PDF fits with resummation scale variations in DIS

V. Bertone, G. Bozzi, F. Hautmann, S. Zenaiev

xFitter study is based on the paper

V. Bertone, G. Bozzi, F. Hautmann "Perturbative RGE systematics in precision observables"

Phys.Rev.D 111 (2025) 7 [arXiv:2407.20842]

xFitter meeting
8 Oct 2025

xFitter study

- The idea is to study the impact of resummation scale variations in a PDF fit using inclusive HERA data with xFitter
- Resummation scale is implemented in APFELxx evolution and interfaced in xFitter, branch `apfelxx_muresum`. It uses branch `DoubleOperator` of APFEL++.
  - parametrised with $\xi = \mu_{Res}/Q \qquad (0.5 < \xi < 2)$

```
DefaultEvolution: proton-evolution
Evolutions:
  proton-evolution:
    ? !include evolutions/APFELxx.yaml
    xi : 0.5
    kmc : 1.37
    QGrid: [50, 0.999, 1000.0, 3]
    QGridAs: [100, 0.999, 1000.0, 3]
  proton-APFEL:
    ? !include evolutions/APFEL.yaml
    kmc : 1.37
    qLimits : [0.999, 1000.0]
    muRoverQ : 1.0
    muFoverQ : 1.0
```

- DIS structure functions are computed using APFEL (FORTRAN) FONLL-C scheme

PDF fit setup

PDF parametrisation and fit settings follow BG paper

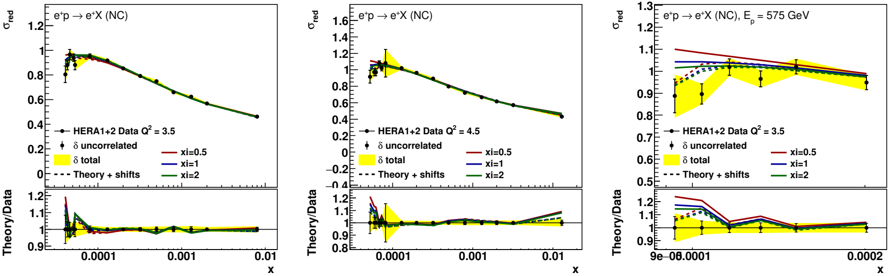
M. Bonvini, F. Giuli, Eur.Phys.J.Plus 134 (2019) 10, 531 [arXiv:1902.11125]

Differences in the fit setup	Setup of Sect. 3, same as [10]	New setup, same as [22]
heavy flavour scheme	TR	FONLL
initial scale μ_0	1.38 GeV	1.6 GeV
charm matching scale μ_c	m_c	$1.12m_c$
charm mass m_c	1.43 GeV	1.46 GeV

- inclusive HERA data, $Q^2 > 3.5(10)$ GeV^2
- NNLO, FONLL-C, $\alpha_S(M_Z) = 0.118$, $m_c = 1.46$ GeV, $m_b = 4.5$ GeV
- starting scale $Q_0 = 1.6$ (3.2) GeV with threshold $k_{m_c} = 1.12(2.2)$
- BG PDF parametrisation:

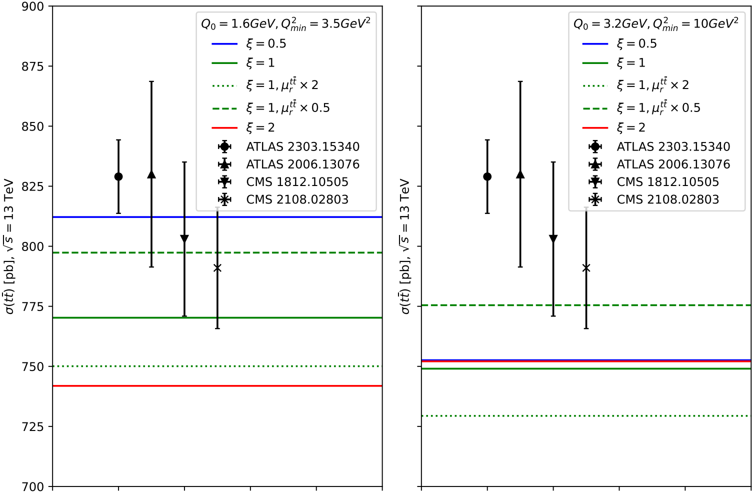
$$xg(x) = A_g x^{B_g}(1-x)^{C_g}(1 + F_g \log(x) + G_g \log^2(x))$$
$$xu_v(x) = A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}(1 + E_{u_v}x^2) + F_{u_v}\log(x) + G_{u_v}\log^2(x)$$
$$xd_v(x) = A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}$$
$$x\overline{U}(x) = A_{\overline{U}} x^{B_{\overline{U}}}(1-x)^{C_{\overline{U}}}$$
$$x\overline{D}(x) = A_{\overline{D}} x^{B_{\overline{D}}}(1-x)^{C_{\overline{D}}}(1 + D_{\overline{D}}x + F_{\overline{D}}\log(x))$$

$\rightarrow$ provides best description of HERA data at NNLO ($\chi^2/dof = 1312/1127$ from BG paper is reproduced, c.f. $\chi^2/dof = 1388/1131$ using HERAPDF parametrisation)



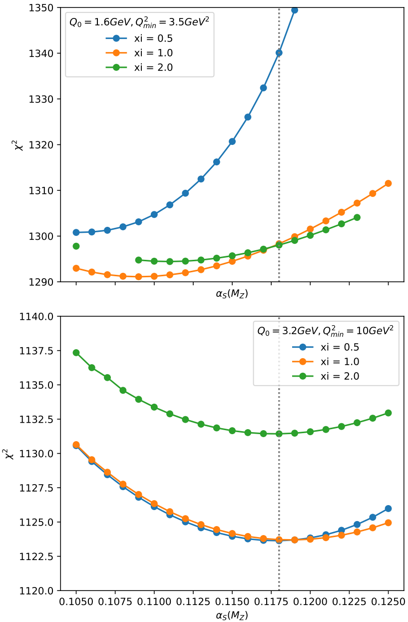
- Largest differences at lowest Q^2, x
- All plots in BACKUP

Impact of ξ variations on predicted $t\bar{t}$ at LHC



- $t\bar{t}$ cross-section at $\sqrt{s} = 13$ TeV is computed at NNLO using HATHOR (xFitter)
- $\sim 5\%$ effect: comparable to experimental data unc. and/or $t\bar{t}$ scale variation unc.
- reduced to $< 1\%$ if $Q_0 = 3.2$ GeV, $Q^2_{min} = 10$ GeV^2 is used

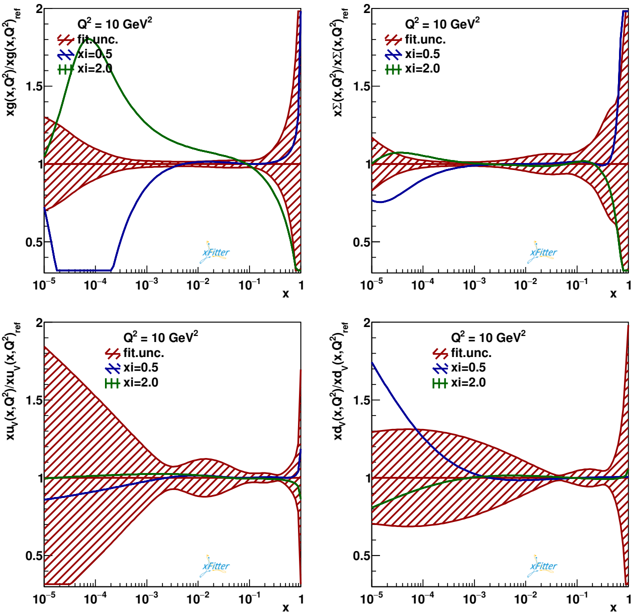
$\alpha_S(M_Z)$ scan with ξ variations



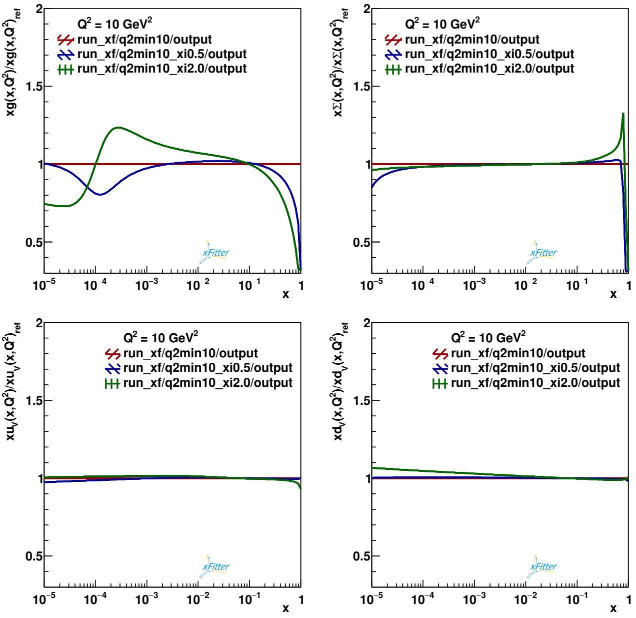
- More stable results with $Q_0 = 3.2$ GeV, $Q^2_{min} = 10$ GeV^2

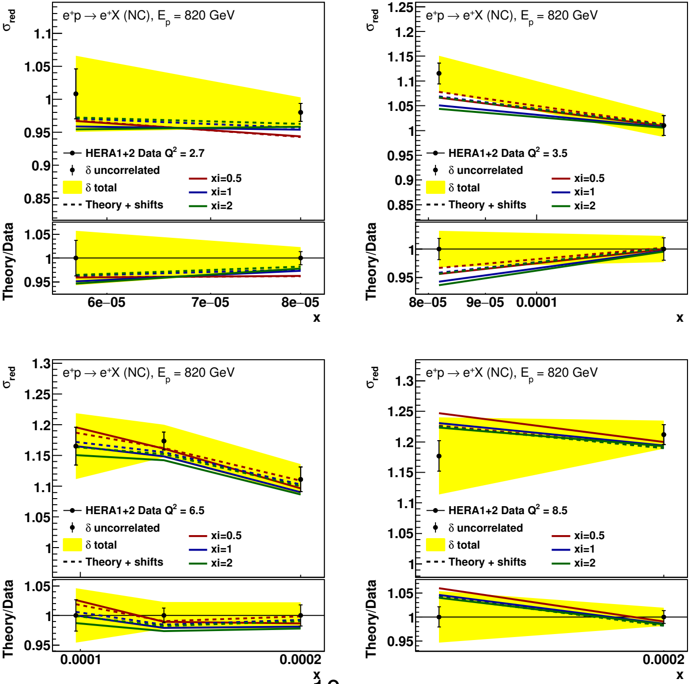
BACKUP

ξ variations ($Q^2 = 10$ GeV^2) [$Q_0 = 1.6$ GeV & $Q^2_{min} = 3.5$ GeV^2]

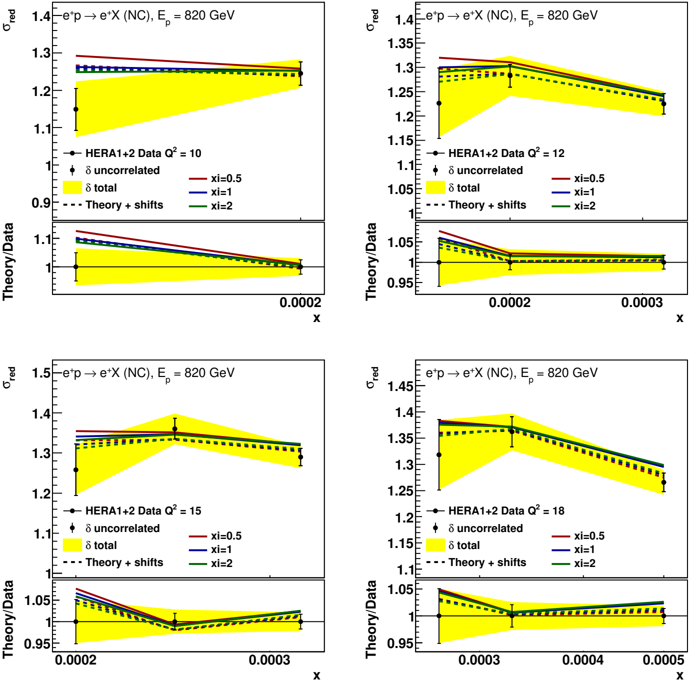


ξ variations ($Q^2 = 10$ GeV^2) [$Q_0 = 3.2$ GeV & $Q^2_{min} = 10$ GeV^2]

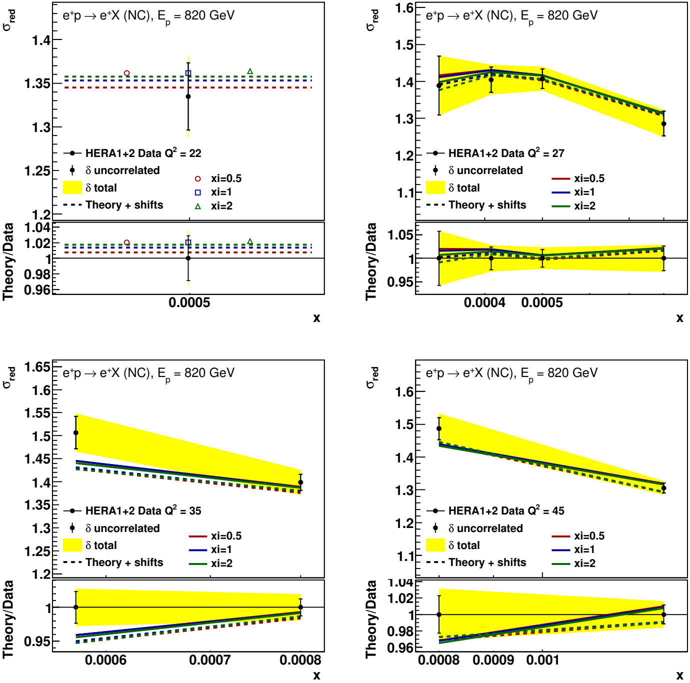




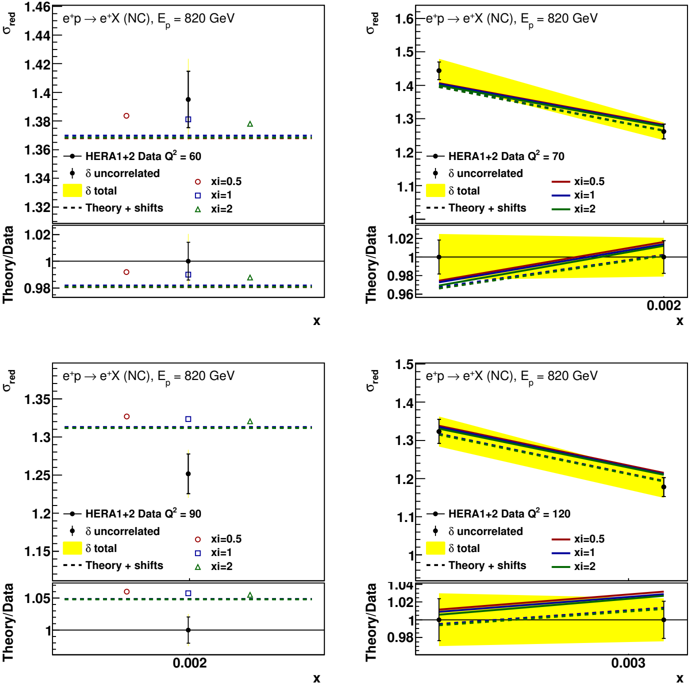
e⁺p → e⁺X (NC), E_p = 820 GeV
HERA1+2 Data Q² = 2.7
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_red
Theory/Data
x
HERA1+2 Data Q² = 3.5
HERA1+2 Data Q² = 6.5
HERA1+2 Data Q² = 8.5



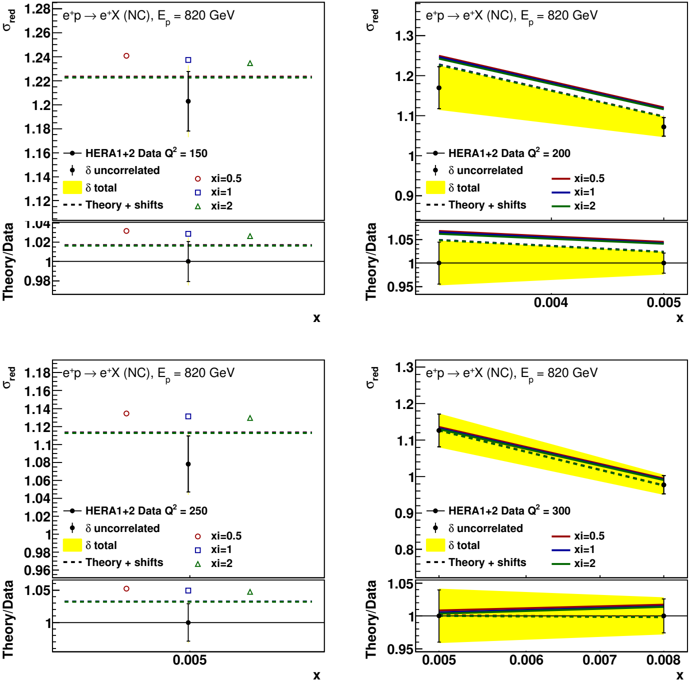
$e^+p \rightarrow e^+X$ (NC), E_p = 820 GeV
σ_{red}
Theory/Data
x
HERA1+2 Data Q^2 = 10
HERA1+2 Data Q^2 = 12
HERA1+2 Data Q^2 = 15
HERA1+2 Data Q^2 = 18
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
0.0002
0.0003
0.0004
0.0005



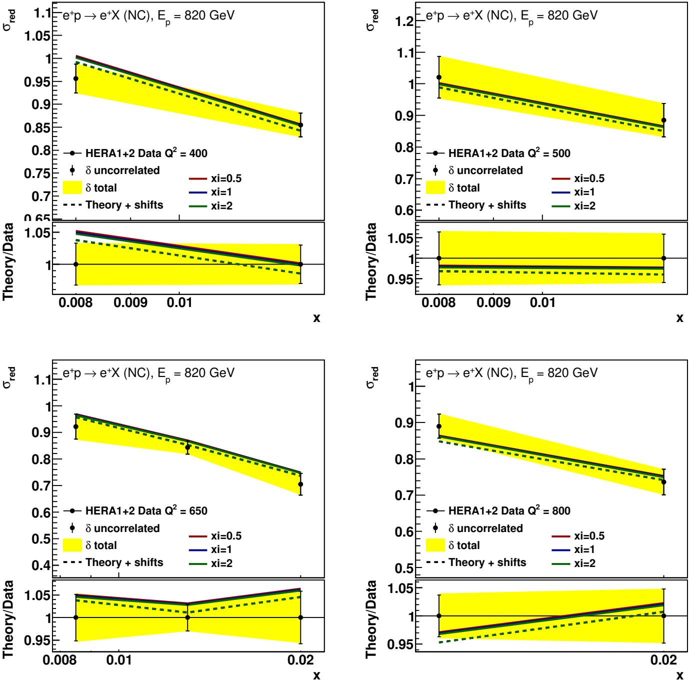
e+p → e+X (NC), Ep = 820 GeV
HERA1+2 Data Q2 = 22
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σred
Theory/Data
x
HERA1+2 Data Q2 = 27
HERA1+2 Data Q2 = 35
HERA1+2 Data Q2 = 45



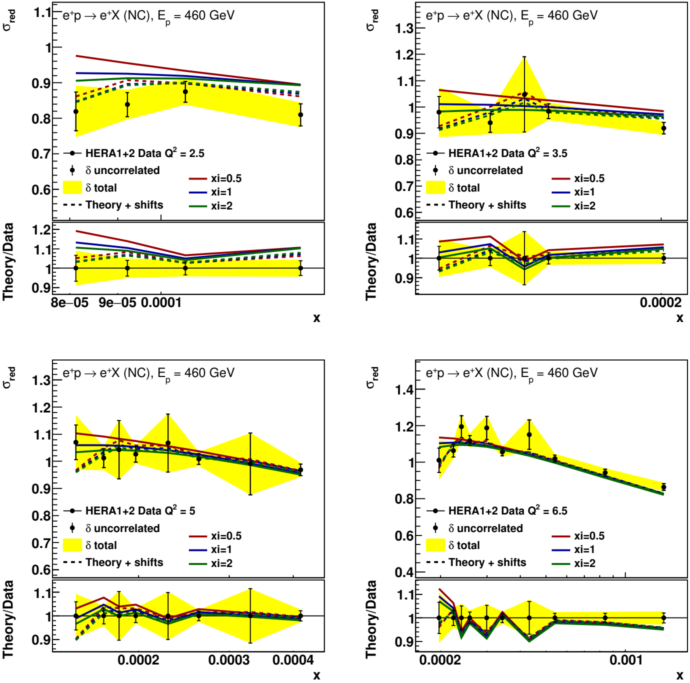
e+p → e+X (NC), Ep = 820 GeV
HERA1+2 Data Q2 = 60
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σred
Theory/Data
x
1.46
1.44
1.42
1.4
1.38
1.36
1.34
1.32
1.02
1
0.98
e+p → e+X (NC), Ep = 820 GeV
HERA1+2 Data Q2 = 70
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σred
Theory/Data
x
1.6
1.5
1.4
1.3
1.2
1.1
1
1.02
1
0.98
0.96
0.002
e+p → e+X (NC), Ep = 820 GeV
HERA1+2 Data Q2 = 90
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σred
Theory/Data
x
1.35
1.3
1.25
1.2
1.15
1.05
1
0.002
e+p → e+X (NC), Ep = 820 GeV
HERA1+2 Data Q2 = 120
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σred
Theory/Data
x
1.5
1.4
1.3
1.2
1.1
1
1.04
1.02
1
0.98
0.003



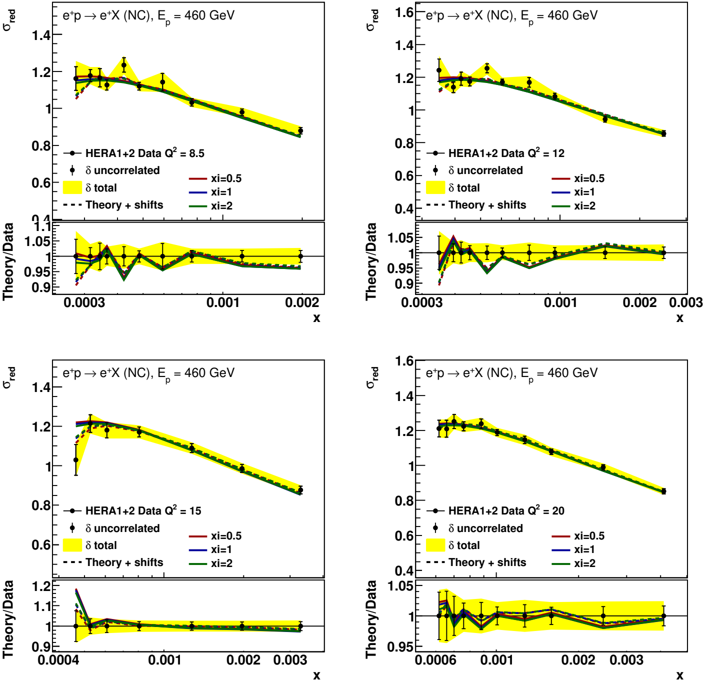
e⁺p → e⁺X (NC), E_p = 820 GeV
σ_red
Theory/Data
x
HERA1+2 Data Q^2 = 150
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
e⁺p → e⁺X (NC), E_p = 820 GeV
σ_red
Theory/Data
x
HERA1+2 Data Q^2 = 200
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
0.004
0.005
e⁺p → e⁺X (NC), E_p = 820 GeV
σ_red
Theory/Data
x
HERA1+2 Data Q^2 = 250
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
0.005
e⁺p → e⁺X (NC), E_p = 820 GeV
σ_red
Theory/Data
x
HERA1+2 Data Q^2 = 300
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
0.005
0.006
0.007
0.008

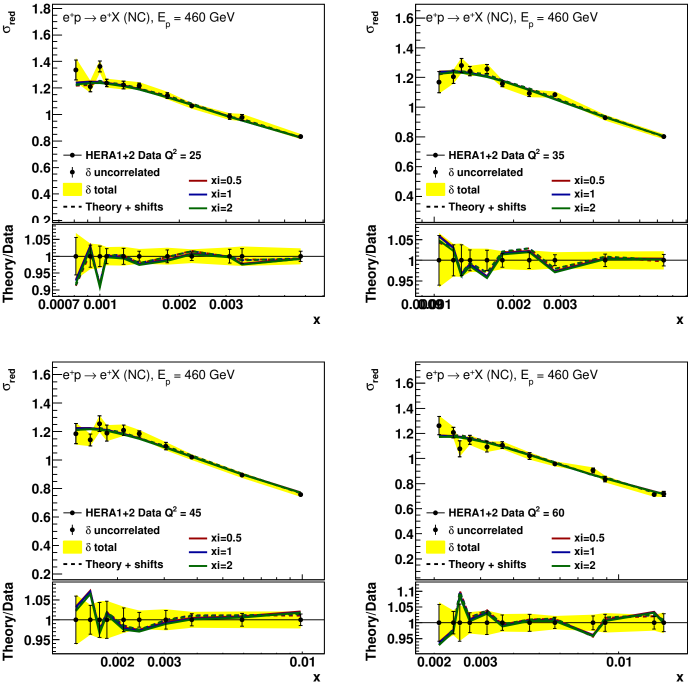


e+p → e+X (NC), Ep = 820 GeV
σred
Theory/Data
x
HERA1+2 Data Q2 = 400
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
HERA1+2 Data Q2 = 500
HERA1+2 Data Q2 = 650
HERA1+2 Data Q2 = 800
0.008
0.009
0.01
0.02

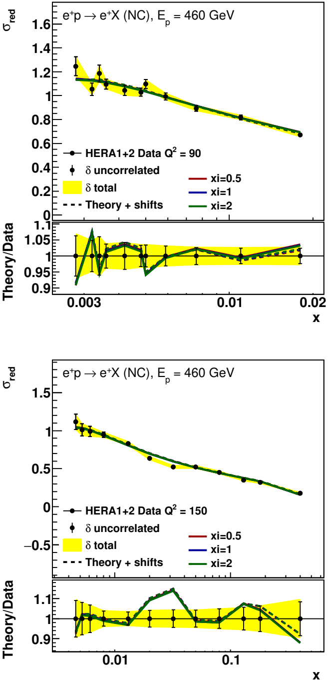


e+p → e+X (NC), Ep = 460 GeV
HERA1+2 Data Q2 = 2.5
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σred
Theory/Data
x
HERA1+2 Data Q2 = 3.5
HERA1+2 Data Q2 = 5
HERA1+2 Data Q2 = 6.5

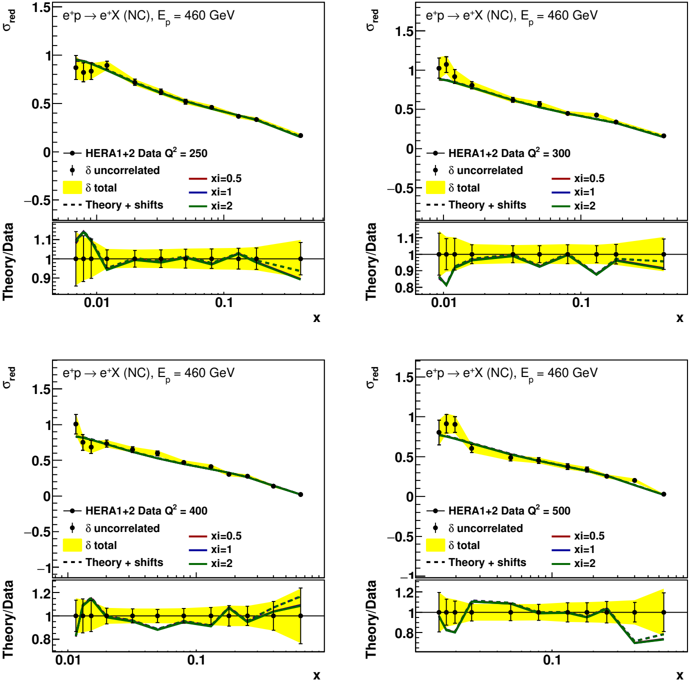

e⁺p → e⁺X (NC), E_p = 460 GeV
HERA1+2 Data Q² = 8.5
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_red
Theory/Data
x
HERA1+2 Data Q² = 12
HERA1+2 Data Q² = 15
HERA1+2 Data Q² = 20

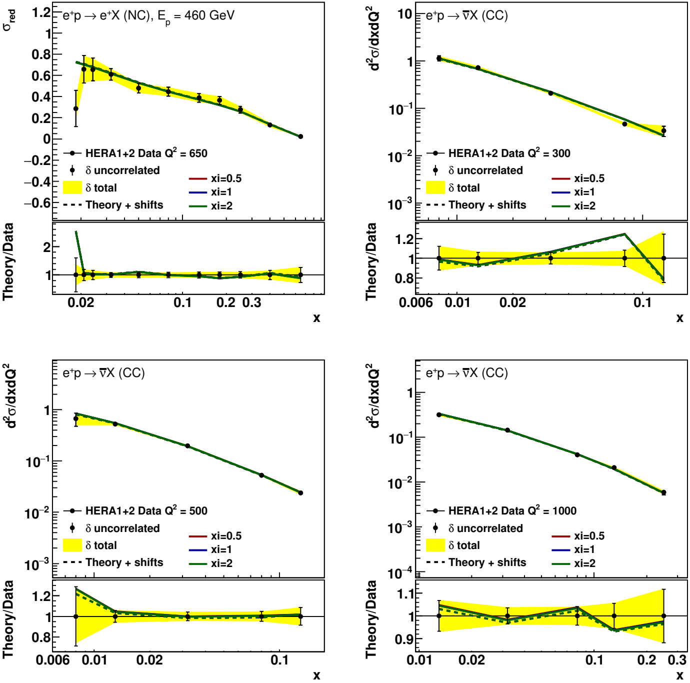


$e^+p \rightarrow e^+X$ (NC), E_p = 460 GeV
HERA1+2 Data Q^2 = 25
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
HERA1+2 Data Q^2 = 35
HERA1+2 Data Q^2 = 45
HERA1+2 Data Q^2 = 60
σ_{red}
Theory/Data
x

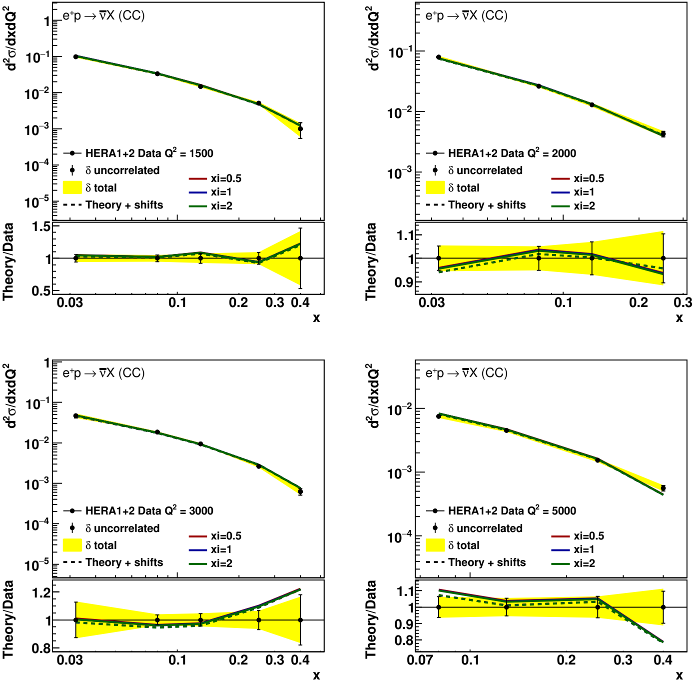


e⁺p → e⁺X (NC), E_p = 460 GeV
σ_red
Theory/Data
x
HERA1+2 Data Q² = 90
HERA1+2 Data Q² = 120
HERA1+2 Data Q² = 150
HERA1+2 Data Q² = 200
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2

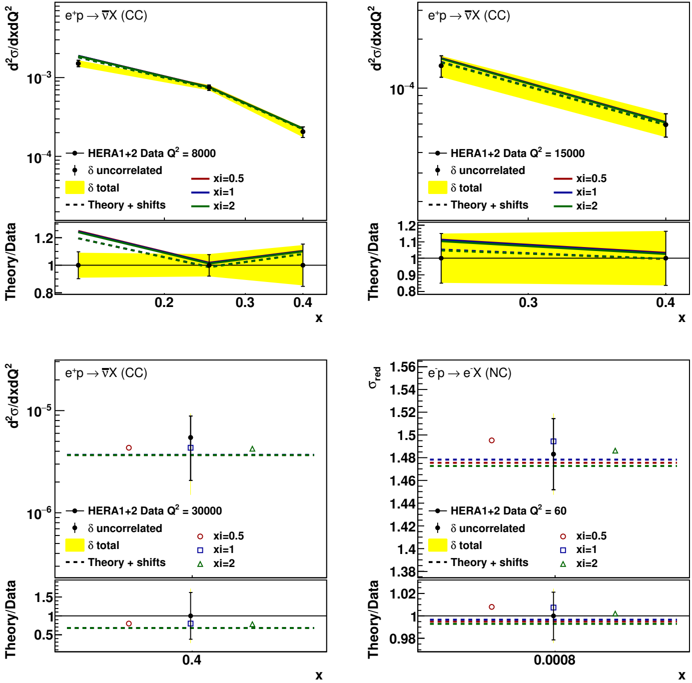

e⁺p → e⁺X (NC), E_p = 460 GeV
HERA1+2 Data Q^2 = 250
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
e⁺p → e⁺X (NC), E_p = 460 GeV
HERA1+2 Data Q^2 = 300
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
e⁺p → e⁺X (NC), E_p = 460 GeV
HERA1+2 Data Q^2 = 400
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
e⁺p → e⁺X (NC), E_p = 460 GeV
HERA1+2 Data Q^2 = 500
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_{red}
Theory/Data
x



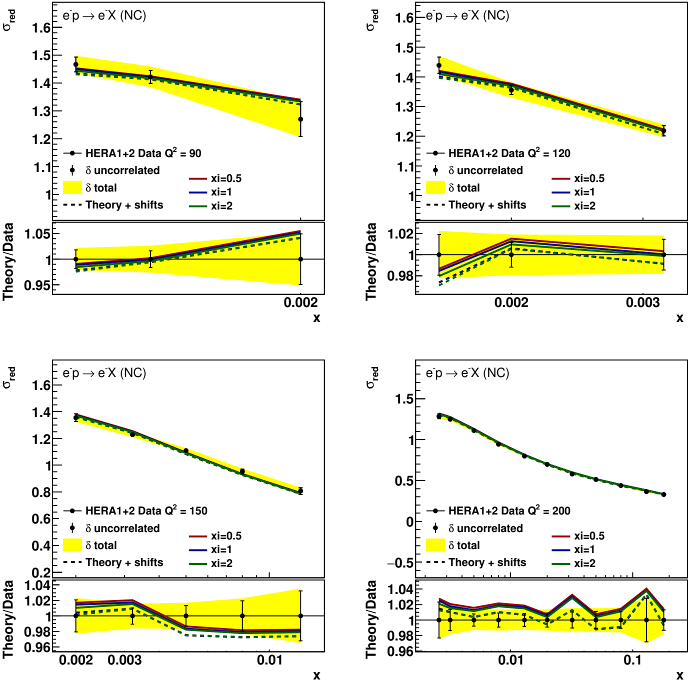
e⁺p → e⁺X (NC), E_p = 460 GeV
σ_{red}
HERA1+2 Data Q^2 = 650
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
Theory/Data
x
e⁺p → $\bar{\nu}$X (CC)
$d^2\sigma/dxdQ^2$
HERA1+2 Data Q^2 = 300
e⁺p → $\bar{\nu}$X (CC)
HERA1+2 Data Q^2 = 500
e⁺p → $\bar{\nu}$X (CC)
HERA1+2 Data Q^2 = 1000



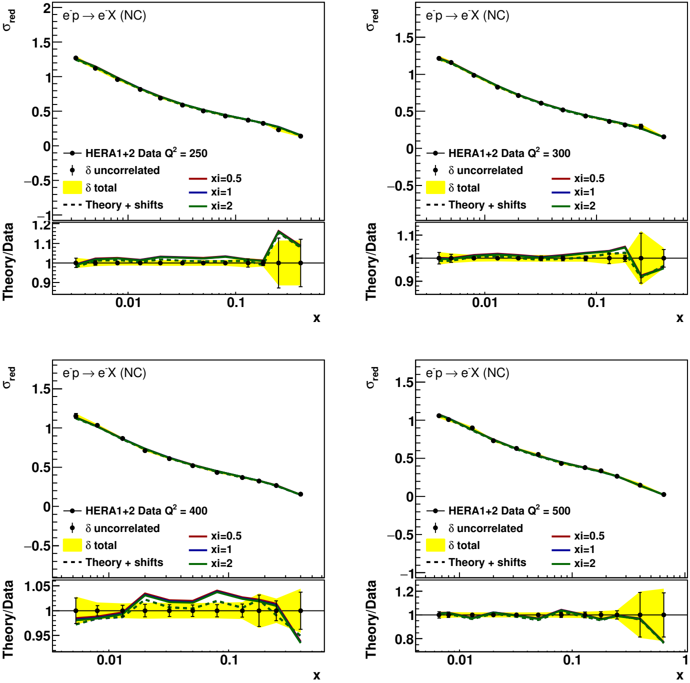
$e^+p \rightarrow \bar{\nu}X$ (CC)
HERA1+2 Data Q^2 = 1500
HERA1+2 Data Q^2 = 2000
HERA1+2 Data Q^2 = 3000
HERA1+2 Data Q^2 = 5000
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
$d^2\sigma/dxdQ^2$
Theory/Data
x



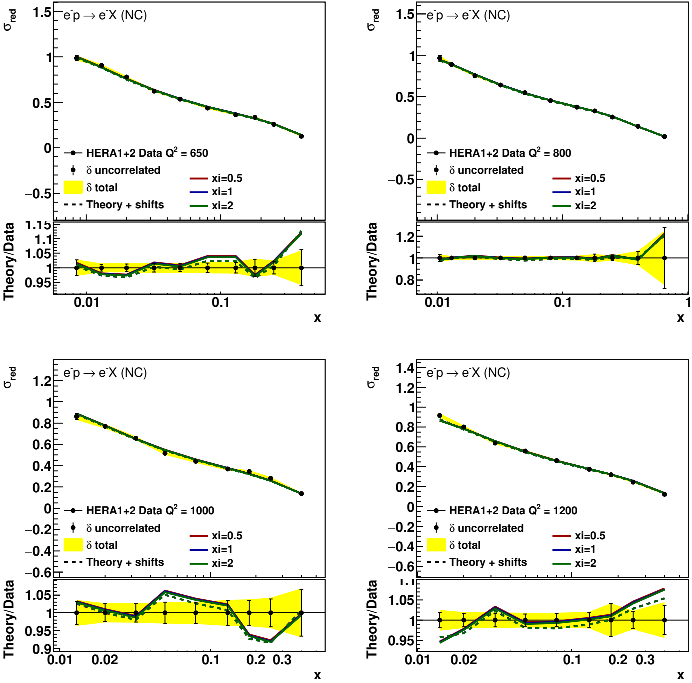
e⁺p → ν̄X (CC)
HERA1+2 Data Q² = 8000
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
d²σ/dxdQ²
Theory/Data
x
e⁺p → ν̄X (CC)
HERA1+2 Data Q² = 15000
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
d²σ/dxdQ²
Theory/Data
x
e⁺p → ν̄X (CC)
HERA1+2 Data Q² = 30000
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
d²σ/dxdQ²
Theory/Data
x
e⁻p → e⁻X (NC)
HERA1+2 Data Q² = 60
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_red
Theory/Data
x



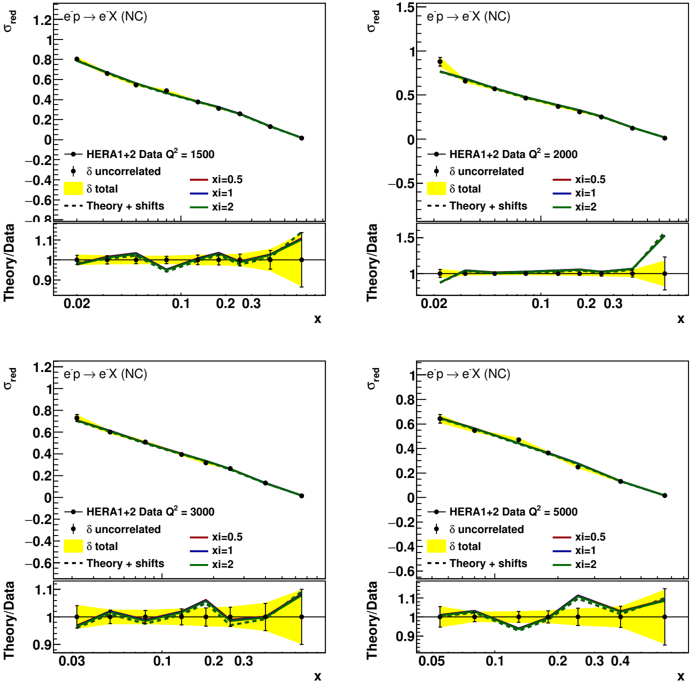
$e^-p \to e^-X$ (NC)
σ_{red}
HERA1+2 Data Q^2 = 90
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
Theory/Data
x
HERA1+2 Data Q^2 = 120
HERA1+2 Data Q^2 = 150
HERA1+2 Data Q^2 = 200

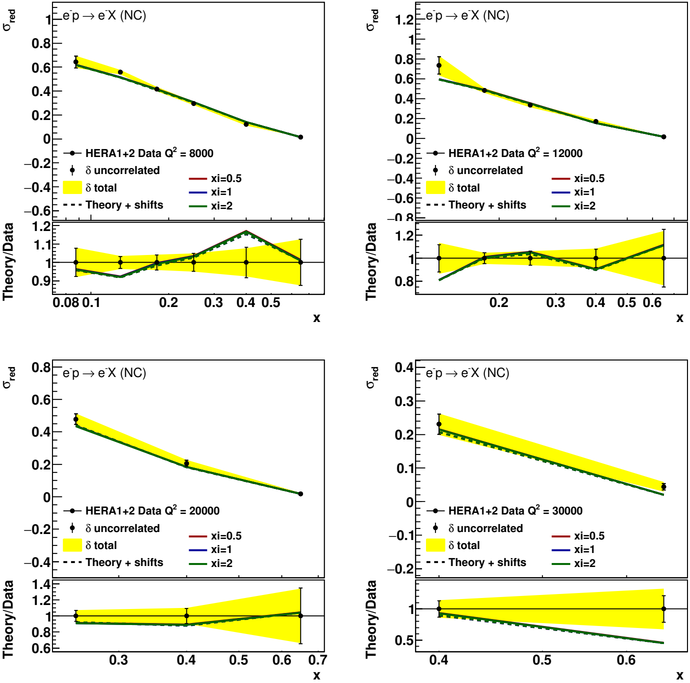


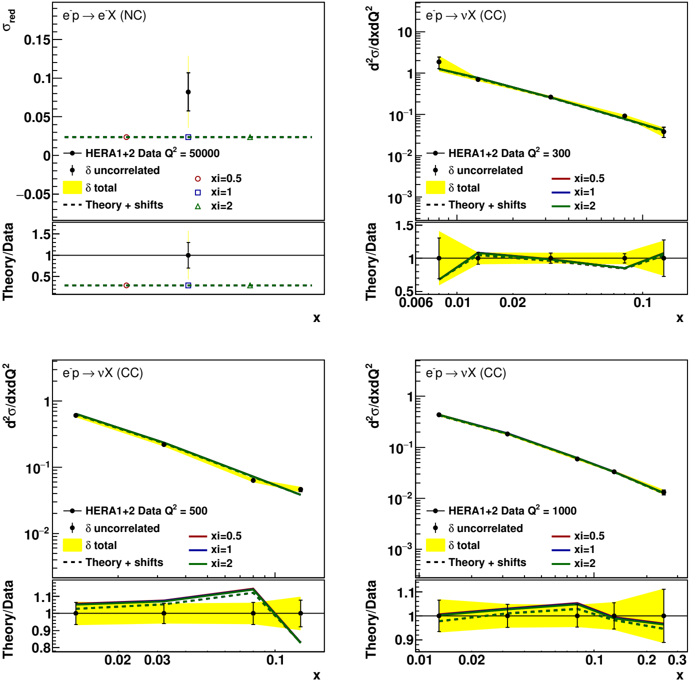
e⁻p → e⁻X (NC)
HERA1+2 Data Q² = 250
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_red
Theory/Data
x
e⁻p → e⁻X (NC)
HERA1+2 Data Q² = 300
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_red
Theory/Data
x
e⁻p → e⁻X (NC)
HERA1+2 Data Q² = 400
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_red
Theory/Data
x
e⁻p → e⁻X (NC)
HERA1+2 Data Q² = 500
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_red
Theory/Data
x



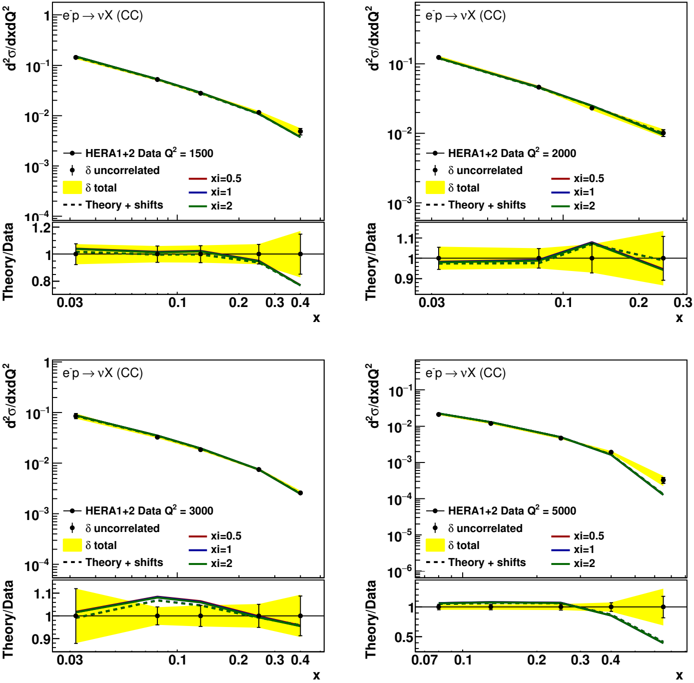
e⁻p → e⁻X (NC)
HERA1+2 Data Q^2 = 650
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
HERA1+2 Data Q^2 = 800
HERA1+2 Data Q^2 = 1000
HERA1+2 Data Q^2 = 1200
σ_{red}
Theory/Data
x


e⁻p → e⁻X (NC)
HERA1+2 Data Q^2 = 1500
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
HERA1+2 Data Q^2 = 2000
HERA1+2 Data Q^2 = 3000
HERA1+2 Data Q^2 = 5000
σ_{red}
Theory/Data
x

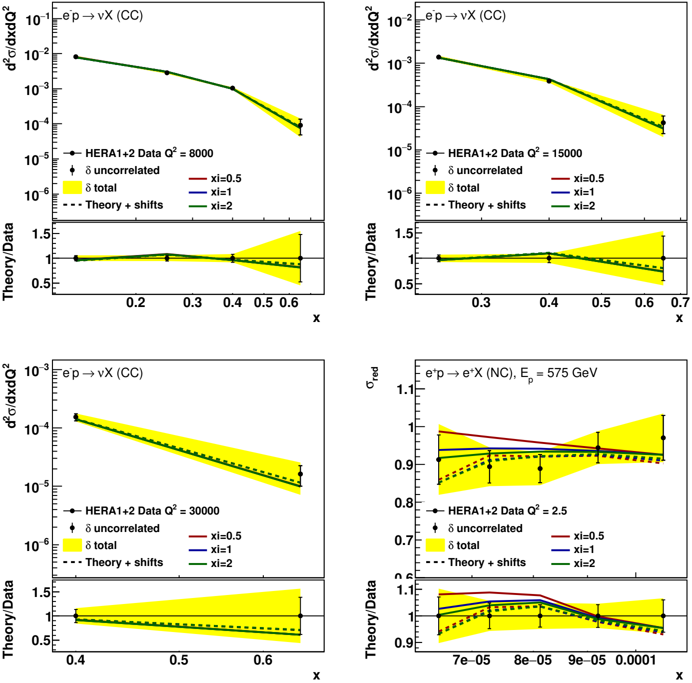

e⁻p → e⁻X (NC)
HERA1+2 Data Q^2 = 8000
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
HERA1+2 Data Q^2 = 12000
HERA1+2 Data Q^2 = 20000
HERA1+2 Data Q^2 = 30000
σ_{red}
Theory/Data
x



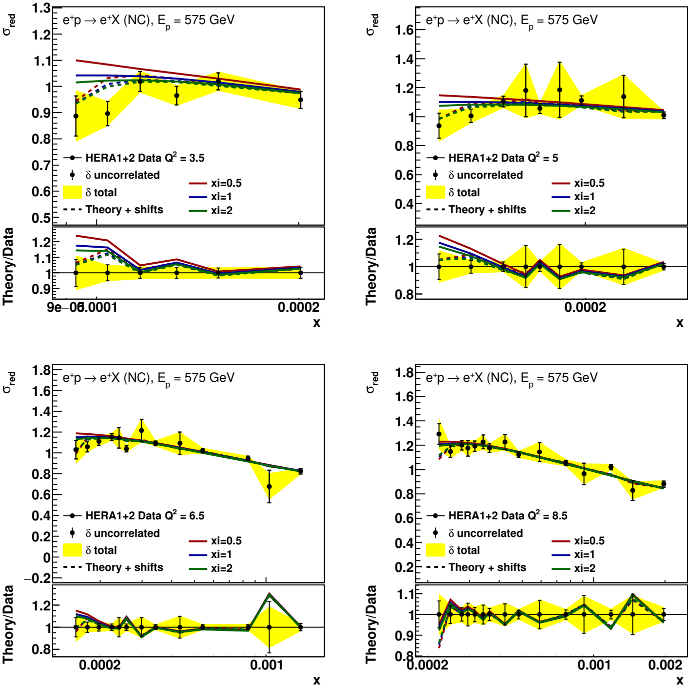
$e^-p \to e^-X$ (NC)
σ_{red}
HERA1+2 Data Q^2 = 50000
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
Theory/Data
x
$e^-p \to \nu X$ (CC)
$d^2\sigma/dxdQ^2$
HERA1+2 Data Q^2 = 300
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
Theory/Data
x
$e^-p \to \nu X$ (CC)
$d^2\sigma/dxdQ^2$
HERA1+2 Data Q^2 = 500
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
Theory/Data
x
$e^-p \to \nu X$ (CC)
$d^2\sigma/dxdQ^2$
HERA1+2 Data Q^2 = 1000
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
Theory/Data
x



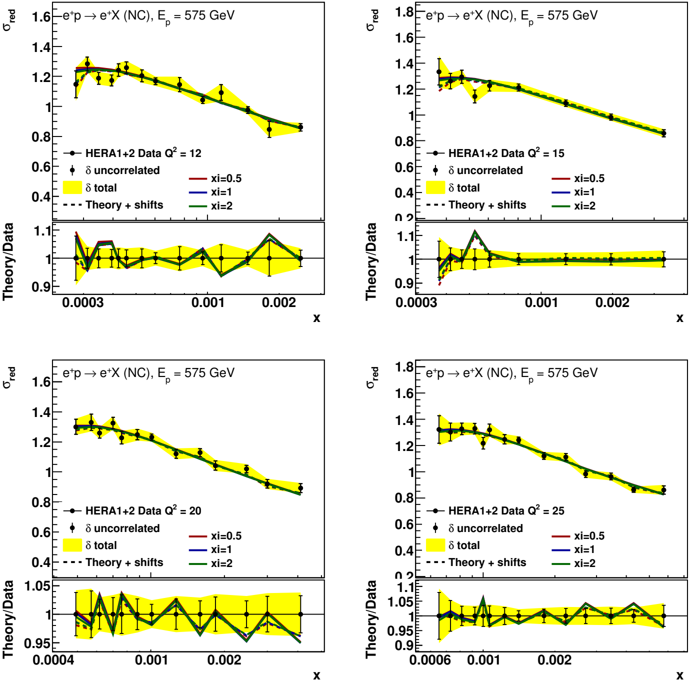
$e^- p \to \nu X$ (CC)
HERA1+2 Data Q^2 = 1500
HERA1+2 Data Q^2 = 2000
HERA1+2 Data Q^2 = 3000
HERA1+2 Data Q^2 = 5000
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
$d^2\sigma/dxdQ^2$
Theory/Data
x



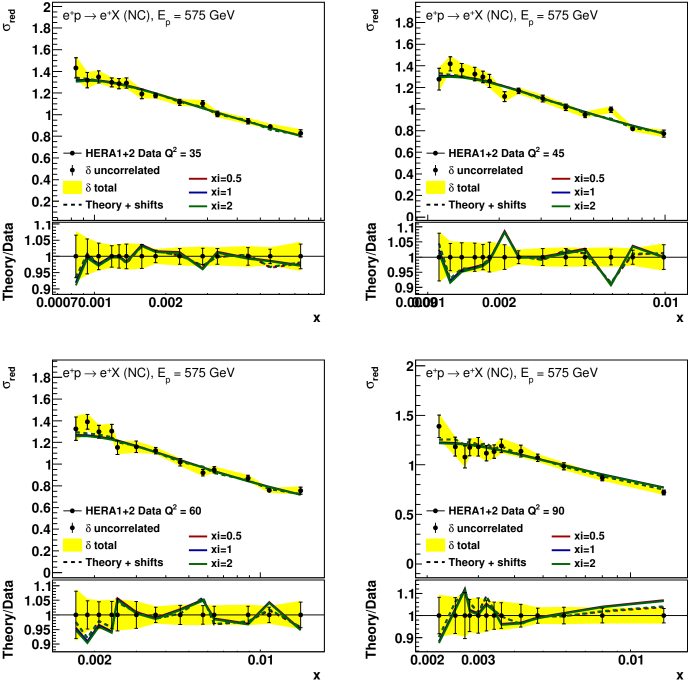
e⁻p → νX (CC)
HERA1+2 Data Q² = 8000
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
d²σ/dxdQ²
Theory/Data
x
e⁻p → νX (CC)
HERA1+2 Data Q² = 15000
e⁻p → νX (CC)
HERA1+2 Data Q² = 30000
e⁺p → e⁺X (NC), E_p = 575 GeV
HERA1+2 Data Q² = 2.5
σ_red



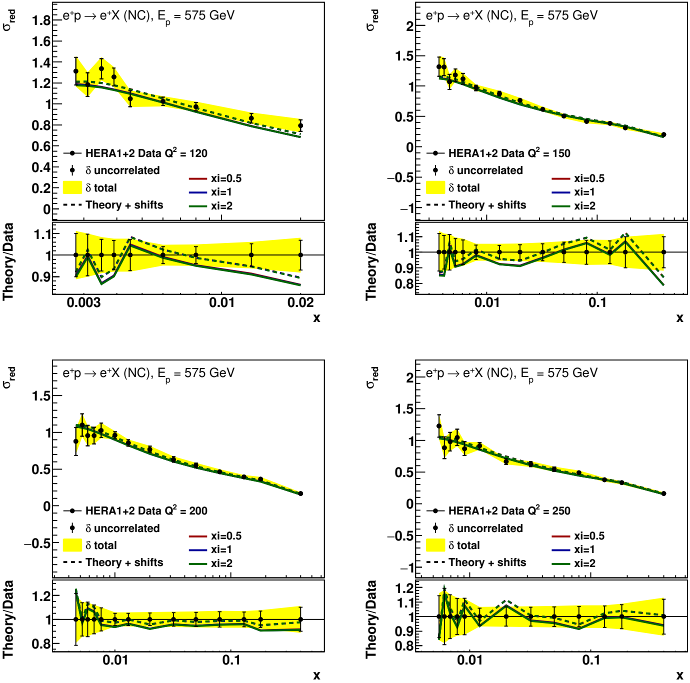
$e^+p \rightarrow e^+X$ (NC), E_p = 575 GeV
HERA1+2 Data Q^2 = 3.5
HERA1+2 Data Q^2 = 5
HERA1+2 Data Q^2 = 6.5
HERA1+2 Data Q^2 = 8.5
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_{red}
Theory/Data
x



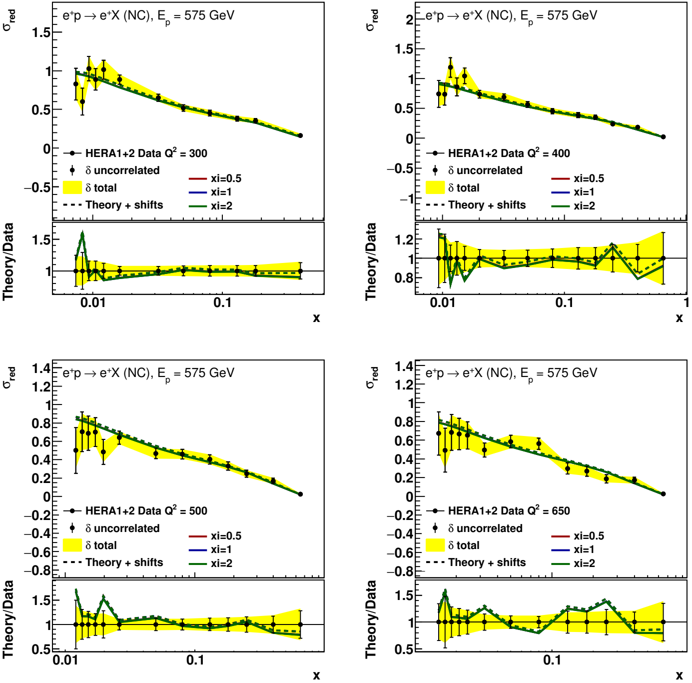
e⁺p → e⁺X (NC), E_p = 575 GeV
HERA1+2 Data Q² = 12
HERA1+2 Data Q² = 15
HERA1+2 Data Q² = 20
HERA1+2 Data Q² = 25
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_red
Theory/Data
x



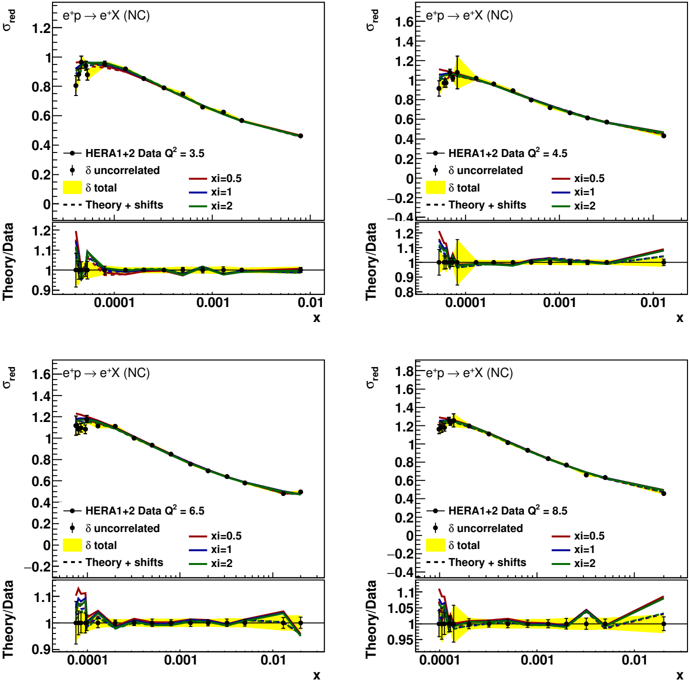
$e^+p \rightarrow e^+X$ (NC), E_p = 575 GeV
HERA1+2 Data Q^2 = 35
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_{red}
Theory/Data
x
0.00070.001
0.002
$e^+p \rightarrow e^+X$ (NC), E_p = 575 GeV
HERA1+2 Data Q^2 = 45
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_{red}
Theory/Data
x
0.002
0.01
$e^+p \rightarrow e^+X$ (NC), E_p = 575 GeV
HERA1+2 Data Q^2 = 60
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_{red}
Theory/Data
x
0.002
0.01
$e^+p \rightarrow e^+X$ (NC), E_p = 575 GeV
HERA1+2 Data Q^2 = 90
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_{red}
Theory/Data
x
0.002
0.003
0.01



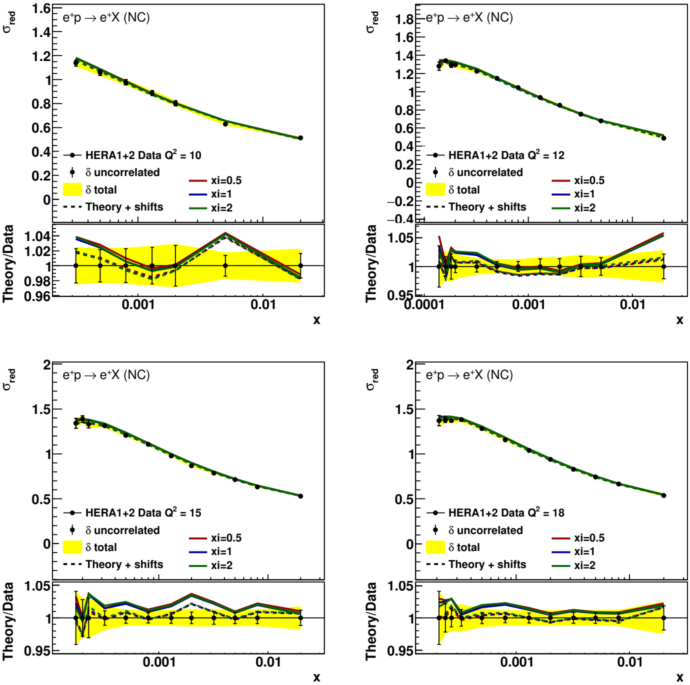
σ_{red}
$e^{+}p \rightarrow e^{+}X$ (NC), E_p = 575 GeV
HERA1+2 Data Q^2 = 120
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
Theory/Data
x
HERA1+2 Data Q^2 = 150
HERA1+2 Data Q^2 = 200
HERA1+2 Data Q^2 = 250



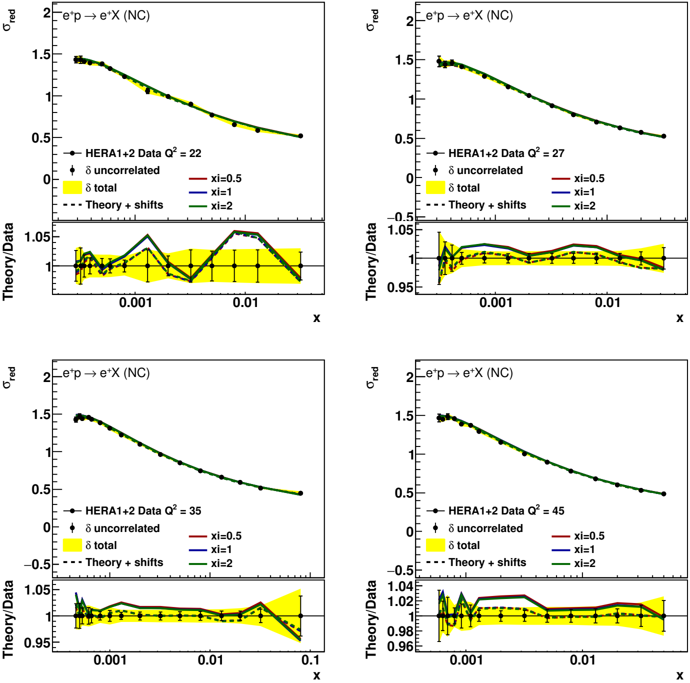
$e^+p \rightarrow e^+X$ (NC), E_p = 575 GeV
σ_{red}
HERA1+2 Data Q^2 = 300
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
Theory/Data
x
HERA1+2 Data Q^2 = 400
HERA1+2 Data Q^2 = 500
HERA1+2 Data Q^2 = 650



e+p → e+X (NC)
HERA1+2 Data Q2 = 3.5
HERA1+2 Data Q2 = 4.5
HERA1+2 Data Q2 = 6.5
HERA1+2 Data Q2 = 8.5
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σred
Theory/Data
x
0.0001
0.001
0.01



e+p → e+X (NC)
HERA1+2 Data Q2 = 10
HERA1+2 Data Q2 = 12
HERA1+2 Data Q2 = 15
HERA1+2 Data Q2 = 18
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σred
Theory/Data
x



$e^+p \rightarrow e^+X$ (NC)
HERA1+2 Data Q^2 = 22
δ uncorrelated
δ total
Theory + shifts
xi=0.5
xi=1
xi=2
σ_{red}
Theory/Data
x
$e^+p \rightarrow e^+X$ (NC)
HERA1+2 Data Q^2 = 27
$e^+p \rightarrow e^+X$ (NC)
HERA1+2 Data Q^2 = 35
$e^+p \rightarrow e^+X$ (NC)
HERA1+2 Data Q^2 = 45

Dataset	xi=0.5	xi=1	xi=2
HERA1+2 NCep 820	77 / 70	72 / 70	72 / 70
HERA1+2 NCep 460	227 / 204	223 / 204	223 / 204
HERA1+2 CCep	37 / 39	37 / 39	38 / 39
HERA1+2 NCem	216 / 159	216 / 159	218 / 159
HERA1+2 CCem	50 / 42	50 / 42	50 / 42
HERA1+2 NCep 575	230 / 254	222 / 254	221 / 254
HERA1+2 NCep 920	413 / 377	402 / 377	404 / 377
Correlated χ^2	82	75	76
Log penalty χ^2	+7.7	+1.8	-2.08
Total χ^2 / dof	1340 / 1127	1298 / 1127	1298 / 1127
χ^2 p-value	0.00	0.00	0.00

Parameter	xi=0.5	xi=1	xi=2
'Adbar'	0.085 ± 0.015	0.080 ± 0.016	0.083 ± 0.016
'Adv'	1.0000	1.0000	1.0000
'Ag'	1.0000	1.0000	1.0000
'Auv'	1.0000	1.0000	1.0000
'Bdbar'	-0.342 ± 0.033	-0.336 ± 0.042	-0.329 ± 0.043
'Bdv'	0.984 ± 0.099	0.985 ± 0.098	0.972 ± 0.097
'Bg'	-0.557 ± 0.041	-0.530 ± 0.044	-0.523 ± 0.042
'Buv'	0.86 ± 0.16	0.84 ± 0.17	0.83 ± 0.18
'Cdbar'	22.7 ± 4.8	23.3 ± 5.0	23.1 ± 5.1
'Cdv'	4.67 ± 0.47	4.66 ± 0.46	4.68 ± 0.46
'Cg'	4.78 ± 0.64	4.72 ± 0.64	5.53 ± 0.69
'Cubar'	11.0 ± 1.4	10.9 ± 1.4	10.9 ± 1.4
'Cuv'	4.51 ± 0.24	4.53 ± 0.22	4.58 ± 0.22
'Ddbar'	35 ± 16	38 ± 17	36 ± 17
'Dubar'	18.8 ± 5.6	19.7 ± 6.0	19.6 ± 5.8
'Euv'	1.5 ± 1.7	1.7 ± 1.8	1.9 ± 1.9
'Fdbar'	0.0789 ± 0.0045	0.0702 ± 0.0079	0.0673 ± 0.0091
'Fg'	0.2278 ± 0.0037	0.2268 ± 0.0042	0.2222 ± 0.0042
'Fuv'	0.378 ± 0.069	0.372 ± 0.068	0.369 ± 0.070
'Gg'	0.01273 ± 0.00045	0.01289 ± 0.00047	0.01254 ± 0.00044
'Guv'	0.064 ± 0.026	0.061 ± 0.026	0.060 ± 0.027
'fs'	0.4000	0.4000	0.4000
Fit status	converged	converged	converged
Uncertainties	migrad-hesse	migrad-hesse	migrad-hesse