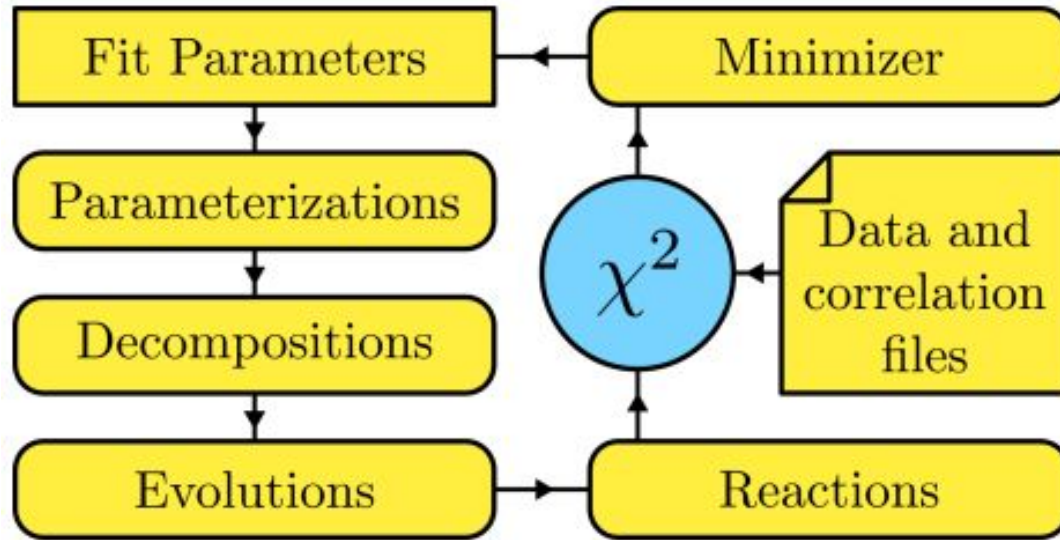


xFitter N^3LO fits

S. Glazov, DIS 2025,
March 2025

What is xFitter



Quantitative comparison of the data and QCD predictions aimed to determine theory parameters involves a number of steps. **xFitter** binds them together.

Status of xFitter project

Example analyses for the last year

Analysis of DIS data at N3LO: settings

Model variations:

q_0^2	1.9 ± 0.2
q_{\min}^2	3.5 ± 1.0
m_c	1.47 ± 0.06
m_b	4.50 ± 0.25
f_s	0.40 ± 0.08

HERAPDF2.0 parameterisation:

$$xg(x, \mu_0^2) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g}$$

$$xu_v(x, \mu_0^2) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} \left[1 + E_{u_v} x^2 \right]$$

$$xd_v(x, \mu_0^2) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}$$

$$x\bar{u}(x, \mu_0^2) = A_{\bar{u}} x^{B_{\bar{u}}} (1-x)^{C_{\bar{u}}} \left[1 + D_{\bar{u}} x \right]$$

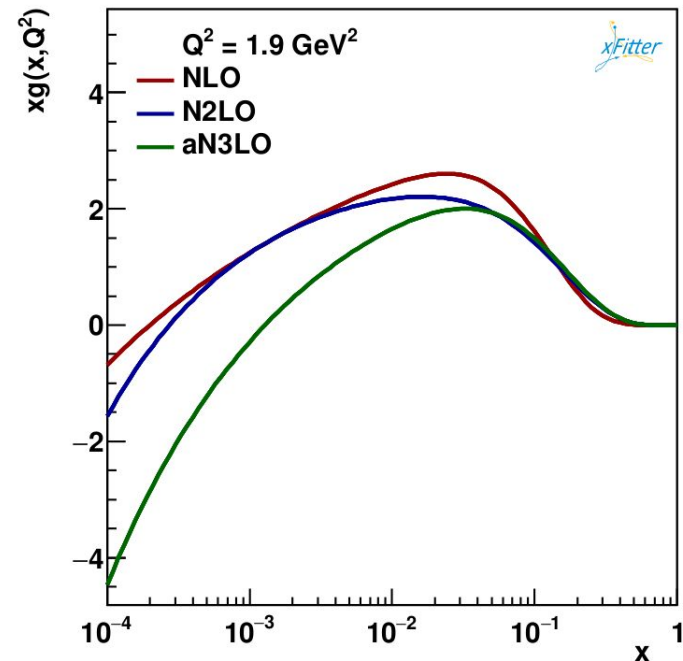
$$x\bar{d}(x, \mu_0^2) = A_{\bar{d}} x^{B_{\bar{d}}} (1-x)^{C_{\bar{d}}} \quad xs(x, \mu_0^2) = x\bar{s}(x, \mu_0^2) = r_s x\bar{d}(x, \mu_0^2) \quad r_s = \frac{f_s}{1-f_s}$$

Alternative gluon parameterisation:

$$xf(x, \mu_0^2) = A x^B (1-x)^C \left[1 + Dx + Ex^2 \right] \left[1 + F \log x + G \log^2 x + H \log^3 x \right]$$

- HERAI+II inclusive (arXiv:1506.06042) and charm+bottom data (arXiv:1804.01019)
- N³LO evolution as implemented in APFEL based on code in HOPPET (refs?)
- N³LO coefficient functions from APFEL checked with HOPPET, **exact** and approximate matching conditions
- FONLL VFNS implemented in xFitter, checked with fortran APFEL for N²LO

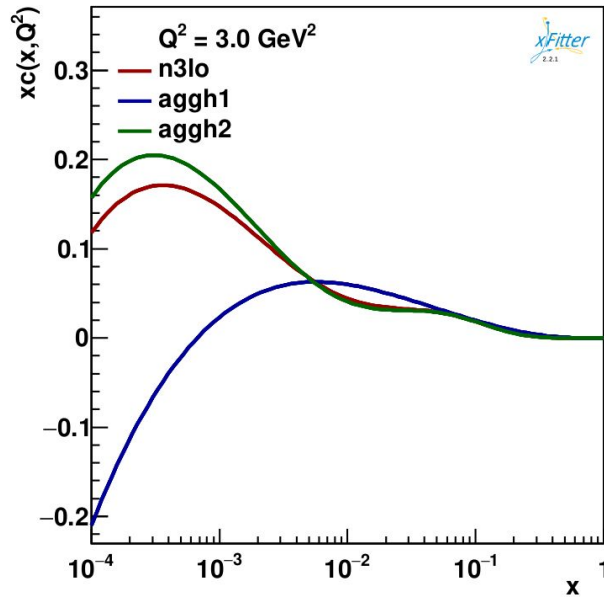
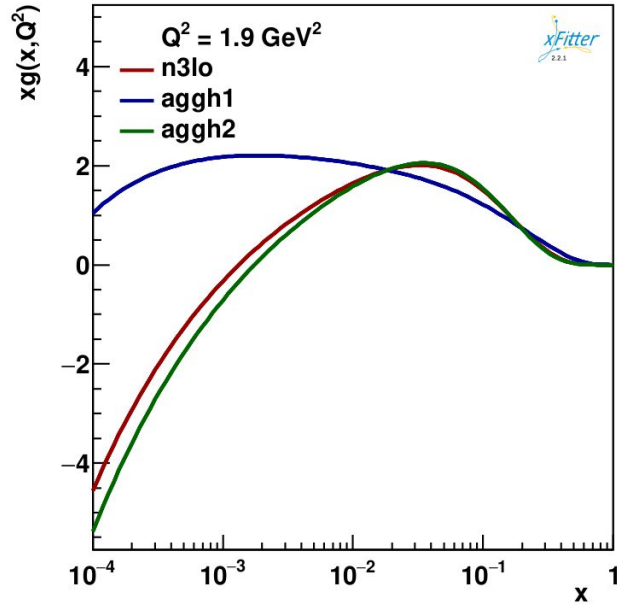
N³LO with approx. matching vs N²LO and N¹LO



Dataset	NLO	N2LO	aN3LO
HERA1+2 NCep 820	68 / 70	67 / 70	68 / 70
HERA1+2 NCep 460	217 / 204	218 / 204	215 / 204
HERA1+2 CCep	43 / 39	44 / 39	48 / 39
HERA1+2 NCem	218 / 159	216 / 159	223 / 159
HERA1+2 CCem	53 / 42	56 / 42	61 / 42
HERA1+2 NCep 575	216 / 254	220 / 254	215 / 254
HERA1+2 NCep 920	424 / 377	452 / 377	476 / 377
Correlated χ^2	77	98	103
Log penalty χ^2	+7.7	+14	+14
Total χ^2 / dof	1325 / 1131	1385 / 1131	1422 / 1131

- Implement approximate N³LO based on <https://arxiv.org/pdf/2207.04739>
- Similar pattern as observed in the past: N³LO gluon is more negative at low x for the starting scale, χ^2 /dof is the worst.

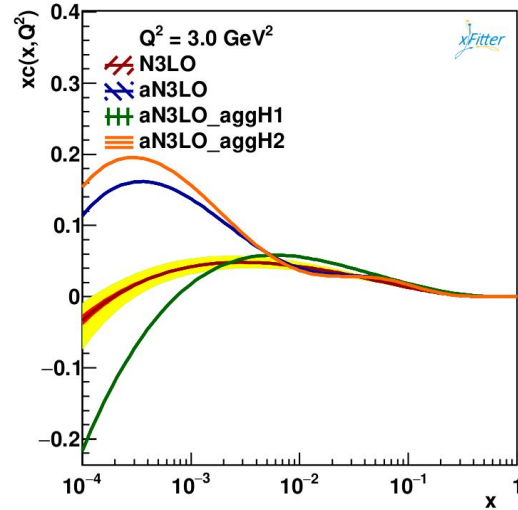
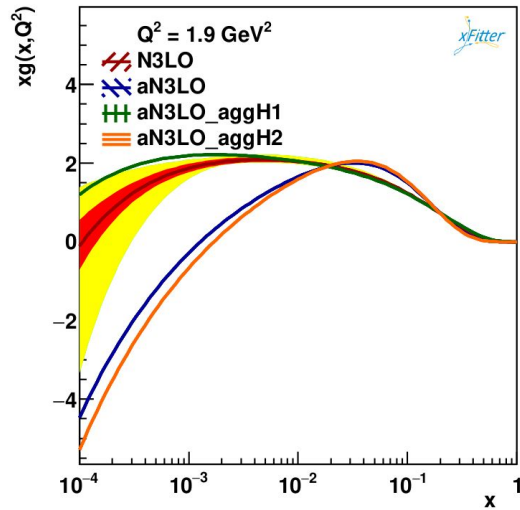
N³LO with approximate matching uncertainties



Approximate matching conditions have uncertainties for transition matrix elements. Large impact on gluon/charm due to $-2000 < a_{gg,H} < -700$

$$A_{gg,H}^{(3)} = A_1 \ln^2(1-x) + A_2 \ln(1-x) + A_3 x^2 + A_4 \ln x + A_5 x + a_{gg,H} \frac{\ln x}{x}$$

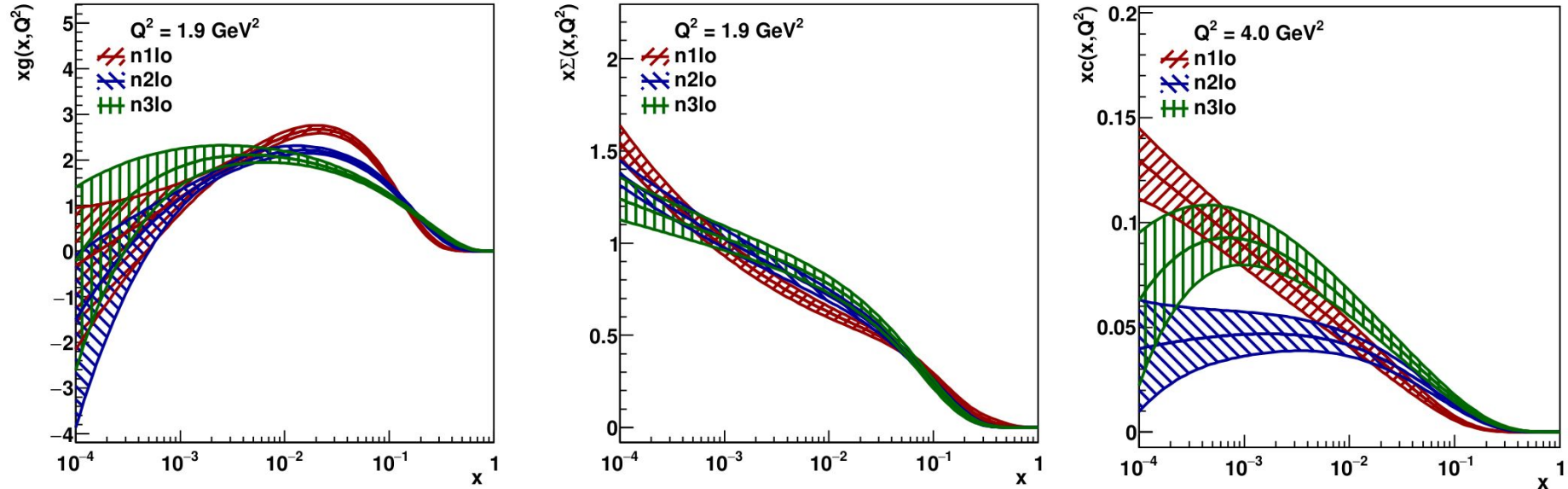
Exact N³LO vs approximate N³LO



Dataset	N3LO	aN3LO
HERA1+2 NCep 820	66 / 70	68 / 70
HERA1+2 NCep 460	222 / 204	215 / 204
HERA1+2 CCep	48 / 39	48 / 39
HERA1+2 NCem	224 / 159	223 / 159
HERA1+2 CCem	59 / 42	61 / 42
HERA1+2 NCep 575	215 / 254	215 / 254
HERA1+2 NCep 920	471 / 377	476 / 377
Correlated χ^2	106	103
Log penalty χ^2	+9.9	+14
Total χ^2 / dof	1422 / 1131	1422 / 1131

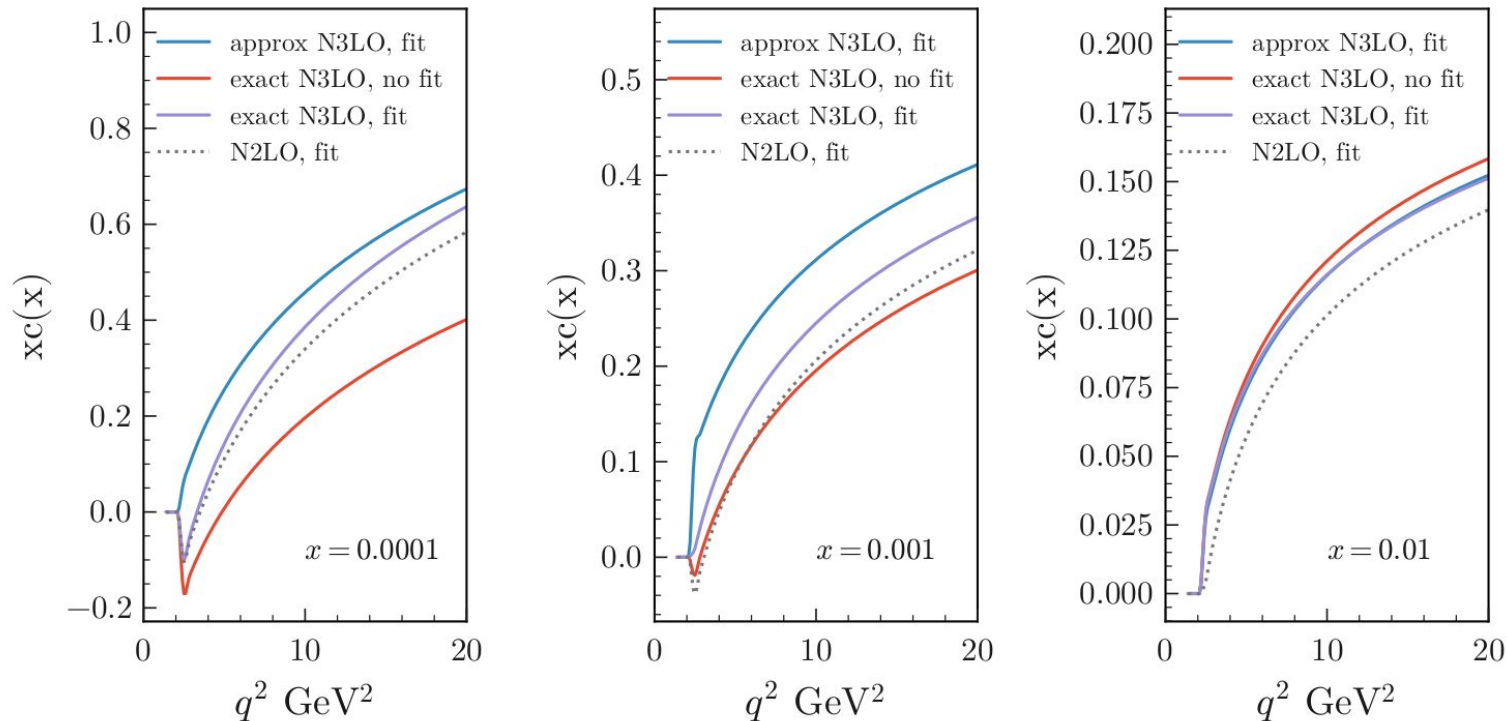
First fit using exact N3LO matching conditions, shown with full experimental plus model uncertainties. Large difference vs aN3LO central fit for the gluon and charm, however within uncertainties of aN3LO. Fit quality to the data is about the same

$N^1\text{LO}$, $N^2\text{LO}$ vs $N^3\text{LO}$ with exact matching



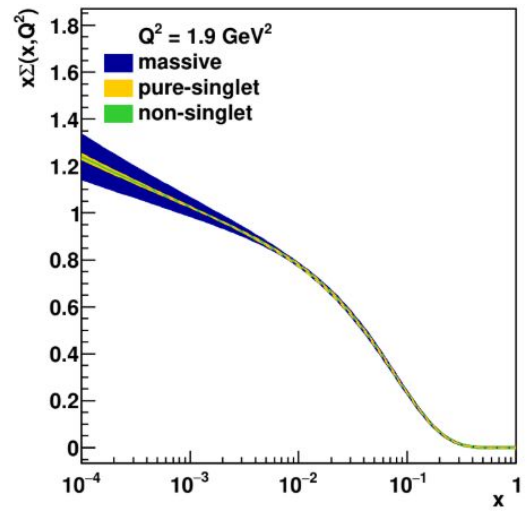
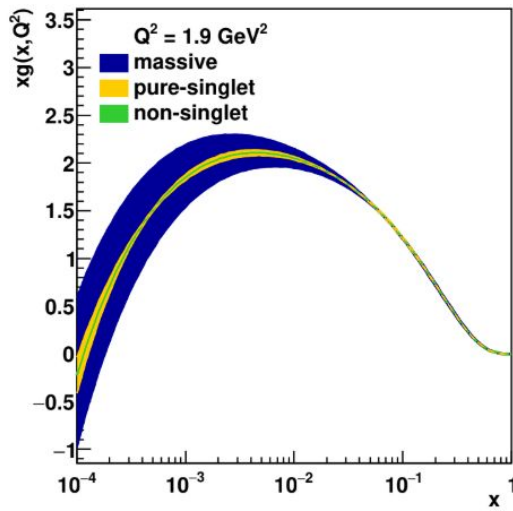
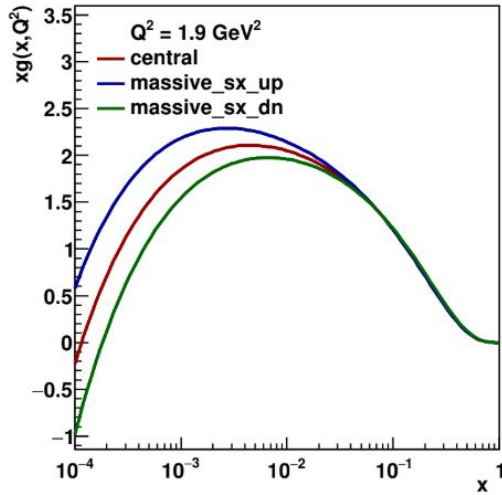
Fits with alternative gluon parameterisation, consistent results to HERAPDF style.
 $N^3\text{LO}$ gluon is the hardest, also largest at small x . For $x=0.01$, all orders agree.

Effect of the matching conditions on charm



Compare $xc(x)$ vs q^2 between approximate and exact N³LO (for the same PDFs), exact N³LO fit and N²LO fit. Significant differences at low x

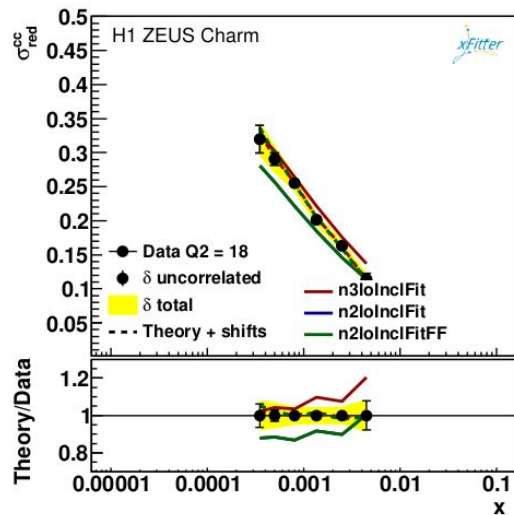
Heavy flavour uncertainties



Additional uncertainties arise from approximate treatment of massive coefficient functions. Implemented based on <https://arxiv.org/pdf/2401.12139>

→significant uncertainty due to small- x parameter variation, dominant compared to other sources. MAYBE DOUBLE COUNTING (discussion with Marco)

F₂ charm predictions (N2LO and N3LO)

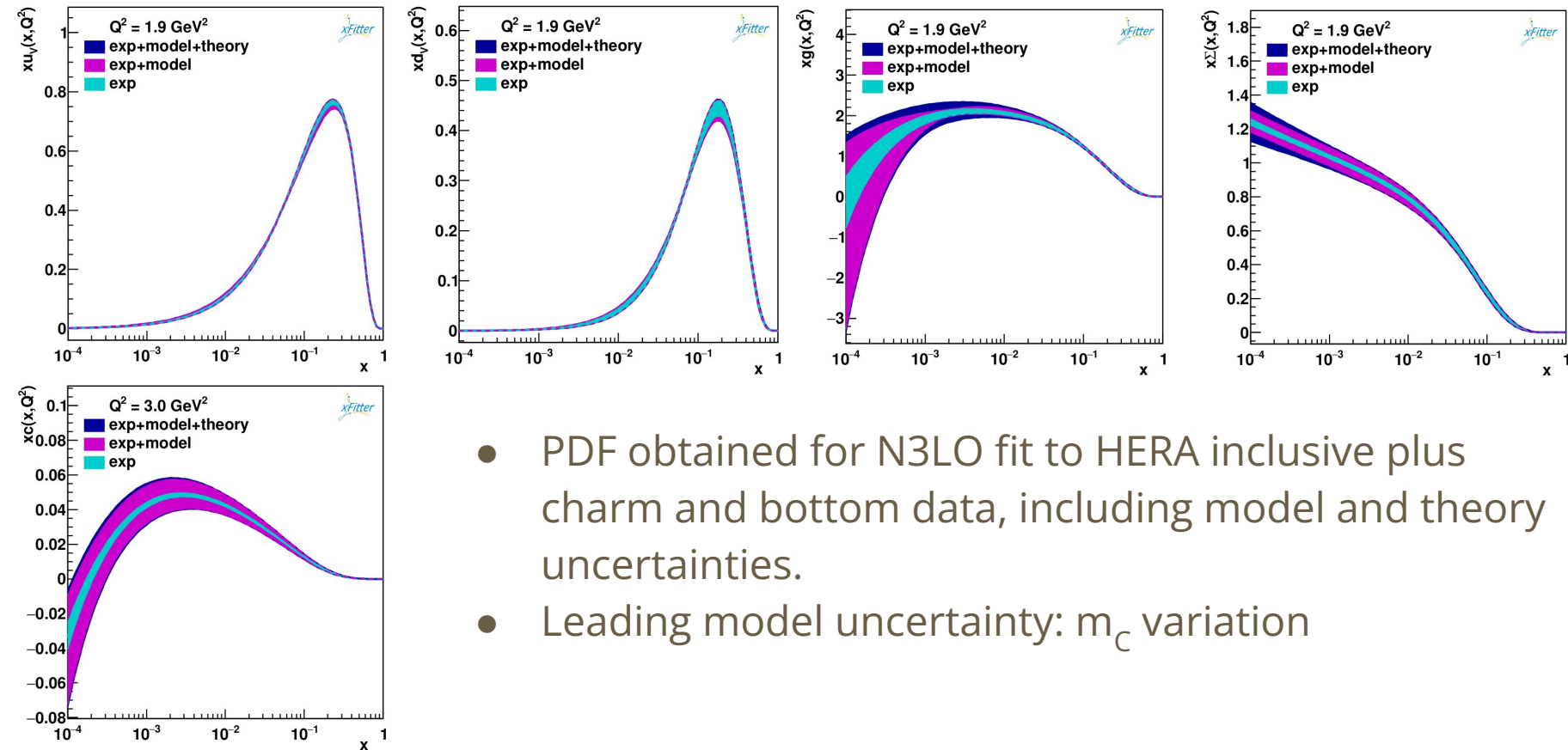


Dataset	n3loInclFit	n2loInclFit	n2loInclFitFF
HERA c	41 / 47	50 / 47	50 / 47
HERA b	15 / 26	20 / 26	20 / 26
Correlated χ^2	43	61	61
Log penalty χ^2	+8.6	-5.35	-5.34
Total χ^2 / dof	108 / 59	125 / 59	125 / 59
χ^2 p-value	0.00	0.00	0.00

- Compare predictions of the N2LO and N3LO fit to the inclusive data vs charm from combined H1ZEUS analysis <https://arxiv.org/abs/1804.01019>
- Better χ^2 /dof for the N³LO fit

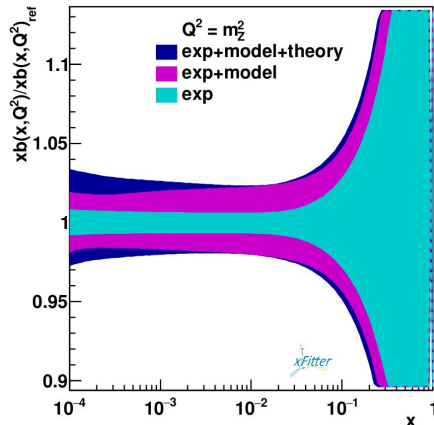
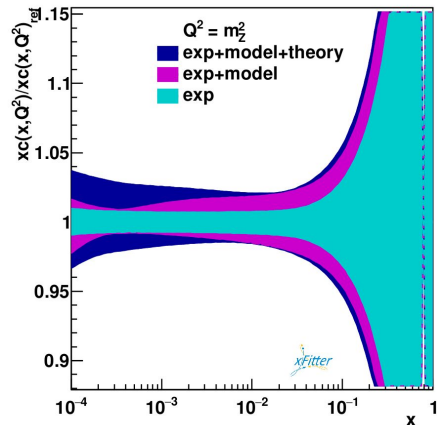
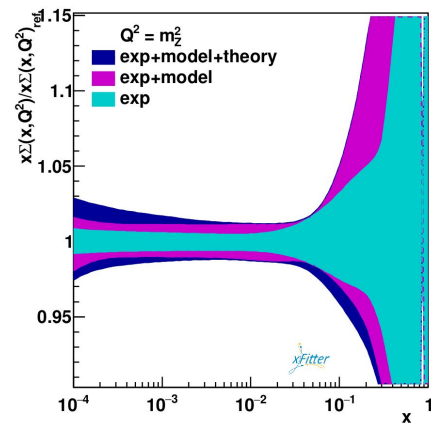
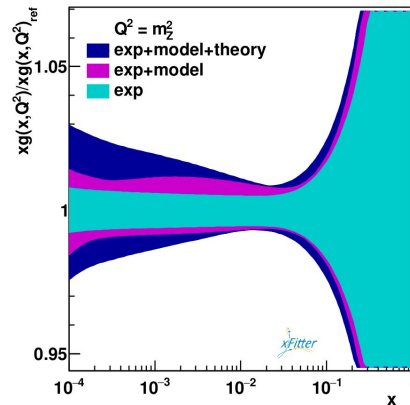
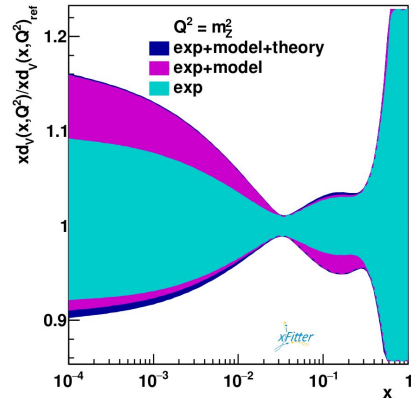
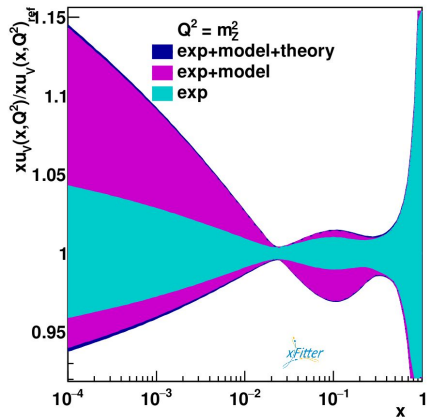
Fitting charm data (N^2LO and N^3LO)

N³LO pdfs with full uncertainties



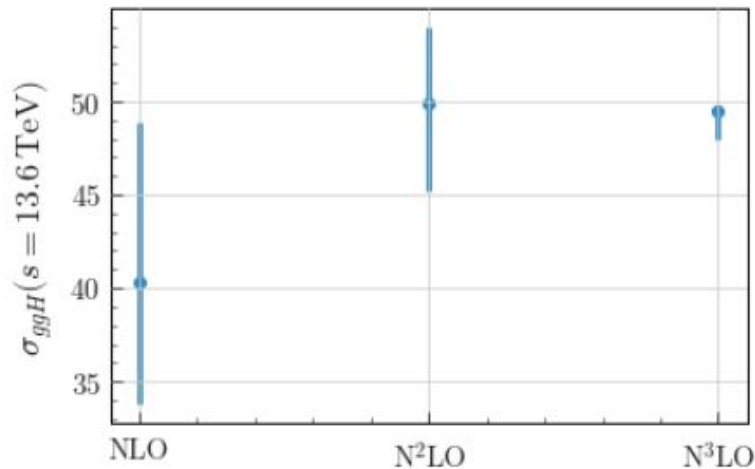
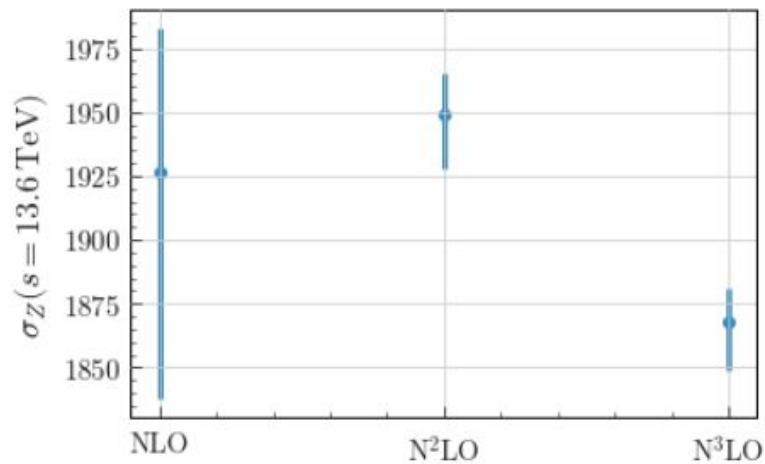
- PDF obtained for N3LO fit to HERA inclusive plus charm and bottom data, including model and theory uncertainties.
- Leading model uncertainty: m_c variation

$N^3\text{LO}$ uncertainties at $Q^2 = M_Z^2$



At higher scales, large uncertainties for valence quarks at low x . Significant theory uncertainty for gluon at $x < 0.01$. Charm and bottom uncertainties from m_c and m_b values

Impact on predictions at the LHC



Predictions at N3LO for ggH, Z. Only scale uncertainties.

Summary and Outlook

- Analysis with consistent N³LO for the evolution and coefficient functions
- First analysis including exact matching conditions
- Residual uncertainties from heavy flavour coefficient functions
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