High Energy neutrino cross-sections

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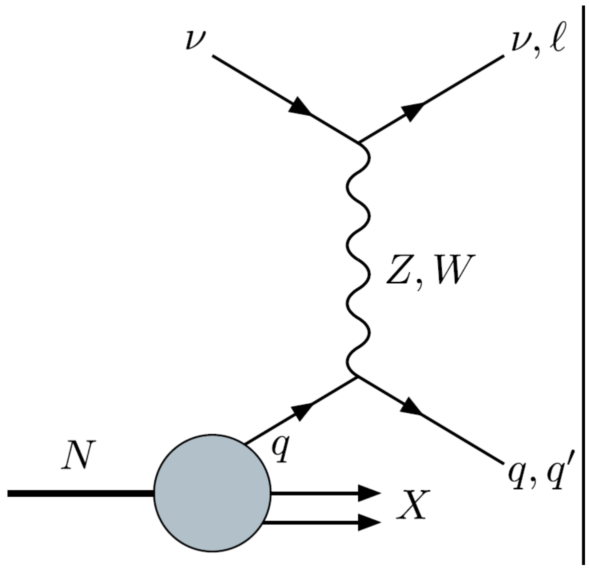
- Predictions of high energy ν and $\bar{\nu}$ CC and NC cross-sections can be made within the conventional framework of NLO QCD using the DGLAP formalism
- This depends on knowledge of the Parton Distribution Functions (PDFs) and their uncertainties

The point is to estimate the uncertainties of conventional predictions in order to see when we have unconventional behaviour

Since high energy neutrino cross sections probe the partons at very low Bjorken x we may expect unconventional behaviour from

- ln(1/x) resummation-- BFKL
- non-linear effects: gluon recombination, saturation
  BK, JIMWLK, colour glass condensate ...
- Need to respect the Froissart bound

ν cross-section are Deep Inelastic Scattering processes which can be described in terms of partons



Double differential cross-section

$$\frac{\mathrm{d}^2\sigma(\nu(\bar{\nu})N)}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{G_F^2 M_W^4}{4\pi(Q^2+M_W^2)^2 x}\sigma_r(\nu(\bar{\nu})N)$$

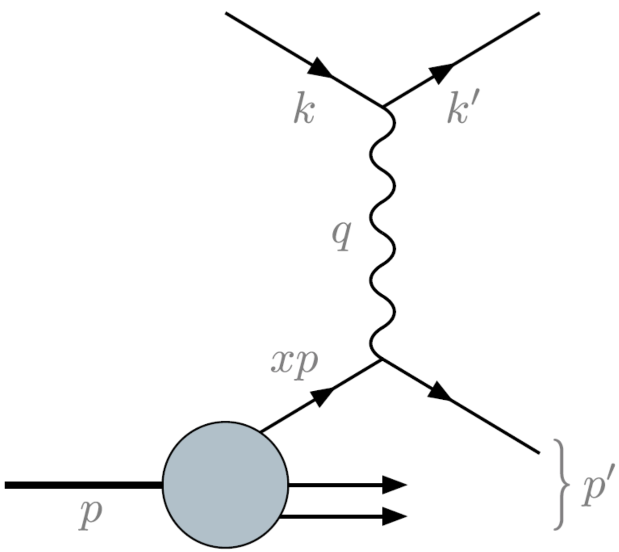
with reduced cross-section

$$\sigma_r(\nu(\bar{\nu})N) = \left[Y_+ F_2^\nu(x,Q^2) - y^2 F_L^\nu(x,Q^2) \pm Y_- x F_3^\nu(x,Q^2)\right]$$

where $Y_\pm = 1 \pm (1-y)^2$.

Total cross-section

$$\sigma = \int \mathrm{d}x \int \mathrm{d}Q^2 \, \frac{\mathrm{d}^2\sigma(\nu(\bar{\nu})N)}{\mathrm{d}x\,\mathrm{d}Q^2}$$



four Lorentz invariants:

- centre of mass energy $\sqrt{s}$
  $s = (p+k)^2$
- momentum transfer
  $Q^2 = -q^2 = -(k-k')^2$
- Bjorken scaling variable
  $x = Q^2/(2p \cdot q)$
- inelasticity
  $y = p \cdot q/(p \cdot k)$

In the Quark-Parton Model the structure functions relate simply to the parton distributions

For neutrino interactions

$$F_2^{\nu} = x(u + d + 2s + 2b + \bar{u} + \bar{d} + 2\bar{c}), \quad xF_3^{\nu} = x(u + d + 2s + 2b - \bar{u} - \bar{d} - 2\bar{c}),$$

and for antineutrino interactions,

And F_L=0 for both

$$F_2^{\bar{\nu}} = x(u + d + 2c + \bar{u} + \bar{d} + 2\bar{s} + 2\bar{b}), \quad xF_3^{\bar{\nu}} = x(u + d + 2c - \bar{u} - \bar{d} - 2\bar{s} - 2\bar{b}).$$

But we need to go beyond the QPM to QCD . We need QCD calculations to at least next-to- leading-order in α_S. The relationship of structure functions to parton distributions gets somewhat more complicated.. but structure functions are still fully calculable from the parton distributions

structure functions

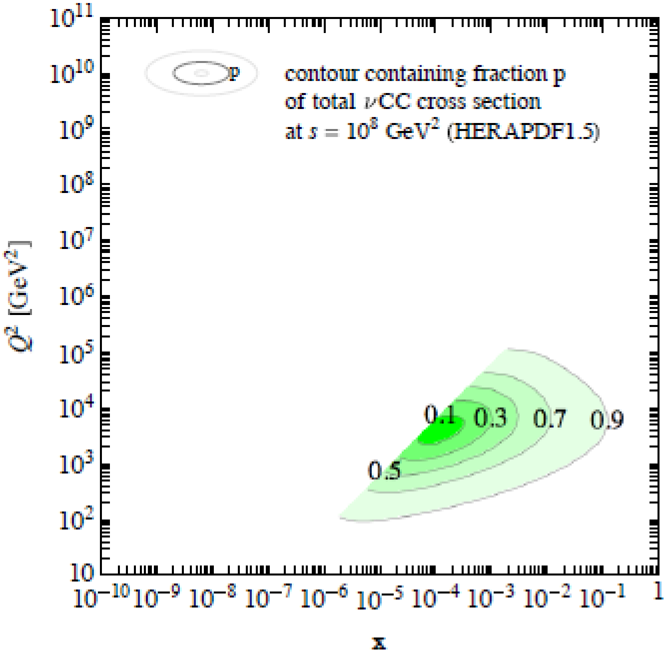
$$F_2 = \int_x^1 \frac{\mathrm{d}\xi}{\xi} \left[\sum_i e_i^2 \, xq_i(\xi, Q^2) C_q\left(\frac{x}{\xi}, \alpha_s\right) + \left(\sum_i e_i^2\right) xg(\xi, Q^2) C_g\left(\frac{x}{\xi}, \alpha_s\right) \right]$$

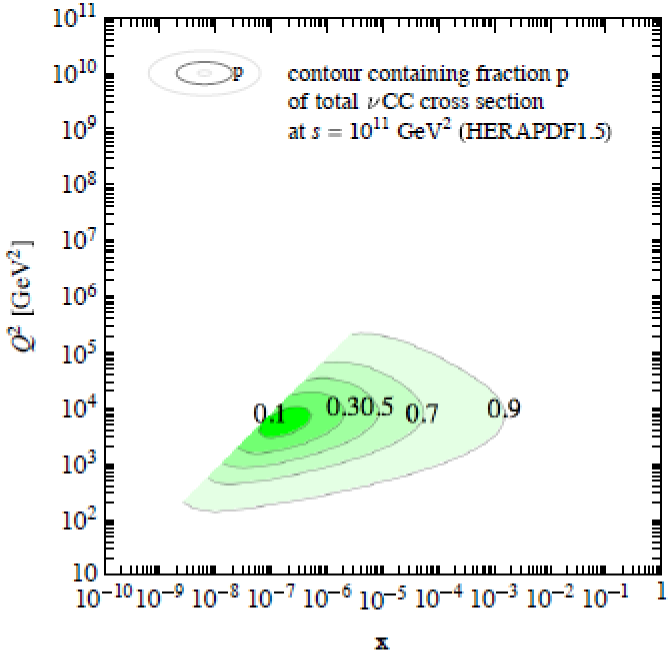
$$F_L = \ldots$$

$$xF_3 = \ldots$$

C_q, C_g: coefficient functions

So we need to know the parton distributions
And we need to know them for low-x
Lower and lower as the neutrino energy gets higher and higher





Kinematic region probed for two neutrino energies

How do we determine parton distributions?

We do not yet have the ability to calculate them – this would involve the non-perturbative part of QCD (e.g. lattice calculations)

However we do know how they evolve with the scale of the probe Q^2

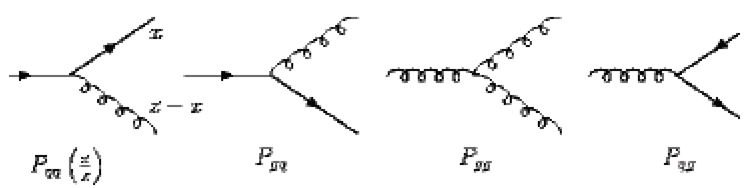
DGLAP evolution

$$\frac{\partial q^{\mathrm{NS}}(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\left(q^{\mathrm{NS}} \otimes P_{qq}\right)$$

$$\frac{\partial \Sigma(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\left(\Sigma \otimes P_{qq} + g \otimes 2n_f P_{qg}\right)$$

$$\frac{\partial \Sigma(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\left(\Sigma \otimes P_{gq} + g \otimes 2n_f P_{gg}\right)$$

Σ and q^{NS} are convenient linear combinations of quark PDFs.



Formally the DGLAP equations re-summ contributions in $\ln Q^2$ ….. See later

So if we know them at one scale Q^2_0 then we can know them at all other scales
What we do is FIT parametrisations at Q^2_0 to Deep Inelastic Scattering data that has already been measured e.g at HERA and earlier fixed-target experiments

How do we determine parton distributions?

Parametrise the parton distribution functions (PDFs) at Q^2_0 (~1-7 GeV^2)

$$xu_v(x) = A_u x^{au} (1-x)^{bu} (1+ \varepsilon_u \sqrt{x} + \gamma_u x)$$

$$xd_v(x) = A_d x^{ad} (1-x)^{bd} (1+ \varepsilon_d \sqrt{x} + \gamma_d x)$$

$$xS(x) = A_s x^{-\lambda s} (1-x)^{bs} (1+ \varepsilon_s \sqrt{x} + \gamma_s x)$$

$$xg(x) = A_g x^{-\lambda g} (1-x)^{bg} (1+ \varepsilon_g \sqrt{x} + \gamma_g x)$$

The light quark sea

These parameters control the low-x shape

These parameters control the middling-x shape- the form of the polynomial can vary

These parameters control the high-x shape

The heavy partons c,b are not parametrized but calculated from boson-gluon fusion.

Parameters Ag, Au, Ad are fixed through momentum and number sum rules – other parameters may be fixed by model choices-

Model choices ⇒Form of parametrization at Q^2_0, value of Q^2_0, cuts applied, heavy flavour scheme, value of m_c,m_b, α_S

→ typically ~15-22 parameters free

Use the DGLAP equations to NLO in QCD to evolve these PDFs to $Q^2 > Q^2_0$

Construct the measurable structure functions by convoluting PDFs with coefficient functions: make predictions for ~3000 data points of Deep Inelastic Scattering cross-sections

Perform χ2 fit to the data

The fact that so few parameters allows us to fit so many data points established QCD as the THEORY OF THE STRONG INTERACTION and provided the first measurements of α_s (as one of the fit parameters)

Data are cross sections for lepton (e,μ,ν, $\bar{\nu}$) hadron (p,n) scattering by both neutral (γ,Z) and charged ($W^{\pm}$) currents: this yields many different combinations of partons

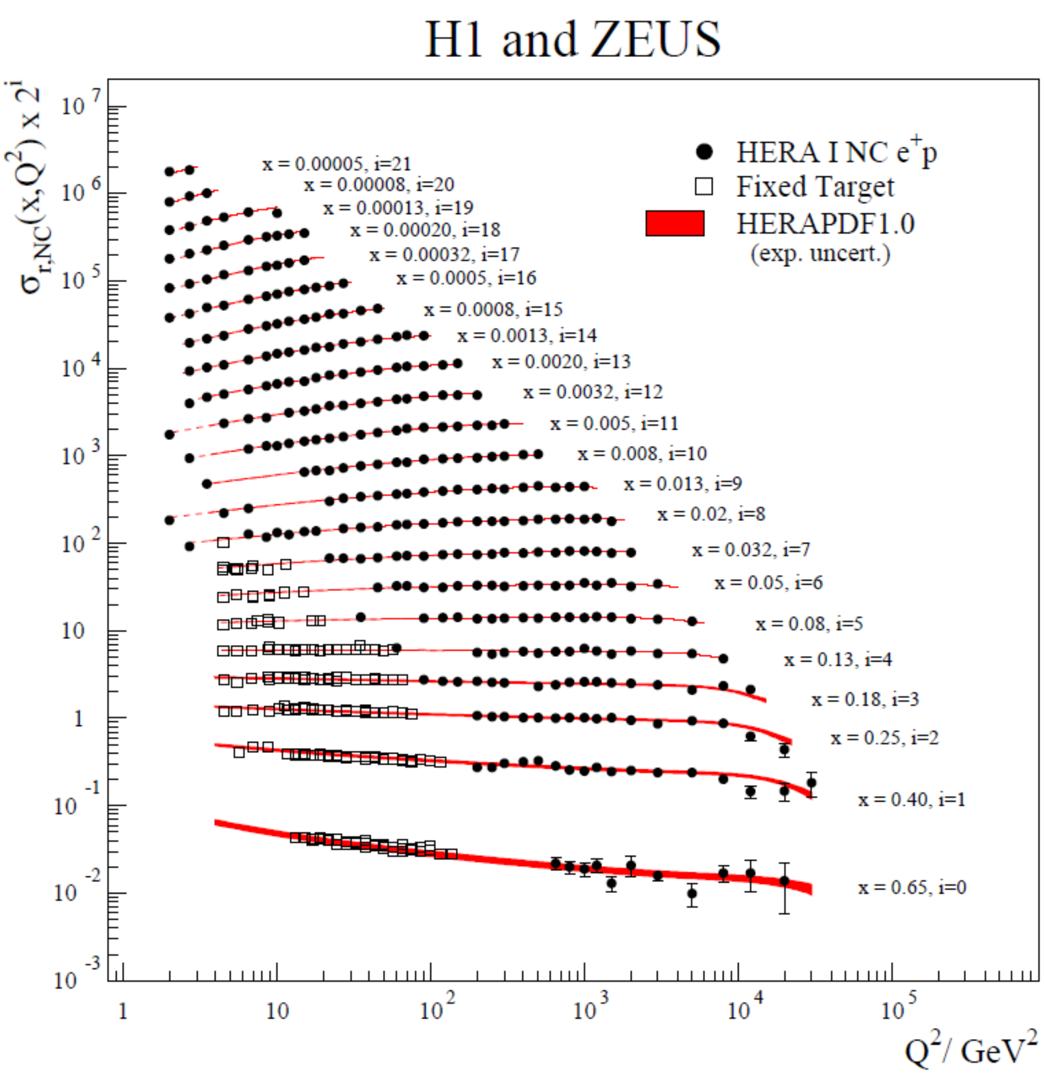
Deep Inelastic Scattering data are now dominated by the HERA data

These also probe to the lowest-x most relevant for high energy neutrino cross sections

Recently there has been a very high accuracy combination of the HERA

Experimental data from ZEUS and H1 (JHEP101(2010)109)

Determinations which do not use these data are significantly out of date.

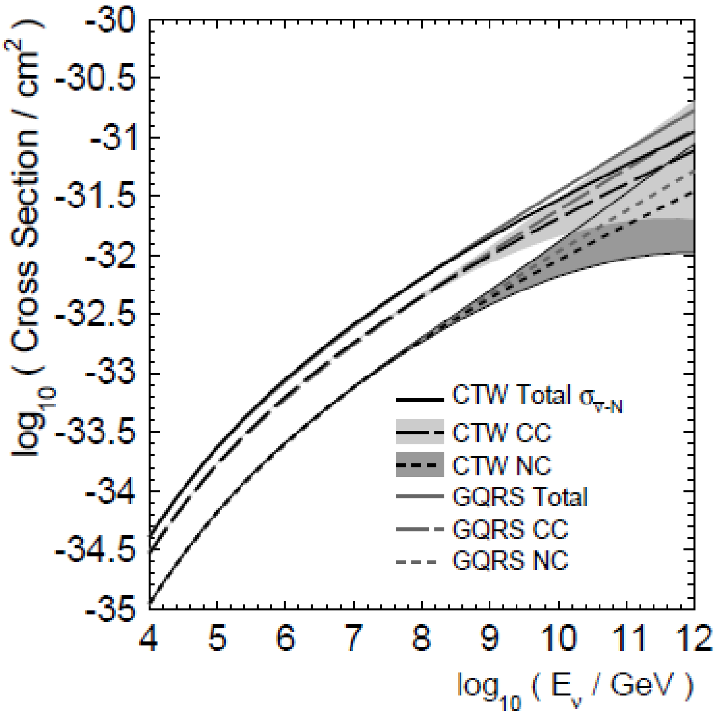


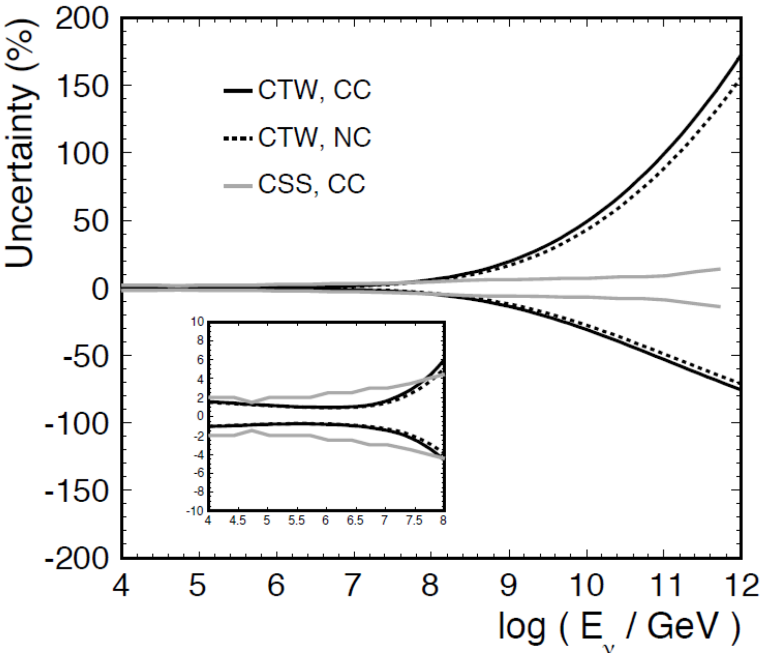
High energy neutrino cross sections were first given by Gandhi et al in 1996 (PRD58(1996)093009)

This used CTEQ4-DIS PDFs and is significantly out of date, since the most extensive and accurate HERA data at low-x were published well after this date.

Even the later work of C-SS (JHEP0901(2008)075) using ZEUS 2005PDFs

and CTW (Phys Rev D83 (2011)113009) using MSTW 2008 PDFs is now out of date





But today discrepancies are more significant for the estimate of uncertainties rather than the central value. We will investigate these differences in the estimate of uncertainties

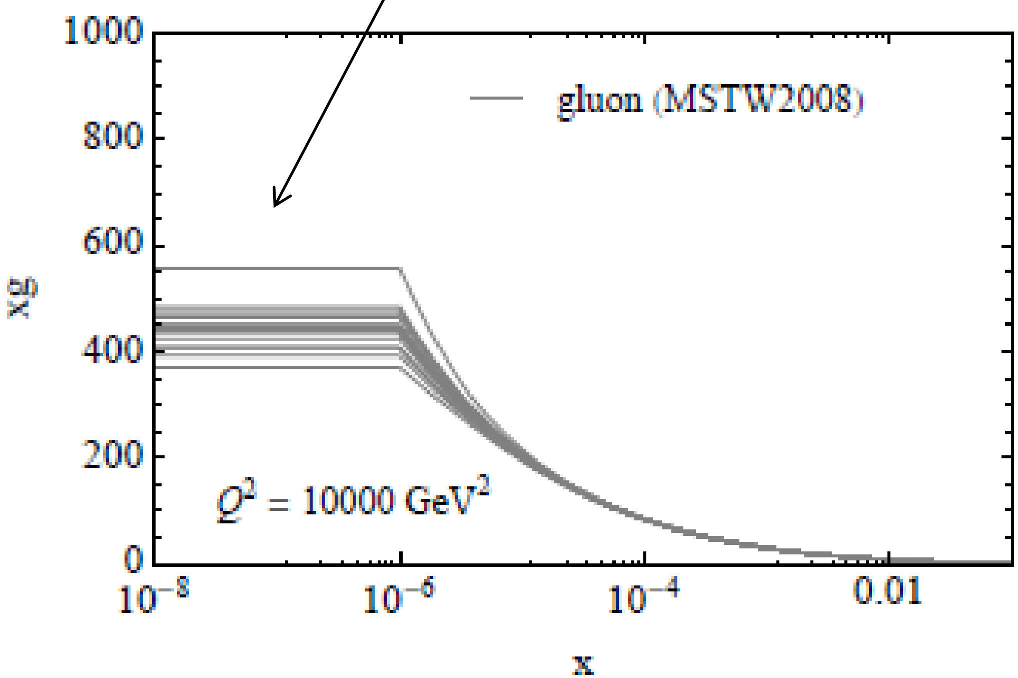
There are modern PDFs available on the LHAPDF data base
Why not pick one and make your own calculation?

Firstly you'll need the corresponding coefficient functions for NLO – LO PDFs are not good enough because LO does not fit the data well..

And many off the shelf generators like PYTHIA make only LO calculations

Secondly you need correct heavy quark treatment-- for c and b- quarks

Thirdly- and most importantly- of the shelf PDFs are available on x,Q^2 grids which don't extend to low enough x for high energy neutrino cross-sections – they freeze for lower x



However- we CAN go to lower x by using the parametrized forms

C-S MS (JHEP08(2011)042) does this for the PDFs HERAPDF1.5 and CT10 with comparison to MSTW2008

PDF uncertainties

experimental uncertainties

- many experimental errors correlated
- correlation matrix diagonalised

$\rightarrow$ linearly independent eigenvectors = variations of best-fit PDF

- can add errors from eigenvectors in quadrature

The 68%CL uncertainties are sometimes set by increased error tolerances e.g $\Delta\chi 2 \sim 50$ rather than $\Delta\chi 2 \sim 1$

model/parameter uncertainities

- some parameters/model assumptions get fixed before fit
- vary these parameters within c.l. interval

$\rightarrow$ variations of best-fit PDF

α_s uncertainties

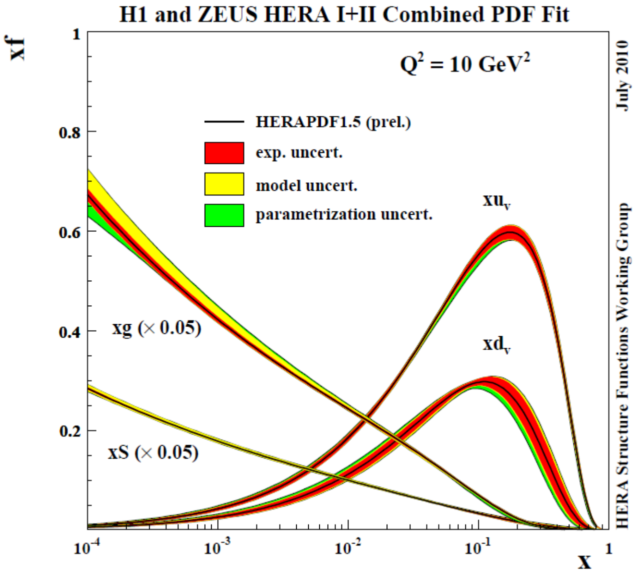
- α_s determines how quickly PDFs rise at low x

Vary values of heavy quark masses
Vary data sets used and cuts imposed on the data sets
Vary form of the parametrisation
Vary the value of α_S

We will investigate the effect of changing assumptions on the predictions for the neutrino cross sections- and we find that most of the variations do not bring more than a few percent variations..

We focus on the few cases which bring larger variations

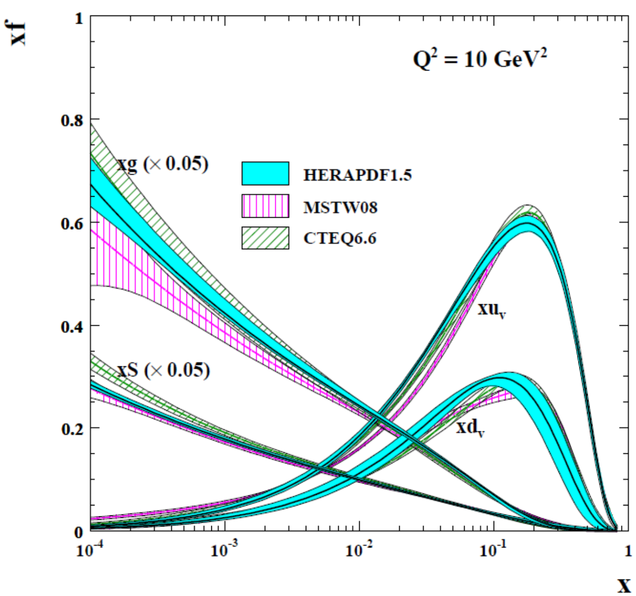
But first let's compare some modern PDFs



HERAPDF 1.5 uses just HERA data

1. a pure proton target so no corrections for nuclear effects and no strong isospin assumptions
2. a very consistent accurate data set so $\Delta\chi2$~1 for 68%CL experimental uncertainties
3. variation of model assumptions and parametrization are also added

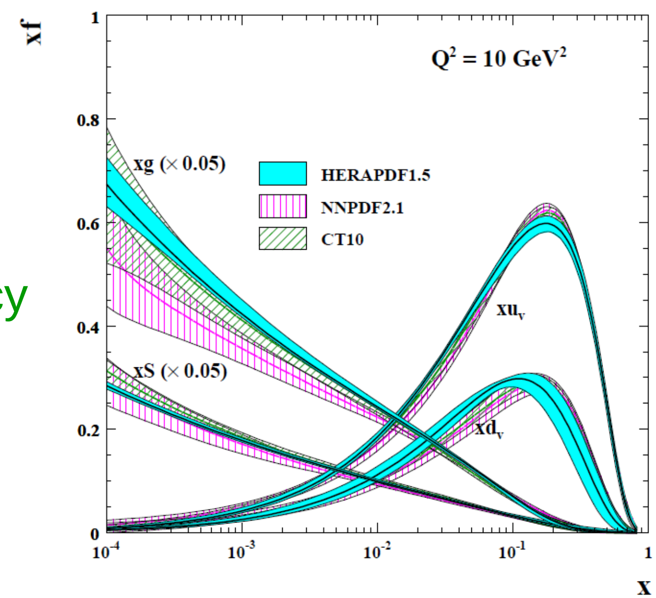
Perhaps it is surprising that there is fair agreement



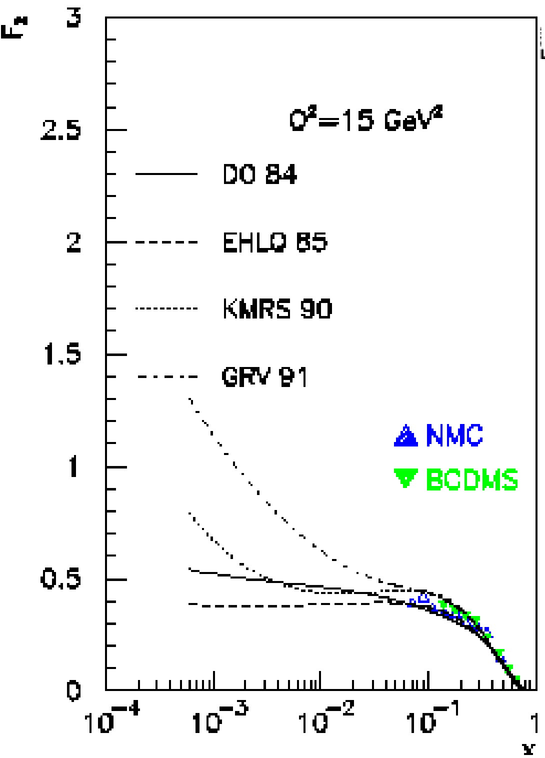
Compare to MSTW, CTEQ/CT and NNPDF for which more data sets are used and $\Delta\chi2$~30 for MSTW, $\Delta\chi2$~60 for CT to allow for data inconsistency

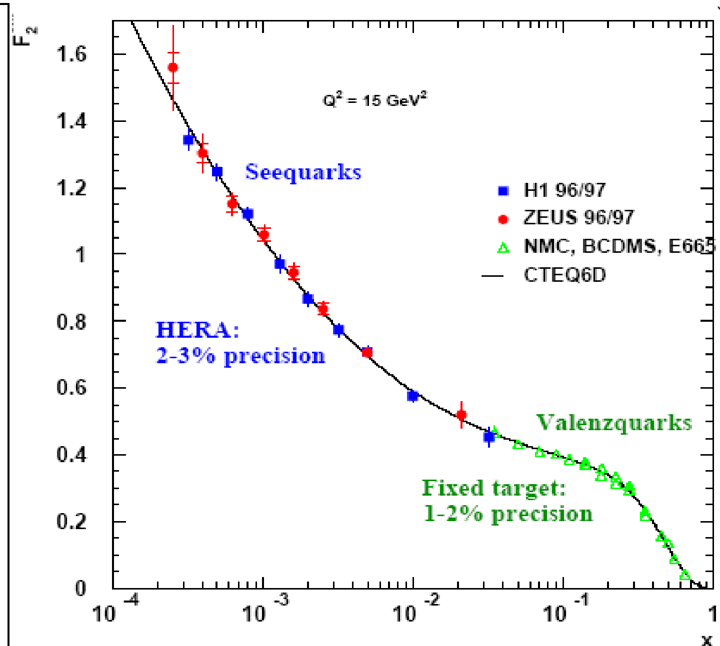
No variation of model assumptions or parametrisation–

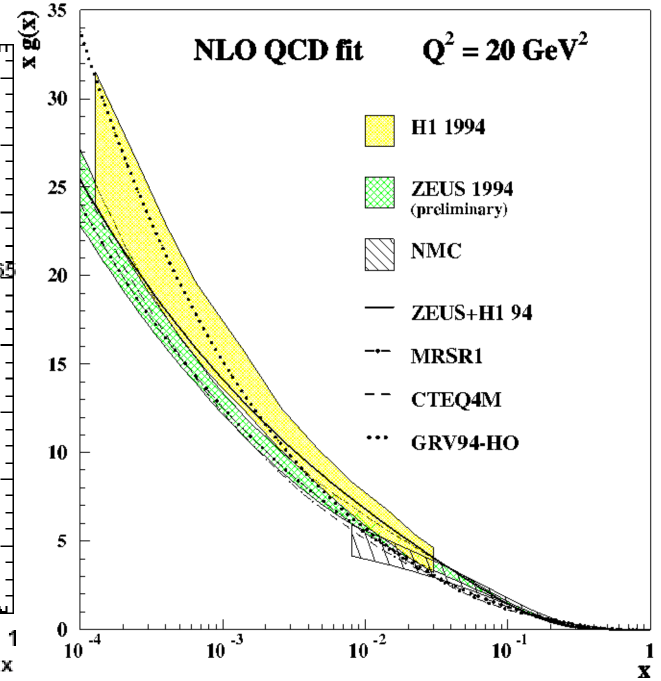
however NNPDF use a neural net (and cannot extrapolate to low x)



What sort of assumptions might be important for high energy neutrino cross sections? Those which affect the parton distributions at low-x --







Before the HERA measurements most of the predictions for low-x behaviour of the structure functions and the gluon PDF were wrong

Most people didn't expect the sharp rise at low-x which was seen in F2, and deduced in the gluon,

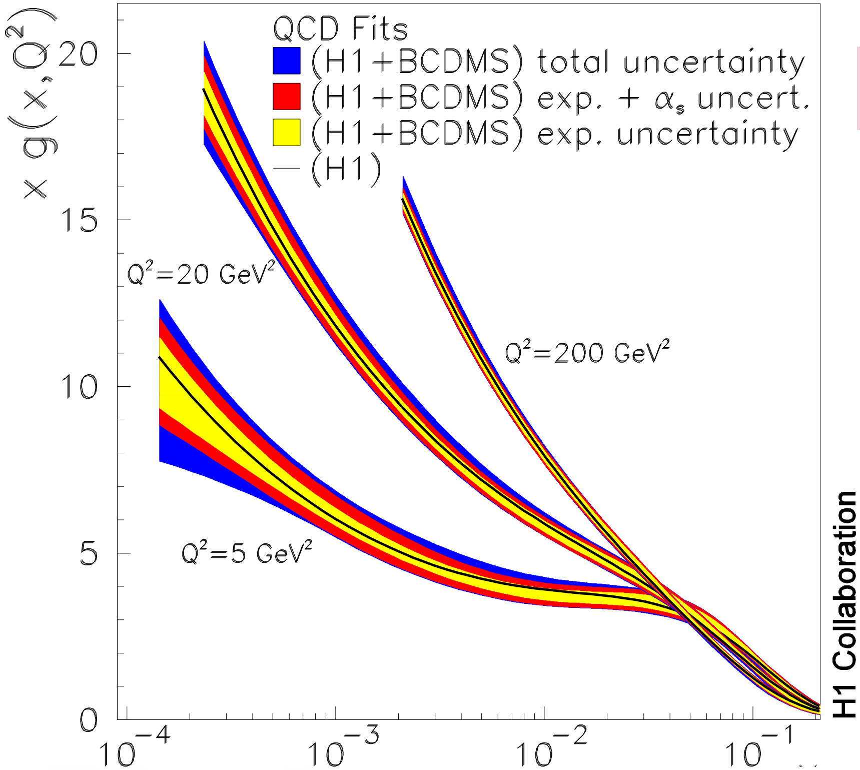
But this IS what pQCD in the DGLAP formailsm predicts!.....

xSea directly from F_2, $F_2 \sim xq$

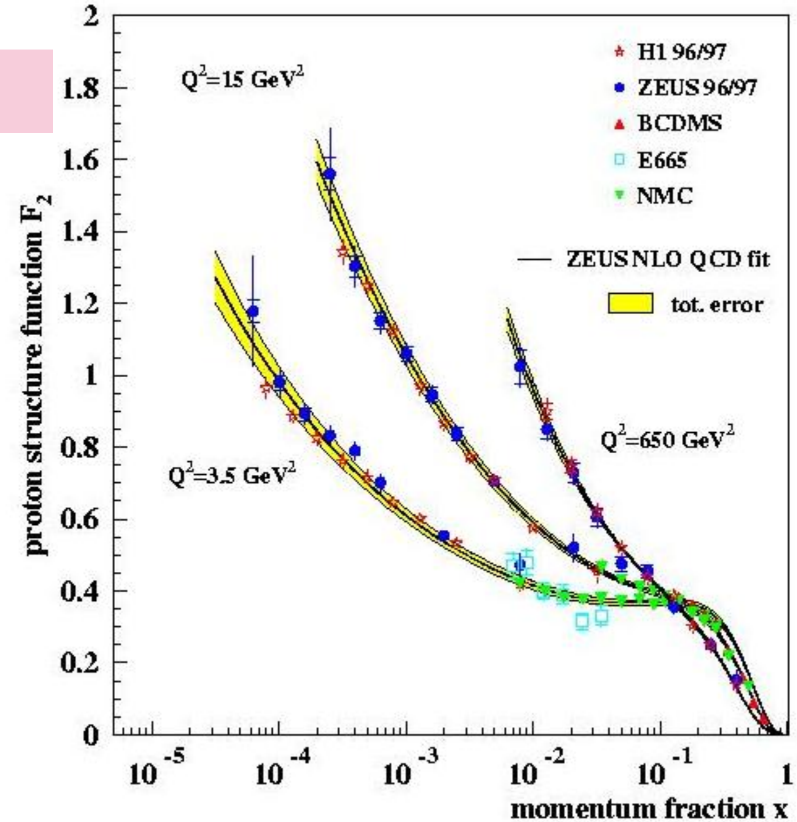
xGluon from scaling violations $dF_2 /d\ln Q^2$

at small-x,

$dF_2/d\ln Q^2 \sim$ Pqg xg



Low-x



$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\left[\Sigma_q P_{gq}(z)\, q(y,Q^2) + P_{gg}(z)\, g(y,Q^2)\right]$$

At small x,

small z=x/y

$$P_{gq} \to \frac{C_F}{z}, \quad P_{gg} \to \frac{2C_A}{z}$$

Gluon splitting functions become singular

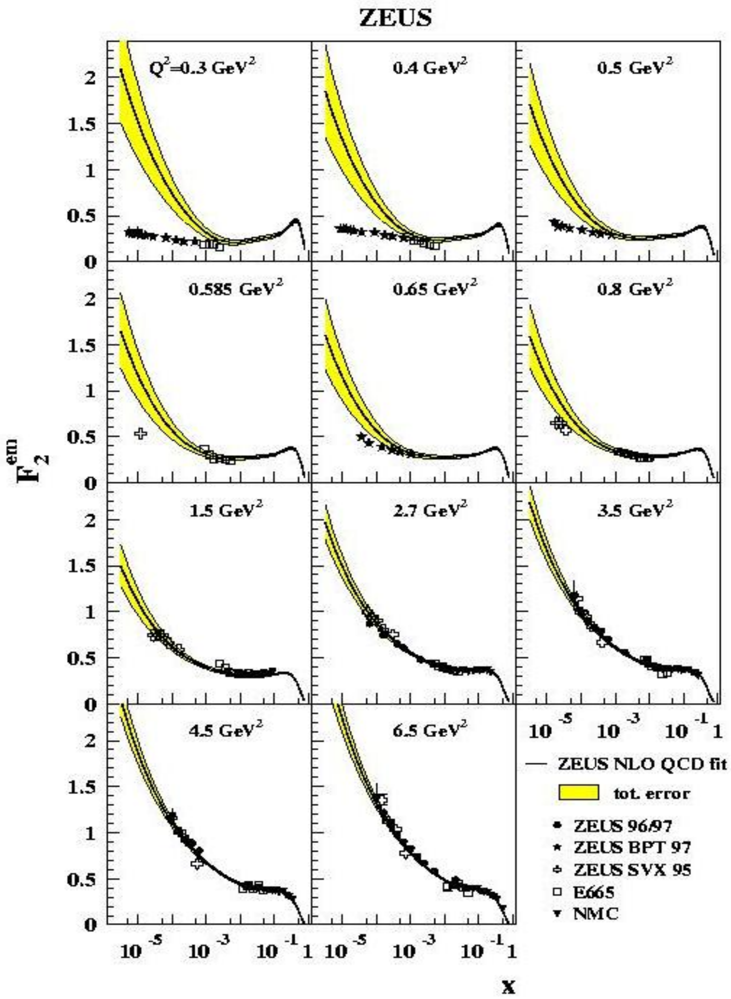
$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\frac{6}{z}\, g(y,Q^2)$$

$xg(x,Q^2) \sim x^{-\lambda_g}$, $\lambda_g = \left(\dfrac{12\ln(t/t_0)}{\beta_0 \ln(1/x)}\right)^{\frac{1}{2}}$, $t = \ln Q^2/\Lambda^2$

$\alpha_s \sim 1/\ln Q^2/\Lambda^2$

A flat gluon at low Q^2 becomes very steep AFTER Q^2 evolution the gluon becomes dominant and generates the sea by $g \to q\bar{q}$ splitting so that **F_2 becomes gluon dominated**

$F_2(x,Q^2) \sim x^{-\lambda_s}$, $\lambda_s = \lambda_g - \varepsilon$



Nevertheless the first results were much steeper than had been anticipated

And it was even more of a surprise to see the second results: F_2 steep at small x - for very low Q^2, $Q^2 \sim 1$ GeV^2

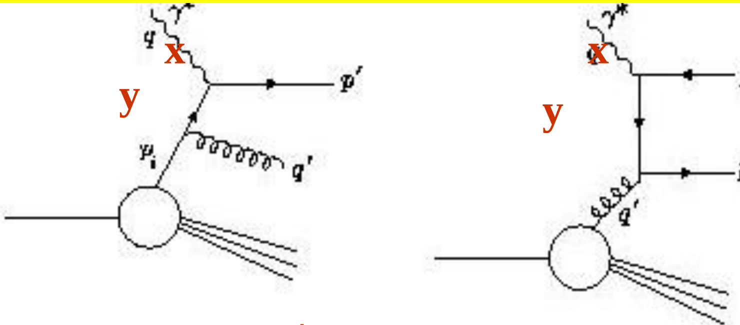
- Should perturbative QCD work? α_s is becoming large - α_s at $Q^2 \sim 1$ GeV^2 is ~ 0.4

- There hasn't been enough lever arm in Q^2 for evolution, but even the starting distribution is steep- the HUGE rise at low-x makes us think

1. there should be ln(1/x) resummation (BFKL) as well as the traditional ln(Q^2) DGLAP resummation- BFKL predicted $F_2(x,Q^2) \sim x^{-\lambda s}$, with λ_s=0.5, even at low Q^2
2. and/or there should be non-linear high density corrections for x < 5 10^{-3}

Is conventional QCD evolution in the DGLAP formalism good enough at low x?

Recap how QCD improves the Quark Parton Model

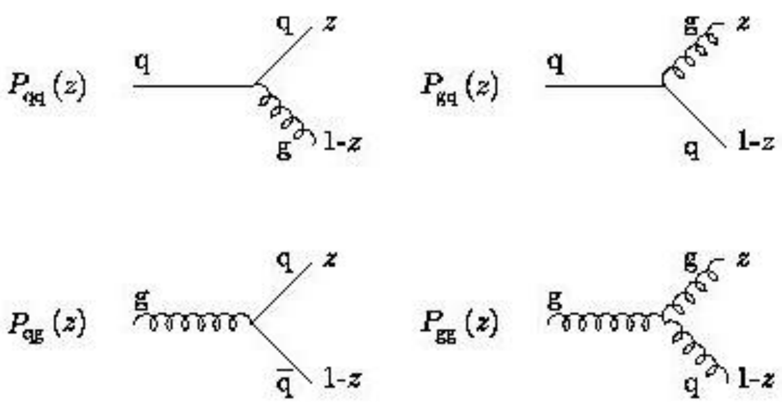


y > x, z = x/y

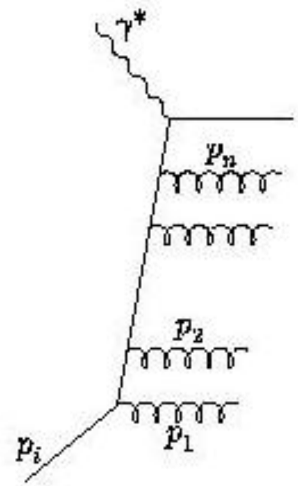
$$\frac{dq(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[Pqq(z)q(y,Q^2) + Pqg(z)g(y,Q^2)\right]$$

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[\Sigma_q Pgq(z)q(y,Q^2) + Pgg(z)g(y,Q^2)\right]$$

The DGLAP parton evolution equations



Note $q(x,Q^2) \sim \alpha_s \ln Q^2$, but $\alpha_s(Q^2) \sim 1/\ln Q^2$, so $\alpha_s \ln Q^2$ is O(1), so we must sum all terms



$\alpha_s^n \ln Q^{2n}$

Leading Log

Approximation

x decreases from target to probe

$x_{i-1} > x_i > x_{i+1} \dots$

p_t^2 of quark relative to proton increases from target to probe

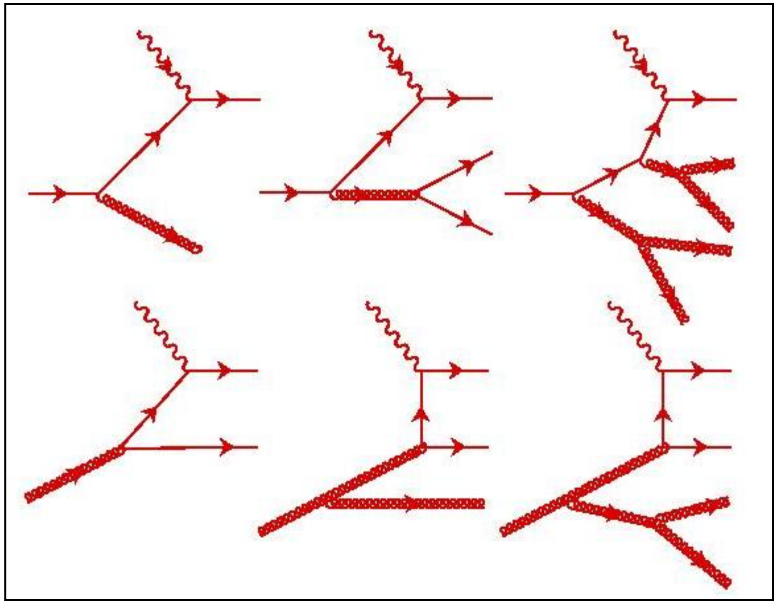
$p_t^2{}_{i-1} < p_t^2{}_i < p_t^2{}_{i+1}$

Dominant diagrams have STRONG p_t ordering

But what about disordered ladders?

Are they always sub-dominant?

What if higher orders are needed?



$$P_{qq}(z) = P^0_{qq}(z) + \alpha_s P^1_{qq}(z) + \alpha_s^2 P^2_{qq}(z)$$

LO NLO NNLO

The splitting functions have contributions
$P^n(x) = 1/x\ [\ a_n \ln^n (1/x) + b_n \ln^{n-1} (1/x) \ldots$

And thus give rise to contributions to the PDF
$\alpha_s^{\ p} (Q^2) (\ln Q^2)^q (\ln 1/x)^{\ r}$

In the DGLAP Leading Log Approximation we are summing p=q for LO, p=q+1 for NLO… What about r?

This may matter at low-x where ln(1/x) becomes large

Summing p=r, p=r+1 (regardless of q) is BFKL resummation.

It corresponds to gluon ladders that are disordered in pt

and at leading order it gives a very steep rise of the low-x gluon and thus neutrino cross sections which rise more steeply than for DGLAP

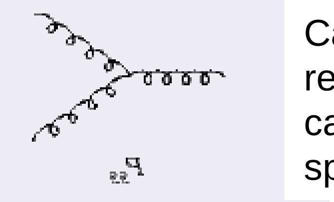
BFKL summation at LL(1/x) $\Rightarrow$ $xg(x) \sim x^{-\lambda}$

Where $\lambda = \dfrac{\alpha_S C_A \ln 2}{\pi} \sim 0.5$

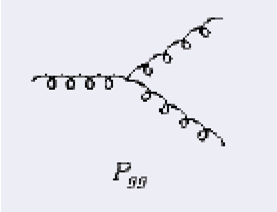
But NLL(1/x) softens this somewhat

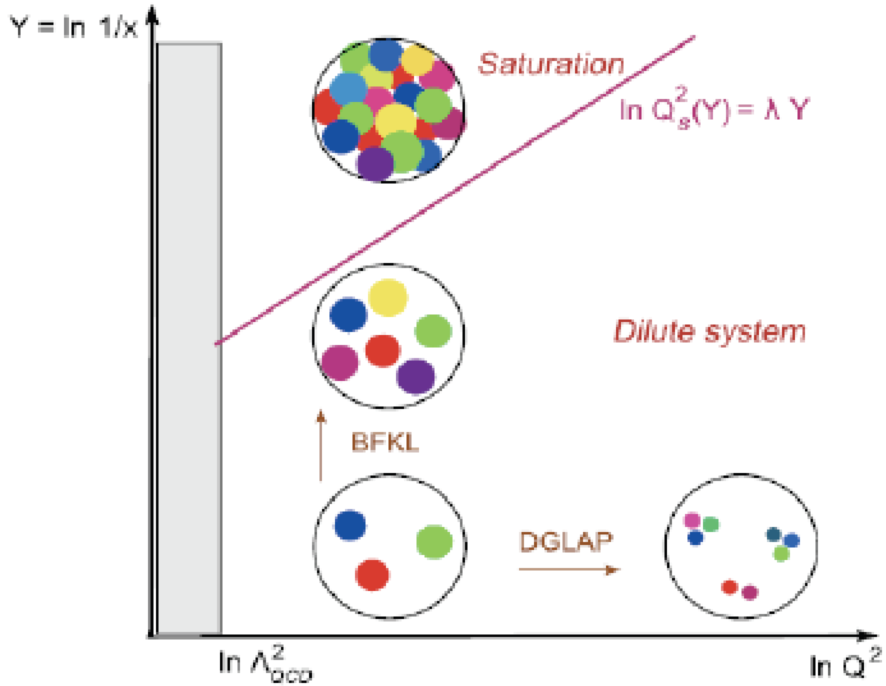
$\Rightarrow$steep gluon even at moderate Q^2

The steeply rising gluon density may lead to an overcrowded nucleon Could gluon recombination become as important as gluon splitting?



Can recombination cancel out splitting?





Might there be gluon saturation?
We would need non-linear evolution equations with dependence on $g(x)^2$
GLR… BK- JIMWLK
Colour glass condensate..

New regimes of QCD

And saturation would help with a further potential problem:
Both DGLAP and BKFL predict

$\Rightarrow xg(x) \sim x^{-\lambda}$ **at low x**

But the rise is steeper for BFKL
This implies $\rightarrow$ $\sigma(\gamma^* p) \sim (W^2)^{\lambda}$ $(W^2=(p+q)^2)$
Such a steep rise may lead to Violation of the Froissart Bound for the neutrino-nucleon cross section
Saturation could tame such a rise

Is there any evidence for that we have reached these new regimes of QCD in DIS? Nothing definitive BUT when you look at the sea and the gluon deduced from the DGLAP formalism at low Q^2 there are odd features

At $Q^2 \sim 1$ GeV^2 -where DGLAP still describes the data--- the gluon is no longer steep at small x – in fact its valence-like or even negative!

The problem is that we are deducing this from limited information

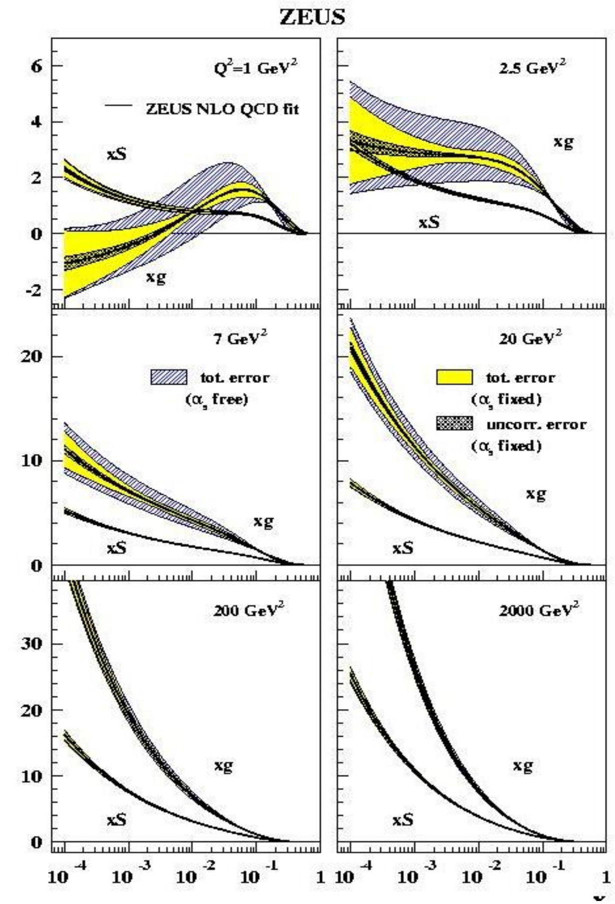
At low-x, we use

$F_2 \sim xq$ for the sea

$dF_2/d\ln Q^2 \sim Pqg\ xg$ for the gluon

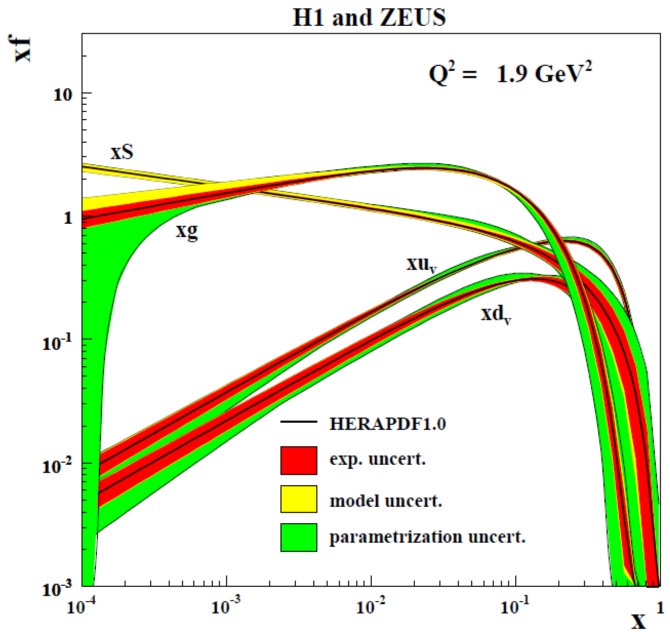
Unusual behaviour of $dF_2/d\ln Q^2$ may come from unusual gluon or from unusual Pqg- alternative BFKL evolution?. Non-linear effects?

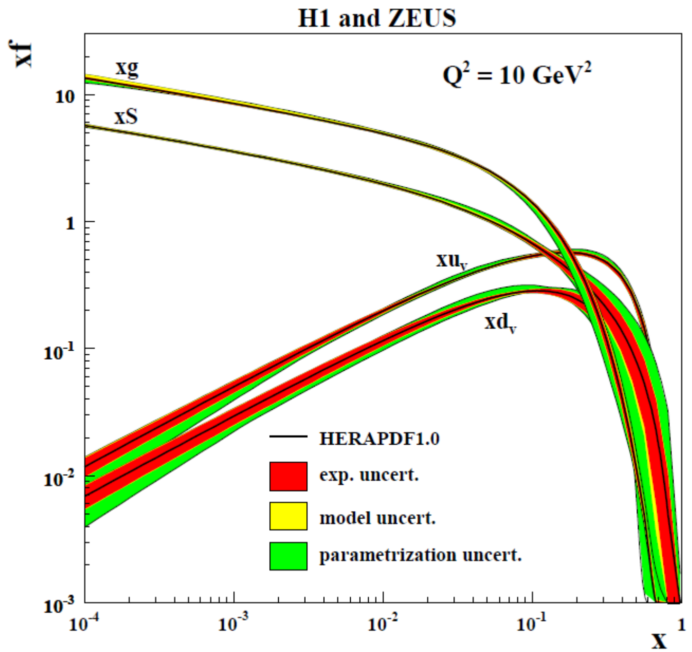
(Note for experts-- measurement of FL has NOT resolved this!)



At HERA as we go to low x we go to low Q^2, perhaps its all non perturbative anyway?... BUT High energy neutrino cross sections probe low x for high Q^2

The need to go beyond DGLAP is NOT firmly established experimentally.
To use high energy neutrino cross sections to establish it we need to know the limits of the DGLAP predictions. This can be hard because sometimes effects beyond DGLAP have been built into the boundary conditions of DGLAP at Q^2_0





This applies to the tendency of the low-x gluon– to become negative and it is NOT a small effect.

Conventional QCD evolution is such that a steep gluon at low-x and moderate Q^2..

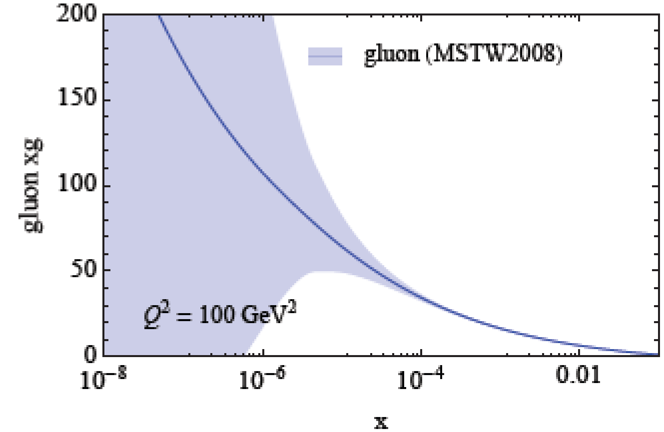
which fits the HERA data well

implies that at low Q^2 the gluon turns over and may even become negative.

This may signal the breakdown of conventional QCD-DGLAP evolution?-

And the need for ln(1/x) resummation (BFKL evolution) or non-linear effects?

MSTW2008 has this tendency to a negative gluon built into the boundary conditions– ie it is part of the parametrisation at the starting scale..
(and that's why the CTW error gets so large)

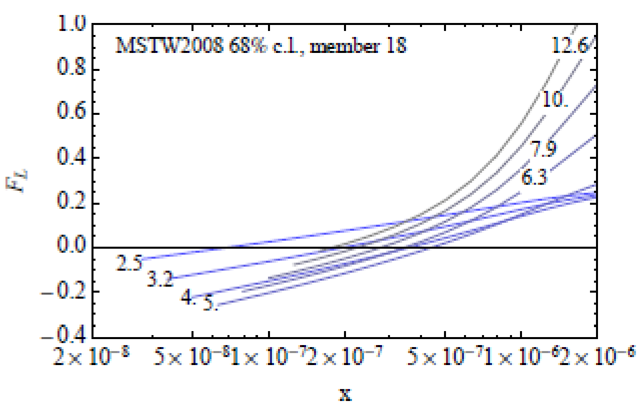


HERAPDF1.5 has a parametrization variation (number 9) which allows us to investigate this tendency to a negative gluon. It also ensures that the negative gluon does not give unphysical predictions for cross secions

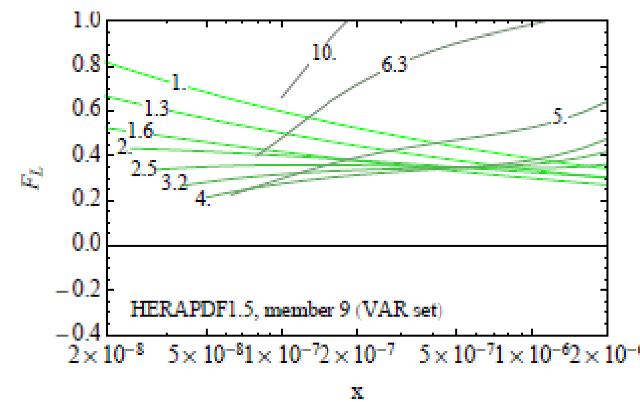
Could the gluon become negative?

at NLO, the gluon **could** become negative

however longitudinal structure function F_L **must** stay positive



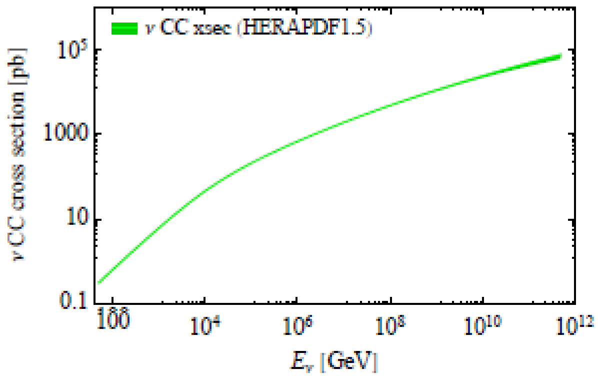
With MSTW2008, F_L **does** go negative!

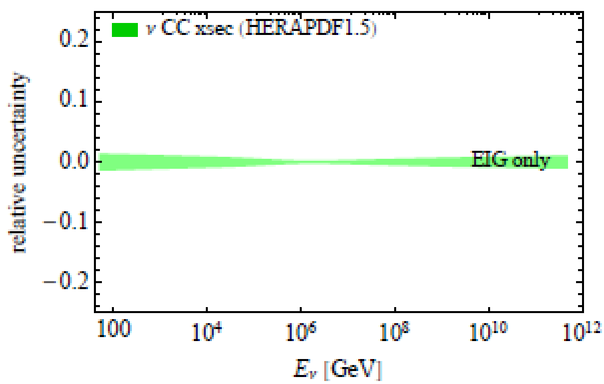


With HERAPDF1.5, F_L **does** stay positive!

Let's look at the predictions for the neutrino cross sections for HERAPDF with and without a negative gluon term

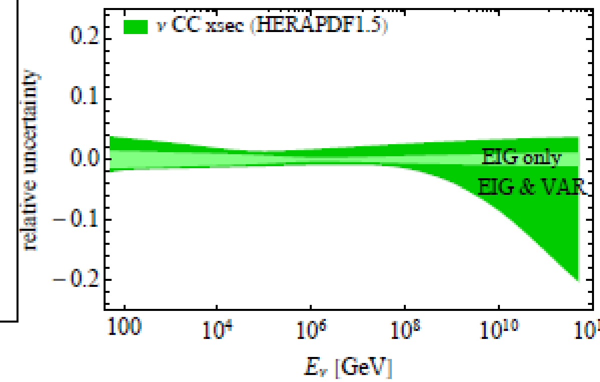
Let's look at the predictions of HERAPDFs



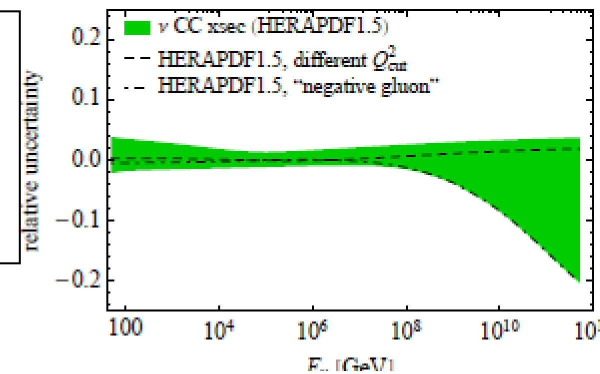


All the experimental errors of the various DIS experiments input to PDF determination yield only ~2% of the error on the neutrino cross-section

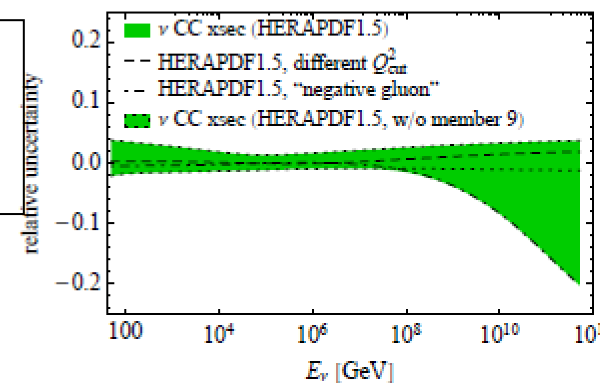
However the variations in assumptions can increase this to up to 20% at the largest neutrino energies



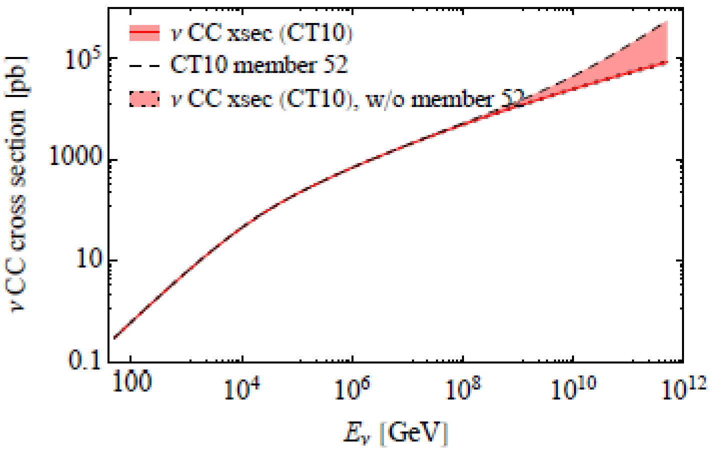
And this is mostly due to the variation which allows a negative gluon (no 9)

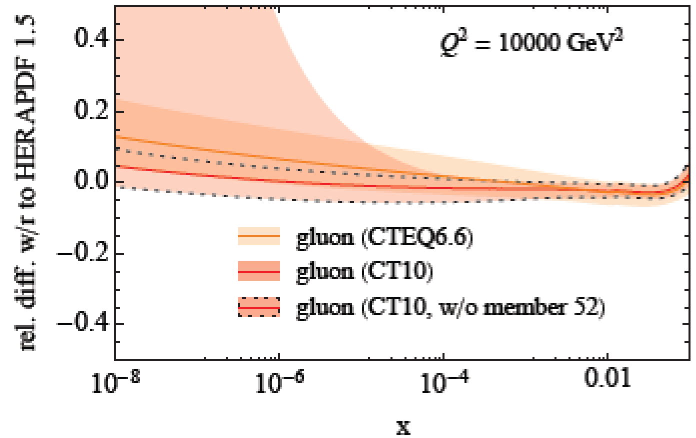


In fact without this the variation is a modest ~4% effect



Now let's look at the predictions of CT10 PDFs

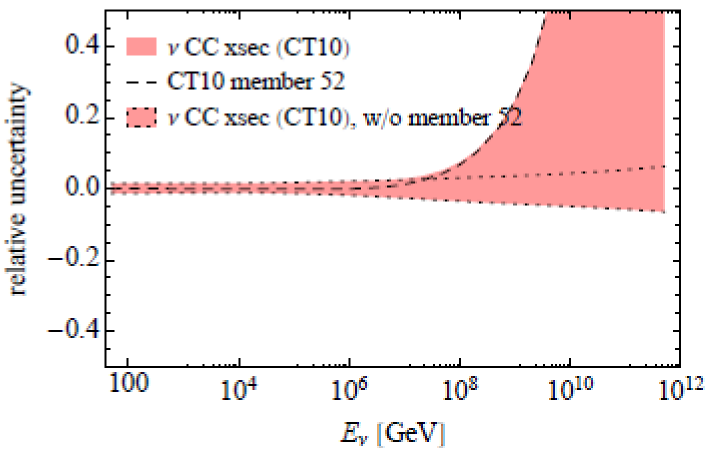




CTEQ do not allow the gluon to become negative

This was not there in previous CTEQ PDFs like CTEQ6.6.
It is somewhat ad hoc --to acknowledge that we really do not know what happens for low-x.. But is this a good idea?
All current PDF shapes will lead to violation of the Froissart bound eventually.. But some of them will get there quicker than others

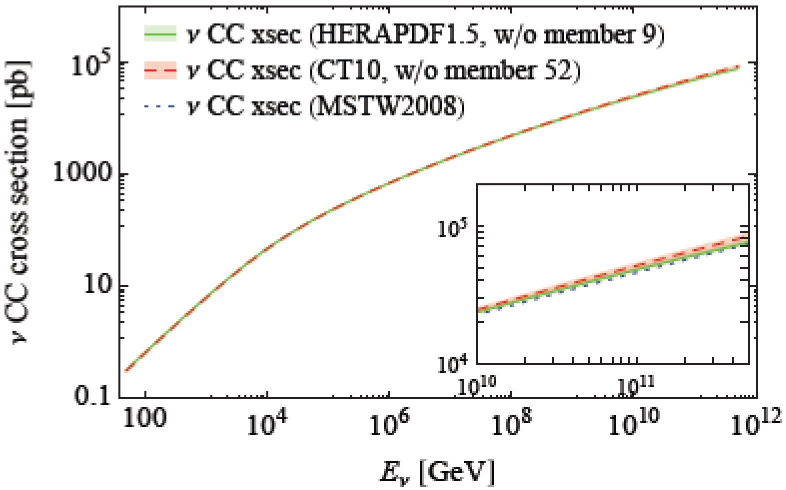
cross-section for member 52 rises $\propto E_\nu^{0.7}$; central member $\propto E_\nu^{0.3}$



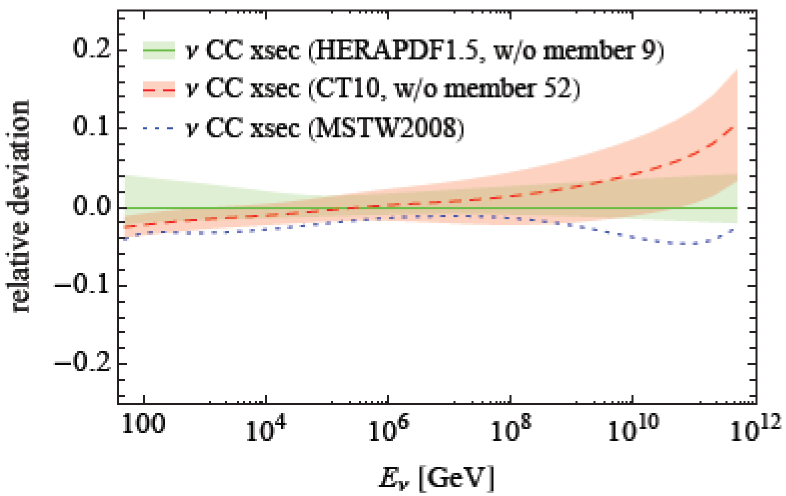
But they have now added a variant (no 52) which has a very steeply rising neutrino cross-section

We need to compare estimates of neutrino cross-sections from calculations going beyond NLO DGLAP (BFKL, non-linear etc) to our best guess from DGLAP

Let’s compare the predictions of HERAPDF1.5, CT10 and MSTW2008 PDFs

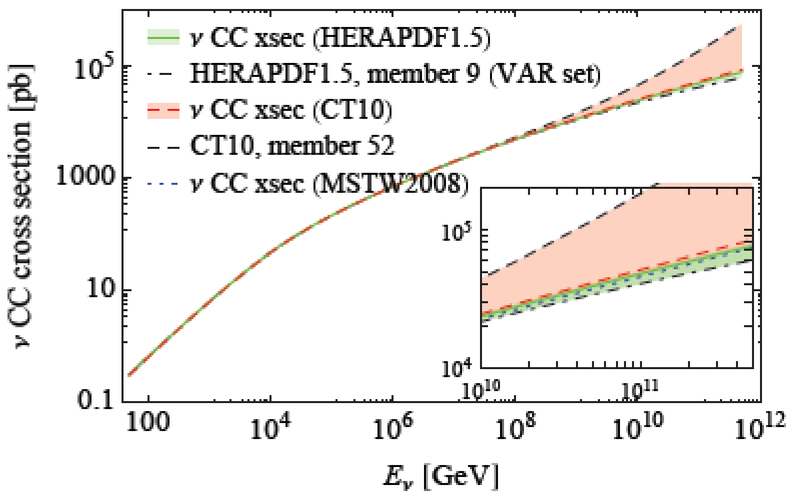


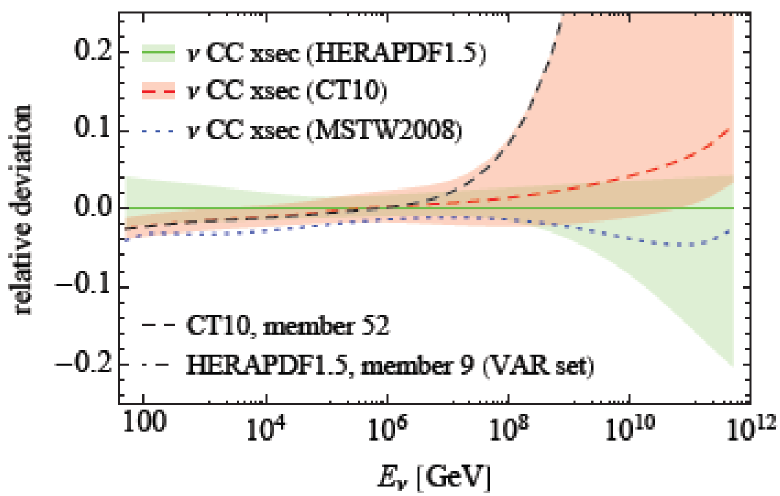
The central value predictions for neutrino cross sections from HERAPDF1.5, CT10 and MSTW2008 are actually in rather good agreement



And even the uncertainty bands are in fair agreement if ‘rogue’ members with either very steeply rising or falling gluons are excluded.

We take the view that our best estimate does not include variants which have a negative gluon or a strong tendency to violate the Froissart bound.





BUT the variation in cross-section values is not so dramatic even if the 'rogue' members are left in

C-S MS (JHEP08(2011)042) gives tables of:

- neutrino and antineutrino Charged Current (CC) and Neutral Current (NC) cross-sections and uncertainties
- for neutrino energies from 50 GeV to $5{\times}10^{11}$ GeV
- for the HERAPDF1.5 predictions both with an without the rogue member 9.
- This is done for both proton and isoscalar targets on http:///www-pnp.physics.ox.ac.uk/~cooper/neutrino

- Differential cross sections also available on request
- These predictions are being incorporated into ANIS

Summary

•Predictions of high energy ν and $\bar{\nu}$ CC cross-sections can be made within conventional framework NLO QCD using the DGLAP formalism
•With systematic accounting for PDF uncertainties
•Including general mass variable flavour treatment of heavy quarks

The point is to estimate how well known conventional predictions are in order to when we really have unconventional behaviour at small-x
BFKL ln(1/x) resummation
non-linear effects
gluon recombination, saturation –
BK,JIMWLK, colour glass condensate
etc

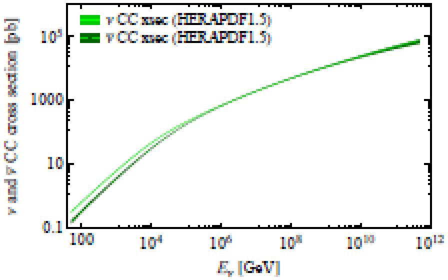
Measurements of ultra-high energy neutrino cross sections could be sensitive to new regimes of QCD

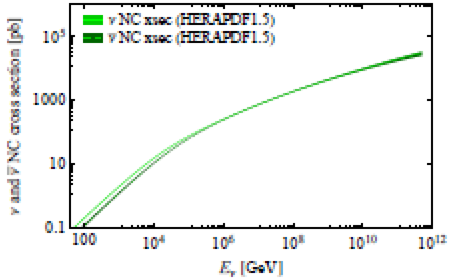
extras

Antineutrino cross-sections are closely similar at high energies because $xF_3^{\nu} \sim -xF_3^{\bar{\nu}}$.

and xF3 contributes with opposite sign in neutrino and antineutrino cross-sections

However there are differences at low energies as we access high-x and the valence quark contribution become important





Low energy regime б ~ E

As neutrino energy decrease the PDF uncertainties decrease since very low-x values are no longer probed.

PDF uncertainties are smallest at $s \sim 10^5$ corresponding to middling x, $10^{-2} < x < 10^{-1}$

PDF uncertainties increase again at lower neutrino energies as we move into the region of large **x**

Where does the information on parton distributions come from?

CC e-p

$$\frac{d^2\sigma(e^-p)}{dxdy} = \frac{G_F^2 M^4_W}{2\pi x(Q^2+M^2_W)^2}[x\,(u+c) + (1-y)^2 x\,(\bar{d}+\bar{s})]$$

CC e+p

$$\frac{d^2\sigma(e^+p)}{dxdy} = \frac{G_F^2 M^4_W}{2\pi x(Q^2+M^2_W)^2}[x\,(\bar{u}+\bar{c}) + (1-y)^2 x\,(d+s)]$$

- The charged currents give us flavour information for high-x valence PDFs

NC e+ and e-

$$\frac{d^2\sigma(e\pm N)}{dxdy} = \frac{2\pi\alpha^2 s}{Q^4} Y_+ \left[F_2(x,Q^2) - \frac{y^2}{Y_+} F_L(x,Q^2) \pm \frac{Y_-}{Y_+} xF_3(x,Q^2)\right], \quad Y_\pm = 1 \pm (1-y)^2$$

$F_2 = F_2^{\gamma} - v_e P_Z F_2^{\gamma Z} + (v_e^2+a_e^2)P_Z^2 F_2^Z$

$xF_3 = - a_e P_Z xF_3^{\gamma Z} + 2v_e a_e P_Z^2 xF_3^Z$

Where $P_Z^2 = Q^2/(Q^2 + M^2_Z)\ 1/\sin^2\theta_W$, and at LO

$[F_2, F_2^{\gamma Z}, F_2^Z] = \Sigma_i\ [e_i^2, 2e_i v_i, v_i^2+a_i^2][xq_i(x,Q^2) + x\bar{q}_i(x,Q^2)]$

$[xF_3^{\gamma Z}, xF_3^Z] = \Sigma_i\ 2[e_i a_i, v_i a_i]\ [xq_i(x,Q^2) - x\bar{q}_i(x,Q^2)]$

So that $xF_3^{\gamma Z} = 2x[e_u a_u u_v + e_d a_d d_v] = x/3\,(2u_v+d_v)$

Where $xF_3^{\gamma Z}$ is the dominant term in xF_3

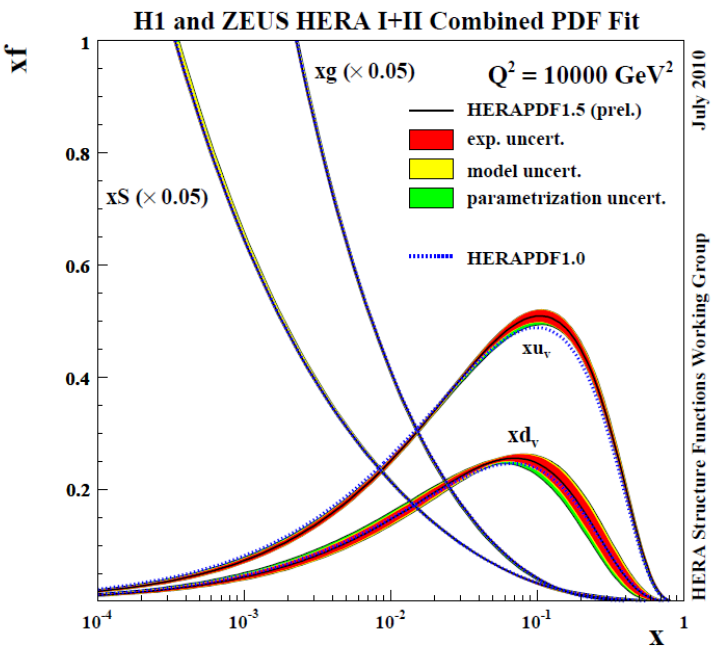
The neutral current F2 gives the low-x Sea

The difference between e- and e+ also gives a valence PDF for x>0.01- not just at high-x

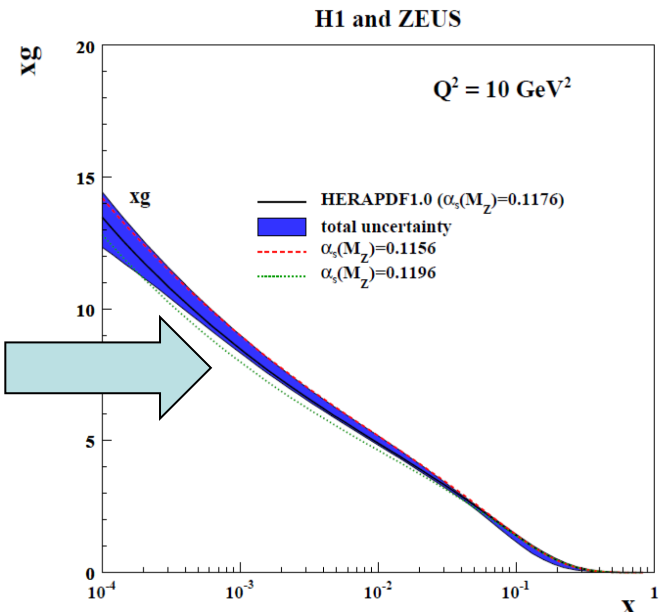
And of course the scaling violations give the gluon PDF

What sort of assumptions might be important at low-x?

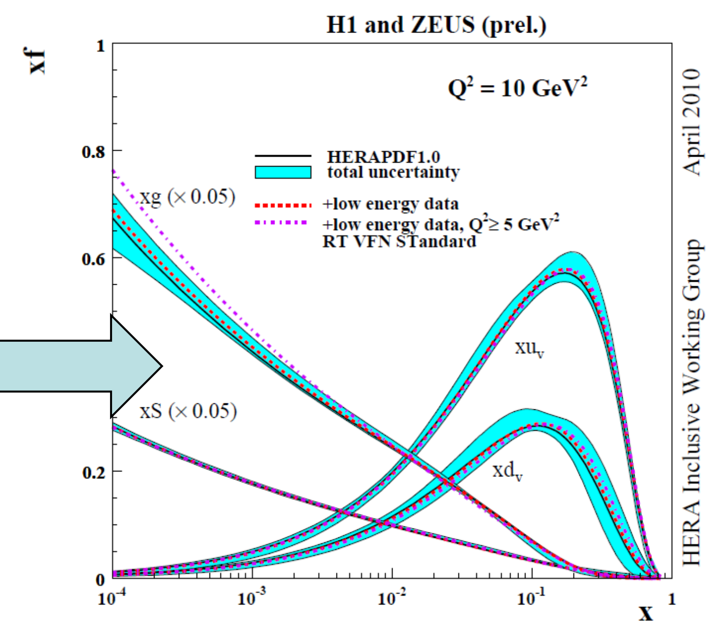
Those which affect the gluon shape at low-x --
since the gluon becomes dominant and generates the sea
by $g \rightarrow q\,\bar{q}$ splitting



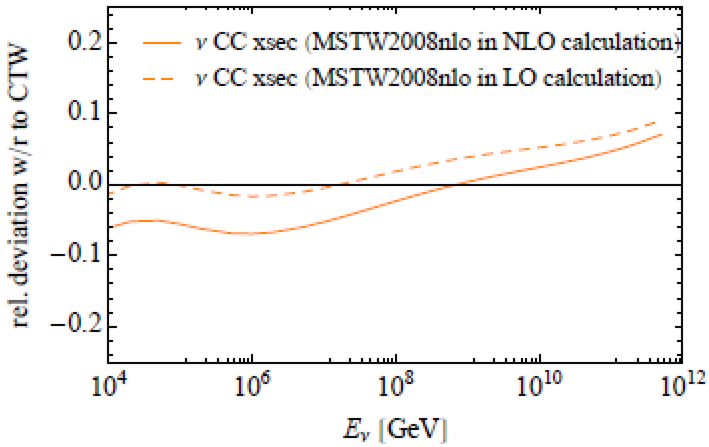
1. the value of $\alpha_S(M_Z)$ could be important--- but this turns out to be a small effect

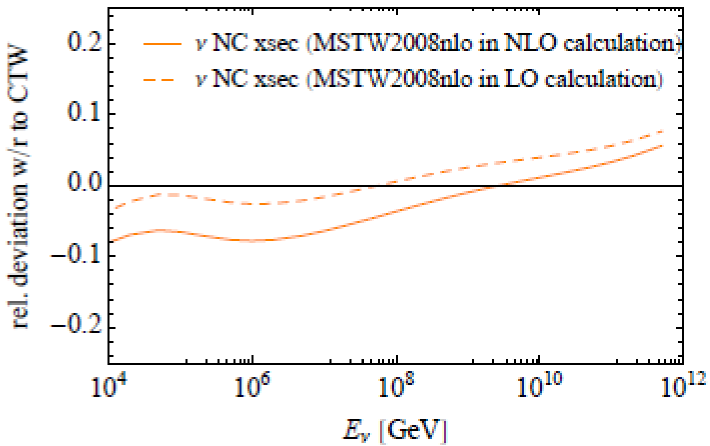


2. the cuts applied to the fitted data could be important -- cutting out low Q^2 data – also cuts out low-x data and this tends to result in a steeper gluon— but this is also a small effect on the neutrino cross-sections

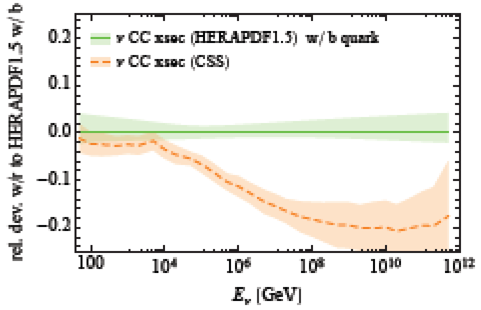


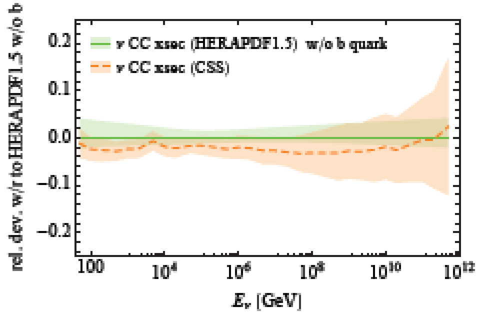
Comparison with CTW

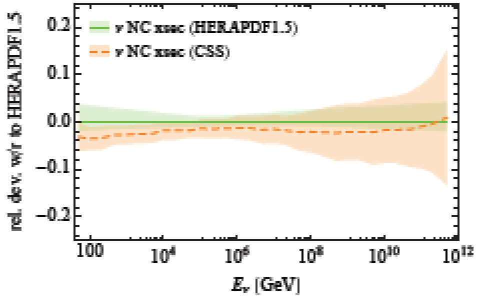




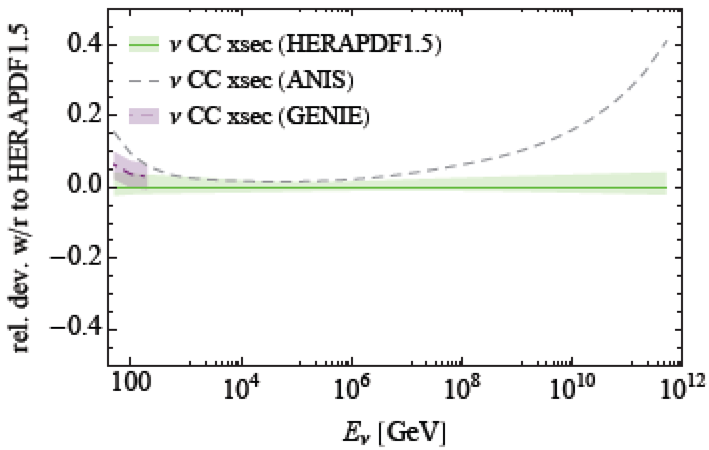
Comparison with CSS

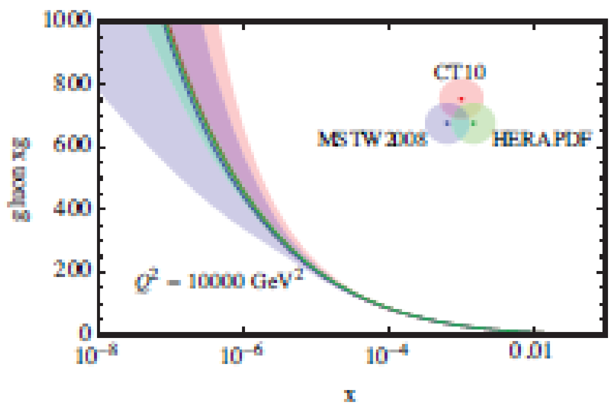


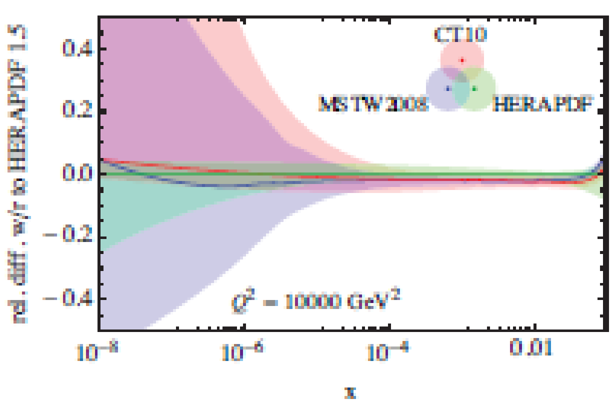




Comparison with ANIS



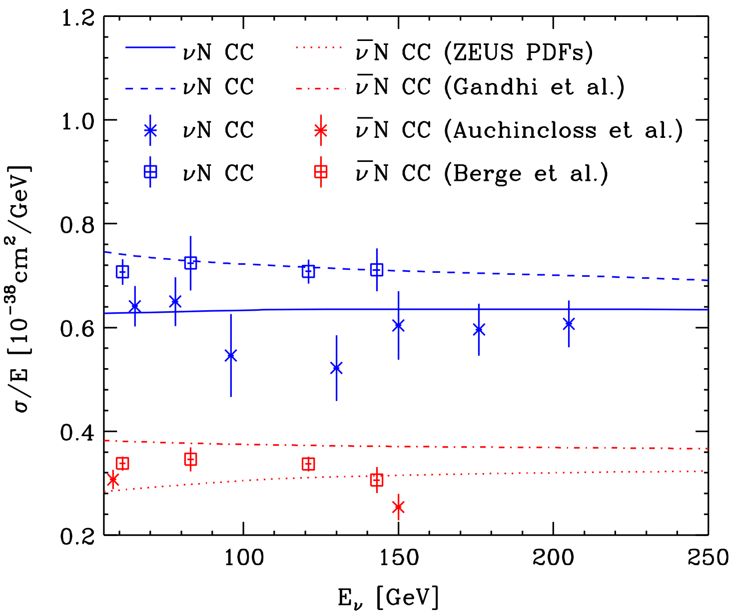


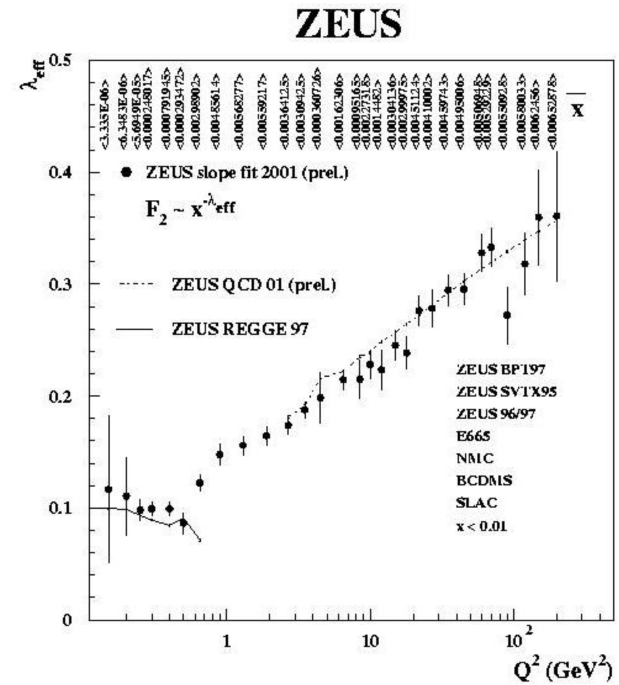


And just for completeness sake lets show the new and the old predictions at very low energy compared to data

Note the perturbative predictions of the present work cannot be use for Q2 < 1 GeV^2 and hence we are missing a fraction of the lowest energy cross-sections. This is most significant in the smaller antineutrino cross-section. Hence no predictions are given for

s < 100 GeV^2 (Eν < 53.3 GeV)




ZEUS
λ_{eff}
ZEUS slope fit 2001 (prel.)
$F_2 \sim x^{-\lambda_{eff}}$
ZEUS QCD 01 (prel.)
ZEUS REGGE 97
ZEUS BPT97
ZEUS SVTX95
ZEUS 96/97
E665
NMC
BCDMS
SLAC
x < 0.01
$\bar{x}$
Q^2 (GeV^2)