966); *ibid.* **35**, 1067 (1972).

A.A. Varfolomeev *et al.*,
Sov. Phys. JETP **42**, 218 (1976).

P. Anthony *et al.*,
Phys. Rev. Lett. **76**, 3550 (1996).

P. Anthony *et al.*,
Phys. Rev. D **56**, 1373 (1997).

H.D. Hansen *et al.*,
Phys. Rev. D **69**, 032001 (2004).

K.K. Andersen *et al.*,
Phys. Rev. D **88**, 072007 (2013).

The “semi-bare" electron interpretation. I

A. I. Akhiezer, N. F. Shul’ga. *High-energy electrodynamics in matter* (Gordon & Breach, Amsterdam, 1996):
“The Coulomb field related to a scattered electron does not appear immediately. [...] During $\Delta t \leq (ck - \vec{k} \cdot \vec{v}_1)^{-1}$ the Fourier components of the potential $\vec{A}$ with the wavevector $\vec{k}$ are, in fact, absent. Since the main contribution to the potential $\vec{A}_1$ is from the wavevectors $\vec{k}$ whose directions are close to that of the velocity $\vec{v}_1$, the time interval where the Fourier components of the scattered electron Coulomb field are absent, will be about

$$\Delta t \sim 2E^2/m^2 c^3 k. \tag{1}$$

In other words, one can say that after scattering, for the time interval (1), an electron is in a ’semi-bare’ state, i.e. to an appreciable extent, without its Coulomb field."

The “semi-bare" electron interpretation. II

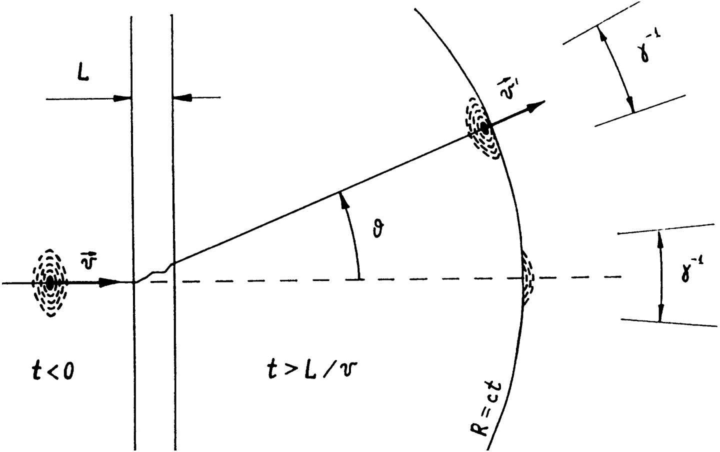


Figure: A figure from: A. I. Akhiezer, N. F. Shul’ga. *High-energy electrodynamics in matter*

The “semi-bare" electron interpretation. III

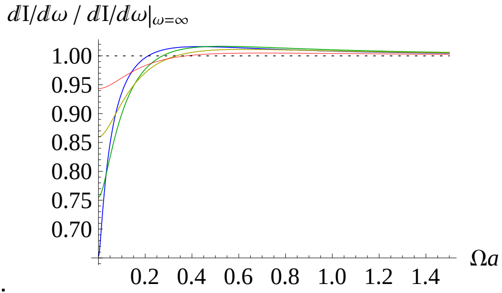
B.M. Bolotovsky and A.M. Serov, Usp. Fiz. Nauk **167** (1997) 1107:
“The total ability of a classical electron to emit radiation is determined by the field asymptotics in the electron location point, which is unaffected by rescatterings."

“Re-dressing" of the “semi-bare" electron. Where is the missing constructive interference?

However, in classical electrodynamics (which must hold as long as $\omega \ll E$) there is an energy sum rule:

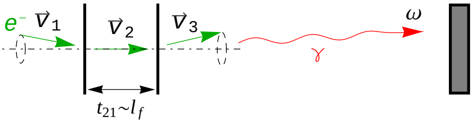
$$\int_{-\infty}^{\infty} dt \frac{dI}{dt} = \sum_{\mathrm{scat}=k} \int_{-\infty}^{\infty} dt \frac{dI_k}{dt} \quad \Rightarrow \quad \int_0^{\infty} d\omega \left(\frac{dI}{d\omega} - \sum_{\mathrm{scat}=k} \frac{dI_k}{d\omega} \right) = 0.$$

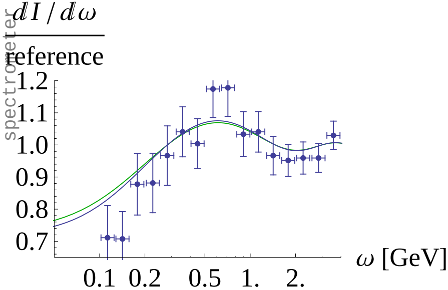
It implies that besides suppression, there must also be a region of spectral enhancement.



J. S. Bell. Nucl. Phys. **8** (1958) 613:
“The reduction in radiation at low frequencies is compensated for by enhancement at very high frequencies. But it must be admitted that the enhancement is mainly in a frequency range where classical theory is quite wrong."

Bremsstrahlung in a two-foil target. Spectral oscillations





Theory:
R. Blankenbecler, Phys. Rev. D **55** (1997) 190.
M. V. Bondarenco and N. F. Shul'ga, Phys. Rev. D **90** (2014) 116007; *ibid.* **95** (2017) 056003.

$\omega \ll E_e$ permits the use of classical electrodynamics.

Experiment (CERN, NA63):
K. K. Andersen *et al.*, Phys. Lett. B **732** (2014) 309.

$$\frac{dI}{d\omega} = \left\langle \frac{dI_1}{d\omega} \right\rangle + \left\langle \frac{dI_2}{d\omega} \right\rangle - \frac{8e^2}{\pi} \int_0^\infty dss \langle G \rangle_1 (s) \langle G \rangle_2 (s) \cos \frac{t_{21}}{l_f(\omega)}(1+s). \quad (2)$$

Functions $\langle G \rangle_{1,2}$ depend on the rms scattering angle in each plate.

Dielectric suppression vs. LPM and photoabsorption. I

M. L. Ter-Mikayelyan. *High Energy Electromagnetic Processes in Condensed Media* (Wiley, New York, 1972):
“The reason for the changes in the Bethe-Heitler formula is the interaction of the emitted quanta with the medium. [...] The whole point comes down to the fact that in the original field equations the photon velocity c has to be replaced with the phase velocity $c/\sqrt{\varepsilon}$ where $\sqrt{\varepsilon}$ is the refractive index for a given frequency.
[...]
A question may arise: is it legitimate to use the dielectric constant of a medium when calculating the radiation of quanta whose wavelength is much smaller than the interatomic distances. This is actually the case as the energy of the emitting particle increases. To justify the use of the conventional dielectric constant, one has to start with microscopic Maxwell equations. By considering the interaction of photons with electrons of the medium [E.Fermi, Rev. Mod. Phys. 1932] (which comes down to the summation of Compton effect chains for scattering at an angle of zero) and averaging over all possible states, we could introduce the dielectric constant of the medium in a consistent manner. These calculations should yield a criterion for the applicability of a macroscopic consideration of the effect. No one has yet performed such a detailed analysis.

Dielectric suppression vs. LPM and photoabsorption. II

An argument can be made that the results obtained by the method described above will not differ from the formulas given in the text. This follows from the fact that the effective length parameter included in the problem is the coherent length, and not the wavelength of the emitted photon. If this is so, then macroscopic electrodynamics can always be applied provided the coherent length is greater than the distance between atoms. For our case, the coherent length for the emission of soft quanta is significantly greater than c/ω_p, where ω_p is the Langmuir frequency. Therefore, one can hope that accurate microscopic calculations for the emission of soft quanta in a transparent medium, taking into account the polarization effect, will confirm the equations obtained above.

Dielectric suppression vs. LPM and photoabsorption. III

Figure: A. Zhukov, M. Bondarenco, M.L. Ter-Mikaelyan. Belgorod (Russia), 2000

Dielectric suppression vs. LPM and photoabsorption. IV

For bremsstrahlung from an ultrarelativistic electron, the coherence length is large, $l_f = (p - p' - k)^{-1} \propto \gamma \gg 1$ but fragile:
Scattering of both the electron and the photon in the medium ruins their mutual coherence, thereby suppressing the radiation intensity."

Besides the bremsstrahlung intrinsic coherence, there is scattering coherence, characterized by the lateral (transverse) length $l_\perp$.
Electron's and photon's lateral coherence length values are vastly different.

The electron's transverse coherence length depends only on the typical transverse momentum transfer $q_\perp$, and is small on the atomic scale:

$$l_\perp^{(e)} = \frac{\hbar}{q_\perp} \sim a_{\mathrm{TF}} \ll \text{Å}.$$

Therefore, electron scattering on different atoms is incoherent,
whereas its scattering on electrons of the same atom is coherent.

Dielectric suppression vs. LPM and photoabsorption. V

On the contrary, photon's transverse coherence length depends on ω, and can be large on the atomic scale, if $\omega \sim \gamma\omega_p$ (as is typical for dielectric suppression):

$$l_\perp = \frac{1}{k_\perp} = \frac{1}{\omega\theta} \sim \frac{\gamma}{\omega} \sim \frac{1}{\omega_p} \gg n_a^{-1/3}.$$

Thus, such a soft photon does not resolve the atomic structure, and is forward-scattered on all the intra-medium atomic electrons coherently.
Then, if the atomic density of the matter is uniform, this mainly produces a phase shift for the photon, rather than an angular spread.
This phase shift proves to be the same as if it was calculated according to the macroscopic dielectric susceptibility of the medium.

$$\varepsilon(\omega) - 1 = \frac{4\pi n_e}{\omega^2} f_{\mathrm{Compt}}(\omega, \theta = 0) \simeq -\frac{\omega_p^2}{\omega^2}, \quad \omega_p = \sqrt{4\pi n_e r_e},$$

$$\omega/k = 1/\sqrt{\varepsilon} > 1 \ (\text{superluminal}).$$

Optimal conditions for observation of pure dielectric suppression

In a quasi-infinite uniform medium, the angle-integral spectrum of bremsstrahlung from one electron per its unit path length is described by Migdal's formula [**?**]:

$$\frac{dI}{d\omega dz} = \underbrace{\frac{1}{1+\gamma^2\omega_p^2/\omega^2}}_{\text{Ter-Mikaelian}} \Phi_M \underbrace{\left[\frac{1}{4\gamma}\sqrt{\omega l_{\text{scat}}}\left(1+\frac{\gamma^2\omega_p^2}{\omega^2}\right)\right]}_{\text{Landau-Pomeranchuk-Migdal}} \frac{dI_{\text{BH}}}{d\omega dz}. \tag{3}$$

In Eq. (3),

$$l_{\text{scat}} = \frac{l}{\gamma^2\left\langle\chi^2(l)\right\rangle} \sim \frac{X_0}{420} \sim \frac{\alpha}{\pi}X_0 \tag{4}$$

is the length, at which typical multiple scattering angles $\left\langle\chi^2(l)\right\rangle \approx \frac{l}{X_0}\left[\frac{13.6\text{ MeV}}{E}\left(1+0.038\ln\frac{l}{X_0}\right)\right]^2$ become commensurable with typical radiation angles, i.e., the inverse Lorentz factor.

$$\frac{dI_{\text{BH}}}{d\omega dz} \approx \frac{4\hbar}{3X_0} \qquad (\hbar\omega \ll E), \tag{5}$$

Pure Ter-Mikaelian regime

If

$$l_{\mathrm{scat}} > l_f,$$

with

$$l_f \sim \gamma^2/\omega \sim \gamma/\omega_p, \tag{6}$$

the LPM factor Φ_M remains close to unity at any ω, and (3) reduces to

$$\frac{dI}{d\omega dz} \approx \frac{1}{1+\gamma^2\omega_p^2/\omega^2}\frac{dI_{\mathrm{BH}}}{d\omega dz}. \tag{7}$$

Here the first factor furnishes a quadratic suppression at low ω (Ter-Mikaelian effect).

DESY TB experiment layout

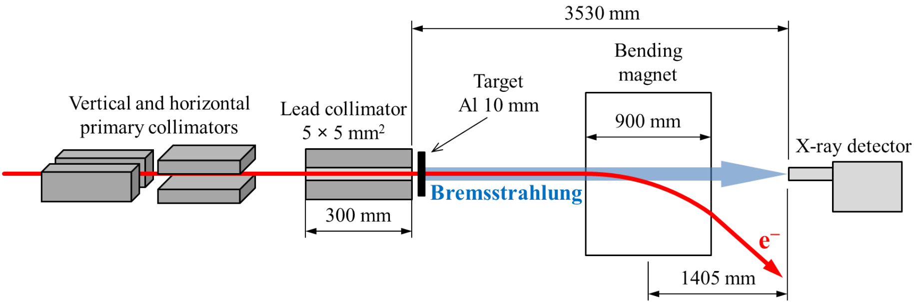


Figure: The layout of our experiment on measurement of bremsstrahlung spectra (2019).

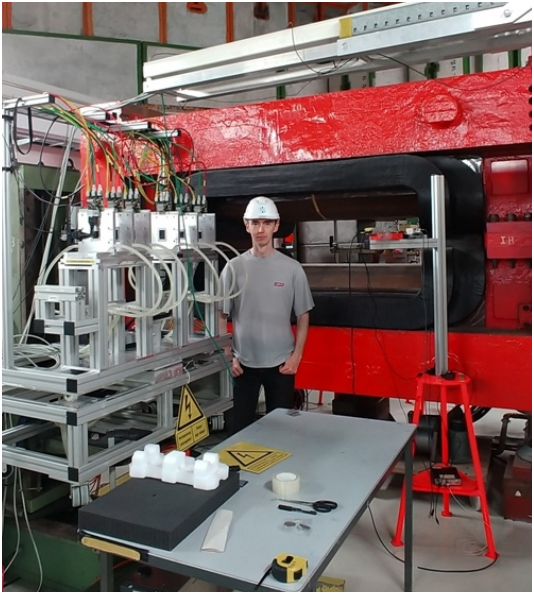


Figure: A. V. Shchagin, M. Bondarenco, DESY-TB, Nov. 2019

CdTe detector

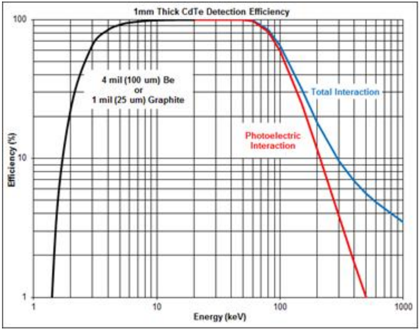


Figure: Schematic course of the Amptek CdTe detector efficiency – not to be applied for reconstruction.

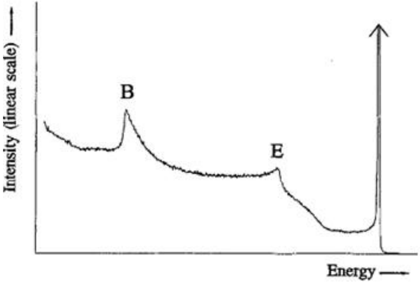
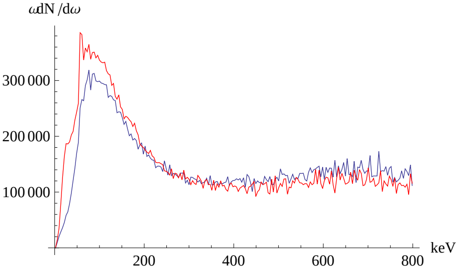
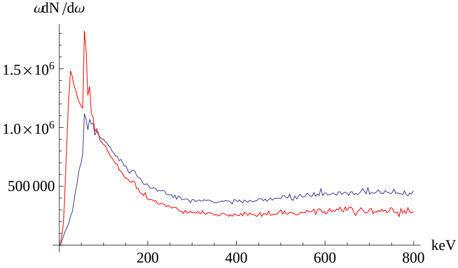


Figure: Spectral nonlocality of detection of a gamma quantum.
E – low-energy edge of the Compton scattering (back-angle scattering).
B – photoelectric peak.

For $\omega \ll m_e = 500$ keV the detector efficiency may be regarded as spectrally local, though poorly known. To eliminate it (get rid of sharp spectral features), it suffices to form a ratio of the bremsstrahlung to background (no target) . Assume that in the background spectrum the dielectric suppression is small.

Radiation spectra (2019)





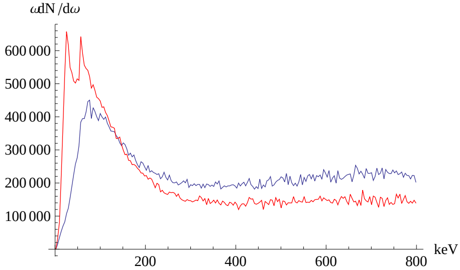


Figure: Measured radiation spectra per unit time, without (left) and with (right) the 1 cm aluminum target. The measurements were performed at electron beam energies 1, 2, and 3 GeV (top to bottom).

Bremsstrahlung to background ratio (2019)

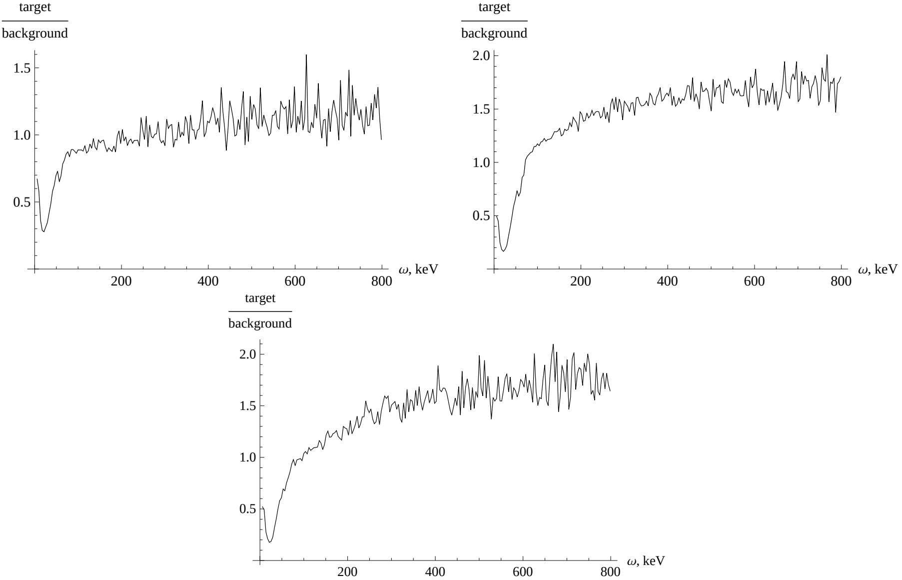
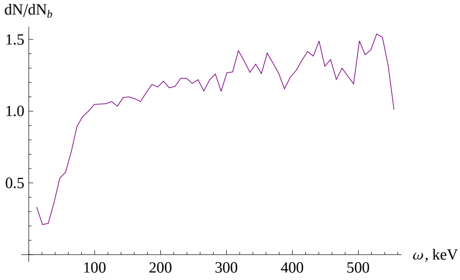


Figure: Ratios of the measured bremsstrahlung and background spectra, for $E = 1$, 2 and 3 GeV.

Bremsstrahlung to background ratio (2023)



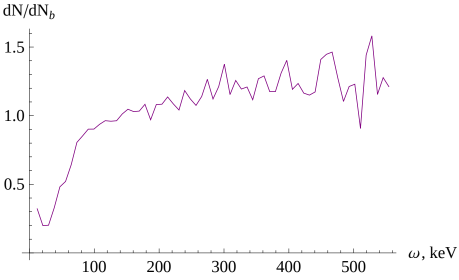


Figure: Ratios of the measured bremsstrahlung and background spectra, for E =2 and 4 GeV. The statistics is lower than in 2019, but for E=4 GeV the change of the spectrum with the electron energy is clearer. To eliminate the energy-independent effect of radiation absorption in the target, an additional reconstruction procedure is needed.

Summary

- Measurements of radiation spectra from 1–5 GeV electrons have been presented herein. They are expected to be the first measurement of the Ter-Mikaelian effect not contaminated by LPM suppression or transition radiation effects.

- At the present stage, our measurements are relative, determining not the Ter-Mikaelian infrared asymptotics $dN/d\omega \underset{\omega\to 0}{\sim} \omega$ itself, but a deviation from the Bethe-Heitler plateau at $\omega \gtrsim \gamma\omega_p$.