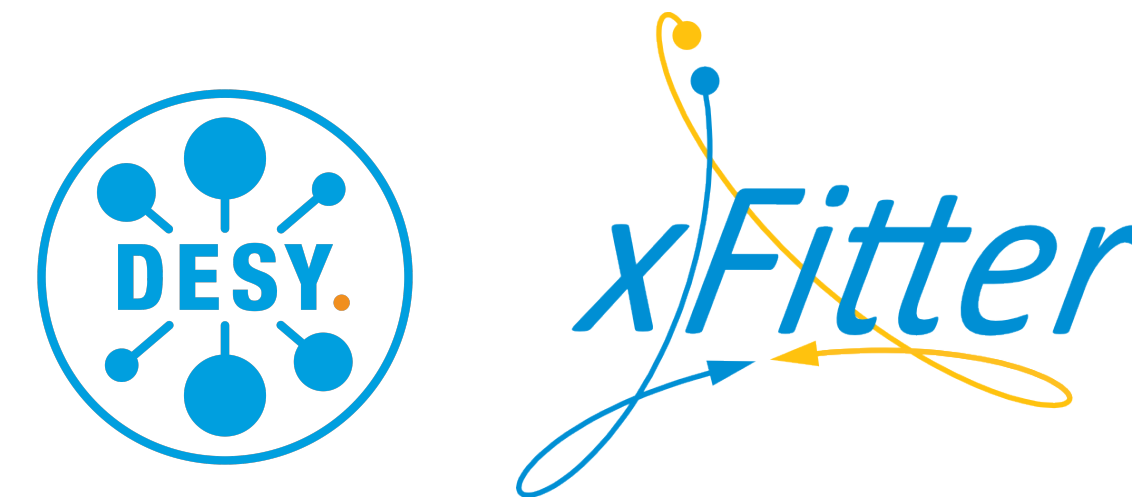


Studies of N3LO Fits to DIS Data Using xFitter

EPS-HEP 2025

Marseille, July 7, 2025

F. Dattola, on behalf of the xFitter Developers' team



N3LO PDF fits with xFitter

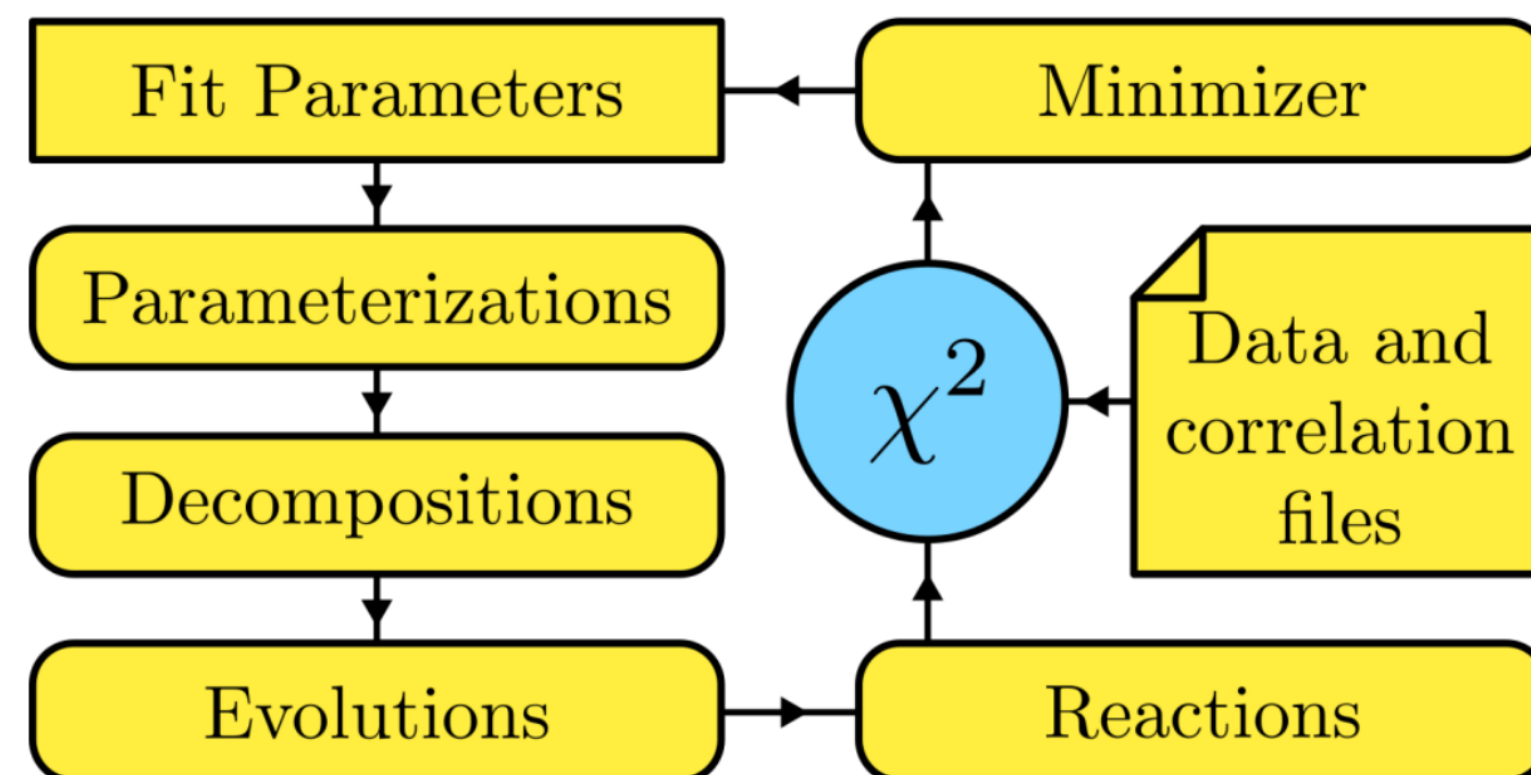
Why and how?

- **Why?** PDF uncertainties are becoming a **bottleneck** for LHC precision measurements

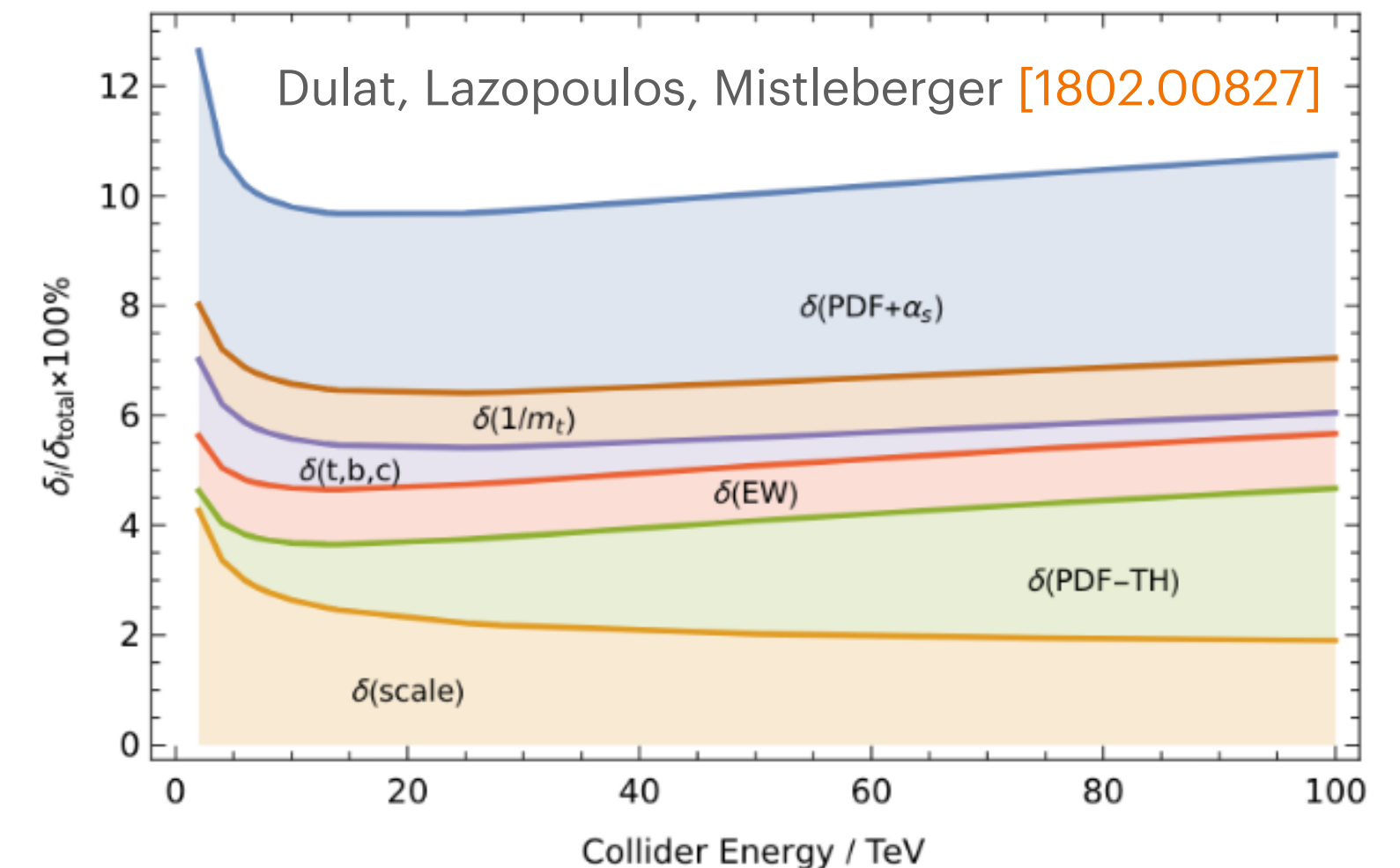
- N3LO cross sections available for Higgs (ggF, VBF, VH) and DY(NC, CC)
→ call for N3LO PDFs to reduce mismatch and PDF uncertainties

- PDF fits at N3LO: **how?**

- **xFitter! Open-source QCD fit framework**



Inclusive Higgs production



- **Project in active development**

- Stable release 2.2.0 ("future freeze")
- Nightly "master" branch validated vs several public results
- Interfaces to ploughshare, hoppet, PineAPPL, CIJET
- N3LO FONLL for APFEL++, DYTurbo interface
- Parallel derivatives in CERES, parallel profiling, parallel asymmetric Hessian uncertainties evaluation



N3LO PDF fits with xFitter

Ingredients for DIS PDF fits at N3LO



- **QCD inputs needed** for N3LO DIS fits can be grouped in 3 categories

- **N3LO (4-loop) DGLAP splitting functions** for PDF evolution

$$Q^2 \frac{df_i}{dQ^2} = P_{ij} \otimes f_j \rightarrow P_{ij}(x, \alpha_s) = \alpha_s P_{ij}^{(0)}(x) + \alpha_s^2 P_{ij}^{(1)}(x) + \alpha_s^3 P_{ij}^{(2)}(x) + \alpha_s^4 P_{ij}^{(3)}(x) + \dots$$

- **N3LO matching conditions** to change number of PDF flavours at heavy quark mass (m_h) thresholds

$$f_i^{n_f+1}(x, Q^2) = [A_{ij}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2)](x)$$

- **N3LO coefficient functions** to determine DIS structure functions

$$F_2(x, Q^2) = \sum_{\alpha, \beta} (C_{\beta, \alpha}^{VF, n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2))$$

Analysis of DIS data at N3LO

Overview

- HERAPDF2.0 parametrization [1506.06042]

$$xu_v(x, \mu_0^2) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} [1 + E_{u_v} x^2]$$

$$xd_v(x, \mu_0^2) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}$$

$$x\bar{u}(x, \mu_0^2) = A_{\bar{u}} x^{B_{\bar{u}}} (1-x)^{C_{\bar{u}}} [1 + D_{\bar{u}} x]$$

$$x\bar{d}(x, \mu_0^2) = A_{\bar{d}} x^{B_{\bar{d}}} (1-x)^{C_{\bar{d}}}, \quad xs(x, \mu_0^2) = x\bar{s}(x, \mu_0^2) = r_s x\bar{d}(x, \mu_0^2) \quad \text{with} \quad r_s = \frac{f_s}{1-f_s} \quad (f_s = 0.4)$$

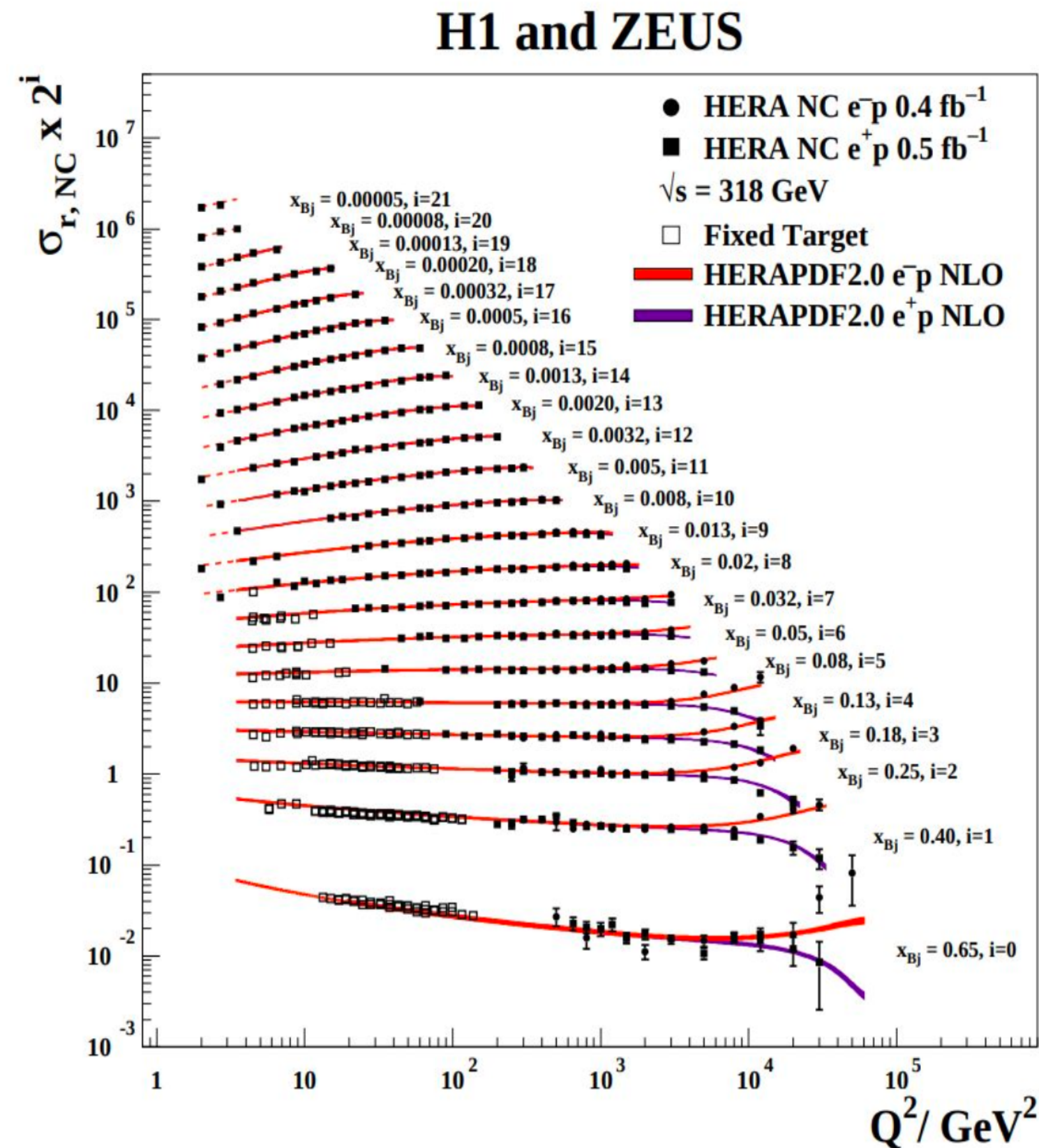
- Alternative gluon parametrisation [1902.11125] $\rightarrow$ $xg(x, \mu_0^2) = A_g x^{B_g} (1-x)^{C_g} [1 + F \log x]$
- HERAI+II** inclusive [1506.06042] and charm+bottom data [1804.01019]
- N3LO evolution** as implemented in APFEL [1708.00911] — based on HOPPET [0804.3755]
- Exact** [DESY 24-037] and **approximate N3LO flavor matching conditions** provided to HOPPET and implemented in APFEL [2404.15711]
- FONLL VFNS implemented in xFitter, checked with fortran APFEL for N2LO

• Model variations

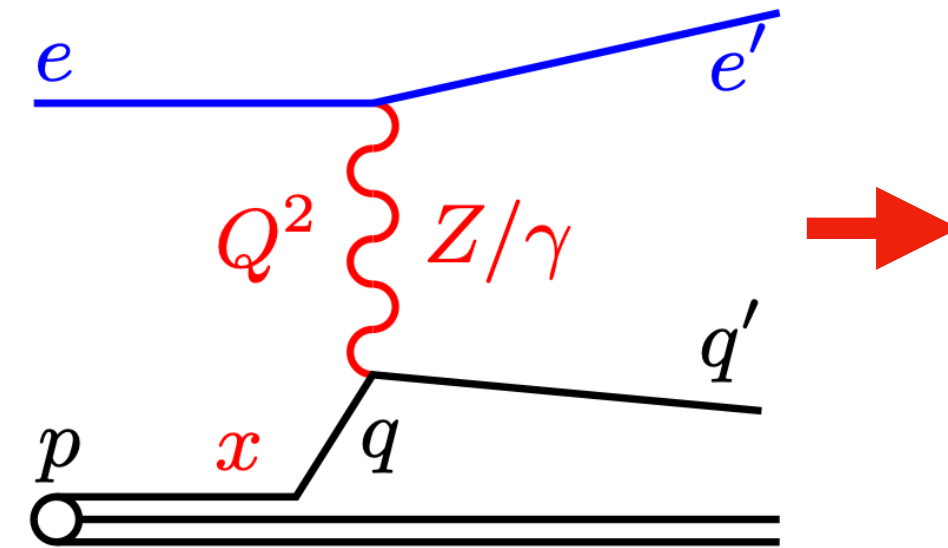
- $Q_0^2 = 1.9 \pm 0.2 \text{ GeV}^2$
- $Q_{\min}^2 = 3.5 \pm 1.0 \text{ GeV}^2$
- $m_c = 1.47 \pm 0.06 \text{ GeV}$
- $m_b = 4.50 \pm 0.25 \text{ GeV}$
- $f_s = 0.40 \pm 0.08$

HERA DIS data

PDF determination



[Eur.Phys.J.C 75 (2015) 12, 580]



$$\frac{d^2\sigma_{e^\mp p}^{NC}}{dx dQ^2} = \frac{2\pi\alpha^2 Y_+}{xQ^4} \left(F_2 - \frac{y^2}{Y_+} F_L \pm \frac{Y_-}{Y_+} x F_3 \right)$$

$$Y_\pm = 1 \pm (1-y)^2$$

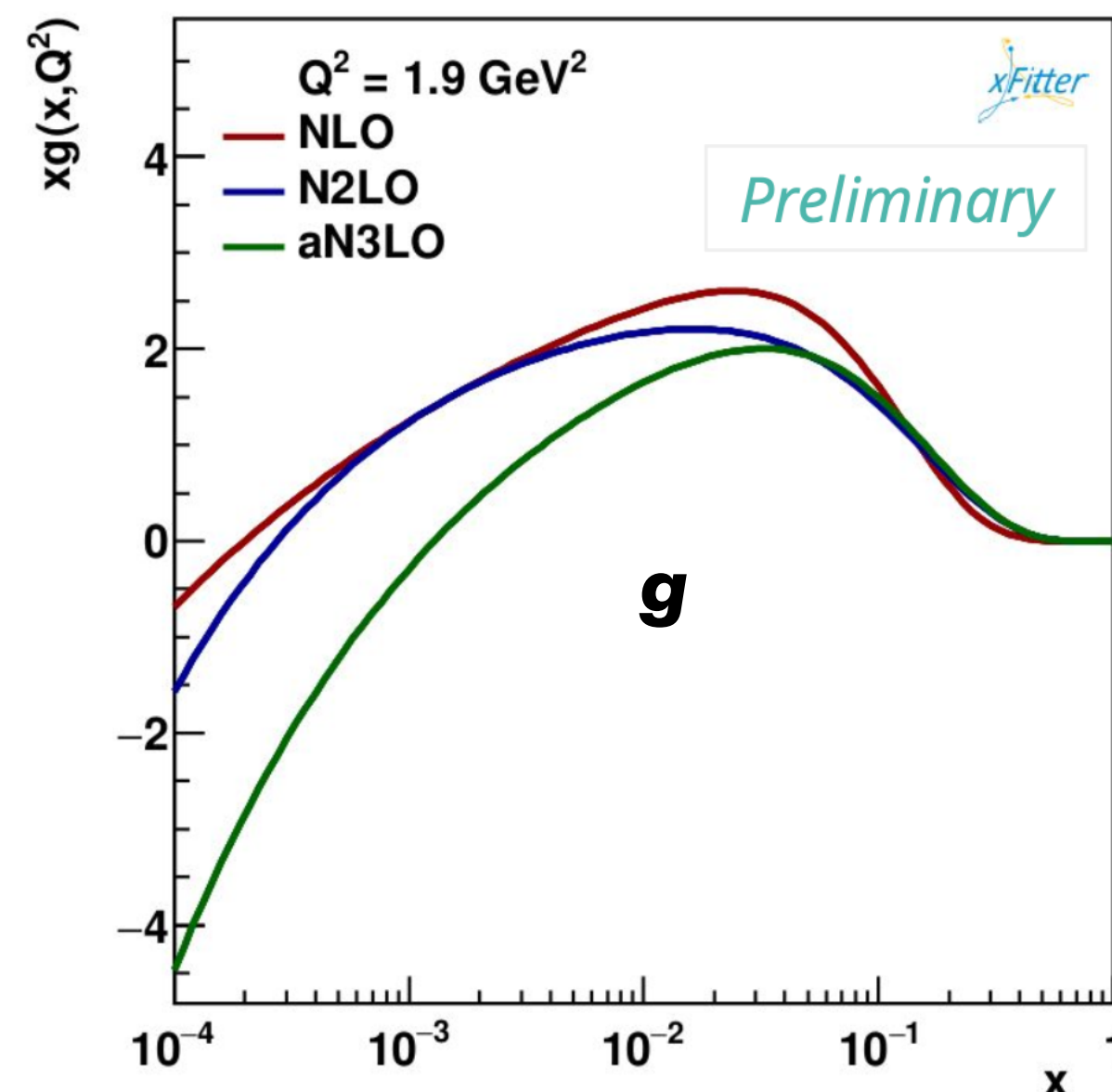
$$F_2 = x \sum e_q^2 (q(x) + \bar{q}(x)), \quad xF_3 = x \sum 2e_q a_q (q(x) - \bar{q}(x))$$

- At HERA dominant contribution photon exchange: **$u(x)$** best determined PDF
- Sizeable **$c(x)$** at low x : up to 30% of F_2
- **Gluon** measured from Q^2 dependence of the structure function F_2
- Lower accuracy at high x , **$d_v(x)$** and **$u_v(x)$** separated using charged-current data

Approximate N3LO

N3LO with approximate matching vs N2LO and NLO

- Compare fits to HERA DIS data at **NLO** and **N2LO** with **approximate N3LO (aN3LO)**



Preliminary			
Dataset	NLO	N2LO	aN3LO
HERA1+2 NCep 820	68 / 70	67 / 70	68 / 70
HERA1+2 NCep 460	217 / 204	218 / 204	215 / 204
HERA1+2 CCep	43 / 39	44 / 39	48 / 39
HERA1+2 NCem	218 / 159	216 / 159	223 / 159
HERA1+2 CCem	53 / 42	56 / 42	61 / 42
HERA1+2 NCep 575	216 / 254	220 / 254	215 / 254
HERA1+2 NCep 920	424 / 377	452 / 377	476 / 377
Correlated χ^2	77	98	103
Log penalty χ^2	+7.7	+14	+14
Total χ^2 / dof	1325 / 1131	1385 / 1131	1422 / 1131

- Approximate N3LO flavour matching conditions based on MSHT20 approximation [\[2207.04739\]](#)

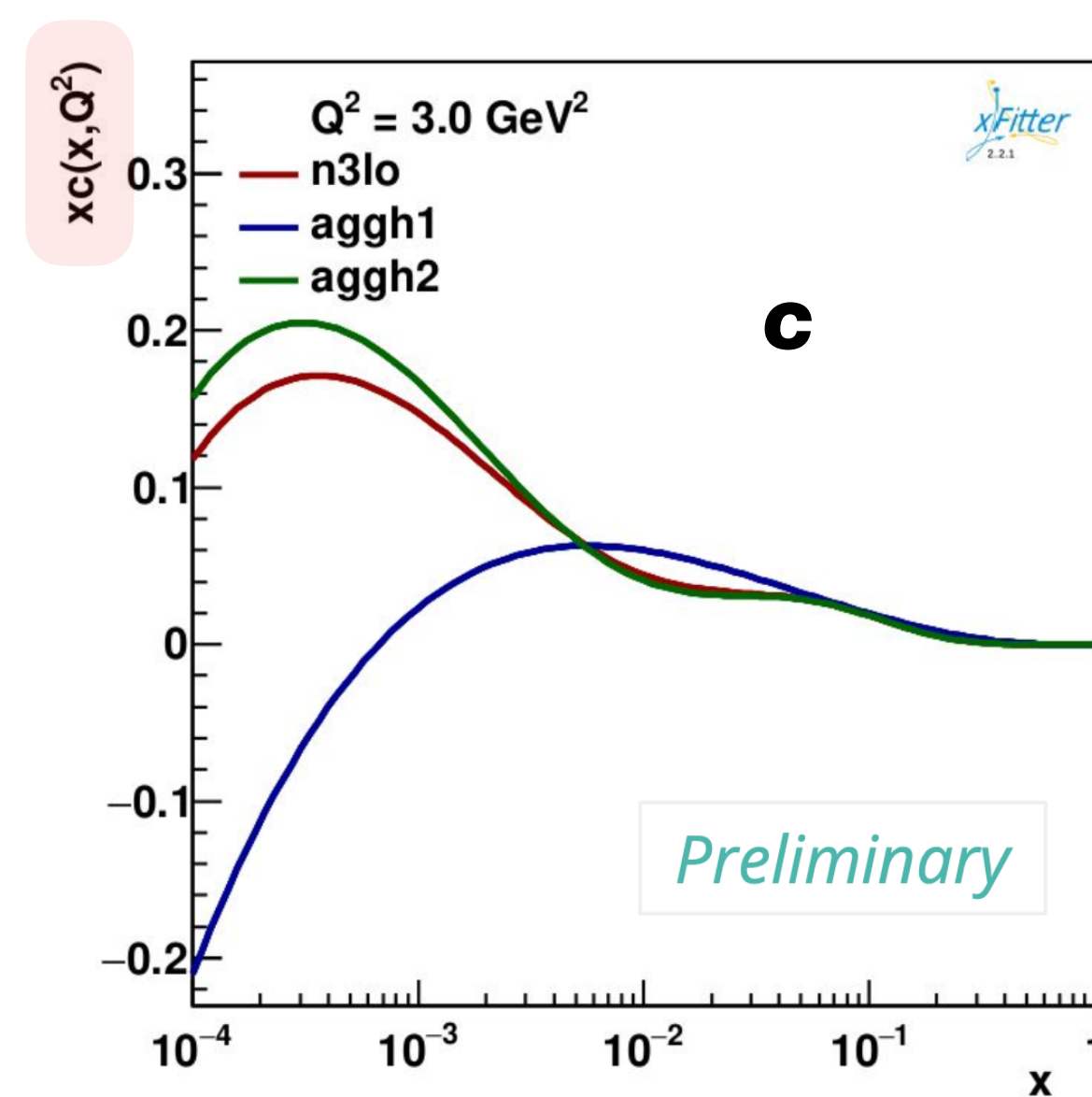
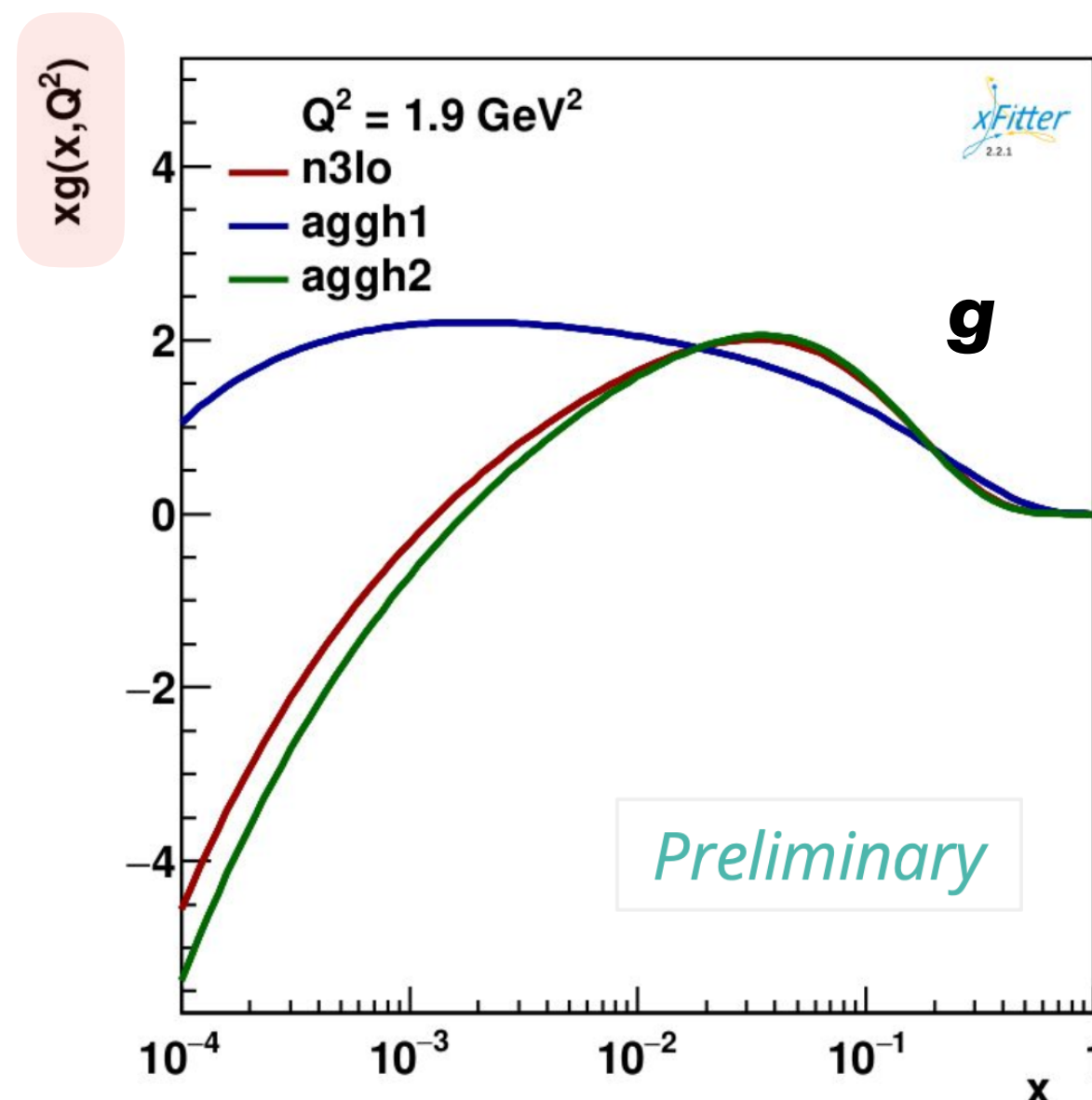
- aN3LO gluon is more negative at low x at the starting scale → **Worst χ^2 / dof**

Approximate N3LO

N3LO with approximate matching vs N2LO and NLO

- Prescription based on MSHT20 allows variations of approximated transition matrix elements accounting for missing corrections

- $A_{gg,H}^{(3)} = A_1 \ln^2(1-x) + A_2 \ln(1-x) + A_3 x^2 + A_4 \ln x + A_5 x + a_{gg,H} \frac{\ln x}{x}$
 $\leftarrow -2000 < a_{gg,H} < -700$
- $-2000 < a_{gg,H} < -700$ induces large effect on gluon/charm at low x (aggh1, aggh2)

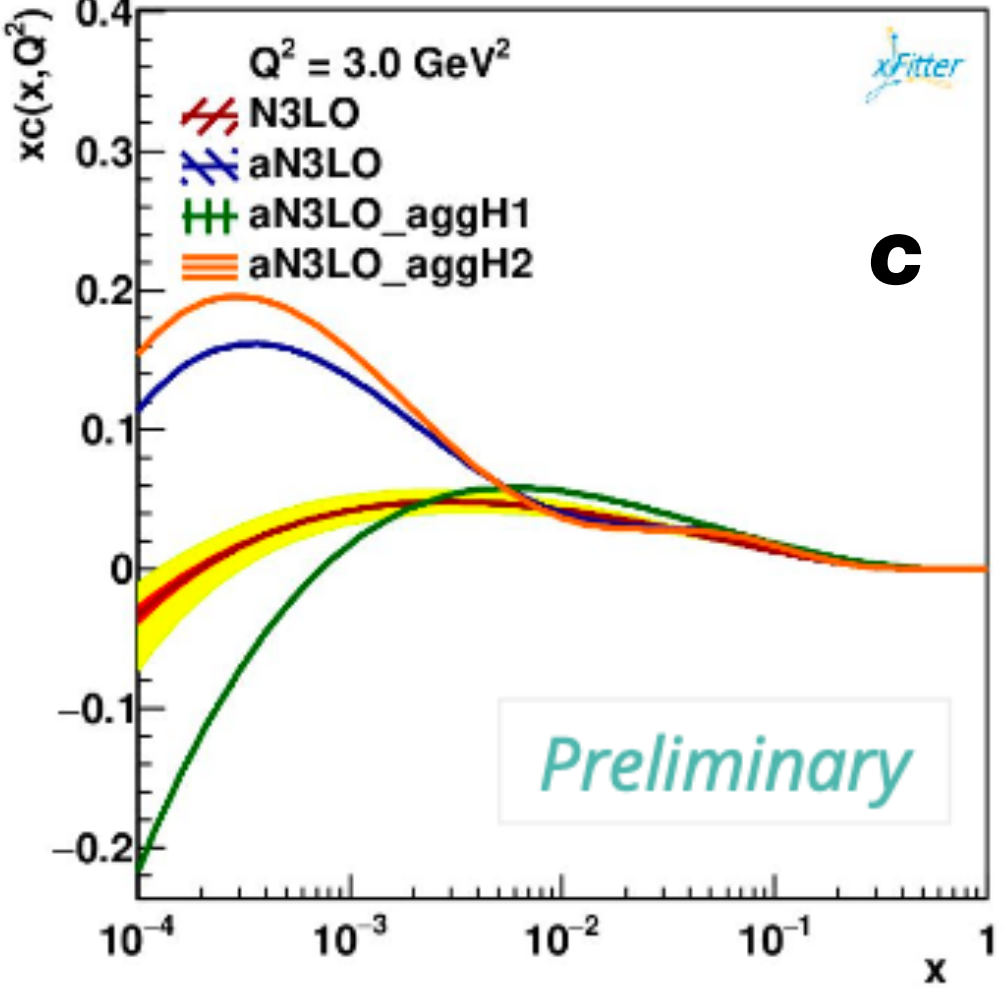
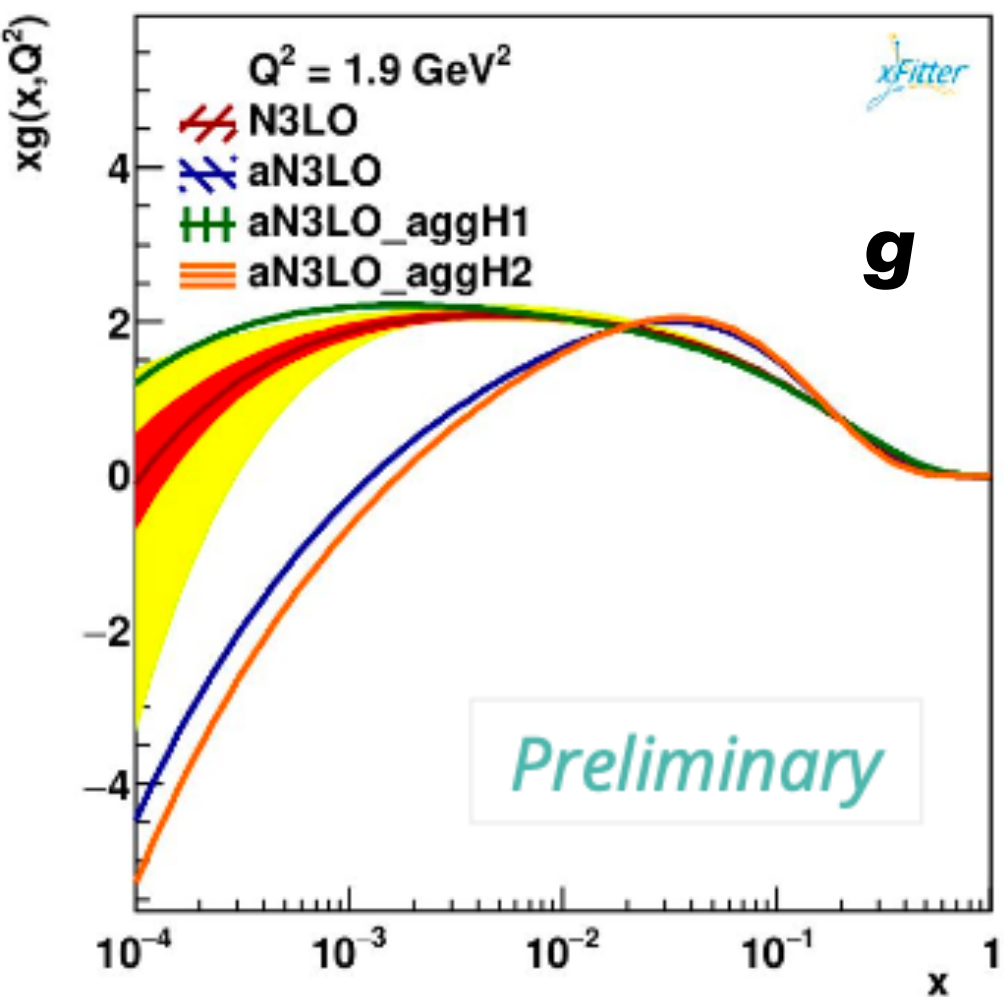


Exact N3LO

Exact N3LO vs approximate N3LO matching

- Fit to HERA DIS data repeated using **exact N3LO matching conditions** and compared to aN3LO

Dataset	Preliminary	
	N3LO	aN3LO
HERA1+2 NCep 820	66 / 70	68 / 70
HERA1+2 NCep 460	222 / 204	215 / 204
HERA1+2 CCep	48 / 39	48 / 39
HERA1+2 NCem	224 / 159	223 / 159
HERA1+2 CCem	59 / 42	61 / 42
HERA1+2 NCep 575	215 / 254	215 / 254
HERA1+2 NCep 920	471 / 377	476 / 377
Correlated χ^2	106	103
Log penalty χ^2	+9.9	+14
Total χ^2 / dof	1422 / 1131	1422 / 1131



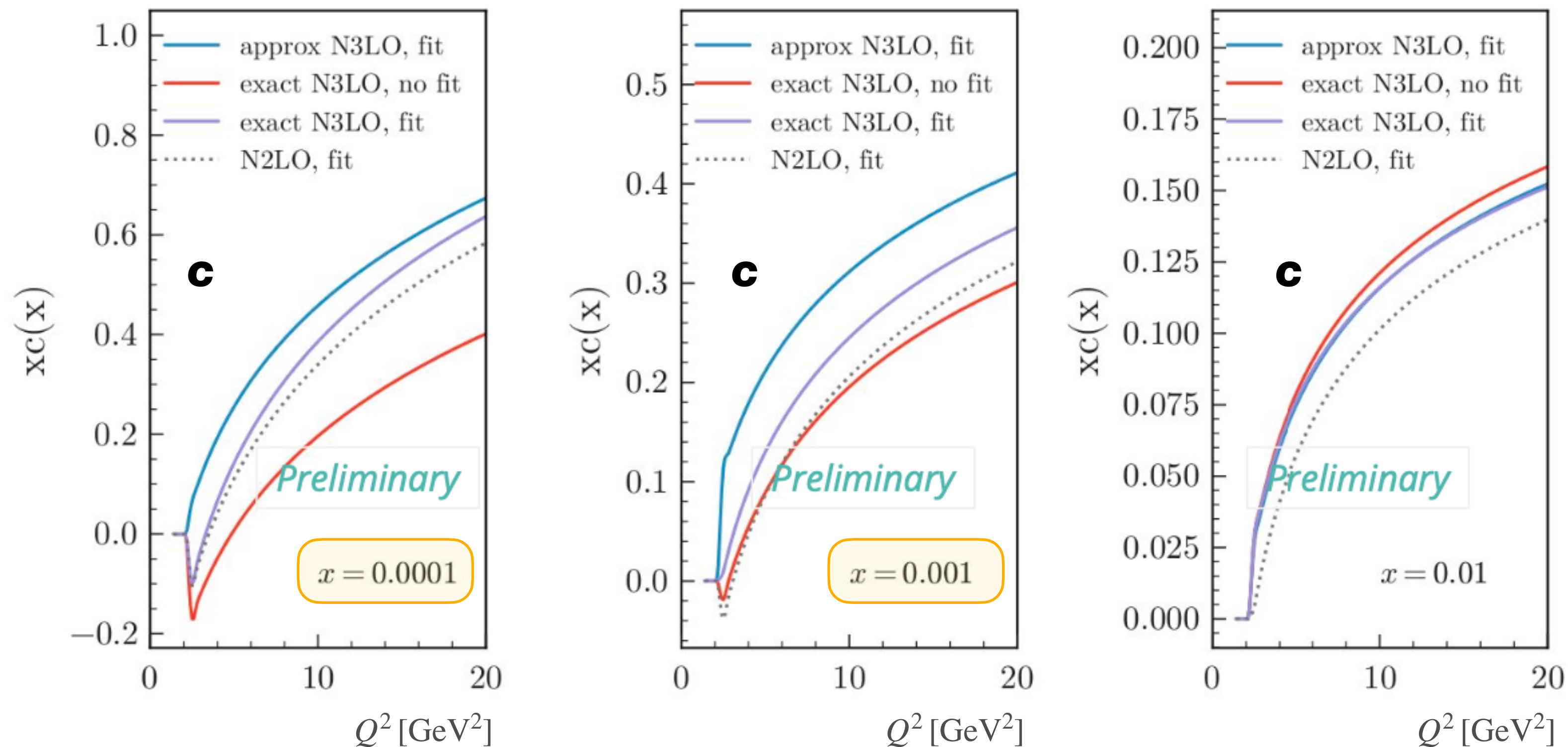
- No sizeable difference in the fit quality

- Exact N3LO fit shown with experimental + model uncertainty bands
- Large difference for exact N3LO gluon and charm at low x compared to aN3LO, but within aN3LO uncertainty (aggH1, aggH2)

Heavy flavour matching at N3LO

Effect of matching conditions on charm: N3LO vs aN3LO and N2LO

- Evaluate $\mathbf{xc(x)}$ vs Q^2 at different Bjorken- x values
- Compare **exact N3LO fit** (and N3LO before fit) with **approximate N3LO** and **N2LO** fits

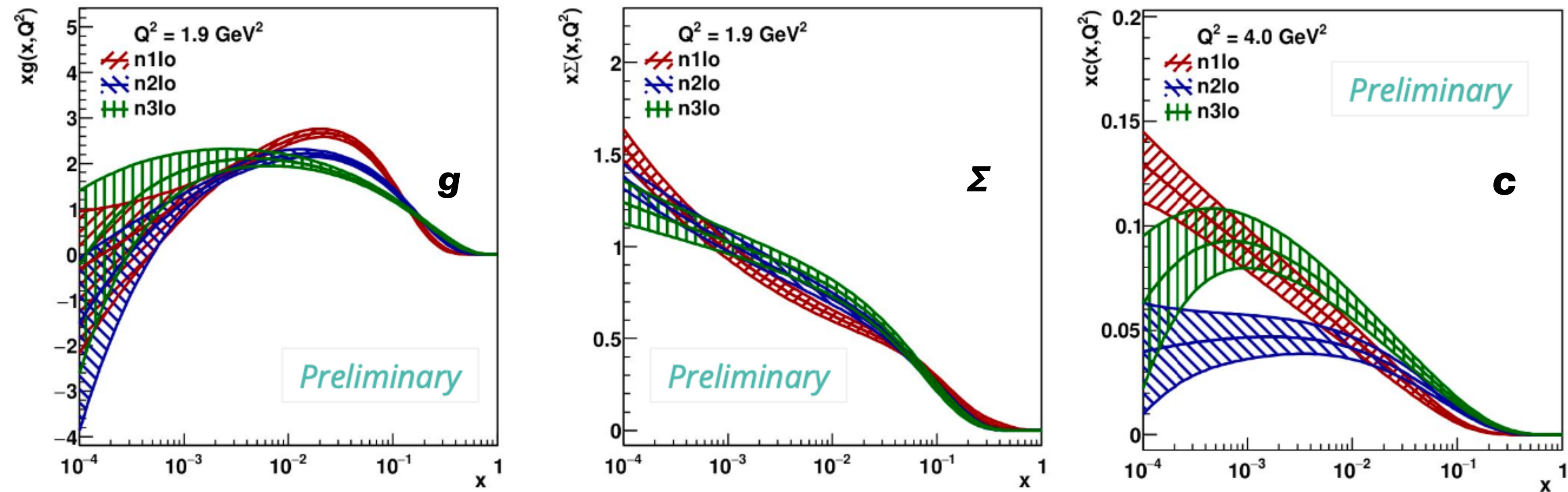


- Significant differences observed at low x

PDFs at exact N3LO

PDFs at N3LO with exact matching vs NNLO and NLO

- xg , $x\Sigma$ and xc PDFs compared for **N3LO**, **N2LO** and **NLO** perturbative accuracy of the PDF fit

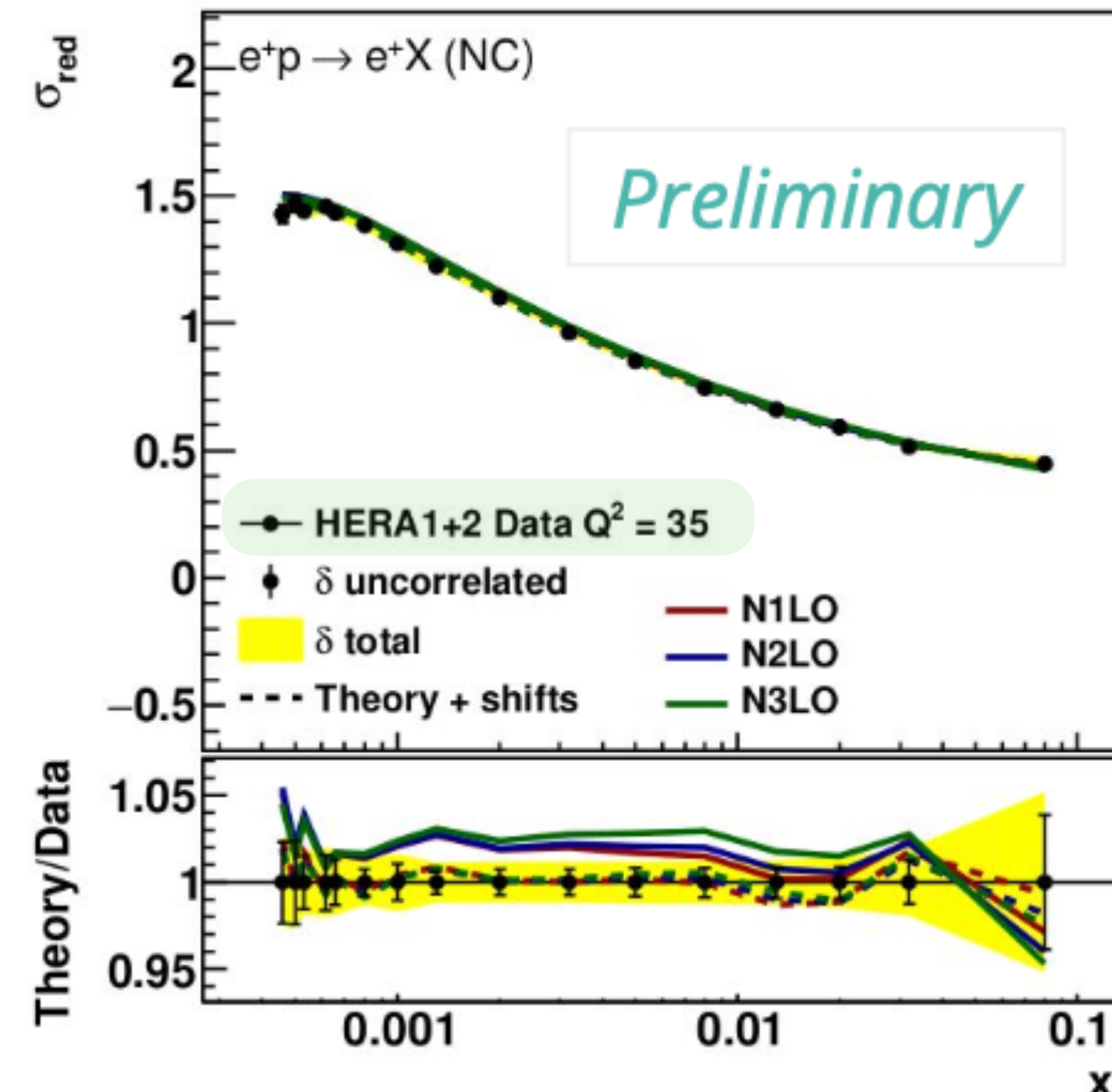
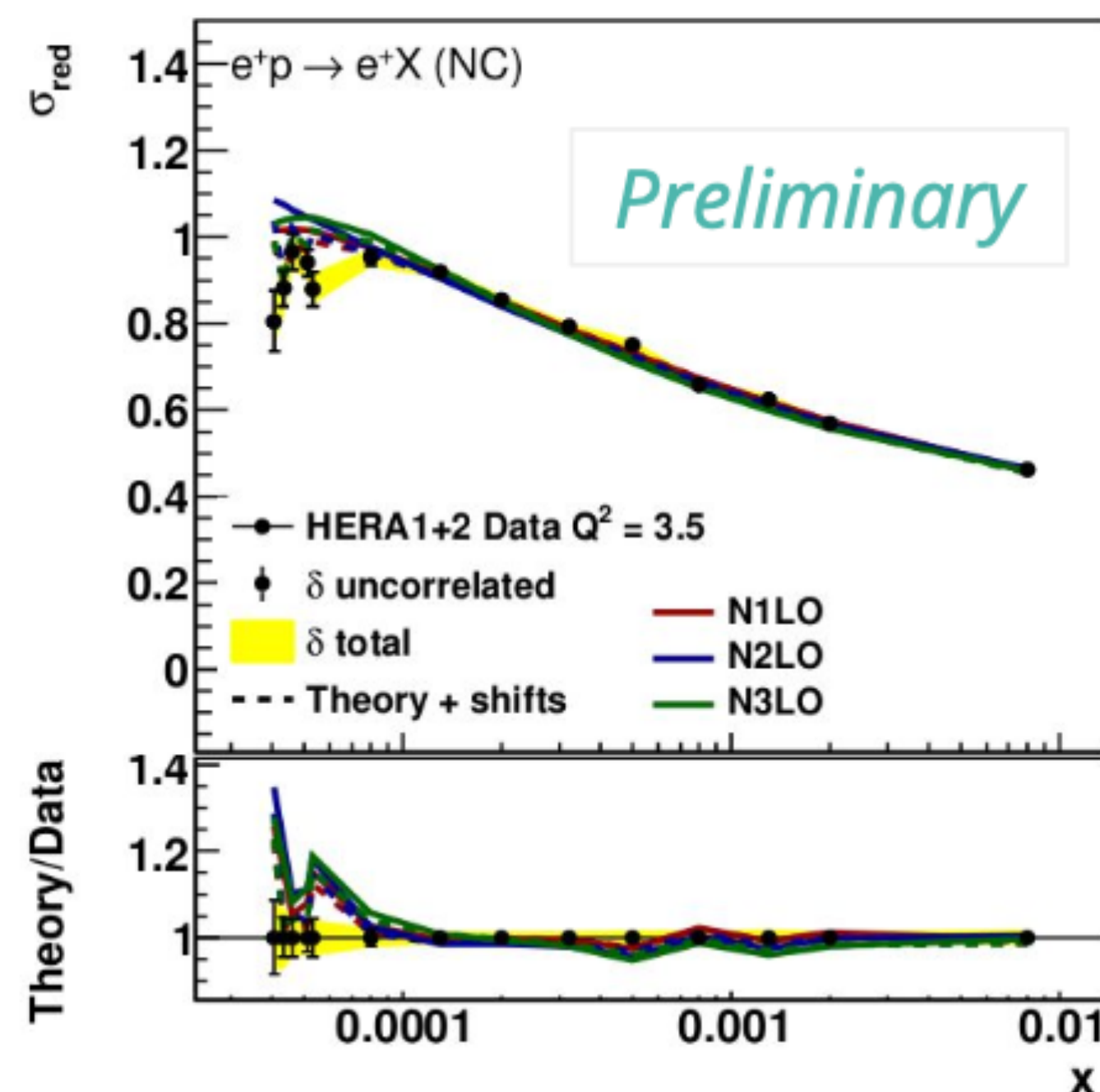


- Fits with alternative gluon parametrisation consistent with HERAPDF style
- N3LO gluon (g) is hardest, also largest at small x
- General good agreement at $x \geq 0.01$

Fits results at exact N3LO

$\sqrt{s}=920$ GeV data

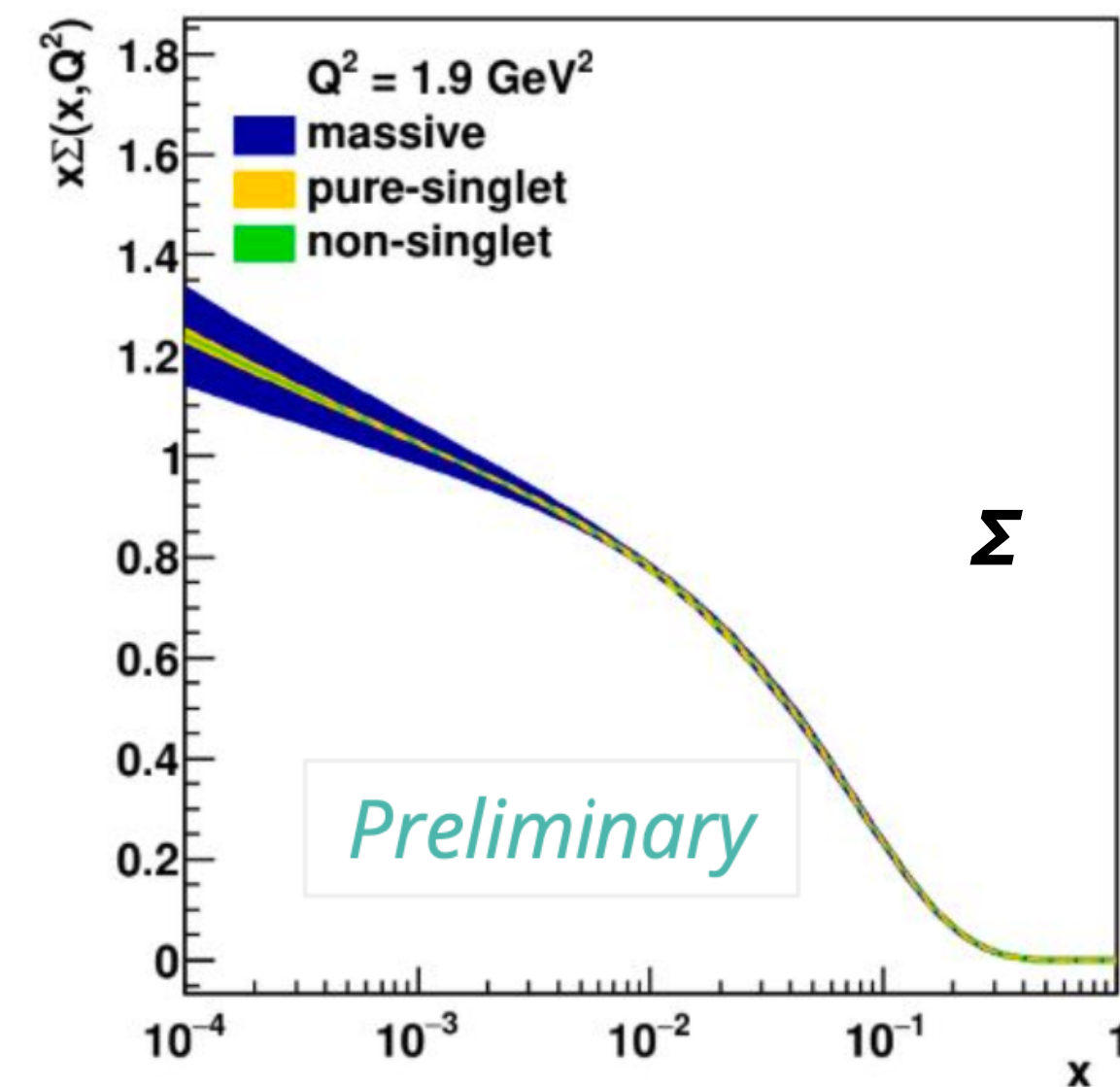
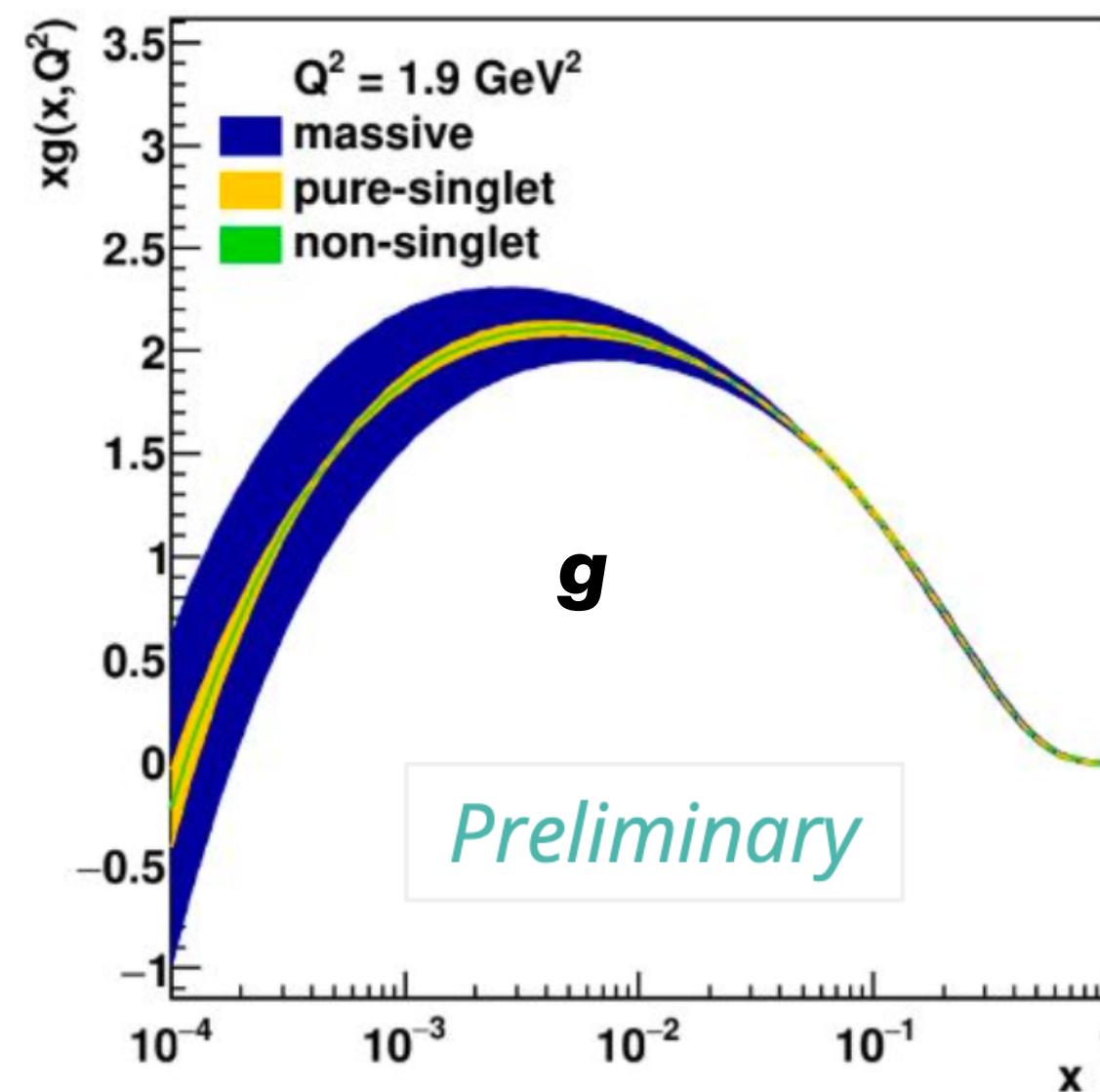
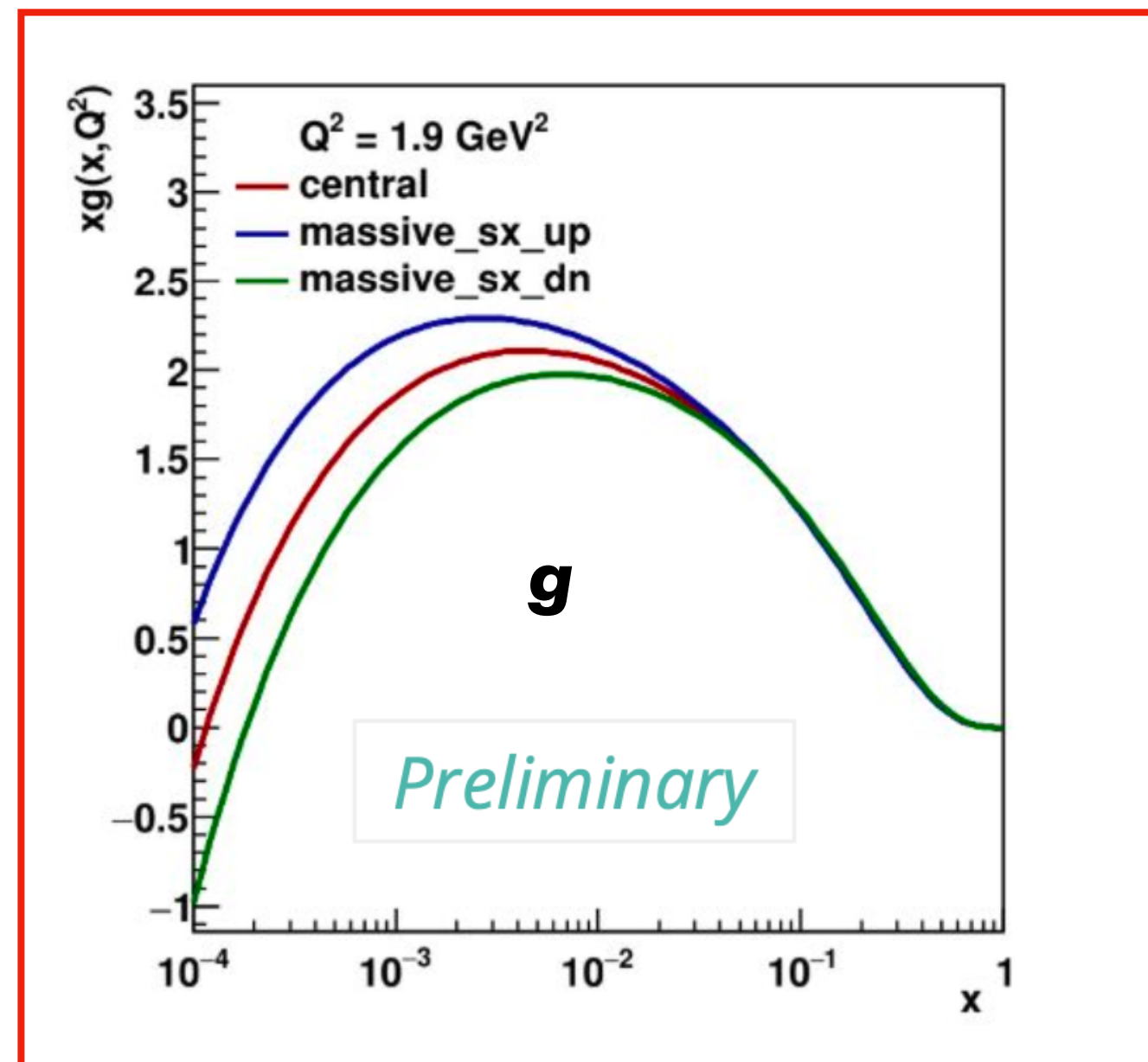
- Most precise HERA data: all fixed-order fits do not describe well data at lowest x / highest inelasticity y
- Slight low- x improvement at N3LO compared to N2LO
 - **need** $\ln(1/x)$ -resummation corrections — **small- x resummation** [1802.00064]
- Enhanced normalisation **excess over data** for N3LO at intermediate $Q^2 = 35 \text{ GeV}^2$



Heavy flavours at N3LO

Massive coefficient functions and related uncertainties

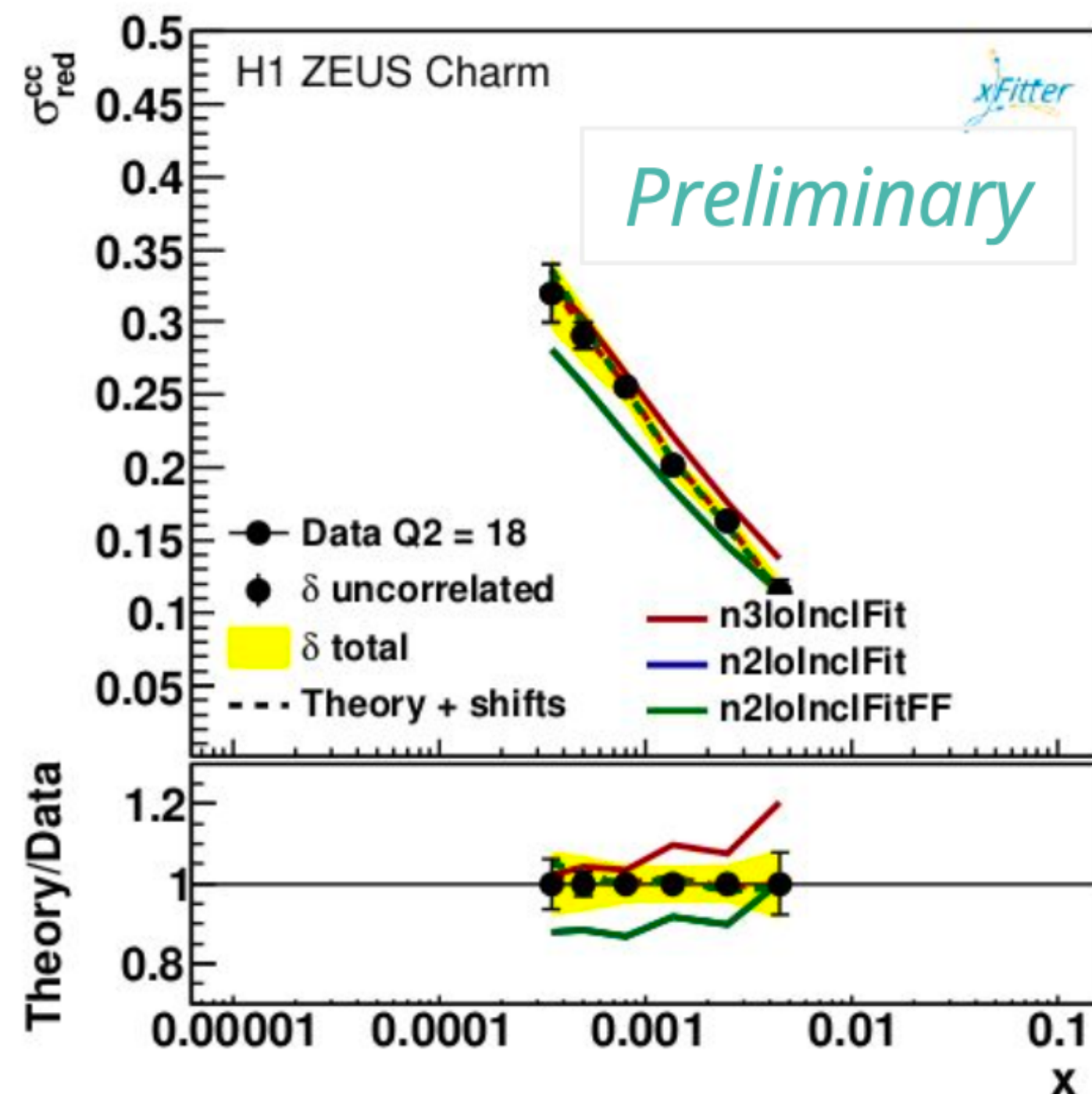
- Approximation of N3LO **DIS massive coefficient functions** in heavy quark production and uncertainties from [2401.12139]
 - Dominant uncertainty from **NLL small-x approximation** of exact coefficient functions
- **Larger uncertainty** compared to variations of **singlet** and **non-singlet splitting functions**



The charm structure function $F_{2,c}$

Predictions from N2LO and N3LO fits

- Compare $F_{2,c}$ predictions for **N2LO** and **N3LO** fits to inclusive HERA data with charm data from H1 and ZEUS combined analysis [1804.01019]



Dataset	<i>Preliminary</i>	
	N ³ LO	N ² LO
HERA c	41 / 47	50 / 47
HERA b	15 / 26	20 / 26
Correlated χ^2	43	61
Log penalty χ^2	+8.6	-5.35
Total χ^2 / dof	108 / 59	125 / 59
χ^2 p-value	0.00	0.00

- Improved χ^2 / dof for the N3LO fit

Overview of PDFs from N3LO DIS fit

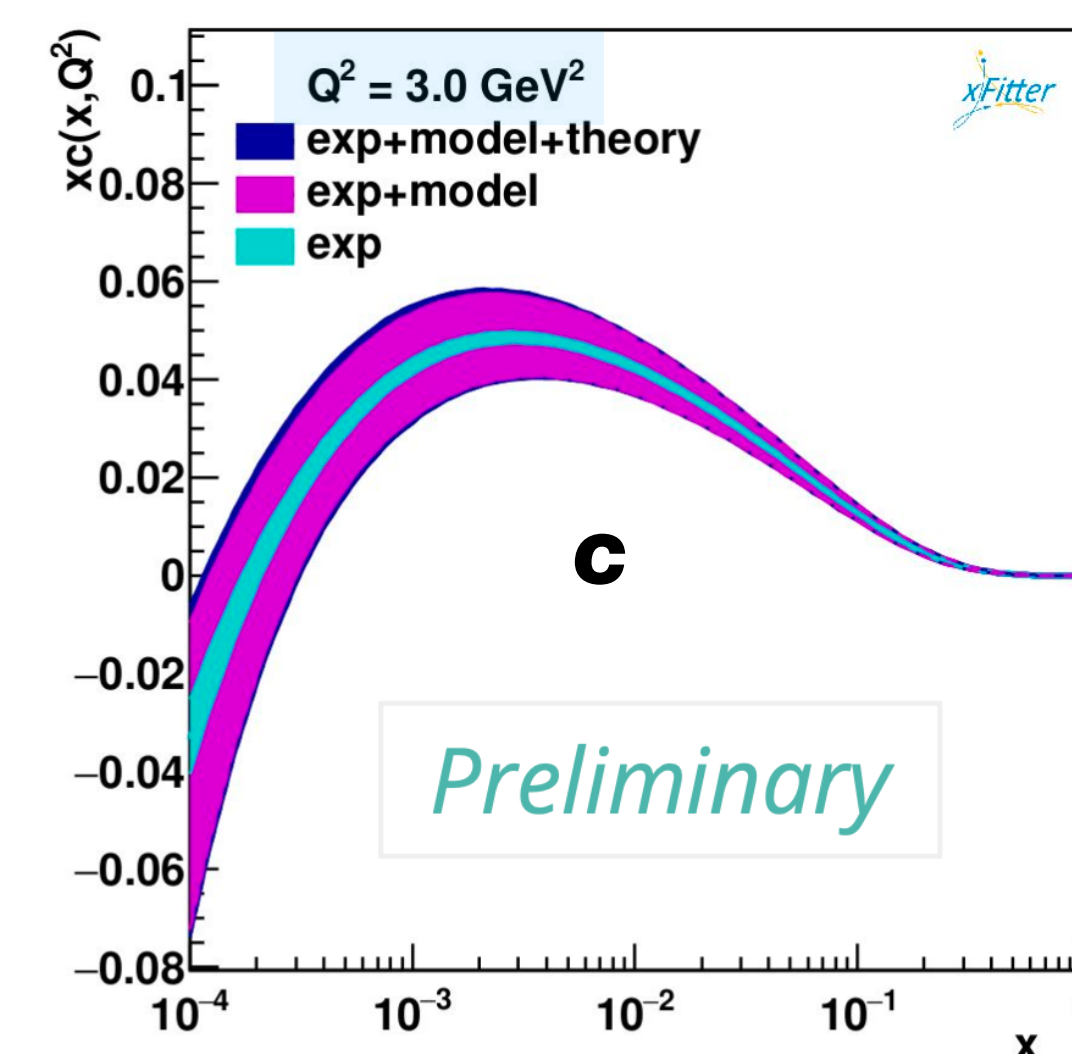
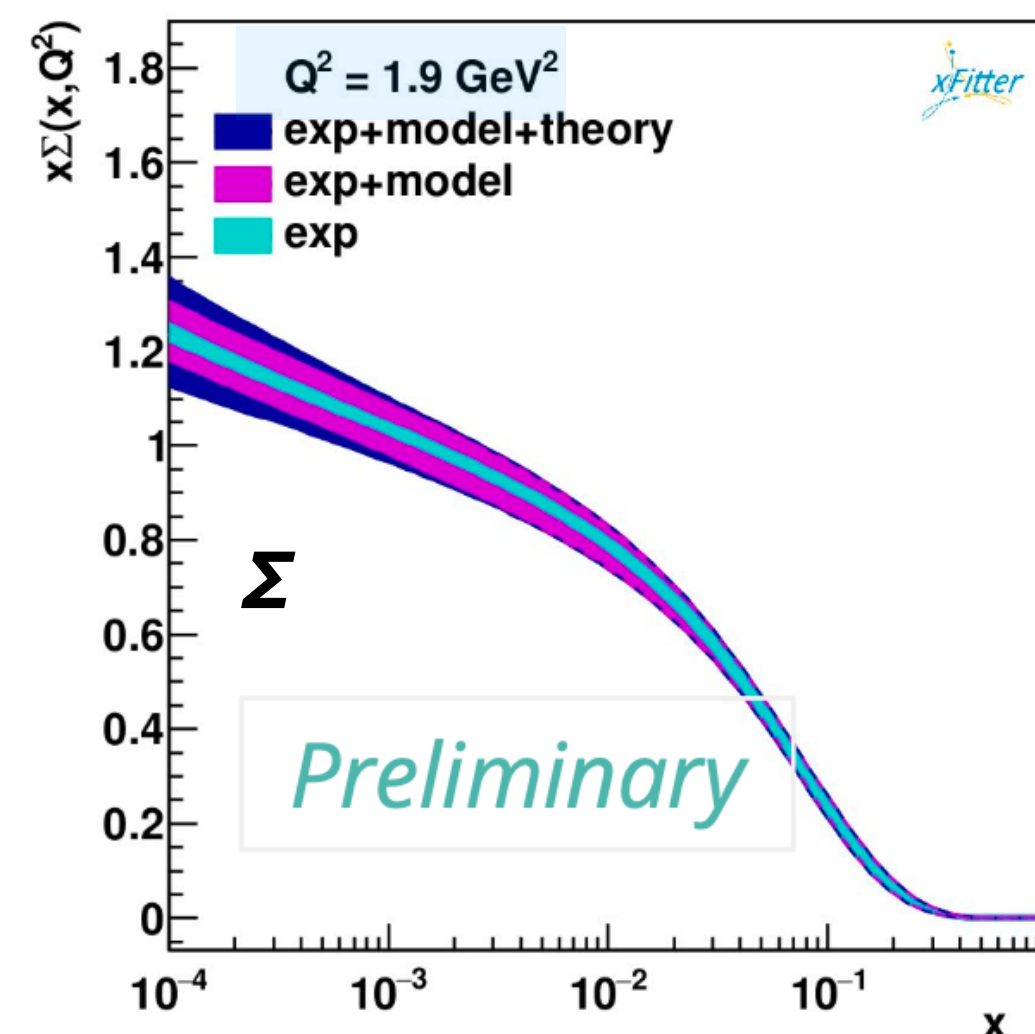
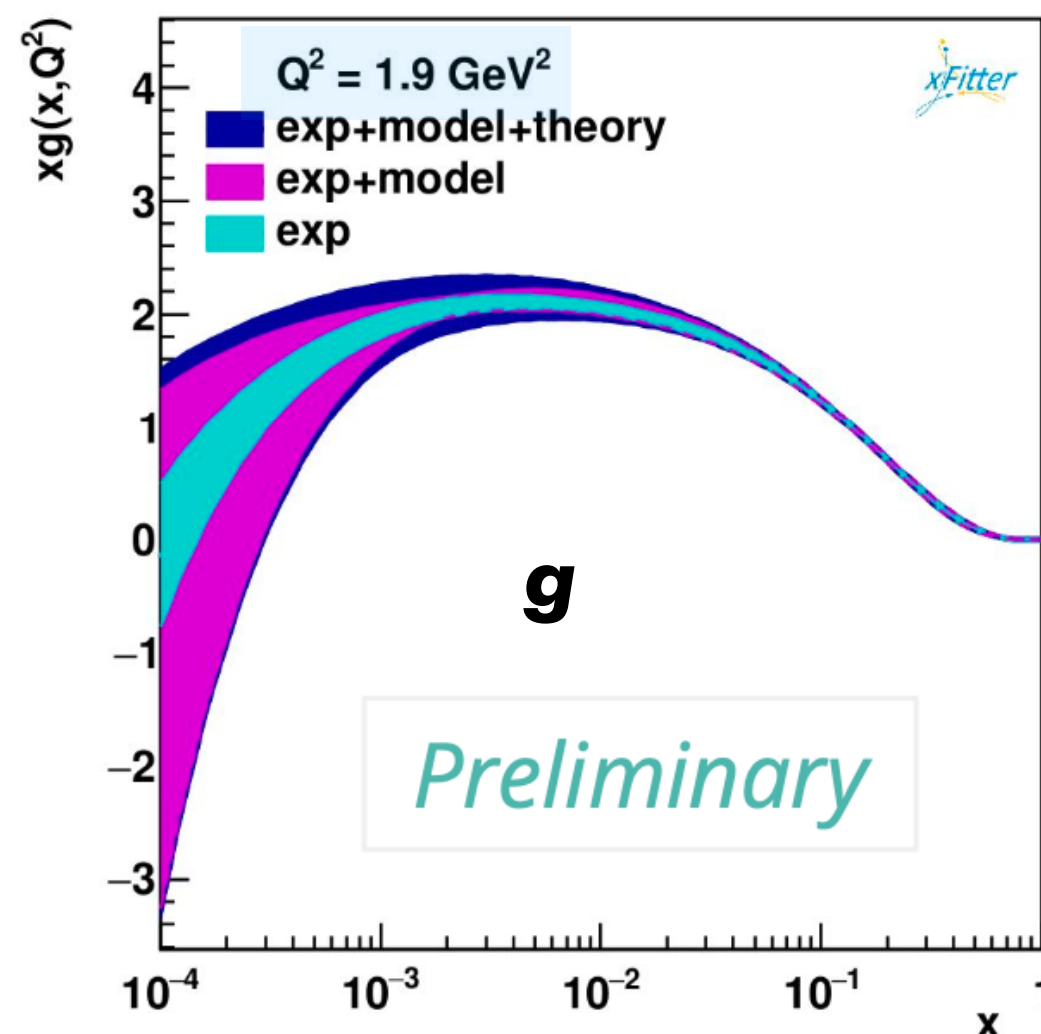
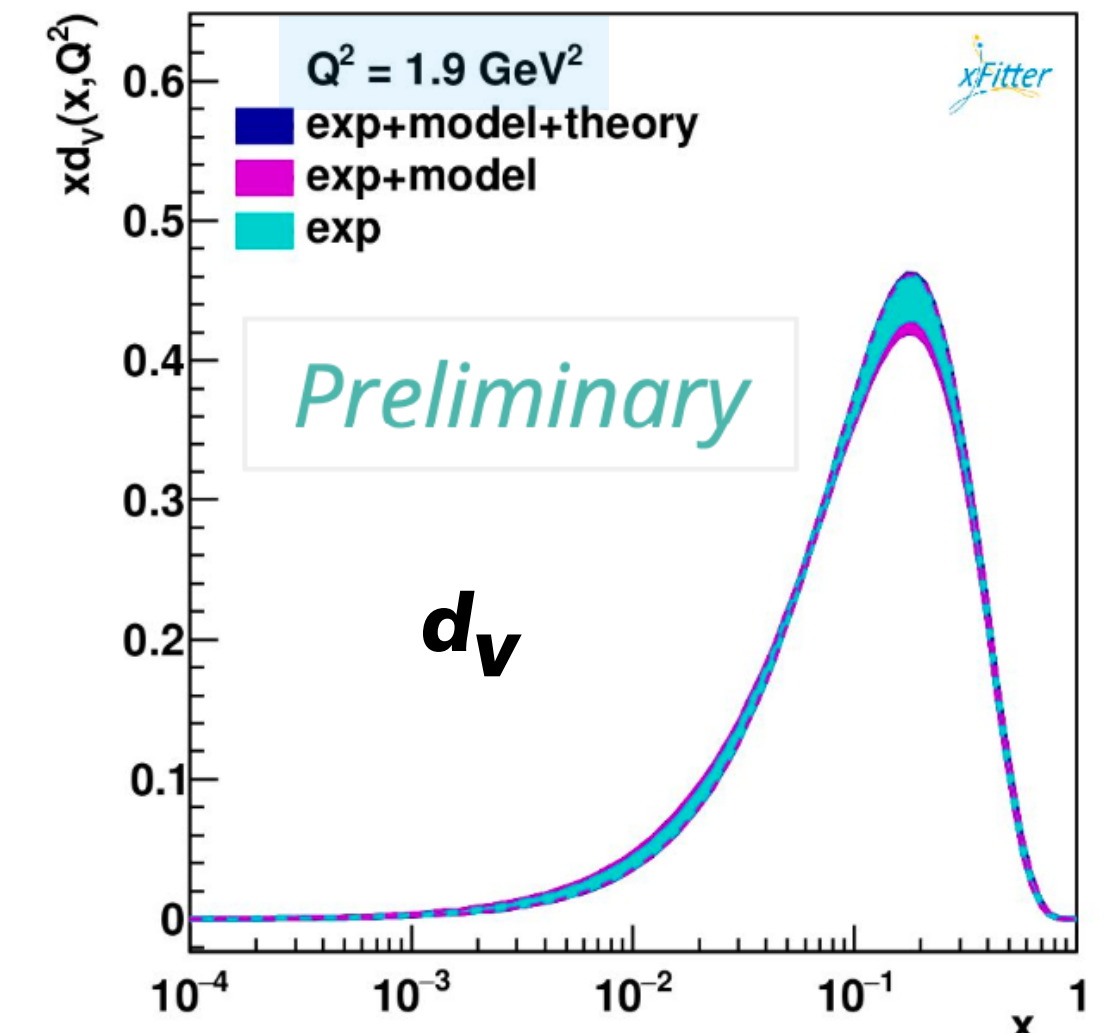
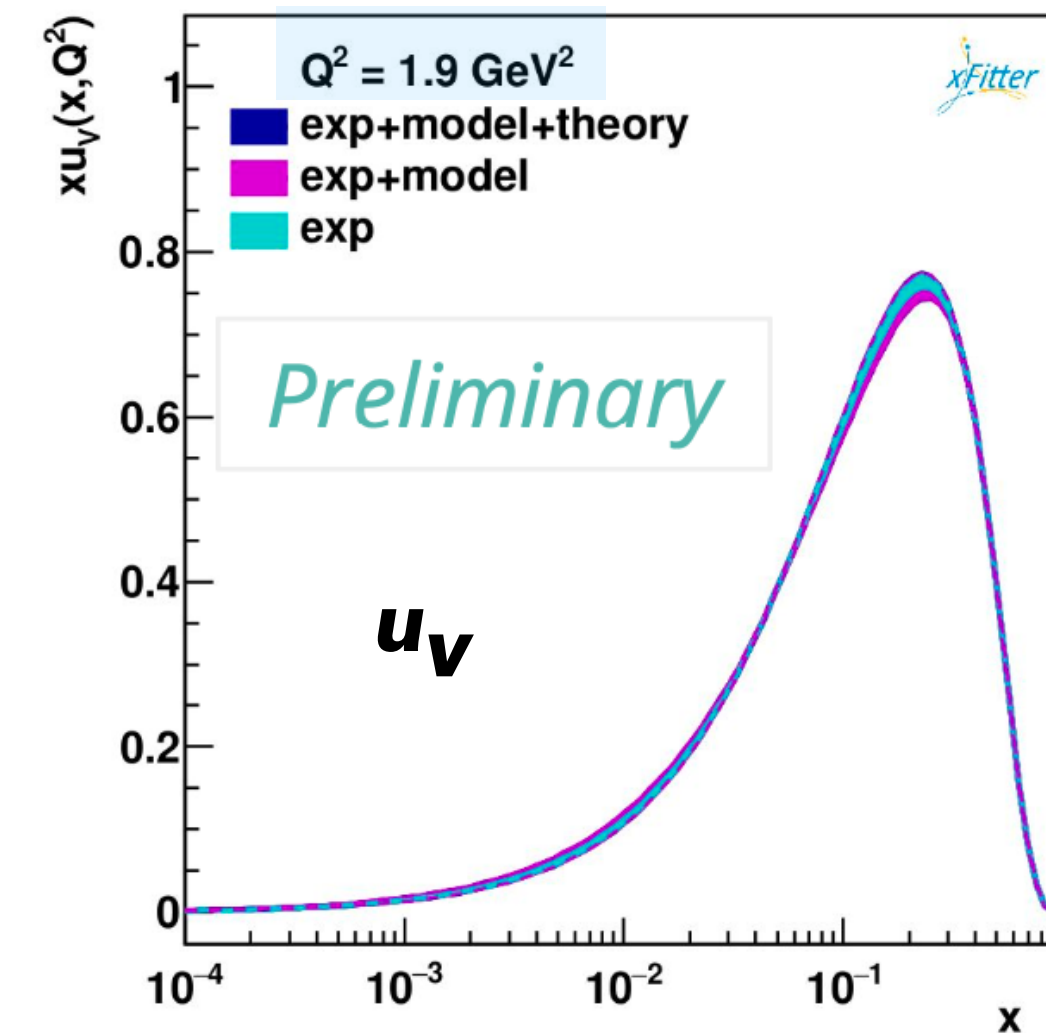
Complete PDF uncertainty *budget*

- PDFs from N3LO fit to HERA inclusive + charm and bottom data

- Model** and **theory uncertainties** included

- PDFs at the starting scale:

- Leading model uncertainty = variation of m_c



Overview of PDFs from N3LO DIS fit

Complete PDF uncertainty *budget*

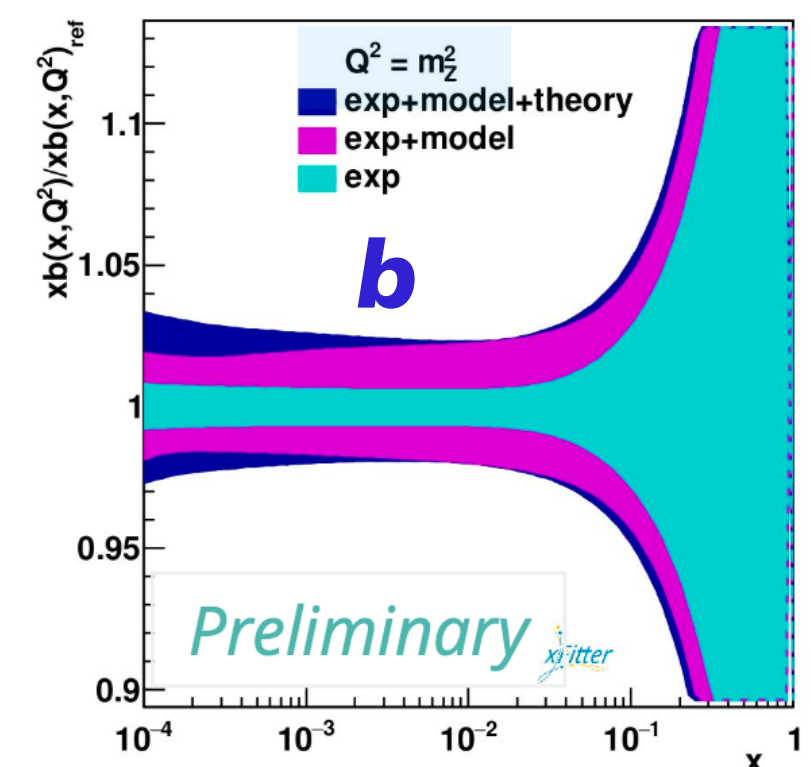
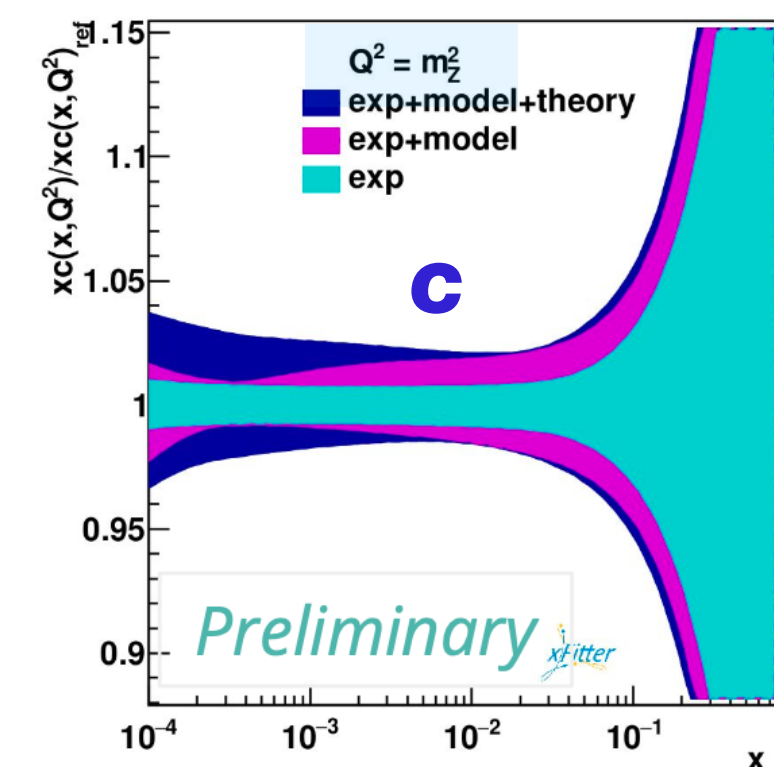
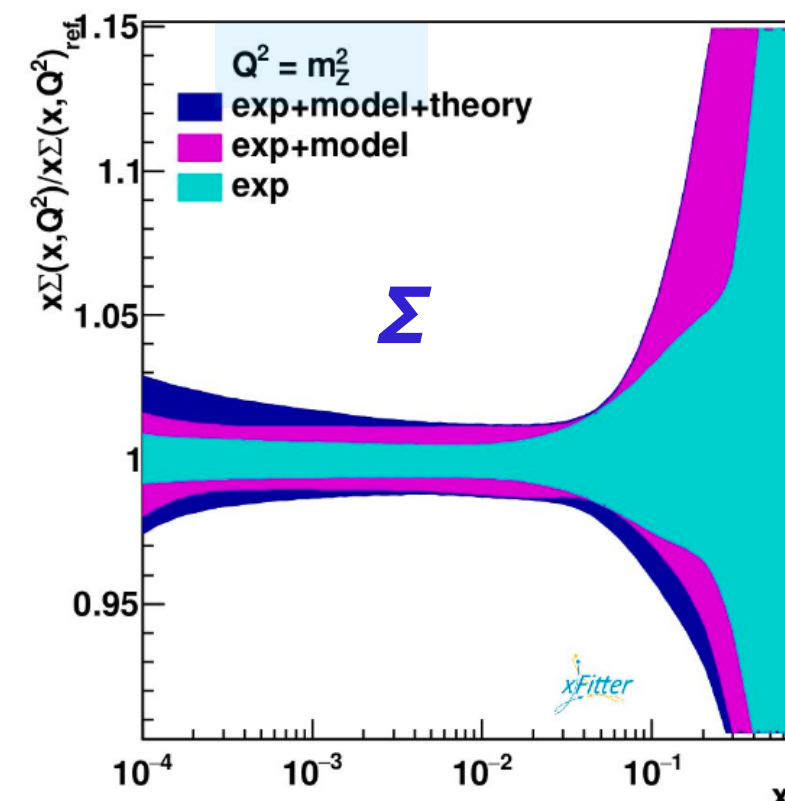
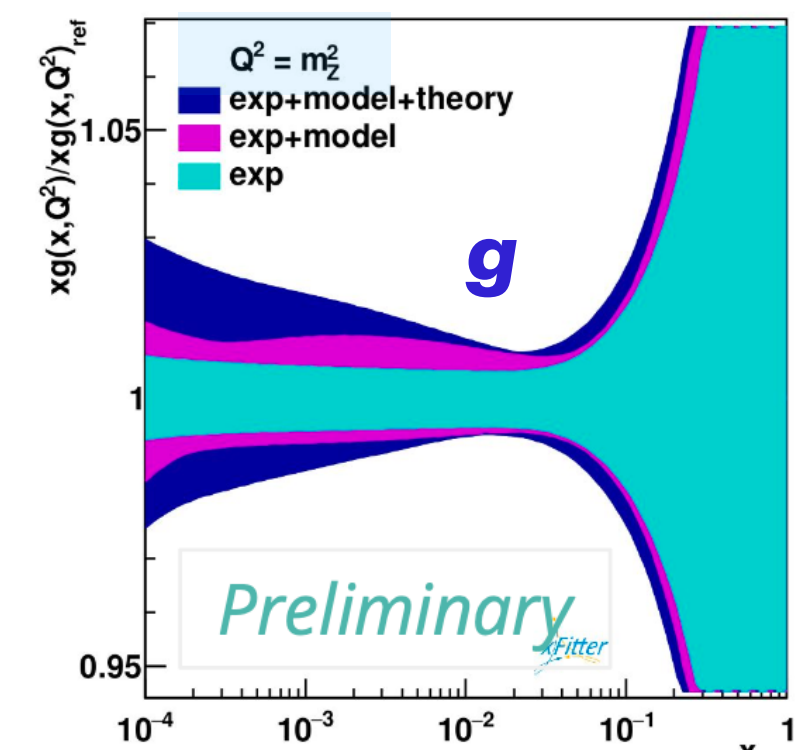
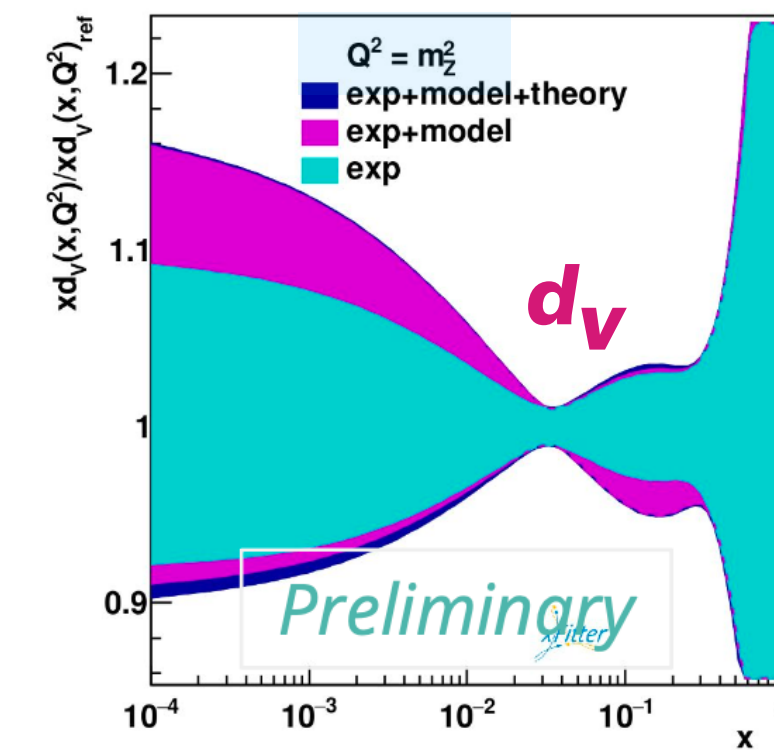
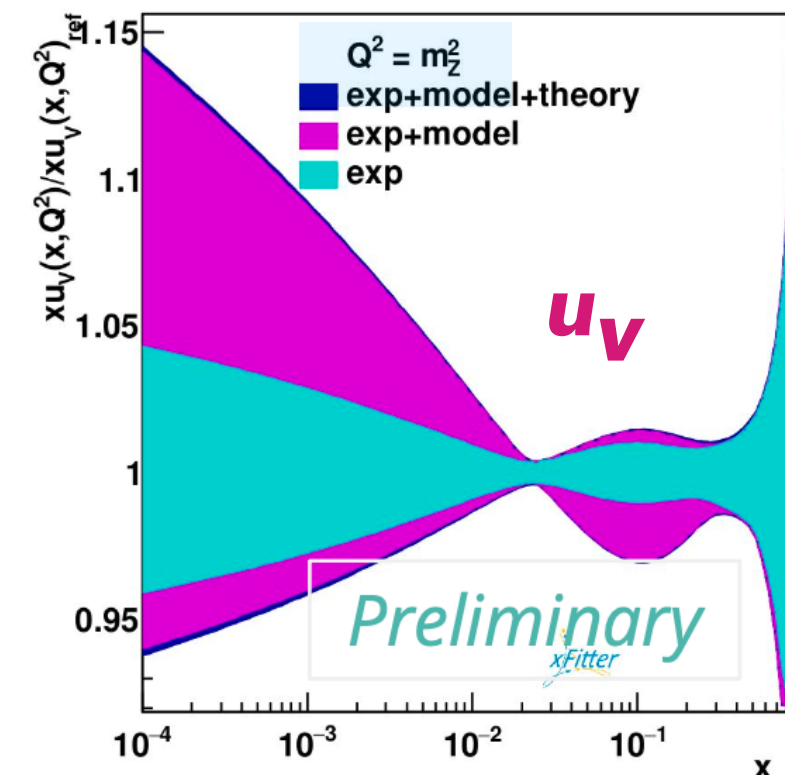
- PDFs from N3LO fit to HERA inclusive + charm and bottom data

- Model** and **theory uncertainties** included

- PDF relative uncertainties at $Q^2 = M_Z^2$

- Large uncertainties for $x d_V(x)$ and $x u_V(x)$ at low x
- Significant theory uncertainty for $x g(x)$ at $x < 0.01$
- $x c(x)$ and $x b(x)$ uncertainties driven by m_c and m_b variations

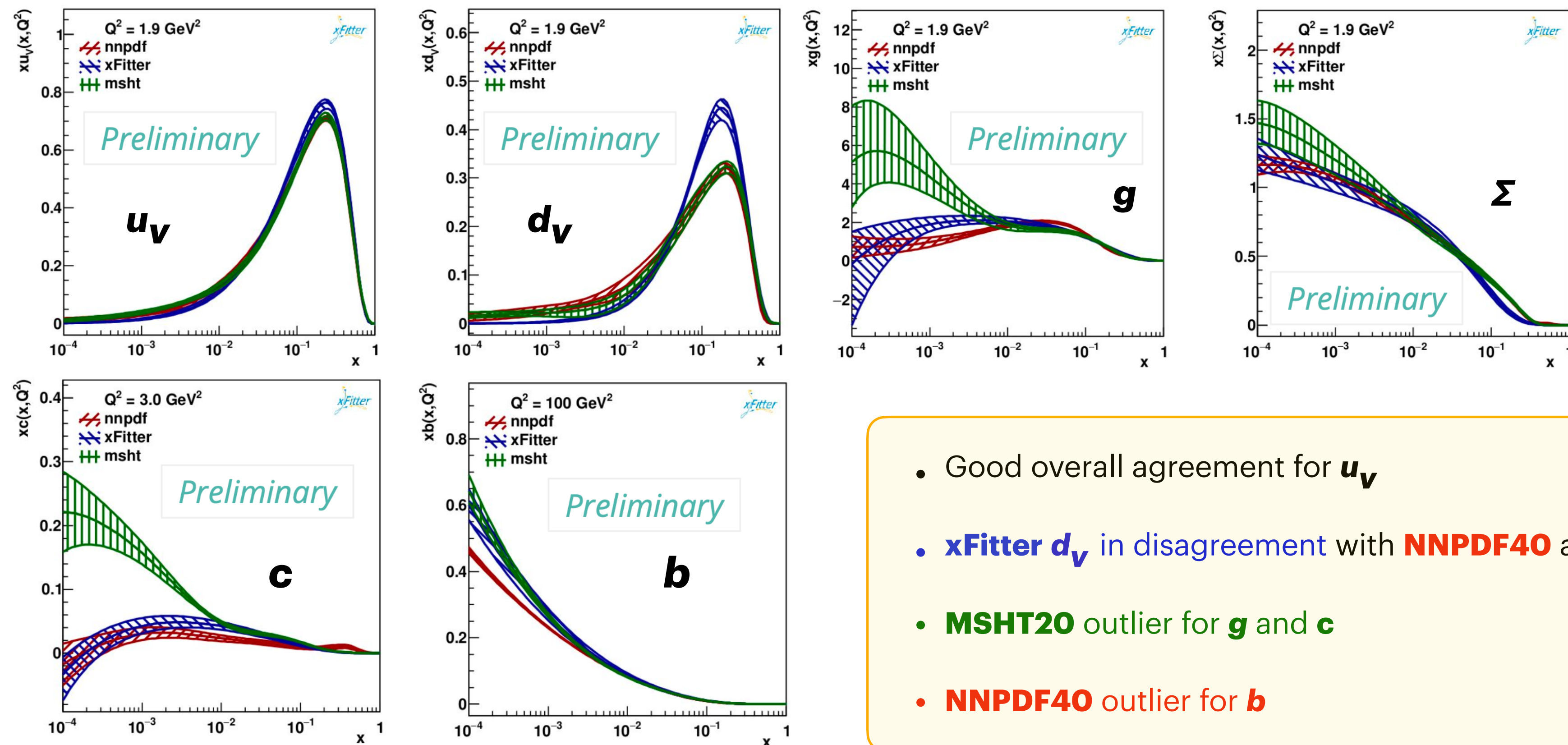
$$Q^2 = M_Z^2$$



Overview of PDFs from N3LO DIS fit

Comparisons to existing aN3LO sets from global PDF fits

- PDFs from N3LO **xFitter fit** to HERA inclusive + charm and bottom data in comparison with **NNPDF40_an3lo** and **MSHT20an3lo**

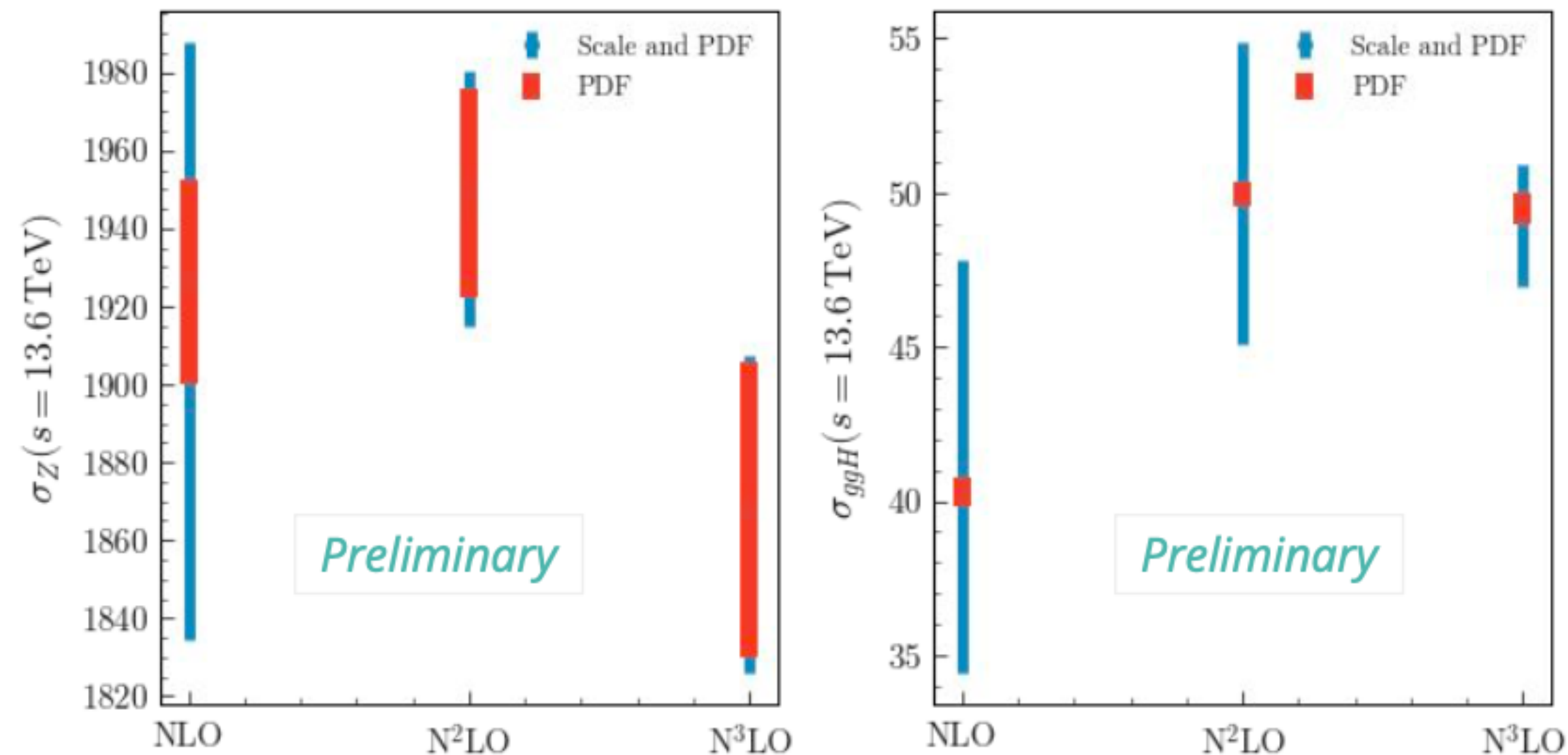


- Good overall agreement for u_v
- xFitter d_v** in disagreement with **NNPDF40** and **MSHT20**
- MSHT20** outlier for g and c
- NNPDF40** outlier for b

xFitter N3LO PDF fit in action

Impact on LHC predictions

- **Put xFitter N3LO PDF fit in action:** compute inclusive cross sections for **gluon-fusion Higgs (ggH)** and **neutral-current DY (Z) production** with the **n3lox**s program [2209.06138] using xFitter PDFs



- Compare predictions at NLO, N2LO and N3LO using PDFs from xFitter fits performed the same perturbative accuracy
- PDF uncertainties for N3LO predictions include theory uncertainties
- Good convergence for ggH , but **PDF uncertainty remains large for Z at N3LO**

Conclusion

Summary and outlook

- xFitter DIS-only analysis with consistent **N3LO** modelling of **evolution** and **coefficient functions**
- **First analysis** including **exact matching** conditions
- Residual uncertainties originating from heavy flavour coefficient functions
- Visible impact on PDF (i.e. largest gluon at small- x) and predictions for the LHC
- Further **studies including small- x resummation** ongoing





Backup