PDF fits with resummation scale variations in DIS

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xFitter study is based on the paper

V. Bertone, G. Bozzi, F. Hautmann “Perturbative RGE systematics in precision observables”

Phys.Rev.D 111 (2025) 7 [arXiv:2407.20842]

xFitter meeting
23 Jul 2025

Valerio Bertone, DIS2025 [link]

RGEs in QCD

- Study of the **solution** of a generic renormalisation group equation (RGE):

$$\frac{d}{d\ln\mu}\ln R(\mu) = \gamma(\alpha_s(\mu))$$

- **Perturbative** anomalous dimension:

$$\gamma(\alpha_s(\mu)) = \frac{\alpha_s(\mu)}{4\pi}\sum_{n=0}^{\infty}\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n \gamma_n$$

- Define the Green's function (evolution operator) such that:

$$R(\mu_1) = G(\mu_1,\mu_2)\otimes R(\mu_2)$$

- We consider effects due to solving the RGE **analytically**, as opposed to a **numerical** solution, by means of expansions in α_s that cause the inequality

$$G(\mu_1,\mu_0)\otimes G(\mu_0,\mu_2) \neq G(\mu_1,\mu_2)$$

- where the identity is violated by ***subleading*** terms.

- Examples of R are: α_s, **PDFs**, **TMDs**, GPDs, running masses, etc.

Introduction of resummation scale

Valerio Bertone, DIS2025 [link]

α_s: solution of the RGE

- How to **parametrise** *allowed*, *i.e.* subleading, violations of the RGE?
- Given the general RGE for the strong coupling truncated to k-th order:

$$\frac{d \ln \alpha_s}{d \ln \mu}(\mu) = \overline{\beta}(\alpha_s(\mu))$$

- The **analytic** N^{k}LL solution from μ_0 to μ can be written in the form:

$$a_s^{\mathrm{N}^k\mathrm{LL}}(\mu) = a_s(\mu_0) \sum_{l=0}^{k} a_s^l(\mu_0) g_{l+1}^{(\beta)}\left(\lambda, \frac{\mu_{\mathrm{Res}}}{\mu}\right) \qquad \lambda = a_s(\mu_0)\beta_0 \ln\left(\frac{\mu_{\mathrm{Res}}}{\mu_0}\right)$$

- Up to NLL one has:

$$g_1^{(\beta)}(\lambda) = \frac{1}{1-\lambda}\,, \qquad g_2^{(\beta)}\left(\lambda, \frac{\mu_{\mathrm{Res}}}{\mu}\right) = \frac{1}{(1-\lambda)^2}\left[-\frac{\beta_1}{\beta_0}\ln(1-\lambda) - \beta_0 \ln\frac{\mu_{\mathrm{Res}}}{\mu}\right]$$

- Notation purposely reminiscent of q_{T}/threshold resummation:
  - introduction of the **resummation scale** μ_{Res} at the level of the solution of the RGE,
  - variations of μ_{Res} around μ provide an estimate of subleading (higher-order) corrections.

Introduction of resummation scale

Valerio Bertone, DIS2025 [link]

α_s: resummation uncertainty

- The resummation scale μ_{Res} allows us to estimate the uncertainty due to the **truncation of the β-function** in analytic solutions.
- How about **numerical** solutions of the RGE?
- We proved that μ_{Res} can be introduced at the level β-function by "standard" scale variations.
- At NLO:

$$\overline{\beta}(\mu) = -a_s(\mu_{\mathrm{Res}})\beta_0 \left(1 + a_s(\mu_{\mathrm{Res}}) \left[b_1 - 2\beta_0 \ln \frac{\mu_{\mathrm{Res}}}{\mu}\right]\right) + \mathcal{O}(\alpha_s^3)$$

- This β-function can be used to estimate missing higher-order corrections also when solving the RGE **numerically**.
- Bottom line:
  - **the evolution of α_s has an uncertainty** (which is often neglected).
  - variations of μ_{Res} estimate this uncertainty in ***both*** analytic and numerical solutions.

Valerio Bertone, DIS2025 [link]

PDFs: solution of the RGE

- We played the same game with PDFs:

$$\frac{d\ln f}{d\ln\mu}(\mu) = \gamma(\alpha_s(\mu))$$

- The **analytic** solution N^{k}LL can be written in the form:

$$f^{\mathrm{N}^k\mathrm{LL}}(\mu) = g_0^{(\gamma),\mathrm{N}^k\mathrm{LL}}\left(\lambda, \frac{\mu_{\mathrm{Res}}}{\mu}\right) \exp\left[\sum_{l=0}^{k} a_s^l(\mu_0) g_{l+1}^{(\gamma)}\left(\lambda, \frac{\mu_{\mathrm{Res}}}{\mu}\right)\right] f(\mu_0)$$

- Up to NLL one finds:

$$g_0^{(\gamma),\mathrm{NLL}}\left(\lambda, \frac{\mu_{\mathrm{Res}}}{\mu}\right) = 1 + a_s(\mu_0)\frac{1}{\beta_0}\left(\gamma_1 - \frac{\beta_1}{\beta_0}\gamma_0\right)\frac{\lambda}{1-\lambda}$$

$$g_1^{(\gamma)}\left(\lambda, \frac{\mu_{\mathrm{Res}}}{\mu}\right) = -\frac{\gamma_0}{\beta_0}\ln(1-\lambda)$$

$$g_2^{(\gamma)}\left(\lambda, \frac{\mu_{\mathrm{Res}}}{\mu}\right) = -\frac{\gamma_0}{\beta_0^2}\frac{\beta_1\ln(1-\lambda) + \beta_0^2\ln\frac{\mu_{\mathrm{Res}}}{\mu}}{1-\lambda}$$

- The NLL **numerical** solution is instead computed using:

$$\gamma(\mu) = a_s(\mu_{\mathrm{Res}})\gamma_0 + a_s^2(\mu_{\mathrm{Res}})\left[\gamma_1 - \beta_0\gamma_0\ln\frac{\mu_{\mathrm{Res}}}{\mu}\right]$$

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- The idea is to study the impact of resummation scale variations in a PDF fit using inclusive HERA data with xFitter
- Resummation scale is implemented in APFELxx evolution and interfaced in xFitter, branch `apfelxx_muresum`. It uses branch `DoubleOperator` of APFEL++.
  - parametrised with $\xi = \mu_{Res}/Q \qquad (0.5 < \xi < 2)$

```
DefaultEvolution: proton-evolution
Evolutions:
  proton-evolution:
    ? !include evolutions/APFELxx.yaml
    xi : 0.5
    kmc : 1.37
    QGrid: [50, 0.999, 1000.0, 3]
    QGridAs: [100, 0.999, 1000.0, 3]
  proton-APFEL:
    ? !include evolutions/APFEL.yaml
    kmc : 1.37
    qLimits : [0.999, 1000.0]
    muRoverQ : 1.0
    muFoverQ : 1.0
```

- DIS structure functions are computed using APFEL (FORTRAN) FONLL-C scheme

Impact of resummation scale variations on structure functions

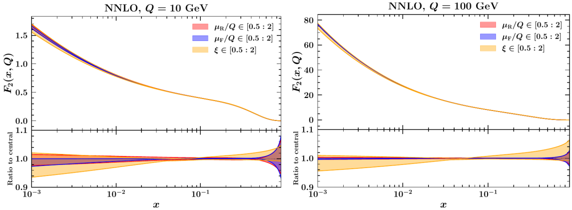
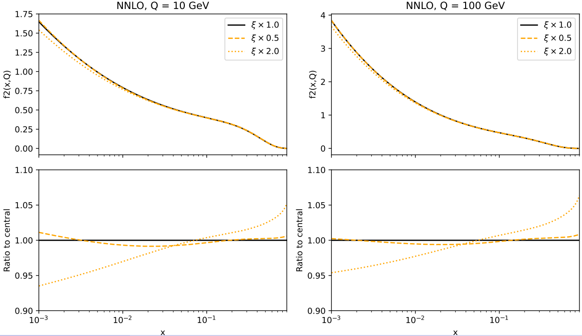


Figure 4: The DIS F_2 structure function at NNLO plotted versus x for two different values of Q. Uncertainty bands associated with variations of renormalisation and factorisation scales, μ_R and μ_F, and resummation-scale parameter, ξ, are also displayed. The lower insets show the predictions normalised to the central-scale curves.



Impact of resummation scale variations on structure functions

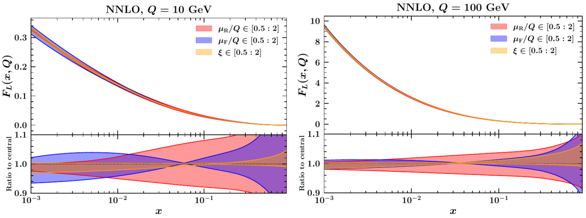
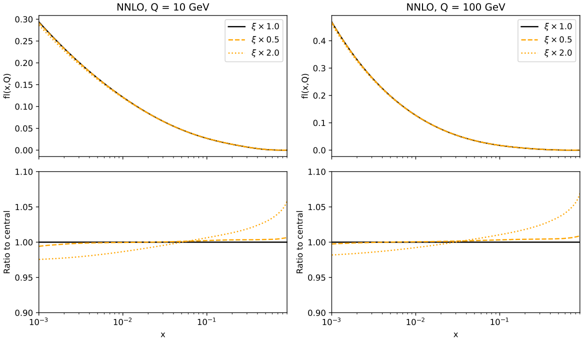


Figure 5: Same as Fig. 4 for F_L.



Impact of resummation scale variations on structure functions

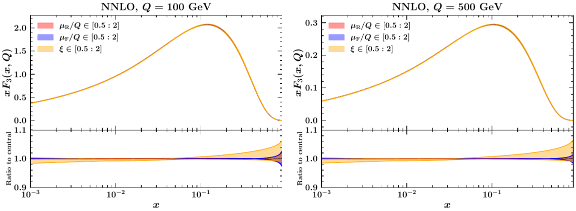
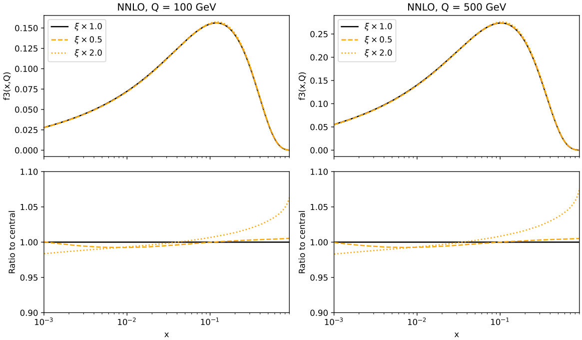


Figure 6: Same as Fig. 4 for xF_3.



PDF fit setup

PDF parametrisation and fit settings follow BG paper

M. Bonvini, F. Giuli, Eur.Phys.J.Plus 134 (2019) 10, 531 [arXiv:1902.11125]

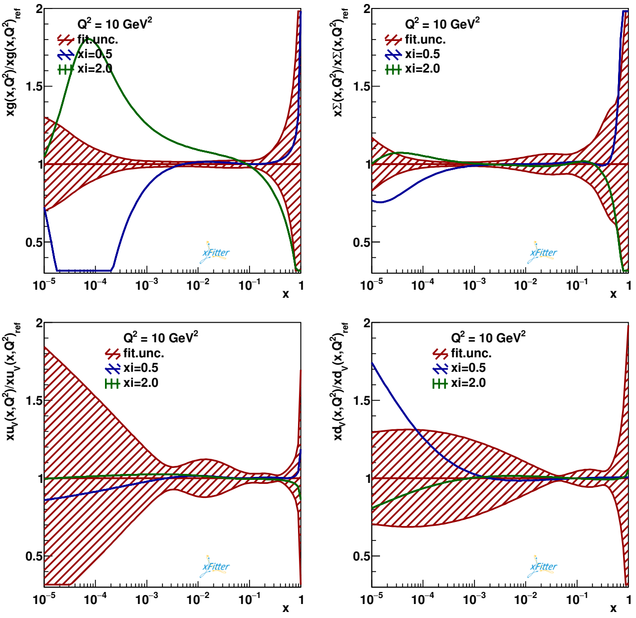
Differences in the fit setup	Setup of Sect. 3, same as [10]	New setup, same as [22]
heavy flavour scheme	TR	FONLL
initial scale μ_0	1.38 GeV	1.6 GeV
charm matching scale μ_c	m_c	$1.12m_c$
charm mass m_c	1.43 GeV	1.46 GeV

- inclusive HERA data, $Q^2 > 3.5$ GeV^2
- NNLO, FONLL-C, $\alpha_S(M_Z) = 0.118$, $m_c = 1.46$ GeV, $m_b = 4.5$ GeV
- starting scale $Q_0 = 1.6$ GeV with threshold $k_{m_c} = 1.12$
  - alternative $Q_0 = 2$ GeV, $k_{m_c} = 1.37$ (consistent with arXiv:2407.20842)
- BG PDF parametrisation:

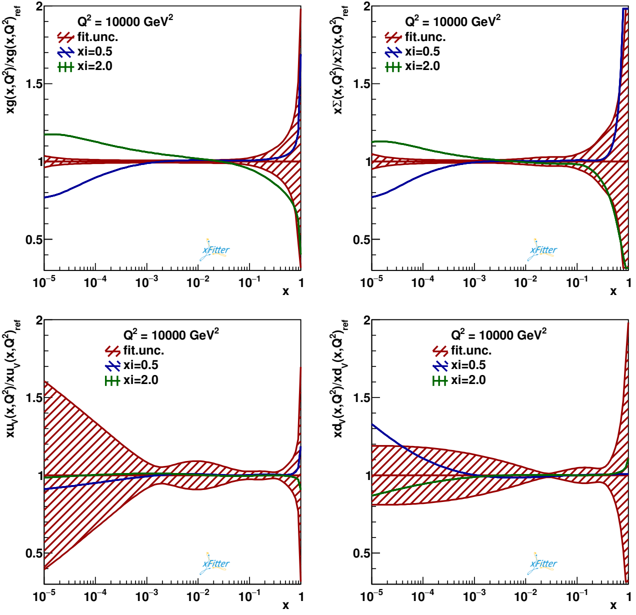
$$xg(x) = A_g x^{B_g}(1-x)^{C_g}(1 + F_g \log(x) + G_g \log^2(x))$$
$$xu_v(x) = A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}(1 + E_{u_v} x^2) + F_{u_v}\log(x) + G_{u_v}\log^2(x)$$
$$xd_v(x) = A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}$$
$$x\overline{U}(x) = A_{\overline{U}} x^{B_{\overline{U}}}(1-x)^{C_{\overline{U}}}$$
$$x\overline{D}(x) = A_{\overline{D}} x^{B_{\overline{D}}}(1-x)^{C_{\overline{D}}}(1 + D_{\overline{D}} x + F_{\overline{D}}\log(x))$$

$\rightarrow$ provides best description of HERA data at NNLO ($\chi^2/dof = 1312/1127$ from BG paper is reproduced, c.f. $\chi^2/dof = 1388/1131$ using HERAPDF parametrisation)

PDF fit results ($Q^2 = 10$ GeV^2) [$Q_0 = 1.6$ GeV]



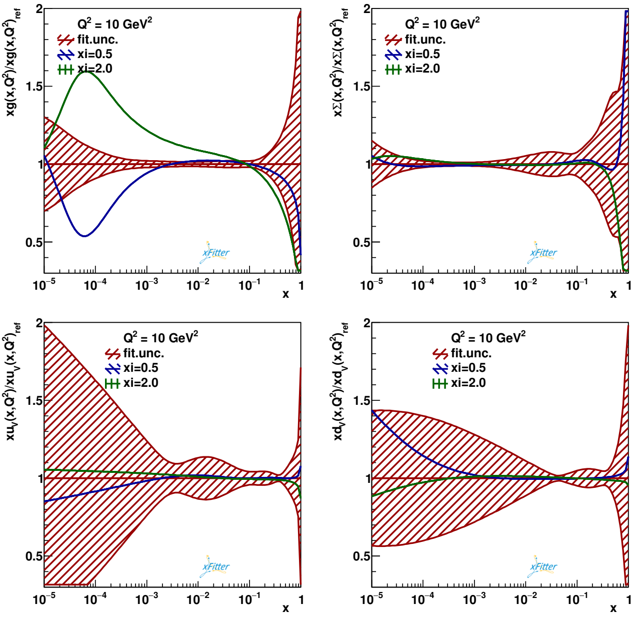
PDF fit results ($Q^2 = 10000$ GeV^2) [$Q_0 = 1.6$ GeV]



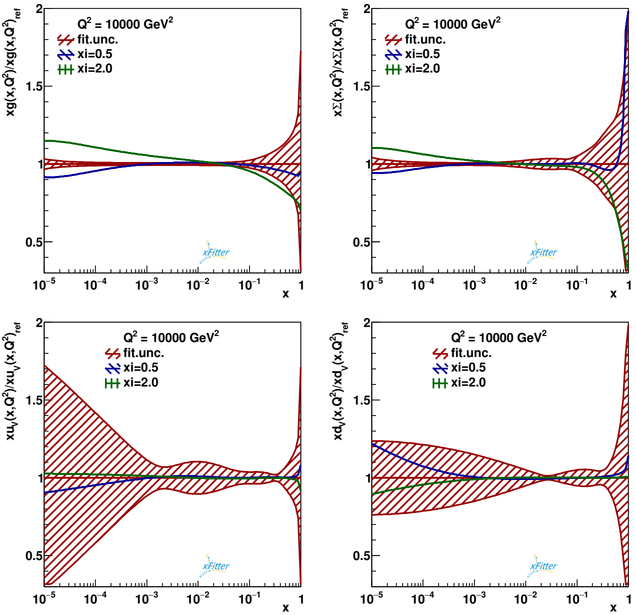
PDF fit results χ^2 table [$Q_0 = 1.6$ GeV]

Dataset	fit.unc.	xi=0.5	xi=2.0
HERA1+2 NCep 820	74 / 70	82 / 70	70 / 70
HERA1+2 NCep 460	224 / 204	233 / 204	225 / 204
HERA1+2 CCep	37 / 39	37 / 39	38 / 39
HERA1+2 NCem	216 / 159	216 / 159	219 / 159
HERA1+2 CCem	50 / 42	50 / 42	50 / 42
HERA1+2 NCep 575	222 / 254	236 / 254	221 / 254
HERA1+2 NCep 920	406 / 377	431 / 377	400 / 377
Correlated χ^2	79	93	74
Log penalty χ^2	+3.1	+14	-4.67
Total χ^2 / dof	1312 / 1127	1391 / 1127	1291 / 1127
χ^2 p-value	0.00	0.00	0.00

PDF fit results ($Q^2 = 10$ GeV^2) [$Q_0 = 2$ GeV]



PDF fit results ($Q^2 = 10000$ GeV^2) [$Q_0 = 2$ GeV]



PDF fit results χ^2 table [$Q_0 = 2$ GeV]

Dataset	fit.unc.	xi=0.5	xi=2.0
HERA1+2 NCep 820	74 / 70	79 / 70	71 / 70
HERA1+2 NCep 460	225 / 204	228 / 204	225 / 204
HERA1+2 CCep	37 / 39	37 / 39	38 / 39
HERA1+2 NCem	216 / 159	215 / 159	218 / 159
HERA1+2 CCem	50 / 42	50 / 42	50 / 42
HERA1+2 NCep 575	221 / 254	224 / 254	221 / 254
HERA1+2 NCep 920	402 / 377	423 / 377	399 / 377
Correlated χ^2	78	89	75
Log penalty χ^2	+3.6	+9.8	-3.37
Total χ^2 / dof	1308 / 1127	1355 / 1127	1293 / 1127
χ^2 p-value	0.00	0.00	0.00

Summary and next steps

- Implemented μ_{Res} variations in xFitter interface to APFEL++
- PDF fit to inclusive DIS HERA data with varied μ_{Res} demonstrated significant impact of these parameter variations on the fitted PDFs
  - largest impact on g and sea quark distributions which exceed fit uncertainties
  - **this was never explored in PDF fits**
- The impact of μ_{Res} variations is reduced if larger starting scale $Q_0 = 2$ GeV is used
- Next steps:
  - explore model (f_s, $m_{c,b}$?) variations
  - explore PDF parametrisation variations (including further variation of Q_0?)