# *Quantum Chromodynamics*

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*FCC-ee*

# *Quantum Chromodynamics*

- QCD Lagrangian (classical part)

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left( \mathrm{i} \not{D} - m_q \right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left( t_a \right)_{ij} A^a_\mu$

- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$, quark masses $m_q$
- Parameters of Lagrangian have no unique physical interpretation
  - radiative corrections require definition of renormalization scheme

## *Challenge*

- Suitable observables for measurements of $\alpha_s$, $m_q$, . . .
  - comparison of theory predictions and experimental data

# Strong coupling constant (I)

## Essential facts

- $\alpha_s(M_Z)$ from $e^+e^-$ data high
- $\alpha_s(M_Z)$ from DIS data low
- NLO QCD theory
- World average 1992
  $\alpha_s(M_Z) = 0.117 \pm 0.004$

| | Process | Ref. | Q [GeV] | $\alpha_s(Q)$ | $\alpha_s(M_{Z^0})$ | $\Delta\alpha_s(M_{Z^0})$ exp. | $\Delta\alpha_s(M_{Z^0})$ theor. | order of perturb. |
|---|---|---|---|---|---|---|---|---|
| 1 | $R_\tau$ [LEP] | [7-10] | 1.78 | $0.318\ ^{+\ 0.048}_{-\ 0.039}$ | $0.117\ ^{+\ 0.006}_{-\ 0.005}$ | $^{+\ 0.003}_{-\ 0.004}$ | $^{+\ 0.005}_{-\ 0.004}$ | NNLO |
| 2 | $R_\tau$ [world] | [2] | 1.78 | $0.32 \pm 0.04$ | $0.118\ ^{+\ 0.004}_{-\ 0.006}$ | – | – | NNLO |
| 3 | DIS [$\nu$] | [3] | 5.0 | $0.193\ ^{+\ 0.019}_{-\ 0.018}$ | $0.111\ ^{+\ 0.006}_{-\ 0.007}$ | $^{+\ 0.004}_{-\ 0.006}$ | 0.004 | NLO |
| 4 | DIS [$\mu$] | [12] | 7.1 | $0.180 \pm 0.014$ | $0.113 \pm 0.005$ | 0.003 | 0.004 | NLO |
| 5 | $J/\Psi, \Upsilon$ decay | [4] | 10.0 | $0.167\ ^{+\ 0.015}_{-\ 0.011}$ | $0.113\ ^{+\ 0.007}_{-\ 0.005}$ | – | – | NLO |
| 6 | $e^+e^-$ [$\sigma_{had}$] | [14] | 34.0 | $0.163 \pm 0.022$ | $0.135 \pm 0.015$ | – | – | NNLO |
| 7 | $e^+e^-$ [shapes] | [15] | 35.0 | $0.14 \pm 0.02$ | $0.119 \pm 0.014$ | – | – | NLO |
| 8 | $p\bar{p} \to b\bar{b}X$ | [11] | 20.0 | $0.136\ ^{+\ 0.025}_{-\ 0.024}$ | $0.108\ ^{+\ 0.015}_{-\ 0.014}$ | 0.006 | $^{+\ 0.014}_{-\ 0.013}$ | NLO |
| 9 | $p\bar{p} \to W\ jets$ | [13] | 80.6 | $0.123 \pm 0.027$ | $0.121 \pm 0.026$ | 0.018 | 0.020 | NLO |
| 10 | $\Gamma(Z^0 \to \text{had.})$ | [5] | 91.2 | $0.133 \pm 0.012$ | $0.133 \pm 0.012$ | 0.012 | $^{+\ 0.003}_{-\ 0.001}$ | NNLO |
| 11 | $Z^0$ ev. shapes | | | | | | | |
| | ALEPH | [7] | 91.2 | $0.119\ ^{+\ 0.008}_{-\ 0.010}$ | | – | – | NLO |
| | DELPHI | [8] | 91.2 | $0.113 \pm 0.007$ | | 0.002 | 0.007 | NLO |
| | L3 | [9] | 91.2 | $0.118 \pm 0.010$ | | – | – | NLO |
| | OPAL | [10] | 91.2 | $0.122\ ^{+\ 0.006}_{-\ 0.005}$ | | 0.001 | $^{+\ 0.006}_{-\ 0.005}$ | NLO |
| | SLD | [6] | 91.2 | $0.120\ ^{+\ 0.015}_{-\ 0.013}$ | | 0.009 | $^{+\ 0.012}_{-\ 0.009}$ | NLO |
| | *Average* | [6-10] | 91.2 | | $0.119 \pm 0.006$ | 0.001 | 0.006 | NLO |
| 12 | $Z^0$ ev. shapes | | | | | | | |
| | ALEPH | [7] | 91.2 | $0.125 \pm 0.005$ | | 0.002 | 0.004 | resum. |
| | DELPHI | [8] | 91.2 | $0.122 \pm 0.006$ | | 0.002 | 0.006 | resum. |
| | L3 | [9] | 91.2 | $0.126 \pm 0.009$ | | 0.003 | 0.008 | resum. |
| | OPAL | [10] | 91.2 | $0.122\ ^{+\ 0.003}_{-\ 0.006}$ | | 0.001 | $^{+\ 0.003}_{-\ 0.006}$ | resum. |
| | *Average* | [7-10] | 91.2 | | $0.123 \pm 0.005$ | 0.001 | 0.005 | resum. |

Table 1: Summary of measurements of $\alpha_s$. For details see text.

2

**Bethke, Catani CERN TH-6484/92**

# Strong coupling constant (II)

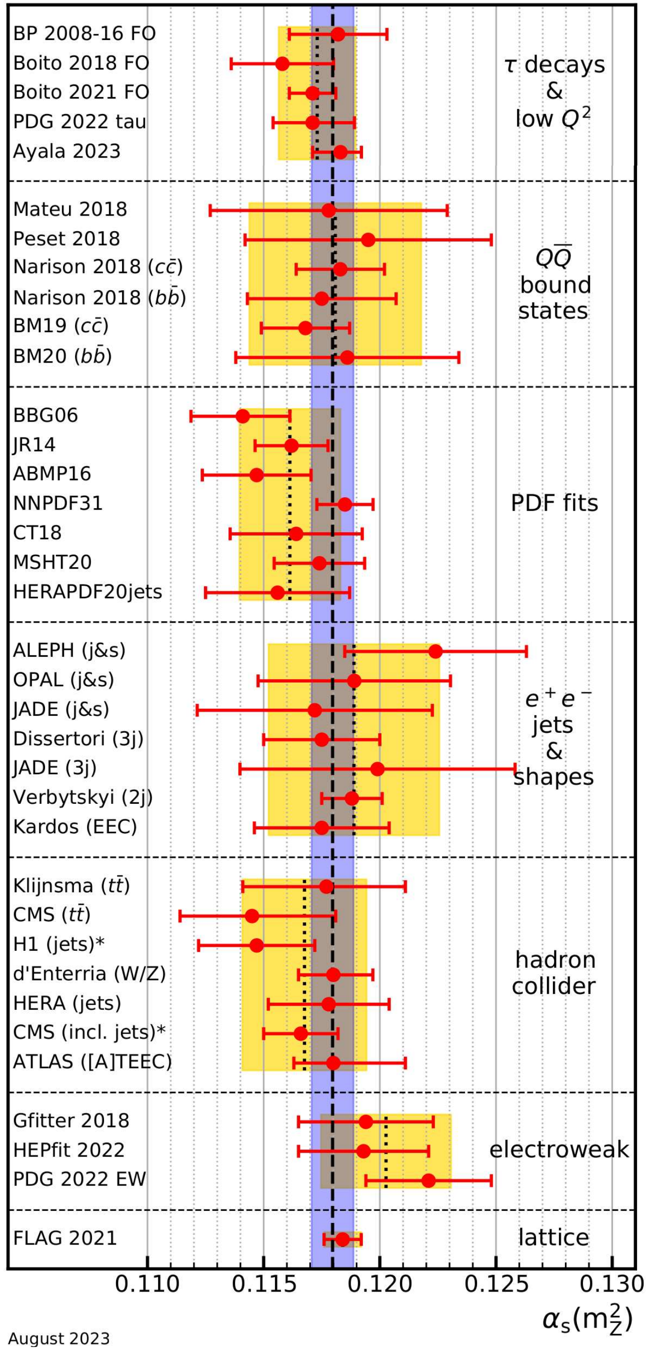
## $\alpha_s$ 2024

- $\alpha_s(M_Z)$ from $e^+e^-$ data high
- $\alpha_s(M_Z)$ from DIS data low
- NNLO QCD theory + lattice

## FCC-ee

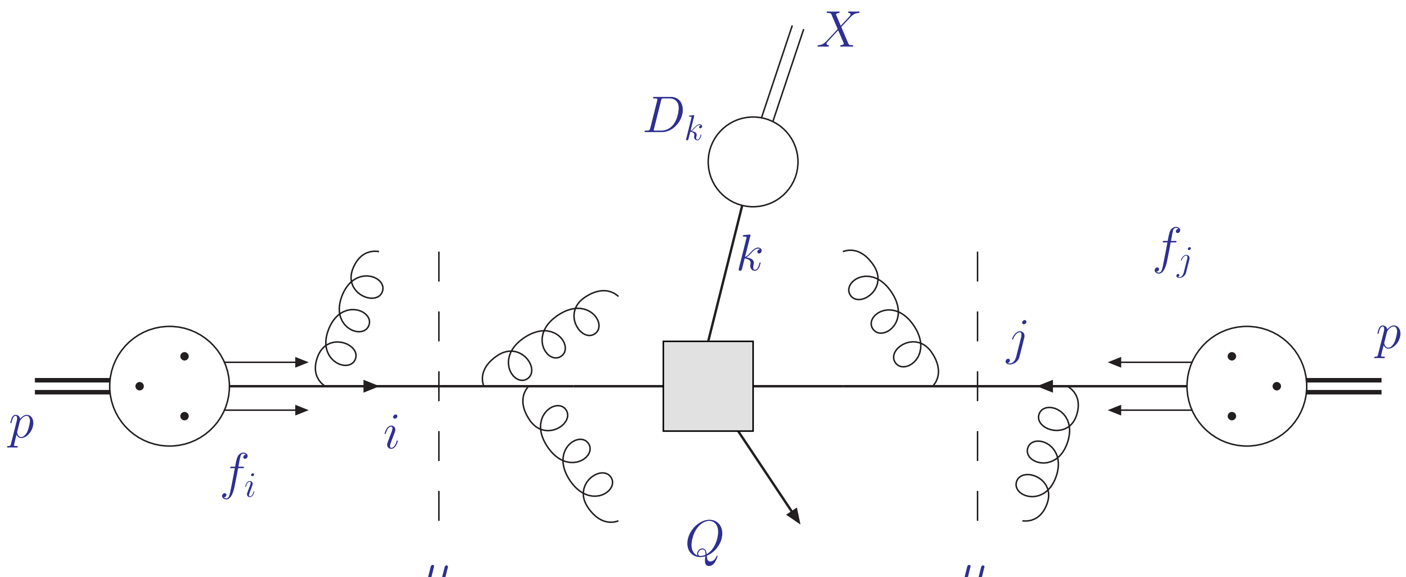
- Potential of FCC-ee:
  $\alpha_s(M_Z)$ at $1\%$ precision

**PDG 2024**



# *Hadron colliders*

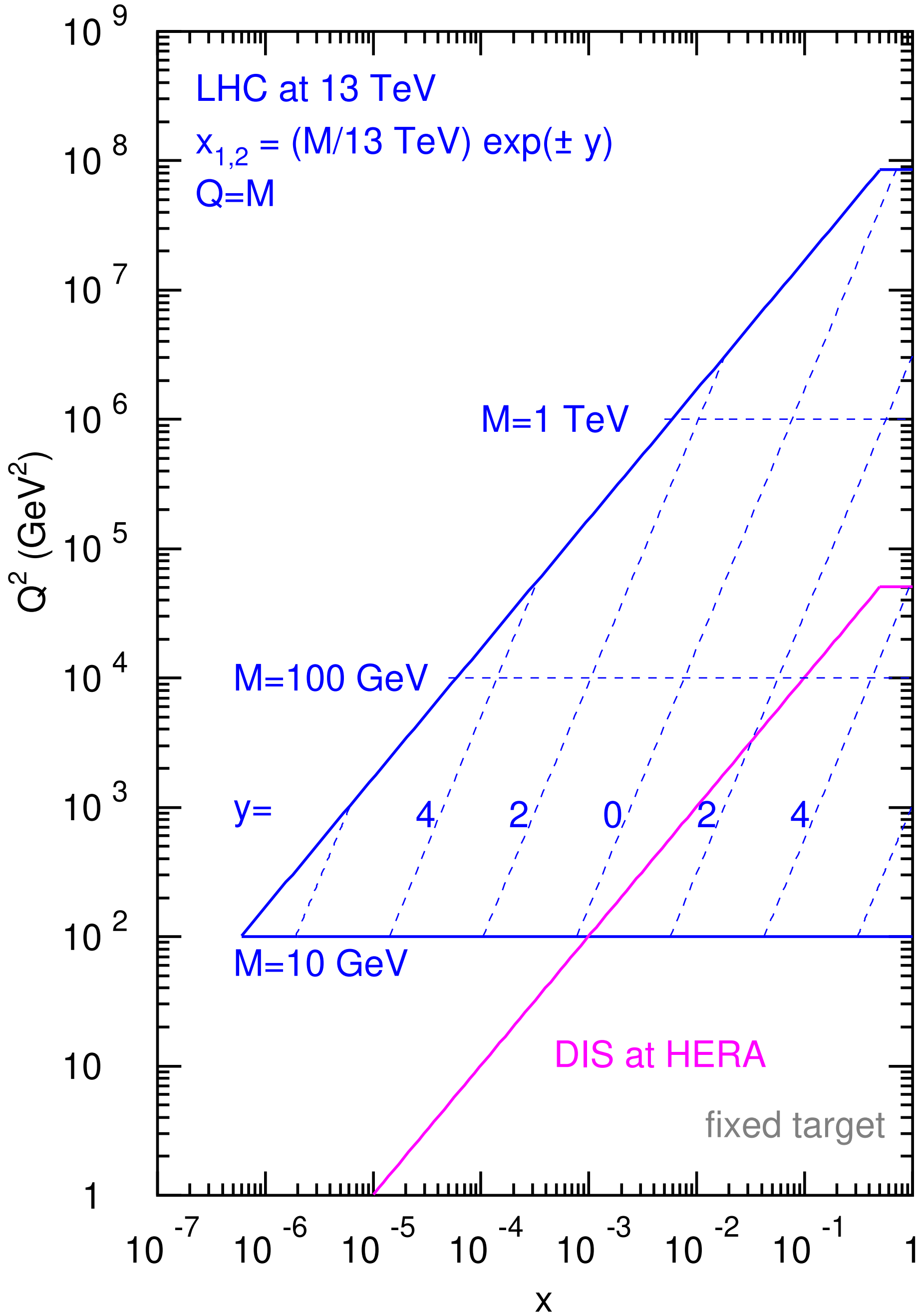
# QCD factorization




$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale $\mu$
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types $i$, $j$ and hadronic final state $X$
- Non-perturbative parameters: parton distribution functions $f_i$, strong coupling $\alpha_s$, particle masses $m_X$
  - known from global fits to exp. data, lattice computations, . . .

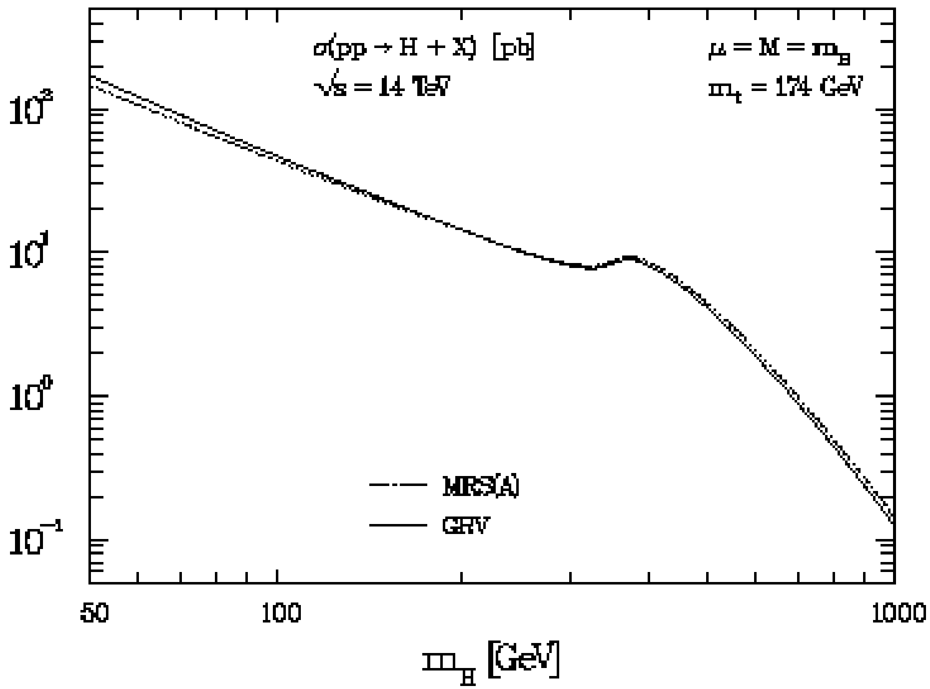
# *Parton kinematics at LHC*




- Information on proton structure depends on kinematic coverage
  - summary of collider kinematics
- LHC run at $\sqrt{s}$ = 13 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics with $x_{1,2} = M/\sqrt{S}\mathrm{e}^{\pm y}$
  - forward rapidities sensitive to small-$x$
  - high scales $Q \simeq 1$ TeV probe large $x \simeq 0.1 \ldots 0.8$

# *Higgs cross section (1995)*

## *NLO QCD corrections*




MRS(A): Martin, Roberts and Stirling, Phys. Rev. D50 (1994) 6734

GRV: Glück, Reya and Vogt, Z. Phys. C53 (1992) 127

**One of the main uncertainties** *in the prediction of the Higgs production cross section is due to the* **gluon density***. [. . . ] Adopting a set of representative parton distributions [. . . ], we find a* **variation of about 7%** *between the maximum and minimum values of the cross section for Higgs masses above* $\sim$ *100 GeV.*

**Spira, Djouadi, Graudenz, Zerwas (1995)**
**hep-ph/9504378**

# *Higgs cross section at LHC*

| | N3LO (δ in %) | | | | N4LOsv (δ in %) | | | |
|---|---|---|---|---|---|---|---|---|
| PDF Name | Central | δ(N3LO) | δ(Scale) | δ(PDF) | Central | δ(N4LO) | δ(Scale) | δ(PDF) |
| ABMP16_5_nnlo [2] | 48.8 | 3.3 | $^{+0.2}_{-3.6}$ | $^{+1.7}_{-1.7}$ | 48.7 | −0.1 | $^{+0.5}_{-2.1}$ | $^{+1.7}_{-1.7}$ |
| ABMPtt_5_nnlo [3] | 48.4 | 3.3 | $^{+0.2}_{-3.6}$ | $^{+1.5}_{-1.5}$ | 48.4 | −0.1 | $^{+0.5}_{-2.1}$ | $^{+1.5}_{-1.5}$ |
| CT18NNLO [4] | 51.3 | 3.5 | $^{+0.3}_{-3.9}$ | $^{+2.8}_{-3.6}$ | 51.3 | −0.1 | $^{+0.5}_{-2.3}$ | $^{+2.8}_{-3.6}$ |
| MSHT20nnlo_as118 [5] | 51.4 | 3.5 | $^{+0.3}_{-3.9}$ | $^{+1.2}_{-1.2}$ | 51.3 | −0.1 | $^{+0.5}_{-2.3}$ | $^{+1.2}_{-1.2}$ |
| NNPDF40_nnlo_as_01180 [6] | 51.7 | 3.5 | $^{+0.3}_{-3.9}$ | $^{+0.6}_{-0.6}$ | 51.7 | −0.1 | $^{+0.5}_{-2.3}$ | $^{+0.6}_{-0.6}$ |
| PDF4LHC21_40 [7] | 51.6 | 3.5 | $^{+0.3}_{-3.9}$ | $^{+0.6}_{-0.6}$ | 51.5 | −0.1 | $^{+0.5}_{-2.3}$ | $^{+0.6}_{-0.6}$ |
| MSHT20an3lo_as118 [8] | 48.7 | 3.5 | $^{+0.3}_{-3.9}$ | $^{+1.9}_{-1.7}$ | 48.7 | −0.1 | $^{+0.5}_{-2.3}$ | $^{+1.9}_{-1.7}$ |
| NNPDF40_an3lo_as_01180 [9] | 50.6 | 3.5 | $^{+0.3}_{-3.9}$ | $^{+0.6}_{-0.6}$ | 50.6 | −0.1 | $^{+0.5}_{-2.3}$ | $^{+0.6}_{-0.6}$ |

- LHC run at $\sqrt{s}$ = 13.6 TeV
  - complete N$^3$LO QCD corrections

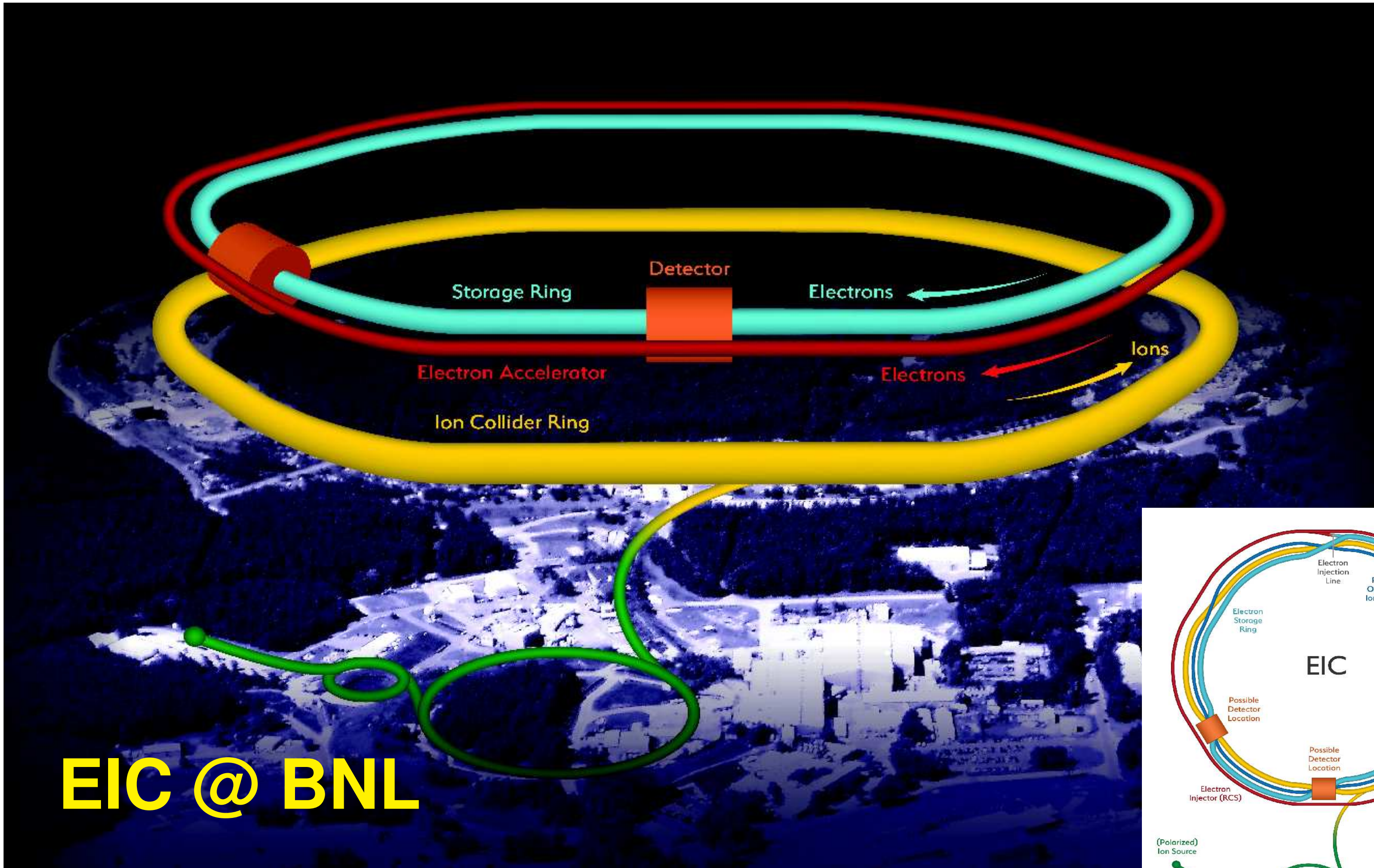
    Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15
  - approximate N$^4$LO QCD corrections Das, S.M., Vogt '20
- Large spread for predictions from different PDFs $\sigma(H) = 48.4 \ldots 51.7$ pb
- *PDF and $\alpha_s$ differences between sets amount to up to $7\%$*
  - significantly larger than residual theory uncertainty due to N$^3$LO QCD and NLO electroweak corrections

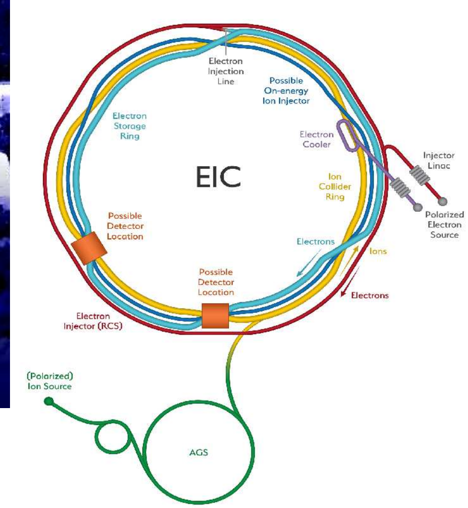
*EIC*

# *Bright future for precision hadron physics*

- Electron-Ion Collider

  *A machine that will unlock the secrets of the strongest force in Nature*

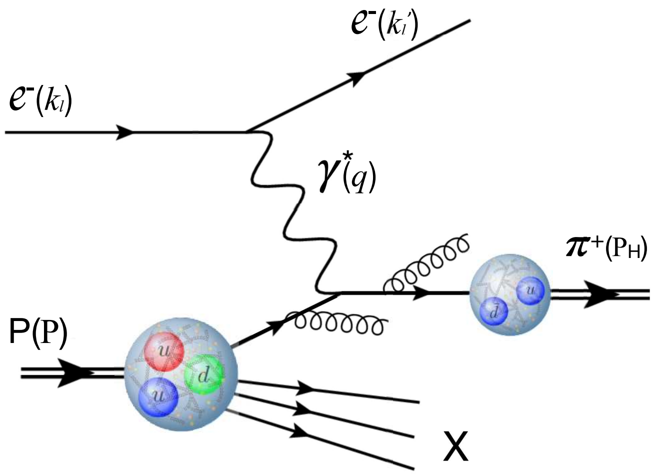


# *Deep-inelastic scattering*

- DIS structure functions
  - unpolarized $F_2$, $F_L$, $F_3$
  - spin dependent $g_1$

- Semi-inclusive DIS
  - production of identified hadrons in DIS
  - multiple hadron species: $\pi$, K, D, p, n, $\Lambda$, . .
  - probe of hadron structure
    in broad kinematic range

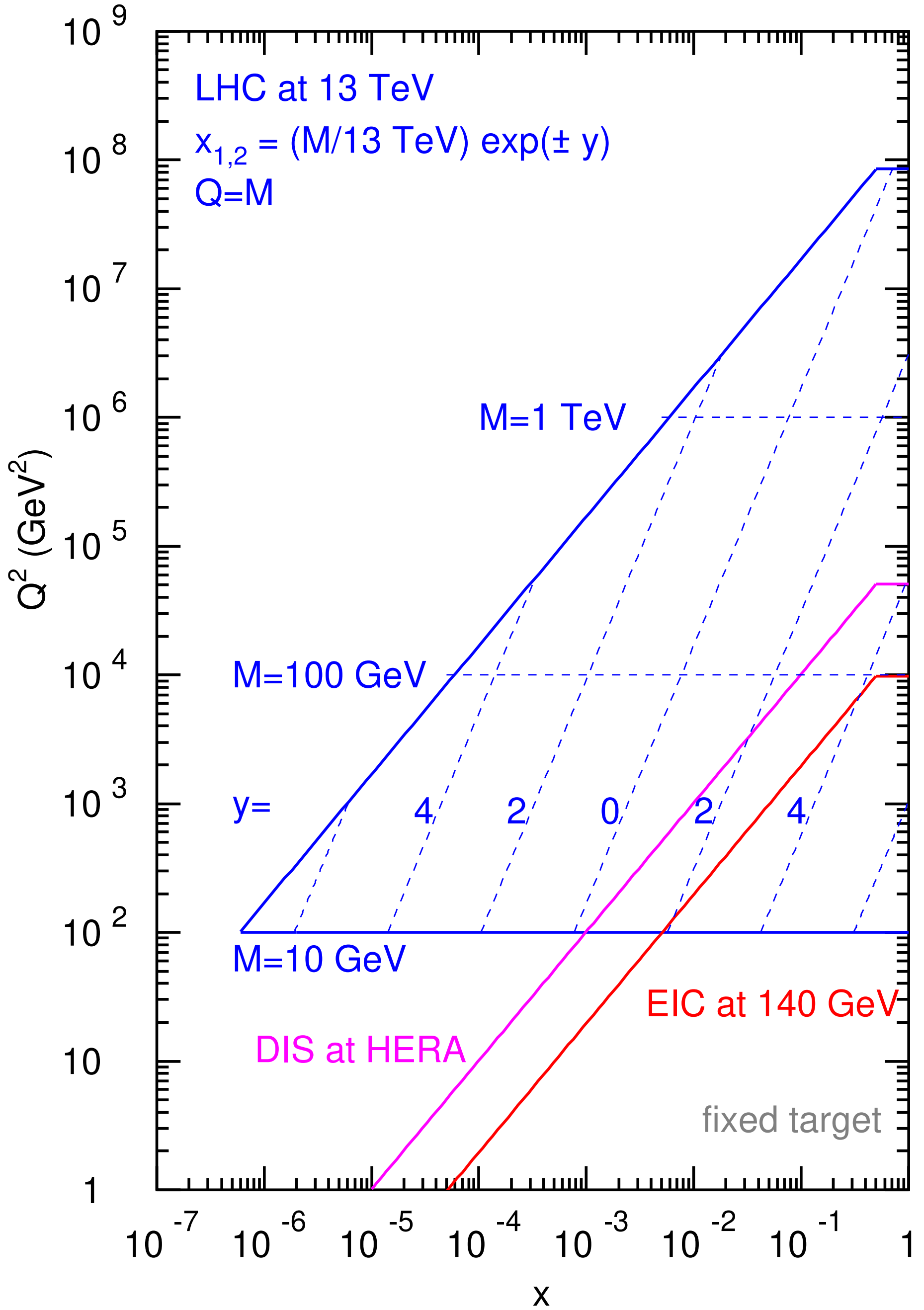



- QCD factorization at scale $\mu^2$

$$\sigma_{\gamma H \to H'} = \sum_{ij} f_{i/H}(\mu^2) \otimes \hat{\sigma}_{\gamma i \to j}\left(\alpha_s(\mu^2), Q^2, \mu^2\right) \otimes D_{H'/j}(\mu^2)$$

  - parton distribution function (PDF) $f_{i/H}(x, \mu^2)$
  - parton-to-hadron fragmentation function (FF) $D_{H'/j}(z, \mu^2)$

# *Parton kinematics at EIC*



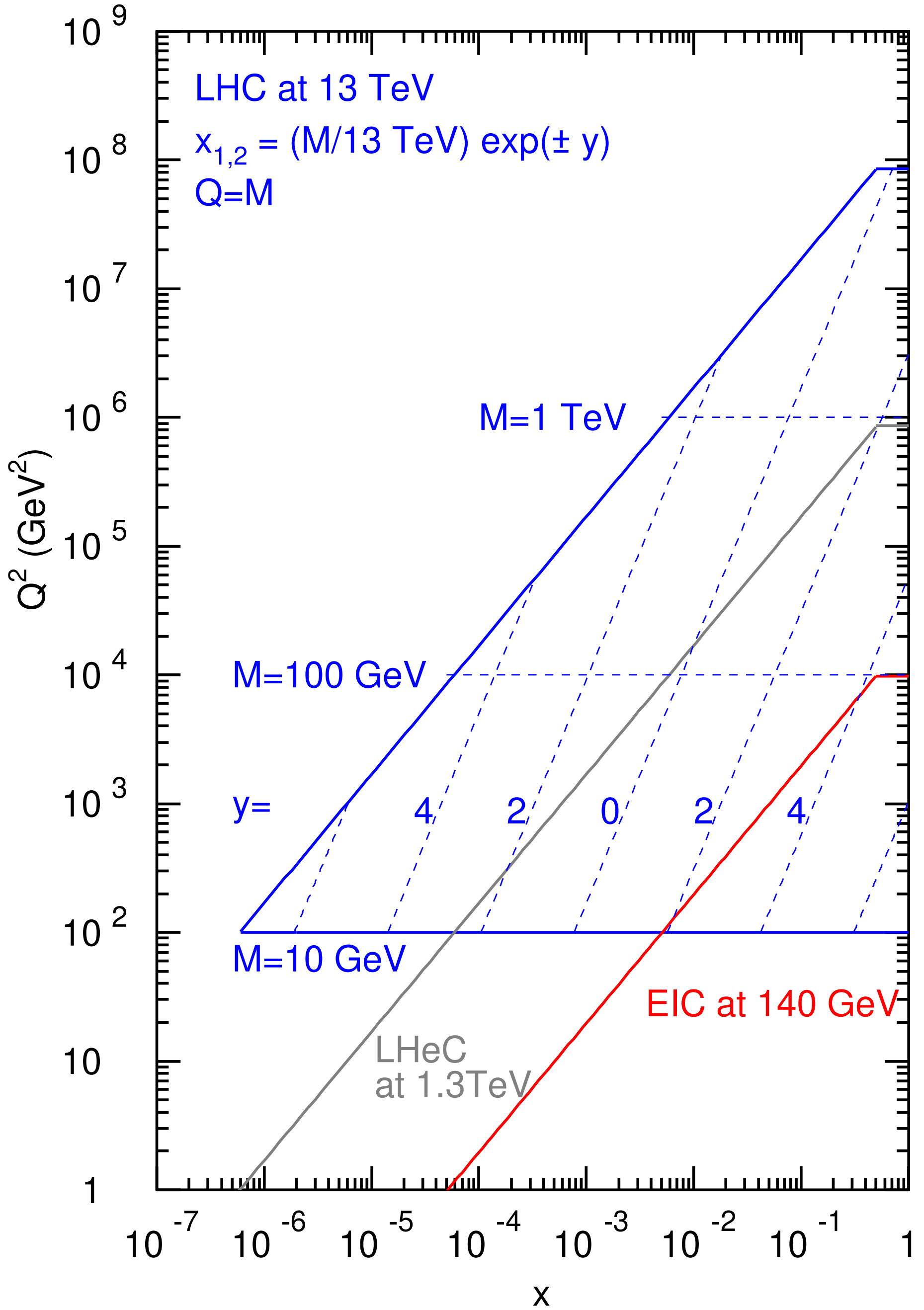

- EIC run at $\sqrt{s}$ = 140 GeV
  - $ep$-collisions at EIC cover large part of phase space relevant for LHC
  - overlap with HERA and fixed target experiments

## *Novel measurements at EIC*

- 3D-images of hadron in position and momentum space (including spin)
- Measurements with unprecedented precision

*LHeC*

# *Parton kinematics at LHeC*



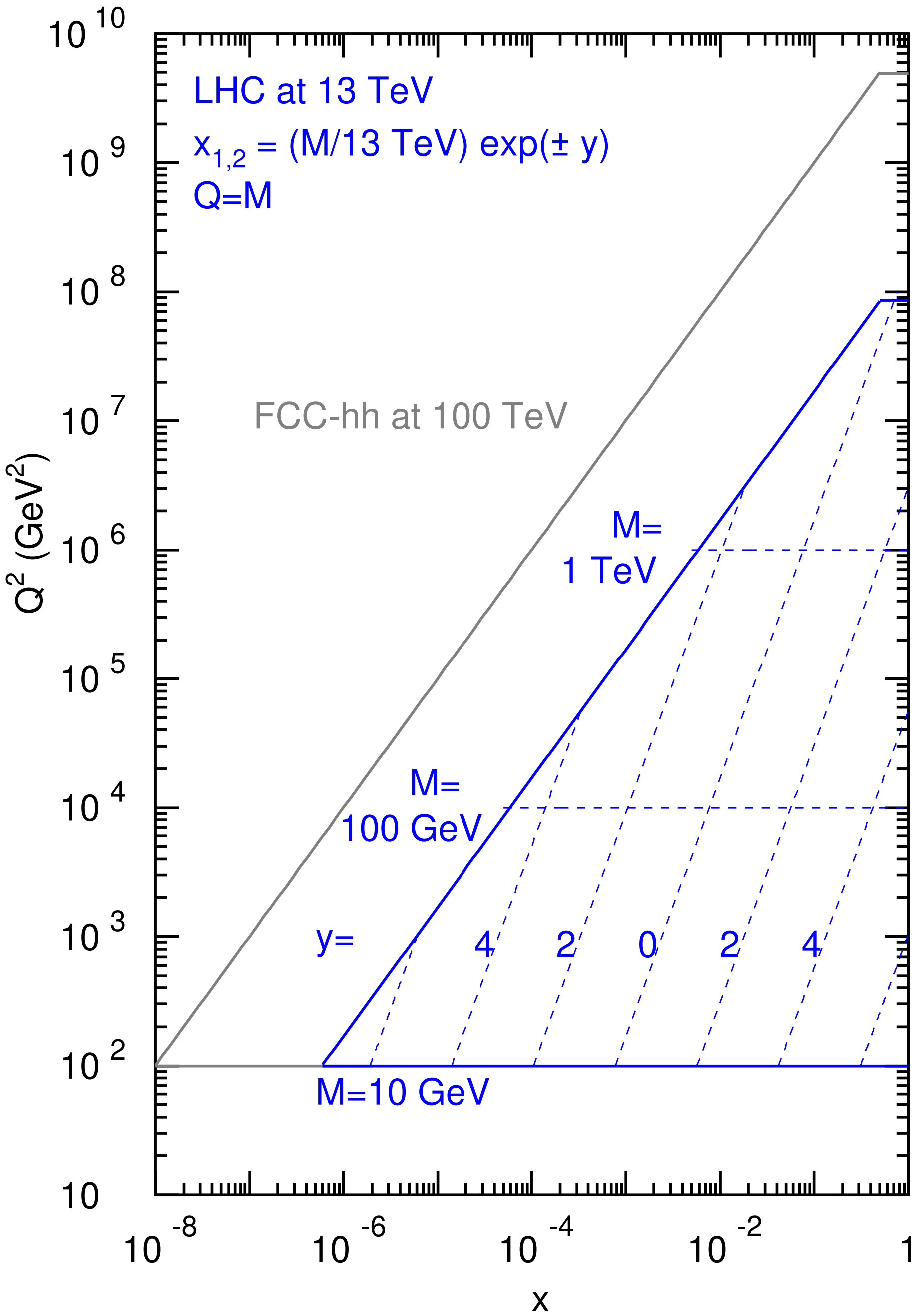

- LHeC run at $\sqrt{s}$ = 1.3 TeV
  - significant reach beyond HERA kinematics

*New QCD regime accessible*

- exploration of parton dynamics at small $x \simeq 10{-}6$

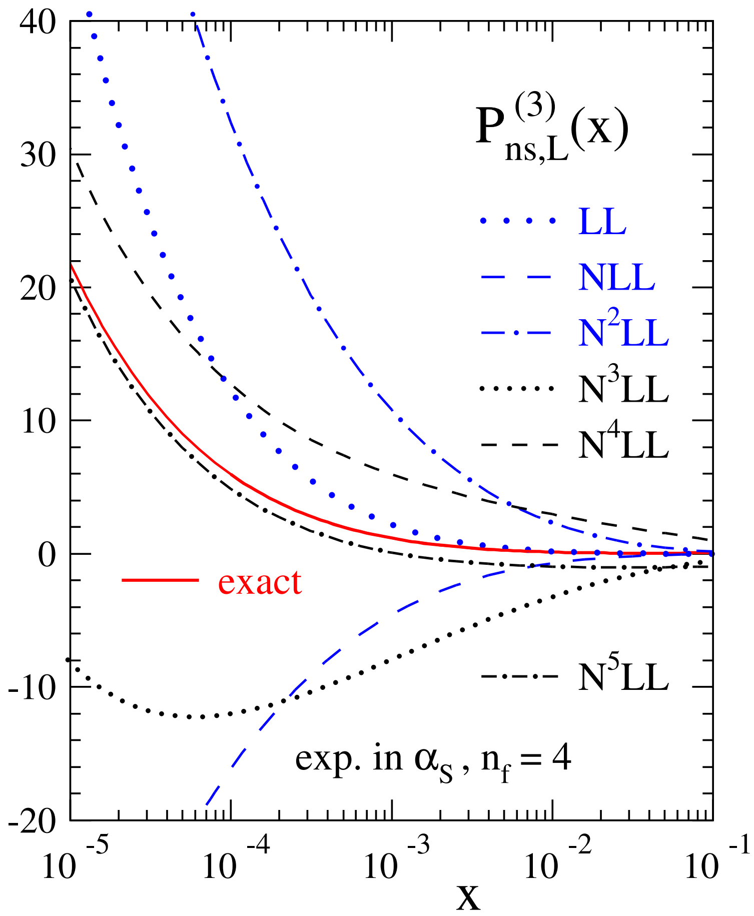
*FCC-hh*

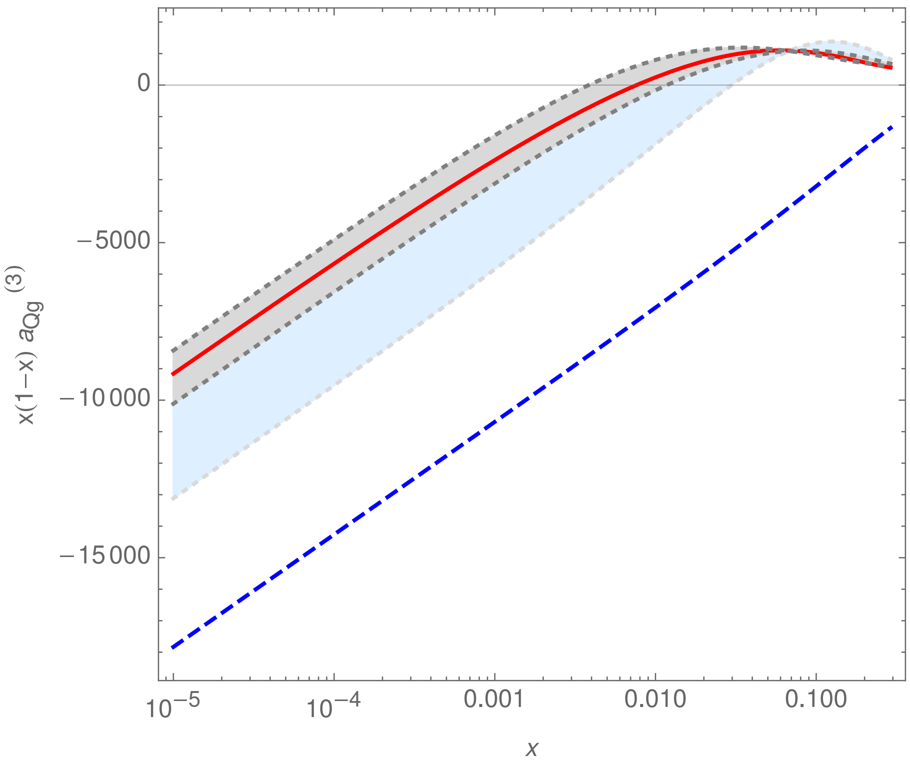
# *Parton kinematics at FCC-hh*




- FCC-hh run at $\sqrt{s}$ = 100 TeV
  - parton kinematics way beyond LHC
    *Watch the scales on axes of plot!*
- Significant sensitivity ot parton dynymics at small-$x$
  - low scales $Q$ and forward rapidities probe $x \simeq 10^{-7}$
  - high scales $Q \simeq 10 \ldots 50$ TeV need PDFs at large $x \simeq 0.1 \ldots 0.8$

# *Small-x behavior*




- Left: 4-loop splitting function $P^{(3),+}_{\mathrm{ns,L}}$ S.M., Vogt, Ruijl, Ueda, Vermaseren '17
- Right: 3-loop heavy-quark operator matrix element $A_{Qg}$ Blümlein et al. '24
- BFKL approach does not give quantitative description
  - sub-leading terms are large
  - need for complete fixed order QCD calculations

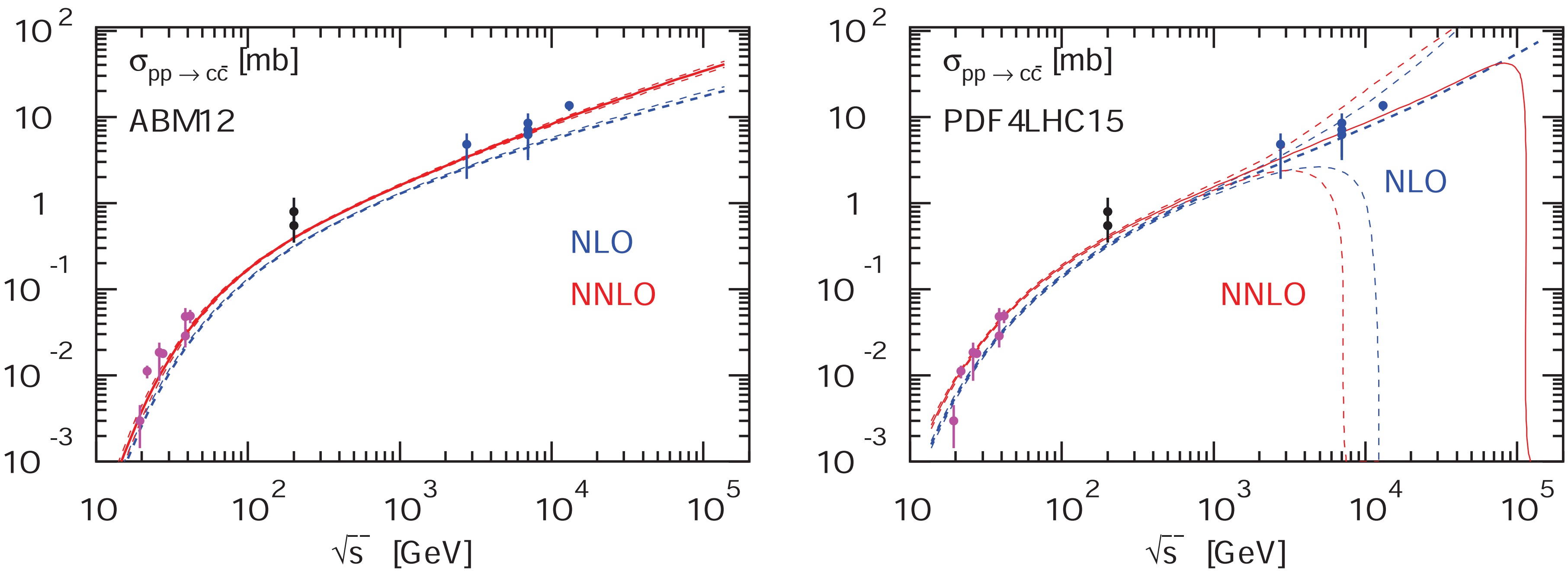
# *Higgs cross section at FCC-hh*

| | N3LO (δ in %) | | | | N4LOsv (δ in %) | | | |
|---|---|---|---|---|---|---|---|---|
| PDF Name | Central | δ(N3LO) | δ(Scale) | δ(PDF) | Central | δ(N4LO) | δ(Scale) | δ(PDF) |
| ABMP16_5_nnlo [2] | 774.0 | 3.6 | $^{+0.9}_{-3.9}$ | $^{+1.8}_{-1.8}$ | 773.4 | −0.1 | $^{+0.0}_{-2.4}$ | $^{+1.8}_{-1.8}$ |
| ABMPtt_5_nnlo [3] | 775.5 | 3.6 | $^{+0.9}_{-4.0}$ | $^{+1.7}_{-1.7}$ | 774.9 | −0.1 | $^{+0.0}_{-2.4}$ | $^{+1.7}_{-1.7}$ |
| CT18NNLO [4] | 802.1 | 3.8 | $^{+1.0}_{-4.3}$ | $^{+4.3}_{-5.0}$ | 801.5 | −0.1 | $^{+0.0}_{-2.6}$ | $^{+4.3}_{-5.0}$ |
| MSHT20nnlo_as118 [5] | 789.5 | 3.8 | $^{+1.0}_{-4.3}$ | $^{+1.4}_{-1.7}$ | 788.9 | −0.1 | $^{+0.0}_{-2.6}$ | $^{+1.4}_{-1.7}$ |
| NNPDF40_nnlo_as_01180 [6] | 805.2 | 3.8 | $^{+1.1}_{-4.3}$ | $^{+0.8}_{-0.8}$ | 804.6 | −0.1 | $^{+0.1}_{-2.6}$ | $^{+0.8}_{-0.8}$ |
| PDF4LHC21_40 [7] | 800.9 | 3.8 | $^{+1.0}_{-4.3}$ | $^{+2.2}_{-2.2}$ | 800.3 | −0.1 | $^{+0.0}_{-2.6}$ | $^{+2.2}_{-2.2}$ |
| MSHT20an3lo_as118 [8] | 794.0 | 3.8 | $^{+1.1}_{-4.3}$ | $^{+3.7}_{-2.7}$ | 793.4 | −0.1 | $^{+0.1}_{-2.6}$ | $^{+3.7}_{-2.7}$ |
| NNPDF40_an3lo_as_01180 [9] | 791.9 | 3.8 | $^{+1.1}_{-4.3}$ | $^{+0.8}_{-0.8}$ | 791.3 | −0.1 | $^{+0.0}_{-2.6}$ | $^{+0.8}_{-0.8}$ |

- FCC-hh run at $\sqrt{s}$ = 100 TeV
- Spread for predictions from different PDFs $\sigma(H) = 774.0 \ldots 805.2$ pb
- PDF and $\alpha_s$ differences between sets amount to up to $4\%$
  - parton dynamics better constrained in this kinematic range

# *Charm-quark hadro-production*

- Charm-quark hadro-production at high energies
  - quark-gluon parton luminositiy dominates




- Left: gluon PDF from ABM12 fit yields $xg(x) \simeq x^a$ with $a \simeq -0.2$
  - predictions compatible with LHC measurements (Alice, ATLAS, LHCb)
- Right: gluon PDF from PDF4LHC15 negative at small-$x$ and low scales
  - negative cross section $\longrightarrow$ invalidates predictive potential

# Summary

- Exploration of new kinematic regimes will probe QCD theory
  - small $x$ (low scales $Q^2$)
  - large $x$ (TeV range high scales $Q^2$)
- Improved measurements of QCD parameters
  - $\alpha_s$
  - $m_q$ (charm, bottom, top)
- Hadron structure
  - proton PDFs
  - nuclear PDFs
  - hadron tomography (spin dependence, $k_T$-dependence, etc)