From amplitudes to correlation functions in $\mathcal{N}=4$ SYM

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Amplitudes versus correlation functions

- ✔ Carry different/supplementary information about gauge theory:

  $G_n = \langle \mathcal{O}(x_1) \dots \mathcal{O}(x_n) \rangle =$ off-shell (anomalous dimensions, structure constants of OPE)

  $A_n = \langle p_1, \dots, p_n | \, S \, | 0 \rangle \; =$ on-shell (S-matrix)

- ✔ Amplitudes are **not** well-defined in $D = 4$ dimensions
  - ✗ Suffer from IR divergences and require a regularization
  - ✗ Part of symmetries (conformal + dual conformal) are broken by IR regulator
- ✔ Correlation functions of half-BPS operators are well-defined in $D = 4$ dimensions
  - ✗ Do not require a regularization for generic positions of operators
  - ✗ Inherits all (unbroken) symmetries of $\mathcal{N} = 4$ SYM
- ✔ They are related to each other in planar $\mathcal{N} = 4$ SYM:

$$\lim_{x^2_{i,i+1} \to 0} \ln G_n(x_i) \; \sim \; 2 \ln A_n(p_i) \, , \qquad p_i = x_i - x_{i+1}$$

This talk:

Correlation functions have a new hidden symmetry in $\mathcal{N} = 4$ SYM (no need for planar limit!)

Allows us to predict correlators/amplitudes at higher loops without any Feynman graph calculations!

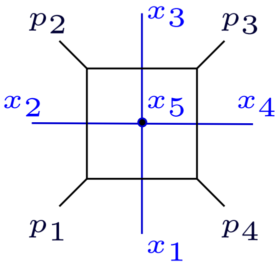
Gluon amplitudes

- Four-gluon amplitude in $\mathcal{N}=4$ SYM at weak coupling $a = g^2 N_c/(8\pi^2)$

$$A_4/A_4^{(\mathrm{tree})} = 1 + a\, st I^{(1)}(s,t) + O(a^2)$$

Scalar box in the dimensional regularization (for IR divergences) with $D = 4 - 2\epsilon$

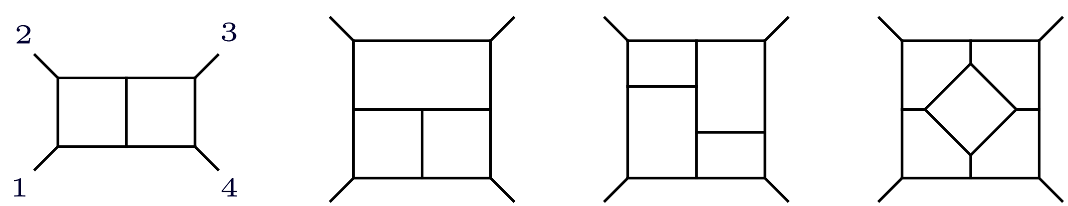
$$I^{(1)}(s,t) =$$



$$\sim \int \frac{d^D x_5}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2}\,, \qquad (x_{12}^2 = x_{23}^2 = x_{34}^2 = x_{41}^2 = 0)$$

Dual variables $p_i = x_i - x_{i+1}$ with $p_i^2 = x_{i,i+1}^2 = 0$

- (Broken) dual conformal symmetry
- All-loop BDS ansatz / AdS prediction / Wilson loop duality
- Explicit expressions for loop *integrands* up to 7 loops [Bourjaily,DiRe,Shaikh,Spradlin,Volovich'12]
- Seemingly increasing complexity of diagrams at higher loops



$h(1,3;2,4)$ $T(1,3;2,4)$ 4-loops 5-loops

Correlation functions

- ✔ Protected superconformal operators made from six real scalars Φ^I

$$\mathcal{O}(x) = \mathrm{Tr}(ZZ)\,, \qquad \tilde{\mathcal{O}}(x) = \mathrm{Tr}(\bar{Z}\bar{Z})\,, \qquad Z = \Phi^1 + i\Phi^2$$

  - ✗ All-loop scaling dimension = tree level dimension
  - ✗ Two- and three-point correlation functions do not receive quantum corrections

- ✔ Simplest correlation function

$$G_4 = \langle \mathcal{O}(x_1)\tilde{\mathcal{O}}(x_2)\mathcal{O}(x_3)\tilde{\mathcal{O}}(x_4)\rangle = G_4^{(0)}\left[1 + 2a\, x_{13}^2 x_{24}^2 g(1,2,3,4) + O(a^2)\right]$$

One-loop 'cross' integral

$$g(1,2,3,4) = \frac{1}{4\pi^2}\int \frac{d^4x_5}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2}\,, \qquad (x_{12}^2, x_{23}^2, x_{34}^2, x_{41}^2 \neq 0)$$

- ✔ Loop corrections to the amplitude and to the correlator involve *the same* integral $g(1,2,3,4)$ but for *different* kinematics: on-shell $x_{i,i+1}^2 = 0$ for A_4 and off-shell $x_{i,i+1}^2 \neq 0$ for G_4

- ✔ *Amplitude/correlation function duality*

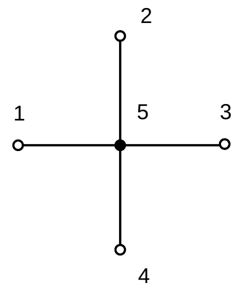
$$\lim_{x_{i,i+1}^2 \to 0} \ln\left(G_4/G_4^{(0)}\right) = \ln\left(A_4/A_4^{(\mathrm{tree})}\right)$$

Understood at the level of integrands in planar $\mathcal{N}=4$ SYM

A hidden symmetry

Examine one-loop correction to the correlator

$$G_4^{(1)} \sim \int \frac{d^4 x_5}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2} \qquad = $$



The corresponding integrand

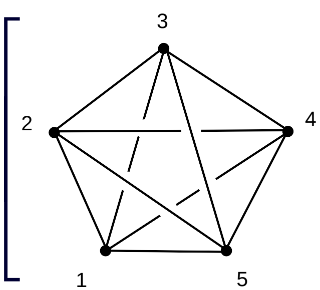
$$[G_4^{(1)}]_{\text{Integrand}} \sim \frac{1}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2}$$

The r.h.s. has S_4 permutation symmetry w.r.t. exchange of the external points $1, 2, 3, 4$

Equivalent form of writing

$$[G_4^{(1)}]_{\text{Integrand}} \sim x_{12}^2 x_{13}^2 x_{14}^2 x_{23}^2 x_{24}^2 x_{34}^2 \times \left[\prod_{i<j} \frac{1}{x_{ij}^2} \right]$$

$$= x_{12}^2 x_{13}^2 x_{14}^2 x_{23}^2 x_{24}^2 x_{34}^2 \times \Bigg[\qquad \Bigg]$$



The second factor in the r.h.s. has the complete S_5 permutation symmetry!

Two loops

- Explicit two-loop calculation: [Eden,Schubert,Sokatchev'00],[Bianchi,Kovacs,Rossi,Stanev'00]

$$G^{(2)} = h(1,2;3,4) + h(3,4;1,2) + h(2,3;1,4) + h(1,4;2,3) + h(1,3;2,4) + h(2,4;1,3) + \frac{1}{2}\left(x_{12}^2 x_{34}^2 + x_{13}^2 x_{24}^2 + x_{14}^2 x_{23}^2\right)[g(1,2,3,4)]^2$$

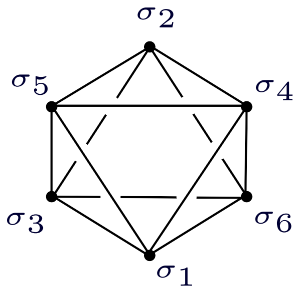
$h(1,2;3,4)-$ 'double' scalar box integral

- Go to a common denominator

$$G_4^{(2)} = x_{12}^2 x_{13}^2 x_{14}^2 x_{23}^2 x_{24}^2 x_{34}^2 \int d^4x_5 d^4x_6 \, f^{(2)}(x_1,\ldots,x_6)$$

- 7 integrals in $G_4^{(2)}$ are described by a single $f-$function

$$f^{(2)}(x_1,\ldots,x_6) = \frac{1}{48}\sum_{\sigma\in S_6} \frac{x_{\sigma_1\sigma_2}^2 x_{\sigma_3\sigma_4}^2 x_{\sigma_5\sigma_6}^2}{\prod_{1\le i<j\le 6} x_{ij}^2} \quad =$$



Has the complete S_6 permutation symmetry !

- Integrand of the correlator has the complete permutation symmetry exchanging the external and integration points *(no need for the planar limit !)* ... Where does it come from?

$\mathcal{N}=4$ stress-tensor supermultiplet

$$G_4 = \langle \mathcal{O}(x_1,y_1)\dots\mathcal{O}(x_4,y_4)\rangle = \sum_{l=0}^{\infty} a^l\, G_4^{(\ell)}(1,2,3,4)$$

Half-BPS operators made of the six scalars (complex null vector, $y^2 \equiv y_I y_I = 0$)

$$\mathcal{O}(x,y) = y_I\, y_J\, \mathcal{O}^{IJ}_{\mathbf{20}'}(x) = y_I\, y_J\ \mathrm{tr}\left[\Phi^I \Phi^J(x)\right]$$

The lowest-weight state of the $\mathcal{N}=4$ stress-tensor (chiral) supermultiplet

$$\mathcal{T}(x,\rho,y) = \exp\left(\rho^a_\alpha\, Q^\alpha_a\right)\mathcal{O}(x,y) = \mathcal{O}(x,y) + \dots + (\rho)^4 \mathcal{L}_{\mathcal{N}=4}(x)$$

The on-shell action of the $\mathcal{N}=4$ theory

$$S_{\mathcal{N}=4} = \int d^4x \int d^4\rho\ \mathcal{T}(x,\rho,y)$$

Compute loop corrections using the method of Lagrangian insertions:

$$a\frac{\partial}{\partial a} G_4 = a\frac{\partial}{\partial a}\int D\Phi\, \mathrm{e}^{-\frac{1}{a}S_{\mathcal{N}=4}[\Phi]}\,\mathcal{O}(x_1,y_1)\dots\mathcal{O}(x_4,y_4)$$
$$= \int d^4x_5\, \langle \mathcal{O}(x_1,y_1)\dots\mathcal{O}(x_4,y_4)\mathcal{L}_{\mathcal{N}=4}(x_5)\rangle$$

The loop correction is determined by integrated 5-point correlation function with insertions of the $\mathcal{N}=4$ SYM action

Method of Lagrangian insertions

The $\ell-$loop correction – (integrated) tree-level correlation function with ℓ insertions of $\mathcal{L}_{\mathcal{N}=4}$

$$G_4^{(\ell)}(1,2,3,4) = \int d^4x_5 \ldots \int d^4x_{4+\ell} \, \langle \mathcal{O}(x_1,y_1) \ldots \mathcal{O}(x_4,y_4) \mathcal{L}(x_5) \ldots \mathcal{L}(x_{4+\ell}) \rangle^{(0)}$$

The operators $\mathcal{O}(x,y)$ and $\mathcal{L}(x)$ into the same supermultiplet!

$$G_4^{(\ell)}(1,2,3,4) \; = \; \int d^4x_5 ... d^4x_{4+\ell} \left(\int d^4\rho_5 \ldots d^4\rho_{4+\ell} \, \langle \mathcal{T}(1) ... \mathcal{T}(1) \mathcal{T}(5) ... \mathcal{T}(4+\ell) \rangle^{(0)} \right)$$

The *integrand* of the loop corrections to the four-point correlation function

$$\left[G_4^{(\ell)}(1,2,3,4) \right]_{\text{Integrand}} = \int d^4\rho_5 \ldots d^4\rho_{4+\ell} \, \langle \mathcal{T}(1) ... \mathcal{T}(1) \mathcal{T}(5) ... \mathcal{T}(4+\ell) \rangle^{(0)}$$

The correlation function $\langle \mathcal{T}(1) ... \mathcal{T}(1) \mathcal{T}(5) ... \mathcal{T}(4+\ell) \rangle$ is symmetric under exchange of points:

- ✔ Integrand reveals a new permutation $S_{4+\ell}$ symmetry involving all the $(4+\ell)$ points
- ✔ The OPE leads to powerful restrictions on the form of the integrand of the correlation function
- ✔ This information is sufficient to unambiguously fix the form of the integrand to all loops

All-loop integrand

Loop corrections to 4-point correlator

$$G_4^{(\ell)}(1,2,3,4) = x_{12}^2 x_{13}^2 x_{14}^2 x_{23}^2 x_{24}^2 x_{34}^2 \int d^4x_5 \dots d^4x_{4+\ell}\, f^{(\ell)}(x_1,\dots,x_{4+\ell})$$

✔ General form of $f^{(\ell)}$ for arbitrary ℓ:

$$f^{(\ell)}(x_1\dots,x_{4+\ell}) = \frac{P^{(\ell)}(x_1,\dots x_{4+\ell})}{\prod_{1\le i<j\le 4+\ell} x_{ij}^2}$$

Can be deduced from the OPE analysis of the tree-level correlator

✔ The polynomial $P^{(\ell)}$ should satisfy the conditions:

- ✗ be invariant under $S_{4+\ell}$ permutations of $x_1, ..., x_{4+\ell}$
- ✗ have a uniform conformal weight $(1-\ell)$ at each point, both external and internal

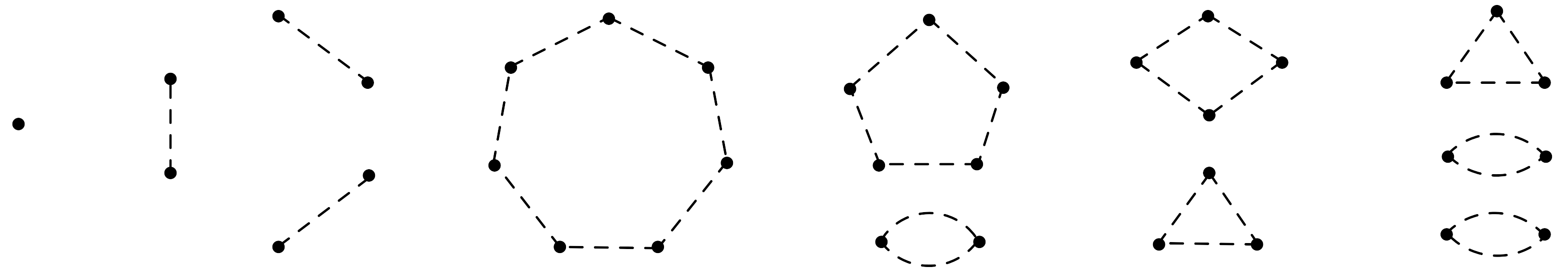
$$P^{(\ell)}(x_i^{-1}) = \prod_{i=1}^{4+\ell} (x_i^2)^{-\ell+1} P^{(\ell)}(x_i)$$

✔ Graph theory solution:

$P^{(\ell)} \mapsto$ ***Multi-graph with*** $(4+\ell)$ ***vertices of degree*** $(\ell-1)$

Properties of the $P-$graphs

$P^{(\ell)}$ = *loop-less multigraph with* $(4+\ell)$ *vertices* $(\ell-1)$ *edges attached to each vertex*



$P^{(1)}$ $P^{(2)}$ $P_1^{(3)}$ $P_2^{(3)}$ $P_3^{(3)}$ $P_4^{(3)}$

Can be easily generated at each loop level ℓ using standard graph-theoretical tools
The number of isomorphism classes (n_ℓ) of the $P-$graphs up to six loops

ℓ	1	2	3	4	5	6
n_ℓ	1	1	4	32	930	189341
$n_\ell^{\rm planar}$	1	1	1	3	7	36
$n_\ell^{\rm rung-rule}$	-	1	1	2	6	23
$n_\ell^{\rm non-rung-rule}$	-	0	0	1	1	13

The vast majority of graphs produce non-planar corrections

Planar topologies ($n_\ell^{\rm planar}$) have an interesting iterative structure, the “rung rule"

The majority of the planar graphs ($n_\ell^{\rm rung-rule}$) can be obtained from lower loops

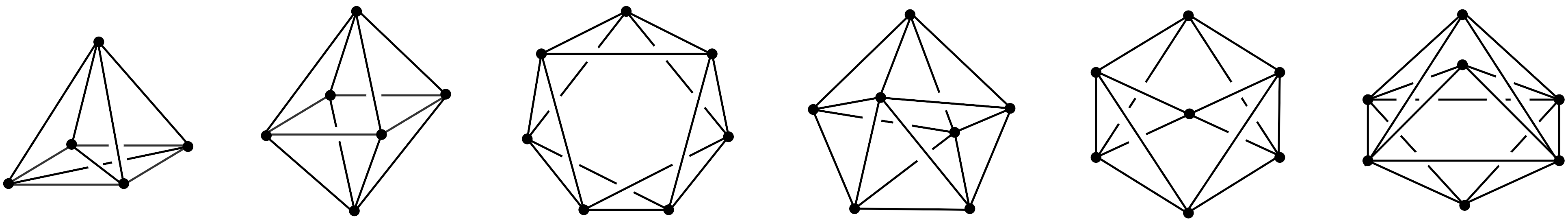
Only a smal number of non-rung-rule planar graphs ($n_\ell^{\rm non-rung-rule}$) require a different approach.

The general form of the integrand

$$f^{(\ell)}(x) = \sum_{\alpha=1}^{n_\ell} c_\alpha^{(\ell)} \frac{P_\alpha^{(\ell)}(x_1,\dots,x_{4+\ell})}{\prod_{1\le i<j\le 4+\ell} x_{ij}^2} = \sum_{\alpha=1}^{n_\ell} c_\alpha^{(\ell)} f_\alpha^{(\ell)}(x_1,\dots,x_{4+\ell})$$

$c_\alpha^{(\ell)}-$arbitrary (rational) coefficients

$$f_\alpha^{(\ell)} \mapsto \textit{ connected graph with } (4+\ell) \textit{ vertices of degree} \ge 4$$



$f^{(1)}$ $\quad f^{(2)}$ $\quad f_1^{(3)}$ $\quad f_2^{(3)}$ $\quad f_3^{(3)}$ $\quad f_4^{(3)}$

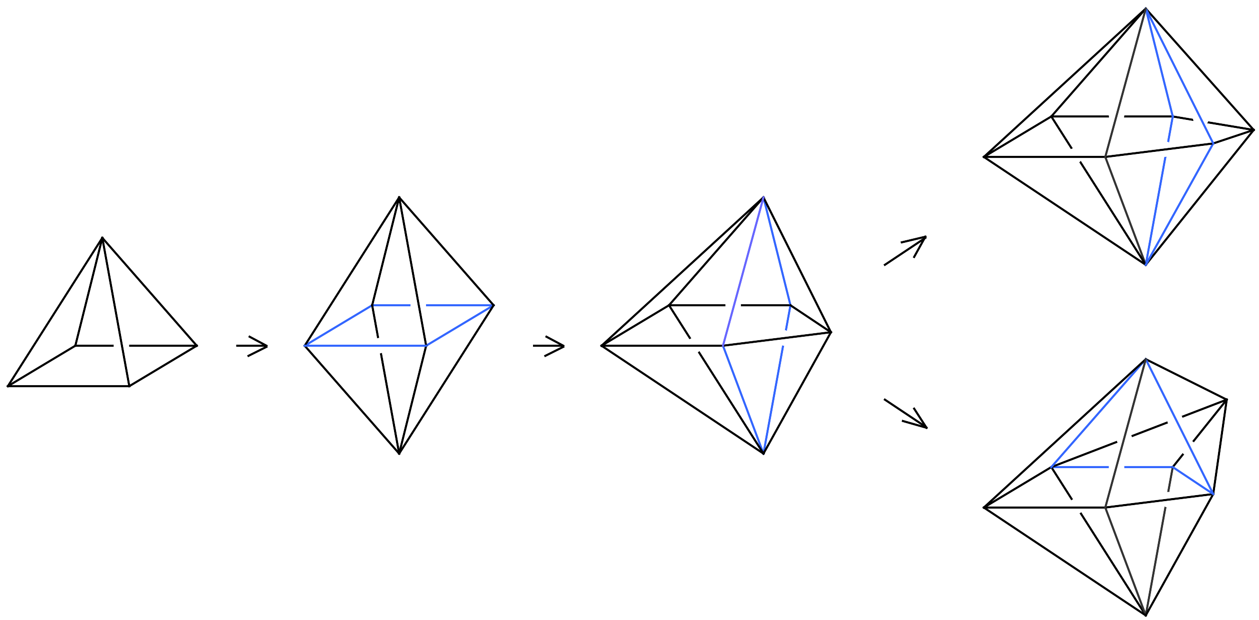
Large N_c scaling of the coefficients $c_\alpha^{(\ell)}$ is determined by the genus of $f-$graphs

$$c[f_2^{(3)}] = O(1/N_c^0)\,, \qquad c[f_1^{(3)}],\, c[f_3^{(3)}],\, c[f_4^{(3)}] = O(1/N_c^2)$$

Planar $f-$graphs have an iterative structure

Rung rule

$f^{(\ell)}$ = Glue $f^{(1)}$ (a square pyramid) to any planar $f^{(\ell-1)}-$ loop graph across rectangle face



From amplitude/correlation function duality, in the light-cone limit

$$\lim_{x^2_{i,i+1}\to 0}\left(1+2\sum_{\ell\geq 1}a^\ell F^{(\ell)}(x_i)\right)=\left(1+\sum_{\ell\geq 1}a^\ell \mathcal{M}^{(\ell)}(p_i)\right)^2$$

this is *precisely* the “rung rule” for the planar four-particle amplitude $\mathcal{M}^{(\ell)}$

The rule also fixes the coefficients of the new “rung-rule” topologies

Starting from 4 loops there are non-rung-rule topologies

OPE constraints

- Correlation function in the like-cone limit $x_{12}^2, x_{23}^2, x_{34}^2, x_{41}^2 \to 0$

$$\ln G_4(1,2,3,4) \sim \Gamma_{\rm cusp}(a) \ln u \ln v + O(a \ln u, a \ln v)\,, \qquad u\,, v \to 0$$

Conformal cross-ratios $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $v = \frac{x_{23}^2 x_{41}^2}{x_{13}^2 x_{24}^2}$

- Examine two-loop integrand

$$\ln G_4 \sim a\, G_4^{(1)} + a^2 \left[G_4^{(2)} - \frac{1}{2} (G_4^{(1)})^2 \right]$$

$$= a \frac{x_{13}^2 x_{24}^2}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2} + a^2 \frac{x_{13}^2 x_{24}^2 [x_{13}^2 (x_{25}^2 x_{46}^2 + x_{45}^2 x_{26}^2) + x_{24}^2 (x_{36}^2 x_{15}^2 + x_{16}^2 x_{35}^2) - x_{13}^2 x_{24}^2 x_{56}^2]}{2 x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2 x_{16}^2 x_{26}^2 x_{36}^2 x_{46}^2 x_{56}^2}$$

Divergences come from integration over x_5 and x_6 approaching the light-like edges, e.g. $x_5 \to x_1 - \alpha x_{12}$

$$x_{5i}^2 \to \alpha x_{1i}^2 + (1-\alpha) x_{2i}^2\,, \qquad 0 \le \alpha \le 1$$

- For the integral to have at most double-log asymptotics $\sim \ln u \ln v$ the polynomial in the numerator should vanish in this limit

- This condition alone fixes all the coefficients $c_i^{(\ell)}$. Checked to 2-, 3-, 4-, 5- and 6-loops.

Permutation symmetry + OPE constraints allow us to construct the integrand of G_4 up to 6 loops!

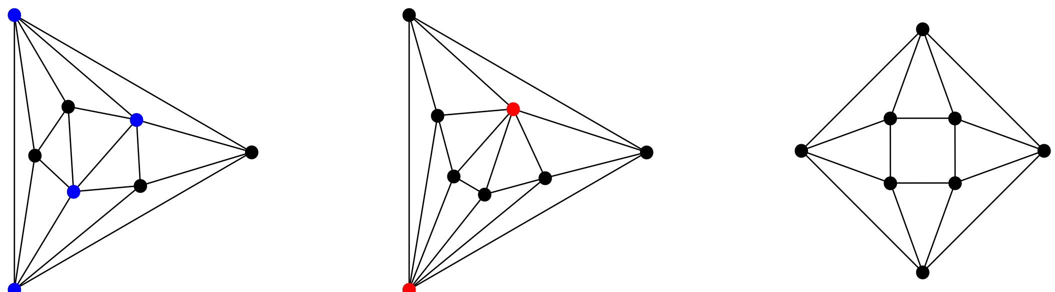
4 loops

$$F^{(4)}(x_1,x_2,x_3,x_4) = \frac{x_{12}^2 x_{13}^2 x_{14}^2 x_{23}^2 x_{24}^2 x_{34}^2}{4!\,(-4\pi^2)^4} \int d^4x_5 d^4x_6 d^4x_7 d^4x_8\, f^{(4)}(x_1,\ldots,x_8)\,,$$

There are in total 32 $f-$graphs: 3 planar and 29 of genus 1

$$f^{(4)} = \sum_{\alpha=1}^{32} c_\alpha^{(4)}\, f_\alpha^{(4)}(x_1,\ldots,x_8) = f_{\mathrm{g}=0}^{(4)}(x_i) + \frac{1}{N_c^2} f_{\mathrm{g}=1}^{(4)}(x_i)\,.$$

Planar four-loop $f-$graphs



$f_1^{(4)}$ $\quad$ $f_2^{(4)}$ $\quad$ $f_3^{(4)}$

The only non-rung-rule graph $f_3^{(4)}$

Correct asymptotics of the planar correlator in the light-cone limit

$$c_1^{(4)} = c_2^{(4)} = 1\,, \qquad c_3^{(4)} = -1$$

4 loops, nonplanar sector

$$\ln G_4 = \ln\left(G_{\mathrm{g}=0} + \frac{1}{N_c^2} G_{\mathrm{g}=1} + \ldots\right) = \ln G_{\mathrm{g}=0} + \frac{1}{N_c^2}\frac{G_{\mathrm{g}=1}}{G_{\mathrm{g}=0}} + \ldots$$

Nonplanar correction starts at 4 loops

$$G_{\mathrm{g}=1} \sim a^4 F^{(4)}_{\mathrm{g}=1}(x_i) = \text{sum of all 32 } f\text{−graphs}$$

A new feature: 3 conformal Gram determinants

$$\sum_{i=1}^{32} (a_k)_i f_i^{(4)}(x_1,\ldots,x_8) = 0\,, \qquad a_{1,2,3} \text{ – lists of 32 coefficients}$$

Correct asymptotics on the light cone leads to

$$F^{(4)}_{\mathrm{g}=1}(x_i) = c_1^{(4)} Q_1(x_i) + c_2^{(4)} Q_2(x_i) + c_3^{(4)} Q_3(x_i) + c_4^{(4)} Q_4(x_i)\,,$$

Q_k – ‘special’ linear combinations of 32 integrals; $c_k^{(4)}$ are arbitrary rational coefficients

Remarkable simplification in the short-distance limit $x_1 \to x_2$ and $x_3 \to x_4$

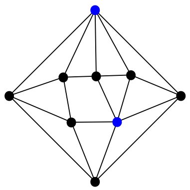
$$\lim_{\substack{x_1\to x_2\\ x_3\to x_4}} Q_{1,2,3,4} = \int \frac{d^4x_5\ldots d^4x_8\, x_{13}^8}{(x_{15}^2 x_{35}^2)\ldots(x_{18}^2 x_{38}^2)}\,\frac{(x_{56}^2 x_{78}^2 + x_{57}^2 x_{68}^2 + x_{58}^2 x_{67}^2)}{x_{56}^2 x_{57}^2 x_{58}^2 x_{67}^2 x_{68}^2 x_{78}^2}$$

Can be evaluated analytically (see Volodya’s talk)

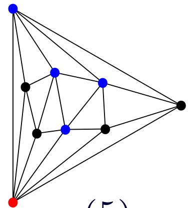
5 loops

$$F^{(5)}(x_i) = F^{(5)}_{\mathrm{g}=0} + \frac{1}{N_c^2} F^{(5)}_{\mathrm{g}=1} + \frac{1}{N_c^4} F^{(5)}_{\mathrm{g}=2}\,,$$

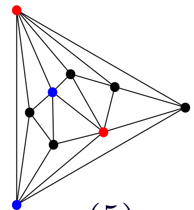
Among 930 five-loop $f-$graphs only 7 are planar



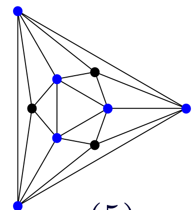
$f_1^{(5)}$



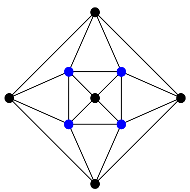
$f_2^{(5)}$



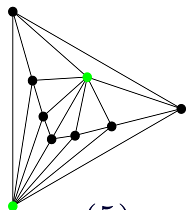
$f_3^{(5)}$



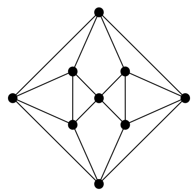
$f_4^{(5)}$



$f_5^{(5)}$



$f_6^{(5)}$



$f_7^{(5)}$

$f_7^{(5)}$ is the only non-rung-rule five-loop graph

Asymptotic behavior of the correlation in the light-cone limit leads to

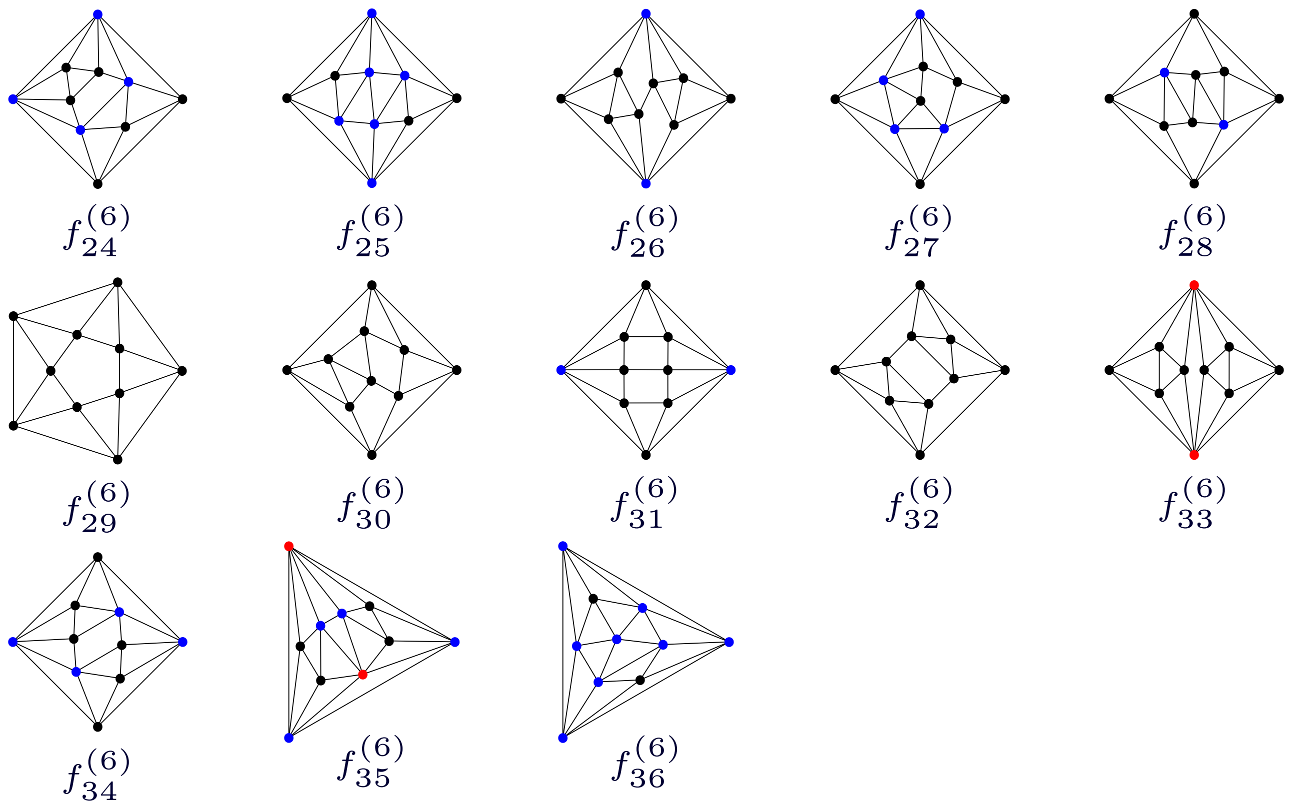
$$-c_1^{(5)} = c_2^{(5)} = c_3^{(5)} = c_4^{(5)} = -c_5^{(5)} = c_6^{(5)} = c_7^{(5)} = 1$$

The first six coefficients are in agreement with the rung rule $c_i^{(5)} = c_i^{(4)}$

6 loops

At six loops there are 189341 different $f-$graphs, but only 36 are planar

23 planar graphs are rung-rule induced, the remaining 13 non-rung-rule graphs are



In fact, only $f^{(6)}_{28}$, $f^{(6)}_{29}$ and $f^{(6)}_{31}$ contribute

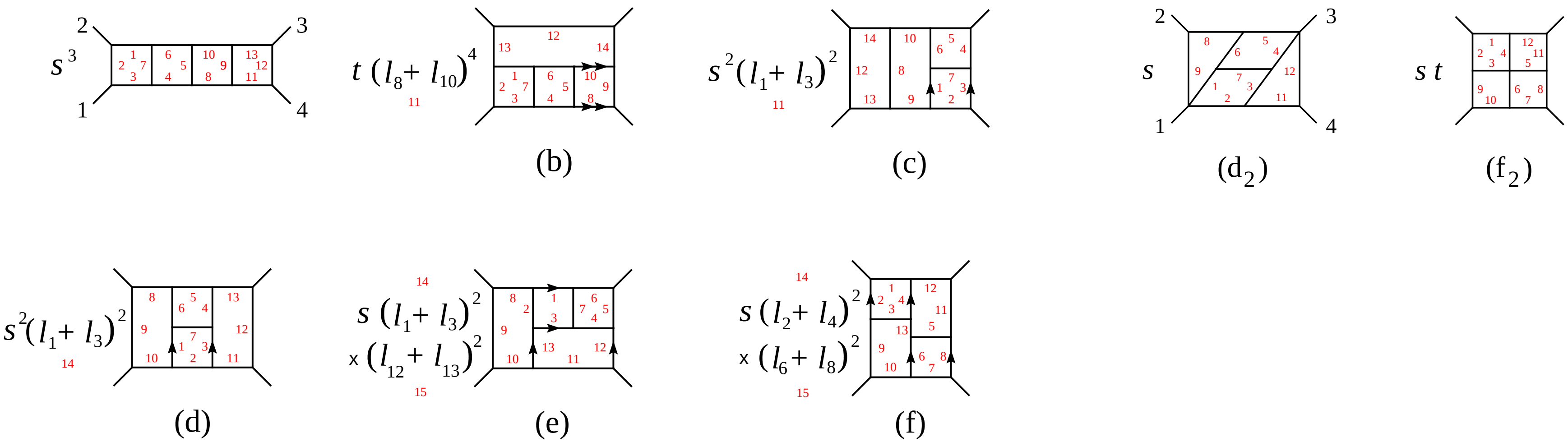
$$c^{(6)}_{28} = c^{(6)}_{31} = 1\,, \quad c^{(6)}_{29} = 2\,, \quad c^{(6)}_{24} = c^{(6)}_{25} = c^{(6)}_{26} = c^{(6)}_{27} = c^{(6)}_{30} = c^{(6)}_{32} = c^{(6)}_{33} = c^{(6)}_{34} = c^{(6)}_{35} = c^{(6)}_{36} = 0$$

Back to the amplitudes: 4-loop 4-gluons

Amplitude/correlator duality

$$M_4^{(4)} = \lim_{x^2_{i,i+1}\to 0}\left[\frac{1}{2}G_4^{(4)} - \frac{1}{4}G_4^{(3)}G_4^{(1)} - \frac{1}{8}(G_4^{(2)})^2 + \frac{3}{16}G_4^{(2)}(G_4^{(1)})^2 - \frac{5}{128}(G_4^{(1)})^4\right]$$

All pseudo-conformal integrals that contribute to four-loop four-point amplitude



Perfect agreement with the known 4-loop result [Bern,Czakon,Dixon,Kosower,Smirnov'06]

$$M_4^{(4)} = \mathcal{I}^{(a)}(s,t) + \mathcal{I}^{(a)}(t,s) + 2\mathcal{I}^{(b)}(s,t) + 2\mathcal{I}^{(b)}(t,s) + 2\mathcal{I}^{(c)}(s,t) + 2\mathcal{I}^{(c)}(t,s) + \mathcal{I}_4^{(d)}(s,t)$$
$$+\mathcal{I}^{(d)}(t,s) + 4\mathcal{I}^{(e)}(s,t) + 4\mathcal{I}^{(e)}(t,s) + 2\mathcal{I}^{(f)}(s,t) + 2\mathcal{I}^{(f)}(t,s) - 2\mathcal{I}^{(d_2)}(s,t) - 2\mathcal{I}^{(d_2)}(t,s) - \mathcal{I}^{(f_2)}(s,t$$

All 15 relative signs/coefficients follow from $c_1^{(4)} = c_2^{(4)} = -c_3^{(4)} = 1$!

Agreement between correlators and amplitudes verified up to 6 loops

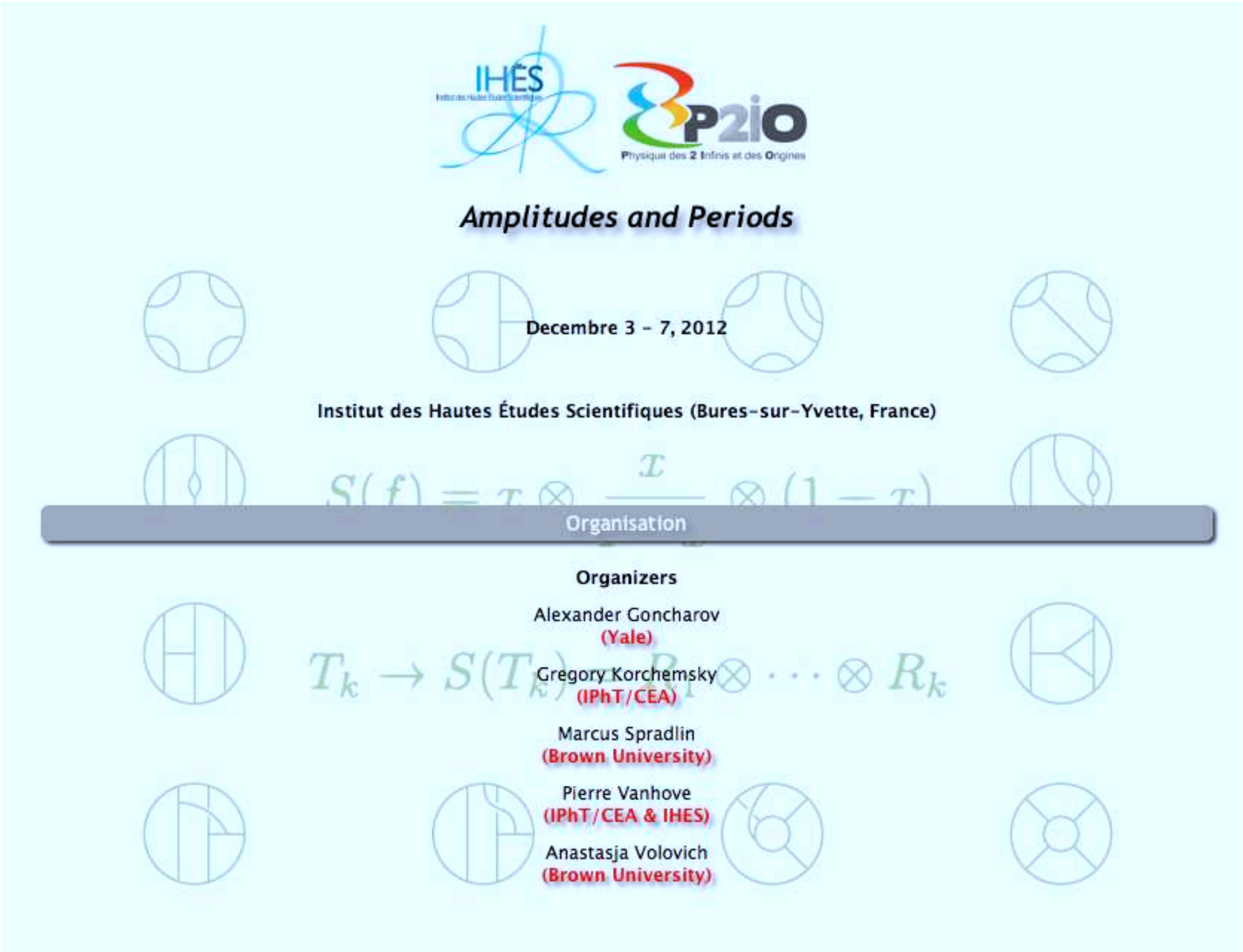
Conclusions

- ✔ The all-loop integrand of 4-point correlator possesses a complete symmetry under the exchange of the four external and all internal (integration) points

- ✔ This symmetry alone + OPE constraints allow us to construct 6-loop integrand of the correlation function in the planar limit (without doing Feynman diagram calculation!)

- ✔ In the light-cone limit, the scattering amplitude/correlator duality predicts the integrand for 4-gluon amplitude

- ✔ In the short-distance limit, the OPE leads to analytical result for the Konishi anomalous dimension at 5 loops [Volodya's talk]



For more information, see `http://www.ihes.fr/~vanhove/QFT2012/`