Lattice Design of Pulsed Synchrotron Rings for a Fermilab Sited Muon Collider

J. S. Berg, [1], K. G. Capobianco-Hogan[†2]

[1]Brookhaven National Laboratory, [2]Stony Brook University

[†] Presenter, kyle.capobianco-hogan@stonybrook.edu

Introduction

- Our goal is to design a lattice for a rapid cycling synchrotron (RCS) chain to accelerate muon and anti-muon beams to their final energy for use in a Fermilab sited muon collider.
- In particular, we seek to determine the maximum energy that such an acceleration chain can reasonably acheive, given our design constraints.
- We assume an injection energy of 173 GeV (top mass) for the first ring.
- For the lower energy rings, we use the Tevatron circumference (6.28 km).
- For higher energy rings, we use a circumference of 15.5 km in order to fit on the Fermilab site.
- We size the appertures to accommodate a 5σ beam.

Beam Size and Excursions

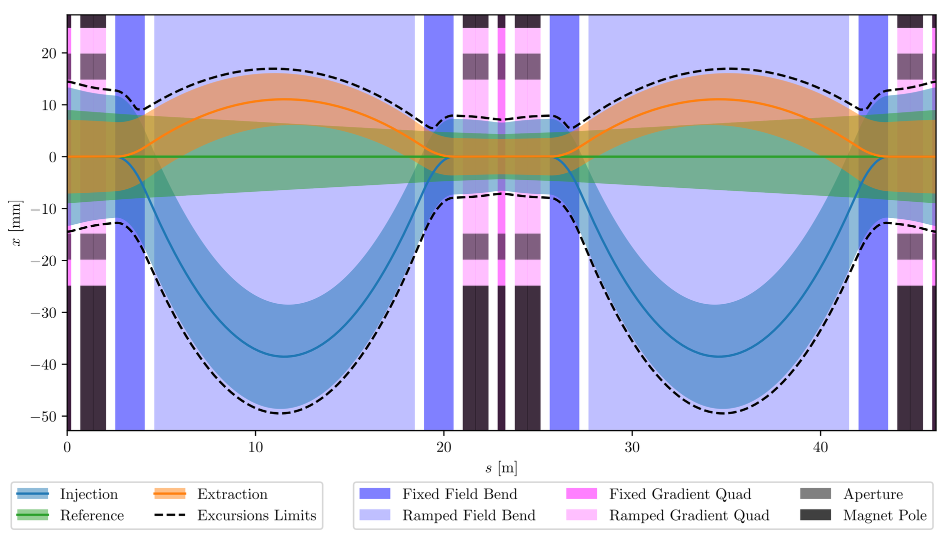
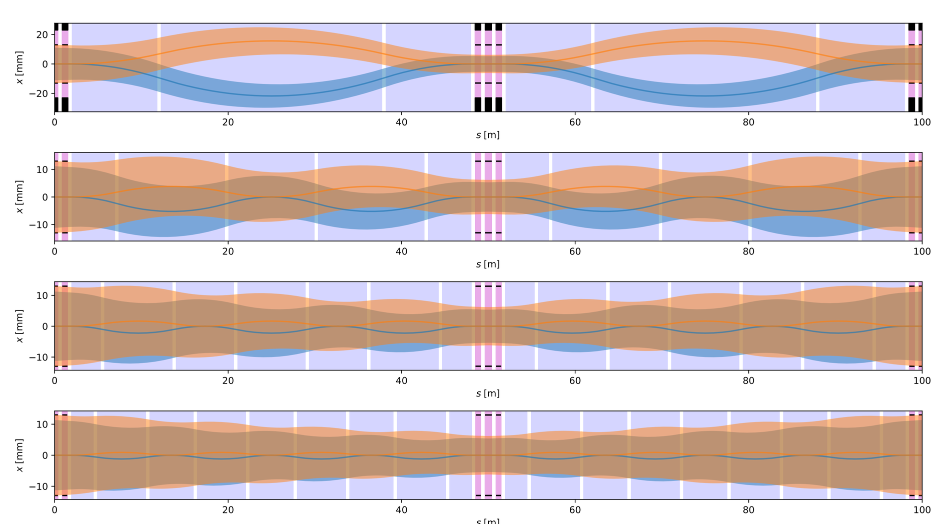


Fig. 1: RCS 2 Arc Cell

Subdivision of Hybrid Bends



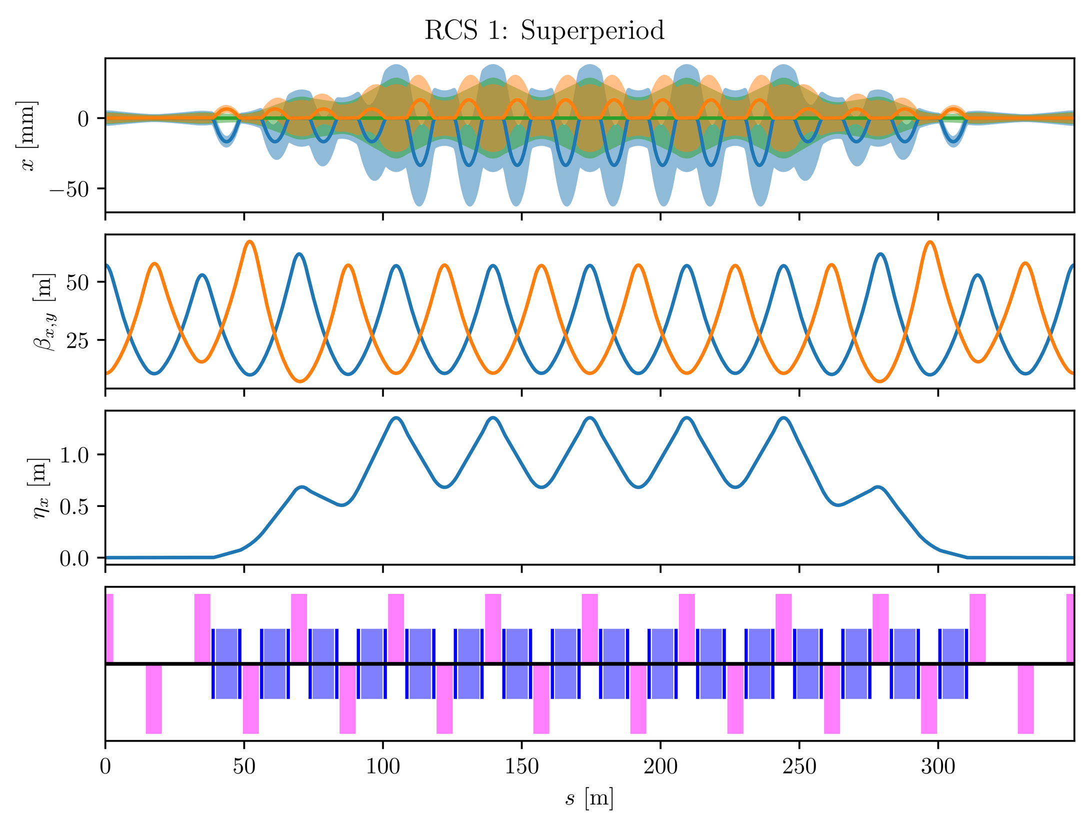
Bends per Half-Cell	E_{inj} [TeV]	E_{ext} [TeV]
1	3.60	5.00
2	3.66	5.00
3	3.72	5.00
4	3.78	5.00

Example of hybrid bend subdivision from an earlier iteration of the lattice design that had neither straight cells (RF) nor dispersion suppression cells.

Acceleration Chain

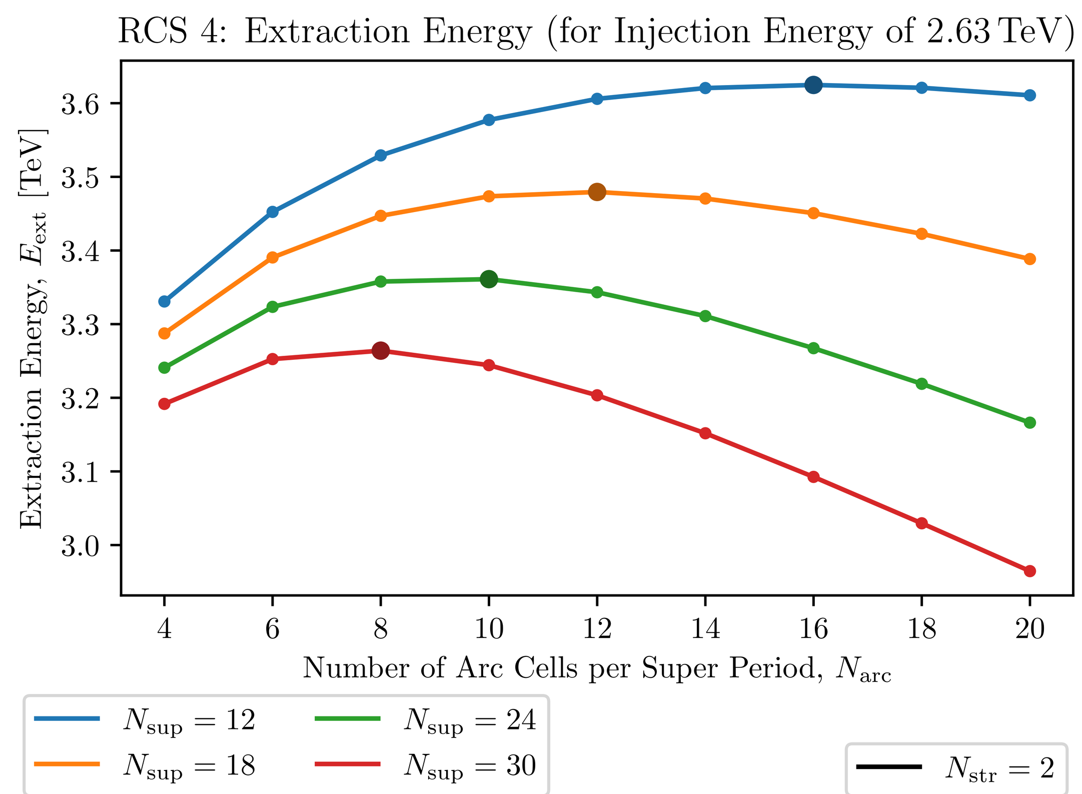
	RCS 1	RCS 2	RCS 3	RCS 4
Injection energy (TeV)	0.173	0.446	1.555	2.624
Extraction energy (TeV)	0.446	1.555	2.624	3.477
Energy ratio	2.578	3.486	1.687	1.325
Number of superperiods	18	24	18	18
Number of arc cells per superperiod	4	8	12	12
Synchronous phase (rad)	0.900	0.591	0.477	0.432
Total RF voltage (GV)	20.221	37.328	34.902	34.138
Superperiod synchrotron tune	0.025	0.010	0.008	0.006
σ_{p_z} at injection (10^{-4})	55.185	33.154	11.821	7.750
$\Delta E_{\text{sup}}/E_{\text{inj}}$ (10^{-4})	40.354	28.957	11.076	6.564
Survival fraction	0.927	0.905	0.959	0.978
Number of turns	21.730	35.768	34.476	27.543
Acceleration time (ms)	0.455	1.849	1.782	1.424
Packing fraction, dipole (%)	33.398	52.500	61.468	62.449
fixed field dipole (%)	7.375	9.663	20.179	29.463
ramped field dipole (%)	26.023	42.837	41.289	32.986

Ring Geometry



- We chose a phase advance per cell of 90° in both x and y to simplify the future inclusion of sextupoles.
- We use a half-bend dispersion suppression scheme.

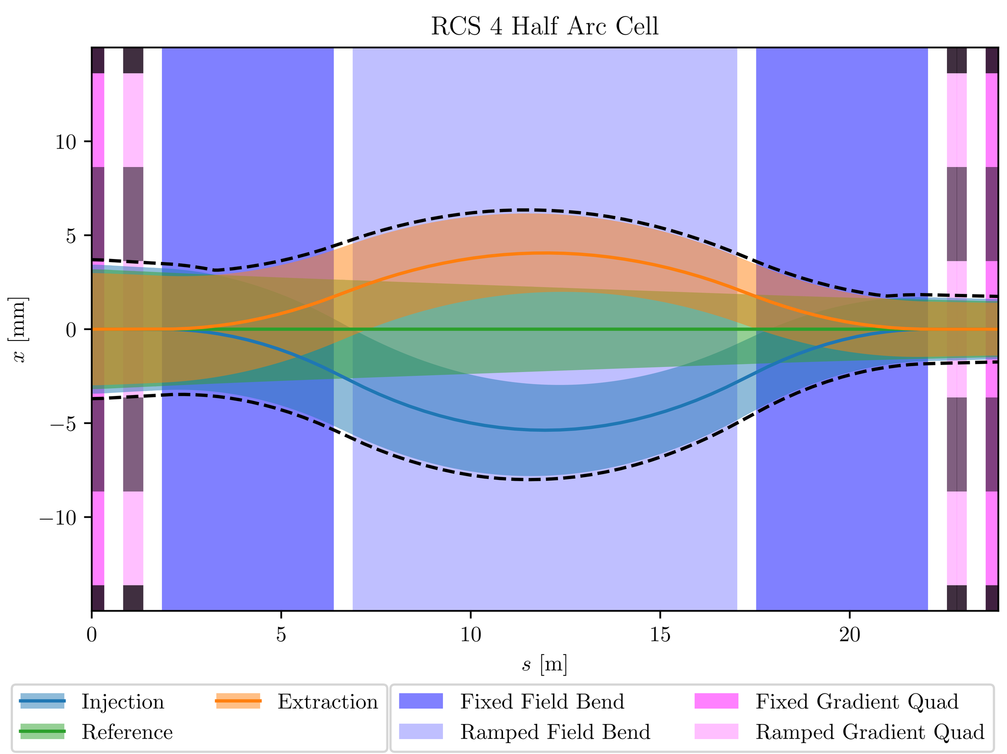
Parameter Scan

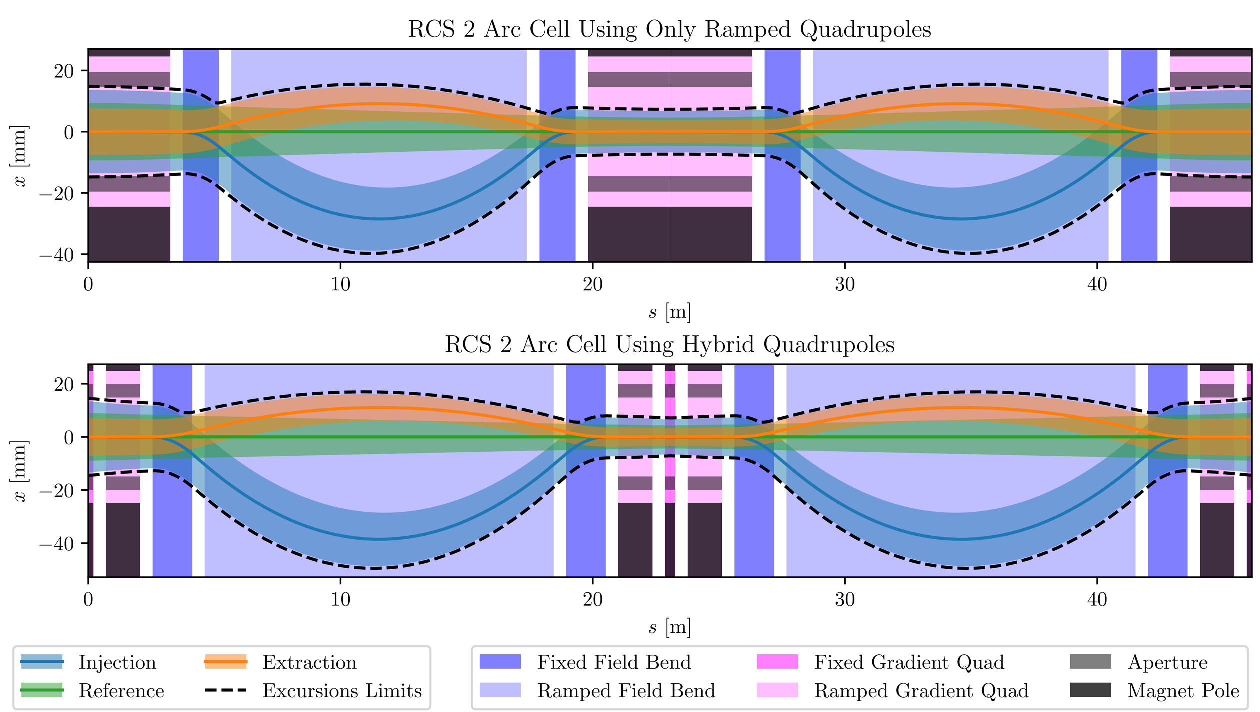


To limit missmatch between beam energy and ramped magnet field, we chose to restrict the maximum relative energy change per superperiod ($\Delta E_{\text{sup}}/E_{\text{inj}}$) to be below σ_{p_z}, which eliminates the $N_{\text{sup}} = 12$ solutions, leading to our choice of $N_{\text{sup}} = 18$ for RCS 4.

"Hybrid" Magnets

Magnets:	Iron Dominated	Superconducting Coil Dominated
Ramp Rate	$O(1\,\text{ms})$	$O(10^4\,\text{s})$
Max Field	1.75 T for dipoles 1.2 T (at pole tip) for quads	14 T for dipoles 12.5 T (at pole tip) for quads
Use Case	ramped field magnets	fixed field magnets





	Ramped	Hybrid
Injection energy (TeV)	0.446	0.446
Extraction energy (TeV)	1.385	1.555
Energy ratio	3.104	3.486
Packing fraction, dipole (%)	45.097	52.500
fixed dipole (%)	8.841	9.663
ramped dipole (%)	36.257	42.837

Acceleration

- In the realtivistic limit, the survival rate for (anti-)muons in a ring is given by $\frac{n_f}{n_i} = \left(\frac{E_f}{E_i}\right)^{-\frac{m_\mu c}{eG_{\text{avg}}\tau_\mu}}$.
- We choose a fixed value of $G_{\text{avg}} = 2\,\text{MV/m}$ for all four RCS rings.
- RF stations are distributed around the ring:
  1. To reduce the mismatch between the beam energy and the field of the ramped magnets.
     - The mismatch can lead to a significant increase in quadrupole apertures.
     - We limit the maximum relative energy change per superperiod ($\Delta E_{\text{sup}}/E_{\text{inj}}$) to be below σ_{p_z}.
  2. So that the synchrotron tune divided by the number of RF stations (the superperiod synchrotron tune) is much less than $1/\pi$.
- We assume cavities operate at 1.3 GHz with 25 MV/m average accelerating gradient.
- We combine the RF cavity parameters with momentum compaction from the lattice to calculate the synchronous RF phase needed to match a 5σ energy spread with an emittance of 0.025 eV s into the RF bucket.

Key Points

- Given our current design constraints, we were able to reach a maximum energy of just over 3.4 TeV with four RCS stages.
- Rather than starting with a final extraction energy of 5 TeV and working back to injection, we start with an injection energy of 173 GeV and work our way up, to see what can be reasonably achieved on the Fermilab site.

Next Steps

- Breaking symetry of hybrid quadrupoles (i.e., using one fixed and one ramped quad for each hybrid) to save space?
- Including sextupoles
- Hybrid bend subdivision (to reduce excursions)
- Tracking simulations
- Improving energy reach by backing off on other parameters (e.g., average gradient).